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Implementation and evaluation of the strategy Pairs Trading for Colombian public debt bonds.

by

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Abstract

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Pair trading is a statistical trading strategy based on the concept of mean reverting; investors select two related assets and establish a relation between them buying the underpriced asset and selling the overpriced. When the market returns to the equilibrium the strategy create profit from the short and long position. The empirical application of this paper proposes the evaluation of three methodologies for the implementation of the pair trading strategy using the information of Colombian public debt bonds. Finding that after applying two methodologies of backtesting stochastic stochastic approach show the best perfomance.

Keywords: pairs trading, spread process, cointegration, distance method, fixed income

Contents

Chapter 1

Introduction

Capital markets are constantly developing quantitative methods to speculate and increase profits, pairs trading it is one of these strategies and has been used since mid-80s [Gatev et all, 2006][1]. Pair trading it is a simple algorithm based on three steps: the identification of two assets that have moved together then follow up the spread between them and when the movement relation change open a long (short) position on the high asset (low asset).

The success of these kinds of strategies is based on the need of guaranteeing the mean reversion process of assets prices. Arbitrage pricing theory (APT) indicates that when two assets have the same risk factors the return of both should to be the same [Vidyamurthy,2004] [2], so the key for a successful pair trading is the selection of: pairs and the threshold to get in (out) of the strategy.

Pairs selection throughout the focus on three main methodologies: Distance method which is based on the statistical relation of the assets identifies the pairs using the sum of squared differences between the two normalized price series [Gatev et all, 1999][3]. The second one is conintegration approach, based on Engle and Granger (1987)[4], this proposal indicates that the time series has to have the following two characteristics in order to find a long term relation: both series have to be integrated of order d and first order combined to create a single time series. Finally the stochastic approach involves the definition of the spread as a latent state variable which follows a Vasicek process [Elliot,2005]

The use of the Pairs strategy has been studied with sufficient proficiency by academics and practitioners; however for fixed income market there is not such abundance of research but it is possible highlight two approximations: the first one related with the use of the return or price of the bonds as price as indicated by Nath [2003][5] and the second with the analysis of interest rate term structure in order to exploit deviations from level, slope and curvature of the yield curve [Chua et all, 2004][6]

The aim of this paper is compare these three methodologies for the most liquid market in Colombia which is the public debt bonds (TES). The paper is organized as follows this introduction as chapter 1, chapter 2 outlines the three methodologies; chapter 3 shows the empirical application for the Colombian market and chapter 4 presents results and conclusions.

Chapter 2

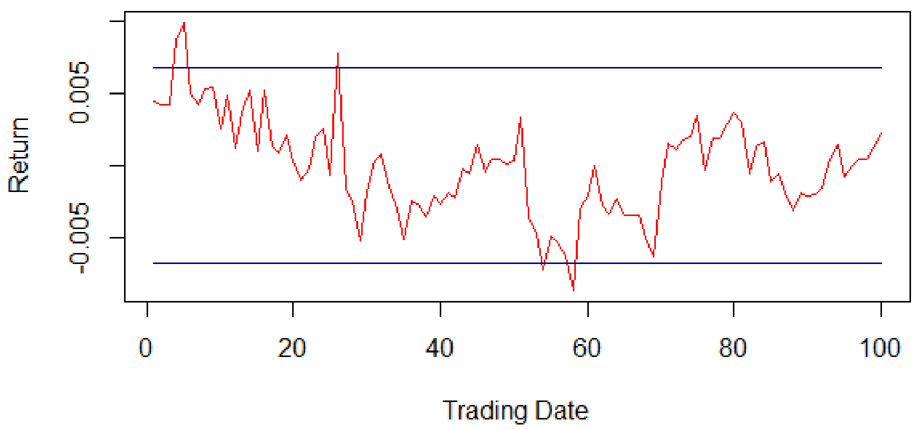
Pairs Trading

Pairs trading is a statistical trading strategy based on the concept of mean reverting; investors select two related assets and establish a relative mean between them, buying the underpriced asset and selling the overpriced. When the market returns to the equilibrium (or media) the strategy create profit from the short and long position [Zhang, 2012][7].

The strategy requires the definition of: the trigger to get in or out of the strategy, the definition of formation period and the execution of the trading strategy and of course the pair selection.

Figure 2.1 indicates as an example of the strategy, the levels where the spread process exceeds some trigger. This means that when the red line it is above the blue line exists a pair opportunity.

FIGURE 2.1: Example of pair trading between TFIT30-TFIT28



A pair trading strategy has two major characteristics that make it interesting for institutions do not want to take major market risks: the first one indicates that the expected return does not depend on the market movement, this means the strategy it is market neutral [Bolgomolov, 2011][8]. The second one is that pairs trading is cash-neutral.

2.1 Distance Method

According with Gatev [1999][3] the selection of a pair respond a combination of assets which minimize the sum of squared deviation of normalized prices (**SSD**). It is simple strategy with a low cost of implementation and for this reason may be the favorite among practitioners.

However this approach has some strong assumptions that could not be real for financial data. The distance method assumes: a static linear relationship between the two assets and prices which are identically independent random variables. One of the advantages of this non-parametric model is the absence of mis-specification and mis-estimation but it does not have forecasting power [Do et all,2006][9].

The trigger and the formation period are arbitrary and according with Gatev [1999][3], the trigger corresponds two historical standard deviations, and formation period should be one year and the trading period six months.

Thus the distance measurement is:

$$SSD_{x,y} = \sum_{i=1}^{T} (S_{x,i} - S_{y,i})^2 \tag{2.1}$$

where:

$SSD_{x,y}$: Sum of squared deviation of normalized prices

$S_{x,i}$: Normalized price asset X

$S_{y,i}$: Normalized price asset Y

and the normalized prices are:

$$S_{xi} = \frac{(P_{x,i} - \mu_x)}{\sigma_x} \qquad S_{yi} = \frac{(P_{y,i} - \mu_y)}{\sigma_y} \tag{2.2}$$

where:

$P_{,i}$: Price of the asset (x or y)

μ: Historical Media

σ: Historical standard deviation

2.1.1 Pairs Formation - Distance Method

The first step in the selection of pairs to evaluate it is the normalization of the asset prices, this means:

$$P^a_n t = \frac{(P_{a,t} - \mu_a)}{\sigma_a} \tag{2.3}$$

With $P_{a,i}$ as the log price of asset a and μ, σ are the mean and standard deviation respectively.

Next it is necessary to establish the sum squared difference, this would be the criteria to select de pairs to test.

$$SSD = \sum (P^a_n - P^b_n)^2 \tag{2.4}$$

Finally the 20 pairs with the lowest sum of squared deviation are going to be selected and to be tested in the pair trading strategy. When an asset is selected to be a part of pair it is not removed from the sample, so it is possible for one asset to belong to more than one pair.

2.1.2 Rules to open or close positions

The price spread for distance method will be:

$$y_t = log(P^a_n t) - log(P^b_n t) \tag{2.5}$$

According with Gatev et all [1999][3], Bogomolov [2010][8], Do et all [2006][9] and Vidyamurthy [2004] [2] the selected trigger to open or close positions will be 2 standard deviations of the price spread this means:

Strategy 1:

$y_t \geq 2\sigma_{yt}$: Sell asset a and buy asset b.

$y_t \leq 2\sigma_{yt}$: Buy asset a and sell asset b.

Open positions will be closed when the spread reach 0.5σ value.Also are going to be evaluated the following triggers 1.5σ and σ. This is: **Strategy 2:**

$y_t \geq 1.5\sigma_{yt}$: Sell asset a and buy asset b.

$y_t \leq 1.5\sigma_{yt}$: Buy asset a and sell asset b.

Strategy 3:

$y_t \geq \sigma_{yt}$: Sell asset a and buy asset b.

$y_t \leq \sigma_{yt}$: Buy asset a and sell asset b.

2.2 Cointegration

Cointegration allows the estimation of the long-term relation of two variables when they have the same integration level, so two non-stationary time series are conintegrated if a linear combination of them is stationary [Engle and Granger, 1987][4].

Conintegration approach fits perfectly for pairs trading concept which try to exploit the short-term deviation from a long-term relation. Short-term deviations are rectified by the error correction which according with Vidyamurthy [2004] [2] correspond to the adjustment of one or both time series to reach the long term relation; this means that unlike the distance method, conintegration has the ability of forecasting based on past information.

The cointegration relation between two assets would be:

$$log(P_{xt}) - \gamma log(P_{yt}) = \mu - \epsilon_t \tag{2.6}$$

Where γ represents the cointegration factor , μ the mean of cointegration relationship and ϵ a new stationary time series which could be tested using Augmented Dickey Fuller (ADF) [10] test on the residuals.

The coefficient γ it obtained using a simple OLS regression, this give us:

$$\gamma = \frac{Cov(log(P_{xt}), log(P_{yt}))}{Var(log(P_{yt})} \tag{2.7}$$

Augmented Dickey Fuller (ADF) is based on the following auxiliary regression establishing that when ρ <1 then the series it is stationary. The hypothesis behind the test are: null hypothesis indicates if $\rho = 1$ and alternative hypothesis indicates $\rho < 1$.

$$\Delta\epsilon = \alpha + \beta t + \rho\epsilon_{t-1} \tag{2.8}$$

Then the τ statistic from the ADF will be:

$$\tau = \frac{\rho}{S.E(\rho)} \tag{2.9}$$

Yakop [2011] [11]suggests that it is necessary should be considering other tests to identify the presence of a unitary root in the series. The suggested tests are: Johansen cointegration test and the Philips Perron (PP) test.

To establish a estacionary relation between the two assets ADF and PP are going to be implemented.

2.2.1 Pairs Formation - Cointegration Approach

Pairs formation under cointegration aproach involves two main steps. The first one it is the estimation of spread process, for this purpose the price spread between assets a and b:

$$y_t = \alpha - log(P_t^a) - \beta log(P_t^b) + \epsilon_t \tag{2.10}$$

Using a simple OLS model the parameters α and β are estimated. The residual vector is tested using Augmented Dickey Fuller (ADF) and Philips Perron test to prove stationarity of the series. Both tests are evaluated at significance level of 5%.Pairs who exceed this confidence level will be used to implement the strategy.

Here it is important to remark that unlike the other two methodologies, the use of cointegration shows the existence of a mechanism of selection of pairs (unit root tests)

2.2.2 Rules to open or close positions

As previously defined the spread between asset prices was estimated by equation [2.10] and the parameters α and β are defined by the OLS estimation. To create a consistent method to compare the methodologies the triggers for cointegration will be the same for distance method:2,1.5 and 1 standard deviation from the price spread process y_t.

Strategy 1:

$y_t \geq 2\sigma_{yt}$: Sell asset a and buy asset b.

$y_t \leq 2\sigma_{yt}$: Buy asset a and sell asset b.

Strategy 2:

$y_t \geq 1.5\sigma_{yt}$: Sell asset a and buy asset b.

$y_t \leq 1.5\sigma_{yt}$: Buy asset a and sell asset b.

Strategy 3:

$y_t \geq \sigma_{yt}$: Sell asset a and buy asset b.

$y_t \leq \sigma_{yt}$: Buy asset a and sell asset b.

The estimated parameters will remain constant throughout the trading period and open positions will be closed when the spread reach 0.5σ value

2.3 Stochastic Spread

According with Elliot et all (2005)[12] if it is possible to establish a mean reverting property of the price spread between two assets it is also possible to expect that this property remains for some time in the future, creating some opportunities for the statistical arbitrage. Do et all (2006)[9] indicates that the observation process would be:

$$y_t = log(P_{at}) - log(P_{bt}) \tag{2.11}$$

Where y_t it is the observed log price spread at time t, and has two main characteristics: it is described by a state-space model and it is guided by a latent state variable x_t which in its discrete version can be write as:

$$x_{t+1} - x_t = \lambda(\mu - x_t)\tau + \sigma\sqrt{\tau\epsilon_{t+1}} \tag{2.12}$$

Where λ indicates the mean reverting speed, μ it is the long term spread mean and σ denotes the standard deviation. In its continuous form x_t would be:

$$dx_t = \lambda(\mu - x_t)dt + \sigma dW_t \tag{2.13}$$

with $\{W_t \,|t \geq 0\}$ as a Brownian motion. Clearly [2.13] shows a Vasicek process which is a special case of Orsetein-Uhlenbeck model. These kinds of models are widely used to describe price process from different assets because they implicitly describe the economic theory of demand and supply: When the price it is too high (low) the demand would decrease (increase) and supply would increase (decrease) until the market arise a new point of equilibrium.

The observation process y_t also could be expressed as the sum of the state variable and a Gaussian noise:

$$y_t = x_t + D\omega_t \tag{2.14}$$

Where ω_t are iid N(0,1) and independent of the ϵ_t and $D > 0$ is a constant measure of errors.

It is possible to re-write the state variable (Eq. 2.12) as:

$$x_{t+1} = A + Bx_t + \epsilon_t \tag{2.15}$$

where A=aτ, B=(1-b)τ and C=$\sigma\sqrt{\tau}$ which are constants and could be obtain through a Kalman Filter.

2.3.1 Kalman Filter

Back to the state-space model previously defined it is possible to define covariance and mean from the state process as:

$$\bar{x}_{t+1} = A + B\hat{x}_t \qquad\qquad \bar{\Sigma}_{xxt} + 1 = B^2\Sigma_{xxt} + C^2 \tag{2.16}$$

where $\bar{x}_t$ and $\bar{\Sigma}_{xxt} + 1$ are defined as:

$$\mu_t = \hat{x}_t = \hat{x}_{t|t} = E(x_t|\mathcal{Y}_t) \tag{2.17}$$

$$\bar{\Sigma}_{xxt} = R_t = E[(x_t - \hat{x}_t)^2|\mathcal{Y}_t)] \tag{2.18}$$

Defining $\mathcal{Y}_t$ as the new arrival of information from the observable variable this is:

$$\mathcal{Y}_t = \sigma\{y_0, y_1, ..., y_t\} \tag{2.19}$$

Then and according to Elliot et all [2005][12] recursively it is possible to obtain the Kalman Gain$\mathcal{K}$ and from there follow the estimation:

$$\mathcal{K}_{t+1} = \frac{\Sigma_{t+1|t}}{\Sigma_{t+1|t} + D^2} \tag{2.20}$$

$$\hat{x}_{t+1} = \hat{x}_{t+1|t+1} = \hat{x}_{t+1|t} + \mathcal{K}_{t+1}[y_{t+1} - \hat{x}_{t+1|t}] \tag{2.21}$$

$$R_{t+1} = \Sigma_{t+1|t+1} = D^2\mathcal{K}_{t+1} = \Sigma_{t+1|t} - \mathcal{K}_{t+1}\Sigma_{t+1|t} \tag{2.22}$$

Finally through the following maximum likelihood function the parameters A,B and C are obtained

$$\begin{aligned} \log L(y) &= \sum_{i=1}^{N} \log p(y_i|Y_{i-1}) \\ &= \frac{N}{2}\log 2\pi - \frac{1}{2}\sum_{i=1}^{N} \log|F_i| + \epsilon' F_i^{-1}\epsilon^i \end{aligned} \tag{2.23}$$

Where:

$F_i = Var(y_i|Y_{i-1})$

$\epsilon = y_i - E[y_i|Y_{i-1}]$

2.3.2 Pairs Formation-Stochastic Approach

After obtaining the parameters A, B and C it is possible to obtain the mean and the standard deviation of the spread between the bonds as follows:

$$\mu_s = \frac{A}{1-B} \qquad \sigma_s = C \tag{2.24}$$

To build the pair it is necesary to obtaining the 20 lowest standard deviations, as in distance method the literature takes as given the pairs and does not describe how to select them.

2.3.3 Rules to open or close positions

For the stochastic approximation, the spread will be defined by the latent state variable x_t described in equation [2.15]. As it was previously defined mean and standard deviation from this process, it is possible to establish some measurement of z-score like:

$$z - score_{stoch} = \frac{x_t - \mu_s}{\sigma_s} \tag{2.25}$$

Using this dispersion measure the pair trading is evalueted

Strategy 1:

$x_t \geq 2z - score_{stoch}$: Sell asset a and buy asset b.

$x_t \leq 2z - score_{stoch}$: Buy asset a and sell asset b.

Strategy 2:

$x_t \geq 1.5z - score_{stoch}$: Sell asset a and buy asset b.

$x_t \leq 1.5z - score_{stoch}$: Buy asset a and sell asset b.

Strategy 3:

$x_t \geq 1z - score_{stoch}$: Sell asset a and buy asset b.

$x_t \leq 1z - score_{stoch}$: Buy asset a and sell asset b.

2.3.4 Backtesting

To perform the backtest of the strategies previously described, it is required establishing criteria to compare the results obtained from the strategy. Sharpe ratio is this measure.

Sharpe Ratio it is a measure of the excess of return for the assumed risk of an asset or portfolio in comparison with a risk free or benchmark portfolio. In this case the risk free rate will be the return of the COLTES basket.

$$\mathbf{SharpeRatio} = \frac{\mathbf{r_{pair}} - \mathbf{r_{COLTES}}}{\sigma_{\mathbf{pair}}} \tag{2.26}$$

Following Campbell et all [2015] [13] two types of tests will be carried out using Sharpe Ratio as an indicator of performance of the strategy as described below:

The first test requires the split of the sample in two groups: The first group will be call as in sample and it is going to be used to select pairs and establish the parameters to describe the spread process this sample contains 293 trading dates. For this sample returns and Sharpe Ratio will be computed as a measure of profitability of the strategy.

The second group will be call as the out of the sample which is used to evaluate how the parameters obtain from the know history of the asset price behaves during a trading period where there is uncertainty, this sample contains 100 trading dates.

This first test to backtest is a trading strategy widely used by for practitioners and academics. However according with Campbell et all [2015] [13] sometimes the use of the results in-sample and out of the sample could create an overfitting of the out of the sample results because it is not true that the results during this periodit are unknown, this can lead to the investor to try to make adjustments for the strategy in order to get better results during the out of the sample evaluation.

The second test used involves the whole sample and also it is based on the Sharpe ratio. This methodology assumed normal returns IID to use the Sharpe ratio as hypothesis test where:

$$\mathbf{T-statistic} = \frac{\mathbf{r_{pair} - r_{COLTES}}}{\sigma_{\mathbf{pair}} * \sqrt{(\mathbf{N})}} \tag{2.27}$$

The correspond p-value will be:

$$p-value = Pr(|r| > T-statistic) \tag{2.28}$$

Null hypothesis for this test is that the tested pair strategies can generate zero or negative returns. Campbell et all [2015] [13] indicates that investors will try N strategies before to accepting that one strategy it is successful this N tries involve: changes of asset, trigger or any other factor that could improve the final result. So it is necessary to create a p-value for the multiple test as:

$$
\begin{aligned}
p - value_M &= Pr(max|r_i|, i....N > T - statistic) \\
&= \prod_{i=1}^{N} Pr(|r| \ > T - statistic) \\
&= 1 - (1 - p - value)^N
\end{aligned}
\tag{2.29}
$$

The main idea it is avoid the overfitting penalizing the use of multiple test reducing the p-value with any new test. But this measure corresponds to the maximum population achieved by the strategy evaluated; clearly in a sample level is expected that the Sharpe Ratio is lower or at best equal to the population. Therefore it is necessary to apply a haircut for the purpose of adjusting the measurement, this will be the haircutted Sharpe Ratio (HSR).

This haircut will be:

$$
haircut = \frac{SharpeRatio - HaircuttedSharpeRatio}{SharpeRatio} \tag{2.30}
$$

Then we can re-write the p-value for multiple test as:

$$
p - value_M = Pr(max|r_i|, i....N > HSR\sqrt(N)) \tag{2.31}
$$

Chapter 3

Results and conclusions

3.1 Results

The data used for this empirical application of pairs trading corresponds to closing clean prices from Colombian public debt bonds (TES). The selected issues to evaluate correspond to those with have mandatory trading operation. The prices are obtained from Bloomberg and cover 393 trading dates.

In order to evaluate correctly trading strategies it is necessary take into account transaction costs that have to be cover by profits generated by the trading activity. For this purpose it is necessary to apply an adjustment to the final result of the strategy. The fee to open or close a position belong 0.008% for operations bigger than COP 5000M according with the Colombian stock exchange (BVC). This means that each time that the strategy was apply the utility it is adjusted by 0.0016%

Threshold to evaluate each method will be 2, 1.5 and 1 times the standard deviations. The idea is to evaluate if given the low costs of the operation of the wholesale fixed income market in Colombia it is possible to increase the return given the possibility of opening and closing positions with greater speed.

The pairs selected for the distance method and the stochastic approximation correspond the 20 lowest measures of dispersion (SSD for distance method and σ_{ij} for stochastic approach. For cointegration method was evaluated, 27 pairs which simultaneously passed the ADF and PP tests.

3.1.1 In sample and Out of the sample results

According with results from table A.1 in sample and out of the sample results from distance method show a positive Sharpe ratio for 95% of the tested pairs using as a trigger 1σ and 1.5σ and for 2σ the proportion it is reduced to 50% and 30% respectively. However if the idea of the investor it is no just to get any positive return instead wants a return that justifies the assumed risk (Sharpe Ratio ¿1) then the proportions seem lowers and as is expected in sample results are better: in average 52% of the tested pairs meet this condition, the average for out of the sample results is 38%.

When the pair is tested using a lower standard deviation the profits from applying the strategy improve. However it does not involve a positive return in all cases. As the value of the trigger to get in a position decreases, the proportion of pairs that provide positive returns increases for the in sample results: 2σ 50% of the pairs show a positive return, for 1.5σ 60% and for 1σ 70% show profits, out sample results indicates that 2σ 40% , 1.5σ 50% and for 1σ 60% show profits.

Cointregration method also shows poorly results in comparison with the returns of the benchmark basket as well as the distance method. In sample results do not exhibit any pair with a Sharpe Ratio bigger than one and out of the sample results shows only three pairs that meet the condition. Table 3.1 exhibits a comparison of the Sharpe ratio from out of the sample results between the common pairs selected by the distance and cointegration method, the best performance of the first method is confirmed for triggers $1.5\ \sigma$ and 1σ.

TABLE 3.1: Sharpe Ratio common pairs Distance and Cointegration Method

	Distance			**Cointegration**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT19-TFIT22	-99.8985	0.8182	0.951	-12.406	-3.719	-3.212
TFIT19-TFIT24	-7.8580	1.3834	0.571	-6.341	-2.314	-1.977
TFIT19-TFIT26	0.0000	1.1338	1.272	-4.165	-4.023	-2.774
TFIT19-TFIT30	1.2154	1.7303	1.184	-11.849	-6.535	-4.052
TFIT22-TFIT26	-10.8855	0.4583	0.722	-8.339	-1.277	-1.092
TFIT22-TFIT28	3.5650	0.9184	1.008	-11.711	-3.018	-1.792
Mean	**-18.976**	**1.073**	**0.951**	**-9.135**	**-3.480**	**-2.483**

The Stochastic approach in general shows better performance than cointegration approach and distance method. Using in sample outcomes 80% of the tested pair give as a result a Sharpe ratio greater than 1 and for the out of sample results the proportion on average was the 48%. One characteristic of the stochastic approach it is that it does not seem affected by the change of trigger level as the other methodologies. From this perspective the performance of this methodology it is superior. Also the comparison of common pairs between distance and stochastic approach will confirm that statement. Table 3.2 for instance indicates that the out of the sample results of pair TFIT22-TFIT26 with distance method it does not reach the level of 1 however with Stochastic approach does it and the return it is the 1.026% .

Table 3.2: Sharpe Ratio common pairs Distance and Stochastic Approach

	Distance			**Stochastic Approach**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT18-TFIT26	0.000	0.962	1.269	-10.107	-10.105	-10.070
TFIT19-TFIT22	-99.899	0.818	0.951	9.831	9.811	9.832
TFIT22-TFIT24	-99.899	0.629	1.355	9.691	9.691	9.691
TFIT22-TFIT26	-10.886	0.458	0.722	-0.188	1.989	1.739
TFIT26-TFIT30	-64.171	0.494	0.198	9.388	8.994	9.784
Mean	**-54.971**	**0.672**	**0.899**	**3.723**	**4.076**	**4.195**

3.1.2 Haircutted Sharpe Ratios

Distance method results showed in Table B1 exhibits that using a simple test, 14 pairs (see Appendix B) with Sharpe Rations greater than 1 and with a confidence level of 95% reject the null hypothesis of zero or negative returns, however as it was describe in chapter 2 is not enough use a simple test to define that the strategy it a success. After applying the haircut to Sharpe Ratio remains 6 pairs with Sharpe ratio greater than 1 only at 2σ trigger, of these only two reject the alternative hypothesis of the test.

Table 3.3: Distance Method -Haircutted Sharpe Ratio

	Distance Method					
	HSR			**p-value**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT18-TFIT24	1.58	0.14	0.00	0.0569	0.4450	0.4994
TFIT22-TFIT30	2.90	0.23	0.00	0.0019	0.4106	0.5000
TFIT19-TFIT22	2.37	0.00	0.00	0.0089	0.5000	0.5000
TFIT19-TFIT26	4.71	0.19	0.00	0.0000	0.4264	0.4998
TFIT19-TFIT28	4.57	0.06	0.00	0.0000	0.4769	0.4993

Performance of the cointegration approach exhibits an improvement for this backtest methodology. Using as a trigger 2 σ for individual test, eleven pairs show Sharpe ratios greater than 1, for 1.5σ there were eight pair and for 1σ only 7 pairs. When the multiple test methodology it is applied nine pairs for the 2σ reject the null hypothesis.

Table 3.4: Cointegration Approach -Haircutted Sharpe Ratio

	Cointegration Approach					
	HSR			**p-value**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT1018-TFIT20	6.741	3.194	2.189	0.0000	0.0007	0.0143
TFIT1018-TUVT25	4.211	0.000	0.000	0.0000	0.5000	0.5000
TFIT1018-TFIT26	4.706	4.137	0.380	0.0000	0.0000	0.3519
TFIT1018-TFIT28	4.706	4.681	0.765	0.0000	0.0000	0.2220
TFIT1018-TUVT33	4.652	0.000	0.000	0.0000	0.5000	0.5000

Stochastic approach exhibits the best performance between the 3 strategies. Using simple test 16 pairs reject the null hypothesis which at first glance seems not to be very different from the results of distance method however the Sharpe ratio are considerably higher. For instance the biggest Sharpe ratio in distance method was 8.38 which belongs to the pair TFIT19-TFIT26 instead stochastic approach gives a 19.75 Sharpe ratio for at least 3 evaluated pairs. Using the multiple test methodology the results of this methodology are clearly superior and 15 of the pairs reject the null hypothesis at any trigger level.

Table 3.5: Stochastic Approach -Haircutted Sharpe Ratio

	Stochastic Spread					
	HSR			**p-value**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT19-TFIT22	4.706	4.706	4.706	0.0000	0.0000	0.0000
TFIT18-TFIT26	5.267	1.982	3.151	0.0000	0.0238	0.0008
TFIT22-TUVT21	6.400	6.940	7.565	0.0000	0.0000	0.0000
TFIT22-TFIT26	7.642	8.691	8.850	0.0000	0.0000	0.0000
TFIT19-TUVT19	6.703	4.752	3.535	0.0000	0.0000	0.0002

3.2 Conclusions

The wholesale public debt market in Colombia presents low transaction costs that could stimulate the use of algorithmic trading strategies such as Pairs Trading, however given is high liquidity it is possible that the returns obtained are not as high as those generated in the stock market or derivative market.

Using the in sample and out of the sample result s as backtest , stochastic approach showed better results than those obtained using the cointegration method or the distance method, also it is the methodology that exhibit less impact for changes in the trigger. This could be a main characteristic to take into the account when the strategy it is implemented in markets whit higher transactional costs like shares or derivatives.

Usually trading strategies are not evaluated behind the out of the sample methodology and in some cases the investor only keeps the results in sample , this could lead major losses when the strategy it is apply in the real world. Before to establish if any strategy it is a real discovery it is necessary to adjust the expected results taking into the account the previous fitting apply during the strategy definition, this would give as a result a conservative risk profile from the expected returns.

Future research on the strategy should consider the use of high frequency data to maximize returns by making use of the intraday volatility of the public debt market, as well as in general it is necessary to establish a pair selection criteria to correct the gap left by the current literature. It is also suggested to analyze this type of strategies for markets such as derivatives and FX where there are greater volatilities and opportunities for increase portfolio returns.

Appendix A

In sample and Out of the sample Results

Table A.1: Results - Distance Method

	Out of the sample						In Sample					
	Returns*			**Sharpe Ratio**			**Returns***			**Sharpe Ratio**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT30-TFIT28	0.121	0.454	1.037	0.244	0.4870	0.87057	0.093	1.165	2.247	-1.557	0.86344	0.3353
TFIT26-TFIT28	-0.048	0.077	-0.960	-14.370	2.6151	-0.722	0.102	-0.246	-0.778	13.621	0.23466	1.262
TFIT26-TFIT30	-0.008	0.442	0.391	-64.171	0.4940	0.19755	-0.065	0.101	-1.150	-10.413	0.99652	0.8068
TFIT18-TFIT24	-0.078	0.362	0.455	-6.218	1.1545	1.01674	0.177	0.288	-1.277	5.625	1.92703	0.8114
TFIT26-TFIT24	-0.005	0.129	-0.044	-81.986	0.8640	0.94144	0.000	1.212	0.843	0.000	0.42831	0.8438
TFIT22-TFIT26	-0.031	-0.247	-0.284	-10.886	0.4583	0.72213	0.354	-0.301	0.405	2.367	0.84413	0.8343
TFIT22-TFIT24	-0.003	0.180	0.405	-99.899	0.6295	1.35544	0.893	0.698	0.616	1.551	0.18926	-0.029
TFIT22-TFIT30	0.084	-0.274	0.070	-0.127	1.6138	1.10332	-0.002	-0.068	1.047	-240.604	1.75819	1.0604
TFIT24-TFIT28	0.046	0.259	-0.248	0.330	1.5735	-0.0085	-0.002	0.239	2.580	-240.604	1.90584	1.3836
TFIT22-TFIT28	0.258	-0.293	-0.012	3.565	0.9184	1.00772	0.000	-0.750	1.477	0.000	0.16218	1.5048
TFIT19-TFIT22	-0.003	-0.031	0.252	-99.899	0.8182	0.95131	0.069	-0.211	1.489	3.554	-0.6247	0.9771
TFIT19-TFIT18	-0.006	-0.047	0.193	-35.794	-4.6212	0.09756	-0.015	-0.527	-0.945	1.275	-1.2846	-0.922
TFIT19-TFIT24	-0.076	0.379	0.731	-7.858	1.3834	0.57124	-0.068	0.442	0.724	-4.322	2.29965	1.1005
TUVT25-TUVT33	-0.234	-0.355	1.802	-2.106	-1.9831	0.88731	-0.058	0.081	3.761	2.609	2.1826	1.2418
TFIT19-TFIT26	0.000	-0.216	0.274	0.000	1.1338	1.27165	0.282	0.629	-0.147	7.242	2.40017	0.9848
TFIT18-TFIT22	0.222	0.016	0.420	1.435	1.5355	1.17661	-0.222	-0.055	0.002	2.624	1.35182	1.2361
TFIT19-TFIT30	-0.141	-0.446	-0.050	1.215	1.7303	1.18387	-0.002	-0.278	1.374	-240.604	2.24721	0.9026
TFIT19-TFIT28	0.242	-0.162	-0.218	2.739	1.6840	0.92415	-0.002	0.160	2.869	-170.425	2.37586	1.446
TFIT18-TFIT24	-0.078	0.362	0.455	-6.218	1.1545	1.01674	0.177	0.288	-1.277	5.625	1.92703	0.8114
TFIT18-TFIT26	0.000	-0.258	-0.339	0.000	0.9615	1.26893	-0.002	0.112	1.665	-240.604	2.27695	1.2948
Average	**0.013**	**0.017**	**0.217**	**-21.000**	**0.730**	**0.792**	**0.086**	**0.149**	**0.776**	**-55.152**	**1.223**	**0.894**

[1]*Percentage values

Table A.2: Cointegration Approach

	Out of the sample						In Sample					
	Returns*			**Sharpe Ratio**			**Returns***			**Sharpe Ratio**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT18-TFIT1018	0.000	0.268	0.168	0.000	2.559	0.561	-0.627	-0.627	0.146	-9.152	-5.179	-0.001
TFIT18-TFIT19	0.000	0.022	-0.301	0.000	-0.830	-9.181	0.020	0.020	-2.915	-5.427	0.147	-6.345
TFIT18-TUVT23	0.000	0.113	-0.624	0.000	0.237	-2.734	-0.070	-0.070	1.766	-13.954	0.369	1.807
TFIT19-TFIT1018	-0.248	-0.111	0.068	-8.517	-8.058	-0.155	-0.861	-0.861	-1.074	-7.089	-5.903	-2.857
TFIT19-TUVT19	-0.190	-0.218	-0.808	-5.139	-9.171	-5.950	0.270	0.270	-2.422	3.681	1.264	-2.616
TFIT19-TFIT20	-0.006	0.100	0.001	-81.081	-0.755	-10.432	-0.033	-0.033	-1.389	-1.676	-1.823	-3.486
TFIT19-TFIT22	-0.038	-0.295	-0.455	-12.406	-3.719	-3.212	-0.069	-0.069	0.388	-12.001	-0.158	-0.108
TFIT19-TFIT24	-0.388	-0.770	-1.117	-6.341	-2.314	-1.977	-0.313	-0.313	-2.396	-3.082	-0.789	-1.540
TFIT19-TUVT25	0.000	0.043	-1.948	0.000	-1.002	-2.689	-1.318	-1.318	-2.764	-7.711	-0.774	-2.497
TFIT19-TFIT26	-0.690	-1.129	-1.995	-4.165	-4.023	-2.774	-0.216	-0.216	1.206	-1.827	2.460	0.327
TFIT19-TFIT30	-0.533	-1.554	-2.468	-11.849	-6.535	-4.052	0.064	0.064	-1.411	-0.107	1.365	-0.341
TFIT19-TUVT33	0.000	-0.544	-4.272	0.000	-3.136	-2.420	-0.638	-0.638	-3.801	-2.765	-0.937	-0.890
TFIT22-TFIT20	-0.236	-0.484	-0.775	-3.247	-2.649	-1.619	-0.650	-0.650	-0.731	-8.189	-4.090	-0.776
TFIT22-TFI24	0.047	0.112	-0.161	-1.215	-0.100	-0.872	0.120	0.120	1.082	-0.414	0.806	0.273
TFIT22-TUVT25	-0.003	0.348	1.013	-113.767	2.090	0.928	-0.858	-0.858	-1.161	-2.620	1.719	-1.707
TFIT22-TFIT26	-0.051	-0.745	-0.633	-8.339	-1.277	-1.092	0.475	0.475	1.726	1.003	0.362	0.420
TFIT22-TFIT28	-0.769	-1.511	-1.162	-11.711	-3.018	-1.792	-0.008	-0.008	0.416	-108.346	0.803	-0.069
TFIT22-TFIT30	-0.030	-0.523	-2.239	-14.389	-3.465	-2.190	1.397	1.397	2.538	3.126	2.069	0.279
TFIT1018-TFIT20	-0.050	-0.395	-0.764	-10.819	-3.585	-1.420	0.716	0.716	2.656	5.729	1.114	3.911
TFIT18-TUVT21	-0.284	-1.523	-3.279	-12.241	-4.082	-3.358	0.150	0.150	-2.153	1.429	-9.882	-2.224
TFIT1018-TUVT23	-1.570	-1.737	-3.165	-5.859	-3.001	-2.544	0.050	0.050	-7.704	-0.144	-6.278	-4.514
TFIT1018-TFIT24	-0.136	-0.375	-1.171	-13.155	-1.324	-2.291	0.046	0.046	2.112	0.740	-0.095	1.386
TFIT1018-TUVT25	-1.083	-1.559	-3.397	-5.930	-2.919	-3.115	1.307	1.307	2.645	3.367	3.738	2.566
TFIT1018-TFIT26	-0.093	-1.655	-2.681	-6.595	-3.840	-2.404	2.763	2.763	4.110	5.652	4.677	2.651
TFIT1018-TFIT28	0.025	-2.361	-4.016	1.685	-3.298	-2.152	2.990	2.990	5.066	7.804	3.846	2.370
TFIT1018-TFIT30	-0.555	-2.576	-4.807	-11.831	-3.351	-2.456	4.067	4.067	3.753	5.457	3.282	1.814
TFIT1018-TUVT33	-1.783	-2.893	-3.765	-5.840	-5.583	-3.724	0.566	0.566	-1.002	5.461	4.437	-0.502
Total	**-8.666**	**-21.949**	**-44.751**	**-13.065**	**-2.820**	**-2.782**	**9.338**	**9.338**	**-1.316**	**-5.224**	**-0.128**	**-0.469**

[2]*Percentage values

Table A.3: Stochastic Approach - Results

	Out of the sample						In Sample					
	Returns*			Sharpe Ratio			Returns*			Sharpe Ratio		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT19-TFIT20	-0.583	-0.583	-0.583	-10.358	-10.358	-10.358	-2.045	-2.045	-2.045	-17.292	-17.292	-17.292
TFIT22-TFIT24	0.676	0.676	0.676	9.691	9.691	9.691	-0.562	-0.562	-0.562	-17.753	-17.753	-17.753
TUVT21-TUVT23	0.350	0.350	0.350	9.403	9.403	9.403	-1.510	-1.510	-1.510	-17.354	-17.354	-17.354
TFIT1018-TFIT20	-0.777	-0.777	-0.777	-10.269	-10.269	-10.269	-2.885	-2.885	-2.885	-17.241	-17.241	-17.241
TFIT1018-TUVT21	-1.976	-1.976	-1.976	-10.106	-10.106	-10.106	0.213	0.213	0.213	15.436	15.436	15.436
TFIT19-TFIT22	1.233	1.105	1.244	9.831	9.811	9.832	4.816	4.738	4.854	17.043	17.042	17.044
TFIT22-TUVT25	-1.125	-0.920	-0.383	-10.186	-10.227	-1.985	7.623	8.898	10.150	4.903	4.418	4.338
TFIT19-TUVT21	0.271	0.318	0.318	1.296	1.494	1.494	6.234	6.734	6.793	5.853	5.654	4.545
TFIT18-TFIT26	-1.947	-1.983	-3.000	-10.107	-10.105	-10.070	5.410	4.519	4.361	5.934	5.420	5.342
TUVT19-TUVT25	2.901	3.011	1.904	9.928	9.931	9.890	2.981	3.418	3.514	16.997	17.013	17.015
TFIT22-TUVT21	-2.255	-2.553	-3.002	-10.093	-10.082	-10.070	7.741	7.875	7.547	10.525	10.866	10.672
TFIT22-TFIT26	0.000	1.026	0.809	-0.188	1.989	1.739	4.521	4.688	4.688	17.038	17.041	17.041
TFIT18-TUVT23	3.518	3.631	3.991	6.943	4.598	4.537	4.261	4.263	4.341	17.033	17.033	17.035
TFIT19-TUVT21	0.271	0.318	0.318	1.296	1.494	1.494	6.234	6.734	6.793	5.853	5.654	4.545
TFIT26-TFIT30	0.341	0.208	0.966	9.388	8.994	9.784	4.370	3.660	4.564	17.035	17.020	17.039
TFIT1018-TFIT22	-0.765	-0.989	-1.441	-10.273	-10.211	-10.145	1.426	1.152	1.535	3.549	3.661	4.286
TFIT19-TUVT19	-1.332	-2.392	-2.340	-10.157	-10.087	-10.089	3.514	4.126	3.562	9.898	9.770	9.966
TFIT20-TFIT24	-2.353	-2.347	-2.453	-10.089	-10.089	-10.085	4.677	4.433	4.693	6.693	6.296	6.380
TFIT1018-TUVT23	-1.624	-1.624	-1.624	-10.129	-10.129	-10.129	-1.296	-1.296	-1.296	-17.393	-17.393	-17.393
TFIT1018-TUVT25	0.512	0.550	0.291	9.592	9.620	9.281	1.057	1.149	0.868	3.381	3.575	3.323
Total	**-4.666**	**-4.951**	**-6.712**	**-1.729**	**-1.732**	**-1.308**	**56.779**	**58.303**	**60.178**	**3.507**	**3.443**	**3.349**

[3]*Percentage values

Appendix B

Haircutted Sharpe Ratios

TABLE B.1: Haircutted Sharpe Ratios - Distance Method

	Sharpe Ratio			p-value			HSR			p-value		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT30-TFIT28	0.570	0.741	0.378	0.28435	0.22936	0.35271	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT26-TFIT28	2.125	2.071	1.336	0.01681	0.01918	0.09071	0.272	0.218	0.000	0.39281	0.41390	0.49991
TFIT26-TFIT30	-3.618	2.138	0.642	0.99985	0.01624	0.26050	0.000	0.287	0.000	0.50000	0.38696	0.50000
TFIT18-TFIT24	2.995	1.978	1.469	0.00137	0.02399	0.07097	1.581	0.138	0.002	0.05689	0.44496	0.49938
TFIT26-TFIT24	-197.290	-0.724	1.215	1.00000	0.76538	0.11210	0.000	0.000	0.000	0.50000	0.50000	0.49999
TFIT22-TFIT26	-1.999	1.009	1.457	0.97719	0.15656	0.07255	0.000	0.000	0.001	0.50000	0.50000	0.49947
TFIT22-TFIT24	1.442	0.697	0.375	0.07469	0.24287	0.35389	0.001	0.000	0.000	0.49957	0.50000	0.50000
TFIT22-TFIT30	3.880	2.080	0.728	0.00005	0.01877	0.23315	2.897	0.226	0.000	0.00188	0.41056	0.50000
TFIT24-TFIT28	2.021	2.174	1.307	0.02164	0.01486	0.09567	0.172	0.328	0.000	0.43152	0.37164	0.49995
TFIT22-TFIT28	1.764	1.277	1.523	0.03883	0.10072	0.06388	0.034	0.000	0.003	0.48635	0.49997	0.49877
TFIT19-TFIT22	3.514	-0.800	1.192	0.00022	0.78805	0.11660	2.371	0.000	0.000	0.00888	0.50000	0.49999
TFIT19-TFIT18	-1.940	0.279	-0.204	0.97382	0.39011	0.58090	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TFIT24	-8.067	1.777	1.291	1.00000	0.03782	0.09840	0.000	0.038	0.000	0.50000	0.48499	0.49996
TUVT25-TUVT33	1.410	0.261	1.060	0.07920	0.39701	0.14463	0.001	0.000	0.000	0.49973	0.50000	0.50000
TFIT19-TFIT26	8.387	2.036	1.391	0.00000	0.02087	0.08207	4.706	0.186	0.001	0.00000	0.42641	0.49979
TFIT18-TFIT22	2.177	0.284	1.325	0.01474	0.38827	0.09252	0.332	0.000	0.000	0.37011	0.50000	0.49993
TFIT19-TFIT30	1.560	2.180	0.871	0.05937	0.01462	0.19180	0.005	0.335	0.000	0.49809	0.36867	0.50000
TFIT19-TFIT28	5.099	1.835	1.480	0.00000	0.03326	0.06938	4.569	0.058	0.002	0.00000	0.47692	0.49928
TFIT18-TFIT24	2.995	1.978	1.469	0.00137	0.02399	0.07097	1.581	0.138	0.002	0.05689	0.44496	0.49938
TFIT18-TFIT26	2.376	1.754	1.596	0.00875	0.03976	0.05524	0.599	0.031	0.007	0.27462	0.48750	0.49715

Table B.2: Haircutted Sharpe Ratios - Cointegration Method

	Sharpe Ratio			p-value			HSR			p-value		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT18-TFIT1018	-20.481	-3.388	-1.532	1.0000	0.9996	0.9373	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT18-TFIT19	0.000	-1.916	2.585	0.0000	0.9723	0.0049	0.000	0.000	0.923	0.00000	0.50000	0.17807
TFIT18-TUVT23	2.244	1.523	1.645	0.0124	0.0639	0.0499	0.415	0.003	0.012	0.33908	0.49877	0.49527
TFIT19-TFIT1018	-9.763	-6.631	-4.130	1.0000	1.0000	1.0000	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TUVT19	-14.509	-2.399	2.319	1.0000	0.9918	0.0102	0.000	0.000	0.517	0.50000	0.50000	0.30243
TFIT19-TFIT20	-3.744	-3.250	-1.184	0.9999	0.9994	0.8817	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TFIT22	-2.850	0.183	0.217	0.9978	0.4274	0.4141	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TFIT24	-1.389	-0.814	-1.551	0.9176	0.7921	0.9396	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TUVT25	2.237	-2.645	-1.491	0.0126	0.9959	0.9320	0.406	0.000	0.000	0.34235	0.50000	0.50000
TFIT19-TFIT26	-1.467	0.988	-0.030	0.9288	0.1615	0.5118	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TFIT30	2.810	2.470	-0.118	0.0025	0.0068	0.5468	1.284	0.742	0.000	0.09950	0.22913	0.50000
TFIT19-TUVT33	-35.877	-3.062	-0.881	1.0000	0.9989	0.8109	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT22-TFIT20	-7.580	-4.453	-1.642	1.0000	1.0000	0.9497	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT22-TFI24	0.728	-0.296	0.388	0.2333	0.6163	0.3491	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT22-TUVT25	5.895	-0.305	0.521	0.0000	0.6196	0.3013	5.670	0.000	0.000	0.00000	0.50000	0.50000
TFIT22-TFIT26	0.802	0.691	0.077	0.2114	0.2447	0.4694	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT22-TFIT28	-0.901	0.532	0.134	0.8163	0.2974	0.4469	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT22-TFIT30	3.984	1.786	0.304	0.0000	0.0371	0.3806	3.041	0.040	0.000	0.00118	0.48385	0.50000
TFIT1018-TFIT20	6.637	4.093	3.391	0.0000	0.0000	0.0003	6.741	3.194	2.189	0.00000	0.00070	0.01429
TFIT18-TUVT21	-13.617	-4.460	-2.188	1.0000	1.0000	0.9857	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT1018-TUVT23	-1.240	-3.954	-6.011	0.8924	1.0000	1.0000	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT1018-TFIT24	0.000	5.027	3.609	0.0000	0.0000	0.0002	0.000	4.470	2.509	0.00000	0.00000	0.00606
TFIT1018-TUVT25	4.837	0.486	-0.524	0.0000	0.3134	0.6998	4.211	0.000	0.000	0.00001	0.50000	0.50000
TFIT1018-TFIT26	13.837	4.782	2.217	0.0000	0.0000	0.0133	4.706	4.137	0.380	0.00000	0.00002	0.35188
TFIT1018-TFIT28	13.795	5.181	2.485	0.0000	0.0000	0.0065	4.706	4.681	0.765	0.00000	0.00000	0.22203
TFIT1018-TFIT30	12.887	3.097	1.377	0.0000	0.0010	0.0843	4.706	1.741	0.000	0.00000	0.04080	0.49983
TFIT1018-TUVT33	5.160	-1.602	-2.405	0.0000	0.9454	0.9919	4.652	0.000	0.000	0.00000	0.50000	0.50000

Table B.3: Haircutted Sharpe Ratios - Stochastic Approach

	Sharpe Ratio			**p-value**			**HSR**			**p-value**		
Tested Pair	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ	2σ	1.5σ	1σ
TFIT19-TFIT20	-19.986	-19.986	-19.986	1.000000	1.000000	1.000000	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT22-TFIT24	19.182	19.182	19.182	0.000000	0.000000	0.000000	4.706	4.706	4.706	0.00000	0.00000	0.00000
TUVT21-TUVT23	18.434	18.434	18.434	0.000000	0.000000	0.000000	4.706	4.706	4.706	0.00000	0.00000	0.00000
TFIT1018-TFIT20	-19.941	-19.941	-19.941	1.000000	1.000000	1.000000	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT1018-TUVT21	-20.087	-20.087	-20.087	1.000000	1.000000	1.000000	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT19-TFIT22	19.746	19.751	19.751	0.000000	0.000000	0.000000	4.706	4.706	4.706	0.00000	0.00000	0.00000
TFIT22-TUVT25	3.797	3.722	3.972	0.000073	0.000099	0.000036	2.778	2.672	3.025	0.00273	0.00377	0.00124
TFIT19-TUVT21	4.668	4.989	4.778	0.000002	0.000000	0.000001	3.982	4.419	4.132	0.00003	0.00000	0.00002
TFIT18-TFIT26	5.607	3.253	4.062	0.000000	0.000570	0.000024	5.267	1.982	3.151	0.00000	0.02376	0.00081
TUVT19-TUVT25	19.682	19.716	19.718	0.000000	0.000000	0.000000	4.706	4.706	4.706	0.00000	0.00000	0.00000
TFIT22-TUVT21	6.405	6.770	7.180	0.000000	0.000000	0.000000	6.400	6.940	7.565	0.00000	0.00000	0.00000
TFIT22-TFIT26	7.229	7.881	7.965	0.000000	0.000000	0.000000	7.642	8.691	8.850	0.00000	0.00000	0.00000
TFIT18-TUVT23	12.833	12.826	19.757	0.000000	0.000000	0.000000	4.706	4.706	4.706	0.00000	0.00000	0.00000
TFIT19-TUVT21	4.668	4.989	4.778	0.000002	0.000000	0.000001	3.982	4.419	4.132	0.00003	0.00000	0.00002
TFIT26-TFIT30	10.405	0.053	10.201	0.000000	0.478821	0.000000	4.706	0.000	4.706	0.00000	0.50000	0.00000
TFIT1018-TFIT22	2.051	2.871	2.818	0.020110	0.002045	0.002415	0.199	1.383	1.298	0.42105	0.08326	0.09711
TFIT19-TUVT19	6.611	5.233	4.341	0.000000	0.000000	0.000007	6.703	4.752	3.535	0.00000	0.00000	0.00020
TFIT20-TFIT24	3.117	3.082	2.632	0.000914	0.001027	0.004250	1.772	1.718	0.997	0.03823	0.04290	0.15931
TFIT1018-TUVT23	-20.149	-20.149	-20.149	1.000000	1.000000	1.000000	0.000	0.000	0.000	0.50000	0.50000	0.50000
TFIT1018-TUVT25	3.043	2.903	3.777	0.001171	0.001846	0.000079	1.656	1.435	2.750	0.04881	0.07564	0.00298

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