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**Intention-Based Economic Theories
of Reciprocity**

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INTENTION-BASED ECONOMIC THEORIES OF RECIPROCITY

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ABSTRACT

In recent years, several experiments have shown individuals exhibit authentic reciprocal behaviour in anonymous one-shot interactions. As reciprocity has been shown to be relevant in several economic fields, there have also been several attempts to model reciprocal behaviour. I review the intention-based models of reciprocity and present an example in teachers management in the public sector in which government offers an incentive scheme to implement a program. The incentive scheme has a prisoner's dilemma structure. In both simultaneous and sequential games, in equilibrium reciprocal teachers may reach other equilibria different from those predicted by the standard theory.

Key words: Game theory, psychological games, Intention-based models, reciprocal behaviour.

JEL classification: C700.

RESUMEN

Recientemente, varios experimentos han mostrado que los individuos exhiben un comportamiento auténticamente recíproco en interacciones anónimas que se dan una sola vez ('one-shot'). En tanto que se ha mostrado que la reciprocidad es relevante en múltiples campos de la economía, han existido varios intentos por modelar el comportamiento recíproco. Este documento revisa los modelos de reciprocidad que se fundamentan en las intenciones y presenta un ejemplo para el caso del manejo de los profesores en el sector público, en el que el gobierno ofrece un esquema de incentivos para la implementación de un programa. Este esquema tiene la estructura del dilema del prisionero. Tanto en los juegos simultáneos como secuenciales, los resultados de equilibrio pueden ser distintos a los que predice la teoría convencional.

Palabras clave: teoría de juegos, juegos psicológicos, modelos basados en intenciones, reciprocidad.

Clasificación JEL: C700.

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1. INTRODUCTION

From several years ago, other social sciences different from economics like psychology, sociology and anthropology have pointed out human beings tend to reciprocate each other. Until recent years it had not been clear whether this behaviour was only caused by some expectations of future rewards or, at least in some cases, it was genuine reciprocal behaviour.

If the first explanation was true, the usual economic hypothesis that individuals behave in a self-interested manner could explain those behaviours. Nevertheless, from two decades ago, several experiments have shown individuals exhibit authentic reciprocal behaviour in anonymous one-shot interactions. For example, in the ultimatum game a pair of individuals has to distribute a fixed sum of money in a sequential move game. The “proposer” has to divide the amount between himself and the second subject. The “responder” can accept or reject the proposed division. If individuals were rational and self-interested, the responder would accept any quantity of money and the proposer would give the smallest possible quantity. However, evidence shows offers lower than 20% are atypical and rejected with a high probability, while offers close to 50% are very common and rarely rejected (Fehr and Fischbacher, 2001).

On the other hand, in the gift-exchange game the proposer (employer) offers a wage to the responder (worker). The worker can either reject or accept it. If the worker rejects both players gain nothing. If the worker accepts she has to exert a costly effort. The higher the effort, both the lower the payoff she gets and the higher the income the employer receives. Under the standard assumptions, the worker will always choose the lowest effort and the employer will only offer the lowest possible wage. Evidence suggests wages are clearly higher than minimum levels and wages and effort have a positive relation (Fehr and Fischbacher, 2001).

Those and other experiments have shown individuals actually reciprocate each other. A reciprocal individual rewards kind behaviour and punishes unkind behaviour. The gift-giving game illustrates the former, sometimes called positive reciprocity, and the ultimatum game the latter (negative reciprocity). Additionally, it has been shown reciprocity can have an important role in some economic fields. In labour economics, questionnaire studies have shown managers are unwilling to cut wages because it can adversely affect work morale. Effectively, wages cuts are considered as an insult by the workers (Bewley, 1995). Besides, Akerlof (1982) suggests reciprocal behaviour can explain why wages remain above the market clearing level. In fact, this is supported by some experiments that have shown reciprocity contributes to the enforcement of contracts, as loyalty and trust are relevant in labour relationships. Further experiments show individuals punish free-riders in public good provision games even if it reduces their own payoffs; material incentives may crowd-out implicit incentives that rely on reciprocal behaviour and reciprocity can explain why in reality contracts are incomplete, among other facts.¹ All these phenomena cannot be explained assuming the self-interest hypothesis.

There have been several attempts to model reciprocal behaviour. In this document I review the so-called intention-based models of reciprocity, particularly the models proposed by Rabin

¹ Fehr and Gächter (2000) survey experimental evidence. Frey (2001) also surveys circumstantial and econometric evidence.

(1993) and Dufwenberg and Kirchsteiger (2001). This approach emphasizes in the fact that reciprocal individuals want to reward kind intentions and to punish unkind intentions. To illustrate these theories I propose two examples in teachers' management. The first one consists in a game that models teachers' strategic behaviour in the following situation: government wants to improve quality of public education for which it intends to implement a program to make better teachers' abilities. Government offers an incentive scheme that has a prisoner's dilemma structure to enforce the program; in such a way that standard game theory will predict both teachers are going to participate. The second game slightly modifies the material payoffs of the first one. I obtain that, in both simultaneous and sequential games, reciprocal teachers may deviate from participation in equilibrium, as they consider participation as an unkind behaviour. Instead no participation is regarded as a kind behaviour. Of course, participation of both teachers may also be an equilibrium when each teacher believes the other is going to participate. In that case, both teachers punish the other's unkind intention.

The text is organized in three sections. In the first one, I provide an overview of the economic theories about reciprocity in order to give a context to the intention-based theories. The second one is divided in several subsections in which I present the examples and the theories mentioned. With expositive purposes I first introduce the example and show the results obtained using the standard theory, and then I provide the model of reciprocity and the new results. Last section offers conclusions.

2. MODELLING RECIPROCITY

In the standard theory self-interest hypothesis is formalized by defining individual preferences solely on the material resources the individual has. One way to model reciprocal behaviour is enlarging the space in which individual preferences are defined to include others' material payoffs or welfare. "When an individual does not only care about the material resources allocated to her but also cares about the relevant reference agents", we will say she has *social preferences* (Fehr and Fischbacher, 2001 p. 2).

In fact, most of the theories that try to model reciprocity introduce it as a social preference. These theories have had into account reciprocity has two elements in nature: it is not only related to the consequences of others' actions but also to the others' intentions. They have focused on one of those elements of reciprocal behaviour. Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) stress the fact that people desire to maintain equity and provide models of inequity aversion. On the other hand, Rabin (1993) and Dufwenberg and Kirchsteiger (2001) emphasize persons want to punish nasty intentions and to reward friendly intentions. Levine (1998) builds a model in which individuals do not respond to intentions but to the type of person they face. The type is determined by the degree of altruism the individual has. Charness and Rabin (2000) and Falk and Fischbacher (2000) develop theories that have elements from both intention-based reciprocity and inequity aversion models. Finally, Segal and Sobel (1999) present an axiomatic treatment of reciprocity and altruism which is compatible with some of the social preferences models of reciprocal behaviour.

It is worthy to point out that some inequity aversion models, which only concern about payoffs distribution, can mimic some predictions of intention-based reciprocity models. However,

though intention-regarding models can be much more difficult to handle than inequity aversion models, experimental evidence suggests people punish others even if punishment does not reduce inequity.³ In the following section I present the pure intention-based economic models of reciprocity.

3. MODELS OF INTENTION-BASED RECIPROCITY

In these models in deciding what action individuals are going to follow they have into account not only their material payoff but also their beliefs about others' kindness. Specifically, individual utility is composed by two parts: a *material payoff*, which is given in terms of some measurable quantity, e.g. money; and a *reciprocity payoff* that she obtains from assessing the others' kindness. So, individuals will do the action that gives them the highest utility regarding both payoffs.

For example, consider the game in Figure 1. It presents a prisoner's dilemma. As usual when individuals only care about their own material payoff the Nash equilibrium is no cooperation for both persons. However, notice that when an individual chooses no cooperation instead of cooperation she is reducing the other's material payoff. So, when one of the agents decides to cooperate, it can be interpreted by the other as a kind action, since the former reduces his payoff and increases the latter's at the same time. If both players have high enough sensitivity to reciprocity concerns, cooperation can be the best option for them.

FIGURE 1

| | | | |
|-----|----|-----------|-----------|
| | | p 2 | |
| | | C | NC |
| p 1 | C | 0.1, 0.1 | -0.1, 0.2 |
| | NC | 0.2, -0.1 | 0, 0 |

It is worthy to point out that beliefs on kindness are formed assessing the other's intentions. If player 1's action increases her payoff and the player 2's payoff simultaneously, player 2 will probably not consider that action as kind. Further, it can happen that even if one player "sacrifices" his material payoff she is to be considered as no kind. For instance, in the game depicted in Figure 1, assume player 2 have no option different from cooperation. Somehow this player is forced to cooperate. So, we have a degenerate game composed by the left column of the game. In this case, player 1 will not believe player 2 is being kind by cooperating, as the latter has no choice.

To illustrate the theories considered in this document, we are going to analyze a qualitative example from the teachers' management. In the next section is posed the basic problem.

² For a complete discussion in this regard, look at Falk and Fischbacher (2000).

3.1. A QUALITATIVE EXAMPLE OF TEACHERS' MANAGEMENT

Assume a government utilizes two teachers in offering public education. There is a teachers' trade union so that if both take the same decision with respect to government policies, government cannot punish them. Assume as well government wants to improve the quality of education offered and hence decide to implement a program that rise teachers abilities.

In order to exert that policy, government brings out an incentive scheme as follows: If both teachers do not participate in the program, government cannot fire them and they continue gaining the same payoff as before, say X . If both teachers enter in the program, they do a higher effort and obtain the same payment X .³ Payoffs cannot be lower than X because otherwise trade union would impede implementation of the governmental program.⁴ Finally, if teachers take different decisions, trade union is not working anymore, so the teacher who does not participate is fired and obtains his reservation utility and the teacher who participates receives a payoff $X + d$ higher than X .

It is also assumed that teachers take their decision simultaneously. The game is depicted in Figure 2. It is easy to see that the incentive scheme has a prisoner's dilemma structure. Government persuades teachers to participate offering a contingent reward d to deter trade union obstructions. Thus, players have an incentive to participate in the program independently from the other's choice. In such a model, if teachers only care about their material payoff, the unique Nash Equilibrium in pure strategies is (participate, participate).

FIGURE 2

| | | | |
|----|----|------------------|-----------------|
| | | T2 | |
| | | np | p |
| T1 | C | x, x | 0, x + δ |
| | NC | x + δ , 0 | x, x |

3.2 INTRODUCING RECIPROCITY

Suppose both teachers regard niceness, so they draw utility from reciprocity concerns. Notice that in this example, as in the first one, when a player attempts to maximize her material payoff reduces the other's payoff. As teachers are reciprocal, they will reward friendly actions and will punish hostile actions. Assume teacher 2 has chosen to participate, so she can obtain either $X + d$ or X . If teacher 1 chooses to participate as well, he not only minimizes teacher 2's payoff (she would obtain X instead of $X + d$) but also maximizes his (he would get X instead of 0). Thus, this action could be considered unkind by teacher 2 and hence she would not be willing to deviate from participation because otherwise she would reward teacher 1.

³ Moral hazard is not an issue here but it should be in a more realistic model.

⁴ In fact, the game structure is preserved even if participation payoffs are higher than X . It would be enough to assume the participation premium to be lesser than d .

Now suppose teacher 1 chooses not to participate, so teacher 2 gets $X + d$ instead of X . In this case, teacher 2 perceives teacher 1 is giving up X for giving her d and hence she could believe teacher 1' action to be kind. In this situation, teacher 2 would be unkind to player 1 if she remains participating. So as teacher 2 is reciprocal she could change her decision (from participation to no participation) if she is better off doing so.

Notice that one player's assessment of the other' kindness depends not only on what the former believes the latter is going to do but also what the former believes the latter believes the former is going to do. To form both beliefs, fairness of intentions is determined assessing the equitability of the final payoffs' distribution with regard to the feasible set of payoffs. Doing so each player will compare utility she gets in both situations: participation brings her a higher material payoff than no participation. Instead, no participation brings her a higher reciprocity payoff than participation. So, if her reciprocity sensitivity is high enough, teacher 2 will decide to give up d of her own payoff for giving teacher 1 X . Doing the same analysis for the other teacher, we obtain that with reciprocal teachers we have two possible equilibria:⁵ (not participate, not participate) and (participate, participate).

But, when (not participate, not participate) will be chosen? It depends on both the notion of fairness and reciprocity sensitivity players have, and the amount of the material payoffs. To see this it is needed to introduce a formal model of reciprocity.

3.3. RABIN (1993)'S MODEL

Rabin (1993) models reciprocity based on psychological games proposed by Geanakoplos, Pearce and Stacchetti (1989) (hereafter GPS). In such games, players' payoffs depend not only on players' actions but also on their beliefs. GPS show that many standard concepts have useful analogues in the framework they develop.

Rabin's goal is to derive psychological games from "material" games. Let us consider a normal form game with two players, player 1 and player 2, who have mixed strategy sets and A_1 , respectively, obtained from pure finite strategy sets S_1 and S_2 . Player i 's material payoff is given by the function $\pi_i : A_1 \times A_2 \rightarrow \Re$.

In order to construct the psychological game, let us assume that when a player chooses her strategy, her subjective utility function will depend on three things: her strategy, her belief about the other's strategy and her belief about the other's belief about her strategy.⁶ Let us call $a_1 \in A_1$ and $a_2 \in A_2$ the strategies of player 1 and player 2, respectively; $b_1 \in A_1$ and $b_2 \in A_2$ player 2's belief about player 1's strategy and player 1's belief about player 2's strategy, respectively; and $c_1 \in A_1$ and $c_2 \in A_2$ player 1's belief about player 2's belief about player 1's strategy and player 2's belief about player 1's belief about player 2's strategy. Observe that although a_i , b_i and c_i belong to the

⁵ In the next section, we will call them fairness equilibria.

⁶ Higher order beliefs can be considered but it is enough to take the first two.

same set, they are different in nature as a_i is a player i's decision, b_j is player j's belief ($j \neq i$) and c_i is a player i's belief.

To incorporate reciprocity (fairness in terms of Rabin) in the model we first need to define a kindness function $f_i(a_i, b_j)$ which measures how kind player i is to player j. If player i believes player j chooses b_j , how kind is player i by choosing a_i ? When player i chooses a_i , is selecting a payoff pair $(\pi_i(a_i, b_j), \pi_j(b_j, a_i))$ from the set of all the feasible payoffs to player j when he chooses b_j . Let us call this set $\Pi(b_j) = \{(\pi_i(a, b_j), \pi_j(b_j, a)) \mid a \in A_i\}$.

How kind player i is being depends on both the point she chooses from $\Pi(b_j)$ and the notion of kindness players have. To express this notion in formal terms, we need to define a function for both player i's kindness to player j and player i's belief about how kind player j is being to her. Rabin (1993) provides some general properties that sort of functions must have. The following payoffs are useful to do that: let $\pi_j^h(b_j)$ be player j's highest payoff in $\Pi(b_j)$, $\pi_j^l(b_j)$ be player j's lowest payoff among the Pareto-efficient points in $\Pi(b_j)$, and $\pi_j^e(b_j)$ be an "equitable payoff" in $\Pi(b_j)$.

The following properties for kindness functions are sufficient conditions for the main result Rabin obtains:⁷

Property 1: A kindness function must be bounded and increasing. A kindness function $f_i(a_i, b_j)$ is bounded and increasing if:

- a. There exists a number N such that $f_i(a_i, b_j) \in [-N, N]$ for all $a_i \in A_i$ and $b_j \in A_j$ and;
- b. $f_i(a_i, b_j) > f_i(a'_i, b_j)$ if and only if $\pi_j(b_j, a_i) > \pi_j(b_j, a'_i)$.

This property rules out the possibility of fairness to generate infinitely positive or infinitely negative utility and brings out a positive association between the player j's payoff and player i' kindness: given b_j , the higher player j' payoff is, the kinder player i is.

Property 2: A kindness function must be a Pareto split. A kindness function $f_i(a_i, b_j)$ is a Pareto split if there exists some $\pi_j^e(b_j)$ such that:

⁷ They are presented as definitions in Appendix A in Rabin (1993), p. 1297. For additional results is also needed to assume kindness function to be affine, but that property is not relevant for our present purposes.

- a. $\pi_j(b_j, a_i) > \pi_j^e(b_j)$ implies that $f_i(a_i, b_j) > 0$; $\pi_j(b_j, a_i) = \pi_j^e(b_j)$ implies that $f_i(a_i, b_j) = 0$; and $\pi_j(b_j, a_i) < \pi_j^e(b_j)$ implies that $f_i(a_i, b_j) < 0$;
- b. $\pi_j^h(b_j) \geq \pi_j^e(b_j) \geq \pi_j^l(b_j)$; and
- c. if $\pi_j^h(b_j) > \pi_j^l(b_j)$, then $\pi_j^h(b_j) > \pi_j^e(b_j) > \pi_j^l(b_j)$

This property says that the *fair* payoff to player j is strictly between the best and the worst Pareto efficient payoffs in $\Pi(b_j)$, provided that Pareto efficient set is not a singleton.

Among the class of functions defined by the previous properties, Rabin picks the following:

Definition 1: Player i's kindness to player j is given by

$$f_i(a_i, b_j) \equiv \frac{\pi_j(b_j, a_i) - \pi_j^e(b_j)}{\pi_j^h(b_j) - \pi_j^{\min}(b_j)}$$

where $\pi_j^{\min}(b_j)$ is the worst possible payoff for player j in $\Pi(b_j)$ and $\pi_j^e(b_j) = \frac{\pi_j^h(b_j) + \pi_j^l(b_j)}{2}$. If $\pi_j^h(b_j) - \pi_j^{\min}(b_j) = 0$ then $f_i(a_i, b_j) = 0$

It is easy to check this function has the general properties presented above: First, $f_i = 0$ if and only if player j receives the equitable payoff. This is so because when $\pi_j^h(b_j) = \pi_j^{\min}(b_j)$ player j always gains the same payoff and there is no kindness issue. Second, $f_i < 0$ when player j's payoff is lesser than the equitable payoff. This happens when either $\pi_j(b_j, a_i)$ is a Pareto-efficient point smaller than the equitable payoff or $\pi_j(b_j, a_i)$ is not an efficient point. Finally, $f_i > 0$ only if player j's payoff is greater his equitable payoff and the Pareto set is not singleton. Notice also the functions take values in the interval $\left[-1, \frac{1}{2}\right]$.

Player i's belief about how nice player j is to her can be expressed as a function $\tilde{f}_j(b_j, c_i)$. This function is formally equal to the previous but it relates the two levels of beliefs considered in the model.

Definition 2: Player i's belief about how kind player j is to her is given by

$$\tilde{f}_j(b_j, c_i) \equiv \frac{\pi_i(c_i, b_j) - \pi_i^e(c_i)}{\pi_i^h(c_i) - \pi_i^{\min}(c_i)}$$

where $\pi_i^{\min}(c_i)$ and $\pi_i^e(c_i)$ have analogue definitions. If $\pi_i^h(c_i) - \pi_i^{\min}(c_i) = 0$ then $\tilde{f}_j(b_j, c_i) = 0$.

Using both functions $f_i(a_i, b_j)$ and $\tilde{f}_j(b_j, c_i)$ we can define a utility function for player i. Doing so, we are assuming players have a shared notion of fairness. This utility function integrates the material payoff and the reciprocity payoff:⁸

$$U_i(a_i, b_j, c_i) \equiv \pi_i(a_i, b_j) + Y_i \tilde{f}_j(b_j, c_i) [1 + f_i(a_i, b_j)]$$

The first term is the material payoff and the second the reciprocity payoff. The constant Y_i reflects how sensitive player i is to reciprocity matters regarding player j and we will assume it is positive. This utility function gathers the main feature about reciprocal behaviour. If player i believes player j is treating her unkindly ($\tilde{f}_j(b_j, c_i) < 0$), she will want to punish him being unkind, that is choosing a_i such that $f_i(a_i, b_j)$ to be low. On the contrary, if player i thinks player j is being nice ($\tilde{f}_j(b_j, c_i) > 0$), she will be nice. Furthermore, the higher $\tilde{f}_j(b_j, c_i)$ is, the more material payoff player i is willing to give up to reward player j. Finally, this utility function has the property that whenever player j is hostile to player i, player i's utility is lesser than her material payoff. That is, an individual is not able to completely recover her welfare taking revenge once other has treated her badly.

These preferences together to the elements already defined for the material game form a psychological game. Using the concept of psychological Nash Equilibrium defined by GPS, Rabin (1993) proposes the following definition,

Definition 3: The pair of strategies $(a_1, a_2) \in (A_1, A_2)$ is a *fairness equilibrium* if, for $i = 1, 2$, $j \neq i$,

- a. $a_i \in \arg \max_{a \in A_i} U_i(a, b_j, c_i)$
- b. $c_i = b_i = a_i$

This notion of equilibrium is analogous to Nash Equilibrium, but applied to psychological games. Condition b. of the definition requires all high-level beliefs to correspond actual behaviour.

⁸ This utility function is slightly different from which Rabin uses. We have added the term in the reciprocity payoff.

Considering again our example, we can calculate the teachers' utility functions regarding reciprocity. Though theory is posed for mixed strategies we only analyze equilibriums in pure strategies. In Figure 3, we can see the utility values once condition b. of fairness equilibrium is satisfied.

FIGURE 3

| | | | |
|----|----|--|--|
| | | T2 | |
| | | np | p |
| T1 | NP | $x + \frac{3}{4} Y_1, x + \frac{3}{4} Y_2$ | $-\frac{3}{4} Y_1, x + \delta + \frac{1}{4} Y_2$ |
| | P | $x + \delta + \frac{1}{4} Y_1, -\frac{3}{4} Y_2$ | $x - \frac{1}{4} Y_1, x - \frac{1}{4} Y_2$ |

First, note that when player i is being unkind to player j , player j 's reciprocity payoff is negative, which reduces his overall utility and introduces incentives to deviate. However, the profile of strategies (participate, participate) is a fairness equilibrium for all values of X and Y_i because response for unkindness is unkindness. Consider now, what happens if player i deviates to no participation. This action increases player j 's reciprocity payoff because he considers player i is being kind. In fact, player j will deviate to no participation strategy if the loss in material payoff, δ , is less than the gain in reciprocity payoff, $\frac{1}{2} Y_j$. The profile (not participate, not participate)

will be a fairness equilibrium whenever $\delta < \frac{1}{2} Y_i$ for $i = 1, 2$. This condition is satisfied when either δ is low enough or Y_i is high enough. If the government gives a reward too little when one teacher participates and the other does not or if both teachers have a strong feeling to reciprocate the other, deviating from participation will be an equilibrium.

This model has been extended to include sequential actions. In principle, such a model would be more adequate to reality because reciprocal actions have an implicit delay. One is kind with somebody that has *been kind*. Besides, "extending the model to sequential games is also essential for applied research" (Rabin (1993), p. 1296), as individuals can change their motivation due to information provided by past decisions.

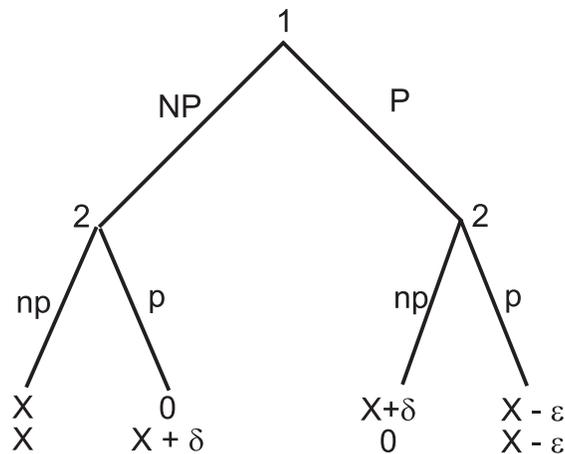
3.4 SEQUENTIAL GAMES

Consider now a slightly modified sequential version of our game of teachers, depicted in Figure 4. Assume there is no trade union anymore, so government can offer a lesser material payoff if both teachers participate in the program, $X - \varepsilon$, $0 < \varepsilon < X$. In the first step, teacher 1 decides whether to participate or not in the program offered and once he has decided, teacher 2 has to take her decision. Assuming further no reciprocity (and perfect and complete information), it can be seen, solving by backward induction, the profile (Participate, participate) will be the

unique Subgame Perfect Nash Equilibrium. Government's strategy to implement the program is completely successful as teachers will always participate.

Let us introduce reciprocity. Suppose 1 chooses do not participate (NP) in the program. Player 2 can choose either X or $X+\delta$ (or mix). Her choice will depend on both her kindness and the belief she has about the 1's intention to choose NP. When teacher 1 chooses NP gives teacher 2 a payoff at least X and at most $X+\delta$. Instead, when teacher 1 chooses P gives teacher 2 a payoff at least 0 and at most $X-\varepsilon$. So 2 will believe 1 is being kind when he chooses NP and if reciprocity payoff is high enough she will choose X instead of $X+\delta$ (or mix). To establish if (NP, np) will be an equilibrium, we have to evaluate what teacher 1 believes when 2 chooses np.

FIGURE 4



It is convenient to point out one difference in the analysis of reciprocity in normal games and extensive games. In normal games teacher 2 will always choose do not participate, provided reciprocity payoff supersedes material payoff. This does not happen in a sequential model because, for instance, once teacher 2 knows teacher 1 has chosen to participate, there is no reason to maintain the decision of do not participate unconditionally. In that case teacher 2 would consider teacher 1 is being hostile and thus she would participate in the program as well. Unlike normal games, in sequential games unconditional np does not occur because player 2 is optimizing in each subgame.

On the other hand, in modelling reciprocity in sequential games it is not plausible to assume players are going to keep their initial beliefs along the game. Player 2's belief about how kind player 1 is being once the latter has decided do not participate is different from the former's belief once the latter has decided to participate. It means it is necessary to analyze changes in beliefs in each node of the game in order to establish equilibrium conditions. Furthermore, it is not possible to consider each subgame separately. Player 2' belief about how nice is player 1, given he has already decided to not participate, depends on which payoffs she would had had if player 1 had decided to participate. Therefore, backward induction cannot be used to obtain the equilibrium. Dufwenberg and Kirchsteiger (2001) (henceforth DK (2001)) provide a concept of

sequential reciprocity which allows them to propose a new solution concept, Sequential Reciprocity Equilibrium (SRE).

3.5 DUFWENBERG AND KIRCHSTEIGER (2001)'S MODEL

As we have said, when reciprocity is incorporated in sequential games it is necessary to distinguish between a player' initial and subsequent belief. Once a subgame has been attained, a player's belief can change and, as kindness depends on belief, kindness may therefore change as well. DK (2001) deal with this by keeping track of how beliefs change when a new subgame is reached and by assuming players' choices take into account the beliefs they hold in the most recently reached subgame. To do that, as Rabin they adopt the psychological games framework; but unlike GPS (1989), who only regards to games where solely initial beliefs can affect player's assessments, DK (2001) propose a notion of reciprocity in which player's beliefs change in each subgame.

Formally, they pose a t -player extensive form game without nature and with perfect recall. Any such a game Γ is described by a finite set of nodes organized in a tree, a collection of information sets, a set of choices available at each decision node, a function assigning each information set to the player who moves at the decision nodes in that set, and a collection of payoff functions assigned to each endnode (Mas-Colell, Whinston and Green (1995)). Let $T = \{1, \dots, t\}$ be the set of players where $t \geq 2$. It is convenient to add new notation to that used in section 3.3, as there are now several players. Let A_i be the set of player i 's behaviour strategies, a_i ; B_{ij} be the set of possible player i 's beliefs about player j 's strategies, b_{ij} ; and C_{ijk} be the set of player i 's belief about player j 's belief about player k 's strategies, c_{ijk} . As in Rabin's model, beliefs are mixed strategies, so we have $B_{ij} = A_j$ and $C_{ijk} = B_{jk} = A_k$. Besides, player i 's material payoff is now given by the function $\pi_i : A \rightarrow \Re$ where $A = \prod_{i \in T} A_i$.

Now, let us proceed to formalize how the player's beliefs change when new subgames are reached. To keep track of how each player's behaviour, niceness and perception of other's niceness differ across subgames, let R be the set of nodes that are starting nodes of all possible subgames in Γ , and let Γ^r be the subgame which starting point is $r \in R$. Let us define the r -part of Γ^r as the set of nodes in Γ^r that do not belong to some proper subgame of Γ^r . For a strategy $a_i \in A_i$, let $a_i(r)$ be the strategy that has the same choices as a_i but assigning a probability equal to 1 to the choices that drive to node r . In an analogous way, define $b_{ij}(r)$ and $c_{ijk}(r)$ for $b_{ij} \in B_{ij}$ and $c_{ijk} \in C_{ijk}$, respectively. Thus, player i decides to play a_i believing other players are playing $(b_{ij})_{i \neq j}$ and believing $(c_{ijk})_{j \neq k}$, whereas in the r -part of the subgame Γ^r , player i is playing $a_i(r) \in A_i$ and believing other players to play $(b_{ij}(r))_{i \neq j}$ and to believe $(c_{ijk}(r))_{j \neq k}$. This means that "even if players initially believe that others mix their choices, the

subsequent perception of kindness is triggered by the actual choice” (DK (2001), p. 8). In terms of our example, consider the proper subgame starting in the player 2’s right side node and call that node r . Player 2 believes player 1 is choosing his strategy as $b_{21} = p'NP + (1 - p')P$. Before 1 plays, *at node r* if is big (1 or near 1) player 2 will think player 1 is kind. However, once r is reached, player 2 does not consider player 1 to be kind anymore. At r , player 2’s belief is $b_{21}(r) = P$.

DK (2001) also change the notion of efficiency suggested by Rabin (1993), which says that the lowest efficient strategy is chosen from $\Pi(b_j)$. They argue that, in a sequential game framework, the set of Pareto-efficient strategies relevant to establish the equitable payoff cannot depend on beliefs, as this can drive us to no existence of equilibrium.⁹ DK (2001)’s efficiency notion can be formulated as

$E_i = \{a_i \in A_i \mid \text{there exists no } a_i' \in A_i \text{ such that for all } r \in (a_i)_{j \neq i} \in \prod_{j \neq i} A_j, k \in T \text{ it holds that}$

$$\pi_k(a_i'(r), (a_j(r))_{j \neq i}) \geq \pi_k(a_i(r), (a_j(r))_{j \neq i}), \text{ with strict inequality for some } (r, (a_j)_{j \neq i}, k)\}$$

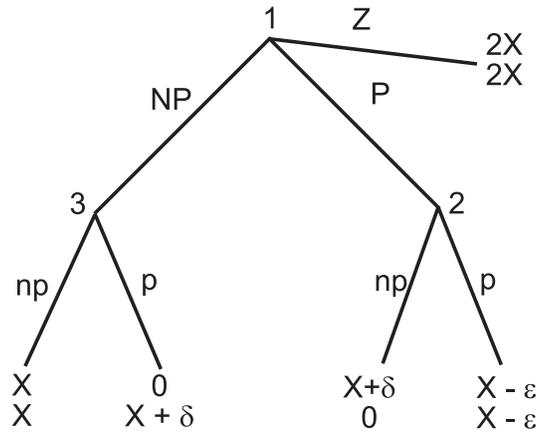
The concept of efficiency has a central role in the intention-based theories. To illustrate this point consider the game depicted in Figure 5. We have the same game of Figure 4 but now player 1 can do an action Z in which both players obtain a payoff $-2X$. Let us suppose player 1 believes with probability one player 2 is playing the strategy np, p . It can be seen 1 believes he selects the material payoff $\pi_2(P, np, p) = x - \varepsilon$ from the feasible set $\{x, x - \varepsilon, -2x\}$. In the game of Figure 4, player 1 would be considered unkind, now are we willing to accept player 1 is being kind due to the mere possibility of Z to be chosen? To rule out this unreasonable consideration we restrict our attention to efficient payoffs in order to determine the equitable payoff. DK (2001) propose the notion of efficiency above to do that.¹⁰

We can define the equitable payoff as $\pi_j^{e_i}((b_i)_{j \neq i}) = \frac{1}{2}(\pi_j^h(a_i(b_i)_{j \neq i}) + \pi_j^l(a_i(b_i)_{j \neq i}))$, which is essentially the same defined in section 3.3. Unlike that one, a subindex i has been added to e to indicate that this is the equitable payoff for i and $\pi_j^l(a_i(b_i)_{j \neq i})$ is now the lowest payoff in E_i .

In turn, kindness, kindness’ belief and utility functions can also be defined in a similar way as before.

⁹ Look at DK (2001) p. 29 for an example of no existence of equilibria due to a belief dependent concept of efficiency.

¹⁰ However, this distinction does not make any difference with respect to our example, because all the strategies are efficient under both concepts.

FIGURE 5

Definition 4: Kindness of player i to another player $j \neq i$ in the r -part of a subgame Γ^r is given by the function $f_{ij} : A_i \times \prod_{j \neq i} B_{ij} \rightarrow \Re$ defined by

$$f_{ij} \left(a_i(r), (b_{ij}(r))_{j \neq i} \right) = \pi_j \left(a_i(r), (b_{ij}(r))_{j \neq i} \right) - \pi_j^{e_i} \left((b_{ij}(r))_{j \neq i} \right)$$

Apart from differences already mentioned, definition 4 is analogous to definition 1. f_{ij} differs formally from f_i in that f_{ij} is not normalized and thus, in principle, it may take values extremely high or low. However, due to we are analysing central points (as we subtract an average from the payoff chosen), it is not expectable to obtain an extreme number, so property 1a in section 3.3 can be hold without large inconveniences. On the other hand, it is straightforward to check definition 4 holds properties 2 and 1b.

Definition 5: Player i 's beliefs about how kind player $j \neq i$ is to i in the r -part of a subgame Γ^r is given by the function: $\tilde{f}_{iji} : B_{ij} \times \prod_{k \neq j} C_{ijk} \rightarrow \Re$ defined by

$$\tilde{f}_{iji} \left(b_{ij}(r), (c_{ijk}(r))_{k \neq i} \right) = \pi_i \left(b_{ij}(r), (c_{ijk}(r))_{k \neq j} \right) - \pi_i^{e_j} \left((c_{ijk}(r))_{k \neq j} \right)$$

This definition is formally equal to the previous. The same comments for f_j with respect to f_i can be done for: \tilde{f}_{iji} in relation to \tilde{f}_j .

Definition 6: Player i 's utility in the r -part of a subgame Γ^r is a function

$U_i : A_i \times \prod_{j \neq i} (B_{ij} \times \prod_{k \neq j} C_{ijk}) \rightarrow \Re$ defined by

$$U_i \left(a_i(r), (c_{ijk}(r))_{k \neq j}, (b_{ij}(r))_{j \neq i} \right) = \pi_i \left(a_i(r), b_{ij}(r)_{j \neq i} \right) + \sum_{j \in T \setminus \{i\}} \left(Y_{ij} f_{ij} \left(a_i(r), (b_{ij}(r))_{j \neq i} \right) \tilde{f}_{iji} \left(b_{ij}(r), (c_{ijk}(r))_{k \neq j} \right) \right)$$

Utility function in Rabin's model has the term $(1 + f_i)$ instead of f_j . For comparison purposes, it has been preferred to keep the functions as alike as those the authors propose. As sensitivity

for reciprocity Y_j is a nonnegative number, reciprocity payoff increases utility if player i believes player j is kind ($\tilde{f}_{iji} > 0$), and reduces utility if player i believes player j is unkind ($\tilde{f}_{iji} < 0$).

Appending this kind of utility functions to an extensive game, we get the tuple $\Gamma^o = (\Gamma, (U_i)_{i \in T})$. DK (2001) call Γ^o a psychological game with reciprocity incentives. There is a notion of equilibrium associated to these games that can be formulated as

Definition 7: The profile $\hat{a} = (\hat{a}_i)_{i \in T}$ is a *Sequential Reciprocity Equilibrium (SRE)* if for all $i \in T$ and for all $r \in R$ it holds that

- a. $\hat{a}_i(r) \in \arg \max_{a_i \in A_i(r, \hat{a}), a_i \in A_i(r, \hat{a})} U_i \left(a_i, b_{ij}(r), (c_{ijk}(r))_{k \neq j, j \neq i} \right)$
- b. $b_{ij} = \hat{a}_j$ for all $j \neq i$
- c. $c_{ijk} = \hat{a}_k$ for all $j \neq i, k \neq j$

where $A_i(r, a)$ is the set of strategies player i can use if she behaves according to $a_i(r)$ at information sets outside the r -part of Γ^r , but is free to choose any alternative in the r -part of Γ^r .

Condition a. says player i maximizes his utility at node r given his beliefs and given that he follows his equilibrium strategy outside the r -part of Γ^r . This entails beliefs to assign a probability one to the sequence of choices that allow r to be reached. Conditions b. and c. says in the equilibrium beliefs are correct and correspond to the actual strategy. DK show every psychological game with reciprocity incentives has at least one SRE. To do that, they first define the size of a subgame as the number of its subgames, then they simultaneously determine equilibrium choices of the subgames with the same size, starting from the smallest (size equal one) until arriving to the complete game.¹¹

In the game that appears in section 3.4, first teacher 1 decides whether to participate or not in the program offered by the government and then teacher 2 does so. We showed there, no reciprocity implies profile (participate, participate) to be the sole Subgame Perfect Nash Equilibrium. How the analysis is affected when teachers are reciprocal? We can find it out using the theory developed in this section. When there is reciprocity between agents, the game becomes a psychological game with reciprocity incentives. Examining SRE for different levels of reciprocity sensitivity we can say:¹²

1. If teacher 2's sensitivity to reciprocity, Y_2 , is low enough, profile (participate, participate) is an

equilibrium behaviour. Specifically this occurs when $Y_2 < \frac{2\delta}{x(\delta + \varepsilon)}$. In this case, each player will

believe the other is going to participate, which will in turn be considered as unkind. Those beliefs render a negative reciprocity payoff to both players and therefore each teacher prefers to participate in the government program. From the previous inequality it can also be seen

¹¹ Demonstration appears in DK (2001) p. 35.

¹² Detailed calculations are included in the Appendix. As the game has two players, we simplify notation, so Y_i , $i = 1, 2$, is agent i 's sensitivity to reciprocity.

that given a sensitivity to reciprocate level for both teachers, Y_1 and Y_2 , the higher δ relative to x and ε is, the more likely both teachers to participate. Government should take this into account in order to make teachers to participate in the program he proposes.

2. If teacher 2's inclination to reciprocate, Y_2 , is high enough, profile (do Not Participate, do not participate) holds in all SRE. Regardless Y_1 , when teacher 2 has a strong inclination to reciprocate, she will obtain a high reciprocity payoff if teacher 1 decides to not participate, so she will play np (instead of p) when teacher 1 does so. Notice player 2 would also get a higher material payoff doing so than that she had obtained if teacher 1 plays P (instead of NP). Teacher 1 knows all this, and thus he will choose to play NP to get a higher material payoff than that he would get if he had played P . This equilibrium behaviour cannot be predicted when we assume no reciprocity. The scheme proposed by government does not work in the way government expects due to reciprocity between teachers.
3. Given a high 2's leaning to reciprocate, it also happens (Participate, participate) to be an equilibrium behaviour when teacher 1 also has a strong tendency to reciprocate. This arises when each player thinks the other is going to play p , as each one expects an unkind action from the other. There are "self-fulfilling expectations".
4. For intermediate values of Y_1 and Y_2 , equilibrium behaviours are mixed strategies. In equilibrium, for player 2 probability of no participation, p , is given by $p = \frac{\varepsilon}{\delta} - \frac{2}{Y_2 x}$. As it can be inferred from previous analysis, this probability increases when Y_2 increases. In addition, p reduces if the ratio between ε and δ decreases and increases if x increases. $\frac{\varepsilon}{\delta}$ can be viewed as the inverse of the incentive government provides to player 2 to participate. Player 2 tries to gain δ (she gains d if (NP, p) is chosen) but she loses ε if (P, p) is chosen. She evaluates how much she can obtain and lose from participation. This evaluation affects p in the way described. On the other hand, an increase in x increases p because *ceteris paribus* it makes less attractive to participate. For player 1, it is not possible to do the same kind of analysis due to parameters affect his probability of no participation, q , in a complex way. In fact, for a given Y_2 , 2's equilibrium behaviour is unique whereas, in general, 1's equilibrium behaviour is not unique for a given Y_1 .¹³

Finally, from the results obtained for this game we can analyze a sequential version of the teacher's game with trade union. In that game $\varepsilon = 0$, so payoffs in profiles (NP, np) and (p, p) are equal to X . The most interesting result in this case is no participation to be an equilibrium behaviour only in mixed strategies. Analysis is as follows. We know teacher 2 will always play p when teacher 1 plays P ,¹⁴ so teacher 2 would get X in this profile. On the other hand, if teacher 1 plays NP , teacher 2 can get either X or $X + \delta$. For (NP, p) to be possible in equilibrium, player 2 has to believe with probability one that player 1 believes player 2 will choose p . But in this

¹³ Look at Remarks 4 and 5 in the Appendix.

¹⁴ Look at Remark 1 in the Appendix

situation, player 2 would obtain X from both (NP, np) and (p, p) and hence there would be no reciprocity issue (reciprocity payoff equal to zero). Therefore, player 2 would prefer to play another strategy, as profile (NP, p) offers player 2 a higher material payoff. A similar analysis can be done for player 1.

4. CONCLUSIONS

Evidence has shown that sometimes people behave in different ways from which is predicted by assuming individuals are self-interested. Furthermore, when persons deviate from self-interested behaviour they do not always try to increase the well-being of others. On the contrary, it has been found individuals usually respond in a kind manner to kind actions and in an unkind manner to unkind actions. In response to these findings, several economic theories have attempted to model reciprocity behaviour. In this document, we have reviewed the so-called intention-based theories of reciprocity, specifically the models made by Rabin (1993) and Dufwenberg and Kirchsteiger (2001).

These theories have received this name because they emphasize people want to punish hostile intentions and to reward nice intentions. To do that, they adopt the psychological games framework developed by Geanakoplos, Pearce and Stacchetti (1989). In this framework individual utility depend not only on strategies but also on beliefs. Rabin (1993) develops a theory for 2-players normal form games and introduces a new equilibrium notion called fairness *equilibrium*. Dufwenberg and Kirchsteiger (2001) in turn extend Rabin's theory dealing with t-players sequential games and present the notion of *sequential reciprocity equilibrium*. The main innovation they do is to keep track of beliefs about intentions as the game evolves. Players maximize their behaviour in each subgame taking into account beliefs about intentions formed in the previous stages. In a particular subgame players use beliefs that comes from the most recently reached subgame.

There are other differences between these models. Rabin (1993) uses a kindness function neutral to units of measure of the stakes, so that kindness cannot infinitely increase or decrease utilities. This also allows individual kindness to reduce as long as payoffs become larger. Instead Dufwenberg and Kirchsteiger (2001) measure kindness in the same units of material payoffs (i.e. money), which has the advantage kindness does not disappear when payoffs rise but the disadvantage it also makes utility to be sensitive to linear transformations as reciprocity payoff is measured in "money squared". Moreover, they differ in the efficiency notion used to define the equitable payoff. Rabin (1993)'s notion depends on beliefs and then it only considers strategies on the equilibrium path; whereas DK (2001) defines inefficient strategies as those that yield a weakly lower payoff for all player (strictly lower for some) than other alternatives in all the subgames. Finally, Rabin (1993) specifies kindness in the utility function in such a way to capture the idea that whenever a player is treated unkindly, her overall utility will be lower than her material payoff (her ability to take payback is not perfect). DK (2001)'s specification does not capture that.

We have also illustrated the theories studied with a simple example in teachers' management. We have proposed an implementation mechanism for a governmental policy when there is a teachers' trade union. Both teachers have to decide to participate (p) or not (np) in a

governmental program. In order to implement the policy, government proposes a game with a prisoner's dilemma structure. Without reciprocal teachers, in both games (normal and sequential forms) there is a unique equilibrium: teachers participate in the governmental program. With reciprocal teachers, we obtain additional results. In the normal form game, there are two fairness equilibria: one in which each teacher is kind to the other and other in which both teachers are unkind. If in equilibrium both teachers are kind to each other, government cannot implement the program.

In the sequential game in turn we have multiple equilibria. We considered two games: a sequential version of the previous one and a game in which there is no trade union and hence government can give a lesser payoff if both teachers participate in the program. Now teacher 2 does not choose np unconditionally as in the normal form, as teacher 2 behaves optimally off the equilibrium path. In both games, conditional "cooperation" can be part of a SRE. However, under trade union np is an equilibrium behaviour only in mixed strategies.

One limitation of the intention-based approach is that one individual only has reciprocal behaviour when other individuals have shown to have kind intentions. Suppose in our example player 2 is constrained to "choose" do not participate. Player 1 will not consider this action as kind because player 2 has no option. In fact, although nowadays there is almost consensus about the existence of reciprocal behaviour, there is still disagreement about the foundations of that behaviour. For instance, other theoretical approaches focus on inequity aversion (Bolton and Ockenfels, 2000) or the type of persons one faces (Levine, 1998). Hence, in an inequity model player 1 will behave kind when there is an inequity issue even if player 2 is forced to choose do not participate. Discussion is opened regarding this point.¹⁵

Another limitation of this approach is that equilibrium analysis is rather complex and there are multiple equilibria due to self-fulfilling beliefs. In the normal form game suggested, for example, both equilibria emerge for this reason, so it is not possible to establish which one is going to occur. On the other hand, even though treatment of beliefs in the sequential model is very innovative it makes difficult to build tractable models.

Finally, despite simplicity of our examples, they suggest it will be worthy to take into account reciprocity in theories that try to model government-teachers' relationships. On one hand, a significant part of literature on reciprocity has shown reciprocal behaviour is relevant in the analysis of employer-employee relationships. It has been documented employers are reluctant to decrease wages in crisis times because they do not want to reduce employees' morale to work. In particular, it would be interesting to find out how reciprocity affects the main results of multiagent settings.¹⁶ On the other hand, some empirical research has shown teachers' trade unions can affect negatively student performances (quality of education) (Hoxby, 1996).

¹⁵ Falk and Fischbacher (2000) show evidence that supports intentions matter. Fehr and Schmidt (2001) survey existing models on fairness and reciprocity

¹⁶ One of the main results in these settings is that under moral hazard, principal can use relative performance of agents to elicit a higher effort (yardstick competition). Cf. Laffont and Martimort (2002).

5. REFERENCES

- Akerlof, George (1982), "Labor contracts as a partial gift exchange", *Quarterly Journal of Economics* 97: 543-569.
- Benabou, Roland and Jean Tirole (2002), Intrinsic and extrinsic motivation, mimeo.
- Bewley, Truman (1995), "A depressed labor market as explained by participants", *American Economic Review* 85, Papers and Proceedings: 250-254.
- Bolton, Gary and Axel Ockenfels (2000), "ERC - A theory of Equity, Reciprocity and Competition", *American Economic Review* 90: 166-193.
- Charness, Gary and Matthew Rabin (2000), Social preferences: some simple tests and a new model, University of California at Berkeley, mimeo.
- Dufwenberg, Martin and Georg Kirchsteiger (2001), A theory of sequential reciprocity, Discussion paper. Center, Tilburg University.
- Falk, Armin and Urs Fischbacher (2000), A theory of reciprocity, Institute for Empirical Research in Economics, University of Zurich, Working Paper, 6.
- Fehr, Ernst and Armin Falk (2001), Psychological Foundations of Incentives, University of Zurich, mimeo.
- Fehr, Ernst and Urs Fischbacher (2001), Why social preferences matter - The impact of non-selfish motives on competition, cooperation and incentives, University of Zurich, mimeo.
- Fehr, Ernst and Simon Gächter (2000), "Fairness and retaliation: The economics of reciprocity", *Journal of Economic Perspectives* 14: 159-181.
- Fehr, Ernst and Klaus Schmidt (2001), Theories of fairness and reciprocity - Evidence and economics applications, University of Zurich, mimeo.
- Frey, Bruno and Reto Jegen (2001), "Motivation crowding-out theory", *Journal of Economic Surveys* 15 (5): 589-611.
- Geanakoplos, John, David Pearce and Ennio Stacchetti (1989), "Psychological games and sequential rationality", *Games and Economic Behaviour* 1: 60-79.
- Hoxby, Caroline (1996), "How teachers' unions affect education production", *The Quarterly Journal of Economics* 671-718.
- Laffont, Jean-Jacques and David Martimort (2002), *The theory of Incentives: The principal agent model*, Harvard University Press.
- Levine, David (1998), "Modelling altruism and spitefulness in experiments", *Review of Economic Dynamics* 1: 593-622.
- Mas-Colell, Whinston and Green (1995), *Microeconomic theory*, Oxford University Press.
- Rabin, Matthew (1993), "Incorporating fairness into game theory and economics", *American Economic Review*, 83 (5): 1281-1302.
- Seabright, Paul (2002), Blood, Bribes, and the crowding-out of altruism by financial incentives University of Toulouse, mimeo.
- Segal, Uzi and Joel Sobel (1999), Tit for tat: Foundations of preferences for reciprocity in strategic settings, University of California at San Diego, mimeo.

APPENDIX

Equilibrium analysis of the Sequential Game

Remark 1: If teacher 1 participates, teacher 2 also participates in every SRE

Note that only the reciprocity payoff can make 2 choose np , as the material payoff *per se* dictates a choice of p for 2. However, for any possible strategy of 2, teacher 2 gets less when 1 chooses P than when he chooses NP . Whatever 1 believes about 2's strategy, 1's choice of P is unkind, and hence 2 must believe that 1 is unkind. Thus the reciprocity payoff as well as the material payoff makes teacher 2 to choose p .

Remark 2: If teacher 1 does not participate, the following holds in all SRE:

- a. If $Y_2 > \frac{2\delta}{\varepsilon x}$, teacher 2 does not participate
- b. If $Y_2 < \frac{2\delta}{x(\delta + \varepsilon)}$, teacher 2 participates
- c. If $\frac{2\delta}{x(\delta + \varepsilon)} < Y_2 < \frac{2\delta}{\varepsilon x}$, teacher 2 does not participate with a probability of $p = 1 + \frac{\varepsilon}{\delta} - \frac{2}{Y_2 x}$

Notice that if 1 does not participate, 2 can give 1 a material payoff of at least 0 and at most x so the "equitable" payoff of 1 is $x/2$. If 2 chooses no participation, 1 receives x . Therefore, 2's kindness of no participation is $x/2$. Similarly, 2's kindness of participation is $-x/2$. In order to calculate how kind 2 believes 1 is after choosing NP we have to specify 2's belief of 1's belief about 2's choice after NP .¹⁷ Denote this by p'' . Then 2's belief about how much payoff 1 intends to give to 2 by choosing NP is $p'' x + (1 - p'')(x + \delta)$, and since 2's payoff resulting from 1's choice of P would be x ,¹⁸ 2's belief about 1's kindness from choosing NP is $p'' x + (1 - p'')(x + \delta) - 0.5 (p'' x + (1 - p'')(x + \delta) + \delta) + x - \varepsilon = 0.5 ((1 - p'') \delta + \varepsilon)$. This implies that when 1 does not participate and the second order belief is p'' , 2's utility of no participation is given by $x + 0.5 Y_2 (x/2) ((1 - p'') \delta + \varepsilon)$, whereas 2's utility of participation is $(x + \delta) - 0.5 Y_2 (x/2) ((1 - p'') \delta + \varepsilon)$. The former is larger than the latter if $Y_2 (x/2) ((1 - p'') \delta + \varepsilon) > \delta$. In equilibrium, the second order belief must be correct. Hence, if in equilibrium 2 does not participate, the condition must hold for $p'' = 1$. This is the case if $Y_2 > \frac{2\delta}{\varepsilon x}$. On the other hand, if in equilibrium 2 participates, that

condition must not hold for $p'' = 0$; This implies that $Y_2 < \frac{2\delta}{x(\delta + \varepsilon)}$ For intermediate values of

$Y_2 \left(\frac{2\delta}{x(\delta + \varepsilon)} < Y_2 < \frac{2\delta}{\varepsilon x} \right)$, neither no participation nor participation can be of an equilibrium. In

¹⁷ In principle we also need 2's belief about 1's behavior. However, after 1 has already chosen NP , 2 already knows what 1 has done, and 2's belief has to be in accordance with her knowledge.

¹⁸ In any SRE player 2 participates after a participation of 1 (Remark 1)

order to have a mixed equilibrium, the utility of no participation must be equal to the utility of participation. This is the case when $Y_2(x/2)((1-p)\delta + \varepsilon) = \delta$. Since in equilibrium the second order belief correct, the actual probability of no participation, p , must be such that the condition is

fulfilled. This implies that $p = 1 + \frac{\varepsilon}{\delta} - \frac{2}{Y_2 x}$.

Notice that probability p equals zero for $Y_2 = 2\delta/(x(\delta + \varepsilon))$, and p equals one $Y_2 = 2\delta/\varepsilon x$. Hence, Remarks 1 and 2 together imply that for a given parameter Y_2 2's equilibrium behaviour is unique. This is, however, in general not true for 1's behaviour which can be characterized by three observations:

Remark 3: If $Y_2 < \frac{2\delta}{x(\delta + \varepsilon)}$, participation is the unique 1's equilibrium behaviour.

Notice that for $Y_2 < \frac{2\delta}{x(\delta + \varepsilon)}$ teacher 2 always participates (Remarks 1 and 2). Hence, only the reciprocity part of the utility function can make 1 to choose NP (the material payoff alone would dictate for 1 to choose P). However, for any second order belief about 1's behaviour 2's strategy of always participating is unkind. Hence, the reciprocity payoff as well as the material payoff makes teacher 1 chooses P .

Remark 4: If $Y_2 > \frac{2\delta}{\varepsilon x}$, 1's equilibrium behaviour is typified by one of the following possibilities:

- a) Teacher 1 does not participate (regardless of Y_1)
- b) $Y_1 > 2/(\varepsilon + \delta)$ and teacher 1 participates
- c) $Y_1 > 2/(\varepsilon + \delta)$ and teacher 1 does not participate with probability $q = 1 - ((xY_1 + 2)/(Y_1(\varepsilon + x + \delta)))$

Note that $Y_2 > \frac{2\delta}{\varepsilon x}$ implies that 2 does not participate when 1 does not participate and participates when 1 participates (Remark 1 and 2). Hence, 1 can give 2 a material payoff of at least $(x - \varepsilon)$ and at most x . Thus, the "equitable" payoff of 1 is $x - (\varepsilon/2)$. If 1 chooses no participation, 2 receives x . Therefore, 1's kindness of no participation is $\varepsilon/2$. Likewise, 1's kindness of participation is $-(\varepsilon/2)$. In order to calculate how kind 1 believes that 2 is we have to specify 1's belief about what 2 believes that 1 will do. Denote by q " this second order belief of 1 choosing NP . Then 1 believes that 2 believes that she gives teacher 1 a material payoff of q " $x + (1 - q$ " $) (x - \varepsilon)$ by choosing her equilibrium strategy. If 2 always does not participates, 1's payoff is q " $x + (1 - q$ " $) (x + \delta)$, whereas if 2 always participates, 1's payoff is q " $0 + (1 - q$ " $) (x - \varepsilon)$. Hence, 1's belief about 2's kindness from choosing np after NP and p after P is given by: q " $x + (1 - q$ " $) (x - \varepsilon) - 0.5 (q$ " $x + (1 - q$ " $) (x + \delta) + q$ " $0 + (1 - q$ " $) (x - \varepsilon) = 0.5 (-\varepsilon - \delta + q$ " $(\varepsilon + x + \delta))$

This implies that when 2 plays the equilibrium strategy and the second order belief is q ", 1's utility of no participation is given by $x + 0.5 Y_1(\varepsilon/2)(-\varepsilon - \delta + q$ " $(\varepsilon + x + \delta))$, whereas 1's utility of participation is $x - \varepsilon - 0.5 Y_1(\varepsilon/2)(-\varepsilon - \delta + q$ " $(\varepsilon + x + \delta))$.

The former is larger than the latter if $\varepsilon + Y_1 (\varepsilon/2) (-\varepsilon - \delta + q) (\varepsilon + x + \delta) = 0$. In equilibrium, the second order belief must be correct. Hence, if in equilibrium 1 does not participate, the condition must hold for $q = 1$, which is always the case.

On the other hand, if in equilibrium 1 participates, the condition must not hold for $q = 0$. This implies that $Y_1 > 2/(\varepsilon + \delta)$. In order to have a mixed equilibrium, the utility of no participation must be equal to the utility of participation.

This is the case when $\varepsilon + Y_1 (\varepsilon/2) (-\varepsilon - \delta + q) (\varepsilon + x + \delta) = 0$. Since in equilibrium the second order belief must be correct, the actual probability of no participation, q , must be such that the condition is fulfilled. This implies that $q = 1 - ((xY_1 + 2)/(Y_1\varepsilon + x + \delta))$

Next we turn to the equilibrium behaviour when 2 is moderately motivated by reciprocity and hence answers a no participate choice of 1 with mixing.

Remark 5: if $Y_2 > \frac{2\delta}{x(\delta + \varepsilon)} < Y_2 < \frac{2\delta}{\varepsilon x}$, 1's equilibrium behaviour is characterized by one of the three following possibilities:¹⁹

- a) $Y_2 > \frac{4\delta}{x(\delta + x)}$ and teacher 1 does not participate
- b) $Y_1 > \frac{x(-4\delta + Y_2(\delta x + x^2))}{\delta^2(2\delta + x)}$ and teacher 1 participates
- c) $Y_1 > \frac{x(-4\delta + Y_2(\delta x + x^2))}{\delta^2(2\delta + x)} > 0$ and teacher 1 does not participate with probability

$$q = \frac{Y_2 \left(2\delta^2 Y_1 + 4x + \delta Y_1 x - x^2 Y_2 - \frac{x^3 Y_2}{\delta} \right)}{Y_1 (-8\delta + 2\delta^2 Y_2 + 3\delta x Y_2 + 2x^2 Y_2)}$$

To see this, notice that $\frac{2\delta}{x(\delta + \varepsilon)} < Y_2 < \frac{2\delta}{\varepsilon x}$ implies that 2 does not participate with probability $p = 1 + \frac{\varepsilon}{\delta} - \frac{2}{Y_2 x}$ when 1 does not participate, and 2 participates when 1 participates (see Remarks 1 and 2). Hence, 1 can give 2 a material payoff of at least $(x - \varepsilon)$ and most $p x + (1-p)(x + \delta)$. Hence, the "equitable" payoff of 1 is $\frac{1}{2}((1-p)\delta - \varepsilon + 2x)$. If 1 chooses no participation, 2 receives $p x + (1-p)(x + \delta)$. Therefore, 1's kindness of no participation is $\frac{1}{2}((1-p)\delta - \varepsilon)$. Similarly, 1's

¹⁹ To obtain the specific right-hand side values of these inequalities we assume . This assumption is no essential and it is made to simplify calculations. Analogous results can be obtained without use it.

kindness of participations is $\frac{1}{2}((1-p)\delta - \varepsilon)$. In order to calculate how kind 1 believes 2 is we have to specify 1's belief about what 2 believes that 1 will do. Denote by q'' this second order belief of 1 choosing *NP*. Then 1 believes that she gives teacher 1 a material payoff of $q''(px + (1-p)0) + (1-q'')(x - \varepsilon)$ by her equilibrium strategy. If 2 always does not participate, 1's payoff is $q''x + (1-q'')(x + \delta)$, whereas if 2 always participates, 1's payoff is $q''0 + (1-q'')(x - \varepsilon)$. Hence, 1's belief about 2's kindness of her equilibrium strategy is $q''(px + (1-p)0) + (1-q'')(x - \varepsilon) - 0.5(q''x + (1-q'')(x + \delta) + q''0 + (1-q'')(x - \varepsilon)) = q''px - 0.5((1-q'')\varepsilon + (1-q'')\delta + q''x)$. This implies that when 2 plays the equilibrium strategy and the second order belief is q'' , 1's utility of no participation is given by $px + \frac{1}{2}Y_1((1-p)\delta + \varepsilon) \left(q''px - \frac{1}{2}((1-q'')\varepsilon + (1-q'')\delta + q''x) \right)$, whereas 1's utility of participation is $x - \varepsilon - \frac{1}{2}Y_1((1-p)\delta + \varepsilon) \left(q''px - \frac{1}{2}((1-q'')\varepsilon + (1-q'')\delta + q''x) \right)$.

The former is larger than the latter if

$$-(1-p)x + \varepsilon + Y_1((1-p)\delta + \varepsilon) \left(q''px - \frac{1}{2}((1-q'')\varepsilon + (1-q'')\delta + q''x) \right) > 0.$$

In equilibrium, the second order belief must be correct. Hence, if in equilibrium 1 does not participate, the condition must hold for $q'' = 1$, that is $px - (x - \varepsilon) + Y_1((1-p)\delta + \varepsilon) \left(px - \frac{1}{2}x \right) > 0$

In general we have a solution for p finding out the roots of the left-hand side quadratic equation. To simplify calculations, let us assume $\varepsilon = \frac{1}{2}x$. In this case the condition holds if $p > 0.5$. This in turn implies that $Y_2 > \frac{4\delta}{x(\delta - x)}$ (see the calculation of p in Remark 2c).

On the other hand, if in equilibrium 1 participates, the condition must not hold for $q''=0$.

Inserting for p and rearranging terms this leads to $Y_1 > \frac{x(-4\delta + Y_2(\delta x + x^2))}{\delta^2(2\delta + x)}$.

In order to have a mixed equilibrium, utility of no participation must be equal to the utility of participation. This is the case when $-(1-p)x + \varepsilon + Y_1((1-p)\delta + \varepsilon) \left(q''px - \frac{1}{2}((1-q'')\delta + q''x) \right) = 0$.

Since in equilibrium the second order belief must be correct, that actual probability of no participation, q , must be such that the condition is fulfilled. Substituting for p this implies that

$$q = \frac{Y_2 \left(2\delta^2 Y_1 * 4x + \delta Y_1 x - x_2 Y_2 - \frac{x_2 Y_2}{\delta} \right)}{Y_1 (-8\delta + 2\delta_2 Y_2 + 3\delta x Y_1 + 2x_2 Y_2)}$$

The other conditions of Remark 5c are necessary to guarantee that q is larger than zero and smaller than 1.

On the other hand, we can also derive the solution for the sequential version of teachers' game with trade union. It is enough to assume $\varepsilon = 0$. Doing so, Remarks 1 and 3 remain unaffected. Remarks 4 and 5a do not hold anymore, as Remark 2a does not. Results are summarized in the following remarks.

Remark 1: If teacher 1 participates (chooses P), teacher 2 also participates in every SRE

Remark 2: If teacher 1 does not participate, the following holds in all SRE:

- a. If $Y_2 < \frac{2}{x}$, teacher 2 participates
- b. If $Y_2 < \frac{2}{x}$, teacher 2 does not participate with probability of $p = 1 - \frac{2}{Y_2 x}$

Remark 3: if $Y_2 < \frac{2}{x}$, participation is 1's unique equilibrium behaviour.

Remark 4: if $Y_2 > \frac{2}{x}$, 1's equilibrium behavior is characterized by: $Y_1 > \frac{x(-2 + xY_2)}{\delta^2}$ and teacher 1 participates.

Remark 5: if $Y_2 > \frac{4}{x}$, 1's equilibrium behavior is characterized by:

- a. $Y_1 < \frac{2xY_2}{\delta(xY_2 - 4)}$ and teacher 1 does not participate with probability $q = \frac{Y_2(\delta^2 Y_1 + 2x)}{Y_1(-4\delta + \delta^2 Y_2 + \delta x Y_2)}$