Universidad del Rosario

Facultad de Economía

1960 - 2010

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LOAN SALES UNDER ASYMMETRIC INFORMATION

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SERIE DOCUMENTOS DE TRABAJO

No. 79

Febrero 2010

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February 2010

Abstract

Loans are illiquid assets that can be sold in a secondary market even that buyers have no certainty about their quality. I study a model in which a lender has access to new investment opportunities when all her assets are illiquid. To raise funds, the lender may either borrow using her assets as collateral, or she can sell them in a secondary market. Given asymmetric information about assets quality, the lender cannot recover the total value of her assets. There is then a role for the government to correct the information problem using fiscal tools.

Keywords: Loan Sales, Asymmetric Information, Liquidity, Securitization.
JEL Codes: G21, G28.

1 Introduction

The first sales of mortgage loans through a securitization mechanism occurred in the 1970s. Since then, the issue of securities based on illiquid assets, as bank loans, has been increasingly widespread. In an asset–backed securitization, a lender sells loans that are converted into securities, which in turn are negotiable in a secondary market. Financial intermediaries are highly interested on selling some of their loans, since this is a way to obtain capital for new investments. Despite the recent financial crisis originated by the securitization of subprime mortgages, loan sales using the asset-backed securitization mechanism is a good alternative for financial intermediaries to raise capital.

*This paper benefited from useful comments by Enrique Gilles, Dario Maldonado, Jorge Ponce, Jean-Charles Rochet and Anna Toldrà. All remaining errors are my own. Toulouse School of Economics and Facultad de Economía, Universidad del Rosario - Bogotá. E-mail: monica.vargas61@urosario.edu.co

Banks have special skills to be the best monitors of debtors (see Diamond (1984, 1991), Besanko and Kanatas (1993)). These financial intermediaries specialize in a monitoring technology, and they have interest on maintaining a business relationship with final debtors. This is true even when banks sell the loans that constitute the main link to borrowers. Indeed, the original lenders continue to be in charge of collecting the principal and interest payments from borrowers and, in many cases, borrowers are not even aware their debts have been sold and converted into securities. The structure of bank loan securitization has been designed to lessen possible moral hazard problems concerning the monitoring of borrowers by lenders.

The loan–backed securitization structure, briefly described in the next section, is known for being a complex process that is often hard to understand for final investors. When buying securities, investors do not have all the information about the quality of the assets they are buying. To correct that information problem, credit rating agencies (CRAs) make evaluations of the assets the securities are based on. The final price of those securities depends on that evaluation and on the structure of the securities (guarantee, payments forms, etc.), so CRAs play a key role on the asset backed securitization. However, with the recent global financial crisis originated by the securitization of subprime mortgages, CRAs have lost credibility and it turns important to find new mechanisms to correct the asymmetric information problem. This paper is concerned with that key question.

The aim of this paper is to explore the use of loan securitization to raise funds when there is asymmetric information between lenders and investors. I have developed a model in which a lender has access to new investment opportunities when having only an illiquid asset: loans. The lender has two alternatives to raise capital: either sell her assets (or part of them) in a secondary market (i.e. to securitize); or borrow from financiers using her assets as collateral. The objective is to find a mechanism through which the lender may find the maximum amount of funds, taking into account that she is the only one who knows the quality of her assets.

Because of the information problem we have obtained that, in both alternatives, the lender cannot recover the total value of her assets. There is then a role for the government: social welfare can be improved using fiscal tools, but only in the loan sales scenario. Therefore, the amount invested by the lender when she has new opportunities can be increased when she sells part of her loans. The government intervention is useless when the lender borrows from financiers.

This paper is mainly related to the literature about asymmetric information in credit markets, liquidity needs, and credit risk transfer (CRT). Concerning the first group of papers, our results coincide with those of DeMarzo and Duffie (1999). They present a liquidity model with asymmetric information, and they find that an optimal strategy for the bank is to retain a portion of the loan in portfolio to diminish the information problem. We have found the same result and we present a mechanism for banks to recover all the value of their assets. Other paper with asymmetric information is Ambrose, LaCour-Little and Sanders (2005). In that article the authors empirically examine whether banks use the asymmetric information problem in their favor to sell riskier loans in the security market. They find that the securitized loans present lower ex-post default risk than the ones retained in bank portfolios. According to them, securitization is a response to bank capital regulations and reputation incentives. In our paper there is not a minimum of capital required and then securitization is only used whith the objective of raising funds.

Concerning the literature about lenders' liquidity needs there is a large number of papers as Diamond and Dybvig (1983), Holmström and Tirole (1998), Diamond and Rajan (2001 and 2005), Freixas, Parigi and Rochet (2003), Rochet (2004) and Gorton and Huang (2004). This paper is closely related to Diamond and Rajan (2001). They analyze the case of a lender who may face a liquidity need (for consumption or for a new investment) when all of her assets are previously granted loans. They assume that loans are illiquid, so the initial terms on the loan contract have to include a liquidity premium. Given that the loan's value cannot be totally recovered, the banker may demand a premium to the entrepreneur and may incorporate contractual terms that allow her to liquidate the entrepreneur's project when she is in need of liquidity, thus there is a liquidation risk for the entrepreneur.

Firms may be liquidated or denied funding because only a too small fraction of their future returns can be paid to outsiders, as in Holmström and Tirole (1998). They show that ex-post unprofitable wealth transfers to these firms can help them survive. While individuals are assumed not being able to commit themselves to make the state-contingent payments, an intermediary can ex-ante hold collateral and thereby commit to make the payments.

In Diamond and Rajan (2001) it is shown that the adverse consequences of illiquidity could be avoided if the lender could borrow the full value of the loan committing to deploy her extractions skills. In their model, the lender may commit by borrowing using demand deposits: a fragile capital structure that is subject to a "run".

The notion of financial assets illiquidity in Diamond and Rajan (2001) is completely related to the specific skills a lender has to collect loan payments. They assume that it is difficult to make lenders commit to collect those payments once they have sold their loans.

This is a main difference with our model because we assume commitment is possible because securitization has been designed to minimize the loss of banker incentives to collect payments. In our model the illiquidity of bank loans is due to asymmetric information between lenders and liquidity providers.

Other paper closely related to ours is Gorton and Huang (2004). In that paper, the authors present a model in which there are illiquid assets that are sold in a secondary market where prices are endogenously determined. Private liquidity supply is socially beneficial, but it is also socially costly since liquidity suppliers could have made more efficient investments ex-ante. As in Holmström and Tirole (1998), and also in our framework, there is a role for the government to improve the social welfare. In the case of Gorton and Huang (2004) the government subsidies distressed banks or firms, in Holmström and Tirole (1998) it plays the role of liquidity provider by issuing bonds. Again, the asymmetry of information we are considering is the main difference of this paper with Gorton and Huang (2004).

Concerning the recent literature about the use of credit risk transfer (CRT) instruments by financial entities, this paper is related to papers analyzing loan sales as Duffie (2007) and Cerasi and Rochet (2008). Notice that among CRT instruments we have loan trading, credit derivatives, asset-backed securities and collaterized debt obligations. Bank loan securitization is considered a CRT instrument when there is a true sale of the loan, so there is CRT to the buyer of the loan.

Cerasi and Rochet (2008) present a model of prudential regulation where banks' main activity is monitoring loans. The authors analyze the implications of two cases: banks suffering a negative solvency shock on loan returns, and banks having new lending opportunities. They show that banks liquidity needs can be solved using CRT instruments as loan sales and credit derivatives. Since bank monitoring is non-observable, those CRT instruments must be combined with optimal capital requirements to improve monitoring incentives. The aim of their model is then to analyze the impact of CRT on the monitoring function of banks and to derive implications for the optimal design of capital requirements.

This paper is organized as follows. In the next section we present a brief description of securitization transactions. We describe the model and the different possible negotiations between the lender and the potential financiers in the third and fourth sections. In the fifth section we present how the government can intervene to improve the social welfare. We finalize, in the sixth section, with some concluding remarks.

2 A Brief Description of ABS Transactions

In a traditional transaction of a bank loan securitization there are at least five parties involved: borrowers, loan brokers, originators (banks), buyers of assets, and investors in the asset backed securities (ABS). The buyer is usually a Special Purpose Entity (SPE). A SPE is established solely to purchase assets and to issue securities against them (e.g. Ginnie Mae in the U.S.).

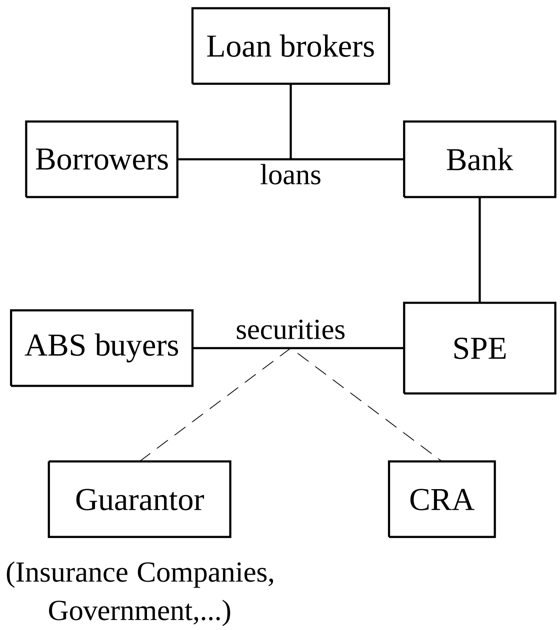

Figure 1: Simple scheme of participants in a bank loan securitization

In a securitization sale, the originator will often make the transfer to the buyer so that it constitutes a "true sale", and the asset is removed from the originator's balance sheet. To guarantee on-time payments to security buyers, the SPE commonly uses a tranching structure as well as guaranteed investment contracts, such as credit enhancements.

Securities receive a qualification from CRAs depending on their structure, guarantees and assets characteristics. Even if the quality of the assets in which securities are based on is not very high, if the ABS has a credible and a high credit enhancement, the qualification will be high. Investors decisions are based mainly upon rating agencies reports.

In a tranching structure, cash flows are split into classes with varying seniorities and absolute priorities. The senior classes, or tranches, are typically rated AAA to A and they have absolute priority in the cash flows over the other classes rated B, C, D, according to their subordination level. The classes absorbing all the initial risks are usually retained by originators to correct possible moral hazard problems.

The tranching structure spreads risks among diversified investors and matches risk with investor risk tolerances. Some papers are concerned with the design of ABS. In particular, Plantin (2003) presents a model in which a bank sells ABS to heterogeneous financial institutions under private information. The institutions differ in their abilities to retail the securities and screen the collateral, that is to find out the collateral quality. In that context, the author shows that tranching arises as the optimal structure for securities because it induces good screeners to specialize on junior tranches.

As a complement to the tranching structure, SPEs may arrange with a third party to provide credit enhancement. A typical credit enhancement for securities is a credit line guaranteed by a third party (another bank, an insurance company, an international agency, a government institution). In case of default the guarantor is obliged to repay the security buyers. Usually, securitized loans have good collateral but recovering their value can take a long time: that is why a credible credit line is necessary.

In some ABS transactions government may play a key role. SPEs can have explicit or implicit government backing (as the Government Sponsored Entities (GSEs) in the US) to sell securities without the credit enhancements. Passmore, Sparks and Ingpen (2002) study the transmission of the government subsidies to SPEs to the mortgage interest rates. They have found that the interest rates of the mortgage loans securitized by a GSE are usually lower than the interest rates of other loans, specially when GSEs behave competitively. In the model presented in the next section, it will be shown that the government can participate in securitization deals using fiscal tools to improve social welfare.

3 The Model

Let us introduce the general framework of the model and a brief discussion of the assumptions. We consider a model with three dates, similar to Diamond and Rajan (2001) and Gorton and Huang (2004). There is a lender, that can be a representative bank, and potential financiers henceforth simply referred to as investors. For simplicity all the agents are risk neutral and they do not discount future cash flows. There is a storage technology that yields \$1 per unit invested. We normalize the lender's initial endowment to \$1. At date 1 potential financiers are endowed with large amounts of money.

At date 0, the lender invests in a loan technology that yields, per unit invested, a high value H with probability π or a low value L with probability $1-\pi$. The investment return occurs at date 2, but information of its value arrives at date 1.

Assumption 1 $\pi H + (1-\pi)L \geq 1$ *and* $0 \leq L \leq 1$.

Assumption 1 means that the bank loan technology is on average profitable, but risky. Observe that this assumption implies that H must be strictly greater than 1.

Assumption 2 *The return of the loan is observable and contractible. However, at date 1 the lender privately observes the realization of the loan return.*

More precisely, at date 2 the realization H or L is observable, but at date 1 only the bank knows the future return of the loan technology. Considering that lenders have special skills to monitor borrowers, we can suppose they are the best informed about the value of their loan assets.

Assumption 3 *The loan investment cannot be liquidated before date 2.*

Assumption 3, together with the asymmetric information structure described in assumption 2, makes illiquid the lender investment at date 1. In Diamond and Rajan (2001) we find a similar assumption: the loans are illiquid before maturity due to specific skills of the lender. The illiquidity of the lender's assets is important in our framework because she may face a high-valued investment opportunity at the interim stage.

Assumption 4 *The lender may have access to a constant returns to scale technology that yields R per unit invested. This occurs at date 1, and with probability θ. The return to this new investment opportunity arrives at date 2 and it is not contractible.*

Assumption 5 $R > 1 > \theta R$.

Assumptions 4 and 5 mean that the new investment opportunity is riskless and profitable. The lender's best choice, however, is always to invest her endowment of \$1 in the loan technology. Observe that whether there is a new investment opportunity at date 1 is independent of the loan investment technology.

Our objective is to find mechanisms that allow the lender to raise the maximum level of funds when she has a new investment opportunity and all her assets are illiquid. In the next section we analyze two different possibilities for raising money: sell the loan, that is to securitize; or borrow from investors. To summarize the framework of the model we present in Figure 2 the sequence of events.

4 The Market for Liquidity

Henceforth we concentrate in the case in which the lender has a new investment opportunity. In this section we analyze the two different ways the lender has to obtain funds for

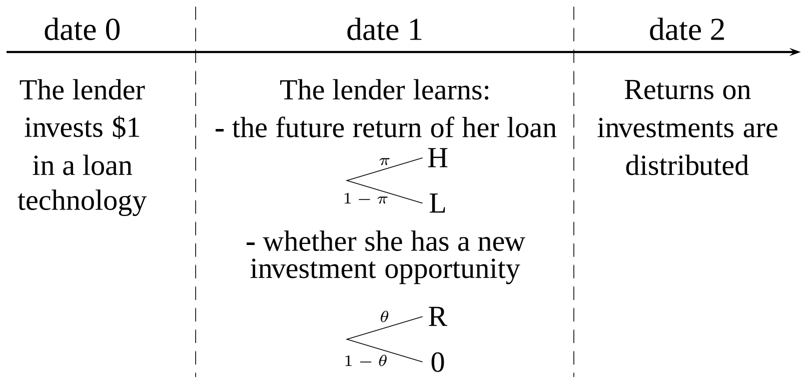

Figure 2: Time line

that new investment. Recall that, to make the model interesting, we include an illiquidity problem. Indeed, banks are sometimes faced to new and good investment opportunities but all their resources are already invested. We suppose, as in Diamond and Rajan (2001), that the lender cannot find liquidity by borrowing against the realization of her new investment opportunity. If that were possible, there would be no liquidity problem. That idea is included in assumption 4.

At date 1 the lender can obtain liquidity in two ways: by selling her loan investment through securitization, or by borrowing.

Assumption 6 *Unsecured credit is not available to the lender, that means she cannot borrow without pledging collateral.*

Assumption 6 implies the lender, whether obtaining funds by securitization or by borrowing, has to use her loan asset. Moreover, by assumption 3, the loan cannot be liquidated at date 1. Besides, there is an asymmetric information problem: at date 1 only the lender has information about the future return of her loan investment.

Without the asymmetry of information, the maximum amount of funds the lender may get is limited by the return of the loan, that is H or L. When the investors have the bargaining power in the negotiation with the lender, they absorb the gains of the new investment. In that case the amount raised by the lender may be H/R or L/R, depending on the future return of the loan.

When the information is not public the lender informs the investors about the value of her loan through a message m, where m is equal to H or L. When the message is equal to L investors believe it, but when it is equal to H investors may be skeptical of its validity because the lender may be lying to raise the maximum amount of money. The asymmetry of information may then generate an inefficiency to the lender when having a loan with a high return as it will be shown. We next analyze and compare the two alternatives the lender has to raise funds.

4.1 Credit Market

In Diamond and Rajan (2001) a lender may have a liquidity shock and there are numerous potential financiers, each with a small endowment. They find that the best way lenders have to raise funds is by issuing deposits. In this subsection we assume potential financiers can be represented by an investor who establishes the terms of the credit he grants to the lender. As a consequence, the investor absorbs the gains of the new technology leaving the lender with the minimum needed such that she is interested in the credit. To avoid confusions, in this part of the analysis we will refer to the lender as the bank.

By assumption 6 the bank has to propose a collateral to the investor when borrowing. Despite the bank is the only one to have information about the future loan return, it has to use it as collateral.

The negotiation between the investor and the bank is the following:

- The investor proposes the terms of the credit.
- The bank accepts or not those terms.
- If the bank accepts, it informs about the future return of the loan sending a message m to the investor, where m is equal to H or L.
  If the bank does not accept, it does not invest in the new technology and it receives the loan return at date 2. This is the status quo scenario.
- The bank receives from the investor an amount of money previously established in the terms of the credit. That amount depends on the message m.
- At date 2 the bank repays the credit, otherwise the investor seizes the loan return.

We assume the investor is unable to perform any kind of screening, so the best he can do is specify the terms of the credit such that the bank send the true message. We call I_m the amount the investor lends to the bank, and F_m the face value of the credit. The terms of the credit specify the values of I_m and F_m for each possible value of m.

The bank accepts the credit terms if the payoff at date 2 of investing in the new opportunity by borrowing is higher than the status quo (i.e. doing nothing). The following inequality represents the participation constraint of the bank when the loan return is H or L.

$$I_m R + [k - F_m]_+ \geq k \quad \text{for} \;\; k = H, L \tag{1}$$

In (1), $I_m R$ is the return of the new investment opportunity, $[k-F_m]_+$[1] is the remainder payoff of the loan investment after paying back the credit to the investor. Recall the loan investment is the collateral to this credit, therefore if the face value F_m is higher than the loan return, the investor gets only the return of the loan.

The investor specifies the terms of the credit forcing the bank to send a true message. For that, the following incentive constraint must be satisfied,

$$k = \arg\max_{m \in H,L} \{I_m R + [k - F_m]_+\} \quad \text{for } k = H, L \tag{2}$$

The terms of the credit $\{I_H, F_H, I_L, F_L\}$ are then found by solving the following problem:

$$(\mathcal{P}_1) \qquad \max_{I_H, F_H, I_L, F_L} \pi(F_H - I_H) + (1-\pi)(F_L - I_L) \tag{3}$$

$$\text{subject to (1), (2) and } F_m \leq m \text{ for } m = H, L$$

The objective function of the problem ($\mathcal{P}_1$) represents the investor's expected return. Restrictions (1) and (2) are the participation and incentive constraint of the bank. The last restriction insures that the face value of the credit is limited by the loan investment return.

The following proposition describes the terms of the credit that are the solution of the problem ($\mathcal{P}_1$).

Proposition 1 *At date 1, the investor proposes a credit to the bank such that no matter the message m sent by the bank, the amount lent is L/R and the corresponding face value is L. In other words, the terms of the contract are $I_H = I_L = L/R$ and $F_H = F_L = L$.*

Proof. See the Appendix.

The investor lends to the bank the same level of liquidity no matter the quality of the bank loan. In turn, the face value is also the same across loan's qualities. The investor extracts the full payoff of the new investment opportunity, this implies his credit has a gross return of R per unit lent. As a consequence, the profit of the bank does not change when it has access to a new investment and, therefore, the bank is indifferent between raising funds at date 1 or not. We could suppose that, if indifferent, the bank would choose to borrow. Indeed, banks are always interested on investing in non-risky projects, even when they do not get monetary benefits, because they can keep clients or they can attract new ones.

[1] $[x]_+$ is equal to x if $x \geq 0$, otherwise it is equal to zero.

Notice that the credit terms are independent of the probability π. This is because at date 1 the bank knows the future return of its loan and the investor designs the credit such that the bank reveals the information it has about its loan. Besides, the credit terms are such that the investor does not lose money in any case. Then the face values F_H and F_L are limited to the lowest return of the date 0 loan, that is L.

4.2 Secondary Market

In the secondary market, the lender finds liquidity by selling a part of her loan investment, that is by issuing securities based on her loan. Recall there is an information problem: at date 1 only the lender knows the future return of her loan. Moreover, we assume that screening by the investors is not possible so the negotiation between the lender and the investors should be based on a message m sent by the lender. The message informs the investors about the loan return, m is equal to H or L.

When the lender sells a part of her loan, the transfer of the loan to the investors has a unit cost of γ. Therefore, the total cost of transferring a fraction q is $q\gamma$. Who pays that cost does not change our results, thus we suppose it is paid by the security buyers (i.e. the investors).

Assumption 7 $\gamma \leq \gamma_m \equiv (1 - \frac{1}{R})L$.

This assumption can be rewritten as $R(L - \gamma) \geq L$. It means that it is good for the lender to sell her low-return loan even taking into account the cost of the assets' transfer. Because L is lower than 1 then γ is also lower than 1.

In what follows, we analyze two possible ways to issue securities. First, we present the case in which the lender directly issues the securities. Second, we consider the case when the lender sells her loan to an Special Purpose Entity which in turn issues the securities.

The bank issues the securities

When investors are many potential financiers with small endowments, the lender proposes the portion of the loan she wants to sell and its corresponding price. Let us call q_m, with $q_m \in [0, 1]$, the fraction of the loan sold by the lender and p_m its corresponding price. The lender then proposes a value for q_m and for the price p_m for m equal to H and L.

When the bank sends the true message, the investors receive $q_m m$ at date 2. When the message is not equal to the future return of the loan, the investors receive $q_H L$ or $q_L H$, depending if the message was H or L. We are interested in a truthful revelation mechanism, i.e. m is indeed equal to the future return of the loan.

When buying part of the loan, investors should get at date 2 at least the price they have paid plus the cost of transferring the assets. Because investors buy a portion q_m of the loan, at date 2 they get $q_m m$ if the message was true. The following inequality represents the participation constraint of the investors for each possible return of the loan.

$$q_k k \geq p_k + \gamma q_k \quad \text{for } \; k = H, L \tag{4}$$

On the same way, the lender is willing to sell part of her loan if this is profitable. The following inequality is the participation constraint of the lender.

$$p_k R + (1 - q_k)k \geq k \quad \text{for } \; k = H, L \tag{5}$$

In (5), $p_k R$ is the return of the new investment and $(1 - q_k)k$ is the return of the part of the loan the lender has retained in her portfolio.

If the lender has a high return loan, she has no interest on lying about the quality of her loan. Besides, if she lies, the investor's payoff will not be negatively affected. However, if the loan return is low, the lender has interest on lying. To avoid that, the investors believe the message sent by the lender if and only if both p_i and q_i ($i = H, L$) satisfy the following lender's incentive compatibility constraint.

$$L = \arg \max_{m=H,L} \{p_m R + (1 - q_m)L\} \tag{6}$$

The lender solves the following problem ($\mathcal{P}_2$) in order to find the portion of the loan to be sold and its corresponding price.

$$(\mathcal{P}_2) \qquad \max_{q_k, p_k} [p_k R + (1 - q_k)k] \tag{7}$$

$$\text{subject to (4), (5), (6) and } q_k \in [0, 1] \text{ for } k = H, L$$

In problem ($\mathcal{P}_2$), the lender maximizes her profit (7) subject to the participation constraints (4) and (5) of the investors and the bank for each possible loan return; the incentive compatibility constraint (6); and the restriction for q_k.

Without the asymmetry of information, the lender would completely sell her loan ($q_k = 1$) at a price $p_k = k$ where k is the future return of the loan. Replacing these values of q and p in problem ($\mathcal{P}_2$), we see the lender's incentive compatible constraint is not satisfied when the return of the loan is low, that is $k = L$. In that case the lender has incentives to lie and she sends a message m equal to H. The next proposition presents the prices and portions of loan to sell such that the lender has no interest to lie.

Proposition 2 *When the lender sells part of her loan in a secondary market, she sells a portion q_k of it at a price p_k where k is the future return of the loan. We have,*

$$q_H = \frac{RL - R\gamma - L}{RH - R\gamma - L} \qquad p_H = (H - \gamma) q_H \tag{8}$$

$$q_L = 1 \qquad p_L = (L - \gamma) \tag{9}$$

Proof. See the Appendix.

Notice that the portions and the prices of proposition 2 are independent of the probability π. This is because the participation constraints of the lender and the investors are not interim but ex-post. Neither the lender nor the investors lose money in any state of the nature.

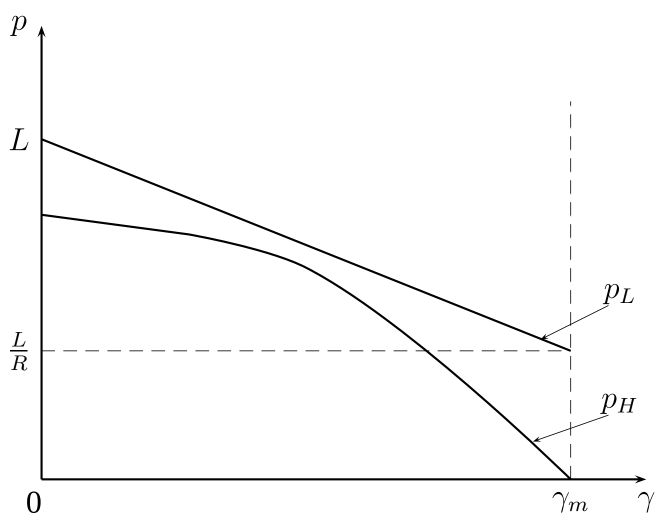

Figure 3: Funds raised by the lender when selling part of her loan

In Figure 3 we represent the prices and the cost of the transaction. We observe that when having high quality loans the lender invests less in the new opportunity compared to the case when she has low quality loans. It sounds paradoxical, but banks with L loans can raise more funds than banks with H loans. In compensation, when having a loan with high return the lender keeps a fraction of it. In contrast, when the loan is L the lender sells all of it without keeping a fraction of it. This can be interpreted as a signal sent by the lender to the investors when the loan return is high.

Corollary 1 *When the lender sells part of her loan in a secondary market, the unit price for the high return loan is equal to $p_H/q_H = H - \gamma$, and for the low return loan it is equal to $p_L/q_L = L - \gamma$.*

With a low return loan the lender can raise more funds than with a high return loan. However, according to corollary 1, the unit price of a loan with high payoff is higher than for a loan with low return. The lender sells her loan at the unit price she would sell it if the information about its return were public. Therefore, the asymmetric information problem affects the amount the lender can obtain but not the unit price.

Up to this point, we have analyzed the performance of two alternatives to raise funds in terms of the amount of money they may collect. Let us compare them now. When having a loan with low return it is better for the bank to go to the secondary market. However, when the loan quality is high, the lender's decision will depend on the cost γ. For small values of γ she gets more in the secondary market, but for high values of γ she gets more in the credit market. Recall Figure 3 and recall that the bank always raise L/R in the credit market.

A Special Purpose Entity (SPE) issues the securities

We analyze now another case we can have when the lender sells her loan in a secondary market. In this subsection we assume that potential financiers can be represented by an investor. The investor buys the loan, or part of it, to issue securities. The investor plays then the role of a SPE (see section 2). Notice that the SPE can be a sort of branch of the bank, or that the bank is shareholder of the SPE. Those cases were analyzed previously in this subsection and now we focus on the case in which the bank and the SPE are independent.

The investor decides the terms of the negotiation, despite the lender has an information advantage. As before, we call q_m, with $q_m \in [0, 1]$, the portion of the loan sold by the lender and p_m its corresponding price. The investor proposes a portion q_m and a price p_m for m equal to H and to L.

Recall that when the lender sends the true message, the investor receives $q_m m$ at date 2. Otherwise the investor receives $q_H L$ or $q_L H$, depending if the message was H or L. The investor decides to buy a part of the loan if it is profitable for him. We present next the investor's participation constraint, which is just equation (4).

$$q_k k \geq p_k + \gamma q_k \quad \text{for } k = H, L \tag{10}$$

In the same way, the lender decides to sell part of her loan whenever this is profitable. We have the same lender's participation constraint as before, that is,

$$p_k R + (1 - q_k) k \geq k \quad \text{for } k = H, L \tag{11}$$

The investor wants the lender to say the truth about the future return of the loan. For that, the following incentive compatibility constraint must be satisfied at the optimum. Notice that we need only one incentive compatibility constraint, as in problem ($\mathcal{P}_2$).

$$L = \arg \max_{m=H,L} \{p_m R + (1 - q_m)L\} \tag{12}$$

To find the portion of the loan to buy and its corresponding price, the investor solves the following problem ($\mathcal{P}_3$).

$$(\mathcal{P}_3) \qquad \max_{q_H, p_H, q_L, p_L} \pi[q_H H - p_H - \gamma q_H] + (1 - \pi)[q_L L - p_L - \gamma q_L] \tag{13}$$

$$\text{subject to (10), (11), (12) and } q_k \in [0, 1] \text{ for } k = H, L$$

In problem ($\mathcal{P}_3$), the investor maximizes his expected profit (13) subject to the participation constraints of the investors (10) and the lender (11); the incentive compatibility constraint (12); and the restrictions for q_H and q_L.

Problems ($\mathcal{P}_2$) and ($\mathcal{P}_3$) only differ in the objective function. The following proposition presents the solution of problem ($\mathcal{P}_3$).

Proposition 3 *When the investor buys part of the lender's loan, he buys a portion q_k of the loan at a price p_k where k is the future return of the loan. We have $q_L = 1$ and,*

(i) if $\pi(RH - R\gamma - L) < H - L$ then $q_H = p_H = 0$ and $p_L = L/R$;

(ii) if $\pi(RH - R\gamma - L) \geq H - L$ and $R(L - \gamma) \geq H$ then $q_H = 1$ and $p_H = p_L = H/R$.

(iii) if $\pi(RH - R\gamma - L) \geq H - L$ and $R(L - \gamma) < H$ then $q_H = \frac{RL - R\gamma - L}{H - L}$, $p_H = \frac{(RL - R\gamma - L)H}{R(H - L)}$ and $p_L = L - \gamma$;

Proof. See the Appendix.

Notice that the prices we have found in this Proposition 3 depend on the probability π, whereas this was not the case in Propositions 1 and 2. Recall that π is the probability that the loan has a high return at date 2. The return of the investor depends on π, that is why final prices depend on the probability π.

Let us briefly discuss Proposition 3. We observe that when the lender has a low return loan, she sells it partially or entirely to raise funds. In contrast, this is not the case when having a high return loan. More precisely, for low values of π, the investor proposes to buy part of the loan but only if the loan return is low. In that case, the lender raises the same amount of money than in the credit market, and the investor absorbs the profitability of the new technology because he pays L/R at date 1 and he receives L at date 2.

When π is high (i.e. the probability of having a high return loan is high) there are two cases depending on whether $R(L-\gamma)$ is higher or lower than H. If we have $R(L-\gamma) \geq H$, it is profitable for the lender to sell her loan at $L-\gamma$ in order to invest in the new technology. This holds even when the loan return is high. Therefore, the investor proposes to buy all the loan, no matter its payoff. The price of the loan is equal to H/R, so in that case the lender is raising more funds for the new investment than what she could have obtained in the credit market.

In turn, when $R(L-\gamma) < H$, it is not profitable for the lender to sell the high return loan at a low price. Therefore, the investor proposes to buy the entire loan if the return is low, but just a part of it if the return is high. When the loan return is low the lender invests in the new opportunity $L-\gamma$ when raising funds in the secondary market, which by assumption 7 is higher than what she can get in the credit market, that is L/R. When the loan return is high, we need to analyze different values for the cost γ to find which market provides more funds to the lender. This is done below.

Figure 4 presents the results of Proposition 3 for different combinations of γ and π, when RL is higher than H.

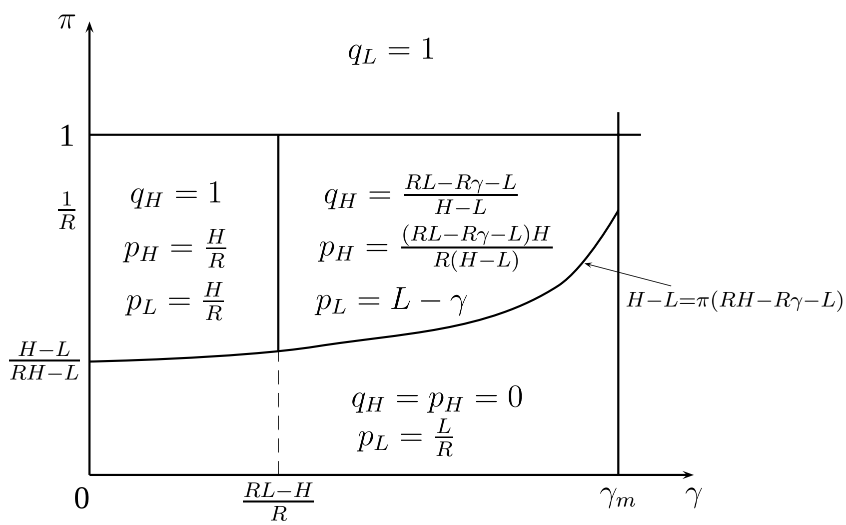

Figure 4: Terms of the loan sale when $RL \geq H$

Notice that when the probability of having a high loan return is high and the cost γ is small, the lender raises H/R no matter the future return of the loan. However, for other values of γ, the lender raises less funds when having a loan with low return, than when having a loan with high return. Then, as in the previous case we have analyzed, it may sound paradoxical but having a high loan return actually diminishes the capacity of the lender to raise funds for new investments. Besides, the lender keeps a part of her high return loan except when the probability π is very high. As it was mentioned before, this can be interpreted as a signal sent to the investor to inform the loan return is high.

Corollary 2 *When $\pi(RH - R\gamma - L) \geq H - L$, the investor proposes to buy part of the high return loan at a unit price equal to $p_H/q_H = H/R$ and to buy the low return loan at a unit price equal to,*

$$\frac{p_L}{q_L} = \begin{cases} L - \gamma & \text{if } L - \gamma < H/R \\ H/R & \text{if } L - \gamma \geq H/R \end{cases}$$

Corollary 2 shows that when it is profitable for the lender to sell part of her high return loan, then its price per unit is always higher or equal than the price per unit of a low return loan. This result is similar to the result of Corollary 1. Besides, in the case of the credit market we find something similar too. Recall in the credit market the bank receives L/R at date 1 and has to pay L at date 2. When the loan return is high, this is equivalent to sell a fraction L/H of the loan at a price L/R, thus the unit price is H/R. In the credit market the price per unit of a low return loan is L/R.

Finally, Figure 5 shows the prices of cases (ii) and (iii) of Proposition 3. For those cases the probability π takes high values.

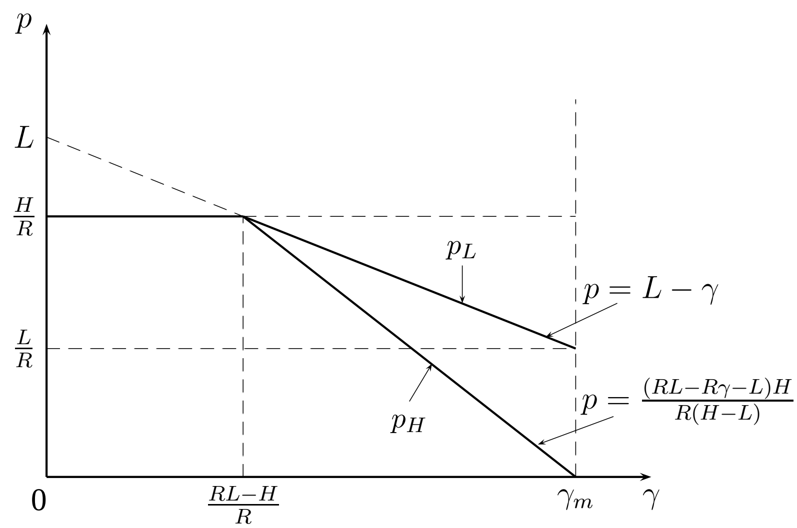

Figure 5: Prices when $\pi(RH - R\gamma - L) \geq H - L$

In Figure 5 it is clearly shown that when having a low return loan, the lender prefers to raise funds in the secondary market rather than in the credit market. Recall the lender raises L/R in the credit market. The lender also prefers the secondary market when her loan return is high, but only if the cost γ is low, otherwise the lender can raise more funds in the credit market.

It is important to highlight the following: a lender with a low return loan raises at least the same amount of money that a lender with a high return loan may obtain, no matter the liquidity market. Therefore, a lender with a high return loan does not invest at date 1 as much as the value of her loan. In the next section we present how the government can participate in the secondary market to improve the date 1 investment of a lender having a high return loan.

5 Government Participation

In the previous section we have studied the terms of the negotiation between the lender and potential financiers in the credit market, as well as in the secondary market. We have shown that no matter the loan return, the lender cannot raise more than L in any of the markets. Moreover, when the lender has a low return loan she can always raise more liquidity than when she has a high return loan. Therefore, the lender invests low amounts of money when she has new opportunities, even when having valuable assets. Those results are consequences of the asymmetry of information between the lender and the investors. In this section we present a mechanism to increase the amount of funds the bank can raise at date 1.

To correct the asymmetric information problem, we propose the government participates in the liquidity market by taxing the lender when she has a new investment opportunity. The tax has to be paid by the lender at date 2 and it depends on the loan return she reports. The tax can play the role of signal to investors: when the loan return is high, the lender can commit to pay a tax at date 2.

We call t_m the tax imposed by the government. Recall m is the loan return reported by the bank at date 1. Observe that t_m can be positive or negative. When it is positive it is a tax and when it is negative it is a subsidy. Then, at date 2 the government taxes or subsidies the lender, depending on the message she has sent.

By introducing the payments t_m we modify the incentive compatibility constraints of the lender. This will allow her to increase the amount of funds she can raise. Let us first analyze how results are modified when the lender sells her loan in the secondary market and the government intervenes. We focus in the case in which the bank issues the securities based on its loan.

When t_k is a tax that have to be paid by the bank at date 2, it must be the case that the lender has enough money to pay it. This is the case if the following limited liability condition is satisfied.

$$R(k-\gamma) \geq t_k \quad \text{for } \ k = H, L \tag{14}$$

Besides, the values of t_H and t_L should be such that at date 0 the bank prefers to invest in the loan technology rather than the status quo. For that, the following participation constraint of the bank should be verified.

$$\theta\{\pi[R(H-\gamma)-t_H]+(1-\pi)[R(L-\gamma)-t_L]\}+(1-\theta)\{\pi H+(1-\pi)L\} \geq 1 \tag{15}$$

If the government imposes transfers t_H and t_L that verify (14) and (15), then the lender solves the following problem $(\mathcal{P}_4)$,

$$(\mathcal{P}_4) \qquad \max_{q_k, p_k}[p_k R + (1-q_k)k - t_k] \tag{16}$$

$$\text{subject to} \quad q_k k \geq p_k + \gamma q_k \tag{17}$$

$$p_k R + (1-q_k)k - t_k \geq k - t_k \tag{18}$$

$$k = \arg\max_{m \in \{H,L\}}\{p_m R + (1-q_m)k - t_m\} \tag{19}$$

$$0 \leq q_k \leq 1 \quad \text{for } \ k = H, L \tag{20}$$

In problem $(\mathcal{P}_4)$ the lender chooses the fraction of loan to sell in the secondary market (q_k), and its price (p_k) so as to maximize her date 2 payoff subject to the participation constraint of the investor (17), to her participation constraint (18), to the incentive compatibility constraints (19) that insure she is not lying about her loan return, and to the conditions for q_k. Notice that the problem $(\mathcal{P}_4)$ differs from problem $(\mathcal{P}_2)$ only in the transfers t_H and t_L. We present next the solution of the lender's problem.

Proposition 4 *When the bank issues securities, and the conditions (14) and (15) are verified, as well as the following conditions:*

$$R(H-L) \geq t_H - t_L \quad \textit{and} \tag{21}$$

$$(RL - R\gamma - L) + t_H - t_L > 0, \tag{22}$$

the bank chooses $q_L = 1$, $p_L = L - \gamma$, $p_H = q_H(H-\gamma)$ *and*

$$q_H = \frac{(RL - R\gamma - L) + t_H - t_L}{RH - R\gamma - L} \tag{23}$$

Proof. See the Appendix.

Some important results arise from this Proposition, let us present them.

Corollary 3 *The lender sells all her loan regardless its return, that is $q_H = q_L = 1$, when $t_H - t_L = R(H - L)$.*

Corollary 4 *Taxing the lender improves the date 1 investment if $t_H \geq t_L$.*

Corollary 3 and Corollary 4 are obtained by comparing the results of Proposition 2 and Proposition 4. If it is to improve the date 1 investment, the government will find convenient to choose t_L equal to 0 and t_H positive. Indeed, the lender would pay a tax only when her loan return is high, and she would never receive a subsidy.

In Figure 6 we show the conditions that t_H and t_L must verify in order to increase the amount of money the bank may obtain at date 1.

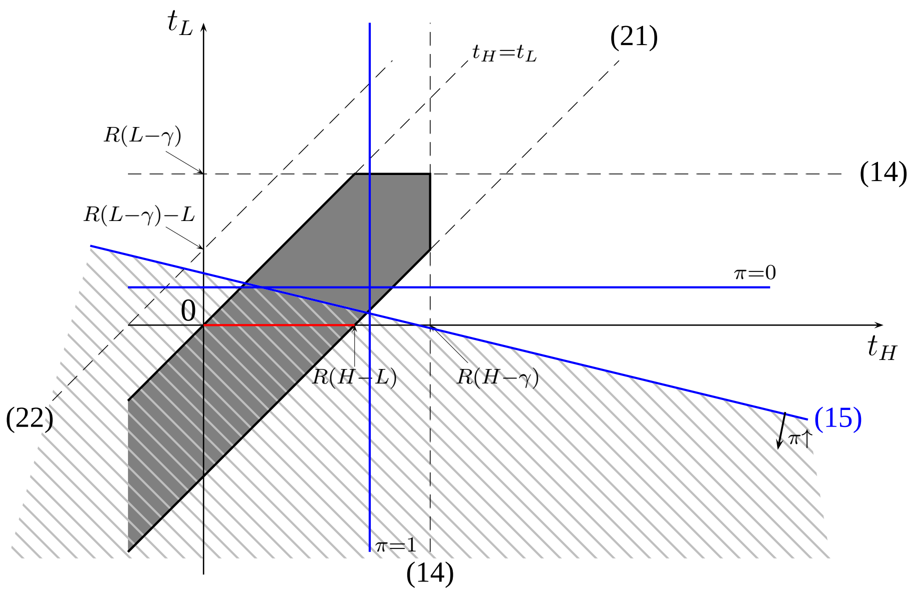

Figure 6: Values of t_H and t_L that improves the date 1 investment

The intersection of the gray and the striped regions represents the combinations of t_H and t_L that improve the amount of money obtained by the lender at date 1. Notice that the participation constraint of the lender (15), represented in blue, moves clockwise as π increases.

Figure 6 shows it is always possible that the government improves the amount invested at date 1 choosing $t_L = 0$, that is without taxing or subsidizing the bank when the loan return is low. This case is represented by the red line. Observe that t_L equal to zero implies that t_H should be positive. We have then the following result.

Corollary 5 *When the loan return is high, the government can always improve the amount of money invested at date 1 by taxing the lender. This holds in the case the lender sells her loan in the secondary market.*

Recall that, without government intervention, the lender informs investors in the secondary market that the loan return is high by retaining a fraction of the loan in her portfolio. With the government participation, in turn, the lender can accept to pay a tax at date 2 instead of keeping part of her loan. In contrast, when the bank borrows from the investors, this is useless.

In the credit market the bank can never get more than L. This is because the corresponding face value of the credit must be lower than the minimum return of the date 0 loan. Recall the incentive compatibility constraint (2) of the bank when borrowing in the credit market. The bank's incentive compatibility constraints, now including t_H and t_L are,

$$I_H R - F_H - t_H \geq I_L R - F_L - t_L \tag{24}$$

$$I_L R + L - F_L - t_L \geq I_H R + [L - F_H]_+ - t_H \tag{25}$$

Therefore, we deduce that F_H must be lower than L because the previous constraints imply $L - F_H \geq [L - F_H]_+ \geq 0$. The investor lends to the bank only if $F_k \geq I_k$ for $k = H, L$, so the amount lent by the investor when the loan return is high, that is I_H, has to be lower than L. Thus the participation of the government does not increase the amount borrowed by the bank at date 1.

Finally, the government may be interested on participating in the secondary market, helping the bank when the new investment is socially useful. Otherwise, there is no reason why the government should participate. Observe that the bank can commit to pay t_H at date 2 to an institution different from the government, but the commitment may not be credible.

6 Concluding Remarks

Loans are illiquid assets that cannot be easily negotiated or transferred. The development of secondary markets have made loan sales possible preserving the role of lenders as collectors of loan payments. Loan sales in secondary markets is a mechanism used by lenders, as banks, to raise funds for new investments. However, when loan buyers do not have a screening capacity to discover the value of the lender assets, the loans cannot be sold at their value. Lenders are the best informed agents about the real value of their assets, and it is difficult to credibly report those values to loan buyers. As long as there is a possibility for the lender to lie about the loan value, investors do not pay a high value for the loans. As a response, lenders retain in their portfolio part of their high valued loans.

Therefore, it is not always possible for lenders to raise an amount of money equal to the value of their assets.

In the context of this paper, loans are illiquid mainly due to the asymmetry of information about their value. Another source of the bank illiquidity in our model is the impossibility of the bank to use the new investment to raise funds. Recall the return of the new investment is not contractible. What we have in mind in this situation is that financial entities as banks have always interest on having available funds for new investments. This is particularly relevant when the banking sector is very competitive, so a bank having available funds can be the winner in a race for a new investment.

To correct the illiquidity of high valued loans, we propose that the government participates taxing the bank when it has a new investment opportunity. We have shown that the tax is a mechanism that allow the lender to credibly announce the value of her loan to create the liquidity. The lender can sell her loan at its value in the secondary market, even when its value is high and this is privately known by the lender. Government participation improves the amount of money raised by the bank because it has the power to commit the bank to behave, even when the new investment return is not contractible.

An important contribution of this paper is that we present a mechanism for the bank to credibly reveal the value of its assets without the participation of a credit rating agency. In the same way, we show how the government can participate to make the secondary market more efficient without subsidizing banks and without given credit enhancement to the security issuer.

Our simple model can be enriched by doing some comparative statics to analyze how the negotiation terms change with the parameters. In the same way, we can improve the model including monitoring of borrowers by lenders. Due to the moral hazard, the lender may have to retain part of her loan to be forced to exert monitoring effort.

Finally, liquidity is not scarce in our framework. It is assumed that there are potential financiers endowed with high levels of money. It can be interesting to change that assumption and to analyze a more competitive liquidity market including other investment opportunities to the financiers, as to invest in government bonds. Therefore, the price of the loans will be affected by the price of the government bonds.

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Appendix

Proof of Proposition 1

The problem $(\mathcal{P}_1)$ is equivalent to the following problem:

$$\max_{I_H, F_H, I_L, F_L} \pi(F_H - I_H) + (1-\pi)(F_L - I_L) \tag{26}$$

$$\text{s.t.} \qquad I_H R \geq F_H \tag{27}$$

$$I_L R + L - F_L \geq I_H R + [L - F_H]_+ \tag{28}$$

$$F_H \leq H \tag{29}$$

$$F_L \leq L \tag{30}$$

With the Kuhn-Tucker conditions it is easy to check that the constraint (28) should be binding. As a consequence (27) and (30) should be binding too, then $F_L^* = L$, $F_H^* = I_H^* R$ and $I_L^* R = F_H^* + [L - F_H]_+$. Analyzing the cases for $[L - F_H]_+$, we obtain that the optimum is achieved when $F_H^* = L$ and $I_H^* = I_L^* = L/R$. □

Proof of Proposition 2.

It is easy to show that when the loan return is L, the price $p_L = L - \gamma$ and $q_L = 1$. Taking into account this, the problem $(\mathcal{P}_2)$ for $k = H$ is equivalent to the following problem:

$$\max_{q_H,t_H} p_H R - q_H H \tag{31}$$

$$\text{s.t.} \quad q_H(H-\gamma) \geq p_H \quad (\mu_1) \tag{32}$$

$$R(L-\gamma) \geq p_H R + (1-q_H)L \quad (\mu_2) \tag{33}$$

$$0 \leq q_H \leq 1 \quad (\mu_3)(\mu_4) \tag{34}$$

The restriction (32) represents the investors participation constraint,(33) represents the incentive compatibility constraint that insures the lender with low return loan is not lying. We call μ_i with $i = 1, ..., 4$ the Kuhn-Tucker positive multipliers. Following we present some of the Kuhn-Tucker conditions that have to be satisfied at the optimum:

$$R = \mu_1 + \mu_2 R \tag{35}$$

$$-H = -\mu_1(H-\gamma) - \mu_2 L - \mu_3 + \mu_4 \tag{36}$$

We analyze three cases: $q_H^* = 1$, $q_H^* = 0$ and $q_H^* \in (0,1)$.

- If $q_H^* = 1$, then $\mu_3 = 0$. $\mu_1 \neq 0$ because when $\mu_1 = 0$ we have $\mu_2 = 1$ and $\mu_4 = -(H-L)$. So $p_H^* = H - \gamma$, but the constraint (33) is not verified. Therefore, at the optimum $q_H^* \neq 1$.

- If $q_H^* = 0$, then $\mu_4 = 0$, $p_H^* = 0$ and $\mu_2 = 0$, $\mu_1 = R$ and $\mu_3 = -(RH - R\gamma - H)$, then at the optimum $q_H^* \neq 0$.

- If $q_H^* \in (0,1)$, then $\mu_3 = \mu_4 = 0$, $\mu_2 = (RH - R\gamma - H)/(RH - R\gamma - L)$ and $\mu_1 = R(H-L)/(RH - R\gamma - L)$. Observe that $\mu_1 \neq 0$ and $\mu_2 \neq 0$ then (32) and (33) should be binding. Finally at the optimum we have

$$q_H^* = \frac{RL - R\gamma - L}{RH - R\gamma - L} \qquad p_H^* = \frac{(RL - R\gamma - L)(H-\gamma)}{RH - R\gamma - L} \tag{37}$$

  By assumption 7 the solution q_H^* is positive and lower than 1. □

Proof of Proposition 3

The problem $(\mathcal{P}_3)$ is equivalent to the following problem:

$$\max_{q_H,p_H,q_L,p_L} \pi[q_H(H-\gamma)-p_H]+(1-\pi)[q_L(L-\gamma)-p_L] \tag{38}$$

$$q_H(H-\gamma)\geq p_H \quad (\eta_1) \tag{39}$$

$$q_L(L-\gamma)\geq p_L \quad (\eta_2) \tag{40}$$

$$p_H R\geq q_H H \quad (\eta_3) \tag{41}$$

$$p_L R-q_L L\geq p_H R-q_H L \quad (\eta_4) \tag{42}$$

$$0\leq q_H\leq 1 \quad (\eta_5)(\eta_6) \tag{43}$$

$$0\leq q_L\leq 1 \quad (\eta_7)(\eta_8) \tag{44}$$

The restriction (39) and (40) represent the investor participation constraint for possible loan return, (41) is the lender's participation constraint when the loan return is high. (42) represents the incentive compatibility constraint that insures the lender with low return loan is not lying. We call η_i with $i=1,...,8$ the Kuhn-Tucker positive multipliers. Following we present some of the Kuhn-Tucker conditions that have to be satisfied at the optimum:

$$\pi(H-\gamma)=-\eta_1(H-\gamma)+\eta_3 H-\eta_4 L-\eta_5+\eta_6 \tag{45}$$

$$-\pi=\eta_1-\eta_3 R+\eta_4 R \tag{46}$$

$$(1-\pi)(L-\gamma)=-\eta_2(L-\gamma)+\eta_4 L-\eta_7+\eta_8 \tag{47}$$

$$-(1-\pi)=\eta_2-\eta_4 R \tag{48}$$

By (46) and (48) we deduce η_3 and η_4 are different from zero. Including (48) in (47) we obtain $\eta_8=\eta_4(RL-R\gamma-L)+\eta_7$, then η_8 is also different from zero. Therefore, restrictions (41) and (42) are binding and $q_L^*=1$. We obtain $p_H^*R=q_H^*H$ and $p_L^*R=q_H^*(H-L)+L$ and $\eta_7=0$.

Following we analyze three possible cases for q_H^*: $q_H^*=0$, $q_H^*=1$ and $q_H^*\in(0,1)$.

- If $q_H^*=0$, then $p_H^*=0$, $p_L^*=L/R$ and $\eta_2=\eta_6=0$. All the constraints of the problem are verified, now we have to find the necessary conditions to ensure that all the Kuhn-Tucker multipliers are positive. We easily obtain $\eta_4 R=(1-\pi)$, then $\eta_8>0$ and $\eta_3 R=\eta_1+1$. Replacing in (45) we obtain

$$\eta_5 R=(H-L)-\pi(RH-R\gamma-L)-\eta_1(RH-R\gamma-H) \tag{49}$$

  We can choose $\eta_1=0$, but to have $\eta_5\geq 0$ and then $q_H^*=0$ we need to have $\underline{H-L\geq\pi(RH-R\gamma-L)}$.

- If $q^*_H = 1$, then $p^*_H = p^*_L = H/R$ and $\eta_1 = \eta_2 = \eta_5 = 0$. To be sure that (40) is verified at the optimum, we need $R(L-\gamma) \geq H$. With that condition we are sure that all the constraints of the problem are verified. Now we have to ensure that all the Kuhn-Tucker multipliers are positive. As in the previous case, we easily obtain $\eta_4 R = (1-\pi)$, then $\eta_8 > 0$ and $\eta_3 R = 1$. Replacing in (45) we obtain $\eta_6 R = \pi(RH - R\gamma - L) - (H - L)$.

  Then to have $q^* = 1$ as a solution, we need to have $\underline{\pi(RH - R\gamma - L) \geq H - L}$ and $\underline{R(L-\gamma) \geq H}$.

- If $q^*_H \in (0,1)$, then $\eta_1 = \eta_5 = \eta_6 = 0$. Replacing those values in the Kuhn-Tucker conditions (45), (46) and (48) we obtain $\eta_3 R = \eta_4 R + \pi$,

$$\eta_4 R(H-L) = \pi(RH - R\gamma - H) \tag{50}$$

$$\eta_2(H-L) = \pi(RH - R\gamma - L) - (H-L) \tag{51}$$

  To have η_2 positive we need $\underline{\pi(RH - R\gamma - L) \geq H - L}$. Then the constraint (40) is binding and we have $q^*_H = (RL - R\gamma - L)/(H-L)$, $p^*_H = q^*_H H/R$ and $p^*_L = L - \gamma$. To insure that $q^*_H \in (0,1)$ we need $\underline{R(L-\gamma) \leq H}$. $\square$

Proof of Proposition 4.

We first find the solution of $(\mathcal{P}_4)$ taking into the taxes t_H, with t_L. It is easy to show that for when the loan return is L, the price $p_L = L - \gamma$ and $q_L = 1$. Taking into account this, the problem $(\mathcal{P}_4)$ for $k = H$ is equivalent to the following problem:

$$\max_{q_H, t_H} p_H R - q_H H \tag{52}$$

$$\text{s.t.} \quad q_H(H-\gamma) \geq p_H \quad (\rho_1) \tag{53}$$

$$R(L-\gamma) - t_L \geq p_H R + (1-q_H)L - t_H \quad (\rho_2) \tag{54}$$

$$p_H R + (1-q_H)H - t_H \geq R(L-\gamma) - t_L \quad (\rho_3) \tag{55}$$

$$0 \leq q_H \leq 1 \quad (\rho_4)(\rho_5) \tag{56}$$

The restrictions (53) represents the investors participation constraint,(54) and (55) represents the incentive compatibility constraints that insures the lender is not lying. We call ρ_i with $i = 1, ..., 5$ the Kuhn-Tucker positive multipliers. Following we present some of the Kuhn-Tucker conditions that have to be satisfied at the optimum:

$$R = \rho_1 + \rho_2 R - \rho_3 R \tag{57}$$

$$-H = -\rho_1(H-\gamma) - \rho_2 L + \rho_3 H - \rho_4 + \rho_5 \tag{58}$$

We analyze three cases: $q_H^* = 1$, $q_H^* = 0$ and $q_H^* \in (0, 1)$.

- If $q_H^* = 1$, then $\rho_4 = 0$. $\rho_1 \neq 0$ because when $\rho_1 = 0$ we have $\rho_3 = \rho_2 - 1$ and $\mu_5 = -(H - L)\rho_2$ which is impossible. So $p_H^* = H - \gamma$, and we need

$$t_H - t_L = R(H - L) \tag{59}$$

- If $q_H^* = 0$, then $\rho_5 = 0$. We have $\rho_1 \neq 0$ or $\rho_2 \neq 0$.

  If $\rho_1 \neq 0$ then $p_H^* = 0$, $\rho_2 = \rho_3 = 0$, $\rho_1 = R$ and $\rho_4 = -(RH - R\gamma - H)$, so this case is not possible.

  If $\rho_2 \neq 0$ then $\rho_3 = 0$ and $p_H^* R = (RL - R\gamma - L) + t_H - t_L$, $\rho_1 = 0$, $\rho_2 = 1$, $\rho_4 = H - L$ and we need to have $0 \geq (RL - R\gamma - L) + t_H - t_L$. Observe that p_H^* is negative in this case, which has no sense except if $t_L - t_H = (RL - R\gamma - L)$.

- If $q_H^* \in (0, 1)$, then $\rho_4 = \rho_5 = 0$. We have that $\rho_1 \neq 0$ and $\rho_2 \neq 0$ because

$$\rho_2 R(H - L) = \rho_1 (RH - R\gamma - H) \tag{60}$$

$$\rho_3 R(H - L) = \rho_1 (RH - R\gamma - L) - R(H - L) \tag{61}$$

  Therefore, constraints (53) and (54) are saturated, so

$$q_H^* (RH - R\gamma - L) = (RL - R\gamma - L) + t_H - t_L \tag{62}$$

$$p_H^* = q_H (H - \gamma) \tag{63}$$

  We need $R(H - L) > t_H - t_L$ to be sure that the constraint (55) is verified and to have $1 > q_H^*$. To have $q_H^* > 0$, we need $(RL - R\gamma - L) + t_H - t_L > 0$. $\square$