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Abstract

In a dynamic competition model where firms initially share half of the market and consumers have switching costs, consumers' sophistication, lifespan and concentration impact the possibility to set collusive prices. I first show that with strategic long-run consumers, collusion is harder to implement than when consumers are not strategic: with sophisticated consumers, a deviating firm can cash-in the rents that a buyer obtains after switching. I then study the consequences of relaxing buyers concentration and show that collusion is then easier to maintain than with non-strategic consumers: with strategic consumers a firm must offer a low price at the moment of deviation as consumers can benefit from increased competition, emerging from an asymmetric market structure, without having to pay switching costs. The paper suggests simple policy recommendations: it does not suffice to educate consumers about the competitive effects of their current purchasing decisions, but central purchasing agencies also need to be promoted.

Keywords: Switching cost, Price collusion; Strategic consumers.

JEL classification: D43; L13; L12

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1 Introduction

In many industries, price competition is affected by the existence of consumers' switching costs that they have to pay if decide to buy from a different supplier.[1] This paper studies how buyer characteristics influence the capacity of firms to tacitly collude so as to set the prices that maximize their joint profits. I find that the ability of a deviant firm to cash-in the rents a buyer obtains after switching is affected by the level of sophistication, lifespan, and concentration of consumers in the market which ultimately affects the possibility to sustain a collusive outcome.

A common interest in the literature of industrial organization has been to study under which circumstances companies, who interact in an infinite repeated game, can sustain a collusive outcome. With a small enough discount rate, [16] and [17] show that firms can tacitly collude in producing equilibrium quantities that maximize joint profits. In this literature, consumers play a passive role and purchase from the company charging the lowest price. The variation that I introduce in the model is that when buyers have switching cost, they choose the firm to make a purchase, taking into account not only the current price but also the prices they expect firms will charge in the future and the eventual cost of switching. With this consumers' "stickiness" to respond to prices, two opposing effects needs to be considered when studying companies' ability to collude tacitly. While, switching costs reduce the incentives to deviate from a collusive agreement, by limiting consumers reactions to price cuts, switching costs also lessen the severity of punishment as firms keep previous buyers captive. [24] concludes that the first effect always dominates, and tacit collusion is harder to sustain in industries when consumers have switching costs. I argue that the type of consumers in the market determines which of the two previous effect dominates.

I construct my analysis using a simple duopolistic competition model where two firms sell the same product to a group of consumers over an infinite number of periods. At the start of the game, a buyer is captive to either firm and endure a switching cost whenever she buys from the

[1] Switching costs are present in many sectors of the economy. For instance, in the IT industry, the use of proprietary formats and the resulting lack of compatibility across platforms creates switching costs. When changing providers, the learning process of a consumer using a new format or technology also originates switching costs.

competitor. Switching costs are invariant and do not depend on the company from which consumers purchase. Consistent with the literature, I assume an environment with short-term contracts, and firms cannot price discriminate.

As I will discuss in Section 4, when consumers are strategic their concentration does not affect the continuation payoffs after deviation. All buyers switch to the deviating firm, and the market structure is reminiscent of a situation with one incumbent and one entrant, who has to pay a cost of entry equal to the level of switching costs. Restricting to a focal equilibrium of the infinitely repeated game, I identify a strong stationary outcome in which a firm with positive market share sets the same price regardless of the history of the game. In equilibrium, no consumer switches and, the company attracting all buyers establishes a price $(1-\delta)S$. The present discounted profits in the punishment path are δS and depend on the interaction between the magnitude of switching costs and the discount factor. [2]

I show that consumers concentration determine firm's profits at the moment of deviation. When buyers are strategic, they are aware of the rents they obtain after switching, and when the market is populated by two buyers, a company can incorporate these rents in his deviating price. The ability to bring future consumers rents into the present, explains why when firms and consumers share the same discount factor, the likelihood to sustain a collusive outcome is neutral on the severity of the retaliation. In equilibrium, the deviating price decreases with switching costs but increase with the discount factor. This justifies why tacit collusion is only an equilibrium with both a small discount factor and large switching costs. Moreover, with strategic consumers, I also need to consider possible deviations by consumers. However, I obtain that consumers' deviations only limit firms' incentives constraint when deviating prices are made arbitrarily close to collusive prices.

These results do not hold when consumers are not concentrated and a continuum of them populates the market. In this case, buyers lack "strategic mass", and can always free-ride on the increased competition emerging from a more asymmetric market structure generated after deviation

[2]This result is reminiscent to the incumbency game presented in [7], where the authors show that firms can only collect the value of switching cost once.

without paying the switching costs. Then, a firm does not appropriate the rents of the switchers, and the deviating price is always smaller than when consumers are concentrated. The construction of the equilibrium is more involved than with two representative buyers. First, I fix consumers' beliefs about the actions taken by the rest of the buyers. This generates a belief system from which I obtain consumers' optimal purchasing decisions. Finally, I check whether the initial belief system is consistent with the equilibrium strategies. I find that the smaller deviating price together with the inability of consumers to individually break a collusive outcome generates a collusive for a wide range of model's parameters. Only when switching costs and the discount factor are sufficiently small, do firms break a collusive outcome.

The previous result is consistent with the discussion initiated by the Office of Fair Trading stating that with a few buyers in the market, the likelihood of successful collusion diminishes. Their argumentation comes from the hardship of co-ordination and difficulty to monitor. It can be difficult to infer that a rival has cut prices just because one buyer has changed supplier. The channel explored in my model is somehow related, as it depends on to the lack of individual consumers to affect the market outcome when they are atomless.

With regards to consumers welfare, the results depend on the equilibrium of the infinite game and buyers' identity. With two consumers, I show that for some values of switching costs, the switchers obtain the same utility either under collusion or when firms deviate from the collusive outcome. The deviating firm incorporates the present discounted utility of the switchers for a broad range of the parameters, and only the non-switchers receive a positive utility. Only when switching costs are sufficiently small, a consumer breaks the collusive outcome, and all buyers obtain a positive utility in equilibrium. When consumers are non-concentrated, all buyers receive a positive utility whenever collusion is not an equilibrium. However, because collusion is sustained for a much larger set of the parameters, buyers' welfare is in general larger in a concentrated market.[3]

To consider firms' strategies alone, Section 5 shuts down the possibility of buyers to behave

[3]Later in section 4.2.3, I show that only for the range of parameters where collusion is not an equilibrium with a continuum of consumers, buyers obtain a larger present discounted utility.

strategically. I study two model variations: non-sophisticated consumers and consumers who love one period and make one time purchase of the product. Non-sophisticated consumers do not anticipate how current purchasing decisions affect future market prices. Then a buyer purchases from the firm offering the lowest price discounted with the level of switching costs. In this setting, the retaliation equilibrium for the case of strategic consumers is still an equilibrium, but I show that firm needs to pay for the switching costs at the moment of deviation. As a result, collusion is not an equilibrium for a broad range of the parameters. With one period lived consumers; old buyers leave the market at the end of each period who are replaced by new ones. Again, buyers in the market purchase from the firm offering the lowest price discounted for the level of switching costs: as with non-sophisticated consumers, the deviating price decreases with the level of switching costs.

When consumers interact with companies only once, the dynamics of the game change dramatically. In all previous models, the infinite game considered had a structure of a stochastic game, in which depending on the actions of firms and consumers, the game changed state. Now I have an infinite repeated game where firms play the same game over and over again. In this case, I show that after deviation, the equilibrium of the continuation game is in mixed strategies. Because consumers have the same switching costs and firms have the same market share, the Nash equilibrium in the continuation game is mathematically equivalent to the price competition model presented in [26]. I adapt his methodology to recover the equilibrium in a duopolistic competition model. I show that the relationship between switching costs and the likelihood to sustain a collusive outcome is not monotone: with small and large switching costs, collusion is easier to sustain than with an intermediary level of switching costs. Compared with the case of strategic buyers, a collusive outcome is, in general, easier to support. Moreover, in equilibrium consumers may change firms many times generating inefficiencies that were not encountered with strategic buyers.

Given that some of the results depend on the assumption that both firms and consumers have the same discount factor. In section 6, I extend my analysis by considering the situation where firms and consumers discount the future differently. When consumers do not put any weight on future utility, the model is equivalent to the non-strategic case. When the discount factors for companies and buyers are positive but not the same, the severity of retaliation has an effect on the ability of firms to sustain a collusive outcome: now the profits at the punishment path and the rents from switchers that a firm appropriates after deviation are no longer interchangeable. Moreover, when consumers have a high valuation for the future, they have more incentives to break the collusive outcome. As a result, tacit collusion is harder to sustain when consumers have a high valuation for the future. Additionally, because consumers incentive constraint bind for sufficient large discount factor, the area where collusion is sustained as an equilibrium is not monotone with the level of switching costs.

These results have clear implications for public policy. First, only after analyzing the type of consumers in the market, antitrust authorities should have a clear stand on the likelihood of firms' coordinated behavior in industries where consumers have switching costs. As a result, a detailed analysis of demand composition is needed in industries where consumers have switching costs rather than a naive "checklist" approach. Moreover, taking alone the value of switching costs is not a good measure to draw implications on the firms' ability to collude. My analysis shows that the magnitude of switching costs may have very different results depending on the type of consumers in the market. For instance, with sophisticated consumers, collusion is never an equilibrium for any value of switching costs when players are sufficiently patient, but collusion is always an equilibrium when consumers are non-concentrated or case to be sophisticated. My paper then advocates to a careful rule of reason approach, which looks in depth at all the factors that can influence the sustainability of collusion in industries where consumers have switching costs. Second, to reduce the probability of collusion, it is not sufficient to inform and instruct buyers how their current purchasing decision affects future market competition in industries with switching costs, but also promote institutions incentivizing consumers to take coordinated purchasing decisions.

The next section briefly discusses the existing literature. All the proofs not included in the main text are in the Appendix.

2 Literature

The main contribution of this paper is to study how consumer's characteristics affect firms incentives to tacitly collude in prices in industries where consumers have switching costs. The price "stickiness" generated by switching costs affects the benefits a company obtains at the moment of deviation as well as the punishment path after deviation. I show that the benefits a firm receives at the time of deviation and in the following punishment path are directly affected by consumers live span, the degree of sophistication and concentration.

The first work discussing the effects that switching costs may have on coordinated practices is [19].This work advocates that the exploitation of locked-in consumers in markets with switching costs makes the noncooperative behavior look collusive. He also informally acknowledges that the significant reduction in prices needed to attract the buyer of the rival makes deviations from the collusive outcome easier to monitor. This increased transparency will make collusion easier to sustain.

Despite this early attempt to identify the effect of switching costs on collusion, the subsequent literature studying competition and switching costs, did not formalize such ideas and focused on analyzing the effect of switching costs on the steady state prices and firms' market shares, [[15], [5], [21], [14], [25]; [13] and [8]].[4] [5] The restriction of these models on strategies depending on the state, represented by firms' stocks of consumers, ruled out any kind of punishment strategies allowing for collusive equilibria. For instance, the formulation in [21] precludes firms using "punishment" strategies which might support non-Markov "collusive" equilibria in an infinite-horizon game. In [25], by restricting to Markovian strategies more collusive equilibria in which firms may earn higher profits by conditioning their prices on the entire history of the game is dismissed. The only theoretical considerations studying coordinated practices of these early models are [15] and

[4]In [8], switching costs "amplify" market competitiveness, making competitive markets more competitive and anti-competitive markets less competitive. [25] and [13] study the effect of switching costs in short and long term equilibrium prices, but do not consider the effects of collusion.

[5]Other dynamic models of switching costs include: [10]; [28]; [12]; [27] and [4]. Earlier models studying the effects that switching costs have on competition considered two-period models, [19], [20], [21],[23] and [9], in which the study of tacit collusion is not possible.

[19]. In [15] switching costs allows firms to divide the market: one firm specializes in its existing customers and the other serves new arrivals. They argue that switching costs and the asymmetric positions of the two firms facilitate tacit collusion. A similar argument is found in [19] who states that switching costs may facilitate collusion by breaking up a market into well-defined submarkets of groups of customers who bought from different firms, and so providing natural “focal-points” for a tacit collusive market division.

In spite of the lack of theoretical analysis, the Office of Fair Trade generated advice about the implications that switching costs can have on the feasibility of coordinated practices. Their experts argue that switching costs require significant price cuts to attract the consumers of the rival. The higher are switching costs, a price cut attracts fewer consumers, and the smaller are the incentive to reduce price. Switching costs may also undermine the severity of any retaliation for deviation from the collusive agreement, as they make punishment difficult and very costly for the punishers. It is harder to punish the deviator by reducing the price, as the “stolen” customers are locked-in. While the first effect makes collusion easier to sustain, the second force moves into the opposite direction. The overall effect is ambiguous and needs of a theoretical model to assess which one of the effects dominates. To this aim, [24] develops an infinite overlapping generation model and focusing in a symmetric stationary Markovian perfect equilibrium concludes that the second effect always dominates and switching costs unambiguously make tacit collusion more difficult to sustain.[6] In my model, I show that one effect may dominate over the other depending on consumers’ characteristics.

Departing from the analysis proposed by [24], I do not consider an overlapping generation model where old consumers leave the market and replaced by uncommitted young consumers. In an overlapping generation model the state changes are exogenous and limits the ability that consumers have to behave strategically. In my model, long-run consumers live infinitely, and they may have different degrees of sophistication. Moreover, by restricting attention to simple strategy profiles, I can analyze the case where consumers are forward-looking. [3] state that the solution proposed

[6] [24] is the first and almost unique theoretical model studying the degree of collusiveness in a market with consumers’ switching costs.

by [24] does not constitute an equilibrium with forward-looking consumers.[7] Finally, by introducing changes in consumer concentration and lifespan, my analysis contemplates how consumers characteristics affect the sustainability of collusive agreements.

In my analysis when consumers interact with firms infinitely, the punishment path emerging after deviation is reminiscent to the study presented by [7], but with one incumbent and one potential entrant.[8] The authors explore the consequences of heterogeneity in switching costs and show that an increase in the asymmetry of switching costs generates lower industry profits. In their paper, the asymmetric role of the players at the beginning of the game, one incumbent and many identical sellers, does not permit to identify the best collusive equilibrium. Such consideration is natural in my model in which the game starts with two incumbents, each serving half of the consumers.

Finally, my analysis with short-run consumers relates to the existing literature on mixed pricing in oligopoly as examined in [26]. With short-run consumers, after deviation from the collusive path, each firm shares half of the market, and the existence of switching costs makes the best response functions fail to be continuous. This gives rise to an equilibrium in mixed strategies. Because our problem is mathematically equivalent to [26], I have adapted his methodology to construct the equilibrium after a firm deviates from the collusive outcome. My result is an specific case in a market with two firms.

3 Model

I consider an infinite horizon game in which a consumer incurs a switching costs $S \in (0, 1)$ when changing supplier. Switching costs are industry compatibilities between products, rather than idiosyncrasies of particular sellers. At the beginning of the game, two identical firms $i = A, B$ share half of the market and produce a non-durable homogeneous good at zero cost. I do not study the process by which each company obtain half of the market, but only the continuation game following

[7] [24] also considers how switching costs affect tacit collusion when firms punishment strategies can only revert to the MPE.

[8] [6], do not restrict attention to a strong stationary outcome and show multiplicity of equilibria which constitutes an application of the Folk theorem in markets with switching costs.

a symmetric market share.

Each consumer has perfectly inelastic demand for one unit of the good, with reservation utility normalized to 1. Only in the beginning of the analysis, I consider the case of two strategic consumers $j = 1, 2$, who live for an infinite number of periods. I will later extend the analysis by considering a continuum of consumers. To focus on firms' strategies, I will also study the case when buyers case to be strategic.

Each period $t = 1, ..., +\infty$, is composed by two stages. In stage 1, both firms simultaneously announce a price. I assume that only short-term contracts can be used, and price discrimination based on purchasing histories is not possible. In stage 2, and when consumers are strategic, they choose the firm they buy, taking into account not only the current price but also the prices that they expect firms to charge in the future and the level of switching costs. If a consumer is indifferent between switching nor not, he decides not to switch. Both companies and consumers have the same discount factor, and I focus attention on the opportunities for price collusion.[9]

4 Strategic consumers

In this section, I show that when consumers are concentrated, that is, the market populated by two representative buyers, a firm deviating from the collusive outcome incorporates the present discounted rent of the switches in his deviating price. To trigger a punishment path, it is necessary that one consumer switches. With a continuum consumers, an individual consumer ceases to have strategic mass. A buyer can then free ride on the increased competition emerging from an asymmetric market without paying switching costs. The deviant firm then has to pay for the switching cost to induce consumers to switch. Consumers are hurt by not being able to take coordinated purchasing decisions as a collusive outcome is easier to materialize.

I first characterize the punishment path of the game following a deviation by a firm or a consumer. I show that the continuation game following a deviation is the same regardless of the number

[9]Later in section 6, I study the case when firms and consumers have different discount factors.

of buyers in the market, and a company gets the same present discounted profit. Later, I show that the equilibrium deviating prices depends on market concentration. I then establish under which parameters tacit collusion is an equilibrium of the infinite game. I begin with two representative consumers and the turn to a continuum of consumers.

4.1 Punishment Path

I proceed to study the punishment path following a deviation from collusion by either a firm or a consumer. To this aim, I restrict attention to stationary strategies where the pair of prices p^I and p^0 characterizes the punishment path, such that: along the equilibrium path and in any subgame a firm having sold to a positive mass of consumers in the previous period always sets a price p^I and a firm with no market share in the previous period sets a price of p^0. This stationary equilibrium establishes a price p^I for any firm having a positive market given a zero market share for the rival. In the section that follows, I show that the equilibrium market structure emerging after deviation is one where a firm sells to all consumers, and the other firm never sells to any consumer.

To obtain the equilibrium prices of this stationary equilibrium, consider a consumer j who always buys from the same firm which has a positive market share. Due to stationarity, regardless of the history of the game, a company with positive market share sets a price p^I. This strategy, gives the consumer a present discounted utility of $U_j^I = (1-p^I)/(1-\delta)$. If this consumer switches and buys from the firm with zero market share, he pays a price of p^0 plus the switching cost S. Because after purchase, this firm has an active mass of consumers she sets price p^I.[10] This strategy gives to consumer j a present discounted utility of $U_j^0 = 1-p^0-S+(\delta/(1-\delta))(1-p^I)$. Then, no consumer j buys from the firm with no market share if

$$U_j^I = \frac{1-p^I}{1-\delta} \geq (1-p^0-S) + \delta\left(\frac{1-p^I}{1-\delta}\right) = U_j^0.$$

[10] In equilibrium, if one consumer switches all will. This again gives a market structure where one firm has the whole market and the other is left with no demand.

Therefore, by setting a price

$$p^I = p^0 - S, \tag{4.1}$$

a firm with a positive market share never loses a consumer to a firm with no market share. Any firm i not selling to any consumer makes zero profits. Then, the lowest price that this company is willing to charge in order to attract the consumer of the rival is equal to

$$\Pi_i^0 = p^0 + \left(\frac{\delta}{1-\delta}\right) p^I = 0 \;\;\rightarrow\;\; p^I = -\left(\frac{1-\delta}{\delta}\right) p^0. \tag{4.2}$$

The first part represents the profit by setting a price p^0 and consumers switch. After switching, the firm has a positive market share and sets a price of p^I. The lower bound on p^0 in (4.2) together with expression (4.1), gives the pair of stationary equilibrium prices $\{p^I, p^0\} = \{(1-\delta)S, -\delta S\}$. Observe that the difference of both prices equals to the level of switching costs, and consumers never switch in equilibrium.[11]

This result constitutes a subgame perfect equilibrium if there is no sub-game when any of the players wants to deviate. Clearly, no firm deviates from this equilibrium. By setting a price above p^I, the firm loses his consumers and does not make any sell. Setting a price below p^I, the company always keeps his buyers captive, but there exist a profitable deviation by increasing the price. Also, there is no profitable deviation by a consumer, when both firms have a positive market share, they charge at least a price of p^I, and the consumer incurs a cost of switching. This result does not depend on the market being populated by two representative consumers or a continuum of them, neither by the degree of consumers' sophistication.

[11] This result is the same as in [7] where they model a game with one incumbent and an infinite number of entrants. [6] show multiplicity of equilibria by restricting the notion of strong stationarity and considering a weak version of stationarity.

4.2 Equilibrium

In this section, I proceed to study under which conditions a collusive outcome can be sustained as an equilibrium of the infinite game. To this end, I consider a simple strategy profile as a rule specifying an initial collusive path and a punishments from any deviation of the initial path.[12] In my model, at the beginning of the game, both firms start in a collusive path, $\mathbf{p}(c)$, and remain there as long as no deviation by neither a company or consumer occurs. Because consumers are sophisticated, they anticipate how current purchasing decisions affect future equilibrium prices. Therefore, I need to consider not only unilateral deviations by firms but also deviation by consumers. Finally, my simple strategy profile prescribes that, after a deviation, the game switches to the punishment path presented in section 4.1 for the rest of the game.[13]

4.2.1 Collusion with two representative consumers

I will show that when the are two representative consumers in the market, the company deviating from the collusive outcome charges a price that incorporates the present discounted utility of switchers. Because firms obtain large profits at the moment of deviation a collusive outcome is difficult to sustain. I also discuss the equilibrium consumers' welfare. In general, non switchers obtain larger rents than switchers.

I define a collusive outcome as usual: each firm sets monopoly price to each of its consumers. Because consumers' demand is inelastic, monopoly price equal to the consumers' reservation price $\mathbf{p}(c) = 1$. Then, a firm who deviates from this collusive path and attracts the consumer of the rival obtains a profit of $\pi_i(p_i^d, c) + \delta\Pi_i(\mathbf{p}(p^I))$. The first part stands for the profits obtained at the moment of deviation; the second part are the profits originated in the punishment path. Hence, no

[12] A path is an infinite stream of one-period action profiles.

[13] [2] shows that there is no loss of generality in restricting to simple strategy profiles. Simple strategies are history-independent in the sense that they specify the same punishment for any deviation of a given player. Contrary to [1], we do not consider optimal penal codes and restricted to the strategies to be stationary. An analysis with optimal penal codes is left for future research.

firm i has incentives to deviate from collusion if

$$\Pi_i(\mathbf{p}(c)) \geq \pi_i(p_i^d, c) + \delta \Pi_i(\mathbf{p}(p^I)). \qquad (IC_f)^{14} \tag{4.3}$$

Following a firm's deviation, a consumer switches to the deviant firm if

$$U_j\left(\mathbf{p}(c)\right) \leq 1 - p_i^d - S + \delta U_j\left(\mathbf{p}(p^I)\right). \qquad (IC_s) \tag{4.4}$$

The left-hand side represents the present discounted utility a consumer obtains in the collusive path. The right-hand side is the utility of a buyer switching to the deviant firm. At the moment of deviation, the consumer pays the price p^d and the switching cost. In the continuation game, because one firm has the whole mass of consumers, a consumer obtains a present discounted utility of $\delta U_j\left(\mathbf{p}(p^I)\right)$. If the consumer does not switch, he expects the firm to charge the monopoly price. Because the worst that can happen to a company is to lose his consumer; if no consumer switches, the best response is to keep on setting the monopoly price, and it is optimal to do so until the consumer switches to the rival. Indeed, it is the switch of the consumer that triggers the punishment path characterized in 4.1. The optimal deviation price is such that the incentive constraint of the consumer is binding, which gives

$$\begin{aligned} p_i^d &= 1 - S + \delta U_j(\mathbf{p}(p^I)) \\ &= \min\left\{\left(1, \frac{(1-\delta)(1-S) + \delta(1-(1-\delta)S)}{1-\delta}\right]\right\}. \end{aligned} \tag{4.5}$$

This price indicates that a firm can cash-in the present discounted rents of the switchers at the moment of deviation. The expression also implies that to attract rival's buyers, a firm does not need to pay for the consumers' switching costs. Indeed, if the level of switching costs is $S \leq \delta/(1-\delta^2)$, a deviating price can be made arbitrarily close to the collusive price.[15] My result contradicts [22]

[14] With the notion of non-cooperative equilibrium, I only consider unilateral deviations by firms or consumers.

[15] It is never an optimal strategy to set a deviating price above consumers' reservation price. Because price discrimination is not possible, setting a deviating price above monopoly generates zero sells to captive consumers, and such

and [18], because a sufficient price cut may not be needed to attack the consumer of the rival.[16]

Introducing the deviation price in (4.5) into firms' incentive constraint, I obtain a necessary condition to implement a collusive outcome. For $S > \delta/(1-\delta^2)$, a necesary condition for collusion is

$$\begin{aligned}
\Pi_i(\mathbf{p}(c)) = \frac{1}{1-\delta} &\geq 2\left(\frac{(1-\delta)(1-S)+\delta(1-(1-\delta)S)}{1-\delta} + \delta S\right) = \pi_i(p_i^d, c) + \delta\Pi_i(\mathbf{p}(p^I)) \\
\iff \frac{1}{1-\delta} &\geq 2\left(\frac{(1-\delta)(1-S)+\delta}{1-\delta}\right) \\
\iff \delta &\leq 1 - \frac{1}{2S}.
\end{aligned}$$

With a level of switching costs $S \leq \delta/(1-\delta)$, a necessary condition for collusion is

$$\begin{aligned}
\Pi_i(\mathbf{p}(c)) = \frac{1}{1-\delta} &\geq 2\left(1+\delta S\right) = \pi_i(p_i^d, c) + \delta\Pi_i(\mathbf{p}(p^I)) \\
\iff \delta &\geq \frac{1}{1+S+\sqrt{1+S^2}}.
\end{aligned}$$

When consumers are strategic, I also need to verify that no buyer decides to break a collusive outcome. With two representative consumers, if one buyer switches to the rival, a company is left with no demand. The continuation game that emerges is the one characterized in section 4.1. Then, a consumer j has incentives to break a collusive outcome if

$$U_j\left(\mathbf{p}(c)\right) \leq 1 - p(c) - S + \delta \times U_j\left(\mathbf{p}(p^I)\right). \qquad (IC_c) \tag{4.6}$$

The left-hand side, stand for the discounted utility of a consumer in the collusive path. In the right-hand side, a consumer pays the collusive price $p(c)$ and the switching costs. The following continuation game gives a present discounted utility of $U_j\left(\mathbf{p}(p^I)\right)$. Then, a consumer breaks

demand reduction is never optimal. Moreover, a price above monopoly could also generate a focal point to consumers to switch to the firm who did not deviate from the collusive outcome.

[16] [22] also claims that a sufficient price cut may facilitate collusion as the profits obtained at the moment of deviation are low.

collusion if

$$0 \leq -S + \frac{\delta\left[1-(1-\delta)S\right]}{1-\delta} \iff \delta \geq \frac{-1+\sqrt{1+4S^2}}{2S}.$$

Analyzing all previous conditions, I see that the incidence of consumers on the sustainability of a collusive outcome depends on the level of switching costs. For $S > \delta/(1-\delta^2)$, a buyer breaks a collusive outcome when companies have the incentives to maintain it. Then, the necessary condition represented by firms' incentive constraint is also sufficient to sustain a collusive outcome. For values of switching costs $S \leq \delta/(1-\delta^2)$, a consumer always have incentives to break a collusive outcome, and the necessary condition to sustain collusion imposed by firms is never fulfilled. With small switching costs, a consumer is willing to pay this cost to generate a more competitive equilibrium in the continuation game. While with large switching costs, it never pays enough to consumers to break a collusive outcome.

The next Proposition summarizes the results.

Proposition 1. *With two sophisticated consumers, collusion is an equilibrium of the infinite game depending on the level of switching costs.*

i) For $S \leq \delta/(1-\delta^2)$, collusion is never sustained in equilibrium. A consumers always switches to break collusion.

ii) Otherwise, collusion is an equilibrium for a discount factor $\delta \leq 1-1/2S$.

Figure 1 displays the set of parameters when collusion is an equilibrium. Firms cooperation happens for large values of switching costs together with a small discount factor. When switching costs are low, I have already argued that buyers are willing to pay this cost to break the collusive outcome. It is only with significant switching costs that buyers do not have incentives to break a collusive outcome. Moreover, with substantial switching costs, the benefits a company obtains in the punishment path are considerable, reducing the retaliation after deviation. However, as the deviating price incorporates the rent of the switchers, larger switching costs also reduces the gains at the moment of deviation. Then, substantial switching costs make deviating from the collusive

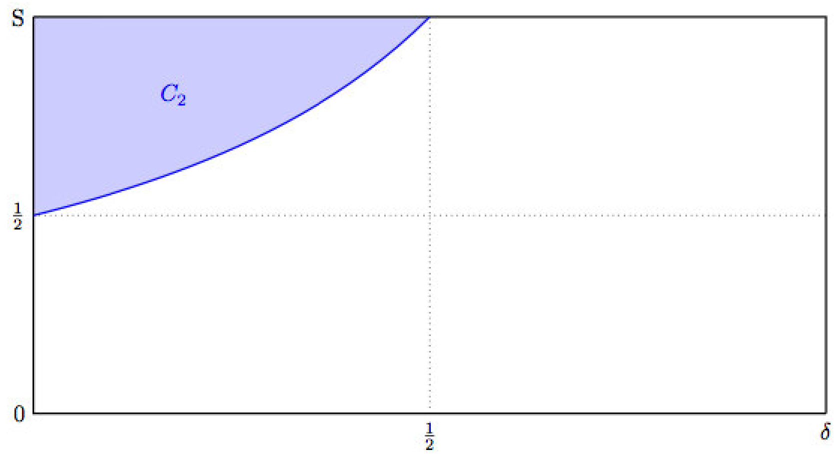

Figure 1: Area of collusion with two sophisticated consumers.

outcome less favorable. Such effect is reinforced with a small discount factor. The value that firms associate with the future is limited, and firms do not benefit from lower retaliation.

With the characterization of the equilibrium. Two aspects need further attention. First, the feasibility of a collusive outcome does not depend on the retaliation after deviation. Second, in equilibrium, switchers get lower utility than not switchers and even in regions where collusion is not an equilibrium, they obtain a zero discounted utility. Later, in section 6, I show that both results depend on the fact that both consumers and firms share the same discount factor.

For $S > \delta/(1-\delta^2)$, the deviant firm incorporates the rents a consumer obtains in the continuation game at the moment of deviation, $p^d = 1 - S + \delta U(\mathbf{p}(p^I))$. Therefore any price variation in the retaliation phase translates into a direct effect on the deviating equilibrium price. This waterbed effect is the result of both firms and consumers having the same discount factor. What a company ceases to gain tomorrow, due to a harsh retaliation, earns it today from a larger deviating price. This effect is not present in the in the infinite overlapping generation model presented in [24]. He shows that because switching costs reduces the severity of punishment, collusion is harder to sustain the larger the level of switching costs.[17] In my model, this neutral effect also stops to exist for a value of switching costs $S \leq \delta/(1-\delta^2)$. This is because it is never optimal to set a deviating price

[17]Because in this section of the model, consumers interact infinitely with firms, and not for only two periods, a firm is able to extract the rents that a consumer obtains after switching.

above the collusive level.

Consumer welfare also depends on the level of switching costs. For a substantial level of switching costs $S > \delta/(1-\delta^2)$, switchers never obtain a positive utility in equilibrium. It is obvious that no consumer collects rents when collusion is an equilibrium. However, because the deviating price incorporates the present discounted utility of switchers, switchers do not get any rent either in an equilibrium where firms break the collusive agreement. Only those consumers who do not switch obtain a present discounted utility equal to the level of switching costs. Therefore, non switchers achieve more rents the larger is the magnitude of switching costs. Only when consumers break a collusive outcome, i.e., $S \leq \delta/(1-\delta^2)$, all consumers obtain rents in equilibrium. Yet, the utility difference between switchers and those who do not switch is still equal to the level of switching costs. I summarize this result in the next Corollary.

Corollary 1. *When collusion is not sustained in equilibrium, the difference in the present discounted utility between switchers and non-switchers is equal to the level of switching costs. Moreover, when* $S > \delta/(1-\delta^2)$ *switchers obtain zero rents in equilibrium.*

4.2.2 Collusion with a continuum of consumers

With a continuum of consumers, individual consumers lack strategic mass. I show that, compared to the previous section, a consumer will not be able to break a collusive outcome. Analytically, this means that consumer incentive constraint will not be considered. Additionally, deviation prices will be lower in equilibrium: now a consumer can free ride on the switchers and benefit from an increased competition after deviation without paying the switching costs. In equilibrium, the deviating firm will not only pay for the switching costs, but will also need to subsidize consumers with an amount equal to the present discounted profits a firm obtains in the continuation game. Both effects makes collusion an equilibrium of the repeated game for a larger set of parameters.

I first study the incentives of an individual buyer to switch to the rival. Because a consumer does not have strategic mass, and cannot affect firms' equilibrium outcome if he switches to the competitor, he incurs to the switching costs without gaining anything, i.e., $1 - p(c) - S + \delta U_j\left(\mathbf{p}(p^0)\right) < 0$. This proves that I can drop the consumers' incentive constraint to break collusion from the analysis.

With the same argument, I can establish under which conditions a consumer switches to a firm deviating from the collusive outcome. To this aim, I first need to assume the belief that a buyer has with respect to the decision of the rest of consumers. Then, μ_j denotes the belief that any consumer j has about the mass of consumers switching to the deviating firm. This belief system, allows me to obtain consumers' optimal strategy. I then check that such strategy is consistent with the belief system previously assumed.

I construct an equilibrium in which any buyer j holds the belief $\mu_j = 1$, that is, after deviation, he expects all consumers are switching to the deviator. Therefore, by switching to the deviant, buyer j enjoys a present discounted utility of $\delta U_j\left(\mathbf{p}(p^I)\right)$. If buyer j does not switch, he gets $\delta U_j\left(\mathbf{p}(p^0)\right)$. Then, any buyer j switches to the deviant firm if

$$1 - p(c) + \delta U_j\left(\mathbf{p}(p^0)\right) \leq 1 - p_i^d - S + \delta U_j\left(\mathbf{p}(p^I)\right). \qquad (IC_s)$$

Because in the optimal deviating price, consumers' incentive constraint is binding, I get

$$\begin{aligned} p_i^d &= 1 - S + \delta\left[U_j\left(\mathbf{p}(p^I)\right) - \left(\mathbf{p}(p^0)\right)\right] \\ &= 1 - (1+\delta)S. \end{aligned} \qquad (4.7)$$

This expression indicates that a firm does not only need to pay the switching costs of consumers to make them switch, but also needs to subside them by the total present discounted profit a firm obtains after deviation. Moreover, for a large enough level of switching costs, i.e., $S > 1/(1+\delta)$, deviating prices are negative. Buyers' lack of strategic mass allows them to free ride on the switching decisions of the rest of consumers without having to pay switching costs. As a result, with a continuum of consumers, switching costs makes consumers very expensive to capture.

As in the case with two buyers, the transfer between future profits to current deviating prices makes the severity of the punishment after deviation irrelevant on the firms' ability to coordinate in prices.[18] Finally, the equilibrium that I have constructed is sequentially rational because in equilibrium all consumers switch to the deviant, and the belief $\mu_j = 1$ held for each one of the buyers is consistent.

Introducing the deviation price into the firms' incentive constraint, collusion is an equilibrium if

$$\begin{aligned} \Pi_i(\mathbf{p}(c)) = \frac{1}{1-\delta} &\geq 2\left(1-(1+\delta)S+\delta S\right) = \pi_i(p_i^d, c) + \delta\Pi_i(\mathbf{p}(p^I)) \\ \iff \frac{1}{1-\delta} &\geq 2(1-S) \iff \delta \geq \frac{1-2S}{2(1-S)}. \end{aligned} \tag{4.8}$$

The next Proposition states the results.

Proposition 2. *With a continuum of consumers, collusion is an equilibrium for a discount factor* $\delta \geq (1-2S)/(2(1-S))$.

Figure 2 illustrates the area where a collusive outcome is sustained in equilibrium. Firms

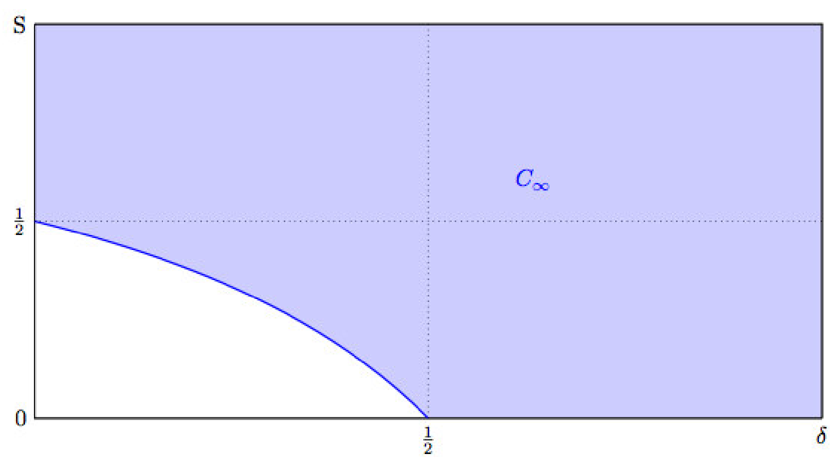

Figure 2: Area of collusion with a continuum of consumers.

only break a collusive outcome when both the level of switching costs and the discount factor are sufficiently small. With large and intermediate switching cost, it is expensive to capture the mass

[18] In section 6, by allowing discount factors to differ between companies and buyers, I show that such neutrality disappears.

of consumers from the rival: the deviant firm needs not only to pay consumers for their switching costs, but also needs to subsidize them by the future profits obtained in the punishment path. Both factors makes collusion easier to sustain.

With respect to the case with two buyers, collusion is always easier to sustain with a continuum of consumers. The next Corollary states the result.

Corollary 2. *A collusive outcome is always easier to sustain with a continuum of consumers than when the market is populated by two buyers.*

4.2.3 Welfare analysis

The different equilibria previously exposed, allows for an analysis of consumers' welfare depending on the buyers' level of concentration. In situations where the collusive outcome is an equilibrium, consumers do not obtain any present discounted utility as the equilibrium price equals to their reservation value. With two consumers, this is illustrated in the north-west region of Figure 3. Figure 3 also depicts two extra regions. The right side of the figure delimited by $S < \delta/(1-\delta^2)$,

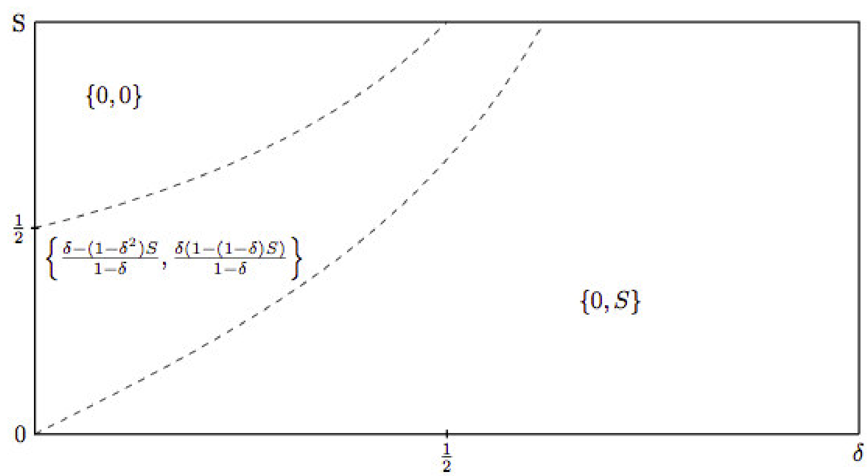

Figure 3: Welfare with two consumers. The first element represents the utility of switchers and the second element the utility of the non-switchers.

illustrates the situation in which collusion is not an equilibrium, but where the deviation prices incorporate the present discounted rent of the switchers. In this case, only the buyer who does not switch obtains a positive utility equal to the value of switching costs. In the middle of the figure, where $S \in \left[1/2(1-\delta), \delta/(1-\delta^2)\right]$, both consumers obtain a positive utility, as they break a collusive outcome. Again the difference of discounted utility between switchers and non-switchers is equal to the value of switching costs.

Figure 4 depicts the situation with a continuum of consumers. The region where collusion is an

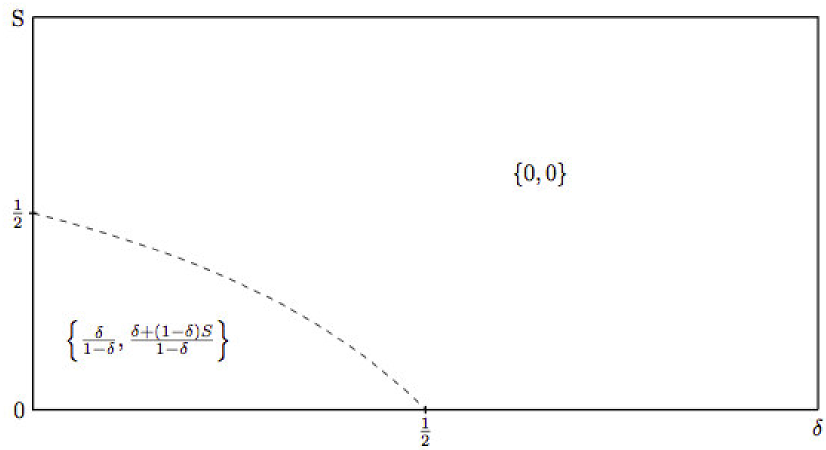

Figure 4: Welfare with a continuum of consumers. The first element represents the utility of switchers and the second element the utility of the non-switchers.

equilibrium, no consumer obtains a positive utility. Only when both the level of switching costs and the discount factor are sufficiently small, firms deviate from the collusive outcome, and consumers obtain a positive utility in equilibrium.

Finally, Figure 5 compares all different regions. In region A, with either concentrated or dispersed consumers, collusion is an equilibrium and no consumer realizes a positive utility. In region B, buyers of a concentrated market get a positive utility, while this is a collusive area with a continuum of consumers. In region C only the no switchers obtain a positive utility with concentrated consumers, the switchers earn zero utility in both market considerations. It is only in region D, where dispersed buyers get a positive utility in equilibrium. In this case, consumers always get larger utility than in a market with concentrated consumers.

This analysis suggests that in order to study how the equilibrium affects consumer welfare in

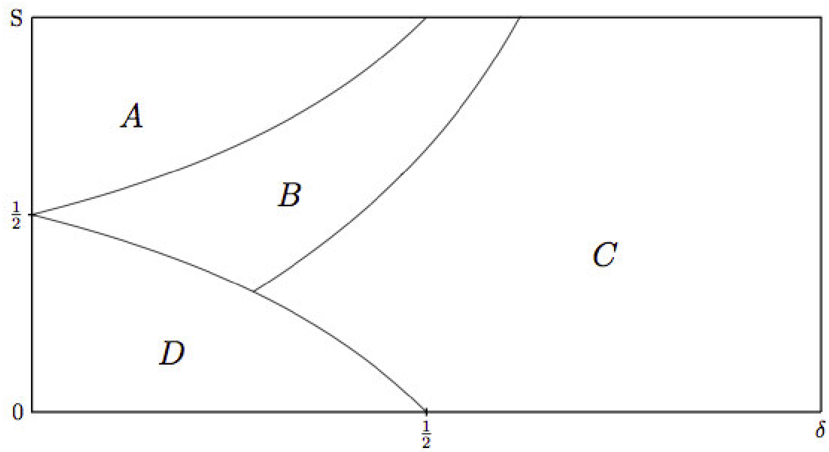

Figure 5: Welfare comparison between concentrated and dispersed consumers.

industries with switching costs, it does not suffice to analyze the regions where collusion can be sustained as an equilibrium of the game. It is also convenient to study the dynamics of competition: in addition to observing the level of competition emerging in the market after a deviation, it is adequate to pay attention to the level potential price deviations.

5 Non-strategic consumers

In this section, to put emphasis solely on firms' strategies, I eliminate the ability of consumers to be strategic. Hence, in section 5.1, I assume that buyers are bounded rational in the sense that they cannot anticipate how their current purchasing decisions affect future equilibrium prices. Later, in section 5.2, I consider that buyers interact with companies only once. The infinite game in these two versions will be very different. With infinitely lived consumers, the infinite game is still a stochastic game in which equilibrium strategies depend past buyers' purchasing decisions. Conversely, when buyers live only for one period, I have an infinite repeated game. The equilibrium in the punishment path will be very different between both games.

5.1 Non-sophisticated consumers

In this section, I show that when consumers are non-sophisticated collusion is sustained by a larger set of parameters. Non-sophisticated consumers do not consider the future effects of their purchasing decisions and buy the product from the firm offering the lowest price once discounted for switching costs. Because the difference in prices in the punishment path is equal to the level of switching costs, the equilibrium presented in section 4.1, is still an equilibrium when consumers do not behave strategically. However, different from the previous analysis, now the firm deviating from the collusive path will have to pay the full switching costs to attract the consumers of the rival. This is consistent with [18], stating that: because firms have to attract the consumers of the rival, they will have to pay the switching costs at the moment of deviation. The analysis that follows is valid regardless of consumers' level of concentration in the market.

Non-strategic consumers will never consider the possibility of breaking a collusive outcome. Analytically, this means that consumers' incentive constraint is dropped from the analysis. Therefore, To sustain a collusive outcome, I only need to verify that firms do not have an incentive to deviate from collusion. This happens when

$$\Pi_i(\mathbf{p}(c)) = \frac{1}{1-\delta} \geq 2\left(p^d + \delta S\right) = \pi_i(p_i^d, c) + \delta \Pi_i(\mathbf{p}(p^I)). \qquad (IC_f)$$

It is easy to prove that a firm deviating from collusion charges a deviating price to attract the consumer of the rival. In the Appendix, I show that the punishment path emerging from a situation when the deviant does not attract the consumer of the rival is in mixed strategies, and such strategy always generate smaller expected payoffs than attracting the buyer of the rival.[19] The maximum deviation price is then arbitrarily close to $p_i^d = 1 - S$. Introducing this expression into firms' incentive constraint, collusion is an equilibrium of the game for a discount factor $\delta \leq (2S - 1 + \sqrt{1-2S})/2S$.

Figure 6 depicts the sets of parameters for a collusive outcome with non-sophisticated consumers.

[19] In the appendix, I show that for low switching costs, any firm obtains strictly higher profits by attracting the consumer of the rival. For high switching costs, the collusive outcome gives higher profits than deviation.

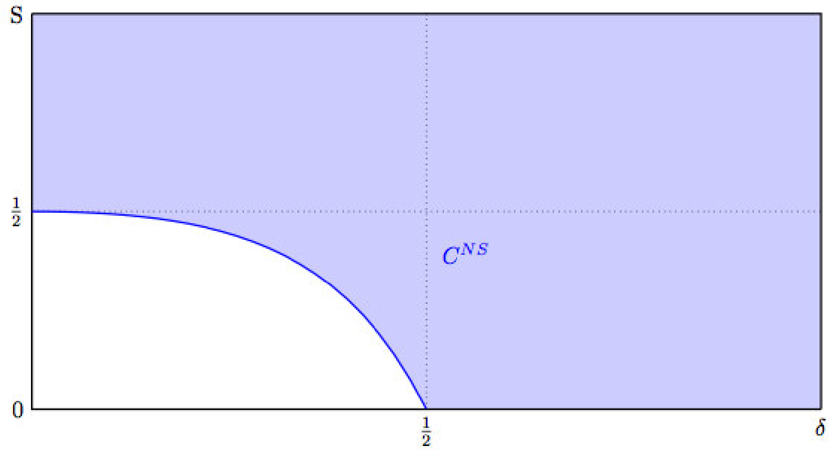

Figure 6: Area of collusion with non-sophisticated consumers.

The figure illustrates that the only region where cooperation in prices is not sustained equilibrium is when both the level of switching costs and the discount factor are small. With a small switching cost, the deviating price increases but the retaliation after deviation is also sever. With a small discount factor, the profits at the moment of deviation are more important than profits at the continuation game, and the fist effect always dominates. As a result deviating from collusion becomes profitable.

Comparing this region of collusion with the case of strategic consumers, observe that the punishment path coincide in both settings. Hence, differences in the ability to collude need to come from the possibility of consumers to break a collusive outcome and the equilibrium profits at the moment of deviation. With two representative strategic buyers, the collusive area is a proper subset of the case with non-strategic consumers. This is because, for $S \leq \delta/(1-\delta^2)$, strategic consumers always break a collusive outcome. For $S > \delta/(1-\delta^2)$, a firm breaking collusion sets a deviating price that incorporates the present discounted utility from switchers. When consumers are non-strategic, the deviating firm always pays the switching costs.

However, with a continuum of consumers, now the collusive region with non-strategic consumers is a proper subset of the area for strategic consumers. In both cases, consumers do not have the ability to break a collusive outcome. The result is then fully driven by the deviating equilibrium price. With strategic consumers, a firm not only have to pay for the switching costs but also needs to offer a consumer the rents that he obtains by free riding on the switching decision of others. This generates a lower equilibrium deviating price and smaller profits at the moment of deviation.

The next Corollary summarizes the discussion.

Corollary 3. *When consumers are concentrated, collusion is harder with strategic than with non-strategic consumers. The contrary happens in a market with a continuum of consumers.*

5.2 Short-run consumers

I change the game presented in section 3 in the following way. At each period, both firms have captive a mass one of consumers who live for only one period. This is the case of an infinite overlapping generation model in which all the members who will belong to a family have learned how to use one of the systems available, and each member will need to pay switching cost when buying the system from the other firm.

I will show that when consumers live for one period only, the punishment emerging in equilibrium is very different from the previous sections. Deviations from the collusive path entail a continuation game where both firms share half of the market. The equilibrium in the continuation game is in mixed strategies and switching happens with positive probability. In equilibrium, the expected payoff of each firm is above the level of switching costs.

Following the structure as in previous sections, I first characterize the punishment path emerging after deviation. Later, I study under which parameters of the model collusion constitutes an equilibrium.

5.2.1 Punishment path

When consumers interact with firms only once, at each period, a firm has captive half of the consumers who pay switching costs when they decide to purchase from the rival. As a result, firms' interaction is an infinite repetition of the same stage game. Then, in each period, a firm faces the tradeoff between charging a large price to extract the rent of captive consumers, with the risk of losing them to the rival, and charging a low price to attract the buyers of the rival. This trade-off generates a discontinuity on firms' profit function and a Nash solution in pure strategies fails to exist. In what follows, I adapt the definition of stationarity in the sense that where no pure strategy equilibrium exists, I look for equilibria in which all firms use the same mixed strategy.

When a company uses a mixed strategy represented by the cumulative distribution $F(p)$, the expected profit of charging any price p, and the rival playing also according to $F(p)$, is:

$$V = p \times [F(min(p+S,1)) - F(max(0,p-S))] + 2p \times [1 - F(min(p+S,1))]\,. \tag{5.1}$$

The first part of the expression represents a situation when the price difference between both companies is below the level of switching costs. In this case, no consumer switches and each firm serves to their mass of captive consumers. In the second part, the price difference is above switching costs and the company setting the lowest price serves the whole market. Arranging terms gives

$$\frac{V}{p} = 2 - [F(min(p+S,1)) + F(max(0,p-S))]. \tag{5.2}$$

This problem is mathematically equivalent to the price competition model presented in [26], and I use his methodology to obtain the equilibrium distribution function $F(p)$. [26] proves the existence of a unique mixed strategy equilibrium where, by an intermediate level of switching costs, the support of prices is equal to twice the level of switching costs. In the Appendix, I characterize the equilibrium distribution function for two firms.[20] The next Lemma states the per-period expected profit that firms obtain in equilibrium.

Lemma 1. *In the unique Nash equilibrium, the per-period expected profit of each firm depends on the level of switching costs:*

[20] In a companion research note, with an intermediate level of switching cost $S \in \left(\sqrt{2}/(1+\sqrt{2}), 1/2\right)$, I obtain a multiplicity of equilibria without assuming any specific tie-breaking rule. I show that a company obtains lower expected payoffs when consumers switch whenever they are indifferent. When consumers are more prompt to switch, firms behave more aggressively leading to lower expected payoffs. In this equilibrium, firms play with a positive probability the lower bound of the support. Conversely, when consumers decide not to switch when they are indifferent, firms play with positive probability the upper bound of the support. This note is available upon request to the author.

i) For large switching costs $S \geq 1/2$, each firm obtain his monopoly profit $V = 1$. Each firm serves to its captive consumers and no consumer switches.

ii) For an intermediate level of switching costs, each firm earns expected profits

$$V = \begin{cases} \left(1+\sqrt{2}\right) S & \textit{for } S \in \left[0, \frac{1}{2+\sqrt{2}}\right], \\ \frac{S+\sqrt{S(4+S)}}{2} & \textit{for } S \in \left(\frac{1}{2+\sqrt{2}}, \frac{1}{2}\right). \end{cases}$$

and consumers switch with positive probability.

This result shows that in equilibrium, with an intermediate level of switching costs, firms' expected profits increase with the level of switching costs. Because after deviation, firms reverse to the Nash equilibrium, then, the severity of retaliation decreases with switching costs. This result is common to all settings previously considered. The difference here is that buyers switch with positive probability; the equilibrium price dispersion allows for a price difference above the level of switching cost. Therefore, switching costs creates a source of inefficiency that was absent when consumers interacted with firms infinitely.[21]

With the expected profits of the punishment path, I proceed to study under which conditions collusion is an equilibrium of the infinite repeated game.

5.2.2 Equilibrium

In this section, I study the equilibrium of the infinite game. Similar to the previous section, there is no need to verify deviations by consumers. When consumers interact only once, they case to be strategic and buy the product from the company offering the lowest price net of switching costs. Therefore, a collusive agreement emerges when no firm wants to unilaterally deviate from collusion.

[21] Remember that with infinitely lived buyers, and because the deviant firm serves the whole market, I find a stationary equilibrium that depends on the limit price charged by the company with no market share.

Firms' incentive constraint is

$$\Pi_i(\mathbf{p}(c)) \geq \pi_i(p_i^d, c) + \delta\Pi_i(F(p)). \qquad (IC_f) \tag{5.3}$$

To attract the consumers from the rival, a company sets a deviation price arbitrarily close to $p^d = 1 - S$. This price, together with the expected payoff in the punishment path in Lemma 1, gives a firms' incentive constraint

$$\frac{1}{1-\delta} \geq 2 \times (1-S) + \left(\frac{\delta}{1-\delta}\right) V. \qquad (IC_f)$$

Because the expected payoff V depends on the level of switching cost, I analyze firm's incentive constraint taking different values of switching costs.

$S = 0$. This is the standard case without switching costs. Retaliation after deviation is the most severe and firms obtain zero profits in the punishment path. Because a company attracts the consumer of the rival by setting a price arbitrarily close to the collusive level, collusion is an equilibrium for a discount factor $\delta \geq 1/2$.

$S \in (0, 1/2)$. The severity of retaliation decreases with switching cost; the expected Nash equilibrium payoff increases with switching costs. However, firms pay the switching cost at the moment of deviation to attack the consumer of the rival. Because the expected payoff in the punishment path changes with different levels of switching costs, I consider various sub-cases. For a low-intermediate level of switching costs $S \in \left(0, 1/(2+\sqrt{2})\right)$, collusion is an equilibrium if

$$\Pi_i(\mathbf{p}(0)) = \frac{1}{1-\delta} \geq 2 \times (1-S) + \delta\left(\frac{(1+\sqrt{2})S}{(1-\delta)}\right) = p_i^d + \delta\Pi_i(F(p))$$
$$\iff \delta \geq \frac{1-2S}{2-(3+\sqrt{2})S},$$

For a high-intermediate level of switching costs $S \in \left(1/(2+\sqrt{2}), 1/2\right)$, collusion is an equilibrium if

$$\Pi_i(\mathbf{p}(0)) = \frac{1}{1-\delta} \geq 2 \times (1-S) + \frac{\delta(S+\sqrt{S(4+S)})}{2(1-\delta)} = p_i^d + \delta \Pi_i(F(p))$$
$$\iff \delta \geq \frac{2(1-2S)}{4(1-S)-(S+\sqrt{S(4+S)})}.$$

$S \geq 1/2$. In the punishment path, firms obtain the same profits than under collusion. Switching costs are too large to generate switching in equilibrium, and competition can only produce collusive prices. At the moment of deviation, a firm trades-off the price reduction that is necessary to attract the buyer from the rival against the increase in demand originated from capturing the consumers of the rival. However, when the level of switching costs is more than half of the consumer's reservation price, the first effect always dominates, and collusion is always an equilibrium of the infinitely repeated game.

In the next Proposition, I collects all previous results.

Proposition 3. *When consumers live for one period only:*

i) Collusion is always an equilibrium of the repeated game for $S \geq 1/2$. Without switching costs, collusion is an equilibrium for a value of the discount factor $\delta \geq 1/2$.

ii) For a low-intermediate level of switching cost such that $S \in \left(0, 1/(2+\sqrt{2})\right]$ collusion is an equilibrium for a discount factor $\delta \geq (1-2S))/\left(2-(3+\sqrt{2})S\right)$, and

iii) For a high-intermediate level of switching costs $S \in \left(1/(2+\sqrt{2}), 1/2\right)$ collusion is an equilibrium for a discount factor $\delta \geq (2(1-2S))\,/\left(4(1-S)-(S+\sqrt{S(4+S)})\right)$.

Figures 7 illustrates the results of the Proposition. With substantial switching costs, collusion is always an equilibrium. The increase in demand at the moment of deviation is not sufficient to pay for the switching costs to capture the consumers of the rival. Hence, deviating from the collusive path never occurs. With an intermediate level of switching costs, when switching cost increase,

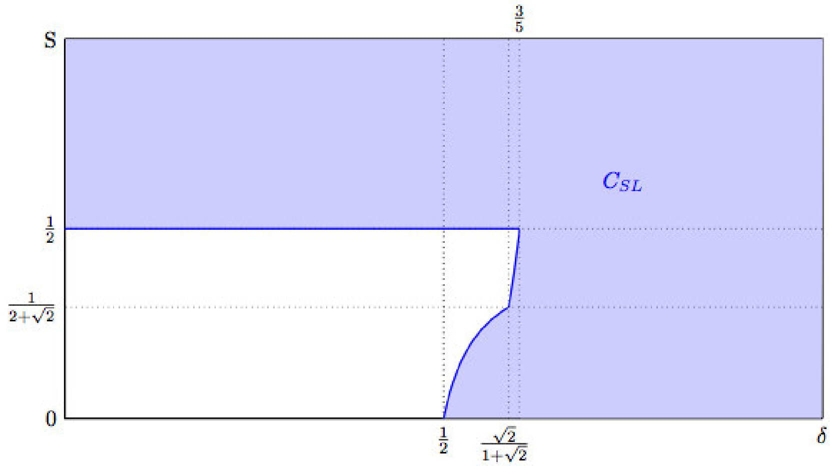

Figure 7: Area of collusion with short-lived consumers.

firms needs to reduce the deviating price to obtain the captive consumers of the rival. However, because the per-period expected profits in the punishment path increase to a larger extent with the level of switching costs, the region where collusion is sustained shrinks.

Comparing this result with the case when consumers interact with firms infinitely, the results depend on consumers' sophistication and concentration. The retaliation after deviation is always less severe when consumers interact only once: competition is stiffer with an asymmetric market structure than when firms have captive half of the market. Then collusion must be easier when consumers live infinitely and are either non-sophisticated or non-concentrated. In the former deviating prices are equal and are smaller in the latter. However, the large deviating price that firms can charge with concentrated consumers always dominates the reduction on the severity of retaliation with one period lived consumers. As a result, collusion is harder to sustain. The next Corollary summarizes the discussion.

Corollary 4. *When consumers live for one period only, collusion is easier that with infinitely lived concentrated consumers. The contrary happens when consumers cease to be strategic or there is a continuum of them.*

6 Different discount factors

In the benchmark model, I consider that firms and consumers share the same discount factor. In this case, the equilibrium in the punishment path have a neutral effect on the likelihood to implement a collusive outcome. In this section, I illustrate the change of results by introducing a different discount factor for firms and consumers. I first explore the situation where two representative buyers populate the market, and later I analyze the case with a continuum of consumers. The situation where consumers have zero discount represents the case with non-strategic consumers.

6.1 Two representative consumers

With two representative consumers, the deviating price in equilibrium incorporated the rents that consumers obtained in continuation game, and lower prices in the punishment path translated into larger deviating prices. In this section, when the firms' discount factor δ_f and consumers' discount factor δ_c do not coincide, this waterbed effect disappears.

The equilibrium in the punishment path is constructed as in section 4.1. Because this path is fully characterized by firms' competitive pressure, equilibrium prices depend only on firms' discount factor $\left\{p^I, p^0\right\} = \{(1-\delta_f)S, -\delta_f S\}$. With the punishment path, I obtain the present discounted utility of consumers. As in the benchmark case, a deviant firm incorporates these rents at the moment of deviation, giving a deviating price

$$p^d = \min\left\{1, \frac{(1-S)(1-\delta_c) + \delta_c(1-(1-\delta_f)S)}{1-\delta_c}\right\}.$$

Then, a collusive outcome emerges in equilibrium if no firm or consumer finds a profitable deviation. I begin with firms' incentive constraint and continue with consumers. The equilibrium deviating prices is lower than collusive prices when $S > \delta_c/(1-\delta_c\delta_f)$, and introducing the deviating price and the discounted profit after deviation into firms' incentive constraint, a necessary condition for collusion is

$$\Pi_i(\mathbf{p}(c)) = \frac{1}{2(1-\delta_f)} \geq 1 - S + \delta_c\left(\frac{1-(1-\delta_f)S}{1-\delta_c}\right) + \delta_f S = \pi_i(p_i^d, c) + \delta \Pi_i(\mathbf{p}(p^I)). \qquad (IC_f)$$
$$\iff \frac{1}{2(1-\delta_f)} \geq \frac{1-(1-\delta_c)S}{1-\delta_c} + (1-\delta_f)S \times \left(\frac{\delta_f - \delta_c}{(1-\delta_f)(1-\delta_c)}\right).$$

This expression yields interesting comparative statics. The sign and intensity on how the incumbency price $(1-\delta_f)S$ affect the sustainability of collusion is measured by the term $\mathcal{D} = (\delta_f - \delta_c)/(1-\delta_f)(1-\delta_c)$. When firms discount the future more than consumers $\delta_f < \delta_c$, a larger incumbency price makes collusion harder to sustain. The contrary happens when consumers value less the future than firms. This result comes from the analysis of the deviation price and continuation payoffs of companies after deviation. When consumers value the future more than firms, the price set at the moment of deviation is substantially large, and this always dominates the small discounted payoffs that firms obtain after deviation. When firms value the future more than consumers, the decrease in the deviating price dominates the larger continuation payoffs after deviation.

A consumer does not have an incentive to break a collusive outcome, if the cost of switching is above the present discounted increase in utility emerging from the punishment path.

$$S \geq \delta_c \times \left(\frac{1-(1-\delta_f)S}{1-\delta_c}\right). \qquad (IC_c)$$

Solving for the level of switching costs gives $S \geq \delta_c/(1-\delta_c\delta_f)$. For lower values of switching costs, the consumer always breaks a collusive outcome, and I do not need to verify firms' incentive constraint for $S < \delta_c/(1-\delta_c\delta_f)$ when the deviating price is made arbitrarily close to the monopoly. Therefore, the next Proposition states when both conditions are satisfied and collusion constitutes an equilibrium

Proposition 4. *When the discount factor of firm and consumers do not coincide, with two buyers,* *collusion is an equilibrium of the game for discount factors*

$$\delta_f \geq \frac{-1 + 2S + \sqrt{1 - 2(1-\delta_c)S}}{2S} \quad \textit{and} \quad \delta_c < \frac{S}{1+\delta_f S} = \bar{\delta}_c.$$

Figure 8 illustrates the area of a collusive outcome as a function of the discount factors and the magnitude of switching costs. With an increase in the consumers' discount factor, the collusive area

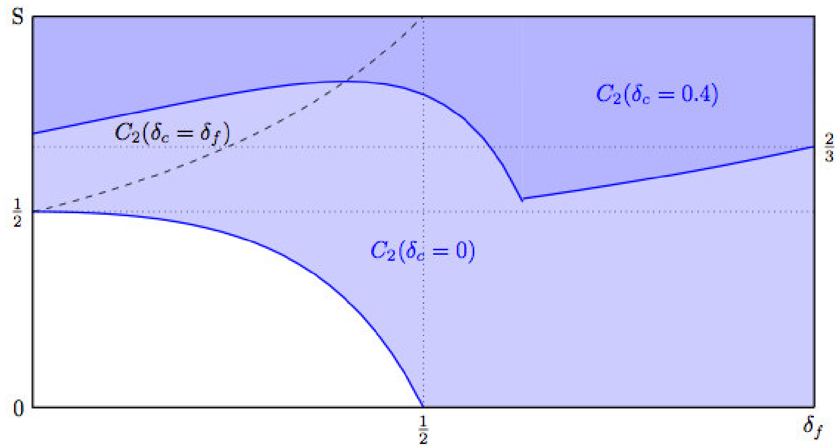

Figure 8: Area of collusion with two consumers and different discount factors.

shrinks. When consumers value more the future, they obtain larger rents after deviation making the deviation price more important. Additionally, because switching cost are paid in the present and rents of the punishment path are enjoyed in the future, the more consumers value the future the more stringent their incentive to break a collusive outcome becomes. In the figure, with $\delta_c = 0.4$ the area of collusion fails to be monotone. This non-monotonicity is explained from the parameter $\mathcal{D}$ in the firms' incentive constraint. The area of collusion is decreasing with δ_f when $\delta_c \geq \delta_f$ and increasing with δ_f when $\delta_c < \delta_f$. Moreover, this pattern is reversed for $0 < \bar{\delta}_c < \delta_f$. In this case, the incentive constraint of consumers is binding, and for larger values of switching costs, consumers deviate from the collusive outcome. For a consumers' discount factor $\delta_c = 0$, the analysis coincide with non-strategic consumers.

6.2 Continuum of consumers

With a continuum of consumers, the deviating price in equilibrium includes a subsidy to consumers equal to the present discounted profits a firm obtains in the continuation game after deviation. Then a lower retaliation translates into a larger subsidy and vise-versa. As before, when firms' and consumers discount factor do not coincide, this water-bed effect disappears.

With a continuum of consumers I do not consider deviation by consumers, and the equilibrium in the punishment path coincides with 4.1. With the punishment path, I obtain the discounted utility of consumers, generating a deviating price

$$\begin{aligned} p_i^d &= 1 - S + \delta_c \left[U_j \left(\mathbf{p}(p^I) \right) - U_j \left(\mathbf{p}(p^0) \right) \right] \\ &= 1 - S + \delta_c \left[\frac{1 - (1-\delta_f)S}{1-\delta_c} - \frac{1+\delta_f S}{1-\delta_c} \right] \\ &= 1 - (1+\delta_c)S \end{aligned}$$

Introducing this price into the firms' incentive constraint, the next Proposition states under which discount factors, collusion is an equilibrium of the game.

Proposition 5. *When the discount factor of firm and consumers do not coincide, with a continuum of consumers, collusion is an equilibrium of the game for the discount factor*

$$\delta_f \geq \frac{(1+\delta_c)S - \sqrt{S\left(2 - (1+\delta_c)(3-\delta_c)S\right)}}{2S}.$$

Figure 9 illustrates the area of a collusive outcome as a function of the discount factors and the magnitude of switching costs. Different from the case with two buyers, now an increase in consumers' discount factor enlarges the area of collusion. The more consumers value the future, the larger needs to be the subsidy to capture them, and the lower are the profits of the firm at the moment of deviation.

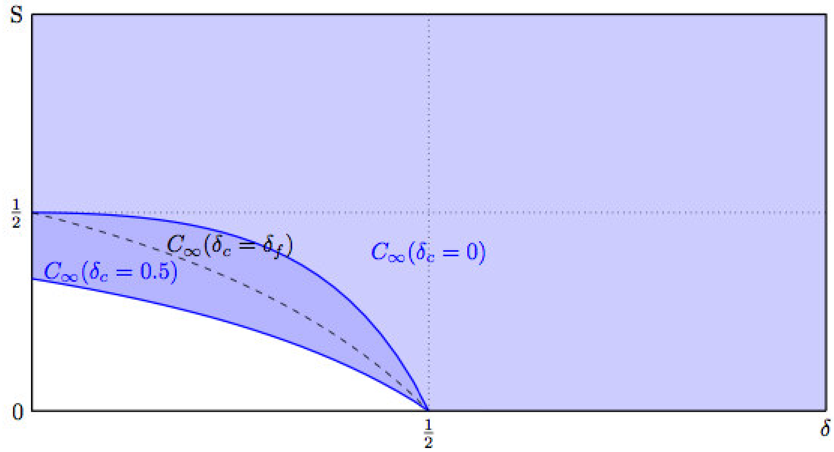

Figure 9: Area of collusion with a continuum of consumers and different discount factors.

7 Conclusion

Much of the existing literature argues that the effect of switching cost on the ability to collude in prices is ambiguous. While switching costs generates a less severe retaliation after deviation, it also increases the cost to attract the consumers of the rival. I provide a theoretical model that quantifies the magnitude of both effects. To the best of my knowledge, the fact that costumers' characteristics have an impact on the ability for firms to coordinate in prices in industries where consumers have switching costs has not been pointed out. Taking into account different types of consumers has enabled me to identify the tradeoffs between retaliation and cost to capture the buyers of the rival, and clearly establish the conditions under which collusion is implemented.

The ability to incorporate the future rents of strategic consumers into firms' deviation price make collusion hard to sustain when consumers in the market are concentrated and have "strategic mass". Consumers' incapacity to influence the equilibrium when they are dispersed allows them to free ride on the purchasing decision of other consumers and a firm needs to pay the full switching costs and the profits of the continuation game to attract them. Then, the low deviation price that firms are able to set explains why collusion is easier to be sustained in this case. Th equilibrium deviating price is also smaller when consumers case to be strategic, and collusion is again easer to sustain.

The results of this paper have clear implications for policy analysis. This article shows that educating consumers on the consequences that current purchasing decisions have on future prices is relevant. However, those measures need to go in hand with mechanisms that help buyers to take coordinated purchasing decisions. I have shown that when consumers do not have "strategic mass", collusion is easily implemented. This paper also highlights in which type of markets a competition authority should be more concerned about potential coordinated practices. For instance, markets where consumers are concentrated and educated, a large level of switching costs should not hinder competition. However, if a large number of dispersed consumers populates the same market, competition is at stake. This may explain why in 2010, normal tariffs for local calls in Sweden were very competitive at a price of 0.029 euros per minute, while in the UK the tariff for the same service at 0.077 euros per minute seemed the result of price coordination (Eurostat, online data code: isoc-tc-prc, Telingen).

This paper may also be tested in the laboratory. In a controlled environment, it is easy to inform participants about the functioning of the market, as well as exogenously change the level of switching costs and the number of potential consumers. While there is an extensive literature in psychology trying to understand the formation of costs when agents change tasks or adapt to new processes, there is no experimental evidence analyzing the effects that exogenous switching costs have on the competitive functioning of a market. Testing some of my theoretical findings in the laboratory may contribute to close this gap.

On the theoretical side, I have used a very stark model in which the market is populated by the same type of consumers who are equally sophisticated and interact with companies for the same number of times. One interesting extension may be to consider a market where consumers while having the same level of switching costs, vary in their degree of sophistication and their interaction with firms. A market populated by consumers having different characteristics, while being more realistic, is more complicated to analyze. I belief that due to the divergent incentives that firms may have with respect to various consumers, the equilibrium after any deviation may be in mixed strategies. Moreover, in the analysis with homogenous consumers, if an action is profitable by a consumer it is beneficial for all. Hence, we did not need to consider any strategy that required coordination by consumers. However, because different types of consumers may take distinct actions, the analysis of the equilibrium after deviation may be more involved.

Another interesting avenue for future research, which has not been taken many attention from a theoretical standpoint, is to endogenize the level of switvchig costs. In our model, and depending on consumers' characteristics, firms may have incentives to either increase or reduce the magnitude of switching costs in order to maintain a collusive outcome. For instance, with strategic consumers, firms may have incentives to raise the level of switching costs in order to sustain a collusive outcome. This may not be required if consumers case to be strategic.[22] Working also with the magnitude of switching costs, it may be interesting to examine the effect of heterogeneity in switching costs among different buyers. In the simpler situation, a positive mass of consumers does not have switching costs. How the decision of uncommitted consumers together with the dispersion of switching costs disrupt the functioning of the market is left for future research.

Finally, while this paper has explored the demand characteristics that would have an effect on the sustainability of collusion, another model may study the supply side characteristics of the market. Questions such as how the number of firms in the market, the possibility of potential entry and differences in production technology, affect the results will be examined in future research.

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A Proofs of Lemmas and Propositions.

Proof. **Lemma 1**.

We characterize the mixed strategy equilibrium of the game where each firm shares half of the market and consumers incur to a switching cost S when buying from the rival. This problem is mathematically equivalent to the competition model presented in [26]. I reconstruct his equilibrium with the case of two competing firms and when consumers have switching costs.[23]

In my candidate mixed strategy equilibrium, firms choose the price distribution function $F(p)$ to maximize expected profits

$$\frac{V}{p} = 2 - [F(min(p+S,1)) + F(max(0,p-S))]. \tag{A.1}$$

Shilony (1977) shows that there exists a unique price distribution function $F(p)$ with a range of the support in $\bar{p} - \underline{p} \leq 2S$. Departing from [26], my strategy to characterize the equilibrium distribution function is by construction. I obtain a form for $F(p)$ when firms set price p belonging to different ranges. Then:

$\underline{p} + S \leqslant p < \bar{p}$. A price in this range gives $F(min(p+S,1)) = 1$ and $F(max(0,p-S)) = F(p-S)$.

Hence, expression (A.1) reduces to $F(p-S) = 1 - V/p$. Arranging terms

$$F(p) = 1 - \frac{V}{p+S} \quad \text{for} \quad \underline{p} \leqslant p < \bar{p} - S.$$

$\underline{p} \leqslant p < \bar{p} - S$. A price in this range gives $F(max(0,p-S)) = 0$ and $F(min(p+S,1)) = F(p+S)$.

Expression (A.1) reduces to $F(p+S) = 2 - V/p$ which gives

$$F(p) = 2 - \frac{V}{p-S} \quad \text{for} \quad \underline{p} + S \leqslant p < \bar{p}.$$

[23] In a companion research note, I show multiplicity of equilibria for some parameter values by not assuming any specific consumers' tie breaking rule.

$\bar{p} - S < p < \underline{p} + S$. The difference between any price belonging to this range and any point in the support is lower than the value of switching costs. No firm loses or gains consumers by setting a price in this range. As a result, it is never a best response to play with positive probability a price in this range. Because the distribution function $F(p)$ needs to be monotone and increasing with p gives a distribution function

$$F(p) = 1 - \frac{V}{\bar{p}} \quad \text{for} \quad \bar{p} - S < p < \underline{p} + S.$$

Collecting all previous results gives a distribution function

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p}, \\ 1 - \frac{V}{p+S} & \text{if } \underline{p} \leqslant p < \bar{p} - S, \\ 1 - \frac{V}{\bar{p}} & \text{if } \bar{p} - S < p < \underline{p} + S, \\ 2 - \frac{V}{p-S} & \text{if } \underline{p} + S \leqslant p < \bar{p}, \\ 1 & \text{if } p \geqslant \bar{p}. \end{cases} \tag{A.2}$$

If we assume that the range of the support is not bigger than twice the vale of switching cost, and no firm plays with a positive mass the extremes of the support, then no firm loses or gains consumers by setting a price $p = \underline{p} + S$. This strategy gives expected payoffs $\underline{p} + S = V$, or equivalently $\underline{p} = V - S$. Similarly, no firm loses or gains consumers by setting a price $p = \bar{p} - S$. The expected payoffs of this strategy are $\bar{p} - S = V$ which gives condition $\bar{p} = V + S$. Indeed, by combining both conditions we obtain a range of the support equal to twice the level of switching costs, i.e., $\bar{p} - \underline{p} = 2S$, and an expected payoff of $V = \underline{p} + S$. Introducing this last result into expression (A.2), I obtain a distribution function

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p}, \\ 1 - \frac{\underline{p}+S}{p+S} & \text{if } \underline{p} \leqslant p < \underline{p} + S, \\ 2 - \frac{\underline{p}+S}{p-S} & \text{if } \underline{p} + S \leqslant p < \underline{p} + 2S, \\ 1 & \text{if } p \geqslant \underline{p} + 2S. \end{cases} \tag{A.3}$$

I proceed to define the support of the distribution function. If a firm sets a price $p = \underline{p} + 2S$, he loses consumers only if the rival sets a price arbitrarily below $\underline{p} + S$. This happens with probability $(\underline{p} - S)/\underline{p}$. Hence, the strategy of setting a price $p = \underline{p} + 2S$ gives the firm expected payoffs

$$\pi(\underline{p} + 2S, F(p)) = (\underline{p} + 2S) \times \left(1 - \frac{\underline{p} - S}{\underline{p}}\right) \iff (\underline{p} + 2S) \times \frac{S}{\underline{p}} = V$$

Because the expected payoff is equal to $V = \underline{p} + S$ then we obtain the lower bound of the support.

$$(\underline{p} + 2S) \times \frac{S}{\underline{p}} = \underline{p} + S \iff \underline{p} = \sqrt{2}S$$

Introducing this result into expression (A.2), fully characterizes the equilibrium price distribution

$$F(p) = \begin{cases} 0 & \text{if } p < \sqrt{2}S, \\ 1 - \frac{\left(1+\sqrt{2}\right)S}{p+S} & \text{if } \sqrt{2}S \leq p < \left(1 + \sqrt{2}\right) S, \\ 2 - \frac{\left(1+\sqrt{2}\right)S}{p-S} & \text{if } \left(1 + \sqrt{2}\right) S \leq p < \left(2 + \sqrt{2}\right) S, \\ 1 & \text{if } p \geq \left(2 + \sqrt{2}\right) S. \end{cases} \tag{A.4}$$

The expected payoff of a firm playing this strategy is equal to $V = \left(1 + \sqrt{2}\right) S$. The upper-bound of the support of the distribution function increases with the level of switching costs. Because it is never optimal strategy to set a price above consumers' reservation, the distribution function is only valid for a value of switching costs in $S \in \big(0, 1/(2+\sqrt{2})\big]$.

I then proceed to characterize the equilibrium distribution function for $S \in \big(1/(2+\sqrt{2}), 1/2\big)$. Because the upper-bound of the support is never larger than the consumer reservation price, by substituting $\bar{p} = 1$, gives a distribution function

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p}, \\ 1 - \frac{V}{p+S} & \text{if } \underline{p} \leqslant p < 1 - S, \\ 1 - V & \text{if } 1 - S < p < \underline{p} + S, \\ 2 - \frac{V}{p-S} & \text{if } \underline{p} + S \leqslant p < 1, \\ 1 & \text{if } p \geqslant 1. \end{cases} \tag{A.5}$$

To obtain the equilibrium distribution, I will conjecture that firms never play with a positive probability the lower-bound of the support, i.e., $F(\underline{p}) = 0$. This implies that $\underline{p} = V - S$. I then show that, if firms play with zero probability the lower bound of the support, then there must be an atom at the upper-bound of the support. To see this, consider that firms play also with zero probability the upper-bound of the support, gives an expected payoff of $V = 1 - S$, and that there must there must be an atom at $\underline{p} + S$ equal to $A(\underline{p} + S) = (1 - 2S(2 - S))/(1 - 2S)$. However, for some of the vales of the switching costs that we are considering, the atom is negative and we reach a contradiction. Moreover, when a firm sets a price equal to $\underline{p} + S$ he does not gain or loses consumers, and a firm will never set this price with positive probability.

Therefore, the only possibility is that there is an atom at the upper bound of the support equal to $A(1) = (S - 1 + V)/(1 - S)$. Finally, a firm setting a price arbitrarily close to $1 - S$ never loses its captive consumer and attracts the consumer of the rival when the rival sets a price equal to 1 which happens with probability equal to $A(1)$. This gives the expected payoffs equal to $V = (S + \sqrt{S(4 + S)})/2$.

Therefore, the equilibrium distribution function equal to:

$$F(p) = \begin{cases} 0 & \text{if } p < \frac{-S+\sqrt{S(4+S)}}{2}, \\ 1 - \frac{S+\sqrt{S(4+S)}}{2(p+S)} & \text{if } \frac{-S+\sqrt{S(4+S)}}{2} \leqslant p < 1-S, \\ 1 - \frac{S+\sqrt{S(4+S)}}{2} & \text{if } 1-S < p < \frac{S+\sqrt{S(4+S)}}{2}, \\ 2 - \frac{S+\sqrt{S(4+S)}}{2(p-S)} & \text{if } \frac{S+\sqrt{S(4+S)}}{2} \leqslant p < 1, \\ 1 & \text{if } p \geqslant 1. \end{cases} \tag{A.6}$$

Mixed strategy equilibrium with non-sophisticated consumers.

The idea of this section is to show that when consumers are non-sophisticated, it is always optimal to attract the buyer of the rival. No firm ever sets a deviation price $p^d = 1-\epsilon$ where $\epsilon < S$. I will show this result by construction. First, I show that the continuation game when the deviant does not attract the consumer of the rival has a solution in mixed strategies. Later, I prove that attracting only the consumers of the rival is always a dominated strategy either by a deviation when a firm obtain the whole market or by not deviating from the collusive path.

When a company fails to attack the consumer of the rival, the continuation game moves to a situation where both firms share half of the market. By an argument similar to section 5, where consumers only live for one period, there does not exist a pure strategy equilibrium, and the equilibrium is mixed strategies.

I restrict attention to a symmetric mixed strategy equilibrium represented by a distribution function $F(p)$. Assuming that the price dispersion is bigger than the magnitude of switching costs allows separating the distribution function into three exclusive regions.[24] With probability $\nu_1 := (1 - F(\min\{p+S, 1\})$ the price difference is above the magnitude of switching costs and the firm appropriates the whole market. The game moves into an "absorbing" state where the punishment path is the same as in section 4.1. With probability $\nu_2 := F(\min\{0, p-S\})$ the firm loses his

[24] I have a stochastic game, in which depending on the action profile, the game translates into different states.

consumers and never obtains any demand. With the probability that the difference in prices is lower than the level of switching costs $\zeta := F(\min\{p+S,1\}) - F(\max\{0,p-S\})$, no customer switches and the game does not change state. Each firm obtains expected discounted payoffs

$$V = \zeta \times (p + \delta V) + \nu_1 \times (2p + 2\delta S)$$

Arranging terms gives

$$V = \frac{\zeta \times p}{1-\delta\zeta} + \frac{\nu_1}{1-\delta\zeta} \times (2p + 2\delta S)\,. \tag{A.7}$$

I solve equation (A.7) using the same procedure as in the proof of Lemma 1. By assuming a price difference of $S < \bar{p} - \underline{p} \leq 2S$, I obtain the distribution function analyzing different price regions.

$\underline{p} + S \leqslant p < \bar{p}$. A price in this range implies $\zeta = 1 - F(p-S)$ and $\nu_1 = 0$. Introducing this into expression (A.7) gives

$$F(p) = 1 - \frac{V}{\delta V + p + S} \quad \text{for} \quad \underline{p} \leqslant p < \bar{p} - S.$$

$\underline{p} \leqslant p < \bar{p} - S$. A price in this range implies implies $\zeta = F(p+S)$ and $\nu_1 = 1 - F(p+S)$, giving a distribution function

$$F(p) = \frac{2(p-S) + 2\delta S - V}{p - S + 2\delta S - \delta V} \quad \text{for} \quad \underline{p} + S \leqslant p < \bar{p}.$$

$\bar{p} - S < p < \underline{p} + S$. The difference between any price and any point in the support is lower than the value of switching costs. Because no firm loses or gains consumers by setting any price in this range, no company sets a price in this range with positive probability. This condition gives a distribution function

$$F(p) = 1 - \frac{V}{\delta V + \bar{p}} \quad \text{for} \quad \bar{p} - S < p < \underline{p} + S.$$

Collecting all the previous results gives

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p}, \\ 1 - \frac{V}{\delta V + p + S} & \text{if } \underline{p} \leqslant p < \bar{p} - S, \\ 1 - \frac{V}{\delta V + \bar{p}} & \text{if } \bar{p} - S \leqslant p < \underline{p} + S, \\ 1 - \frac{S - p + (1-\delta)V}{p - S + \delta(2S - V)} & \text{if } \underline{p} + S \leqslant p < \bar{p}, \\ 1 & \text{if } p \geqslant \bar{p}. \end{cases} \tag{A.8}$$

I obtain the bounds of the support and the expected profit by looking at candidate potential discontinuities. I first conjecture that there is no atom at $\bar{p}$ as no firm will ever reach the state when it obtains all consumers. This implies that

$$\begin{aligned} F(\bar{p}) = 1 \iff & 1 - \frac{S - \bar{p} + (1-\delta)V}{\bar{p} - S + \delta(2S - V)} = 1 \\ \iff & \bar{p} = (1-\delta)V + S \end{aligned} \tag{A.9}$$

The same argument implies that firms do not play $p = \underline{p} + S$ with positive probability. This gives

$$\begin{aligned} 1 - \frac{V}{\delta V + \bar{p}} = 1 - \frac{S - \underline{p} - S + (1-\delta)V}{\underline{p} + \delta(2S - V)} \iff & \frac{V}{V + S} = \frac{(1-\delta)V - \underline{p}}{\underline{p} + \delta(2S - V)} \\ \iff & \underline{p} = \frac{V(V + S - 3\delta S)}{2V + S} \end{aligned} \tag{A.10}$$

I argue that if a firm plays $p = \underline{p} + S$ with zero probability, then it also has to play $\underline{p}$ with zero probability. To see this, consider that a firm plays $\underline{p}$ with positive probability, then there is a profitable deviation by the other company to name the price $\underline{p} + S$ with positive probability as well.

By setting price $\underline{p} + S$, the firm loses no customers and increases his expected payoff. But this contradicts that no firm names with positive probability price $\underline{p} + S$.

Then, the distribution function $F(p)$ is atomless. This gives $\underline{p} = (1-\delta)V - S$, and also a difference in the extremes of the support equal to twice the magnitude of switching costs, $(\bar{p} - \underline{p}) = 2S$. With this condition, I easily obtain the expected discounted profit of any company playing according to this mixed strategy equilibrium.

$$(1-\delta)V - S = \frac{V(V + S - 3\delta S)}{2V + S} \iff V = \frac{S(1-\delta) + \sqrt{(2 + (\delta - 4)\delta)S^2}}{1 - 2\delta}.$$

However, because the upper-bound of the cannot be larger than the consumers' reservation price. The pervious analysis is only valid for a value of switching cost $S \in \left(0, \bar{S}\right]$, where $\bar{S} \equiv 1 - \left((\delta - 1)^2\right) / \sqrt{(\delta - 1)^2 (2 + (\delta - 4)\delta)}$. For larger levels of switching costs, we set $\bar{p} = 1$. As shown in the proof of Lemma 1, the equilibrium dictates that there is no atom at the upper bound of the support. This condition gives the expected discounted profits

$$F(\bar{p} = 1) = 1 \iff 1 - \frac{S - 1 + (1-\delta)K}{1 - S + 2\delta S - \delta K} = 1,$$

Arranging terms gives

$$K = \frac{1 - S}{1 - \delta}.$$

Having characterized the expected discounted profit of a firm not attracting the consumer of the rival, I show that such strategy is always a dominated strategy. The expected payoffs for each deviation strategy are

$$\Pi^{no} = (1 - \epsilon) + \delta K,$$

$$\Pi^{at} = 2 \times (1 - S) + 2\delta S,$$

where the first and the second expressions stand for not attracting the mass of buyers from the rival and attracting them respectively. Because the expected discounted profit of firms depend on the magnitude of switching costs, I consider the two possible ranges of switching costs separately

For $S \leq \bar{S}$. Not attracting the customer of the rival is a profitable strategy if

$$\begin{aligned}\Pi^{no} \geq \Pi^{at} &\iff 1+\delta\times\left(\frac{S(1-\delta)+\sqrt{(2+(\delta-4)\delta)S^2}}{1-2\delta}\right) \geq 2(1-S)+2\delta S \\ &\iff \delta\sqrt{(2+(\delta-4)\delta)S^2} \geq 1-2(S+\delta)+\delta S(5-3\delta)\end{aligned}$$

and this is never fulfilled for any δ and the level of switching costs considered.

For $S > \min\left\{\bar{S}, 1\right\}$. I find a region where it is optimal not to attract the customer of the rival

$$\Pi^{no} \geq \Pi^{at} \iff 1+\delta\left(\frac{1-S}{1-\delta}\right) \geq 2(1-S)+2\delta S \iff S \geq \frac{1-2\delta}{2+\delta(2\delta-3)} = \widehat{S}$$

because $\widehat{S} < \bar{S}$. However, staying in the collusive path always generate larger profits

$$\frac{1}{1-\delta} > 1+\delta\left(\frac{1-S}{1-\delta}\right) \iff S > 0.$$

I then conclude that not attracting the mass of buyers for the rival at the moment of deviation is a dominated strategy.