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Abstract

We study competition in two sided markets with *common network externality* rather than with the standard inter-group effects. This type of externality occurs when *both groups benefit*, possibly with different intensities, from an *increase* in the size of one group and from a *decrease* in the size of the other. We explain why common externality is relevant for the health and education sectors. We focus on the symmetric equilibrium and show that when the externality itself satisfies an homogeneity condition then platforms' profits and price structure have some specific properties. Our results reveal how the rents coming from network externalities are shifted by platforms from one side to other, according to the homogeneity degree. In the specific but realistic case where the common network externality is homogeneous of degree zero, platform's profit do not depend on the intensity of the (common) network externality. This is in sharp contrast to conventional results stating that the presence of network externalities in a two-sided market structure increases the intensity of competition when the externality is positive (and decreases it when the externality is negative). Prices are affected but in such a way that platforms only transfer rents from consumers to providers.
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1 Introduction

The theory of two-sided markets has been developed in recent years to investigate market structures in which two groups of agents interact via platforms; see for instance Rochet and Tirole (2006). The central theme of this literature is the presence of network externalities, which occur when the benefit from joining a platform for individuals of a given group depends on the size of membership (and/or usage) of the other group (Armstrong, 2006). Prominent examples of sectors in which such inter-group externalities occur range from credit cards, media and software to dating clubs.

We consider a two-sided market with an externality of a different nature. We shall refer to it as a "common network externality". This type of externality occurs when *both groups benefit*, possibly with different intensities, from an *increase* in the size of one group and from a *decrease* in the size of the other. Such externalities are relevant in a number of two-sided markets. For instance, in the health care sector, hospitals compete for patients on one side and for providers on the other side (see Pezzino and Pignatoro, 2008). It is a conventional assumption that the quality of health care depends on the providers' "workload". This is documented, for instance, by Tarnow-Mordi *et al.* (2000) who use UK data to show that variations in mortality can be explained in part by excess workload in the intensive care unit. Accordingly, health care quality is frequently related to the provider/patient ratio; see Mc Gillis Hall (2004). In other words, it increases when the number of health care professionals increases (for a given number of patients) but decreases when the number of patients increases (for a given number of providers). Both sides benefit from a higher quality albeit for different reasons and possibly with different intensity. This is quite obvious on the patients' side, where one can expect a higher quality to translate into a improvement in patients' health state (or at the very least into a reduction in waiting lines for appointments, *etc...*). Physicians benefit from a higher quality through a reduction in their workload[1], or indirectly, through their altruism (or simply job satisfaction).[2]

[1] See for instance Fergusson-Paré (2004) for the nursing workload. Griffin and Swan (2006) also find a strong relationship between nurses' workload and quality of health care.

[2] See, *e.g.*, Choné and Ma (2009).

Similar issues may arise in the education sector. Colleges or universities compete for students on one side and for professors on the other side. The quality of education depends on the pupil/teacher ratio and one can expect both sides to benefit from a higher quality. This is confirmed by surveys in which parents and teachers declare that they prefer a smaller class size (Mueller *et al.*, 1988). Furthermore, lower pupil/teacher ratios are associated with higher test scores for the children (see for instance Angrist and Lavy, 1999) and a smaller class size tends to increase average future earnings (Card and Krueger, 1992). On the other side, teachers enjoy an improved job satisfaction and a lower workload as the pupil/teacher ratio decreases.[3]

In this paper, we revisit the Armstrong's framework with a *common network externality* rather than with the standard inter-group effects. Two platforms compete in prices on two distinct Hotelling's lines. The common externality enters the preferences of both groups as a quality parameter. Each group values the common externality with (possibly) different intensities but the underlying notion of quality that matters (the functional form that specifies quality) is the same for both groups. We focus on the symmetric equilibrium and show that when the externality is specified by an homogeneous function, price structure and platforms' profit present some special features. First, network externalities affect prices in a cumulative way: the price on one side of the market depends on the sum of the externality terms on both sides of the market. Second, the effect on one side's price is, partially or entirely, shifted to the other side of the market. The extend of this shifting depends on the sign of the homogeneity degree of the common network externality (or more precisely, the degree of homogeneity of the function that relates quality to the membership on both sides). Third, competition intensity is also affected by this homogeneity degree. Specifically, the homogeneity degree determines the impact of the common network externaltiy on the platforms' profits. Finally, our results have particularly strong implication for the education and health sectors, where quality is known to mainly depend on consumer/provider ratio *i.e.,* the common externality is homogeneous of degree zero. In this case, platforms' profits do

[3]Buckingham (2003) finds that a reduction of class size slightly increases achievment, but also increases teachers work conditions by lightening their workload and easing classroom managment.

not depend on the intensity of the (common) network externalities. This property is in sharp contrast to the results obtained so far in the two-sided market literature. One of the major findings which has been reiterated in many settings is that the presence of network externalities in a two-sided market structure increases the intensity of competition when the externality is positive (and decreases it when the externality is negative).[4] We show that in a context of common network externality of degree zero, this is not the case. Under this assumption, prices are affected by the externality but in such a way that platforms only transfer rents from one group to the other. Roughly speaking, some rents due to the common network externality are extracted from the "consumers' side" and transferred to "providers". Furthermore, we show that for nonzero degrees of homogeneity, the conventional result can be generalized in an intuitively appealing way. When the degree of homogeneity of the common network externality is positive, platforms' profits decrease in the externality parameters. We can think about this case as that where the global impact of the externality is positive. A negative degree of homogeneity yields exactly the opposite result.

Before proceeding, let us have a closer look at the relationship of our paper to the existing literature. As pointed out by Rochet and Tirole (2003), the two-sided literature is at the intersection between multi-product pricing and network theories. The main focus of this paper lies on the second aspect. Several types of network externalities have been analyzed in the two-sided markets literature. The standard one is the *inter-group* network externality which we have mentioned above. It has also been pointed out that negative *intra-group* network "externality" can occur in equilibrium. This may be the case when members of a given group compete with each other. An additional member on one side then not only creates a positive inter-group externality but, at the same time, it can adversely affect welfare of the other members of the considered group.[5] For instance, in Bardey and Rochet (2009), health plans compete for policy holders on one side and for physicians on the other side. When a health plan enlists more physicians, this directly increases welfare of its policy holders. However, at the

[4] See, *e.g.,* Armstrong (2006).
[5] Most of time, this effect occurs because it increases the number of competitors.

same time, it may tend to attract riskier policy holders who place a higher value on the diversity of physicians. The induced adverse selection problem can be seen as a negative intra-group network "externality" that occurs on the policy holders' side.

These intra-group effects are of course strictly speaking not externalities as they operate through the price system. However, some recent papers have also considered proper negative intra-group network externalities. Belleflamme and Toulemonde (2007) develop a model where agents value positively the presence of members of the other group, but may value negatively members of their own group. For instance, both advertisers and consumers benefit from a large representation of the other group (positive inter-brand externality) but advertisers are in competition for eyeballs (negative intra-brand externality). Kurucu (2008) analyses a matching problem in which agents on one side prefers more agents on the other side but less on their own side. Such a configuration of externalities can occur for matrimonial or job matching agencies.

Our paper is inspired by Belleflamme and Toulemonde (2007) and Kurucu (2008) from whom we borrow the presence of negative intra- and positive inter-groups network externalities. However, we combine the same ingredients in a different way. In our framework, an additional consumer generates a negative intra-group and a positive inter-group network externality. Roughly speaking, the utility of a consumer is increasing in the number of providers and is decreasing in the number of the other consumers affiliated with the same platform. On the providers' side, network externalities work on the opposite direction. In other words, the utility of a provider is increasing in the number of providers affiliated to the same platform (positive intra-group network externality), while it is decreasing in the number of consumers present on the other side (negative inter-group network externality). The combination of these two characteristics leads to our concept of *common network externalities*: both groups benefit, possibly with different intensities, from an increase in the size of one group and from a decrease in the size of the other group.

The paper is organized as follows. Section 2 lays out the model. In Section 3, we determine the equilibrium and study its properties. Some illustrations are provided in Section 4.

2 Model

Consider two platforms $j = \{1, 2\}$ located at both endpoints of the Hotelling's segment. They compete for two groups of agents $i = \{A, B\}$ of mass 1 (group A) and m (group B) respectively. Agents of each group are uniformly distributed over an interval of length 1. The utilities of both groups exhibit quadratic transportation costs with parameters t_A and t_B respectively. For the sake of simplicity, we shall refer to members of group A as "customers" while group B individuals are considered as "providers". We shall return to this interpretation later.

The utility of a group A individual (a customer), located at z, who patronizes platform j (consumes one unit of its product) is given by

$$V = \overline{V} + \gamma q_j - P_j - t_A (z - x_j)^2 ,$$

where P_j denotes platform j's price, while γ measures the preference intensity for a quality q_j. An individual of group B (a provider), located at y, who works (a given number of hours) for platform j has utility

$$U = \overline{U} + \theta q_j + w_j - t_B (y - x_j)^2 ,$$

where w_j denotes the wage paid by platform j, while θ is the preference for quality q_j. Without loss of generality, reservation utilities are equal to zero. Consequently, the constants $\overline{V}$ and $\overline{U}$ denote the gross utility on sides A and B; they are assumed to be sufficiently large to ensure full coverage on both sides of the market. Platforms maximize profits and simultaneously set their price/wage vectors (P_j, w_j), $j = 1, 2$.

Let n_j^i denote the *share* of type $i = A, B$ individuals affiliated with platform $j = 1, 2$, while N_j^i denotes the *number* of affiliates. With our normalizations we have $N_j^A = n_j^A$ and $N_j^B = mn_j^B$. The quality offered by platform j depends on its number of affiliates in both groups and is determined by

$$q_j = f(N_j^A, N_j^B) = f\left(n_j^A, mn_j^B\right) .$$

This function specifies what we refer to as a "*common network externality*" and which is defined as follows.

Definition 1 *A common network externality, described by the function* $q_j = f(N_j^A, N_j^B)$*, occurs when both sides value, possibly with different intensities, the same network externality.*

An important feature of this definition is that the functional form f is the same on both sides. In other words, customers and providers agree on the ranking of quality levels. However, the taste for quality (measured by γ and θ) can differ between customers and providers. Since A refers to the consumer side, while index B is used for the provider side we assume $\partial f/\partial N_i^A < 0$ and $\partial f/\partial N_i^B > 0$.[6]

Prominent examples of such a common externality can be found in the health care and education markets. In the hospital sector, for instance, on can think of n_j^A as representing the number of patients while mn_j^B stands for the number of physicians. Alternatively, n_j^A can be interpreted as the number of students while mn_j^B stands for the number of teachers. In both of these cases one would expect quality to increase with mn_j^B and to decrease with n_j^A. A formulation often used in the literature on education and health is given by $q_j = \left(cmn_j^B/n_J^A\right)^\delta$, where c and δ are constants. With this specification the quality offered by a hospital or a university depends upon provider/patient or teacher/student ratio, and the function f is homogenous of degree 0.[7] More generally, one can assume that the function specifying the quality is homogenous of degree k, which may or may not be positive. For instance when quality is specified by

$$q_j = \left(N_j^B\right)^\beta / \left(N_J^A\right)^\alpha, \tag{1}$$

f is homogenous of degree $\beta - \alpha$. We do *not* impose this assumption when determining the equilibrium in the next section. However, it will turn out that the equilibrium has

[6]While this assumption reflects the spirit of our definition of the common network externality, it is of no relevance to our formal analysis. Specifically, Propositions 1, 2 and 3 do not rely on this assumption. However, the assumption is important for the interpretation of our results (and their economic content). It is also key to understanding the difference between our framework and Kurucu (2008) and Belleflamme and Toulemonde (2007). In our setting, the utility functions on both sides, $V^A(N_A, N_B)$ and $U^B(N_A, N_B)$ are both decreasing in N_A and increasing in N_B. In their framework (but with our notations), $V^A(N_A, N_B)$ is increasing in N_B but decreasing in N_A while $U^B(N_A, N_B)$ is increasing in N_A but decreasing in N_B.

[7]Krueger (2003) provides a cost-benefit analysis of class size reduction. He shows that the internal rate of return of a class size reduction from 22 to 15 students is around 6%.

specific properties when the common externality is homogenous of degree k. We shall focus more particularly on the realistic case $k = 0$ which has some strong implications.

Using subscripts to denote the derivatives of f with respect to its first and second arguments (N_j^A and N_j^B respectively) and applying Euler's law yields the following property.

Property 1 *When a common network externality is homogenous of degree k then $N_j^A f_A(N_j^A, N_j^B) + N_j^B f_B(N_j^A, N_j^B) = kf(N_j^A, N_j^B)$.*

3 Equilibrium

First, we characterize the demand functions (market shares) on both sides. Then, we determine the price equilibrium and study the properties of the corresponding allocation.

3.1 Demand functions

On group A's side, the marginal consumer indifferent between two platforms is determined by

$$\tilde{z} = \frac{1}{2} + \frac{1}{2t_A} \left[\gamma (q_1 - q_2) - (P_1 - P_2) \right],$$

while in group B, the marginal provider is given by

$$\tilde{y} = \frac{1}{2} + \frac{1}{2t_B} \left[\theta (q_1 - q_2) + (w_1 - w_2) \right].$$

As both sides are fully covered, demand levels are equivalent to market shares. On side A, we have $n_1^A = \tilde{z}$ and $n_2^A = 1 - \tilde{z}$, while on side B, $n_1^B = \tilde{y}$ and $n_2^B = (1 - \tilde{y})$. Defining the quality differential between platforms as

$$g\left(n_1^A, mn_1^B\right) = f\left(n_1^A, mn_1^B\right) - f\left(1 - n_1^A, m(1 - n_1^B)\right) = q_1 - q_2,$$

the demand functions are determined by the following system of implicit equations

$$n_1^A = \frac{1}{2} + \frac{1}{2t_A} \left[\gamma g\left(n_1^A, mn_1^B\right) - (P_1 - P_2) \right], \tag{2}$$

$$n_1^B = \frac{1}{2} + \frac{1}{2t_B} \left[\theta g\left(n_1^A, mn_1^B\right) + (w_1 - w_2) \right]. \tag{3}$$

Let $\phi = (\gamma, \theta, t_A, t_B, m)$ denote the vector of exogenous parameters. Equations (2)–(3) define the demand levels of platform 1, $n_1^A(P_1, P_2, w_1, w_2, \phi)$ and $n_1^B(P_1, P_2, w_1, w_2, \phi)$, as functions of both platforms price/wage vectors and of the exogenous variables[8]. With full market coverage on both sides, demand levels of platform 2 are then also fully determined and given by $n_2^A = 1 - n_1^A$ and $n_2^B = 1 - n_1^B$.

Totally differentiating (2)–(3), solving and defining $g_A = \partial g/\partial N_1^A$, $g_B = \partial g/\partial N_1^B$ with

$$\Phi = \left(1 - \frac{\theta}{2t_B} m g_B\right)\left(1 - \frac{\gamma}{2t_A} g_A\right) - \frac{\gamma\theta g_A m g_B}{4t_A t_B},$$

yields the following properties of the demand functions:

$$\frac{\partial n_1^A}{\partial \gamma} = \frac{g\left(n_1^A, m n_1^B\right)}{2t_A\Phi}\left(1 - \frac{\theta}{2t_B} m g_B\right), \tag{4}$$

$$\frac{\partial n_1^B}{\partial \gamma} = \frac{g\left(n_1^A, m n_1^B\right)\theta g_A}{4t_A t_B\Phi}, \tag{5}$$

$$\frac{\partial n_1^A}{\partial P_1} = -\frac{\left(1 - \frac{\theta}{2t_B} m g_B\right)}{2t_A\Phi}, \tag{6}$$

$$\frac{\partial n_1^B}{\partial P_1} = -\frac{\theta g_A}{4t_A t_B\Phi}, \tag{7}$$

$$\frac{\partial n_1^A}{\partial w_1} = \frac{\gamma m g_B}{4t_A t_B\Phi}, \tag{8}$$

$$\frac{\partial n_1^B}{\partial w_1} = \frac{\left(1 - \frac{\gamma}{2t_A} g_A\right)}{2t_B\Phi}. \tag{9}$$

These properties are used in the next subsection to determine the market equilibrium.

3.2 Equilibrium prices and allocation

Platform 1 maximizes its profit with respect to P_1 and w_1 and solves

$$\max_{P_1, w_1} \Pi_1 = P_1 n_1^A(P_1, P_2, w_1, w_2, \phi) - m w_1 n_1^B(P_1, P_2, w_1, w_2, \phi).$$

[8] We assume throughout the paper that demands are well defined and unique for any price levels. When $q_j = m n_j^B - n_j^A$, it is straightforward that demands are uniquely defined. Appendix A shows that it is also the case when $q_j = \left(m n_j^B / n_J^A\right)$.

The first-order conditions are given by

$$\frac{\partial \Pi_1}{\partial P_1} = n_1^A + P_1 \frac{\partial n_1^A}{\partial P_1} - m w_1 \frac{\partial n_1^B}{\partial P_1} = 0, \tag{10}$$

$$\frac{\partial \Pi_1}{\partial w_1} = -m w_1 \frac{\partial n_1^B}{\partial w_1} - m n_1^B + P_1 \frac{\partial n_1^A}{\partial w_1} = 0. \tag{11}$$

The first two terms of equations (10) and (11) represent the traditional marginal income tradeoff, while the third terms capture the two-sided market feature. Specifically, an increase in the price charged on one side of the market also affects the demand on the other side. Equations (10) and (11) determine platform 1's best-reply functions: $P_1 = \widetilde{P}_1(P_2, w_2, \phi)$ and $w_1 = \widetilde{w}_1(P_2, w_2, \phi)$. Platform 2's best-reply functions $P_2 = \widetilde{P}_2(P_1, w_1, \phi)$ and $w_2 = \widetilde{w}_2(P_1, w_1, \phi)$ can be determined in a similar way by the maximization of Π_2. Solving these best-reply functions yields the Nash equilibrium $[P_1^*, w_1^*), (P_2^*, w_2^*)]$.

In the remainder of the paper, we concentrate on symmetric equilibria in which both platforms charge the same prices, pay the same wages and equally split the market on both sides ($n_1^A = n_2^A = 1/2$ and $n_1^B = n_2^B = 1/2$) so that quality levels are also identical ($g = 0$). To determine the symmetric equilibrium we solve (10) and (11). The derivatives of n_1^A and n_1^B that appear in these expressions are given by equations (6)–(9); with $n_1^A = 1/2$ and $n_1^B = 1/2$, they are all well determined and the problem reduces to the solution of a system of linear equations.[9]

Using the Cramer's rule, we obtain

$$P_1 = \frac{\frac{1}{2}\left[\frac{\partial n_1^B}{\partial w_1} + m \frac{\partial n_1^B}{\partial P_1}\right]}{D}, \qquad w_1 = \frac{\frac{1}{2}\left[m \frac{\partial n_1^A}{\partial P_1} + \frac{\partial n_1^A}{\partial w_1}\right]}{mD}, \tag{12}$$

where

$$D = \left[-\frac{\partial n_1^A}{\partial P_1} \frac{\partial n_1^B}{\partial w_1} + \frac{\partial n_1^A}{\partial w_1} \frac{\partial n_1^B}{\partial P_1} \right]. \tag{13}$$

[9] The derivates depend on n_1^A and n_1^B (which are by definition set at 1/2) but not directly on P_1 and w_1. The underlying reason for this simplification is that for the determination of demands only *differences* in prices and wages matter; see expressions (2)–(3).

Substituting from (6)–(9) and rearranging yields

$$\frac{1}{2}\left[\frac{\partial n_1^B}{\partial w_1}+m\frac{\partial n_1^B}{\partial P_1}\right]=\frac{1}{2}\left[\frac{2t_A-\gamma g_A-\theta m g_A}{4t_At_B\Phi}\right], \tag{14}$$

$$\frac{1}{2}\left[m\frac{\partial n_1^A}{\partial P_1}+\frac{\partial n_1^A}{\partial w_1}\right]=\frac{1}{2}\left[\frac{-2t_B+\theta m g_B+\gamma g_B}{4t_At_B\Phi}\right], \tag{15}$$

$$D=\frac{1}{4t_At_B\Phi}, \tag{16}$$

where g_A and g_B are evaluated at $(1/2, m/2)$. Substituting (14)–(16) into (12), simplifying and defining $g_A^*=g_A(1/2,m/2)$ and $g_B^*=g_B(1/2,m/2)$ establishes the following proposition:

Proposition 1 *Symmetric equilibrium prices are given by*

$$P_j^*=t_A-\frac{1}{2}(\gamma+m\theta)\,g_A^*, \tag{17}$$

$$w_j^*=-t_B+\frac{1}{2}(\gamma+m\theta)\,g_B^*,\ \ \forall j=1,2. \tag{18}$$

Observe that this proposition provides a closed form solution with explicit expression for the equilibrium prices. To interpret these expressions recall that per our assumption on f, we have $g_A^*<0$ and $g_B^*>0$. As usual, on both sides, platforms take advantage of transportation costs to increase their markup. Network externalities, on the other hand, affect prices in a more interesting way. Because of their specific nature *i.e.* common externality, their impact is "cumulative", as is reflected by the factor $(\gamma+m\theta)$ in the second term. In other words, network externalities affect prices on the side where they occur by $(\gamma+m\theta)\,g_i^*$, and they are partially or entirely shifted to the other side by $(\gamma+m\theta)\,g_h^*$, with $h\neq i$.

Using (17) and (18) we can now express equilibrium profits as

$$\begin{aligned}\Pi_j^*&=\frac{1}{2}\left(P_j^*-mw_j^*\right)\\&=\frac{1}{2}(t_A+mt_B)-\frac{1}{4}(\gamma+m\theta)\left(mg_B^*+g_A^*\right),\end{aligned} \tag{19}$$

so that

$$\frac{\partial\Pi_j^*}{\partial\theta}=m\frac{\partial\Pi_j^*}{\partial\gamma}=-\frac{1}{4}\left(mg_B^*+g_A^*\right). \tag{20}$$

Equation (20) establishes the following proposition:

Proposition 2 *The impact of individual valuations of quality γ and θ on (symmetric) equilibrium profits is described by*

$$m\frac{\partial \Pi_j^*}{\partial \gamma} = \frac{\partial \Pi_j^*}{\partial \theta} \lesseqqgtr 0 \qquad \textit{if and only if} \qquad -g_A^* \lesseqqgtr mg_B^*, \forall j = 1, 2.$$

Proposition 2 shows that the impact of the externality (or, more precisely of the relevant preference parameters) on profits depend on the relative strength of the externalities created by the membership on the two sides. To interpret this proposition, we shall concentrate on the case where the common externality is homogenous of degree k. According to Property 1 we then have $N_1^B f_A + N_1^A f_A = kf$. Moreover, at a symmetric equilibrium, we have:

$$\begin{aligned} g_A &= 2f_A, \\ g_B &= 2f_B, \end{aligned}$$

so that

$$mg_B^* + g_A^* = 4kf\left(\frac{1}{2}, \frac{m}{2}\right).$$

Proposition 3 *When f homogenous of degree k the symmetric equilibrium implies $mg_B^* + g_A^* = 4kf\left(\frac{1}{2}, \frac{m}{2}\right)$, so that*

$$sign\left(\frac{\partial \Pi_j}{\partial \gamma}\right) = sign\left(\frac{\partial \Pi_j}{\partial \theta}\right) = -sign\left(k\right), \qquad j = 1, 2 \tag{21}$$

Proposition 1 has shown that network externalities have a cumulative effect on prices. Proposition 3 shows how profits and competition intensity are affected. When the degree of homogeneity of the common network externality is positive, platforms' profit decrease in the externality's parameters. In other words, the common network externality increases the competition intensity between platforms. This outcome occurs because in this case, platforms have to pay a higher relative price w_j^*/P_j^* on providers' side. Recall that we have a positive externality created by one side and a negative externality generated by the other side. We can think about the case of $k > 0$ as that where the global impact of the network externalities is positive. In other words, if we increase membership on both sides in the same proportion, f (and thus quality) increases. From that perspective we can think of our finding as a generalization of the conventional result in the literature (relating profits and intensity of competition to the sign of the externality).

When $k < 0$, on the other hand, we have a negative global externality which brings about extra profits for the platforms. The wage paid to providers continues to increase in the network externality parameters. However, this increased cost is now more than fully shifted to the consumers. This is because quality is more sensitive to the number of consumers–and recall that quality decreases with the number of consumers. Consequently, the common network externality tends to reduce the intensity of competition on the consumers' side. The firms are then able to extract more extra rents from the consumers than they have to concede to the providers.

Finally, let us consider the special case in which the homogeneity degree is equal to 0 *i.e.* $k = 0$.

Corollary 1 *For a symmetric equilibrium,* $\forall j = 1, 2$ *with* f *homogenous of degree* 0 *we have* $mg_B^* + g_A^* = 0$*, so that*

$$\frac{\partial \Pi_j}{\partial \gamma} = \frac{\partial \Pi_j}{\partial \theta} = 0.$$

In that case, the intensity of preferences for quality (and thus the intensity of the externality) has no impact on equilibrium profits. Specifically, profit levels are the same when the externality does not matter at all (in which case $\gamma = \theta = 0$) as when one or both of these parameters are positive. In other words, a common network externality that is homogenous of degree zero, has no impact on the intensity of competition which is in stark contrast with conventional results obtained in the two-sided market literature (for alternative forms of externalities). The expressions for the prices (17) and (18) make it clear why this result emerges. Assuming $g_B^* > 0$ (providers produce a positive externality) the externality in itself (or an increase in its valuation on either side) increase "rents" on the providers' side: wages are increased. However, this increase in wages has no impact on profits because it is entirely shifted to consumers: the price increase exactly matches the increase in wages.

4 Examples and illustrations

To illustrate the results and provide some additional insight, we shall now present the full analytical solution for three special cases and give a numerical illustration for one of them. First, we consider the case there the externality simply depends on the ratio between membership on both sides (so that f is homogenous of degree zero). Then we consider a setting with different degrees of homogeneity. Finally, we provide an example for the non homogenous case.

When $q_j = \left(mn_j^B / n_j^A \right)^\delta$, the common network externality is homogeneous of degree zero. Proposition 1 implies that equilibrium prices are given by

$$P_j^* = t_A + 2(\gamma + m\theta)\,\delta m^\delta,$$
$$w_j^* = -t_B + 2(\gamma + m\theta)\,\delta m^{\delta-1}.$$

With this price structure, it is clear that platforms only transfer rents from "consumers" to "providers" and have equilibrium profits independent of the network externalities. To confirm this, note that with this specification (19) reduces to

$$\Pi_j^* = \frac{1}{2}(t_A + mt_B), \qquad j = 1, 2,$$

which does not depend on γ or θ. It is worth noticing that (as discussed in the introduction) this functional form (with quality depending on the ratio), has interesting applications for education and heath care sectors.

To illustrate this case numerically, we consider the following parameters' values: $t_A = 0.3, t_B = 0.4, \gamma = 0.3, \theta = 0.5, m = 0.05$ and $\delta = 1$. The symmetric equilibrium leads to $P_i = 0.3325$ and $w_i = 0.25$ with a level of profit $\Pi_i = 0.16$. The following figure represents the profit function of firm 1 as a function of admissible[10] P_1 and w_1 evaluated at symmetric equilibrium prices $P_2 = 0.3325$ and $w_2 = 0.25$. One can see that the symmetric equilibrium constitutes effectively a global best reply for platform 1 (and thus by symmetry for platform 2). Though of limited scope this observation is interesting. As usual in two-sided market models, second-order condition are quite

[10] By admissible, we mean that the demands lie between 0 and 1 and that profits are positive.

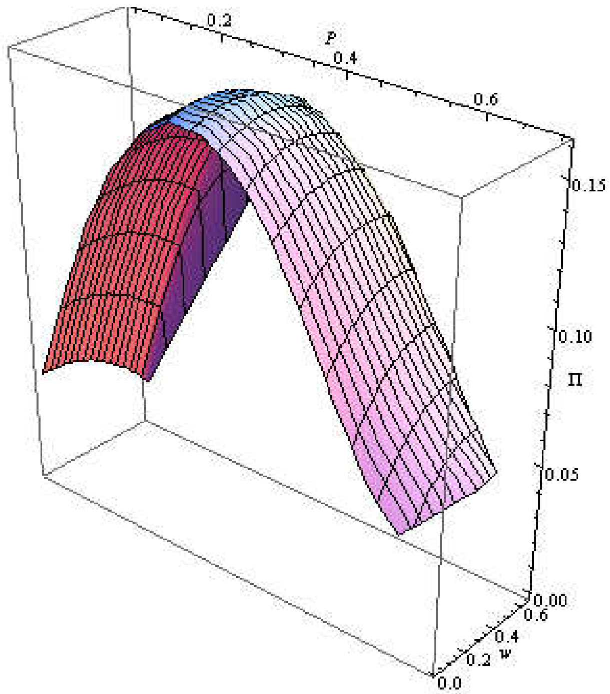

Figure 1: Profit function of platform 1 (when prices of platform 2 are set at their equilibrium levels).

complex in our setting. The example shows that at least for a simple specification of f, the existence of the equilibrium is not a problem.

We now turn to the more general case where f is specified by (1) so that the externality is homogenous of degree $k = \beta - \alpha$. The nice feature about this specification is that it shows that negative levels of k do not have to be ruled out. Equilibrium prices and profit levels are given by $(j = 1, 2)$

$$P_j^* = t_A - \left(\frac{1}{2}\right)^{\beta-\alpha-1} \alpha m^\beta (\gamma + m\theta), \tag{22}$$

$$w_j^* = -t_B + \left(\frac{1}{2}\right)^{\beta-\alpha-1} \beta m^{\beta-1} (\gamma + m\theta), \tag{23}$$

$$\Pi_j^* = \frac{1}{2}(t_A + mt_B) - \left(\frac{1}{2}\right)^{\beta-\alpha} m^\beta (\gamma + m\theta)(\beta - \alpha). \tag{24}$$

Not surprisingly, network externalities affect profits according to the sign of $k = \beta - \alpha$. When $\alpha > \beta$, quality offered by the platforms are relatively more sensitive to the number of consumers than to the number of providers. Therefore, platforms can charge a higher relative price on the consumers' side for the quality provided without transferring all the network externalities rents to the providers (negative global externality).

Finally, let us consider a case where the homogeneity property does not hold at all. For instance, think about a case where quality depends positively on the ratio $\left(mn_j^B/n_j^A\right)$ but also depends positively on the "volume" of consumers treated by the provider[11]. In such a case, we have $q_j = \left(mn_j^B/n_j^A\right) + cn_j^A$ with $c > 0$ small enough to ensure that we continue to have a negative intra-group externality on the consumers' side. Equilibrium prices, wages and profits are now given by

$$
\begin{aligned}
P_j^* &= t_A + 2\left(\gamma + m\theta\right)\left(m - \frac{c}{2}\right), \\
w_j^* &= -t_B + 2\left(\gamma + m\theta\right)\left(m\right), \\
\Pi_j^* &= \frac{1}{2}\left(t_A + mt_B\right) - \left(\gamma + m\theta\right)c.
\end{aligned}
$$

It is worth noticing that because of the parameter c (which reduces the intensity of the negative intra-group externality on the consumers' side) the common network externality does not satisfy the homogeneity property. Then, platforms' profit are reduced in equilibrium because they charge a lower price on the consumers' side. This lower price is not outweighed by a lower wage paid on the providers' side.

5 Conclusion

This paper has examined a market in which two platforms compete for consumers and for providers in the presence of a common network externality. For a given price structure, consumers and providers value the same index of quality (albeit possibly with different intensities). This index depends positively on the number of providers, but negatively on the number of consumers. We have shown that the symmetric equilibrium has some specific properties. First, the common network externality has a cumulative effect on prices: its effect on one side's price is, partially or entirely, shifted to the other side of the market. Second, when the quality index is specified by a homogenous function, the

[11] This assumption makes sense for a hospital. The quality of care can depend on the volume of patients treated.

sign of the homogeneity's degree determines the net impact of the externality, *(i),* on prices on both sides of the market and, *(ii),* on competition intensity. In the specific, but empirically appealing case where the homogeneity's degree is equal to zero, the presence of the common network externality has no impact on equilibrium profits; the price increase on one side of the market is totally shifted to the other side.

Our analysis could be extended in at least two different ways. First, it would be interesting to consider the case where the market is not fully covered (on one or two sides). When the severity of their illness is not high enough, some of the potential patients may decide not to consume medical (hospital) services at all. On the providers' side, some of the physicians may prefer to remain self-employed. A second possible extension concerns the type of hospitals that compete in the market for patients. For instance, for-profit providers could coexist with not-for-profit or physician's-owned hospitals. This would admittedly not be a trivial extension because in the case of mixed oligopolies one can no longer concentrate on symmetric equilibria. These issues are on our research agenda.

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Appendix

A Uniqueness of demand functions

In the ratio case we have $q_1 = mn_1^B/n_1^A$. From(3), one obtains

$$g\left(n_1^A, mn_1^B\right) = \frac{2t_B}{\theta}\left[n_1^B - \frac{1}{2} - \frac{\Delta w}{2t_B}\right],$$

where $\Delta w = w_1 - w_2$.

Substituting this expression in (2) yields:

$$n_1^A = \frac{1}{2} + \frac{\gamma}{t_A}\frac{t_B}{\theta}\left[n_1^B - \frac{1}{2} - \frac{\Delta w}{2t_B}\right] - \frac{\Delta P}{2t_A}, \tag{25}$$

where $\Delta P = P_1 - P_2$. It is worth noticing that if n_1^B is unique, then n_1^A is also unique.

From (3), one has:

$$n_1^B\left[1 - \frac{\theta m}{2t_B}\left(\frac{1}{n_1^A} + \frac{1}{1-n_1^A}\right)\right] = \frac{1}{2} + \frac{1}{2t_B}\left(-\frac{\theta m}{1-n_A^1} + \Delta w\right),$$

Multiplying both sides of this equality by $n_1^A\left(1-n_1^A\right)$ yields:

$$n_1^B\left[n_1^A\left(1-n_1^A\right) - \frac{\theta m}{2t_B}\right] = \frac{1}{2}\left[n_1^A\left(1-n_1^A\right)\left[1 + \frac{\Delta w}{t_B}\right] - \frac{\theta}{t_B}n_1^A\right].$$

Substituting (25) gives n_1^B as a solution to a third degree polynomial equation. Some additional tedious computations show that only one of the three solutions is a real numbers.[12]

[12] The market share belongs to $[0,1]$. If the solution of the polynomial is negative or superior to 1, it means that we are in presence of a corner solution at the demands level.