Product Compatibility as an Strategy to Hinder Entry Deterrence

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Abstract

In many markets, firms produce and sell complementary components that form a product system. This paper studies the effects of compatibility in product advertisement and entry decisions in a differentiated product market. While advertising enhances the ability of consumers to mix and match components closer to their preferences, more advertising does not always generate larger welfare. In my model, an incumbent uses advertising to increase the prospects of market competition with the objective to deter potential entry. However, under some parameters, entry deterrence does not occur when products are made compatible. With compatible products, the incumbent either obtains large benefits from accommodation or equilibria when all consumers are aware of the existence of the available products emerge. In this latter case, the amount of advertising cannot be further expanded to protect the incumbent’s monopolistic position. As a result, policies in favor of compatibility may encourage entry and generate larger levels of advertisement.

Keywords: Product compatibility; Informative advertising; Entry deterrence; Market structure

JEL classification: D21, D43, L13, L15

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1 Introduction

A common interest in the literature of industrial organization has been to study the benefits from compatibility. Most of the analysis has focused on identifying the potential demand and supply economies of scale that compatibility brings about. For instance, enhanced compatibility allows the interchangeability of complementary products, permitting potential buyers to obtain a product closer to their preferences. Compatibility also grants the interchangeability of different product parts facilitating mass production. The literature also identifies strategic effects of compatibility. In their seminal work, Matutes and Regibeau (1988), argue that firms will make their components compatible to shift the industry demand upwards and make the market more profitable.\(^1\) However, their strategic considerations have been limited to the assumption that buyers are fully informed about all products, and the market structure is exogenous. Building on Matutes and Regibeau (1988), I study how compatibility choices affect the information trough advertisement that buyers obtain in the market, and its effect on the equilibrium market structure. My model identifies an additional strategic use of compatibility, one in which compatibility impedes an incumbent to use advertising as an entry deterrence strategy.

An example fitting my model is the capsule system popularized by Nespresso\(^2\). By the beginning of the decade, Nespresso\(^2\) had a dominant position, part of which came as a result of the existing patents which generated problems of compatibility from capsules produced by other suppliers.\(^3\) Additionally, in many countries, this dominant position was further enhanced by an aggressive advertising campaign to invoke curiosity among coffee drinkers who were unfamiliar with the brand. For instance, in Australia Nespresso\(^2\) implemented an advertising campaign in

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\(^1\)Economides (1989) obtains similar results in a more general setting. Kim and Choi (2015) revisits Economides (1989) and show a fundamental conflict of interests between consumers and producers over the standardization decision. Consumers prefer standardization with full compatibility because it offers more variety which confers a better match with their ideal specifications. However, firms are likely to choose the minimal compatibility to maximize product differentiation and soften competition.

\(^2\)Information about facts and figures of Nespresso\(^2\) can be found at https://www.nestle-nespresso.com/about-us/facts-and-figures.

\(^3\)A technician of Coffee machines states that: “the needles in the Nespresso\(^2\) models are so thin that they are not able to pierce the capsule of some competitors and in the extreme case, will crush the capsule.”
which consumers narrated their experience when first tried Nespresso®.\textsuperscript{4} Not after Nespresso’s® patents were revoked in different countries, new firms began creating coffee pods fully compatible with the Nespresso® system (“Nespressos bitter taste of defeat”, 24 April 2013, The Financial Times).\textsuperscript{5} A story of success is the case of PODiSTA®, the first company in Australia to offer Nespresso compatible pods. PODiSTA® found people stating that Nespresso® pods were too weak, and first launched pods with a more intense coffee flavor. Product variety also increased by the offering of a coffee machine fully compatible with Nespresso’s® pods.\textsuperscript{6}

With this example, I develop a simple duopolistic model with a potential entrant and an incumbent who produce and sell two differentiated components, needed in equal proportions to create a product system.\textsuperscript{7} To study firms’ interaction, I consider the following game. First, the entrant can invest in the design of its components to allow compatibility with the incumbent. After product design, the entry decision occurs and firms engage in advertising to give awareness of their goods to potential buyers. Together with advertising, firms also set prices for each of their components simultaneously. I allow the incumbent to use its first-mover advantage to announce an advertising intensity before entry occurs. My model explores the incumbent’s strategy to oversupply advertisement to deter entry. I show that, under some model parameters, such strategy is only feasible when products are not compatible. Then, a potential entrant may have incentives to produce compatible products to impede an effective entry deterrence strategy.

To study the equilibrium in different market configurations, section 3 presents the case when only the incumbent is active in the market. As I later discuss in section 4.2, a monopolistic market

\textsuperscript{4}The marketing and communication manager at Nespresso® Oceania, Nicole Parker, states that: “the reason to show hot beautiful and emotive sepia imagery showing coffee drinkers reveling the moment they first tried Nespresso was to attract new drinkers to try it”. Pictures and experiences will be displayed across consumer touch points including boutiques, trade point of sale, print, outdoor and online channels (“Nespresso puts Australian customers up front in first ever local campaign”, 30 September, AdNews).

\textsuperscript{5}More information can be found in Legal Post “Nestle loses patent on a Nespresso single-serve coffee machine”, November 26, 2013. Before this ruling, Nespresso® was told to share information about its coffee machines so that rival firms could more easily make compatible coffee capsules or pods.

\textsuperscript{6}By offering four varieties of coffees, five flavors of hot chocolate, and a range of sugar-free Poddies for kids, PODiSTA® has increased its range to ensure pods are accessible to all age groups, including those who don’t drink coffee (“Our Story”, PODiSTA® https://podista.com.au).

\textsuperscript{7}In the coffee example, a system or final product is obtained with the combination of a coffee machine and coffee capsules. To be consistent with the assumption of equal proportions, I will consider the unit of a capsule to be the number consumed in the life-cycle of the coffee machine.
may be the result of either large entry cost or an effective entry deterrence strategy. With a monopolistic incumbent, equilibrium prices generate inefficiencies as some consumers who receive an advertisement, and obtain a net surplus from consumption, are left unserved. Due to the lack of targeting in advertising, a monopolist finds it optimal to extract rents from consumers with preference closer to its product specification rather than selling to all its potential demand. The equilibrium intensity of advertisement increases with consumers’ valuation as the profits from purchase raises.

The bulk of my analysis, with the interaction of two active firms, is presented in section 4. With an exogenous level of product compatibility, I obtain that prices and advertising intensity increase with compatibility. While equilibrium prices decrease with advertising, because consumers can shop around as they are aware of both systems, with compatible products competition materializes at the component level rather than at the system level, and firms have fewer incentives to decrease prices. In my model, the increased advertising generated by compatibility always has a second order effect on the reduction of competition that compatibility brings about. How efficient is the advertising technology and the fixed cost of entry determine the profits of the entrant and the subsequent equilibrium market structure.

When the magnitude of the fixed entry cost is not sufficient to block entry, section 4.2 identifies an entry deterrence strategy consisting in announcing an oversupplying of advertisement before entry occurs. Contrary to Schmalensee (1983), the strategic implications of advertising align with those in productive capacity investment. In my model, more advertising by the incumbent increases competition and makes it more difficult for a potential entrant to generate profits. I later endogenize compatibility decisions and show that the feasibility of an entry deterrence strategy depends on product compatibility. When products are made compatible, two effects lessen the ability of using advertisement to deter entry. First, more compatible components reduce competition, and more advertising does not suffice to overcome the prospects of entry gains. Second, with compatible components, equilibria with fully informed consumers arise, and the incumbent can not further
expand his level of advertising to protect his monopoly position.

The welfare analysis is presented in section 5. For a given market structure, my model identifies different sources of inefficiencies. A monopolistic incumbent oversupplies advertising only when used as an entry deterrence strategy and undersupplies it otherwise. With two active firms, even with relatively substitutable products, the level of product compatibility determines whether advertising is either over or undersupplied. Advertising provides customers with the possibility to shop around and better combine components closer to their preferences. This expands the market as consumers become aware of the existing products. While the full market expansion and the better match of buyers is not taken into account by firms, generating advertising to be undersupplied, advertisement also permits companies to steal consumers away from the rival. As the profits for consumer increase with compatibility so does the incentives to capture consumers from the rival. This last effect may generate advertising to be oversupplied.

Before presenting the formal model, the next section briefly discusses the related literature.

1.1 Literature

The main contribution of this paper is to study the effects of product compatibility on the equilibrium market structure when firms use advertisement to inform consumers about their products. I show how product compatibility affects the information that buyers obtain in the market which shapes the incumbent’s incentives to deter entry. In my model, product compatibility either increases the profits of accommodation or renders the entry deterrence strategy unfeasible.

The first work to consider the link between compatibility and entry deterrence is Tirole (1987). He shows that firms only have an incentive to produce compatible components as long as they do not try to exclude their rival from the market. Preserving product incompatibility is a strategy to deter entry because it reduces industry demand and makes the incumbent respond to entry with more aggressive pricing than if compatibility prevailed. While the link between compatibility and product competition is preserved in my model, the channel that I consider is how compatibility
affects the information that consumers obtain through advertisement and the subsequent equilibrium market structure.

By studying the link between advertising and potential entry, my model relates to the theoretical literature regarding advertising as an entry deterrence strategy like in Bain (1965); Nelson (1970, 1974); Schmalensee (1974); Spence (1980); Cubbin (1981); Rajeev (1994) and Bonnano (1988).\textsuperscript{8} In those early papers, advertising alone is not sufficient to deter entry. In Schmalensee (1974), advertising generates a barrier to entry only with the presence of both demand and costs asymmetries.\textsuperscript{9} In Spence (1980), output together with advertising is needed to create economies of scale to credibly deter entry. In the later set of papers, by changing the preferences of consumers, the persuasive element of advertising is used to deter potential competitors. Then, in Cubbin (1981), advertising increases the monopoly power through a reduction in the price elasticity of demand, which jeopardizes the competition from a potential entrant to obtain consumers. In Rajeev (1994), advertising generates brand loyalty, and the subsequent switching costs make it difficult for an entrant to get a positive market share. In Bonnano (1988), following an increase of advertising by the incumbent, the entrant needs to match the amount of advertising to avoid her product to be perceived as inferior. Departing from these models, advertising does not affect consumers’ valuation but is used as a mean by which firms can inform consumers of their consumption alternatives.

By considering elements of informative advertising, my work relates to Schmalensee (1983) and Ishigaki (2000). In Schmalensee (1983), an incumbent’s investment in advertising increases the set of customers not tempted by an entrant’s product. The incumbent is then less eager to expand output or lower the price in response to entry because such strategy involves giving up secure profits that could be earned on sales to loyal customers. As a result, an incumbent’s investment in advertising makes entry more attractive by signaling the entrant a friendlier accommodation. Therefore, an optimal deterrence strategy involves under-advertising. This feature is similar to the

\textsuperscript{8}Other models consider advertising as a signal of market profitability as Milgrom and Roberts (1983,1986), Kihlstrom and Riordan (1984), Bagwell and Ramey (1987) and Baldani and Masson (1981).

\textsuperscript{9}The author argues that if the existing firms and the entrant can produce equally effective advertising and equally desirable products it is hard to see why any restrictions to entry will exist.
Farrell and Shapiro’s (1988) switching cost model, in which the established firm may choose to mine its installed base of customers allowing the entrants to serve new customers. Also in Schmalensee (1983), an established firm’s goodwill leads a firm to price high and allow entry. A similar result is found in Ishigaki (2000) where he shows that the incumbent is unable to strategically deter entry via informative advertising in a homogenous product market. Contrary to these works, by considering a differentiated product market where firms produce two complementary components, I find that an oversupply of advertisement is used to deter entry. My results also contrasts with Fudenberg and Tirole (1984) where more investment makes the incumbent soft.\textsuperscript{10}

My work also relates to the literature on horizontal foreclosure and commitment. Most of this literature, starting with Dixit (1980), has studied how an established firm pre-entry decisions can influence the prospective entrant’s view of what will happen if he enters, and how a company may try to exploit this possibility to its advantage. In Nalebuff (2000) and Choi and Stefanadis (2001), an incumbent decides to bundle their products as a way to deter entry. Nalebuff (2000) point out that, while bundling intensifies price competition, it also gives a larger market share to the bundling firm. The prospects of a small market deter entry of potential competitors. In an innovation process when an incumbent monopolist faces the threat of entry in all complementary components, Choi and Stefanadis (2001) show that tying makes the prospects of successful entry less certain, which discourages rivals from investing and innovating.\textsuperscript{11} I explore the strategy of a potential entrant to make his products compatible with the incumbents to hinder the latter the use of advertising as means to deter entry.

Finally, my welfare analysis complements the extensive literature studying the effects of informative advertisement on welfare. Staring with Butters (1977), the literature have shown that

\textsuperscript{10}This is the so called fat-cat effect. In Fudenberg and Tirole (1984), the incumbent’s optimal price is an increasing function of investment in advertising because the ads lower the elasticity of demand for the product, so that more advertising makes the incumbent soft. By being soft, the fat cat encourages its rival to be less aggressive. This strategy involves overinvestment in order to optimally accommodate entry.

\textsuperscript{11}In Carlton and Waldman (2002) forcing a bundle product on consumers denies the competitor sales when it only has the complement product. The sales reduction can be enough to make entry unprofitable. In Whinston (1990) by tying the monopolist reduces the sales of its tied good market competitor, thereby lowering his profits below the level that would justify continued operation.
the market-determined level of advertising is excessive (Grossman and Shapiro (1984); Stegeman (1991); Soberman (2004) and Hamilton (2009)). Grossman and Shapiro (1984) demonstrate that the market level of advertising excess the social optimal when brands are sufficiently close substitutes. In Hamilton (2009), when advertising by one brand reaches existing buyers of the rival brand, it invites consumers away from the rival, an activity that is less valuable to society than to individual firms. In my model, whether the market level of advertisement is excessive or not depends on the choice of product compatibility and the equilibrium market structure rather than the degree of product differentiation as considered in other studies.

In the next section, I present the formal model and I later proceed with the analysis.

2 Model

I consider a market for system goods in which two firms produce two components \( x \) and \( y \), that generate value when combined in equal proportions. My model adopts a variation of Matutes and Regibeau (1988) in which I endogenize the market structure and consumers are only able to locate specific brands when they see an advertisement.

In the market, consumers have heterogeneous preferences over the characteristics of each element summarized by her location \((x, y)\) in the product space of a Hotelling square of length 1. I assume a unit mass of consumers uniformly distributed and an inelastic demand for one unit of the system generating a gross utility of \( V \). Firms’ components are maximally differentiated. Then, the incumbent locates at the origin of the Hotelling square, and the potential entrant locates at the point of coordinates \((1,1)\). The distance \(d_{ij}\) from the ideal \(i\)'s component characteristics and the \(j\)'s firm specification generate a transportation costs normalized to 1 as illustrated in Figure 1.

To simplify the analysis I assume that, under competition, the equilibrium prices are suffi-
Assumption 1. The gross utility of consumption is sufficiently large to guarantee a covered market under competition.

The game in played in three stages. In the first stage, the potential entrant makes a decision regarding its product design. I assume that it can invest in a blueprint, with a cost of $\Gamma$, that will allow to make his components compatible with the incumbent. The compatibility choice by the entrant does not affect the intrinsic characteristics of the components produced, but it allows consumers to assemble systems combining components manufactured by different firms. After product design, entry is considered generating a fixed costs of $F$. In stage two, firms decide simultaneously on the advertising reach and the prices for each component. Advertisement is truthful and convey information about the existence of products and the attributes contained in each brand. In stage three, consumers who have been exposed to advertising make their purchasing decisions.\(^{15}\)

\(^{14}\)The fixed cost of entry may be the cost needed to develop the product or to set up the facilities to produce. \(^{15}\)The only way consumers receive information about the products is with advertising. Buyers have a passive role as they do not search for or experiment with products for which they have not seen advertising. In Meurer and Stahl (1994) consumers observe prices while firms decide whether to inform them about product characteristics. In Bester and Petrakis (1995) consumers know that two firms exist and the price of the product in their region but only learn the price of the other firm once they receive an advertisement.
Firms send independent advertising messages and do not have the ability to target advertisements towards consumers located at a particular point in the Hotelling square. Hence, the advertising reach $\phi$, generating expenditure $E(\phi, \alpha) = \phi^2/\alpha$, makes a fraction $\phi$ of the target population exposed to the message. The parameter $\alpha$ stands for the effectiveness of the advertising technology. In my model, I allow for advertising to give a first-mover advantage to the incumbent as it can announce an intensity of advertising $\phi_F^I$ before the entry decision.\footnote{Advertisement announcements were present in 2013 when \textit{Nespresso} launched an aggressive advertising campaign in which club members were interviewed to express their impressions when they first tried a \textit{Nespresso} cup.} I assume an implicit cost of advertisement announcement such that the incumbent will only find it profitable to engage in such strategy when the resulting market structure is a monopoly.\footnote{This assumption is necessary to maintain the nature of competition. After an advertisement announcement, the incumbent becomes a Stackelberg leader and he always obtains larger payoffs than Cournot competition. As I will discuss in section 4.2, I want to disregard the effect of reduced competition emerging in a Stackelberg environment, but only consider the case where advertising is used as an entry deterrence strategy.}

The timing of the game is summarized in:

$t_0$: Product design by the potential entrant.

$t_0'$: The incumbent decides whether to announce advertisement or not.

$t_1$: Entry decision.

$t_2$: If entry occurs, firms compete in advertising intensity and prices.

$t_3$: Consumers make their purchasing decisions.

Finally, when entry happens, I look for a symmetric subgame-perfect Nash equilibrium where each firm maximizes its profit and each consumer takes optimal purchasing decisions. In the next section, I consider the case when only the incumbent is active in the industry. Later, I study competition. All proofs not included in the main text are in the Appendix.
3 Monopoly

In this section, only the incumbent is active in the market. Later in section 4, I identify the conditions leading to a monopolistic market. With a single firm, I show that, prices and the advertising intensity increase with consumers’ valuation. However, a single incumbent never sets prices to serve all buyers who receive an advertisement: because advertisement is not targeted, the market power of a monopolist generates inefficiencies as some advertisement is wasted.

With a single firm, a consumer only buys the system from the incumbent. She effectuates a purchase if the value of consumption is above preference costs - the distance from consumer location and the incumbent’s product specification - and the equilibrium prices, i.e., \( V \geq (d_x^x + d_y^y) + (p_x^I + p_y^I) \). With this consumers’ purchasing rule, the incumbent sets prices and advertising reach to maximize its profits

\[
\Pi_I = \phi_I \left( p_x^I + p_y^I \right) D_{II}(p_x^I, p_y^I) - E(\phi_I, \alpha).
\]  

(3.1)

Restricting attention to the case where consumers’ valuation is above the maximal preference cost, i.e., \( V > 2 \), the incumbent sells to all buyers who receive an advertisement by setting a price for each of the components equal to \( p = (V - 2)/2 \). I show that this price is never optimal: the monopolist always benefits from increasing the price of each component by an amount \( \epsilon \), even if this generates a loss in demand equal to

\[
\frac{1}{2} \left( \int_{1-\epsilon}^{1} \left( \int_{1-\epsilon}^{1} dx \right) dy \right) = \frac{\epsilon^2}{2}.
\]

To see this, by setting a price for each component equal to \( (V - 2)/2 + \epsilon \) the incumbent obtains

\[
\Pi_I(V, \epsilon) = \phi_I \left[ (V - 2 + 2\epsilon) \left( 1 - \frac{\epsilon^2}{2} \right) \right] - E(\phi_I, \alpha),
\]

(3.2)

and by maximizing with respect to \( \epsilon \) gives \( \epsilon^M(V) = (1/6) \left[ 2 - V + \sqrt{28 + (V - 4)V} \right] \). As a result, the incumbent maximizes profits by increasing the price of the system that will serve to all consumers
by an amount $2\epsilon^M(V)$. Because this magnitude is a decreasing function on consumers’ valuation and converges to zero from the right as consumers’ valuation grows to infinity

$$\lim_{V \to \infty} \left[ \frac{1}{6} \left( (2 - V) + \sqrt{28 + (V - 4)V} \right) \right] = 0^+,$$

confirms that a monopolist never sets prices to serve all potential buyers who receive an advertisement. A monopolist benefits in exploiting consumers with a location closer to its product specification and forgoes the benefits that could be generated from buyers who have received an advertisement but have preferences further away. This result identifies an inefficient use of advertising as some consumers aware of the good produced by the monopolist do not make a purchase. The inability of firms to target advertisement explains this result.

Because consumers are uniformly distributed in the Hotelling square, it is optimal to set the same price for each component, i.e., $p^I_x = p^I_y = p^M = (V - 2)/2 + \epsilon^M(V)$. Using the magnitude of $\epsilon^M(V)$, the first order condition of equation (3.1) with respect to advertising, and using a cost of advertisement of the form $E(\phi_I, \alpha) = \phi^2/\alpha$, the equilibrium prices and advertising intensity are introduced in the next Proposition.

**Proposition 1.** A monopolistic incumbent sets the same price for each component $p^I_x = p^I_y = p^M$, where

$$p^M = \frac{1}{6} \left[ 2(V - 2) + \sqrt{28 + (V - 4)V} \right],$$

(3.3)

and an advertising intensity of $\phi^M = \min \left\{ \alpha p^M(1 - (\epsilon^M(V))^2/2), 1 \right\}$.

Both prices and advertising intensity increase with consumers’ valuation. Absent competition, the incumbent passes-on the increase on consumers’ valuation to prices, and a higher profit per consumer gives incentives to boost the advertising reach. Because ads are independently sent to consumers, the equilibrium price is not a function of the advertising level, which increases with a more efficient advertising technology.
The profits from an incumbent monopolist $\Pi^M = 2p^M \phi^M D^{M}_{II}(\cdot) - (\phi^M)^2/\alpha$ are

$$\Pi^M = \alpha (p^M)^2 \left(1 - \frac{\epsilon^M(V)^2}{2}\right),$$  \hfill (3.4)

with an advertising technology generating an advertising reach $\phi^M < 1$, and profits

$$\Pi^M = 2 (p^M)^2 \left(1 - \frac{\epsilon^M(V)^2}{2}\right) - \frac{1}{\alpha},$$  \hfill (3.5)

when $\phi^M = 1$ resulting from an advertising technology $\alpha > 2/p^M (2 - \epsilon^M(V)^2)$. From expressions (3.4) and (3.5), the larger the effectiveness parameter $\alpha$, the cheaper it is to inform consumers and the larger the profits the incumbent obtains. With competition, the efficiency on the advertising technology determine the competitive pressure in the market affecting the equilibrium prices. I later show that a more effective advertising technology may generate lower profits under competition.

4 Competition

I now turn to the main theme of the article: the compatibility and entry decision of a potential competitor. In this section, I first study the consequences of competition with an exogenous compatibility level, and I derive the circumstances under which the incumbent uses advertising to deter entry. I later show that an entry deterrence may not occur under product compatibility.

With competition, advertising generates heterogeneity among consumers in addition to their consumption preferences. Then, with probability $\phi_I \phi_E$, a buyer receives an advertising message from both firms and becomes selective as she can “mix and match” components produced by different companies. With probability $\phi_I (1 - \phi_E) + \phi_E (1 - \phi_I)$, a consumer is only exposed to one advertising message and becomes captive to the firm she has received the advertisement from. With probability $(1 - \phi_I) (1 - \phi_E)$ a consumer is uninformed and cannot make any purchase.

Regarding purchasing decisions, a buyer receiving one advertisement takes the same purchasing
decision rule as presented in the previous section. A buyer receiving both advertisements, selects components, potentially from different firms, to maximize the utility

\[ V - (d_i^x + d_j^y) - p_i^x - p_j^y - z \times 1(i \neq j) \quad i, j = I, E, \] (4.1)

where the parameter \( z \in [0, 1] \) represents the level of compatibility. Incompatibility generates a direct utility loss if the consumer purchases components produced by different firms. With the normalization of the transportation costs, \( z = 1 \) represents full incompatibility and \( z = 0 \) stands for complete compatibility.

To solve for the equilibrium of the game, I work with backward induction. Then, with a given level of compatibility \( z \), I find the equilibrium prices and advertising reach. I later establish under which conditions, will the incumbent engage into an entry deterrence strategy. Finally, I study the choice of compatibility.

4.1 Prices and Advertising

Under competition, to characterize the equilibrium prices and advertising reach, I first construct the demand for each group of consumers. To this aim, I use the assumption that consumers’ valuation is large enough to guarantee a market that is fully-covered.

When the market is fully-covered, the demand for consumers who receive one advertisement is equal to one. The demand for selective consumers - aware of the existence of both systems - depends on prices and the level of product compatibility. Figure 2 illustrates the demand for the different systems available as a function of prices and level of compatibility. The first underscript stands for the demand of component \( x \) and the second for component \( y \). Then, \( D_{ij} \) is the consumers’ demand for component \( x \) produced by firm \( i \) and component \( y \) produced by firm \( j \). The diagonal line in the figure depends on the level of compatibility between the components produced by different firms. Then, with full incompatibility the diagonal line divides the full square, and selective consumers can only purchase a system with components produced by the same firm. With
complete product compatibility the diagonal line disappears, and consumers can mix and match different firms’ components according to their preferences. With a uniform distribution of buyers, the demand for selective consumers equals

\[
D_{EE} = \int_0^1 \left( \int_0^{1/2 + \frac{p_I^y - p_E^y + z}{2}} dy \right) dx \\
+ \int_0^{1/2 + \frac{p_I^y - p_E^y + z}{2}} \left( \int_0^{1/2 + \frac{p_I^y - p_E^y + z}{2}} dy + \int_{1/2 + \frac{p_I^y - p_E^y + z}{2}}^{x} dy \right) dx \\
= \frac{1}{4} \left[ (1 + p_I^y - p_E^y + z)(1 + p_I^y - p_E^y + z) - 2z^2 \right],
\]

and

\[
D_{EI} = \int_0^{1/2 + \frac{p_I^y - p_E^y + z}{2}} \left( \int_0^{1/2 + \frac{p_I^y - p_E^y + z}{2}} dy \right) dx = \frac{1}{4} \left[ (1 + p_I^y - p_E^y - z)(1 - p_I^y + p_E^y - z) \right].
\]

Similar calculations give the demand for the systems \(D_{II}\) and \(D_{IE}\).

With competition, the profits for each firm depends on its proportion of selective and captive consumers.
consumers

\[\Pi_i = \phi_i \phi_j ((p^x_i + p^y_i)D_{ii} + p^x_i D_{ij} + p^y_i D_{ji}) + \phi_i(1 - \phi_j)(p^x_i + p^y_i) - E(\phi_i, \alpha), \text{ for } i, j = E, I. \quad (4.4)\]

Because, firms undertake advertising and pricing decisions simultaneously, the first order conditions on prices and advertising generate a sufficient condition for optimality. The pricing conditions \(p^x_E\) and \(p^y_E\) for the entrant are

\[
\phi_E \phi_I \left[ D_{EE} + D_{EI} + (p^x_E + p^y_E) \frac{\partial D_{EE}}{\partial p^x_E} + p^x_E \frac{\partial D_{EI}}{\partial p^x_E} + p^y_E \frac{\partial D_{IE}}{\partial p^x_E} \right] + \phi_E (1 - \phi_I) = 0,
\]

\[
\phi_E \phi_I \left[ D_{EE} + D_{IE} + (p^x_E + p^y_E) \frac{\partial D_{EE}}{\partial p^y_E} + p^y_E \frac{\partial D_{IE}}{\partial p^y_E} + p^x_E \frac{\partial D_{IE}}{\partial p^y_E} \right] + \phi_E (1 - \phi_I) = 0,
\]

and introducing the demand functions (4.2) and (4.3) yields

\[
\phi_E \left[ (2 - \phi_I) + (p^x_I - 2p^x_E) \phi_I + (p^y_I - 2p^y_E) \phi_I z \right] = 0,
\]

\[
\phi_E \left[ (2 - \phi_I) + (p^y_I - 2p^y_E) \phi_I + (p^x_I - 2p^x_E) \phi_I z \right] = 0.
\]

The same pricing conditions are obtained for the incumbent but changing the underscripts. Manipulating the previous expressions give the condition \(p^x_I - p^y_I = 2(p^x_E - p^y_E)\). Restricting attention to a symmetric equilibrium in which firms set the same price for each component gives \(p^x_i = p^y_i = p^D_i\).

This condition together with the first order condition specifying the optimal level of advertising give an equilibrium characterized by the system

\[p^D \phi^D (1 + z) = (2 - \phi^D), \quad (4.5)\]

and

\[p^D \times (2 - \phi^D) = E_\phi(\phi^D, \alpha). \quad (4.6)\]

The market advertising condition in (4.6) indicates that the marginal private costs equal to the
marginal private revenue. With an increase in the advertising reach, a firm informs on average one more consumer. With probability $\phi$, the consumer is already informed by the rival, and the company makes a sale only half of the time. With probability $1-\phi$ a buyer becomes captive and purchases the whole system. As a result, the increase in expected sales becomes $2 \times (\phi^D/2 + (1-\phi^D)) = 2 - \phi^D$.\footnote{This condition coincides with Hamilton (2009). However, in my model the equilibrium price is affected by the level of product compatibility.}

Combining expressions (4.5) and (4.6), I obtain the equilibrium condition

$$E_\phi(\phi^D, \alpha) = \frac{(2 - \phi^D)^2}{\phi^D(1 + z)}.$$ (4.7)

The effects of changes in compatibility and advertising technology on the equilibrium advertising and prices are obtained by direct manipulation of this equilibrium condition. I show that these effects hold true for a fairly general advertising technology where I only need to assume decreasing returns on both advertising reach and advertising effectiveness together with $E_{\phi\alpha}(\cdot) < 0$. I later use the functional form $E(\phi, \alpha) = \phi^2 / \alpha$ to obtain a closed solution for prices and advertising.

An increase in the advertising effectiveness $\alpha$ expands the equilibrium advertising reach $d\phi^D/d\alpha > 0$ (in (4.6), and assuming that $E_{\phi\alpha}(\cdot) < 0$, the left-hand side decreases with $\alpha$, and the equality is recovered from an increase in the advertising intensity). In equilibrium a higher proportion of consumers become aware of the product system. With the equilibrium condition in (4.7) less compatibility decreases the equilibrium advertising intensity $d\phi^D/dz < 0$ (the left-hand side of (4.7) decreases with $z$, and a reduction on the advertising intensity restores the equilibrium). With less compatible components, competition for selective consumers intensifies: firms compete with the whole system rather than with separate components. To reduce such competitive pressure, firms respond by lowering advertising reach which decreases the proportion of selective consumers - the price elastic segment of the market. The effect on the equilibrium prices are less direct to obtain and the next Lemma states the result.

**Lemma 1.** With competition, equilibrium prices decrease with advertising effectiveness but increase.
with compatibility.

\[
dp^D/dz < 0, \quad dp^D/d\alpha < 0.
\]

The reduction in prices with advertisement effectiveness comes from the pro-competitive effect of advertising though improved information. Advertising leads to more elastic demand: by providing information, advertising expands consumer choices and generates a larger proportion of selective consumers, the price elastic part of the market, with respect to captive consumers, the inelastic part of the market. Firms set higher equilibrium prices when they sell more compatible components. Compatibility weakens firms’ incentives to cut prices as the rival captures some of the benefits with those consumers purchasing one of its components.\(^{19}\) However, greater compatibility also generates higher advertising intensities, enhancing consumers’ information which boost competition. In the appendix, I show that the change of equilibrium prices with respect to compatibility is explicitly characterized by

\[
dp^D/dz = \underbrace{-\phi^D (2 - \phi^D)}_{\text{direct effect}} + \underbrace{-\frac{d\phi^D}{dz} \times 2(1 + z) \phi^D (1 + z)^2}_{\text{indirect effect}},
\]

and the indirect effect coming form the change in advertisement is always of second order. Therefore, equilibrium prices always increase with product compatibility.\(^{20}\)

Introducing the parametric advertisement cost function, \(E(\phi, \alpha) = \phi^2/\alpha\), into condition (4.7) an explicit solution for prices and advertisement is presented in the next Proposition.

**Proposition 2.** *With a level of compatibility \(z\), competition in the market generates an advertising*

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\(^{19}\)This result contradicts Christou and Vettas (2006). They show that the measure of marginal consumers, those who are indifferent between two alternatives, is usually smaller under incompatibility, which implies that the marginal gains from a price cut are smaller when firms sell incompatible products. Kim and Choi (2015) finds market configurations that shorten the market boundaries and generate more differentiated product systems. This softens price competition and increases profits.

\(^{20}\)In Soberman (2004), increasing informative advertising alone can lead to both higher or lower prices. In his model, the direction of this relationship depends on the level of product differentiation between competing firms. Einhorn (1992) shows that in a duopolistic model, in which system components and consumer tastes differ with regards to vertical characteristics, the sum of each producer’s component prices is (weakly) higher with compatible components.
reach of $\phi^D = \min \left\{ \frac{2\alpha}{\alpha + \sqrt{2\alpha(1+z)}}, 1 \right\}$. The equilibrium price for each component is

$$p^D = \begin{cases} \sqrt{\frac{2}{\alpha(1+z)}} & \text{for } \phi^D < 1, \\ \frac{1}{1+z} & \text{for } \phi^D = 1. \end{cases} \tag{4.8}$$

This equilibrium gives rise to different market configurations depending on the level of information consumers receive. The following Corollary asserts that the equilibrium market configuration depends on the efficiency of the advertising technology.

**Corollary 1.** With a large advertising efficiency, i.e., $\alpha \geq 2(1+z)$ there is a “fully-informed” equilibrium in which both firms set $\phi^D = 1$ resulting in all consumers in the market being selective. Otherwise, there is a “partially-informed” equilibrium in which firms set $\phi^D < 1$, and selective and captive consumers coexist in the market. In a ”partially-informed” market there are also potential buyers who do not effectuate any purchase because they do not receive any advertisement.

The equilibrium prices and advertising reach generate a duopolistic profit of

$$\Pi^D = \frac{\alpha - (1+z)}{\alpha(1+z)} \tag{4.9}$$

in a “fully-informed” market, and

$$\Pi^D = \frac{4\alpha}{\left(\alpha + \sqrt{2\alpha(1+z)}\right)^2}, \tag{4.10}$$

when the market is “partially-informed”. The equilibrium profits of my model are consistent with the results of Grossman and Shapiro (1994). Lower advertising costs reduces profits as a result of an increased competition only when my market is “partially-informed”. In a “fully-informed” equilibrium, a better advertising technology only generates cost reductions keeping constant the competitive pressure.
4.2 Entry Deterrence

Most of the existing models in Industrial Organization establish that the fixed cost of entry has a direct effect on the entry decision and the equilibrium market structure of an industry. In addition to fixed costs, by requiring that firms need to advertise their products in the market, I show that entry also depends on the effectiveness of the advertising technology. Advertising determines the competitive pressure in the market and affects the ability of the entrant to generate profits. Contrary to existing models of entry deterrence and informative advertisement, in my model the incumbent may announce an oversupply of advertising to signal aggressive competition and deter the entry of a potential competitor.

With the duopolistic profits obtained in the previous section, entry is blocked depending on the effectiveness of the advertising technology and the fixed costs of entry. With $\alpha < 2(1 + z)$ entry does not occur for $F \geq 4\alpha / \left(\alpha + \sqrt{2}\alpha(1 + z)\right)^2$. When the advertising technology is sufficiently effective, i.e., $\alpha \geq 2(1 + z)$, such that the market is “fully-informed”, entry does not happen for an entry fixed cost of $F \geq (\alpha - (1 + z)) / \alpha(1 + z)$. As a result, the incumbent is the only active firm in the market and the equilibrium is the one characterized in section 3.

I proceed to show that only in situations when the market is “partially-informed”, the incumbent may have incentives to announce an oversupply of advertising with the objective to deter entry. An increase in the advertising reach generates more potential buyers to be informed about the existing products, boosting the competitive pressure in the market. With more competition, the entrant obtains less profits per consumer and advertises less in equilibrium. The ability of the incumbent to announce an advertising intensity before entry occurs is a way in which an incumbent can exploit its leadership role.

To deal with commitment, I consider that an announcement of advertising not fulfilled generates serious problems of reputation. For instance, in 2013 Nespresso® Oceania announced an advertising campaign in which photos and interviews were taken to club members around Australia with the aim to invoke curiosity among coffee drinkers unfamiliar with the brand. The media was talking
about this advertising campaign far before it was implemented. In addition, with an announcement of advertising, the nature of competition changes. The incumbent becomes the Stackelberg leader and reduces the profits of the entrant compared to Cournot competition. To focus only on the use of advertisement as an entry deterrence strategy, I assume some cost associated to advertising announcements such that the incumbent finds it optimal to make such announcements only when it successfully deters entry.

The amount of advertising reach $\tilde{\phi}_I$ that successfully deters entry solves $\Pi^D_E(p_I, p_E, \tilde{\phi}_I, \phi_E | z, \alpha) = F$, where $p_I$, $p_E$ and $\phi_E$ maximize the profits for the incumbent and the entrant given $\tilde{\phi}_I$. The first order condition with respect to prices gives

$$p^I_I = p^I_I = p^I_I = \frac{4\tilde{\phi}_I + \phi_E(2 - 3\tilde{\phi}_I)}{3\phi_E(1 + z)}, \quad (4.11)$$

and

$$p^E_E = p^E_E = p^E_E = \frac{4\phi_E + \tilde{\phi}_I(2 - 3\phi_E)}{3\phi_E(1 + z)}, \quad (4.12)$$

for the incumbent and the entrant respectively. Introducing this price conditions into the first order condition of the entrant’s advertising reach gives

$$\frac{\partial \Pi^D_E(p_I, p_E, \tilde{\phi}_I, \phi_E | z, \alpha)}{\partial \phi_E} = \tilde{\phi}_I [2p_E D_{EE}(p_I, p_E) + p_E (D_{EI}(p_I, p_E) + D_{IE}(p_I, p_E))] + (1 - \tilde{\phi}_I)2p_E - E_{\phi_E}(\phi_E, \alpha) = 0$$

$$\Rightarrow \frac{(\phi_E(4 - 3\tilde{\phi}_I) + 2\tilde{\phi}_I)^2}{9\phi_E^2(1 + z)} = \frac{2\phi_E}{\alpha}.$$  \quad (4.13)

The advertising reach from the incumbent such that entry is deterred makes the profits from the entrant equal to zero

$$\Pi^D_E(p_I, p_E, \tilde{\phi}_I, \phi_E | z, \alpha) = F \iff \frac{(\phi_E(4 - 3\tilde{\phi}_I) + 2\tilde{\phi}_I)^2}{9\phi_I\phi_E(1 + z)} = \frac{\phi_E^2}{\alpha} + F. \quad (4.14)$$
Using expressions (4.13) and (4.14), I explicitly derive the entrant’s advertising reach \( \phi_E = \min \{1, \sqrt{\alpha F}\} \), which increases with the advertising effectiveness and the fixed costs of entry. A more effective advertising technology boosts the advertising intensity. Larger entry costs reduces the amount of advertising that the incumbent needs to deter entry, and this incentivises the use of advertising by the entrant. Because entry is effectively deterred, the entrant never becomes active in the market and the advertising intensity \( \phi_E \) is never set on the equilibrium path. I consequently replace \( \phi_E \) for \( \phi_E^{Out} \) for the rest of the analysis, where the upper-script stands for out-of-equilibrium.

Introducing \( \phi_E^{Out} \), into (4.14), the intensity of advertisement \( \tilde{\phi}_I \) is implicitly characterized by

\[
\frac{\left(\sqrt{\alpha F}(4 - 3\tilde{\phi}_I) + 2\tilde{\phi}_I\right)^2}{18\tilde{\phi}_I\left(\sqrt{\alpha F}(1 + z)\right)} = F. \tag{4.15}
\]

The comparison between the equilibrium advertising intensities under entry deterrence against the advertising reach when entry is accommodated is introduced in the next Lemma.

**Lemma 2.** With entry deterrence, the incumbent announces an advertising reach larger than competition. The entrant’s out-of-equilibrium advertising reach is smaller, i.e., \( \tilde{\phi}_I > \phi_D > \phi_E^{Out} \).

This result differs from Bonnano (1988), in which following an increase of advertising by the incumbent, the entrant responds by matching the amount of advertising to avoid his product to be inferiorly perceived. Contrary to Bonnano, I consider informative instead of persuasive advertisement. In my model, when the incumbent increases his amount of advertising, competition in the market intensifies, and the lower profits per consumer, makes the entrant to reduce its advertising. Divergent levels of advertising will also generate different equilibrium prices, i.e., \( p_I > p^D > p_E^{Out} \).

This is because when the incumbent advertises more than the entrant, the former obtains a larger proportion of captive consumers, the inelastic part of the market, and sets a higher price accordingly. The opposite happens for the entrant.

The advertising intensity that effectively deters entry does not depend on consumers’ valuation, but on the advertising intensity and fixed entry costs. Then, for a sufficiently large \( V \), the mo-
nopolistic intensity of advertisement $\phi^M$ is larger than $\tilde{\phi}_I$. As a result, the incumbent gains from announcing $\phi^M$ instead of $\tilde{\phi}_I$. Therefore, the announcement of advertising that effectively deters entry is $\phi^F = \max\left\{\phi^M, \tilde{\phi}_I\right\}$. The next Proposition states under which conditions the incumbent engages into an entry deterrence strategy.

**Proposition 3.** There exist an entry deterrence equilibrium such that:

i) For $\alpha < 2(1 + z)$, and

$$\theta(F, \alpha, z) := \frac{\left(\sqrt{\alpha F} (4 - 3\phi^F_I) + 2\phi^F_I\right)^2}{18\phi^F_I \left(\sqrt{\alpha F (1 + z)}\right)} - F,$$

the incumbent announces an advertising reach $\phi^F_I = \max\left\{\phi^M, \tilde{\phi}_I\right\}$, and with fixed entry costs $F \in \left(\theta(F, \alpha, z), 4\alpha / (\alpha + \sqrt{2\alpha(1 + z)})\right)^2$ entry is effectively deterred.

ii) There exist a level of advertising efficiency $\tilde{\alpha}(F, z)$ such that, for $\alpha \in [\tilde{\alpha}(F, z), 2(1 + z))$, then $d\theta(F, \alpha, z)/d\alpha > 0$ and $d\theta(F, \alpha, z)/d\alpha < 0$ for $\alpha < \tilde{\alpha}(F, z)$.

The Proposition asserts that an entry deterrence strategy only happens when the advertising technology is sufficiently inefficient. For an advertising technology such that $\alpha \geq 2(1 + z)$, the market is fully-informed, and the incumbent cannot further increase advertising to deter entry. The fixed costs of entry and the advertising technology determine the equilibrium market structure. Figure 3 illustrates under which parameters entry is blocked, deterred or accommodated. When the effectiveness of the advertising technology is below $2(1 + z)$, entry will be blocked for a relatively large costs of entry, deterred for intermediate costs and accommodated otherwise. The function $\theta(F, \alpha, z)$, separating the regions where entry is either deterred or accommodated decreases when advertising technology is very inefficient and increases otherwise. With a very small $\alpha$, advertising is expensive and competition is weak. Then, the incumbent has a hard time to expand its advertising reach to deter entry. As the advertising technology improves, the cheaper it becomes to expand advertising and entry is successfully deterred for a larger range of fixed costs. However, with a sufficiently efficient advertising technology, i.e., $\alpha \in [\tilde{\alpha}(F, z), 2(1 + z))$, the region where entry is
deterred decreases with the efficiency of advertising. This is because the equilibrium advertising reach is closer to generate a fully-informed market, and the incumbent can only announce an advertising reach such that all consumers are aware about its product. For the region where entry deterrence disappears, i.e., $\alpha \geq 2(1 + z)$, a further increase on the advertising efficiency expands the region where entry is accommodated. When all consumers are informed about the existing products in the market, the degree of competition does not depend on the advertising technology, and a further increase in efficiency reduces costs.

My result complements the findings of Groomsman and Shapiro (1984). In their model improvements in advertising technology results in exit, and fewer brands in the market. Here, a more effective advertising technology generates more competition enlarging the region where entry is blocked. However, in my model, a more effective technology also provides more information to consumers in equilibrium which thwarts the ability of the incumbent to use informative advertising as an entry deterrence strategy. Moreover, in situations when the market is fully informed, as a result of an improvement on the advertising technology, the region when entry happens increaser with further improvements of advertising.

The study of the effects of compatibility on the feasibility of an entry deterrence strategy and
the equilibrium market structure is considered in the next section.

### 4.3 Compatibility Choice

This section studies the decision of the potential entrant to make his components compatible with the incumbent. I show that making products compatible difficult the ability of the incumbent to use advertisement as an entry deterrence strategy. Compatible products reduce market competition and generate a larger equilibrium advertising reach. Both effects makes the use of advertising to deter entry less effective.

For simplicity I assume that if the entrant makes its components compatible with the incumbent it must pay a fixed cost $\Gamma$. Then, the entrant makes its product compatible when the benefits from compatibility are larger than the costs, i.e.,

$$\Delta^D(\alpha) = \Pi^D_E(z = 0 \mid \alpha, F) - \Pi^D_E(z = 1 \mid \alpha, F) \geq \Gamma.$$

As the advertising technology determines the equilibrium advertising reach, different values in the advertising efficiency, generate the gain from compatibility

$$\Delta^D(\alpha) = \begin{cases} 
\frac{8\alpha^2(\sqrt{\alpha}(2+\sqrt{2})+1)}{((\alpha+\sqrt{2}\alpha)(\alpha+\sqrt{4}\alpha))}^2 & \text{for } \alpha < 2, \\
\frac{(\alpha-1)(2\sqrt{4}\alpha+\alpha)-4}{\alpha(2\sqrt{4}\alpha+\alpha)} & \text{for } \alpha \in [2, 4), \\
\frac{1}{2} & \text{for } \alpha \geq 4.
\end{cases}$$

The profits from compatibility fail to be monotone with the advertising technology. This comes from the effect that the equilibrium advertising reach has on competition. For a low advertising efficiency, i.e., $\alpha < 2$, the gains from compatibility decrease with the effectiveness of advertising: in a “partially-informed” market, a more efficient advertising technology increases advertising reach reducing the effect that compatibility has on lowering competition. Under compatibility, the market is “fully-informed” for an advertising technology $\alpha \geq 2$. In this case, advertising does not increase with a more efficient technology, and it only reduces advertising costs. While increases in advertising with a more efficient advertising technology are still present under incompatibility. The profits of compatibility are stable for $\alpha \geq 4$, as changes in compatibility do not affect the advertising reach.
Figure 4 illustrates the entrant’s compatibility decisions as a function of the advertising technology.

To study how the choice of compatibility reduces the ability to use advertising as an entry deterrence strategy, I consider the case that the cost of compatibility is low enough such that regardless of the advertising technology, it is beneficial to choose compatibility, i.e., $\Delta^D(\alpha) > \Gamma$. Figure 5 illustrates the changes in the different regions. In what follows, I split the analysis on the changes of the equilibrium regions that compatibility brings about, in which the market is “fully-informed”, i.e., $\alpha \geq 2$, and where it is “partially-informed”, i.e., $\alpha < 2$.

When the entrant makes his components compatible with the incumbent, the former commits to lower competition after entry. With larger profits per consumer, firms have more incentives to expand their advertising reach, and equilibria where the market is “fully-informed” emerge for a larger set parameters. In this case, the incumbent cannot further increase its advertising intensity to deter entry. The region of entry deterrence move to the left generating an increase in accommodation of

$$\Delta (AC - ED)_{FI} = \int_2^4 \left( \frac{4\alpha}{(\alpha + \sqrt{4\alpha})^2} - \theta(F, \alpha, z = 1) \right) d\alpha.$$  

Because competition generates larger profits for the entrant, there is also a substitution in the region

Figure 4: Entrant’s compatibility decisions as a function of the advertisement efficiency and the fixed costs of compatibility.
Figure 5: Equilibrium regions as a function of the advertisement efficiency and the fixed costs of entry when $\Delta^D(\alpha) > \Gamma$.

When entry is blocked in favor of accommodation

$$\Delta (AC - EB)^{FI} = \int_{2}^{\infty} (\Delta(\alpha) - \Gamma) \, d\alpha - \Delta (AC - ED)^{FI}.$$ 

When the market is “partially-informed” regardless of the level of compatibility, i.e., $\alpha < 2$, Figure 5 illustrates a movement of all the regions upwards. A less competitive market as a result of compatibility reduces the region of fixed cost for which entry is blocked. As a result, in order to keep its monopolistic position, the incumbent will have to oversupply advertising for values of fixed costs that generated negative profits for the entrant absent compatibility. The decrease in the region of blocked entry in favor of entry deterrence is

$$\Delta (ED - EB)^{PI} = \int_{0}^{2} \left( \frac{4\alpha}{(\alpha + \sqrt{2}\alpha)^2} - \Gamma - \theta(F, \alpha, z = 0) \right) \, d\alpha.$$ 

However, the decrease in competition renders unprofitable for the incumbent to increase its advertising reach for low enough entry fixed cost. As a result, the region in which entry is accommodated expands by

$$\Delta (AC)^{PI} = \int_{0}^{2} (\theta(F, \alpha, z = 0) - \theta(F, \alpha, z = 1)) \, d\alpha.$$
Ultimately, the changes in the equilibrium regions will crucially depend on the actual compatibility costs. Because the gains from compatibility are not monotone, the entrant may decide to make its product compatible only under some parameters of the advertising technology. As a result, some of the variations previously displayed may disappear.

5 Welfare

In this section, I compare the equilibrium advertising levels with the social optimal. Following Hamilton (2009), I take a partial view of advertising, in which I compare the equilibrium advertising outcome to the social optimal conditional on the equilibrium prices.\(^{21}\) Moreover, I compare social welfare against the equilibrium with respect to the advertising reach only, taking the market structure as given.\(^{22}\)

5.1 One active firm

With the monopolistic price, the conditional probability that a consumer buys the product after receiving an advertisement is \((1 - \epsilon^M(V)^2)/2\). Conditional on a sale, the incumbent obtains a surplus of \(2p^M\) and the consumer surplus is \(V - 2p^M - PC\). The amount \(PC\) stand for the preference costs standing for the difference between consumers’ preferences and the firms’ product specification. Because with a monopolistic incumbent consumers who are furthest away from the product specification do no effectuate a purchase, the average preference cost is

\[
PC = \int_0^{\epsilon^M(V)} \left( \int_0^{\epsilon^M(V)} (x + y) dy \right) dx = \int_0^{\epsilon^M(V)} \left( \int_0^{\epsilon^M(V)} \left[ xy + \frac{y^2}{2} \right] dx \right) \\
= \int_0^{\epsilon^M(V)} \left[ \frac{x^2\epsilon^M(V)}{2} + \frac{x(\epsilon^M(V))^2}{2} \right] = (1 - \epsilon^M(V))^3.
\]

Aggregate welfare is taken to be the sum of consumer and producer surplus, and under a

\(^{21}\)The social optimal advertising intensity depends on the efficient prices, equal to the marginal cost of production which are different from the equilibrium prices.

\(^{22}\)My analysis does not consider the optimal market structure.
monopolistic incumbent this is equal to

\[ SW = \phi \left( 1 - \frac{\epsilon^M(V)^2}{2} \right) \left[ 2p^M + \left( V - 2p^M - (1 - \epsilon^M(V))^3 \right) \right] - E(\phi, \alpha). \]

The first part in the bracket stands for the producer surplus and the second part for consumer surplus. Observe that those are multiplied by the probability of the consumer being informed (\( \phi \)) and served \((1 - \epsilon^M(V)^2)/2\).

The change in social welfare resulting from a change in advertising is

\[
\frac{\partial SW}{\partial \phi} = 2p^M \left( 1 - \frac{\epsilon^M(V)^2}{2} \right) - E_{\phi}(\phi, \alpha) + \left( 1 - \frac{\epsilon^M(V)^2}{2} \right) \left( V - 2p^M - (1 - \epsilon^M(V))^3 \right) \tag{5.1}
\]

An increase in advertising that informs one more consumer induces a purchase of frequency \((1 - \epsilon^M(V)^2)/2\). This generates an increase in profits of \(2p^M\) and an increase in consumer surplus of \((V - 2p^M - (1 - \epsilon^M(V))^3)\). The sum of those surpluses minus the marginal cost of advertising gives the marginal social return to advertising.

To assess the efficiency of the monopolistic advertisement outcome, I evaluate the social welfare change in equation (5.1) at the monopolistic advertisement level

\[
\left. \frac{\partial SW}{\partial \phi} \right|_{\phi = \phi^M} = \left( 1 - \frac{\epsilon^M(V)^2}{2} \right) \left( V - 2p^M - (1 - \epsilon^M(V))^3 \right) > 0.
\]

A monopolist undersupplies advertising because it does not internalize the total gain of consumers from purchasing a product system after they obtain an advertisement, but only the private benefits of a purchase. As in previous studies, a monopolist undersupplies purely informative advertising as only the market size effect is at work in the industry.\(^{23}\)

However, in my the model, an incumbent may announce advertisement as an entry deterrence strategy. This effect may attenuate the undersupply of advertising or it may even generate adver-

\(^{23}\)Models where advertisement is undersupplied with a monopoly are Groomsman and Shapiro (1984) and Hamilton (2009).
tisement to be oversupplied. To see this, Proposition 3 establishes the amount of advertisement needed to effectively deter entry, i.e., $\phi^F_I = \max \left\{ \phi^M, \tilde{\phi}_I \right\}$. Only when the amount of advertising is above the monopolistic level $\phi^F_I = \tilde{\phi}_I$, can advertising be oversupplied. Using expression (5.1) but substituting for $\phi = \tilde{\phi}_I$ gives
\[
\left. \frac{\partial SW}{\partial \phi} \right|_{\phi = \tilde{\phi}_I} = \left[ 2p^M \left( 1 - \frac{\epsilon^M(V)^2}{2} \right) - E_{\phi}(\tilde{\phi}_I, \alpha) \right] \left[ \frac{1}{2} \left( 1 - \frac{\epsilon^M(V)^2}{2} \right) (V - 2p^M - (1 - \epsilon^M(V)^3)) \right].
\]

The second effect is the same as before. Because $\tilde{\phi}_I > \phi^M$, the marginal cost of advertising are larger than the marginal profits and $E_{\phi}(\tilde{\phi}_I, \alpha) > 2p^M (1 - (\epsilon^M(V)^2/2))$. This makes the first part of the expression to be negative. Because $\partial p^M/\partial V > 0$ and $E_{\phi}(\cdot) > 0$, advertising may be oversupplied for a sufficiently small consumption value and a sufficiently convex advertising technology.

5.2 Two active firms

Under complete coverage all consumers who receive at least one advertising purchase the good from which they obtain a gross value of consumption $V - 2p^D$. Under competition total preference costs depends on the extend to which consumers at each location are aware of the firms’ products. Moreover, consumers who are fully informed and purchase products from different firms incur to an additional compatibility cost.

With probability $2\phi^D (1 - \phi^D)$ consumers only receive one advertisement message and incur to an average preference cost of
\[
PC_c = \int_0^1 \left( \int_0^1 (x + y)dy \right) dx = \int_0^1 \left( \int_0^1 \left[ xy + \frac{y^2}{2} \right] \right) dx = \left[ \frac{x^2}{2} + \frac{x}{2} \right] = 1. \tag{5.2}
\]

With probability $(\phi^D)^2$, a consumer receives both advertising messages. Figure 6 represents the
consumption bundles, generating an average preference cost of

$$PC_s(z) = 2(a + b + c + d) = 2 \left[ \int_0^{1+z} \left( \int_0^{1-z} (x+y)dy \right) dx + \int_0^{1-z} \left( \int_0^{1-z} (x+y)dx \right) dy \right]$$

$$+ 2 \left[ \int_0^{1-z} \left( \int_0^{1-z} (x+y)dy + \int_0^{1-z} (x+y)dx \right) dx \right]$$

$$= 2 \left( \frac{(1-z)^3}{8} + \frac{1-z^2}{8} + \frac{z(1-z)(3-z)}{8} + \frac{z^2(3-z)}{6} \right) = \frac{1}{2} + \frac{z^2(3-2z)}{6}. $$

Consumers who are aware of both brands will also incur to compatibility costs if they purchase products produced by different firms. Total incompatibility costs are

$$IC = (D_{IE} + D_{EI}) z = \frac{z(1-z^2)}{2}, \quad \text{(5.3)}$$

where $D_{IE}$ and $D_{EI}$ stand for the mass of consumers who purchase brands produced by different firms.
Combining these terms with industry profits, aggregate welfare is given by

\[ SW = \phi^2 \left( V - 2pD - \left( \frac{1}{2} + \frac{z^2(3 - 2z)}{6} \right) - \frac{z(1 - z^2)}{2} \right) + 2\phi(1 - \phi) \left( V - 2pD - 1 \right) \]

\[ + \phi(2 - \phi)2pD - 2E(\phi, \alpha). \]

This welfare measure relates to that employed in Hamilton (2009), with the exception of the compatibility costs. Consumers who are aware of both products reduce their preference cost by purchasing products from different firms, even if this entails some extra costs associated to compatibility.

The change in social welfare from a change in advertising frequency is

\[ \frac{\partial SW}{\partial (2\phi)} = (1 - \phi) \left( V - 2pD - 1 \right) + \phi \left( \frac{1}{2} - \frac{z^2(3 - 2z)}{6} - \frac{z(1 - z^2)}{2} \right) + (1 - \phi)2pD - E(\phi, \alpha), \]

and evaluating the market change at the market advertising level gives

\[ \frac{\partial SW}{\partial (2\phi)} \bigg|_{\phi = \phi^D} = (1 - \phi^D) \left( V - 2pD - 1 \right) + \phi^D \left( \frac{1}{2} - \frac{z^2(3 - 2z)}{6} - \frac{z(1 - z^2)}{2} \right) - \phi^DpD. \] (5.4)

As in Hamilton (2009), the efficiency of the advertising allocation depends on the sum of three terms. The first term is the market-size effect. A small increase in the advertising reach, informs a consumer, who was previously uninformed with probability \((1 - \phi^D)\). New consumption happens in the economy generating an increase in social welfare of \(V - 2pD - 1\). The second term stands for the matching effect. As consumers have heterogeneous preferences, there is a social benefit deriving from the improved matching of consumers and brands that occurs when consumer information is more complete. Then, with probability \(\phi^D\) a consumer who was partially informed is now aware of both products and can mix and match components produced by different firms. While not creating new consumption value, preference cost decrease as consumers can purchase components closer to their preferences. Finally, the last term stands for the stealing effect generated by consumers, who are now aware of both brands, decide to buy from the rival.
The market size and the matching effect causes informative advertising to be undersupplied. Firms do not incorporate the full gains from trade in their advertising decisions. As a result, whether advertising is over or under supplied in equilibrium depends on the business stealing-effect which shapes firms’ incentive to provide (costly) advertising. An individual firm does not account for the profit reduction at other firms as it increases its advertising intensity and captures customers from its rivals. In my model, the business stealing effect depends on the level of product compatibility.

A small levels of compatibility generates market competition translating into lower equilibrium prices, giving a small profit per purchase. Because consumers trend to buy the whole system from a firm rather than separate components, consumers’ tastes for both components needs to be aligned to the companies’ system specification to attract them. These results in the business stealing effect being small, and adverting is undersupplied. As products become more compatible, prices increase as a result of a less market competition. Firms compete over individual components rather than the whole system. The resulting business stealing effect increases and eventually dominates the market and matching effect. This results in advertising to be oversupplied in equilibrium. The next Proposition introduces the result.

**Proposition 4.** *Informative advertising is oversupplied with compatible products and undersupplied when products are made compatible.*

This result complements the findings of Hamilton (2009) who shows that advertisement is undersupplied when brands are sufficiently substitutable, and oversupplied otherwise. In my model, brands are relatively substitutable which will generate advertising to be undersupplied in equilibrium. However, in a market where firms produce different components that form a product system, compatibility decisions need to be taken into account to establish whether advertising is either oversupplied or undersupplied.
6 Conclusion

This paper has studied the strategic interaction of an incumbent and a potential entrant who produce two heterogeneous components that form a product system. I identify the use of informative advertisement as an effective tool to deter entry. As a response, a potential entrant may decide to make its products compatible with the incumbent to signal lower competition. Compatibility makes the entry deterrence strategy through advertising more complicated. Because compatibility expands the intensity of advertising, equilibria with a “fully-informed” market emerges for a larger set of parameters. As a result, the incumbent cannot increase its advertising reach to deter entry. Compatibility also reduces competition, and the incumbent finds it profitable to accommodate entry for a larger set of entry fixed costs.

My model then gives a rationale for current managerial strategies in which for markets where advertising is key, firms design products compatible with the leading brands. After Nespresso’s patents were revoked, many firms in different countries began to produce and sell machines and coffee pods fully compatible with Nespresso. Podista is the leading brand in Australia and The Capsoul in Spain. The model also shows that an increases in the advertising technology such that reaching potential consumers in the market is more effective, is similar to the effect of product compatibility. This is because an equilibrium where all consumers are aware of both brands emerge, and the incumbent is not able to use advertising as an entry deterrence device.

Relative to the analysis of the efficient advertising provision, my results depend on the emerging market structure. With a single firm in the market, advertising is generally undersupplied and oversupplied only when used as an entry deterrence strategy. With two firms, the equilibrium level of advertising crucially depends on the level of product compatibility. I have shown that even with relatively enough substitutability between brands, advertisement may be oversupplied when products are made compatible. An individual firm does not account for the profit reduction at other firms as it increases its advertising intensity and captures customers from its rivals.

Some of the results in my model depend on the assumption that consumers are not able to
undertake an active search behavior and that firms restrict by offering uniform prices to all consumers. First, with active consumers a “loss leader” advertising strategy, where firms advertise a single component at a loss, may emerge (Chen and Rey, 2012; Ellison, 2005). If with a “loss leader” strategy competition reduces, firms may have higher incentives to increase advertising. Then, product design through compatibility choices may loses its capacity to adjust the level of competition in the market and its power to difficult the entry deterrence strategy. Relaxing the restriction on a uniform price, firms may price discriminate by introducing a discount to consumers who buy the whole system from the same firm. As a result, competition will be enhanced. Then, making products compatible will no longer be credible as a device to soften competition. As a result, the market will be characterized by lower levels of compatibility and advertising intensity. This analysis will be addressed in future research.

References


Proof of Proposition 1.

The monopoly price is obtained by introducing $\epsilon^M(V) = (1/6) \left[ 2 - V + \sqrt{28 + (V - 4)V} \right]$ into $p^M = (V - 2)/2 + \epsilon^M(V)$. To obtain the intensity of advertisement, the first order condition of the
profit function (3.2) with respect to $\phi$ gives

$$\frac{\partial \Pi_I(V, \epsilon)}{\partial \phi} = 2p^M D_M^M - 2\phi^M / \alpha = 0 \Rightarrow \phi^M = \alpha p^M \left(1 - \frac{(\epsilon^M(V))^2}{2}\right).$$

Because the intensity of advertisement cannot be larger than one, we obtain the result expressed in the Proposition.

**Proof of Lemma 1.**

To study the effect of compatibility on equilibrium prices, I first characterize the change of advertising reach with compatibility. Then, by differentiating the equilibrium condition (4.7) with respect to compatibility gives expression

$$E_{\phi\phi}(\cdot) \times \frac{d\phi_D}{dz} = \frac{-2 \left(2 - \phi_D\right) \left(\phi_D(1 + z)\right) \times \frac{d\phi_D}{dz} - \left[(2 - \phi_D)^2 \left(1 + z\right) \times \frac{d\phi_D}{dz} + \phi_D\right]}{(\phi_D(1 + z))^2}.$$

Arranging terms I obtain

$$- \frac{d\phi_D}{dz} = \frac{\phi_D \left(2 - \phi_D\right)^2}{(1 + z) \left(4 + (\phi_D)^2 \left(E_{\phi\phi}(\cdot)(1 + z) - 1\right)\right)}.$$

Now, by differentiating the equilibrium price condition (4.5) with respect to compatibility gives

$$\frac{dp_D}{dz} = \frac{- \left(\phi_D(1 + z)\right) \times \frac{d\phi_D}{dz} - \left[(1 + z) \times \frac{d\phi_D}{dz} + \phi_D\right] \left(2 - \phi_D\right)}{(\phi_D(1 + z))^2},$$

\[\text{direct effect}\]

$$= \frac{-\phi_D \left(2 - \phi_D\right) + \left[\frac{d\phi_D}{dz} \times 2(1 + z)\right]}{(\phi_D(1 + z))^2}.$$

\[\text{indirect effect}\]

While the direct effect is always negative, the indirect effect is positive due to (A.1). Therefore, a necessary and sufficient condition for the equilibrium price increases with compatibility is that

$$\frac{d\phi_D}{dz} < \frac{\phi_D \left(2 - \phi_D\right)}{2(1 + z)},$$

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which can be shown to be always satisfied. Substituting the left hand side by (A.1) gives

\[
\frac{\phi^D(2 - \phi^D)^2}{(1 + z)(4 + (\phi^D)^2)(E_{\phi^D}(\cdot)(1 + z) - 1)} < \frac{\phi^D(2 - \phi^D)}{2(1 + z)} \iff E_{\phi^D}(\cdot) > -\frac{(2 - \phi^D)}{1 + z},
\]

and due to the convexity of the advertising costs, the previous expression is always true.

Differentiating the equilibrium price condition in (4.5) with respect to the effectiveness parameter \( \alpha \) gives

\[
\frac{dp^D}{d\alpha} = -\left(\phi^D(1 + z)\right) \times \frac{d\phi^D}{d\alpha} - \left((1 + z)(2 - \phi^D)\right) \times \frac{d\phi^D}{d\alpha} = \frac{2 \times d\phi^D}{(\phi^D)^2(1 + z)},
\]

and the result stated in the Lemma comes directly from the observation that the equilibrium advertising intensity increases with \( \alpha \).

**Proof of Proposition 2.**

The symmetric equilibrium presented in the Proposition requires the market to be covered. To obtain the parameters for consumers’ valuation to ensure that this is the case, I work with the equilibrium condition in (4.7). Under competition, a necessary condition for full covered market is that the captive consumer who is furthest away makes a purchase, i.e., \( V \geq 2 + 2 \times p^D \). A lower bound for consumers’ valuation satisfying this condition is

\[
V \geq 2 + \frac{2(2 - \phi^D)}{\phi^D(1 + z)} = \frac{2 \times (2 + \phi^D z)}{\phi^D \times (1 + z)} = V(\phi^D, z).
\]

Because a selective consumer always have lower preference costs, this necessary condition is also sufficient. In addition, under complete coverage, firms may have an incentive to deviate from the pure strategy equilibrium by charging the full reservation price \( p = (V - 2)/2 \). This strategy renders
unprofitable when consumers’ valuation stays below an upper bound

\[
\frac{1}{2}(\phi^D)^22p^D + \phi^D(1 - \phi^D)2p^D - E(\phi^D, \alpha) \geq 2\phi^D(1 - \phi^D) \left(\frac{V - 2}{2}\right) - E(\phi^D, \alpha)
\]

\[\iff V \leq \frac{4 - \phi^D \times (\phi^D - 2z(1 - \phi^D))}{(1 - \phi^D) \times \phi^D \times (1 + z)} = \tilde{V}(\phi^D, z).\]

It is easy to verify that for all values of \(z\), it is the case that \(\tilde{V}(\phi^D, z) > \hat{V}(\phi^D, z)\). For values outside this range, the equilibrium may be in mixed strategies in prices and advertising reach. For an exhaustive analysis of an incomplete covered market with differentiated products and informative advertising, I refer to the work of Hamilton (2009).

**Proof of Lemma 2.**

I will proceed by construction. I first show that for all of the parameters \(\alpha\) and \(F\), such that there is an deterrence strategy, I find that \(\phi^D > \phi^D_{Out}\). With this result, and using the left-hand-side of the equilibrium advertising reach in expressions 4.13 and 4.7, I demonstrate that \(\tilde{\phi} > \phi^D\).

The entrant’s out-of-equilibrium advertising reach is \(\phi^D_{Out} = \min\left\{1, \sqrt{\alpha F}\right\}\), and from Proposition 2 the advertising reach in equilibrium is \(\phi^D = \min\left\{1, 2\alpha / \left(\alpha + \sqrt{2\alpha(1 + z)}\right)\right\}\). For the values of \(\alpha\) in which both variables are below unity,

\[
\phi_E \leq \phi^D \iff \sqrt{\alpha F} \leq \frac{2\alpha}{\alpha + \sqrt{2\alpha(1 + z)}} \iff \alpha F \leq \frac{4\alpha^2}{\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2} \iff F \leq \frac{4\alpha}{\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2},
\]

where the last condition state the range of fixed cost that are necessary for entry to be deterred. With this value of fixed costs, it is easy to verify that \(\partial\phi^D / \partial \alpha > \partial\phi^E / \partial \alpha\) and \(\phi^D\) reaches the upper-bound for a larger set of parameters \(\alpha\).

I proceed to show that \(\tilde{\phi} > \phi^D\). I prove this result by contradiction. With the use of the left
hand sides from expressions 4.13 and 4.7, and setting \( \tilde{\phi}_I = \phi^D \), I will reach a contradiction if

\[
\frac{\left( \sqrt{\alpha F} (4 - 3\phi^D) + 2\phi^D \right)^2}{9\alpha F \phi^D (1 + z)} \geq \left( \frac{2 - \phi^D}{\phi^D (1 + z)} \right)^2. \tag{A.2}
\]

Operating, I obtain

\[
\frac{\left( \sqrt{\alpha F} (4 - 3\phi^D) + 2\phi^D \right)^2}{9\alpha F (1 + z)} \geq \left( \frac{2 - \phi^D}{(1 + z)} \right)^2 \iff \frac{\sqrt{\alpha F} (4 - 3\phi^D) + 2\phi^D}{3\sqrt{\alpha F}} > 2 - \phi^D \iff 4\sqrt{\alpha F} + 2\phi^D \geq 6\sqrt{\alpha F} \iff \phi^D \geq \sqrt{\alpha F}, \tag{A.3}
\]

and this is true from the previous analysis. Therefore, when \( \phi^D > \phi^D_E \), the incumbent sets a larger advertising reach, i.e., \( \tilde{\phi}_I > \phi^D \).

**Proof of Proposition 3.**

To show that for a sufficient advertising efficiency and gross consumption value, the monopolistic advertising reach is above \( \tilde{\phi}_I \), I make use of the first order conditions rather than the expressions explicitly defining the advertising intensity. Then, under a single incumbent the first order condition with respect to advertising is

\[
2p^M D^M_{II} = E_{\phi}(\cdot),
\]

and

\[
\frac{\left( \sqrt{\alpha F} (4 - 3\tilde{\phi}_I) + 2\tilde{\phi}_I \right)^2}{9\alpha F \tilde{\phi}_I (1 + z)} = E_{\phi}(\cdot),
\]

for the entry deterrence. Because the right hand side of both expressions coincide and increases with advertising reach, the monopolistic amount of advertisement is larger when

\[
2p^M D^M_{II} \geq \frac{\left( \sqrt{\alpha F} (4 - 3\tilde{\phi}_I) + 2\tilde{\phi}_I \right)^2}{9\alpha F \tilde{\phi}_I (1 + z)}.
\]
From Proposition 1, and because $\epsilon > 0$, a sufficient condition is $\phi^M > \tilde{\phi}_I$ is

$$V - 2 \geq \frac{\left(\sqrt{\alpha F(1 - 3\tilde{\phi}_I)} + 2\tilde{\phi}_I\right)^2}{9\alpha F\tilde{\phi}_I(1 + z)},$$

where the left hand side increases with $V$ and the right hand side decreases with $\alpha$.

I now proceed to show that it is always profitable for the incumbent to engage into an entry deterrence strategy. Because $\phi^F = \max\left\{\phi^M, \tilde{\phi}_I\right\}$, I consider each one of the cases separately.

i) $\phi^F = \phi^M$. The incumbent sets the monopoly advertising reach to deter entry. As shown in section 3, the monopoly profits is $\Pi^M > \alpha(V - 2)^2/4$ then a sufficient and necessary condition for entry deterrence to be profitable is

$$\Pi^M > \frac{\alpha(V - 2)^2}{4} \geq \frac{4\alpha}{\left(\alpha\sqrt{2\alpha(1 + z)}\right)^2} = \Pi^D \iff V \geq \frac{4 + 2\left(\alpha + \sqrt{2\alpha(1 + z)}\right)}{\alpha + \sqrt{2\alpha(1 + z)}}.$$

Because the left hand side is increasing with consumers valuation $V$, a sufficient condition is obtained by using the lowest valuation $V = 2\left(2 + \phi^D z\right)/\left(\phi^D(1 + z)\right)$ obtained in the proof of Proposition 2. Then, the previous condition is

$$\frac{2\left(2 + \phi^D z\right)}{\left(\phi^D(1 + z)\right)} \geq \frac{4 + 2\left(\alpha + \sqrt{2\alpha(1 + z)}\right)}{\alpha + \sqrt{2\alpha(1 + z)}} \iff 2\left(2 + \frac{2\alpha z}{\alpha + \sqrt{2\alpha(1 + z)}}\right) \geq \frac{\alpha + \sqrt{2\alpha(1 + z)}}{\alpha + \sqrt{2\alpha(1 + z)}} \iff \frac{\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}{\alpha(1 + z)} + \alpha z \geq \frac{4 + 2\left(\alpha + \sqrt{2\alpha(1 + z)}\right)}{\alpha + \sqrt{2\alpha(1 + z)}} \iff 2\alpha\sqrt{2\alpha(1 + z)} \geq \alpha\sqrt{2\alpha(1 + z)} \iff 2 \geq 1,$$

which is always fulfilled.
ii) $\phi_I^F = \tilde{\phi}_I$. In this case, the incumbent needs to set an advertising reach larger than $\phi^M$ to deter entry. Because the amount $\tilde{\phi}_I$ cannot be larger than one, a sufficient condition for the incumbent to engage into an entry deterrence strategy is when $\tilde{\phi}_I = 1$, and the incumbent gets $\Pi^M(\tilde{\phi}_I) > \Pi^M(\tilde{\phi}_I = 1) = \langle \alpha(V - 2) - 1 \rangle / \alpha$. Then, a sufficient and necessary condition for the entry deterrence strategy to be profitable for $\alpha < 2(1 + z)$ is

$$\Pi^M(\tilde{\phi}_I) > \frac{\alpha(V - 2) - 1}{\alpha} \geq \frac{4\alpha}{\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2} = \Pi^D$$

$$\iff V \geq \frac{4\alpha^2 + (2\alpha + 1)\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}{\alpha \left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}.$$ 

As before, substituting $V$ for $V = 2\left(2 + \phi^D z\right) / \left(\phi^D(1 + z)\right)$, the previous sufficient condition becomes

$$\frac{2 \left(2 + \phi^D z\right)}{(\phi^D(1 + z))} \geq \frac{4\alpha^2 + (2\alpha + 1)\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}{\alpha \left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}$$

$$\iff \frac{2 \left(2 + \frac{2\alpha z}{\alpha + \sqrt{2\alpha(1 + z)}}\right)}{\left(\frac{2\alpha(1 + z)}{\alpha + \sqrt{2\alpha(1 + z)}}\right)} \geq \frac{4\alpha^2 + (2\alpha + 1)\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}{\alpha \left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}$$

$$\iff \frac{2\alpha(1 + z) + 2\sqrt{2\alpha(1 + z)}}{\alpha(1 + z)} \geq \frac{4\alpha^2 + (2\alpha + 1)\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}{\alpha \left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2}$$

$$\iff \left(2\sqrt{2\alpha(1 + z)} - (1 + z)\right)\left(\alpha + \sqrt{2\alpha(1 + z)}\right)^2 \geq 4\alpha^2(1 + z)$$

$$\iff \left(\frac{\sqrt{2\alpha}}{2\sqrt{1 + z}} - \frac{1}{4}\right)\left(1 + \sqrt{\frac{2(1 + z)}{\alpha}}\right)^2 \geq \alpha.$$ 

This condition is fulfilled for a sufficiently large $\alpha$. In claim that when the advertising technology is sufficiently inefficient, i.e., a small $\alpha$, then $\tilde{\phi}_I < 1$ and the sufficient condition is relaxed.
For $\alpha \geq 2(1 + z)$, then $\phi^D = 1$ and the condition becomes

$$
\Pi^M(\hat{\phi}_I) > \frac{\alpha(V - 2) - 1}{\alpha} \geq \frac{\alpha - (1 + z)}{\alpha(1 + z)} = \Pi^D \iff V \geq \frac{1}{1 + z} + 2.
$$

Substituting $V$ for $V = 2(2 + z)/(1 + z)$, the previous sufficient condition becomes

$$
2(2 + z)/(1 + z) \geq \frac{1}{1 + z} + 2 \iff 2(2 + z) \geq 2(1 + z) + 1 \iff 1 \geq 0.
$$

Finally, to see the form of $\theta(F, \alpha, z)$, I differentiate this expression with respect of $\alpha$.

$$
\frac{d\theta(F, \alpha, z)}{d\alpha} = \left[\left(\frac{\sqrt{\alpha F}(4 - 3\phi^F_I) + 2\phi^F_I}{18F\phi^F_I(1 + z)} + 2\sqrt{\alpha F}(4 - 3\phi^F_I) - 2\phi^F_I\right)\right] \frac{18F\phi^F_I(1 + z)}{2\sqrt{\alpha F}(4 - 3\phi^F_I - 2\phi^F_I)^2}.
$$

The first part of the numerator in brackets and the denominator are always positive. Then, the function $\theta(F, \alpha, z)$ is either increasing or decreasing in $\alpha$ depending on the magnitude $(\sqrt{\alpha F}(4 - 3\phi^F_I) - 2\phi^F_I)$.

For $\alpha \in [\hat{\alpha}(F, z), 2(1 + z))$ where

$$
\hat{\alpha}(F, z) := \frac{(2\phi^F_I/(4 - 3\phi^F_I))^2}{F},
$$

the function $\theta(F, \alpha, z)$ increases with $\alpha$, and decreases otherwise.

**Proof of Proposition 4.** Making use of equation (4.5) and setting $z = 0$, expression (5.4) can be written as

$$
\frac{\partial SW}{\partial (2\phi)} \bigg|_{\phi = \phi^D} = (1 - \phi^D) \left(V - 2\frac{(2 - \phi^D)}{\phi^D} - 1\right) + \phi^D \frac{2 - \phi^D}{\phi^D} - \phi^D \frac{2 - \phi^D}{\phi^D}. \quad (A.4)
$$

Substituting $V$ at the boundary where the market is fully covered $V = 2(2 + \phi^D z)/(\phi^D(1 + z)) =$
4/\phi^D, obtained in the proof of Proposition 2, expression (A.4) becomes
\[
\frac{\partial SW}{\partial (2\phi)} \bigg|_{V=V} = (1 - \phi^D) \left( \frac{4}{\phi^D} - 2 \frac{(2 - \phi^D)}{\phi^D} - 1 \right) + \phi^D \frac{2 - \phi^D}{2\phi^D} = \frac{\phi^D - 2}{2} < 0.
\]

The market equilibrium involves an inefficient high level of advertising in the neighborhood of $V$. Because $\partial p^D/\partial z < 0$ the business stealing effect decreases the less compatible products are. When products are made incompatible, $z = 1$, advertising is always undersupplied. This is because
\[
\frac{\partial SW}{\partial (2\phi)} \bigg|_{V=V} = (1 - \phi^D) \left( \frac{2(2 + \phi^D)}{2\phi^D} - 2 \frac{(2 - \phi^D)}{2\phi^D} - 1 \right) + \phi^D \frac{2 - \phi^D}{2\phi^D} = \frac{\phi^D}{3} > 0.
\]