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MEASUREMENT PROBLEM. CAN WE SOLVE IT?

Hernando Zuleta

SERIE DOCUMENTOS DE TRABAJO

No. 67
Agosto 2009

IF FACTOR SHARES ARE NOT CONSTANT THEN WE HAVE A MEASUREMENT PROBLEM. CAN WE SOLVE IT?*

Hernando Zuleta
Universidad del Rosario
American University in Bulgaria
hernando.zuleta@gmail.com

Abstract

Recent evidence show that factor shares, if properly measured, are far from constant. Moreover, the shares of natural resources and raw labor seem to be negatively correlated with income per capita while the share of human and physical capital is positively correlated with income per capita. Now, if factor shares are not constant then (i) growth accounting exercises rely on a false assumption and (ii) there is a measurement problem. The effect that changes in factor shares have on output depend on the relative abundance of factors and, for this reason, it is necessary to have correct measures. We propose an empirical methodology to solve the measurement issue and estimate TFP growth.

JEL Codes: O11, O30, O41

Keyword: Factor Shares, Production Function, Measurement.

** The findings, recommendation, interpretation and conclusion expressed in this paper are those of the authors and not necessarily reflects the view of the Department of Economics of the Universidad del Rosario*

1. Introduction

The idea that labor income share is roughly constant, namely, the Cobb-Douglas-Kaldor paradigm¹, produced important consequences in the area of economic growth. Almost all of the literature on growth accounting assumes that the elasticity of output with respect to capital is constant and concludes that the major part of economic growth is explained by growth in TFP¹ (see Easterly and Levine, 2002; Solow, 1957 or Young, 1994 among others).

Recent empirical work shows that the Cobb-Douglas-Kaldor paradigm is not really supported by the data. Kahn and Lim (1998) show that the shares of equipment, production workers and non production workers have clear trends. Blanchard (1997) observes the share of labor decreases in continental Europe after the 80s and suggests that the reason of such decline may be technological bias. Other authors calculate the income share of reproducible factors (human and physical capital) and not reproducible factors and it turns out that the later is positively correlated with the income level (see Krueger, 1999; Caselli and Feyrer, 2007; Zuleta, 2008a; Sturgill, 2009 and Zuleta, Parada and Campo, 2009)².

Additionally, the variability of factor shares may have different effects on output depending on the factor abundance of the economy. If the income share of abundant factors is growing the effect of these changes on the income level is positive but if income share of abundant factors is decreasing the effect of these changes on the income level is negative.

In this paper we propose an empirical methodology to estimate the correct measures of factor shares. In the next section we explain why measurement is an issue when factor shares are not constant. In the third section we present the methodology. In the last section we present the conclusions.

2. Factor Shares and Measurement

¹ See Cobb and Douglas (1928) and Kaldor (1961).

² In the last decades some authors revisited the theory of biased innovations and challenged the Cobb-Douglas-Kaldor paradigm. Zeira (1998) provides a model of technological innovations that reduce labor requirements and find that innovations are only adopted in countries with high wages. Seater (2005), Zeira (2006), Peretto y Seater (2007) and Zuleta (2008b) among others, present models where the scarcity of not reproducible factors generates incentives to adopt technologies that reduce the need of the these factors. In a market economy, this type of technological change affects factor income shares in such a way that the share of reproducible factors is positively correlated to the income level of the economy.

To illustrate the importance of correct measures for the factors consider the simplest Cobb-Douglas technology with two factors: K and L. Output per worker, $y = \frac{Y}{L}$, can be expressed

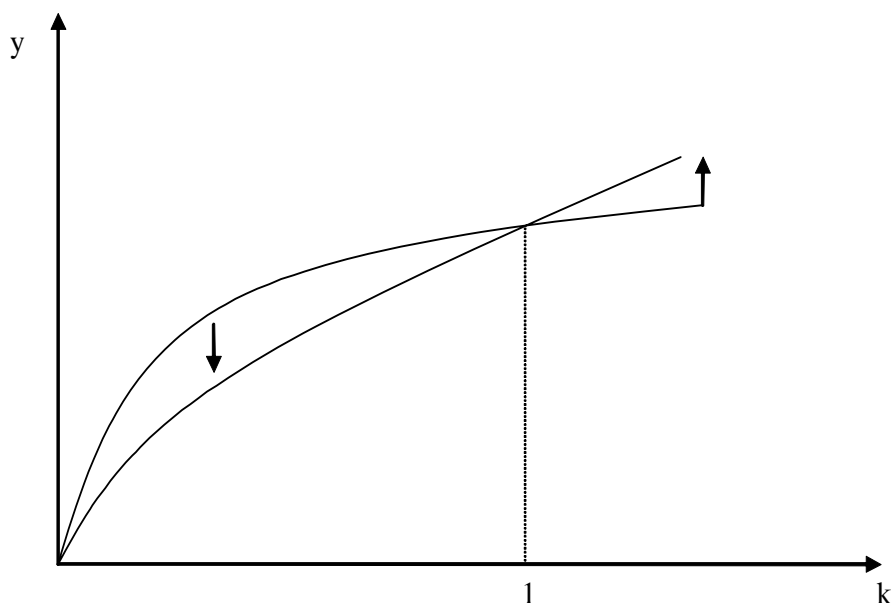
as a function of capital per worker $k = \frac{K}{L}$:

$$y = Ak^\alpha$$

Now, suppose that there is an increase in the share of capital. The effect of the change in factor shares on income per worker depends on the relative abundance of capital,

$$\frac{\partial y}{\partial \alpha} = Ak^\alpha \ln k$$

Therefore, if $k > 1$ the effect is positive and if $k < 1$ the effect is negative.



3. How to solve the problem?

For simplicity consider a production function with only two factors:

$$(1) \quad Y_t = A_t K_t^{\alpha_t} L^{1-\alpha_t}$$

Where Y is total income, K is physical capital and L is raw labor. Note that factor shares have the subindex t , namely they are variables not parameters.

a. The Methodology assuming away factor augmenting technological change.

Differentiating equation 1,

$$(2) \quad \frac{\Delta Y_t}{Y_t} = \left\{ \frac{\Delta A_t}{A_t} + \alpha_t \frac{\Delta K_t}{K_t} + (1 - \alpha_t) \frac{\Delta L_t}{L_t} \right\}$$

Now define

$$(3) \quad S_t = \frac{\Delta Y_t}{Y_t} - \left(\alpha_t \frac{\Delta K_t}{K_t} + (1 - \alpha_t) \frac{\Delta L_t}{L_t} \right)$$

The variable S in equation 3 is the Solow residual. Now, from equations 2 and 3 it follows that the Solow residual is not only TFP growth. The Solow residual includes biased technological change:

$$(4) \quad S_t = \frac{\Delta A_t}{A_t} + \Delta \alpha_t \ln \left(\frac{\phi_K K_t}{\phi_L L_t} \right)$$

Define $\tilde{S}_t = S_t - \Delta \alpha_t \ln \left(\frac{K_t}{L_t} \right)$

So equation 4 can be rewritten

$$(5) \quad \tilde{S}_t = \frac{\Delta A_t}{A_t} + \Delta \alpha_t \ln \left(\frac{\phi_K}{\phi_L} \right)$$

Therefore, the reduced form to be estimated is the following,

$$(6) \quad \tilde{S}_t = C_0 + C_1 \Delta \alpha_t + \rho_t$$

Where $\frac{\Delta A_t}{A_t} = C_0 + \rho_t$, $C_1 = \ln \left(\frac{\phi_K}{\phi_L} \right)$.

Therefore, this methodology allows us to identify the correct measures of factors per worker

$$\frac{\phi_K}{\phi_L}.$$

Finally, combining the previous results with equation 1 and taking logs we get

$$(7) \quad \log Y_t = \log A_t + \alpha_t \log \left(\frac{\phi_K K_t}{\phi_L L_t} \right) + \log(\phi_L L_t)$$

We also know that $\log A_t = \log A_0 + \log(1 + C_0 + \rho_t)$. Therefore, we can define

$$\tilde{Y}_t = \log Y_t - \alpha_t \left(\log \left(\frac{\phi_K K_t}{\phi_L L_t} \right) \right) - \log(1 + C_0 + \rho_t) - \log L_t \text{ so that equation 7 can be written}$$

as $\tilde{Y}_t = \log(A_0 \phi_L)$. Therefore, we can also identify the initial level of TFP multiplied by the correct measure of labor.

b. What if there is labor augmenting or capital augmenting technological change?

We assume that there is factor augmenting technological change and the rate of technological change is constant. Equation 2 becomes

$$(2A) \quad \frac{\Delta Y_t}{Y_t} = \left\{ \begin{aligned} & \frac{\Delta A_t}{A_t} + \alpha_t \frac{\Delta A_{K_t}}{A_{K_t}} + (1 - \alpha_t) \frac{\Delta A_{L_t}}{A_{L_t}} \\ & + \alpha_t \frac{\Delta K_t}{K_t} + (1 - \alpha_t) \frac{\Delta L_t}{L_t} + \Delta \alpha_t \ln \left(\frac{\phi_{K,t} K_t}{\phi_{L,t} L_t} \right) \end{aligned} \right\}$$

Where $\frac{\Delta A_{K_t}}{A_{K_t}}$ and $\frac{\Delta A_{L_t}}{A_{L_t}}$ are, respectively, capital and labor augmenting technological change.

Equations 4 and 5 becomes

$$(4A) \quad S_t = \left\{ \frac{\Delta A_t}{A_t} + \alpha_t \frac{\Delta A_{K_t}}{A_{K_t}} + (1 - \alpha_t) \frac{\Delta A_{L_t}}{A_{L_t}} + \Delta \alpha_t \ln \left(\frac{\phi_{K,t} K_t}{\phi_{L,t} L_t} \right) \right\}$$

$$(5A) \quad \tilde{S}_t = \left\{ \Delta \alpha_t \ln \left(\frac{\phi_{K,0}}{\phi_{L,0}} \left(1 + \frac{\Delta A_{K_t}}{A_{K_t}} - \frac{\Delta A_{L_t}}{A_{L_t}} \right) \right) + \frac{\Delta A_t}{A_t} + \frac{\Delta A_{L_t}}{A_{L_t}} + \alpha_t \left(\frac{\Delta A_{K_t}}{A_{K_t}} - \frac{\Delta A_{L_t}}{A_{L_t}} \right) \right\}$$

Therefore, the reduced form to be estimated is the following,

$$(6) \quad \tilde{S}_t = C_0 + C_1 \Delta \alpha_t + C_2 \alpha_t + \rho_t$$

Where

$$C_1 = \ln \left(\frac{\phi_K}{\phi_L} \left(1 + \frac{\Delta A_{K_t}}{A_{K_t}} - \frac{\Delta A_{L_t}}{A_{L_t}} \right) \right), \quad C_2 = \left(\frac{\Delta A_{K_t}}{A_{K_t}} - \frac{\Delta A_{L_t}}{A_{L_t}} \right) \text{ and } C_0 + \rho_t = \frac{\Delta A_t}{A_t} + \frac{\Delta A_{L_t}}{A_{L_t}}.$$

Therefore, we can identify the correct measures of factors per worker $\frac{\phi_K}{\phi_L}$, the difference

between capital augmentin and labor augmenting technological change $\frac{\Delta A_{K_t}}{A_{K_t}} - \frac{\Delta A_{L_t}}{A_{L_t}}$

and the sum of neutral plus labor augmenting technological change $\frac{\Delta A_t}{A_t} + \frac{\Delta A_{L_t}}{A_{L_t}}$.

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