Essays on International Trade and Economic Growth

A Dissertation

Presented to the Faculty of Economics

of

Universidad del Rosario

in Candidacy for the Degree of

Doctor of Philosophy in Economics

by:

IADER GIRALDO SALAZAR

Dissertation Director:

DR. Fernando Jaramillo Mejía

June 2015

To my wife, my parents, my brothers and my in-laws, all of whom have helped immensely in the building of this life project.

Acknowledgments

I am deeply appreciative of the numerous people who have supported my work and continually encouraged me throughout the writing of this dissertation. Without their time, thoughtful feedback and patience I would not have been able to complete this dissertation.

First of all, I wish to express my gratitude to my supervisor, Fernando Jaramillo Mejia PhD, for his invaluable role as both a teacher and advisor. In particular, I sincerely appreciate his astute advice and continual encouragement.

I would like to thank Thierry Verdier for his advice during my internship at the Paris School of Economics; his pertinent recommendations have contributed to a great extent to the development of this research. In addition, I wish to thank Hubert Kempf, Matthieu Crozet and Cecilia García-Peñalosa, all of whom were generous with their time during my stay in Paris and when listening to this thesis and providing me with various clear suggestions.

I would also like to thank the professors of the Faculty of Economics at the Universidad del Rosario and the participants of the different seminars in which the articles that comprise this thesis were first exposed; the resulting recommendations have significantly improved the development of the present dissertation.

Finally, I would like to thank my wife and my family for their patience and comprehension during this long period of study, without their support this project had not have been possible.

Contents

Chapter 1

Introduction

At present, we are witnessing globalization as a truly worldwide phenomenon. Trade agreements among differing countries, a reduction in trade costs, the mobility of production factors, the free flow of information and so on are all proof of the present day era of globalization. Countries are trading with one another more and more every day and the effects of international trade on economies represent a central discussion in all economic spheres.

In spite of increasing trade around the world and the promotion of globalization by multilateral organisms such as WTO and IMF, the effects of international trade are not yet clear. Economics literature concerning the effects of international trade on economic growth and welfare remains ambiguous in terms of both theoretical models and empirical research. The present thesis tries to contribute to the theoretical debate surrounding the effects of dynamic international trade, focusing in particular on the implications for economic growth, welfare and changes in the preferences of individuals.

This dissertation consists of three articles that double as chapters. In the first chapter I develop an international trade model with the Home Market Effect, with differences in income and productivity between countries and sectors. The inclusion of non-homothetic preferences allows the inclusion of an original channel in the determination of international trade effects, the demand composition. The model allows for the identification of static effects of international trade through three main determinants: population size, productivity levels and demand composition. Interactions among these channels determine the trade effects in terms of industrialization and the welfare of the countries that trade.

In the second chapter I analyze international trade effects on economic growth. I consider an endogenous economic growth model in an open economy with the Home Market Effect and non-homothetic preferences. The implications of such modelling allow for an understanding of the heterogeneity of dynamic international trade effects. Convergence or divergence in the economic growth of trade partners depends on the similarity of countries in terms of income, productivity and demand composition, and the technology transfer between them. The more similar the countries are, the more likely the convergence. Nevertheless, welfare can improve or decline after trade depending on convergence or divergence in the income levels of the countries.

Finally, in the third chapter I consider a model of dynamic international trade in order to analyze the effects of international trade on the preferences of the agents and the implications for economic growth. The model used is based on the Home Market Effect with external habit formation (catching up with the "Joneses") and learning by doing in production. I find that the historical composition of consumption within the countries determines industrialization levels after trade. The consumption habits of the countries converge at the same level and composition shows the interrelation of consumption preferences under trade. In spite of this convergence in consumption preferences, income levels may converge or diverge among trade partners depending on the historical composition of consumption, supernumerary income and productivity levels. The added effect of convergence in the habits of consumption and convergence or divergence in income levels generates different results for the welfare levels of countries after trade, sometimes where the autarky is strictly preferred to trade.

This thesis proceeds as follows. Each chapter corresponds to an article. Every article contains an introduction that references the literature review and presents some stylized facts about the specific topic of each article. After the introduction, each article outlines the fundamentals of the model, the model under a closed and open economy, and the welfare effects in relation to the autarky scenario.

Chapter 2

Productivity, Demand and the Home Market Effect.

2.1 Introduction

In an increasingly globalized world, bilateral and multilateral trade agreements occur more and more frequently and do so among a larger variety of countries. Relationships between developed countries in the European Union, the ascent of the BRIC countries (Brazil, Russia, India and China) in the world market, and treaties between developed and developing countries, such as NAFTA, are becoming more frequent. Within this context, a study of the effects of international trade on well-being is of great relevance. New trade theory and the performance of countries newly liberalized for world trade suggest the importance of certain questions, such as, why are trade effects different between countries and what are the main variables that determine whether trade effects are positive or negative?

In this article, we consider the aforesaid questions through a general equilibrium model of bilateral trade with Home Market Effect (HME). In contrast to the standard literature of HME (Krugman 1980,1991, etc.), we introduce additional mechanisms by which trade increases or decreases Gross Domestic Product (GDP) and well-being. This allows us to identify and analyze the most suitable trade partners for an economy. Indeed, where non-homothetic preferences are at play, income differences between countries and differences in productivity between sectors and between countries contribute to the structure of demand inside each country. At the same time, in a model with HME, demand acts over the effects of trade liberalization on welfare and the industrialization of countries.

The model proposed herein analyses interactions between supply-side variables, like productivity, and demand-side factors, like the size and the composition of internal demand. It identifies three mechanisms through which HME acts: population size, demand composition and levels of productivity. The consequences of international trade in terms of industrialization - as evident under positive transportation costs - can be analyzed through the interplay of these three mechanisms. In fact, this article shows that population size, demand structure and productivity levels determine the level of industrialization generated after international trade. In addition, we discuss the effects of international trade on welfare, which are positive whenever the global market of manufacturing increases after trade.

Traditional models of international trade focus on the supply side. In contrast, the new theory of international trade, particularly that of Krugman (1980), takes into account the effects of demand on trade. The Home Market Effect establishes that the market size of a closed economy determines its trade patterns and industrial development. This is an important factor that is first mentioned by Linder (1961). This approach has allowed for the identification of agglomeration and dispersion effects generated by trade, which show the positive and negative effects of international trade on economic performance, depending on the size of the market of each economy. Helpman and Krugman (1985).

The HME was first proposed by Corden (1970) and then extended by way of formal changes in a seminal article by Krugman (1980). Further modifications have mainly been carried out by the same author and presented in Helpman and Krugman (1985). The literature surrounding HME focuses on population size as a demand element in determining patterns of trade and the industrial distribution of countries, showing transportation costs as the crucial variable. Although this literature does not exclude the possibility of additional mechanisms, it does not give sufficient importance to these and assumes the size of the country as being the only channel through which demand determines trade patterns.

The traditional HME suggests that the most densely populated countries concentrate the production of manufacturing internationally. However, in contrast with reality, China and India would have benefited the most from trade relations at the international level. In particular, China and India have populations 4.32 and 3.95 times larger than that of the United States respectively.[1] However, trade between these countries has not allowed full specialization in the manufacturing industry for the former examples (as predicted by the traditional HME), and much less any specialization in the other sectors for a commercial counterpart.

In the presence of HME, the number of agents in an economy is a fundamental variable in determining trade patterns and the distribution of industrial production among countries that trade. However, there are additional variables that complement this, and which therefore contribute to the final effects of international trade on GDP. Income and competitiveness differentials determine the composition of demand for countries, but these variables are shelved in HME standard modeling because this theory assumes homothetic preferences. With the inclusion of the variables analyzed in our model, the size

[1] World Bank data 2013.

of the market for industrial goods is limited by both the number of agents that comprise it and the ability to generate demand in relation to their commercial counterpart.

The importance of the components of demand can be justified by Engel's law (Engel 1857), which establishes that the income elasticity of demand for food is less than one (Banks, Blundell, and Lewbel, 1997). Accordingly, the higher an agent's income, the lower the proportion of food spending. There is a solid empirical bibliography that confirms this law (Hamilton 2001, Banks, Blunder and Lewbel, 1997; etc.). Consumer surveys about individual consumer spending in the United States show that spending on food in 1946 was to the order of 24% of total consumption, while in 2011 it was only 7% (Bureau of Labor Statistics, 2014). This relationship is important in an international context due to the heterogeneity in income levels between countries. Indeed, in terms of per capita the level of income in the ninth decile of the distribution of countries is 72 times higher than in the first.[2] This element, clearly differentiated, has direct consequences for the patterns of demand in each country. The countries with low levels of income must have a structure of demand mainly concentrated in vital consumer goods. In contrast, countries with higher levels of income principally demand manufactured goods. The higher the level of income, the higher the demand for manufactured items like cars, computers, cell phones and so on. Mitra and Trindade (2005); Bohman and Nilsson (2007); Dalgin, Mitra and Trindade (2008).

This article is based on empirical evidence regarding the importance of the composition of demand in patterns of trade and specific specializations within countries (Markusen 1986; Dalgin, Mitra and Trindade 2008). This evidence shows the presence of HME between different partner countries and the interactions among supply and demand elements (Davis and Weinstein 1996; Davis, Hanson and Weinstein 2003; Xiang 2004). Indeed, Yu (2005) shows the differentiated effects of HME after including symmetrical transportation costs for both goods and differences in the elasticity of substitution. Chung (2006) shows the importance of demand composition in the determination of HME. Crozet and Trionfeti (2008) show the non-linearity of HME. Huang and Huang (2011) demonstrate the possibility of reversing HME with a technological advantage in production, based on a sample of six types of industry. Indeed, the evidence supports the importance of building a good indicator of HME.

Less conclusive estimates about the presence of HME, such as Davis (1998) and Antweile and Trefler (2002), reveal the presence of additional channels that are not taken into account by the traditional model. Even the lack of a robust HME effect may be due to the omission of key channels in the determination of the structure of demand and thus due to the omission of key explanatory variables.

The most common procedure for including the determinants of demand in international trade models entails the incorporation of non-homothetic preferences. Generally, the model assumes a utility

[2] Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, July 2012.

function with a minimum consumption of vital goods (agricultural). After exceeding this threshold of survival, income differentials change the level of demand for manufactured goods in each country, thus relating the composition of demand in each region to income levels. Furthermore, the addition of differences in productivity between countries allows us to consider the effects of technological change on income and prices, and as a consequence of the composition of demand and competitiveness in each nation. This strategy is used by Stokey (1991) and Matsuyama (2000) in a product cycle model with a North-South asymmetrical countries scenario. Zweimuller (2000) and Foellmi and Zweimuller (2006) also use the same procedure to link the distribution of income to the composition of demand and economic growth.

The effects of trade between symmetric countries (North-North or South-South), as well as asymmetric countries (North-South), can be studied by the model proposed herein. In addition, studying HME through different channels gives robustness to empirical exercises that seek to establish the presence of HME in international trade. Indeed, when it is only the population size that is included in an econometric exercise, there exists a bias due to omitted variables. The model shows that the inclusion of additional variables that determine the structure of demand, such as productivity among countries and among sectors, differences in income and the composition of the population, allow for a more robust analysis of the effects of HME in relation to international trade.

This article consists of six sections, including the introduction. The second section presents the characteristics of the model, the third shows the effects of an open economy, the fourth exposes the alternative HME in the model, the fifth presents comparative statics, and the sixth section concludes.

2.2 The Model

We start from the basic structure of the theory of the Home Market Effect, as presented by Krugman (1980), but we break the homothetic preferences assumption and add the Stone-Geary utility function. In addition, differences in productivity between sectors and between countries are used.

We assume the presence of two regions, domestic and foreign ($*$),[3] independent of size. There are two types of good: homogeneous (X), which represents agricultural goods and presents constant returns to scale in production, and heterogeneous (Y), which represents manufactured goods and exhibits increasing returns to scale in production. The varieties of heterogeneous good are horizontally differentiated *à la* Dixit-Stiglitz, and the firms in this sector maximize their benefits under monopolistic competition. Labor (L) is the only existing factor of production and is mobile among sectors but immobile among countries.

With the idea of modeling the effects of demand composition on the internal market in a simple

[3] Hereafter the variables corresponding to foreign have the superscript $*$.

way, we use the Chung (2006) strategy. This assumes that the number of people consuming differs from the number of people producing; countries have the same amount of labor ($L = L^*$), but their populations (N and N^*) may be different. So it is supposed that domestic households offer one unit of work for each resident ($N = \gamma L = L$), while foreign households offer ($\frac{1}{\gamma^*}$), meaning ($N^* = \gamma^* L^*$).[4]

Intuitively, γ captures the demographic and redistribution factors that affect the relative demand for diversity goods in comparison to homogeneous goods. According to this modification, it is possible to interpret γ as the proportion of the population that earns an income.

The consumption side assumes that all households demand goods and that they symmetrically demand each variety of heterogeneous good (Y). Households in both countries have the same non-homothetic utility function.

$$U = (X - \overline{X})^{\alpha} Y^{1-\alpha} \tag{2.1}$$

$$\text{With } Y = \left(\sum_{i=1}^{n} y_i^{\sigma} \right)^{\frac{1}{\sigma}}, \; 0 < \sigma < 1, \quad n = \text{ number of varieties consumed} \tag{2.2}$$

Where $\overline{X}$ is the minimum consumption (of survival) of the homogeneous good,[5] and X is the consumption of this same good beyond the threshold of survival. Y is the aggregate consumption of all n varieties of heterogeneous good and y_i is the consumption of the ith variety.

Both goods use the same factor of production, namely labor. The production of homogeneous goods, and all varieties of the heterogeneous sector, is performed with the same function of production in both countries. The homogeneous goods sector has the following production function:

$$Q_x = L_x A_x \tag{2.3}$$

In equilibrium it should be equal to added demand for this good.

$$NX = D_x = Q_x = L_x A_x \tag{2.4}$$

Where Q_x is the aggregate production of a homogeneous good, D_x is the aggregate demand for the homogeneous good, L_x is the amount of labor used in the production of this good, and A_x is the productivity in this sector. The cost function for the heterogeneous goods sector is given by:

$$l_i = \frac{\mu}{A_y} + \frac{\beta Q_i}{A_y} = \frac{\mu}{A_y} + \frac{\beta D_i}{A_y} \quad i = 1, 2...n \quad \text{where } D_i = N y_i \tag{2.5}$$

Where Q_i and D_i are, respectively, the aggregate supply and demand of the i-th variety, l_i is the amount of labor used in the production of each variety, and A_y is productivity in this sector. Moreover, μ and β are the parameters of fixed costs and variable costs, respectively.

[4] $\gamma^* > 1$

[5] This consumption is equal for all countries, indicating that everybody needs the same minimum consumption of food to survive.

Finally, the full-employment condition is assumed, meaning that:

$$L = L_X + L_Y = \frac{D_X}{A_X} + \sum_{i=1}^{n} (\frac{\mu}{A_y} + \frac{\beta D_i}{A_y}) \tag{2.6}$$

2.2.1 Closed Economy

Consumer

Agents maximize their utility function (2.1) subject to the budget restriction. With the aim of introducing differences in the incomes of agents, as differentiated by Chung (2006), this model differentiates between members of the household who work and those who only consume. It is assumed that in one of the countries each worker supports γ additional agents that only consume; they are part of the total population but not of the employed population, and they do not receive any income. The population is proportional to the number of employees, $N = \gamma L$.[6].

$$MaxU = (X - \overline{X})^{\alpha} Y^{1-\alpha} \text{ s.t. } P_x X + P_y Y = \frac{w}{\gamma} \tag{2.7}$$

After having been normalized by the total population (N), the consumer maximization program defines the optimal quantities demanded of good X, and the aggregate demand of all varieties of heterogeneous good Y.

$$Y = \frac{1-\alpha}{P_y} \left(\frac{w}{\gamma} - P_x \overline{X} \right) \tag{2.8}$$

$$X = \frac{\alpha}{Px} \left(\frac{w}{\gamma} - P_x \overline{X} \right) + \overline{X} \tag{2.9}$$

Where P_x is the price of a homogeneous good, P_y is the price index of heterogeneous goods, which is an aggregate price of each variety's price. The optimization process defines the demand of each variety of heterogeneous good, which is determined by aggregate spending on such goods, the price of each variety i, and the sum of the price of all the n varieties.

$$y_i = \frac{p_i^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{w}{\gamma} - P_x \overline{X} \right)}{P_y^{\frac{\sigma}{\sigma-1}}} \tag{2.10}$$

$$\text{Where } \; P_y = \left(\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} \tag{2.11}$$

P_y is the index price for the heterogeneous good that is found from y_i and its implications on the aggregate demand for heterogeneous goods (2.2). This index is established as an aggregate of prices of different varieties, weighted by the degree of substitutability between them.

[6] γ represents the relationship between the population and labor. The dependence factor.

Producer

In the production of homogeneous goods there exists a competitive environment, thus implying an equilibrium with zero profit. At the same time, the price of the homogeneous good has been established as a numeraire. As a result, the wages (w) in the homogeneous goods sector are exogenous and equal to productivity:

$$P_x = 1 = \frac{w}{A_x} \tag{2.12}$$

A direct consequence of the last equation is that the per capita income, in terms of homogeneous goods, is completely determined for productivity in this sector and the dependence factor γ.

$$\frac{wL}{N} = \frac{A_x L}{N} = \frac{A_x}{\gamma} \tag{2.13}$$

For the heterogeneous goods sector, the presence of a large number of varieties implies that the price decision of each firm has virtually no effect on the marginal utility of income. Therefore, the function of demand for each variety (2.10) is such that the price elasticity of demand ($\epsilon_{y,p}$) of each of the varieties is constant and exogenous:

$$\epsilon_{y,p} = -\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} \tag{2.14}$$

$$\epsilon_{y,p} = \frac{1}{(1-\sigma)} \tag{2.15}$$

In the monopolistic competition scenario for which the production of such goods is inscribed, there is an explicit relationship between price elasticity and marginal cost, which maximizes benefits for the firms that produce some of the varieties of heterogeneous good.

$$p_i\left(1 - \frac{1}{\epsilon_{y,p}}\right) = Cmg \tag{2.16}$$

$$p = p_i = \frac{\beta w}{\sigma A_y} \tag{2.17}$$

From (2.17), the price of each variety is defined by parameters, being constant for all varieties. Inserting the prices into the zero benefits condition, determined by the free entry and exit of firms, it is possible to find the production of each variety of heterogeneous good, which is equal to the total demand for each variety.

$$\pi_i = pD_i - \left(\frac{\mu}{A_y} + \frac{\beta D_i}{A_y}\right) w = 0 \tag{2.18}$$

$$D = D_i = \frac{\mu\sigma}{(1-\sigma)\,\beta} \tag{2.19}$$

In the last equation, it is possible observe that the quantity demanded of and produced for each variety is independent of the productivity rate and country size. As is typical in models with monopolistic competition and preferences *á la* Dixit-Stiglitz, increases in productivity are reflected in a rise in the number of varieties in demand but not in the amount demanded of each, Romer (1990).

Finally, from the full-employment condition (2.6) one can obtain the number of varieties of heterogeneous good present in this economy.

$$L_y = \sum_{i=1}^{n} \left(\frac{\mu}{A_y} + \frac{\beta D_i}{A_y} \right) \tag{2.20}$$

$$n = \frac{L_y \left(1-\sigma\right) A_y}{\mu} \tag{2.21}$$

The amount of labor used in the heterogeneous sector is the total available workforce minus the quantity used in the production of homogeneous goods.

$$L_y = L - L_x \tag{2.22}$$

$$L_y = L - N \left(\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{A_x} \overline{X} \right) \tag{2.23}$$

Replacing (2.23) and (2.21) it is possible to find the number of varieties as a function of the parameters of the model, the amount of the available productivity factor, and the productivity within each sector of the economy.

$$n = \frac{\left(L - N \left(\frac{\alpha}{\gamma} + \left(\frac{1-\alpha}{A_x} \right) \overline{X} \right) \right) \left(1-\sigma\right) A_y}{\mu} \tag{2.24}$$

The Equation (2.24) can be rewritten in the following way:

$$n^A = \left(\frac{A_x}{\gamma} - \overline{X} \right) \frac{(1-\sigma)(1-\alpha)}{\mu} \frac{A_y}{A_x} N \tag{2.25}$$

$$n^A = (A_x - \gamma \overline{X})(1-\sigma)(1-\alpha) \frac{A_y}{A_x} \frac{L}{\mu} \tag{2.26}$$

For the last equation (2.25), it can be noted that the number of varieties (n) produced in a country corresponds to its level of industrialization.[7] Basically, this depends on three particular elements. First, the comparative advantages ($\frac{A_y}{A_x}$), defined by the ratio between the productivity of the two sectors of the economy. This ratio determines the competitiveness of a country, according to its relative production advantages in one of the two sectors. Second, the population size of the country, in the sense of the standard theory of HME. Third, the supernumerary income ($A_x - \gamma \overline{X}$), which determines the purchasing power of workers in terms of heterogeneous goods. These elements persist in the open economy and determine the effects of international trade.

Additionally, the degree of industrialization that is determined by the number of varieties produced

[7] The level of industrialization can also be defined by the amount of labor available in the manufactured goods sector (Desdoigts & Jaramillo, 2009).

in the manufacturing sector also determines welfare levels.

$$U^A = \left(\alpha(\frac{A_x}{\gamma} - \overline{X})\right)^\alpha \left(\frac{(1-\alpha)}{n^{\frac{\sigma-1}{\sigma}} \frac{\beta A_x}{\sigma A_y}} \left(\frac{A_x}{\gamma} - \overline{X}\right)\right)^{1-\alpha} \tag{2.27}$$

$$U^A = \alpha^\alpha (1-\alpha)^{1-\alpha} \left(\frac{\sigma}{\beta}\right)^{1-\alpha} \left(n^A\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \left(\frac{A_y}{A_x}\right)^{1-\alpha} \left(\frac{A_x}{\gamma} - \overline{X}\right) \tag{2.28}$$

The greater the degree of country industrialization, the greater the welfare. Similarly, the same variables that determine the level of industrialization of a country affect welfare levels. In summary, the levels of productivity in the sector of manufactured goods, the population size and the supernumerary income define both the degree of industrialization of a country and its welfare.

2.3 Open Economy

In this section, we extend the model to an open economy scenario, which establishes the basis for the HME model. There are two trading countries that differ in population size (N) and productivity in each of the sectors (A_x y A_y).

Assuming costless international trade in the homogenous good (X),[8] its price is equalized in the two countries. This price will be taken as a numeraire ($P_x = P_x^* = 1$). There are transportation costs associated with the heterogeneous goods trade, which are modeled as "iceberg" transportation costs. In particular, it is supposed that a τ portion of the transported goods arrives, while $(1-\tau)$ is lost in transit. Including this relationship of costs to prices in the international market, the prices of each variety of heterogeneous good are as follows:

$$\text{Domestic}\begin{cases} p = p \\ \widehat{p^*} = \frac{p^*}{\tau} \end{cases}, \quad \text{Foreigner}\begin{cases} p^* = p^* \\ \widehat{p} = \frac{p}{\tau} \end{cases} \tag{2.29}$$

Therefore, the consumption of national varieties differs from the varieties imported due to price differences. The representative home maximizing program is then modified in relation to the varieties of heterogeneous domestic and foreign goods.[9] The aggregate consumption of varieties of heterogeneous good is no longer represented by (2.2), but it becomes an aggregate of both domestic and foreign varieties that differ in price $Y' = \left(\sum_{i=1}^n y_i^\sigma + \sum_{j=1}^{n^*} y_j^{*\sigma}\right)^{\frac{1}{\sigma}}$. Therefore, the budget constraint is now defined by:

$$\sum_{i=1}^n p_i y_i + \sum_{i=1}^n \widehat{p_j^*} y_j^* - (1-\alpha)\left(\frac{w}{\gamma} - P_x\overline{X}\right) \tag{2.30}$$

[8] This is a simplified assumption that is widely used (Helpman & Krugman 1985; Krugman 1991, etc.) and which does not affect the essential argument of the model.

[9]

$$\mathcal{L} = \left(\sum_{i=1}^n y_i^\sigma + \sum_{j=1}^{n^*} y_j^{*\sigma}\right)^{\frac{1}{\sigma}} - \lambda\left(\sum_{i=1}^n p_i y_i + \sum_{i=1}^n \widehat{p_j^*} y_j^* - (1-\alpha)\left(\frac{w}{\gamma} - P_x\overline{X}\right)\right)$$

Where y_i is the demand for each domestic variety and y_j^* is the demand for each foreign variety. The budget restriction at the foreign level is symmetric to this.

As a result of the maximization, we find the ratio between the demand for domestic and foreign varieties to be a function of the price ratio of these,

$$\frac{y_j^*}{y_i} = \left(\frac{p_i}{\widehat{p_j^*}}\right)^{\frac{1}{1-\sigma}} \tag{2.31}$$

The local demand for each variety of heterogeneous domestic and foreign good (y_i and y_j^*), resulting from the maximization program of the domestic agent, is defined by the proportion of revenue earmarked for heterogeneous goods demand and the price of each variety weighted by the addition of the prices of all available varieties around the world.

$$y_i = \frac{p_i^{\frac{1}{\sigma-1}}(1-\alpha)\left(\frac{w}{\gamma} - P_x\overline{X}\right)}{\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} + \sum_{j=1}^{n^*} \widehat{p}_j^{*\frac{\sigma}{\sigma-1}}} \tag{2.32}$$

$$y_j^* = \frac{\hat{p}_j^{*\frac{1}{\sigma-1}}(1-\alpha)\left(\frac{w^*}{\gamma} - P_x\overline{X}\right)}{\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} + \sum_{j=1}^{n^*} \widehat{p}_j^{*\frac{\sigma}{\sigma-1}}} \tag{2.33}$$

The new basket of varieties available worldwide that enters into the aggregation of heterogeneous goods Y', modifies its index price, similarly affecting the proportion of income available for the consumption of such goods. Performing the same procedure as that used with the price index in a closed economy, the free-trade index, depends on the price of existing domestic and foreign varieties of heterogeneous goods:

$$P_{Y'} = \left[\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} + \sum_{j=1}^{n^*} \widehat{p}_j^{*\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} \tag{2.34}$$

Equation (2.31) is the ratio between the consumption of domestic and foreign varieties in terms of the ratio between prices. In order to determine the world equilibrium we need to add the quantities of the goods used for the transportation of products. The demand rate for foreign heterogeneous goods, in terms of the domestic (θ) and corresponding rates for foreign goods (θ^*) is equal to:

$$\theta = \frac{y_j^*}{y_i} = \left(\frac{p_i}{p_i^*}\right)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \tag{2.35}$$

$$\theta^* = \frac{y_i}{y_j^*} = \left(\frac{p_i^*}{p_i}\right)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} = \left(\frac{p_i}{p_i^*}\right)^{-\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \tag{2.36}$$

After determining the ratio of demand for varieties among foreign and domestic varieties, one can define individual demand patterns for heterogeneous goods in each country, which are restricted by the proportion of expenditure for manufacturing consumption. The national demands for domestic and foreign heterogeneous goods are:

$$y_i = \left(\frac{p_i}{P_{Y'}}\right)^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{\frac{w}{\gamma} - P_x\overline{X}}{P_{Y'}}\right) \tag{2.37}$$

$$y_j^* = \left(\frac{\widehat{p_j^*}}{P_{Y'}}\right)^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{\frac{w}{\gamma} - P_x\overline{X}}{P_{Y'}}\right) \tag{2.38}$$

2.3.1 Producer

Using equations (2.37) and (2.38) it is easy to show that the elasticity of demand for exports is the same as in a closed economy for heterogeneous goods ($\frac{1}{1-\sigma}$). Therefore, transportation costs have no effect on the pricing policy of the firm. This result shows that the domestic and foreign prices of each variety of heterogeneous good remain the same as under autarky, in their respective local markets.

$$p = \frac{\beta w}{\sigma A_y} \qquad \wedge \qquad p^* = \frac{\beta w^*}{\sigma A_y^*} \tag{2.39}$$

Given the characterization of monopolistic competition in the market of heterogeneous goods, every variety of this type of good is only produced by one firm.[10] The number of varieties produced in each region is determined in the first instance by productivity in this sector, the amount of labor force used in the production of these goods (L_y and L_y^*) and the model parameters.

$$n = \frac{L_y\,(1-\sigma)\,A_y}{\mu} \quad \wedge \quad n^* = \frac{L_y^*\,(1-\sigma)\,A_y^*}{\mu} \tag{2.40}$$

With regard to the homogeneous good, the equalization of prices at the international level sets a relationship of proportionality between the wages and agricultural productivity of both countries.

$$w = A_x \ \text{ and } \ w^* = A_x^* \tag{2.41}$$

In accordance with the equation (2.41), the per capita incomes, in terms of the homogeneous good, are the same as in a closed economy.

$$\frac{wL}{N} = \frac{A_x}{\gamma} \text{ and } \frac{w^*L^*}{N^*} = \frac{A_x^*}{\gamma^*}$$

2.4 The Home Market Effect

The presence of increasing returns to scale in the production of heterogeneous goods, and transportation costs generated for its trade at the international level, create an incentive to produce such goods in the "biggest market", thus taking advantage of economies of scale and minimizing transportation costs

[10] The only way in which the results are modified in relation to the closed economy is if the wages between countries differ, a central element in the section below.

(Krugman 1980, 1991, etc.). In this sense, and according to the purposes of this article, the "largest market" is not only determined by the number of agents in a country, but also by their productivity and per capita supernumerary incomes. In other words, the demand effect, through the purchasing power and the level of competitiveness of the agents, constitutes a market.

Starting with two countries that possess the established features, aggregate demand for heterogeneous goods in each country is the sum of the domestic and foreign demand for this type of good, that is, the domestic consumption of heterogeneous goods plus exports of these kinds of good (2.42 and 2.43).[11]

$$npD = \frac{n}{n+\left(\frac{p^*}{p}\right)\theta n^*}(1-\alpha)\left(\frac{w}{\gamma}-\overline{X}\right)N+\frac{\theta^* n}{\theta^* n+\left(\frac{p^*}{p}\right)n^*}(1-\alpha)\left(\frac{w^*}{\gamma^*}-\overline{X}\right)N^* \quad (2.42)$$

$$n^*p^*D = \frac{\theta n^*}{\left(\frac{p}{p^*}\right)n+\theta n^*}(1-\alpha)\left(\frac{w}{\gamma}-\overline{X}\right)N+\frac{n^*}{\left(\frac{p}{p^*}\right)\theta^* n+n^*}(1-\alpha)\left(\frac{w^*}{\gamma^*}-\overline{X}\right)N^* \quad (2.43)$$

Aggregate demand for heterogeneous goods in a closed economy, which is only determined by the proportion of domestic spending dedicated to this type of good, is now determined by a combination of variables regarding the economies that are trading. In particular: a) the proportion of spending on such goods $(1-\alpha)$; (b) the demand rate among domestic and foreign varieties, that is, ultimately, a price ratio (fractions depending on (n and θ)); (c) the supernumerary income of the agents; and (d) the total population. Additionally, the productivity in each of the two sectors plays a fundamental role in the demand for such goods through the real income of workers. On the one hand, the productivity of the homogeneous good sector determines wages, demarcating agents' revenues and the costs of firms, while the productivity of the heterogeneous sector determines the price of each variety.

Solving (2.42) and (2.43) obtains the relationship between the number of varieties produced domestically against those produced overseas as a measure of HME, which is determined by the interactions between supply and demand elements that are additional to those presented in the traditional approach.

$$\frac{pn}{p^*n^*} = \frac{\left[\frac{\left(\frac{A_x}{\gamma}-\overline{X}\right)N(1-\theta)}{\left(\frac{A_x^*}{\gamma^*}-\overline{X}\right)N^*(1-\theta^*)}\right]-\theta}{1-\theta^*\left[\frac{(1-\theta)\left(\frac{A_x}{\gamma}-\overline{X}\right)N}{(1-\theta^*)\left(\frac{A_x^*}{\gamma^*}-\overline{X}\right)N^*}\right]} \quad (2.44)$$

Equation (2.44) is a novel result in the theory of international trade with increasing returns to scale. In the first instance, it is evident that the HME presented through the varieties rate of the heterogeneous goods produced in each country depends on the same elements as its traditional version, the parameter (θ), which mainly includes the effects of the trade frictions, particularly transportation costs. However, in equation (2.44) we identify other channels by which the ratio $(\frac{n}{n^*})$, and therefore HME, can be changed.

[11] D is determined by the zero profit condition $D_i = D = \frac{\mu\sigma}{(1-\sigma)\beta}$

The term between the brackets of equation (2.44) collects most of the different effects evident in this relationship. The first fraction of this term $\left(\frac{\frac{A_x}{\gamma}-\overline{X}}{\frac{A_x^*}{\gamma^*}-\overline{X}}\right)$, corresponds to the relative supernumerary income, which is a direct consequence of the non-homothetic preferences assumption and relates to the purchasing power of the agents. The second term corresponds to the relationship with population sizes $\left(\frac{N}{N^*}\right)$, which shows the effects of the ratio among the sizes of the markets, in the standard form of HME. Finally, the last expressions in parentheses convey the degree of competitiveness of the markets according to their productive advantages, weighted by existing trade frictions $\left(\frac{1-\theta}{1-\theta^*}\right)$. In global terms, the expression reflects the relationship between the relative sizes of the demands of the two countries, which is determined by population size, the purchasing power of agents and the degree of competitiveness of these.

The disparity between $(\gamma \neq \gamma^*)$ reflects the difference in per capita income $(\frac{A_x}{\gamma})$, which will modify the demand for heterogeneous goods in each country, affecting the number of varieties of heterogeneous goods produced in each region. This channel identifies differences in the purchasing power of the residents of a market. It is hoped that countries with greater purchasing power demand a higher proportion of heterogeneous goods, creating an incentive for the establishment of firms in this market, which will increase the number of varieties produced.

Variation in the productivity of both sectors is another channel through which international trade can affect the degree of industrialization of a country. Variations in productivity in the homogeneous good sector alters wages, generating two contrary effects within the economy that result in a differentiated aggregate effect. The first is an expenditure effect, which alters the level of revenue dedicated to the purchase of heterogeneous goods. The second is a cost effect, which changes the prices for each variety of these goods. These two effects act in opposite directions, and so the result of an increase of A_x, in terms of the number of varieties produced, depends on the magnitude of each one of these effects. Furthermore, variations in the productivity of heterogeneous goods modify the prices of such goods and thus the number of varieties in demand, leading to alterations in the number of varieties produced in each country. Productivity within both sectors figures strongly in the case of comparative static, which is developed in the next section.

In order to simplify the initial analysis, equal productivity among the sectors of each country is assumed $(A_x = A_y = A)$, but it differs between countries $(A \neq A^*)$. In this way, there exists a scenario in which relative income and productivity vary between countries. The final effects of trade will therefore depend on the three fundamental channels described in this article, which produce HME: population size, relative income differences and differences in productivity. The implications of the assumptions presented for the general result (2.44) are verified in the following equation:

$$\frac{n}{n^*} = \frac{\frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}\frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*}} = \frac{Z-\tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}Z} \tag{2.45}$$

where

$$Z = \frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*} \tag{2.46}$$

This equation presents the effects of the different channels on the HME through the number of produced varieties of heterogeneous good. The Z variable in the equation (2.45) corresponds with the supernumerary income ratio (the centerpiece of this result, since it depends on the three channels in question) and the ratio of population size. The Z variable collects the different channels in the model, so that supernumerary income is affected by the country's productivity levels, the dependence factor (γ) and population size. Increased productivity or a reduced dependence factor increases per capita income levels, creating a demand effect. This in turn stimulates the production of more varieties of heterogeneous good in the country with a higher income. This is so because it boosts the size of demand, which allows it to exploit economies of scale. Countries with a greater supernumerary income, as caused by any of the channels presented in this case, will then have a higher real income, which increases the economic market size, directly affecting the number of varieties of heterogeneous good produced in the economy.[12]

In HME function (2.45), the interval of incomplete specialization, where both countries produce two types of good, occurs when the (Z) variable belongs to the interval $\left(\tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}}\right)$. The greater the transportation costs and the lower the economies of scale, the greater the range of incomplete specialization. Outside this interval of the variable, the full specialization of the partners takes place. A trade balance in the heterogeneous goods of a domestic country is obtained from the demands of these goods from domestic and foreign countries.

$$TB_Y = \frac{\theta^* n}{\theta^* n + n^*}(1-\alpha)\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^* - \frac{\theta n^*}{n+\theta n^*}(1-\alpha)\left(\frac{A}{\gamma}-\overline{X}\right)N \tag{2.47}$$

The behavior of the trade balance in the range of incomplete specialization depends on exogenous variables (productivity, dependence factors, proportion of income devoted to spending on heterogeneous goods and transportation costs), and the number of varieties of heterogeneous good produced in each country. When the countries have the same supernumerary income and population size $\left(Z = \frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*} = 1\right)$, both produce the same number of varieties of heterogeneous good, presenting equilibrium in the trade balance of manufacturing. Out of equilibrium, the performance of the trade

[12] In the annex we show that, even when international trade reduces the degree of industrialisation in the countries, the welfare of the representative agent improves because the international market offers a greater number of varieties.

balance depends on the number of varieties produced in each country (2.48), which is directly defined by the relationship between the supernumerary incomes of the countries and population size (2.45). The country with a higher per capita income will produce more varieties than the other, and will experience a trade balance surplus in heterogeneous goods at this interval.

$$TB_Y = \frac{\left(A^* - \gamma^* \overline{X}\right)(1-\alpha)L\tau^{\frac{\sigma}{1-\sigma}}}{\tau^{\frac{\sigma}{1-\sigma}}n + n^*}(n - n^*) \tag{2.48}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \tag{2.49}$$

If $Z > \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}}$, the trade between the two countries will involve a full specialization in heterogeneous goods domestically, and in homogeneous goods overseas. On the other hand, if $Z < \tau^{\frac{\sigma}{1-\sigma}}$, the trade between the two countries will take on a full specialization in heterogeneous goods overseas, and in homogeneous goods domestically. The more similar (different) the traded countries are ($Z \approx 1$) in terms of population size, income and productivity, the higher (lower) the probability of incomplete specialization, intra-industry trade (inter-industry trade).

The effects of (Z) on the number of varieties produced in each country can be determined analytically and show a positive relation. The country with a higher income will be the largest producer of heterogeneous goods within a bilateral trade relationship, with positive transportation costs:

$$\frac{d(\frac{n}{n^*})}{dZ} = \frac{1 - \tau^{\frac{2\sigma}{1-\sigma}}}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}}\left(\frac{\left(\frac{A}{\gamma} - \overline{X}\right)N}{\left(\frac{A^*}{\gamma^*} - \overline{X}\right)N^*}\right)\right)^2} > 0 \quad \text{given } \tau < 1 \tag{2.50}$$

Given the characteristics of the HME function (2.45), this can be represented in a graph, as shown below.

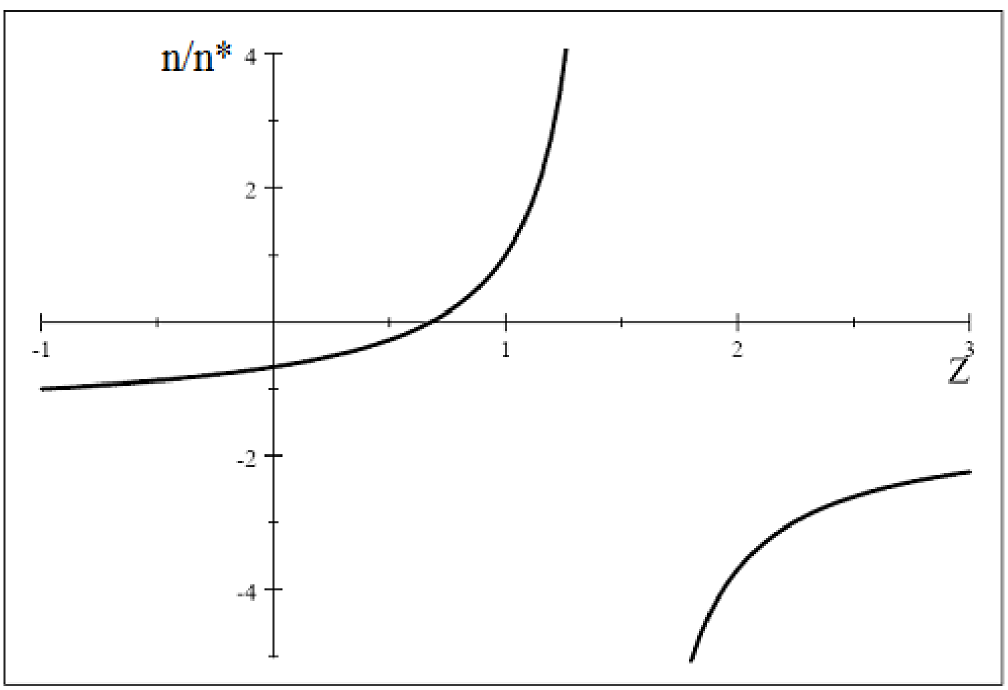

Figure 1: Population size, supernumerary income and HME

The graph is similar to the traditional HME. It shows the variable Z and the channels involved, while defining the number of varieties produced by each country. Located on the right of the asymptote are the cases of complete specialization in heterogeneous goods on domestic production after trade implementation. Similarly, the points to the left of the intercept demarcate the overseas cases of complete specialization for such goods. The interval between the intercept and the asymptote is the area of incomplete specialization and illustrates the case in which both countries are equal ($Z = 1$) and produce the same number of varieties.

Is important to highlight that the HME is determined by the Z variable and not only by population size, as in traditional models. It shows the importance of the demand composition in the results of the trade. More than country size, economic market size is key in the sense of the purchasing power of agents, which determines the consequences of international trade on the degree of industrialisation for countries that trade. International trade increases industrial production in relation to autarky levels if the relative supernumerary income is sufficently greater in relation to the transportation cost.

On the other hand, via this same model it is possible to determine the trade implications for welfare. The relationship between utility under an open economy and autarky is such that it will only depend on the number of varieties to which the country has access after and before trade, *ceteris paribus*.

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \tau^{\frac{\sigma}{1-\sigma}}}{n_A} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \tag{2.51}$$

The outcome in terms of welfare depends on two effects: the first is the number of varieties produced domestically after trade, in relation to the number produced under autarky, and the second is the number of additional varieties that are accessed after trade in relation to those available under autarky. These two effects can go in the same direction or in opposite directions, depending on whether specialization exists or not, and the impact of trade on the production of heterogeneous goods. However, using the definitions about the number of varieties produced under autarky and under trade, we find that welfare is better under trade than under autarky. This means that in the static scenario exposed in this model, trade is strictly preferred to autarky.

$$\frac{U}{U_A} = \left(1 + \tau^{\frac{\sigma}{1-\sigma}} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} > 1 \tag{2.52}$$

2.5 Comparative Statics

From (2.44), it is possible to generate the different comparative statics that enable the identification of the different channels through which HME may occur after trade liberalization, and they demonstrate that the outcome of international trade depends on the specific characteristics of the partner countries.

2.5.1 Variations in Population Size

Assume the absence of a homogeneous good, that the population is equal to the number of workers, that productivity is equal between the countries, and that transportation costs are positive ($\alpha = 0$, $L = N$, $\overline{X} = 0$ and $\theta > 0$). With these assumptions we achieve the classic results obtained by Krugman (1980), who presents the relationship between population sizes as determinant of the number of varieties of heterogeneous good produced in each country. HME is determined by the population size of each country ($Z = \frac{N}{N^*}$). The graphic representation of this scenario is illustrated in figure 1, where HME is determined by the values that take the variable Z.

$$\frac{n}{n^*} = \frac{Z - \theta}{1 - \theta Z} = \frac{\frac{N}{N^*} - \theta}{1 - \theta\left(\frac{N}{N^*}\right)} \tag{2.53}$$

Given $\left(\frac{dZ}{dN} > 0\right)$, the larger a country in terms of population, the greater the number of varieties of heterogeneous good being produced. It is clear that the size of the population, as Krugman (1980) states, is an important channel in the determination of HME, but it is not the only element that comes into this determination because, as we shall see later on, both the purchasing power and level of productivity in each of the sectors complement the channels through which the size of a market becomes a determinant of the type of product that one country trades (HME).

2.5.2 Variations in Relative Income

Assume the existence of a homogeneous good with a minimum level of consumption $\left(\alpha \neq 0 \text{ and } \overline{X} > 0\right)$, the population differs from the number of workers in each economy ($L \neq N$, with $N = \gamma L$), and the other variables are equal between countries, while HME is obtained from the relative demand of the market. Given the above assumptions, per capita income differentials determine the structure of the demand and so delimit the heterogeneous good varieties produced in each country after trade liberalization.

According to the hypothesis, the prices of every variety are the same in both countries, therefore $\theta = \theta^* = \tau^{\frac{\sigma}{1-\sigma}}$. By definition $N = \gamma L$, assuming $\gamma = 1$ for domestic and $\gamma^* > 1$ for foreign, the relationship presented in (2.44) is defined in the following way:

$$\frac{n}{n^*} = \frac{Z - \tau^{\frac{\sigma}{1-\sigma}}}{1 - Z\tau^{\frac{\sigma}{1-\sigma}}} \quad \text{with} \quad Z = \frac{\left(A - \overline{X}\right) N}{\left(\frac{A}{\gamma^*} - \overline{X}\right) N^*} \tag{2.54}$$

(2.54) has the same functional form of the standard HME, which in this case is presented through other channels and is represented in figure 1. The Z variable determines the complete or incomplete specialization of countries at the same intervals set out in the general case. There is a point where the income is equal for both countries ($\gamma = \gamma^*$), and they therefore produce the same number of varieties while maintaining a balanced trade for such goods. However, outside of this point the trade balance in terms of heterogeneous goods exhibits a particular behavior that is determined by the number of varieties produced in each country (2.55), which in turn is determined by the level of income of each region. Therefore, the country with higher levels of income (fewer γ) will have a trade balance surplus in manufacturing.

$$TB_Y = \frac{(A-\gamma^* X)(1-\alpha)L}{\tau^{\frac{\sigma}{1-\sigma}} n + n^*}(n-n^*) \tag{2.55}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \ \wedge \ n > n^* \Leftrightarrow \gamma^* > \gamma \tag{2.56}$$

In equation (2.50) we show that $\frac{d(\frac{n}{n^*})}{dZ} > 0$, and so the effect of the increases in the variable γ^*, which contains the differentials of per capita income against a foreign partner, is positive.[13] Accordingly, the higher the per capita income of countries, the greater the number of varieties produced.

$$\frac{dz}{d\gamma^*} = \left(\frac{\left(A_x - \overline{X}\right)\overline{X}}{\left(A_x - \gamma^* \overline{X}\right)^2} \right) > 0 \ \ if \ A_x > \overline{X} \tag{2.57}$$

Proposition 1 *: In a world characterized by the presence of homogeneous and heterogeneous goods, where productivity is equal among countries and among sectors, and each country has a different level of supernumerary income, after trade the country with the higher relative income (less γ) will produce a greater number of varieties in the heterogeneous goods sector. At the same time, the country with a higher level of income will have a trade surplus in this sector and its industrial production will be greater than under autarky.*

The result presented in *proposition 1* goes in the same direction as the issues raised in the introduction and as that of the argumentation of the overall result. Countries with higher levels of supernumerary income spend the bulk of their income on elaborate items, which increases market size for this type of good, and so it becomes attractive to establish firms in this sector of production in order to take advantage of economies of scale.

In terms of welfare, the results can be obtained by comparing levels of utility under autarky and in an open economy (2.52). The welfare levels under an open economy are superior to those under autarky because of the greater number of varieties available to agents. International trade increases the varieties available around the world, which allows for an increase in the levels of utility for both countries in relation to their situation under autarky.

[13] This is true so long as worker remuneration is higher than the survival consumption of the agricultural good; this is one of the assumptions made in the present paper.

2.5.3 Productivity

Total productivity

Equalizing the labor-force sizes of the countries, and assuming equality between the population and the number of employees ($N = L$), productivity differences among countries are entered. Productivity differs between countries but productivity among sectors is equal for each country ($A_x = A_y = A$ and $A \neq A^*$). Incorporating the assumptions above into the general equation (2.44), the following expression is reached, which is also represented in figure 1:

$$\frac{n}{n^*} = \frac{Z - \tau^{\frac{\sigma}{1-\sigma}}}{1 - \tau^{\frac{\sigma}{1-\sigma}} Z} \quad \text{con} \quad Z = \frac{\left(A - \overline{X}\right)}{\left(A^* - \overline{X}\right)} \tag{2.58}$$

Differences in productivity, as they are shown, allow for the inclusion of supply and demand effects within the relationship of the varieties of heterogeneous goods. The demand effect dominates through differences in income, and can even generate a complete specialization via productivity differentials. The external position of each economy in the range of incomplete specialization behaves similarly to the previous case. When the productivity factor is equal between countries, they produce the same amount of varieties and achieve a trade balance for differentiated goods. However, when productivity differs, that trade balance depends on the number of varieties produced (2.59), as directly defined by the productivity factor. A more productive country will have a positive trade balance in heterogeneous goods.

$$TB_Y = \frac{\left(A - \overline{X}\right)(1-\alpha)L\tau^{\frac{\sigma}{1-\sigma}}}{\tau^{\frac{\sigma}{1-\sigma}} n + n^*}(n - n^*) \tag{2.59}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \ \wedge \ n > n^* \Leftrightarrow A > A^* \tag{2.60}$$

Given $\frac{d(\frac{n}{n^*})}{dZ} > 0$, the effect of variations in the productivity factor on the number of varieties of heterogeneous goods produced in each country can be determined analytically, showing their direct relationship. In this way, the most productive country within the trade relationship will be the largest producer of heterogeneous goods.[14]

$$\frac{d(z)}{dA} = \left(\frac{1}{A^* - \overline{X}}\right) > 0 \quad si \ A^* > \overline{X} \tag{2.61}$$

Proposition 2 *: Bilateral trade in countries that only differ in their productivity factors, these being equal between sectors, means that the country with higher productivity produces a superior number of varieties of heterogeneous good in relation to its trade partner and its autarky production. At the same time, the country with higher productivity will have a trade surplus in the manufacturing sector.*

Proposition 2 goes in the same direction mentioned above. The productivity channel, such as it arises in this case, raises the supernumerary income in the more productive country, creating a demand

[14] It is always true that productivity is higher than the minimum level of consumption of homogeneous goods.

effect that leads to an increase in the market of heterogeneous goods, making the establishment of firms within this sector in the said country attractive. Countries with high levels of productivity will have high levels of income, which increases the number of varieties among the heterogeneous goods produced.

The current result exposes agglomeration and dispersion effects posed by the standard theory of HME, but via changes in productivity. However, the demand effect is much higher, resulting in the agglomeration effect dominating the dispersion. More productive countries will generate better remunerations for workers (income effect), while the cost effect is cancelled due to a reduction in this via productivity in equal magnitude to the increase in wages. Therefore, a more productive country will have a greater weight of heterogeneous goods in the composition of the individual demand, leading the producers of such goods to becoming established in this market in order to exploit economies of scale.

In terms of welfare, the result compared with autarky is determined in the same way by (2.52). The increase in global demand for heterogeneous goods generates greater varieties, which raises the levels of welfare for countries with respect to their situation in a closed economy.

Comparative advantage in heterogeneous goods

Detailing a little more regarding the implications of changes in productivity, this is a singular case in which there are variations between regions and sectors. Initially, the effects of productivity variations in the heterogeneous goods sector are examined, when these differ between countries and within the homogeneous goods sector, *ceteris paribus*. In a formal way $A_y \neq A_y^* \neq A_x$; $A_x = A_x^*$, then $z = 1$. Defining $\eta_y = \frac{A_y}{A_y^*}$ as the relationship pertaining to productivity in the manufacturing sector. The inclusion of these assumptions in (2.44) generates the following output:

$$\frac{n}{n^*} = \frac{\eta_y \left(H - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \right)}{1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} H} \tag{2.62}$$

where

$$H = \frac{1 - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}}{1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \tag{2.63}$$

As in the previous cases, the functional form is maintained, although the variables involved are clearly different. H defines the positive range of the function, which presents the possibility of incomplete specialization $\left(\eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right)$, as figure 2 shows.

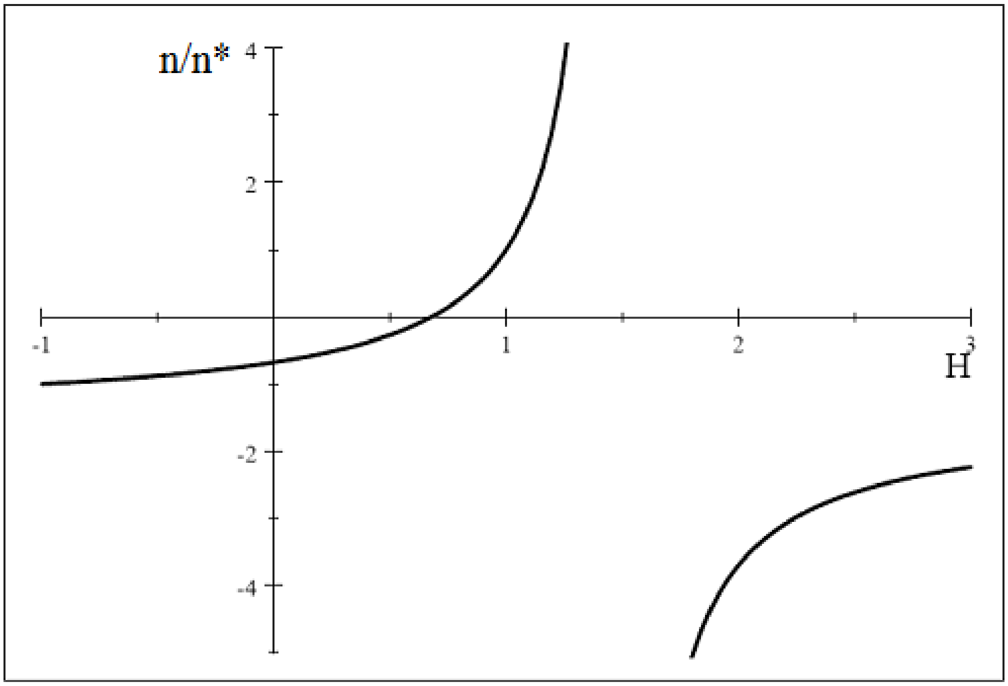

Figure 2: HME and the competitiveness factor

The trade balance of the heterogeneous goods in this interval is again determined by the number of varieties produced in each country, and also by the relationship with productivity in this sector (2.64). Similarly, at the point where both economies have the same productivity, they produce the same number of varieties and present an external equilibrium in this sector.

$$TB_Y = (\eta_y^{\frac{1}{1-\sigma}} - \eta_y^{\frac{-1}{1-\sigma}}) \frac{\tau^{\frac{\sigma}{1-\sigma}}(1-\alpha)\left(\frac{A_x^*}{\gamma^*} - \overline{X}\right) N^*}{\left(1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}\right)\left(1 - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}\right)} \tag{2.64}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \Leftrightarrow \eta_y > 1 \tag{2.65}$$

Equation (2.62), presents the competitiveness factor H and the relationship between productivities (η_y, which represents the price ratio) as determinants of HME. The way in which these variables relate can be verified analytically using the following expressions.

$$\frac{\partial \frac{n}{n^*}}{\partial H} = \frac{\eta_y \left(1 + \tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} H\right)^2} > 0 \tag{2.66}$$

$$\frac{\partial\left(\frac{n}{n^*}\right)}{\partial \eta_y} = \frac{\left(H - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}\right)}{1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} H} + \frac{\left(\frac{1}{1-\sigma}\right)\left(\eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} + \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} H^2 - 2\tau^{\frac{2\sigma}{1-\sigma}} H\right)}{\left(1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} H\right)^2} > 0 \tag{2.67}$$

Competitiveness factor H and the relationship between productivities among the heterogeneous goods η_y are directly related to the quotient of varieties produced between countries. The effects of variations in the productivity of heterogeneous goods are channeled via prices of this type of good. In this way, both supply and demand effects are presented. The first reduces costs for firms in the more productive country, after producing the same with less labor, increasing the number of varieties offered by the added open market. The second, the price effect, increases the number of varieties in demand, given the lower price of each. Finally, the differentiation in the factor of competitiveness with respect to the relationship for productivity is:

$$\frac{dH}{d\eta_y} = \frac{\left(\frac{1}{1-\sigma}\right)\left(\eta_y^{\frac{-2+\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_y^{\frac{\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} - 2\tau^{\frac{2\sigma}{1-\sigma}}\eta_y^{-1}\right)}{\left(1-\eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)^2} > 0 \tag{2.68}$$

The result of (2.68), related to (2.66), determines the aggregate effect of costs, which is presented as positive due to increased efficiency in manufacturing output. At the same time, the effect of the relationship between productivities is direct (2.67).[15] Effects in the same direction create a positive aggregate effect. The more productive the country in the manufacturing sector, the greater the number of varieties that it produces.

$$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_y} > 0 \Leftrightarrow H \in \left(\tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}}\right) \tag{2.69}$$

Proposition 3 *: Trade between countries that only differ in productivity regarding the heterogeneous goods sector means that the country with greater productivity in this sector produces a higher number of varieties of heterogeneous good in relation to its trade partner and its autarky production. The greater the productivity in the heterogeneous goods sector, the greater the number of varieties produced. The effect on the trade balance of the more productive country is also positive.*

This proposition contributes to a delimiting of the effects of productivity as the channel of competitiveness among economies that trade. Thus productivity in the heterogeneous goods sector directly affects the size of the market and may strengthen HME. The result is clear that in the interest interval, where there exists incomplete specialization, the agglomeration effect dominates, so a direct relationship between comparative advantage in the heterogeneous goods sector and the number of varieties produced in the country is present.

Similar to the above cases, it is possible to determine the effects on welfare by comparing the levels of utility between an open economy and autarky. In this case, the expression that determines these effects takes into account the implications of differences in productivity on the relative prices of the

[15] These results are always true under the interest interval.

domestic varieties with respect to the foreigner.

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \left(\frac{\tau}{\eta_y} \right)^{\frac{\sigma}{1-\sigma}}}{n_A} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} = \left(\frac{1 - \tau^{\frac{2\sigma}{1-\sigma}}}{1 - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} > 1 \qquad (2.70)$$

The welfare levels under an open economy are superior to those under autarky because of the greater number of varieties available to agents.

Comparative advantage in homogeneous goods

The other way to see particular differences in productivity between countries and between regions is through productivity in the homogeneous goods sector. Maintaining all other variables equal, countries only differ in productivity in relation to homogeneous goods, which in turn differs from productivity in relation to heterogeneous goods $\left(A_x \neq A_x^* \neq A_y \text{ and } A_y = A_y^*\right)$. Defining $\eta_x = \frac{A_x}{A_x^*}$ as the relationship between productivity for homogeneous goods in the two countries obtains the following result:

$$\frac{n}{n^*} = \frac{\eta_x^{-1} \left[\left(\frac{A_x - \overline{X}}{\frac{A_x}{\eta_x} - \overline{X}} \right) \left(\frac{1 - \eta_x^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}}{1 - \eta_x^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right) - \eta_x^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \right]}{1 - \tau^{\frac{\sigma}{1-\sigma}} \eta_x^{\frac{-1}{1-\sigma}} \left(\frac{A_x - \overline{X}}{\frac{A_x}{\eta_x} - \overline{X}} \right) \left(\frac{1 - \eta_x^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}}{1 - \eta_x^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right)} = \frac{\eta_x^{-1} \left(ZH - \eta_x^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \right)}{1 - \tau^{\frac{\sigma}{1-\sigma}} \eta_x^{\frac{-1}{1-\sigma}} ZH} \qquad (2.71)$$

The functional form persists and ZH defines the interval of incomplete specialization $\left(\eta_x^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\tau^{\frac{\sigma}{1-\sigma}} \eta_x^{\frac{-1}{1-\sigma}}} \right)$ through different channels. This result is presented in figure 3, which shows a graphic representation similar to the previous cases but with different implications.

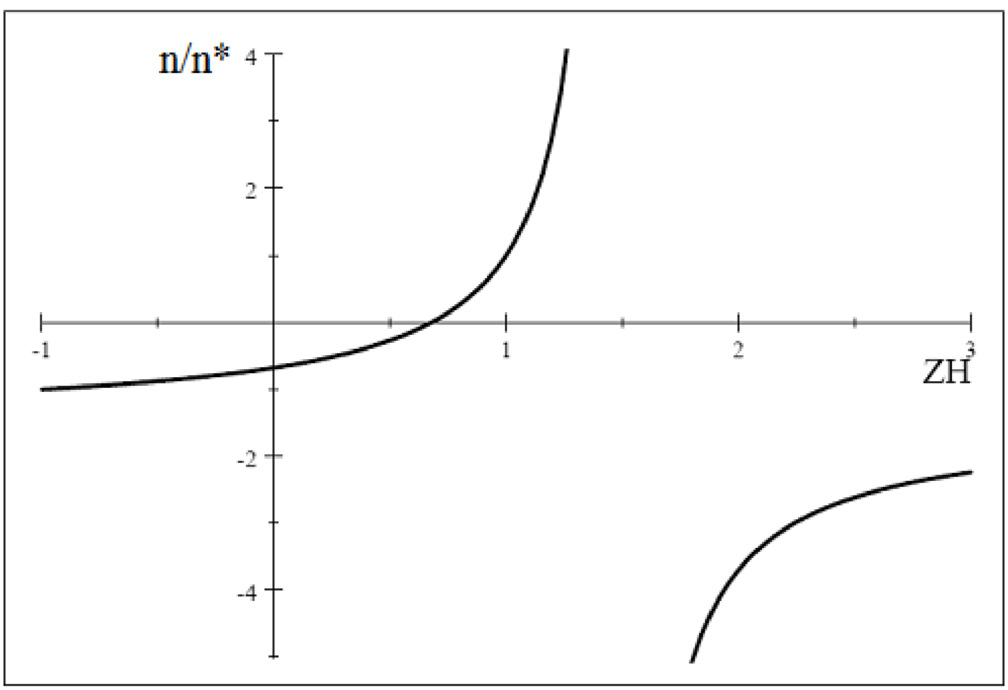

Figure 3: HME supernumerary income and competitiveness factor

This time, the behavior of the trade balance in this interval is given by the number of varieties produced in each country and the relationship between productivities for homogenous goods (wages) (2.72). Similarly, there is a case of balanced trade in this sector when the countries have the same level of productivity in the sector in question.

$$TB_Y = \frac{\tau^{\frac{\sigma}{1-\sigma}}\left(A_x^* - \overline{X}\right)(1-\alpha)L}{\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}n + n^*\eta_x^{-1}}\left(\eta_x^{\frac{-1}{1-\sigma}}n - \eta_x^{\frac{1}{1-\sigma}}n^*\right) \tag{2.72}$$

$$TB_Y > 0 \Leftrightarrow \frac{n}{n^*} > \eta_x^{\frac{2}{1-\sigma}} \tag{2.73}$$

The last term in (2.71) incorporates the factor of competitiveness among countries H, the supernumerary income ratio Z, and the ratio of productivity between countries in the homogeneous goods sector $\eta_x = \frac{A_x}{A_x^*}$, as determinants of HME. This expression allows for an analytical verification of how these determinants relate to the ratio of varieties between countries.

$$\frac{\partial \frac{n}{n^*}}{\partial z} = \frac{\eta_x^{-1}H\left(1-\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}ZH\right)^2} > 0 \tag{2.74}$$

$$\frac{\partial \frac{n}{n^*}}{\partial H} = \frac{\eta_x^{-1}Z\left(1-\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}ZH\right)^2} > 0 \tag{2.75}$$

$$\frac{\partial\left(\frac{n}{n^*}\right)}{\partial \eta_x} = \frac{-\eta_x^{-2}\left(ZH - \eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)}{1-\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}ZH} - \frac{\eta_x^{-1}}{1-\sigma}\left(\frac{\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_x^{\frac{-2+\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}Z^2H^2 - 2\eta_x^{-1}\tau^{\frac{2\sigma}{1-\sigma}}ZH}{\left(1-\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}ZH\right)^2}\right) < 0 \tag{2.76}$$

Competitiveness factor H and relative supernumerary income Z relate directly to the ratio of varieties produced between countries, while the relationship of productivity in the homogeneous goods sector η_x relates indirectly. However, ending the differentiation:

$$\frac{dz}{d\eta_x} = \left(\frac{\frac{A_x}{\eta_x^2}\left(A_x - \overline{X}\right)}{\left(\frac{A_x}{\eta_x} - \overline{X}\right)^2}\right) > 0 \;\; if \;\; A_x > \overline{X} \tag{2.77}$$

$$\frac{dH}{d\eta_x} = \frac{\frac{-1}{1-\sigma}\left(\eta_x^{\frac{\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_x^{\frac{-2+\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} - 2\eta_x^{-1}\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)^2} < 0 \tag{2.78}$$

The result presented in (2.77), combined with (2.74), allows for the identification of the aggregate effect of demand, which is the same as that presented in case three, variations in the productivity factor. Similarly, the result (2.78), related to (2.75), determines the aggregate effect of costs, which is negative by the increase in the remuneration of labor.

The results in this case are ambiguous, since two contrary effects coexist. On the one side there is an income effect through the increase in wages that increases the demand, and therefore raises the proportion of the heterogeneous goods demanded. This effect from the demand side increases the number of varieties produced in the more productive country. On the other side is the cost effect, which arises from increases in wages after increases in productivity, and rises one to one the remuneration of labor, thus reducing the number of varieties produced in the more productive country because of the high costs of production and the tendency to specialize in the production of homogeneous goods. This ambiguity in the relationship shows the presence of the agglomeration and dispersion effects of the HME referenced above, and presents the existence of a trade-off between them, allowing any final result depending on the magnitudes of each.

$$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_x} = \underset{(-)}{\frac{\partial\left(\frac{n}{n^*}\right)}{\partial\eta_x}} + \underset{(+)}{\frac{\partial\left(\frac{n}{n^*}\right)}{\partial Z}}\frac{dZ}{d\eta_x} + \underset{(-)}{\frac{\partial\left(\frac{n}{n^*}\right)}{\partial H}}\frac{dH}{d\eta_x}$$

Proposition 4 *: The trade between countries that only differ in their productivity in the homogeneous goods sector generates opposite effects on the number of varieties of heterogeneous good produced in each country, the aggregate outcome being dependent on the magnitude of the effects presented. On the one side, the demand effect stimulates the production of more varieties given increases in wages, but on the other side, the cost effect reduces the number of varieties produced because the cost of production is higher.*

Proposition 4 contributes, from an alternative angle, to the delimitation of a competitiveness channel as a determinant of the market size of the economies that trade. The presence of contrary effects adds ambiguity to the aggregate result, presenting the result that productivity in this sector reinforces the HME when the agglomeration effect dominates; or, on the contrary, it weakens when the dispersion effect is predominant.

To reduce ambiguity in the results, a simulation was executed in order to determine the values of the parameters within which the dispersion or agglomeration effect dominates. Table 1 shows the values that the parameters must take for the agglomeration effect to dominate the dispersion effect. The first row presents the values that should take the transportation cost parameter if we use the standard values of substitution elasticity among varieties. Similarly, the second row presents the values that should take the substitution elasticity parameter among varieties if we use the standard values of transportation costs.

$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_x} > 0$	$\sigma = 0.4 \wedge \tau < 0.025$	$\sigma = 0.8 \wedge \tau < 0.38$	$\sigma = 0.9 \wedge \tau < 0.59$	Standard σ Values
$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_x} > 0$	$\tau = 0.7 \wedge \sigma > 0.95$	$\tau = 0.8 \wedge \sigma > 0.973$	$\tau = 0.9 \wedge \sigma > 0.9998$	Standard τ values

Table 1: Parameter values, agglomeration dominates dispersion effect

These results suggest that the dispersion effect dominates the agglomeration effect in situations with economic sense in the value of the parameters. The agglomeration effect would only dominate in cases in which the transportation costs or the elasticity of substitution between varieties were extraordinarily high, cases in which international trade is possibly not established. This result justifies the fact that some industries will migrate to developing countries with low levels of productivity and wages. This is the case for industries in China or India, where an increase in the number of firms is due more to low-wage labor than to a large and effective demand for heterogeneous goods. When the dispersion effect dominates, it is possible to produce far from the larger markets and assume the costs of international trade, due to the low production costs of less productive countries.

As in other cases, the effect on welfare in relation to levels of utility could be set under autarky. The following expression exhibits this relationship, including differentials in productivity in the homogeneous goods sector.

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \left(\tau \eta_x\right)^{\frac{\sigma}{1-\sigma}}}{n_A} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} = \left(\frac{1 - \tau^{\frac{2\sigma}{1-\sigma}}}{1 - \eta_x^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} > 1 \tag{2.79}$$

Similar to the previous cases, the effect on welfare is always positive in relation to autarky when the size of the global market for heterogeneous goods increases after trade. Regardless of the outcome of the trade in relation to the industrialization of countries, welfare increases along with the number of varieties to which people have access.

Comparative statics summary

The comparative statics show how the different variables affect the number of varieties produced in each country. This is evidence of the existence of different channels in HME determination. Interactions among channels from the demand and supply side determine HME in a distinct way. The next table summarizes the effects of each determinant on HME. The first column shows the relationship between each determinant and the quotient of varieties produced for each country (the sign of the first differentiation). The second column presents what happens to the HME when two countries start to trade and each variable becomes greater at the domestic rather than foreign level. Finally, the third column presents the trade effect on industrialization (number of varieties produced) in relation to autarky.

Variable	$\frac{\partial n}{n^*}$	**HME**	**Industrialization**
Population Size N	>0	↑	↑
Dependence Factor γ	<0	↓	↓
Total Productivity A	>0	↑	↑
Comparative Advantage Heterogeneous Goods A_y	>0	↑	↑
Comparative Advantage Homogeneous Goods A_x	<0	↓	↓

Table 2: Variable effects on HME[16]

The demand size directly affects the number of varieties produced. Population size, the dependence factor and total productivity,[17] define the supernumerary income, which determines the purchasing power of the agents and, indeed, the size of the market for each country. The interplay of these variables determines the demand size in each country, and so, the larger the demand size of the country, the larger the number of varieties of heterogeneous good that it produces.

The supply side affects the number of varieties produced in different ways. Total productivity and the comparative advantage in heterogeneous goods directly impact the number of varieties produced; these allow for higher production in the heterogeneous goods sector and raise the real income of the agents. On the other hand, the comparative advantage in homogeneous goods reduces the number of varieties produced because it increases production costs and promotes specialization in agricultural production.

HME is generated through the interaction of supply and demand side variables. These variables determine the comparative advantages and the demand size of the countries. The results of international trade in terms of the industrialization of countries depends on the different determinants of HME and their interplay. The effects of international trade on welfare are always positive, relative to levels of autarky (*ceteris paribus*).

2.5.4 Variations in Productivity and Supernumerary Income

With the intention of illustrating the empirical implications of the model, we present a simple exercise which varies population size, relative income and total productivity. For this purpose, the United States is taken as the domestic economy and six other countries/regions as the foreign economies. The results of a hypothetical bilateral trade agreement between the domestic economy and each of the foreign regions is analyzed under the exposed model (equation (2.45), which combines the first three cases of

[16] The comparative advantage variable in the homogeneous good sectors is assumed under economic interpretation in the value of the parameters.

[17] When productivity is equal among sectors, total productivity determines the wages of the workers.

comparative statics). There is evidence of the importance of these three channels (population size, purchasing power and productivity level) for trade outcomes and the presence of HME. In addition, the exercise allows us to highlight the apparent paradoxes of international trade, as in the case of commercial relations between large countries in terms of population size, such as China and India, and economically large countries like the United States. Similarly, it would explain why some regressions in search of the HME are not so robust.

Country/Region (∗)	N^*/N	γ	$\gamma(p)$	Productivity	n/n^*	$n/n^*(p)$	$\triangle U^*$	$\triangle U$
Europe	1.62	2.28	1	69.7	1.95	1.94	23.3%	25.8%
Australia	0.07	1.99	1	74.3	1.76	1.76	56.1%	8.2%
Japan	0.41	2.01	1	63.9	2.18	2.17	36.8%	15.1%
China	4.32	1.74	1.43	12.4	4.24	4.25	28.3%	20.7%
India	3.95	2.88	3.03	8.3	4.42	4.42	39.2%	12.6
Nepal	0.09	3.34	2.34	3	4.64	4.63	114.3%	5%

Tabla 3: Results of hypotetical trade between the U.S. and six countries/regions.

The exercise involves three key variables: population size, number of people employed and level of productivity. The first two determine the value of γ, which is 2.17 for the case of the United States. Foreigners are presented in the third column of table 3.[18] Similarly, the level of labor productivity per worker is taken as the productivity variable; the value for the United States is 93.3 and for other countries/regions is shown in the fifth column of the same table.

The estimation of trade effects on these scenarios through traditional HME derives from situations of complete specialization,[19] however, the results obtained from the exposed HME (sixth column) show that beyond the physical size of the market, HME is determined by economic size. The number of people that constitute a market is important in determinating the effects of trade, but equally important is their purchasing power, which for the case study is determined by the level of productivity and the supernumerary income. In a more graphic way, countries like China and India, with a population almost four times larger than that of the United States, would not achieve concentration in the manufacturing industry after trading with the latter, as traditional HME puts it. In contrast, the United States would present a high concentration of the industry of manufactured goods, due to their large purchasing power being fundamentally related to high levels of productivity.

More generally, it may be seen that trade results are less asymmetrical as the gap in purchasing power among the regions closes, that is, the more similar their productivity and their γ. The results of the trade between the United States and Europe or Australia do not have a concentration of this

[18] Data from the World Bank and APO Productivity Data Book 2012.

[19] Bilateral trade would result in a complete specialization in heterogeneous goods in Europe, China and India; while in the other cases, the United States would concentrate the production of such goods.

type of industry, as is the case with other countries, including the case of Nepal, where the gaps are big enough that the heterogeneous goods sector ends up being suppressed.

Parallel to the previous exercise, an additional simulation is performed by modifying the definition of (γ), not as a relationship between population and employees, but, as the ratio between the population and the people above the poverty line.[20] This is done in order to present the robustness of the findings when income distribution is entered. The results are presented in the columns accompanied by (p), and demonstrate consistency with those presented previously. The effects are virtually identical and underline the importance that the levels of productivity play in the determination of the demand structure in each market.

The last columns of table 3 show the effects of bilateral trade on the welfare of different regions. The result is found from (2.51) and shows the variation of the utility under free trade in relation to autarky. The welfare in all regions increases considerably, regardless of the effects on the domestic industry of heterogeneous goods. That is, while the heterogeneous United States goods industry increases in all cases, the utility of all the countries/regions is much better after the trade agreement due to the increased number of varieties available, which immediately raises welfare levels. The country that increases in welfare the most is Nepal, which in turn is the most different from the United States in terms of the variables in question; this means that the increase in the number of varieties will be much more representative than in other economies.

In the case of the United States, it also benefits from trade in terms of welfare, mainly with Europe, with which its welfare increases by 25.8%; given the access to a greater number of varieties. With other countries the increase in welfare is in the order of 8 to 20%. With Nepal, the variation is only 5% due to the reduced number of additional varieties that this country can offer. In the way that the model predicts, the outcome of international trade is a positive-sum game in which everyone wins. However, it should be clarified that these results have been generated in a static environment, and that the conclusions may be modified in a dynamic setting.[21]

2.6 Conclusions

The traditional literature on HME has focused primarily on the number of agents that make up a market as a determinant of demand for trade between countries, and despite acknowledging the existence of additional channels it has not focused enough on these. This article contributes to the exploration of some of these additional channels in the determination of the effects of international

[20] We take the criterion of the population living on less than 2 US dollars a day from the World Bank.

[21] The application of this open economy model to an endogenous economic growth model is part of the subsequent article on the research agenda.

trade, in terms of the industrialization and welfare of participating countries.

The alternative way in which HME has been modeled, with non-homothetic preferences and differences in productivity, shows the importance of three channels in the determination of international trade effects and HME: population size, relative income and productivity levels. Thus the effects of international trade on the industrialization of countries depends on the way in which these three channels interact, depending on the particularities of the countries participating in the trade.

The greater the population size, relative income, total productivity levels and productivity in the heterogeneous goods sector, the greater the number of varieties produced by a country after trade in relation to its trade partner and autarky production. On the other side, the greater the productivity in the homogeneous goods sector, the fewer the number of varieties produced by a country after trade in relation to its trade partner and autarky production. In terms of the industrialization of countries, international trade can be positive or negative according to each particular scenario. However, the effects on welfare are always positive, relative to levels of autarky (*ceteris paribus*).

Such results have become one of the dimensions of international trade that can be analyzed by different countries when determining the nations with which it should trade. However, the static nature of the results opens up research into the dynamic effects of trade and to future extensions of this model.

2.7 Appendices

2.7.1 Appendix 1

In this section we demonstrate that welfare levels are always better after trade in relation to autarky levels. The next equation relates the utility levels under trade and under autarky:

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \tau^{\frac{\sigma}{1-\sigma}}}{n_A}\left(\frac{p}{p^*}\right)^{\frac{\sigma}{1-\sigma}}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \tag{2.80}$$

Using the equations (2.25) and (2.44)

$$\frac{n}{n^A} = \left(\frac{\frac{n}{n^*}}{\frac{n}{n^*} + \theta \frac{p^*}{p}}\right) + \left(\frac{\theta^* \frac{n}{n^*}}{\frac{p^*}{p} + \theta^* \frac{n}{n^*}}\right)\frac{1}{Z} = \frac{1}{1-\theta^*} - \frac{\theta}{1-\theta}\left(\frac{1}{Z}\right) \tag{2.81}$$

$$\frac{n^*}{n^A} = \left[\left(\frac{\theta}{\frac{n}{n^*}\frac{p}{p^*} + \theta}\right) + \left(\frac{1}{1 + \theta^* \frac{n}{n^*}\frac{p}{p^*}}\right)\frac{1}{Z}\right]\frac{p}{p^*} = \theta\left(\frac{1}{(1-\theta)Z} - \frac{\theta^*}{1-\theta^*}\right) \tag{2.82}$$

Entering the final equations in (2.80) we will have:

$$\frac{U}{U_A} = \left(\frac{1-\theta\theta^*}{1-\theta^*}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \tag{2.83}$$

Given $\theta < 1 \Rightarrow U > U^A$ welfare is always better after trade in relation to autarky.

Bibliography

[1] Antweiler, W., & Trefler, D. (2002). Increasing Returns and All That: A View from Trade. American Economic Review, 93-119.

[2] Banks, J., Blundell, R., & Lewbel, A. (1997). Quadratic Engel Curves and Consumer Demand. The Review of Economics and Statistics, 527-539.

[3] Bohman, H., & Nilsson, D. (2007). Income Inequality as a Determinant of Trade Flows. International Journal of Applied Economics, 40-59.

[4] Chung, C. (2006). Non-homothetic Preferences and the Home Market Effect: Does Relative Market Size Matter? Georgia Institute of Technology.

[5] Corden, W. (1970). A Note on Economies of Scale, the Size of the Domestic Market and the Pattern of Trade. Studies in International Economics.

[6] Crozet, M., & Trionfetti, F. (2008). Trade Costs and the Home Market Effect. Journal of International Economics, 309-321.

[7] Dalgin, M., Mitra, D., & Trindade, V. (2008). Inequality, Non-homothetic Preferences, and Trade: A Gravity Approach. Southern Economic Journal, 747-774.

[8] Davis, D. (1998). The Home Market, Trade, and Industrial Structure. American Economic Review, 1264-1276.

[9] Davis, D., & Weinstein, D. (1996). Does Economic Geography Matter For International Specialization? NBER Working Paper.

[10] Davis, D., & Weinstein, D. (2003). Market Access, Economic Geography and Comparative Advantage: An Empirical Assessment. Journal of International Economics, 1-23.

[11] Desdoigts, A., & Jaramillo, F. (2009). Trade, Demand Spillovers, and Industralization: The emerging global middle class in perspective. Journal of International Economics, 248-258.

[12] Engel, E. (1857). Die Produktions- und Consumtionsverhältnisse des Königreichs Sachsen. Zeitschrift des Statistischen Büreaus des Königlich Sächischen Ministeriums des Innern, 8-9.

[13] Foellmi, R., & Zweimüller, J. (2006). Income Distribution and Demand-Induced Innovations. The Review of Economic Studies, 941-960.

[14] Fujita, M., Krugman, P., & Venables, A. (2000). Economía Espacial. Las ciudades, las regiones y el comercio internacional. Barcelona: Ariel S.A.

[15] Hamilton, B. (2001). Using Engel's Law to Estimate CPI Bias. American Economic Review, 619-630.

[16] Hanson, G., & Xiang, C. (2004). The Home Market Effect and Bilateral Trade Patterns. American Economic Review, 1108-1129.

[17] Helpman, E., & Krugman, P. (1985). Market Structure and Foreing Trade. Cambridge: MIT Press.

[18] Huang, D.-S., & Huang, Y.-Y. (2011). Technology Advantage and Home Market Effect: An Empirical Investigation. Journal of Economic Integration, 81-109.

[19] Krugman, P. (1980). Scale Economies, Product Differentiation, and the Pattern of Trade. American Economic Review, 950-959.

[20] Krugman, P. (1991). Increasing Returns and Economic Geography. The Journal of Political Economy, 483-499.

[21] Linder, B. (1961). An Essay on Trade and Transformation.

[22] Markusen, J. (1986). Explaining the Volume of Trade: An eclectic approach. American Economic Review, 1002-1011.

[23] Matsuyama, K. (2000). A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand complementarities, income distribution, and North-South trade. The Journal of Political Economy, 1093-1120.

[24] Mitra, D., & Trindade, V. (2005). Inequality and Trade. The Canadian Journal of Economics, 1253-1271.

[25] Romer, P. (1990). Endogenous Technological Change. Journal of Political Economy, 71-102.

[26] Stokey, N. (1991). The Volume and Composition of Trade Between Rich and Poor Countries. The Review of Economic Studies, 63-80.

[27] Yu, Z. (2005). Trade, Market Size, and Industrial Structure: Revisiting the Home Market Effect. The Canadian Journal of Economics, 255-272.

[28] Zweimüller, J. (2000). Inequality, Redistribution, and Economic Growth. Empirica, 1-20.

Chapter 3

The Implications of International Trade on Economic Growth.

3.1 Introduction

The relationship between international trade and economic growth is a major topic of discussion in economic theory. The existence of a positive relationship between these two variables is one of the central proposals upon which the so-called Washington Consensus is based, as well as, and in particular, the policies of multilateral organisms of credit and economic development, such as the World Bank and the International Monetary Fund (Williamson 1990, Williamson 2000, Dollar 2005, etc.). A wide spectrum of trade agreements made the world over are explained by the assumption that international trade generates positive effects for economic growth and welfare. However, recent theoretical and empirical papers have shown in a formal way that both positive and negative results are the outcomes of trade, and that therefore the relationship remains ambiguous. International trade effects can be positive or negative depending on the countries engaged in trading. Singh (2010).

The aim of this article is to determine under which circumstances international trade increases economic growth and under which conditions it does not. This objective seeks to avoid general and ambiguous questions about the effects of international trade on economic growth and move instead towards a particular field in which the characteristics that generate a positive relationship are defined. In short, what are the elements that determine the effects of international trade on economic growth? What are the dynamic results for welfare?

This problem is analyzed through an endogenous growth model with "learning by doing" in production and an international trade model based on the existence of the Home Market Effect (HME) with non-homothetic preferences. HME is generated through economic market size, as shown in Giraldo (2015), and thus allows for interactions between elements of demand, such as population size and the purchasing power of agents, and elements of offer, such as productivity between sectors. This structure is brought to a dynamic field in a model of endogenous growth with knowledge spillovers and learning by doing in production. The results present new findings for the effects of international trade, as defined by the characteristics of the associated countries and the commercial legislation of the trading countries.

In contrast to standard models of international trade effects on economic growth and some models that use HME, the model herein gives particular relevance to demand-side variables in the determination of the dynamics of the model. Most of the models that use HME in dynamic environments only utilise the static effects of demand, but the dynamic is addressed by recourse to supply variables, as in standard models. In contrast, the present model allows for the interplay of supply and demand side elements in the dynamic determination of variables because of the assumption of non-homothetic preferences. The dynamic of the model is determined by supply and demand variables but it is addressed through demand variables.

Knowledge spillovers, transportation costs and differences in the purchasing powers of countries are key variables in the results of trade relations. The model shows how a commercial partnership between very different countries leads to a divergence between them, while trade between similar countries may lead to a converging growth path and a stationary equilibrium, that is, conditional convergence. Contrary to the results presented in static models of international trade, and in some models that relate this to economic growth, the results of this relationship are not always positive for welfare. In particular, although levels of welfare initially increase after trade, the scenarios that present a divergence in growth also present a divergence in welfare in the absence of knowledge spillover. In these particular scenarios, autarky is strictly preferable to trade.

The results obtained with the model are consistent with widely known stylized facts about economic growth around the world. During the last two centuries, the global economy has been characterized by a meaningful economic growth rate - which began after the Industrial Revolution - the expansion of international trade, and a convergence in income (and productivity) within developed countries and divergences in developing countries. Maddison (1983), Williamson (2002), Baldwin, Martin, and Ottaviano (2001), Acemoglu, Johnson and Robinson (2005).

Empirical research has found conflicting results that prevent the presentation of definitive answers about the effects of international trade on economic growth.[1] Endogeneity problems in estimations, errors in the measurement of economic policy variables, and sample selection bias are some of the

[1] The most important research in this area includes: Dollar (1992), Ben-David (1993), Sachs and Warner (1995), Edwards (1998), Frankel and Romer (1999), Rodriguez and Rodrik (2001), Chang , Kaltani, and Loayza (2009).

arguments that have been presented in the empirical field as the causes of such inconsistencies in the results. Singh (2010).

Theoretical literature regarding the effects of international trade on economic growth has been present from classical to contemporary times. The general framework in which this theoretical discussion takes place today is summarized in textbooks such as Grossman and Helpman (1997), Barro and Sala-I-Martin (2004) and Acemoglu (2009). Although there are a large number of theoretical writings about the relationship in question, these models currently comprise a combination of endogenous growth models, in differing variations, and monopolistic competition trade models.

Most of this literature is built from the supply side, showing how levels of productivity in each country and the possibilities of a transfer of technology after trade agreements play central roles in the results of these models. For example, Rivera-Batiz and Romer (1991), Young (1993) and, more recently, Gancia & Zilibotti (2005) present variations of the model by Romer (1990) and show how these are applied to an open economy, where dynamic gains are not presented via integration among symmetric economies; static gains are shown, raising the welfare of both countries. However, these conclusions are not preserved when the intertemporal dynamic among asymmetric countries is considered, since the models show a pattern of specialization. The authors do mention that the model does not fit the reality of asymmetric countries due to the absence of determinant variables, such as product cycle and knowledge spillover.

Aghion and Howitt (2005) have built a model of endogenous growth based on innovation quality and not quantity, as has also been done by the above-mentioned authors. The results show a trade-off in the dynamic gains of trade between innovation and amount of skilled labor with size of innovation, degree of competition in the market and possibilities of imitation. In addition, the authors avoid presenting absolute results and state that the introduction of additional variables could modify the results, as is the case with the development of financial markets and property rights legislation.

This general framework of analysis, in which modern theories of economic growth are used to study the relationship between trade and economic growth from the supply side, has served as a basis of analysis for some authors who study particular aspects of the aforementioned relationship. For example, Ventura (1997) uses the trade-growth relationship to explain conditional convergence among Asian countries during the postwar period. The results show the convergence hypothesis, but the effects of trade on growth are conditioned in relation to parallel policy decisions. Thoenig and Verdier (2003) present a model whereby the incentives for innovation are carried out to prevent exporter firms from being easily plagiarized by the firms of other countries, with monopoly time protection provided for each innovation. The results of this model are determined by legislation regarding property rights, again showing the importance of this variable in the frameworks of these models. Galor and Mountford (2008) model the trade-growth relationship and its implications for the demographic transition of countries, as well as the direct effects on economic growth and income distribution. The result predicts the negative effect of trade on growth, but this excludes other variables, such as education, strength of institutions or the level of knowledge spillover that international trade might generate, all of which could change the findings.

The generality of models linking international trade with economic growth is based on the theory of comparative advantage or specific factors. The implementation of dynamic models in an open economy based on the theory of the Home Market Effect is quite scarce. However, this strategy for modeling international trade allows for an analysis of the effects of supply-side and demand-side variables on the dynamic effects of international trade.

From the literature related to the concerns of the present article, the work of Martin and Ottaviano (1999) stands out. They make use of HME to build a model of industrial localization within an endogenous growth environment, with knowledge spillover and transaction costs being the determinants of the location of a firm and, therefore, the rate of economic growth in global regions. Baldwin, Martin and Ottaviano (2001) use this same strategy to show that after the specialization generated by trade liberalization, large countries tend to grow rapidly, while small countries are left behind with a slower rate of growth. Similarly, Kind (2002) does not find any concrete results regarding the effects of trade on growth, leaving an ambiguity to be solved by additional parameters such as transportation costs, knowledge spillover or some form of trade friction. Even if these models use HME to introduce demand elements into the determination of trade effects, the dynamics of the models are dominated by supply side variables, just as they are in standard economic growth models. Contrary to this, the present model allows for interactions between the supply-side and demand-side variables and gives high relevance to the demand-side variables in the determination of the dynamic of the model. The dynamic effects of international trade depend on the economic sizes of the markets that are trading and the purchasing powers of the agents from each country.

This article consists of this introduction and three additional sections. The second section exposes the characteristics of the model, the third develops the model in an open economy with its static and dynamic implications, and the fourth section concludes.

3.2 The Model

The model is based on the basic structure of HME with non-homothetic preferences, Giraldo (2015), and the dynamic is modeled from an endogenous growth model. This model allows for the study of the intertemporal implications of international trade on economic growth. The fundamentals of the model involve a dynamic Stone-Geary utility function, a two-sector economy, productivity differences among countries and the equality of these among sectors.

Two regions are assumed, domestic and foreign ($*$).[2] There are two sectors. First, a sector that produces a homogeneous good (X), which represents agricultural goods, and presents constant returns to scale in production. Second, a manufacturing sector that produces a set of heterogeneous goods (Y), with increasing returns to scale in production. The varieties of heterogeneous good are horizontally differentiated *à la* Dixit-Stiglitz and the firms in this sector maximize their benefits under monopolistic competition. Labor (L), is the only existing factor of production and is mobile among sectors but immobile among countries.

Following Chung (2006), countries differ in terms of amount of labor (L and L^*) and population size (N and N^*), but these variables are constant over time. The number of people who consume is different from the number of people who produce. It is thus supposed that domestic households offer $\frac{1}{\gamma}$ of labor for each resident ($N = \gamma L$) while foreign households offer $\frac{1}{\gamma^*}$, meaning ($N^* = \gamma^* L^*$). This allows for the simple entering of differences in per capita income between countries in a scenario in which wages are equal.

Intuitively, γ captures the demographic and redistribution factors that affect the relative demand for heterogeneous goods in comparison to that for homogeneous goods. Through this modification it is possible to interpret γ as the proportion of the population that earns an income.

I assume that all households demand both types of good and symmetrically demand each variety of heterogeneous good (Y). Households in both countries have the same utility function with non-homothetic preferences.

$$U_t = \int_0^\infty e^{-\rho t} u_t dt \tag{3.1}$$

$$\text{With } u_t = \alpha \ln\left(X_t - \overline{X}\right) + (1-\alpha)\ln Y_t \tag{3.2}$$

$$\text{and } Y_t = \left(\int_1^{n_t} y_{it}^{\sigma} di\right)^{\frac{1}{\sigma}}, \; 0 < \sigma < 1, \quad n = \text{ the number of varieties consumed} \tag{3.3}$$

Where $\overline{X}$ is the minimum consumption (of survival) of the homogeneous good and X_t is the consumption of this good at time t, beyond the threshold of survival. Y_t is the aggregate consumption of all n varieties of heterogeneous good at time t and y_{it} is the consumption of the i-th variety at every moment.

Both goods use the same factor of production, namely labor. The production of each good is determined by the amount of labor used and its productivity. The production of homogeneous goods and all varieties of the heterogeneous goods sector is conducted with the same production function in

[2] Hereafter, the variables corresponding to foreign have the superscript $*$.

both countries. The homogeneous goods sector has the following production function:

$$NX_t = D_{xt} = L_{xt}A_t \tag{3.4}$$

Where D_x is the aggregate demand of the homogeneous good, L_x is the amount of labor used in the production of such goods, and A_t is the productivity. The cost function in the heterogeneous goods sector is given by:

$$l_{it} = \frac{\mu}{A_t} + \frac{\beta D_{it}}{A_t} \quad i = 1, 2...n \quad \text{where } D_{it} = Ny_{it} \tag{3.5}$$

D_{it} is the aggregate demand of the i-th variety, l_{it} is the amount of labor used in the production of each variety and A_t is the productivity at time t. Moreover, μ and β are the parameters of fixed and variable costs respectively.

Technological progress only occurs in the production of heterogeneous goods, which operates by way of a learning by doing process specific to each country, as Krugman (1987) and Lucas (1988) explain. The evolution of productivity depends on both the domestic manufacturing sector of production, and a proportion of the productivity of this sector abroad, representing knowledge spillover from the outside towards the domestic economy. In the case of autarky, this final factor is equal to zero ($\delta = 0$).

$$A_t = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right) \, ds \tag{3.6}$$

Where K_s and K_s^* are the levels of knowledge of each economy, which increase with the production of heterogeneous goods, thus:

$$K_s = \int_{1}^{n_t} y_{it} di \ \text{ and } K_s^* = \int_{1}^{n_t^*} y_{jt} dj \tag{3.7}$$

Finally, the full-employment condition is assumed:

$$L = L_{Xt} + L_{Yt} = \frac{D_{Xt}}{A_t} + \sum_{i=1}^{n} \left(\frac{\mu}{A_t} + \frac{\beta D_{it}}{A_t}\right) \tag{3.8}$$

3.2.1 Equilibrium in a Closed Economy

Consumer

The intratemporal optimization problem of agents is conventional, maximizing (3.2) subject to its budget restriction. As previously mentioned, the methodology presented by Chung (2006) is used to enter income differences. This model differentiates between the members of the household who work and those who only consume. In more formal terms, each worker in the home has (γ) additional agents under its responsibility that only consume. The dependency ratio being $\frac{N}{L} = \gamma$.

$$Max \ u_t = \alpha \ln \left(X_t - \overline{X}\right) + (1 - \alpha) \ln Y_t \ \text{ s.t. } P_{xt}X_t + P_{yt}Y_t = \frac{w_t}{\gamma} \tag{3.9}$$

The optimal demands of the agricultural good X_t and the aggregate manufacturing goods Y_t result from the optimization program of every agent at every moment of time.

$$Y_t = \frac{1-\alpha}{P_{yt}} \left(\frac{w_t}{\gamma} - P_{xt}\overline{X} \right) \tag{3.10}$$

$$X_t = \frac{\alpha}{P_{xt}} \left(\frac{w_t}{\gamma} - P_{xt}\overline{X} \right) + \overline{X} \tag{3.11}$$

The optimal demands at every moment in time depend on supernumerary income, which is weighted by price and which in the case of manufactured goods is an index price established by the price of each of the existing varieties. The continuation of the optimization process allows for a determination of the demand of each of the varieties of heterogeneous good at every moment, which depends on the supernumerary income and the relative price of each variety.

$$y_{it} = \frac{p_{it}^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{w_t}{\gamma} - P_{xt}\overline{X} \right)}{P_{yt}^{\frac{\sigma}{\sigma-1}}} \tag{3.12}$$

$$\text{Where} \quad P_{yt} = \left(\sum_{i=1}^{n_t} p_{it}^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} \tag{3.13}$$

P_{yt} is the index price of heterogeneous goods, which is established as an aggregate of the prices of all varieties that exist in every moment of time, weighted by the degree of substitution between them.

Producer

The production of homogenous goods is supposed under perfect competition. This implies that the equilibrium price is equal to the labor cost. The price of this good is established as a numeraire, so productivity determines salary levels in this economy.

$$P_{xt} = 1 = \frac{w_t}{A_t} \tag{3.14}$$

I assume monopolistic competition in the production of heterogeneous goods. Since every variety of heterogeneous good uses the same technology of production, the price of each of the varieties is the same, and it is determined by wages, productivity, and fixed and variable cost parameters.

$$p_t = p_{it} = \frac{\beta w_t}{\sigma A_t} \tag{3.15}$$

Replacing the last equation in the zero-benefits condition, determined by the free entry and exit of firms in the manufacturing sector, I find the aggregate production of each variety, which is equal to its total demand at each instant of time.

$$D_t = D_{it} = \frac{\mu\sigma}{(1-\sigma)\,\beta} \tag{3.16}$$

Finally, from the full-employment condition (3.8) it is possible to obtain the number of varieties of heterogeneous good present in the economy at each time t.

$$n_t = \frac{L_{yt}\left(1-\sigma\right)A_t}{\mu} \tag{3.17}$$

The full-employment condition determines the amount of labor used in the heterogeneous goods sector L_{yt}, which is equal to the total available labor force (L), minus the quantity used in the production of homogeneous goods $\left(L_{xt} = N\left(\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{A_t}\overline{X}\right)\right)$. Accordingly, the number of varieties produced at every moment can be rewritten as:

$$n_t = (A_t - \gamma\overline{X})(1-\sigma)(1-\alpha)\frac{N}{\mu}\frac{1}{\gamma} \tag{3.18}$$

This expression shows how the number of varieties of heterogeneous good produced in the domestic market - which we assume to be the level of industrialization of a country - is determined by demand and supply factors. The dependency ratio and the productivity levels determine the supernumerary income, which represents the purchasing power of the agents in each country. The population size determines the market size in the standard way. At the same time, the dynamic is determined by a learning process generated in the production of manufactured goods. Thus the productivity variation rate is proportional to the amount of labor used in this sector.

$$\dot{A}_t = K_t = n_t y_i = n_t\left(\frac{\mu\sigma}{\left(1-\sigma\right)\beta}\right) = (A_t - \gamma\overline{X})(1-\alpha)\frac{N}{\gamma}\frac{\sigma}{\beta} = (A_t - \gamma\overline{X})(1-\alpha)L\frac{\sigma}{\beta} \tag{3.19}$$

The productivity growth rate is obtained from this last equation:

$$\frac{\dot{A}_t}{A_t} = \frac{n_t}{A_t}\left(\frac{\mu\sigma}{\left(1-\sigma\right)\beta}\right) \tag{3.20}$$

The dynamics of the economy under autarky are determined by equations (3.20) and (3.18). Welfare levels in a closed economy, which depend mainly on the number of varieties available for agent consumption and productivity levels, can be determined via the previous result. The level of intertemporal utility in this scenario is:

$$U_t = \int_0^\infty e^{-\rho t}u_t dt = \int_0^\infty e^{-\rho t}\left[\alpha\ln\left(\alpha\left(\frac{A_t}{\gamma} - \overline{X}\right)\right) + (1-\alpha)\ln\left(\frac{(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)}{n_t^{\frac{\sigma-1}{\sigma}}\frac{\beta}{\sigma}}\right)\right]dt \tag{3.21}$$

3.3 Open Economy

After establishing the model implications under autarky, it is presented in an open economy setting. The international trade result is determined by the HME with non-homothetic preferences presented in Giraldo (2015), plus the dynamic effects generated according to the model presented. In general, the number of varieties produced in each country is determined by HME, and the dynamic effects are determined by the levels of learning in each country and knowledge spillovers from foreign technologies.

Assuming costless international trade for homogenous good (X), its price is equalized in the two countries. This price is taken as a numeraire ($P_x = P_x^* = 1$), so that productivity determines the salary levels for each economy in the same way as in a closed economy.

The international trade of heterogeneous goods generates positive transportation costs, which are modeled as iceberg costs.[3] Following Giraldo (2015), in the presence of positive transportation costs for the heterogeneous goods trade, market size is determined by three basic elements: population size, relative income and productivity levels. In turn, after trade liberalization the economy with the greater market size gathers the majority of production of the varieties of heterogeneous good. In accordance with the international trade model, the aggregate demand for heterogeneous goods in each country is the sum of the domestic and foreign demand for this type of good:

$$n_t p_t D_t = \frac{n_t}{n_t + \theta \frac{p_t^*}{p_t} n_t^*}(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)N + \frac{\theta^* n_t}{\theta^* n_t + \frac{p_t^*}{p_t} n_t^*}(1-\alpha)\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^* \quad (3.22)$$

$$n_t^* p_t^* D_t = \frac{\theta n_t^*}{n_t \frac{p_t}{p_t^*} + \theta n_t^*}(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)N + \frac{n_t^*}{\theta^* \frac{p_t}{p_t^*} n_t + n_t^*}(1-\alpha)\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^* \quad (3.23)$$

Where $\theta = \left(\frac{p}{p^*}\right)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}$ is the demand rate for foreign heterogeneous goods in terms of the domestic ones, and $\theta^* = \left(\frac{p}{p^*}\right)^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}$ is the corresponding foreign rate.

The previous equations are given a HME dynamic equation (3.24), which is determined by the relationship between the demand elements mentioned above and the evolution of productivity levels in each country.

$$\frac{n_t}{n_t^*} = \frac{\frac{\left(\frac{A_t}{\gamma} - \overline{X}\right)N}{\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1 - \tau^{\frac{\sigma}{1-\sigma}} \frac{\left(\frac{A_t}{\gamma} - \overline{X}\right)N}{\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^*}} \quad (3.24)$$

The last equation shows HME in terms of the number of varieties produced in each country. HME is determined for the interplay of demand and supply elements. After trade, heterogeneous goods production in each country depends on supernumerary income, population size and level of productivity. The interactions between these aforementioned variables yield different trade scenarios with or without complete specialization in each countries' production.[4]

Given that both countries have the same production technologies and the same learning functions, one can determine the contemporary effects of international trade and its implications in the long term. With the learning function of the economy, which presents knowledge spillover, $\delta > 0$, in the case of

[3] The "iceberg cost" supposes that a τ portion of transported good arrives, and that $(1-\tau)$ is lost in transit.

[4] For more details, see Giraldo (2015).

an open economy, it is possible to identify the function of productivity evolution and thus the number of varieties produced in the manufacturing sector at each time.

$$A_t = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right) \, ds \tag{3.25}$$

In order to simplify the model, productivity is redefined in relation to agricultural survival consumption, weighted by the relationship between people who integrate the home and who are part of the labor force (supernumerary income) $\widehat{A_t} = A_t - \gamma \overline{X}$. This redefinition of variables reduces the mathematical processes and allows one to introduce a new state variable with all the determinants of intertemporal market size (population, workforce, purchasing power and, of course, productivity), allowing it to facilitate a dynamic analysis. So productivity is determined as:

$$\widehat{A_t} = A_t - \gamma \overline{X} = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right) ds - \gamma \overline{X} \tag{3.26}$$

Differentiating over time, it is possible to obtain the rates of adjustment of productivity in domestic and foreign markets respectively:

$$\dot{\widehat{A_t}} = K_t + \delta K_t^* = n_t \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{3.27}$$

$$\dot{\widehat{A_t^*}} = K_t^* + \delta K_t = n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{3.28}$$

The productivity growth rate in each country can then be obtained by dividing (3.27) and (3.28) by their respective productivities, which in turn determines the other growth rates of the economy:

$$\frac{\dot{\widehat{A_t}}}{\widehat{A_t}} = \frac{n_t \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t}} \tag{3.29}$$

$$\frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = \frac{n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t^*}} \tag{3.30}$$

I define the relative productivity of the two countries as the ratio between their productivities $H_t = \frac{\widehat{A_t}}{\widehat{A_t^*}}$. The rate of growth of this variable is then the subtraction among domestic and foreign productivity growth rates. The growth rate of relative productivity is determined by productivity levels, the number of varieties produced in each country and the spillovers between countries.

$$\frac{\dot{H_t}}{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = \frac{n_t \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t}} - \frac{n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t^*}} \tag{3.31}$$

These variable changes must be brought to the other equations of the model. After the respective replacements in the HME equation (3.24):

$$\frac{n_t}{n_t^*} = \frac{\frac{\widehat{A_t} N\gamma^*}{\widehat{A_t^*} N^*\gamma} - \tau^{\frac{\sigma}{1-\sigma}}}{1 - \tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{A_t} N\gamma^*}{\widehat{A_t^*} N^*\gamma}} \tag{3.32}$$

After computations, this equation may be defined as a function of the ratio of varieties produced and the gap in net productivity levels. Defining $\widehat{n_t} = \frac{n_t}{\widehat{A_t}}$, and $\widehat{n_t^*} = \frac{n_t^*}{\widehat{A_t^*}}$:

$$\frac{\frac{n_t}{\widehat{A_t}}}{\frac{n_t^*}{\widehat{A_t^*}}} = \frac{\widehat{n_t}}{\widehat{n_t^*}} = \frac{\left(\frac{N\gamma^*}{N^*\gamma} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t}\right)}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}} H_t \frac{N\gamma^*}{N^*\gamma}\right)} \tag{3.33}$$

This last equation is the HME equation in terms of the net productivity gap. The HME equation is now dynamic and it is determined for the same variables as in its static version (3.24) at each moment in time, Giraldo (2015). Similar to the HME equation, one could also redefine (3.22 and 3.23) in these terms:

$$\widehat{n_t} = \left(\frac{1}{1 + \frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t}} \frac{N}{\gamma} + \frac{\tau^{\frac{\sigma}{1-\sigma}}}{\tau^{\frac{\sigma}{1-\sigma}} + \frac{1}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t}} \frac{1}{H_t} \frac{N^*}{\gamma^*}\right) \frac{(1-\alpha)(1-\sigma)}{\mu} \tag{3.34}$$

$$\widehat{n_t^*} = \left(\frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + \tau^{\frac{\sigma}{1-\sigma}}} H_t \frac{N}{\gamma} + \frac{1}{\tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + 1} \frac{N^*}{\gamma^*}\right) \frac{(1-\alpha)(1-\sigma)}{\mu} \tag{3.35}$$

Consequently, there is a system with three equations and three unknowns that determines the equilibrium and the evolution of these open economies. Synthesizing, the equations of the system are:

$$\frac{\widehat{n_t}}{\widehat{n_t^*}} = \frac{\left(\frac{N\gamma^*}{N^*\gamma} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t}\right)}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}} H_t \frac{N\gamma^*}{N^*\gamma}\right)} \tag{3.36}$$

$$\widehat{n_t^*} = \left(\frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + \tau^{\frac{\sigma}{1-\sigma}}} H_t \frac{N}{\gamma} + \frac{1}{\tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + 1} \frac{N^*}{\gamma^*}\right) \frac{(1-\alpha)(1-\sigma)}{\mu} \tag{3.37}$$

$$\frac{\dot{H_t}}{H_t} = \left(\widehat{n_t}(1-\delta H_t) - \widehat{n_t^*}\left(1 - \frac{\delta}{H_t}\right)\right)\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{3.38}$$

Solving the system, the dynamics of the model can be found. The first two equations of the system are entered into the dynamic equation of productivity differences between countries, and the dynamic equation of the net productivity gap is found:

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\widehat{n_t}}{\widehat{n_t^*}}(1-\delta H_t) - \left(1 - \frac{\delta}{H_t}\right)\right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{3.39}$$

The parameter of the demand relation between foreign and domestic heterogeneous goods $\theta = \tau^{\frac{\sigma}{1-\sigma}}$, is determined by the assumptions of transportation costs and productivities. $\phi = \frac{N\gamma^*}{N^*\gamma} = \frac{L}{L^*}$ is defined as a new variable. Thus the equation can be written in the following way:

$$\frac{\dot{H}_t}{H_t} = \left[\frac{\left(\phi - \frac{\theta}{H_t}\right)}{(1 - \theta H_t \phi)} (1 - \delta H_t) - \left(1 - \frac{\delta}{H_t}\right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{3.40}$$

The growth rate of the net productivity gap (the relative supernumerary income between countries) determines the short-term and long-term effects of international trade on economic growth. This growth is determined by the level of the net productivity gap H_t, knowledge spillover δ, and the relative labor force between countries ϕ. In short, interactions between supply and demand variables establish the evolutions that will occur for the two economies after trade.

From the growth rate of the net productivity gap (equation 3.40) the dynamics of the other variables of the model can be deduced. The steady equilibrium properties (existence, unicity and stability) are shown in the appendices and are described in the following propositions.

Proposition 5 *: There is one unique stationary equilibrium possible in this economy, the stability of which depends on the expansion path that crosses this and the value of which is determined by:*

$$H_t^{Equilibrium} = \frac{-\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right) + \sqrt{\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)^2 + \frac{4}{\phi}}}{2} \tag{3.41}$$

The proposition proves the existence of the long-term equilibrium and presents the steady state value of the relative supernumerary income of the economies that are trading. There is one unique steady state in which the net productivity gap remains constant and its growth rate is equal to zero. Knowledge spillover, transportation costs and relative labor force between countries determine the value of the net productivity gap in the long run $H^{Equilibrium}$.[5]

Proposition 6 *: There is only one convergent expansion path of the state variable H, the net productivity gap between countries, which guarantees the stability of the stationary equilibrium. This expansion path satisfies the following characteristics:*

$$\frac{\dot{H}_t}{H_t} = Convergent \Leftrightarrow \delta > \theta \ \wedge \frac{\delta}{\theta} > \phi \tag{3.42}$$

There is only one convergent expansion path that guarantees the stability of the stationary equilibrium. This path only exists in the cases in which the next two conditions are achieved: first, knowledge spillovers between the countries (δ) are greater than the relationship of their demands (θ), which is a price ratio determined by transportation costs and the substitutability between varieties. Second,

[5] In other words, the last proposition summarizes the main information that provides the functional form of the state variable H, which is a quadratic function, presenting two roots that could become points of the stationary equilibrium. However, only the positive root makes economic sense and it is established as the only equilibrium present in the function. The stability of this equilibrium is determined by each of the possible expansion paths that cross it.

the relative labor force between the countries is lower than the ratio of the above two variables ($\frac{\delta}{\theta}$). Synthesizing then, the equilibrium is stable only if the knowledge spillovers are large enough in relation to transportation costs and the relative labor force between countries.

Figure 1 represents the stable equilibrium for the convergent expansion path and the unstable equilibrium for one of the divergent expansion paths with progressive specialization by low levels of knowledge spillover.

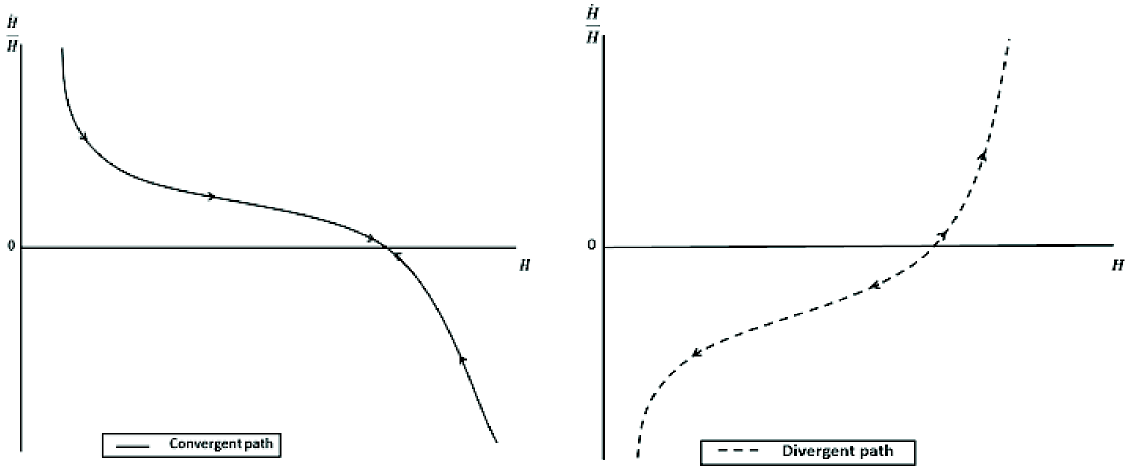

Figure1:Convergent and divergent path of H

Convergence in productivity between countries after trade can be achieved for similar countries in market size and productivity, whenever there is a high level of knowledge spillover between the participating economies. The more similar (different) the countries that trade are, the more (less) likely a convergence in the net productivity gap. This result is known in the economic literature as conditional convergence and it is a stylized fact that has been amply demonstrated by different authors.[6]

When the parameters do not meet the conditions described by *proposition 2*, the equilibrium is unsteady (see figure 1) and the paths are divergent. This divergence is generated by the trade association among very different countries or the absence of knowledge spillovers that increase production learning after trade liberalization.

On the one hand, if the countries that trade are very different, the net productivity gap will be large and it will generate a complete specialization after trade due to HME. The country with a greater supernumerary income will specialize in the production of heterogeneous goods and increase productivity through the learning by doing process, while the partner country will specialize in homogenous goods and only improve productivity via knowledge spillovers. The more different the countries, the more likely the divergence. On the other hand, the absence of knowledge spillovers limits the learning

[6] Barro and Sala-i-Martin (1992) and Sala-i-Martin (1996) provide some of the most cited articles for this empirical estimation.

process among economies after trade, so the country with higher levels of productivity will begin to specialize in heterogeneous goods production until it achieves complete specialization for such goods. The greater the knowledge spillovers, the greater the likelihood of convergence between countries.

In figure 1, the solid line represents the case in which $\delta > \theta$ and $\frac{\delta}{\theta} > \phi$, where the expansion path converges to a stationary equilibrium. The growth rates of productivity among the associated countries are equal and converge to a dynamically stable equilibrium. The dotted line represents the divergent path ($\theta > \delta$), where there also exists an equilibrium but it is unstable. Here the productivity of partner countries tends to diverge over time, generating a complete specialization in the production of manufactured goods in one of the countries involved in the trade. The production in which a country specializes depends on the conditions of the countries at the moment that trade starts. The country with a greater supernumerary income specializes in manufactured goods, while its partner specializes in the production of homogeneous goods.

The results on the state variable show the determinants of international trade effects on economic growth under this analysis framework. First, the economic market size of the countries that are trading, which is defined through the relative supernumerary income, determines the international trade effects on economic growth in the same way as that of the conditional convergence theory - similar countries have similarly steady states. Second, the knowledge spillovers.The more knowledge spillovers there are the easier the convergence among countries that trade.[7]

3.3.1 Complete Specialization

Complete specialization scenarios come from trading between very asymmetrical countries in their supernumerary incomes or from a low level of knowledge spillovers. The mechanism operates through HME and HME produces a concentration of heterogeneous goods production in the more productive country.[8] The productivity functions then change to a new regime of complete specialization. When the relationship of productivities H_t is greater than the value of the asymptote after trade, or the parameters value generates an equilibrium point greater than the vertical asymptote, scenarios of complete specialization occur. In such cases, when countries are very different the country with a greater economic market size specializes in the production of manufactured goods, raises productivity levels and therefore expands the productivity gap with the trading partner. At the same time, the commercial counterpart specializes in the production of homogeneous goods, thereby reducing its growth. The convergence or divergence in the growth rate depends on the existence of knowledge spillovers, yet divergence is always present in the levels of income and productivity.

[7] Some authors relate the knowledge spillovers with the property rights institutions or some institutional features, Acemoglu (2009).

[8] This means the country with the greater supernumerary income.

The other scenarios of complete specialization are presented when the countries are not very different in their supernumerary incomes but the transportation costs are greater than the levels of knowledge spillover ($\theta < \delta$), and/or the quotient of spillovers on transportation costs is less than the relative labor force between countries $\left(\frac{\delta}{\theta} < \phi\right)$. In these cases, trade generates incomplete specialization between countries in the short term but the net productivity gap is expanded through time. The income and productivity levels are more distant each time, until they find a long-term steady state in the new regime of complete specialization.

The dynamic of complete specialization scenarios is determined by equations (3.22) and (3.23). Complete specialization implies that the production of varieties of heterogeneous good is zero in the country specialized in the production of homogeneous goods. By inserting these assumptions into the equations and applying them to the same notation of supernumerary income ($\widehat{A_t}$ y $\widehat{n_t}$) the results are:

Under complete specialization in heterogeneous goods at the domestic level:

$$\widehat{n_t} = (1-\alpha)\left(\frac{N_t}{\gamma} + \frac{N^*}{\gamma^* H_t}\right)\left(\frac{1-\sigma}{\mu}\right) \text{ and } \widehat{n_t^*} = 0 \tag{3.43}$$

Under complete specialization in heterogeneous goods at the foreign level:

$$\widehat{n_t} = 0 \text{ and } \widehat{n_t^*} = (1-\alpha)\left(\frac{N_t H_t}{\gamma} + \frac{N^*}{\gamma^*}\right)\left(\frac{1-\sigma}{\mu}\right) \tag{3.44}$$

The equation (3.43) represents the number of varieties produced in terms of productivity levels for the case of complete specialization in heterogeneous goods in the domestic market, while (3.44) corresponds to the case of complete specialization in heterogeneous goods abroad. The growth rate of the state variable H_t, in scenarios of complete specialization, can be defined in order to identify the results in the best way. However, there exist two different scenarios for this variable depending on the presence, or not, of knowledge spillovers.

Complete specialization with knowledge spillover ($\delta > 0$)

In the presence of knowledge spillover ($\delta > 0$), the two possible scenarios under complete specialization are:

$$\frac{\dot{H_t}}{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = (1-\delta H_t)\left(L + \frac{L^*}{H_t}\right)\frac{(1-\alpha)\,\sigma}{\beta} \text{ With } \widehat{n_t} > 0 \text{ and } \widehat{n_t^*} = 0 \tag{3.45}$$

$$\frac{\dot{H_t}}{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = -\left(1 - \frac{\delta}{H_t}\right)(LH_t + L^*)\frac{(1-\alpha)\,\sigma}{\beta} \text{ With } \widehat{n_t} = 0 \text{ and } \widehat{n_t^*} > 0 \tag{3.46}$$

The equation (3.45) corresponds to the case of complete specialization in heterogeneous goods at the domestic level and the equation (3.46) is the analogous version for the foreign level.

The country that specializes in heterogeneous goods grows according to the number of varieties produced, while the country that specializes in homogeneous goods grows according to knowledge spillovers. After trade, the net productivity gap increases because one country stops producing heterogeneous goods and the other starts to produce all sets of varieties. Consequently, the growth rate of the net productivity gap is positive. Nevertheless, in the long term the economies arrive at a steady state under the complete specialization regime. In this steady state, both countries grow at the same rate but their levels of income and productivity differ.

Proposition 7 *: If knowledge spillovers are present and the countries that trade are very different in their supernumerary incomes ($H_t > \frac{1}{\theta\phi} \vee H_t^{Equilibrium} > \frac{1}{\theta\phi}$), or the parameters value does not meet the conditions of proposition 2, there is complete specialization in the steady state. In this steady state there exists convergence in the growth rate but not at the level of the variables. The countries grow at the same rate, but with different levels of productivity and income.*[9]

The country that specializes in homogeneous goods grows at the same rate as the country that specializes in heterogeneous goods because of the presence of knowledge spillovers. In the long run, the growth rate is the same but the levels of productivity and income are different; this is conditional convergence.

Complete specialization without knowledge spillover ($\delta = 0$)

In the absence of knowledge spillover ($\delta = 0$), the two possible scenarios under complete specialization are:

$$\frac{\dot{H_t}}{H_t} = \left(L + \frac{L^*}{H_t}\right)\frac{(1-\alpha)\,\sigma}{\beta} \text{ With } \widehat{n_t} > 0 \text{ and } \widehat{n_t^*} = 0 \tag{3.47}$$

$$\frac{\dot{H_t}}{H_t} = -\left(LH_t + L^*\right)\frac{(1-\alpha)\,\sigma}{\beta} \text{ With } \widehat{n_t} = 0 \text{ and } \widehat{n_t^*} > 0 \tag{3.48}$$

Equation (3.47) corresponds to the case of complete specialization in heterogeneous goods at the domestic level and the equation (3.48) is the analogous equivalent for the foreign level.

Without knowledge spillovers, the net productivity gap increases after trade, and the countries diverge in growth rates and levels of income and productivity. The country that specializes in heterogeneous goods grows according to the number of varieties produced, while the country that specializes in homogeneous goods stops growing because it does not produce heterogeneous goods and technology transfer is absent.

The last two scenarios show how the growth rate of the net productivity gap between countries is less in scenarios with knowledge spillover than those without. It shows how knowledge spillovers are a fundamental source of technology transfer between countries and how they contribute to reducing the

[9] Under complete specialization at the domestic level, the long-term net productivity gap is $H^{ss} = \frac{1}{\delta}$. Under complete specialization at the foreign level, the long-term net productivity gap is $H^{ss} = \delta$.

gap between asymmetric countries. It has been explained in both development theory and economic history how developed (United States, England, Japon, etc.) and emerging economies (China, Taiwan, Singapore, etc.) have taken advantage of knowledge spillovers to increase technology transfers and reduce the productivity gap for the greatest economies, Chang (2001).

These scenarios of complete specialization, as well as the other cases mentioned above, have direct implications for the dynamics of productivity of the countries, as well as for the different economic variables that compose the model, determining their path and their levels at every moment in time. The expansion path of the state variable of the net productivity gap in the last cases of complete specialization is divergent, and so supernumerary income, productivity and production are divergent too.

3.3.2 Welfare

In this section, I analyze the effects of international trade on welfare. Welfare, in the case of an open economy, is determined by the following expression, which relates to different variables present in the model. The state variable $\widehat{A_t}$, which drives the dynamics of the model and determines the number of varieties produced in each country, as well as the supernumerary income in each region; furthermore, the earnings for diversity in the products, represented by the number of varieties of manufactured goods available after trade.

$$U_t = \int_0^\infty e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t} \left\{ \alpha \ln \left(\alpha \left(\frac{A_t}{\gamma} - \overline{X} \right) \right) + (1-\alpha) \ln \left(\frac{(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)}{\left(n_t \left(\frac{\beta}{\sigma}\right)^{\frac{\sigma}{\sigma-1}} + n_t^* \left(\frac{\beta}{\sigma\tau}\right)^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}}} \right) \right\} dt \tag{3.49}$$

$$U_t = \Omega + \int_0^\infty e^{-\rho t} \left[\ln \widehat{A_t} - (1-\alpha) \frac{\sigma - 1}{\sigma} \ln \left(n_t + n_t^* \left(\frac{1}{\tau} \right)^{\frac{\sigma}{\sigma-1}} \right) \right] dt \tag{3.50}$$

$$\text{Where } \Omega = \alpha \ln \alpha + (1-\alpha) \ln (1-\alpha) - \ln \gamma - (1-\alpha) \ln \left(\frac{\beta}{\sigma} \right) \tag{3.51}$$

Looking for a comparative framework for the effects of international trade, the results of trade in terms of welfare can be contrasted with welfare levels under autarky. Rewriting the intertemporal utility equation at autarky:

$$U_t^A = \Omega + \int_0^\infty e^{-\rho t} \left[\ln \widehat{A_t^A} - (1-\alpha) \frac{\sigma - 1}{\sigma} \ln \left(n_t^A \right) \right] dt \tag{3.52}$$

The subtraction between the two intertemporal utilities leads to the following expression, which presents the welfare differential in terms of the evolution of the productivity level and the number of available varieties of heterogeneous good.

$$U_t - U_t^A = \int_0^{\infty} e^{-\rho t} \left[\ln\left(\frac{\widehat{A_t}}{\widehat{A_t^A}}\right) + (1-\alpha)\frac{(1-\sigma)}{\sigma}\ln\left(\frac{n_t}{n_t^A} + \frac{n_t^*}{n_t^A}\left(\frac{1}{\tau}\right)^{\frac{\sigma}{\sigma-1}}\right)\right] \tag{3.53}$$

The variation in the intertemporal utility - which is a discounted sum of intratemporal variations - basically depends on two differentiated effects. First, the effects of the available varieties of heterogeneous goods, which increases utility levels after trade, as shown in Giraldo (2015). This effect is always positive.

$$Varieties\ Effect = (1-\alpha)\frac{(1-\sigma)}{\sigma}\ln\left(\frac{n_t}{n_t^A} + \frac{n_t^*}{n_t^A}\left(\frac{1}{\tau}\right)^{\frac{\sigma}{\sigma-1}}\right) \tag{3.54}$$

Second, the productivity effect (or supernumerary income effect), which is a direct result of trade effects on the variable of the net productivity gap between countries.

$$Productivity\ Effect = \ln\left(\frac{\widehat{A_t}}{\widehat{A_t^A}}\right) \tag{3.55}$$

This productivity effect can go in any direction, depending on the effects of international trade on the net productivity gap H_t. Thus the productivity effect depends directly on the convergence or divergence scenario, which entails a trading relationship. Convergent scenarios after trade will generate a positive dynamic effect that raises the level of welfare. Divergent scenarios after trade will produce a dynamic effect that is positive in countries that specialize in the production of heterogeneous goods. However, the dynamic effect on countries that specialize in the production of homogenous goods will be positive or negative, depending on the presence, or not, of knowledge spillovers.

Figure 2 shows the simulation of trade effects on welfare for the domestic country by comparing utility levels under trade and autarky in three possible scenarios with positive knowledge spillovers. Trade instantaneously increases utility in relation to autarky levels in the three simulated scenarios. However, in time, the effects of international trade on economic growth are only completely positive for a convergence scenario, or a divergence scenario with complete specialization in heterogeneous goods for the domestic country.

For the scenario in which trade generates divergence with complete specialization in heterogeneous goods for the foreign country, the levels of welfare are worse in the domestic country after the first period but they start to improve after about twenty periods. The effect of the first period represents the standard static effect of greater available varieties, which improves welfare instantaneously. The negative effect of the following periods demonstrates the productivity effect, which is negative and greater than the varieties effect for these periods due to the loss of a manufactured sector for the domestic country. However, productivity growth through knowledge spillovers allows for an increase in productivity levels for the domestic country and counteracts the negative productivity effect until welfare after trade starts to become better than under autarky. In the long run, welfare is better under trade than under autarky in any scenario with positive knowledge spillovers.

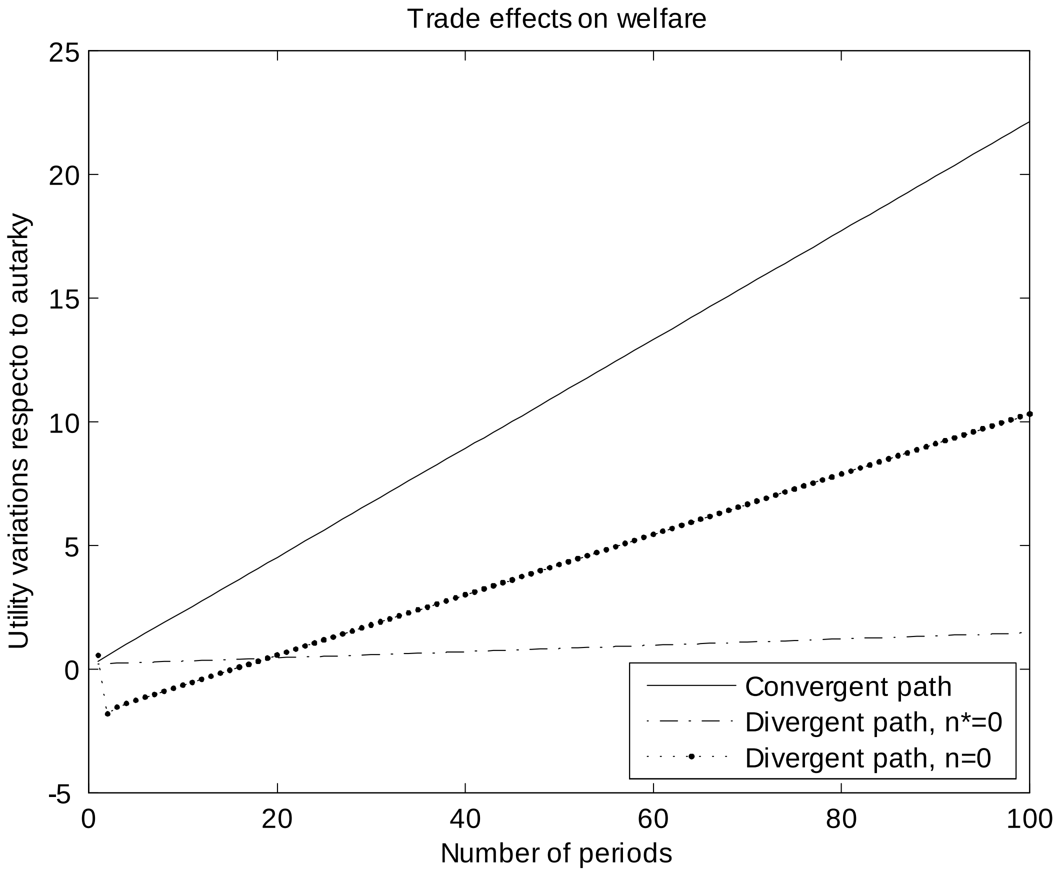

Figure 2: Welfare levels in relation to autarky with positive knowledge spillover

Figure 3 shows the simulation under complete specialization without knowledge spillover between countries after trade. There are two possible divergent scenarios, with complete specialization in heterogeneous goods production at the domestic or foreign level. Trade produces an instant positive effect in both scenarios (the static varieties effect). However, in time, the effects of international trade on welfare are only positive for a divergent scenario with complete specialization in heterogeneous goods at the domestic country level. For the case in which trade generates divergence with complete specialization in heterogeneous goods at the foreign country level, the levels of welfare will deteriorate over time in relation to autarky levels in the domestic market.

The effects are clear and show the importance of knowledge spillovers in the dynamic effects of trade on economic growth. Without technology transfer, trade only carries implications for the number of available varieties in the global market, while productivity levels are self-determined in each market. As a result, supernumerary income depends on the types of goods in which each country specializes.

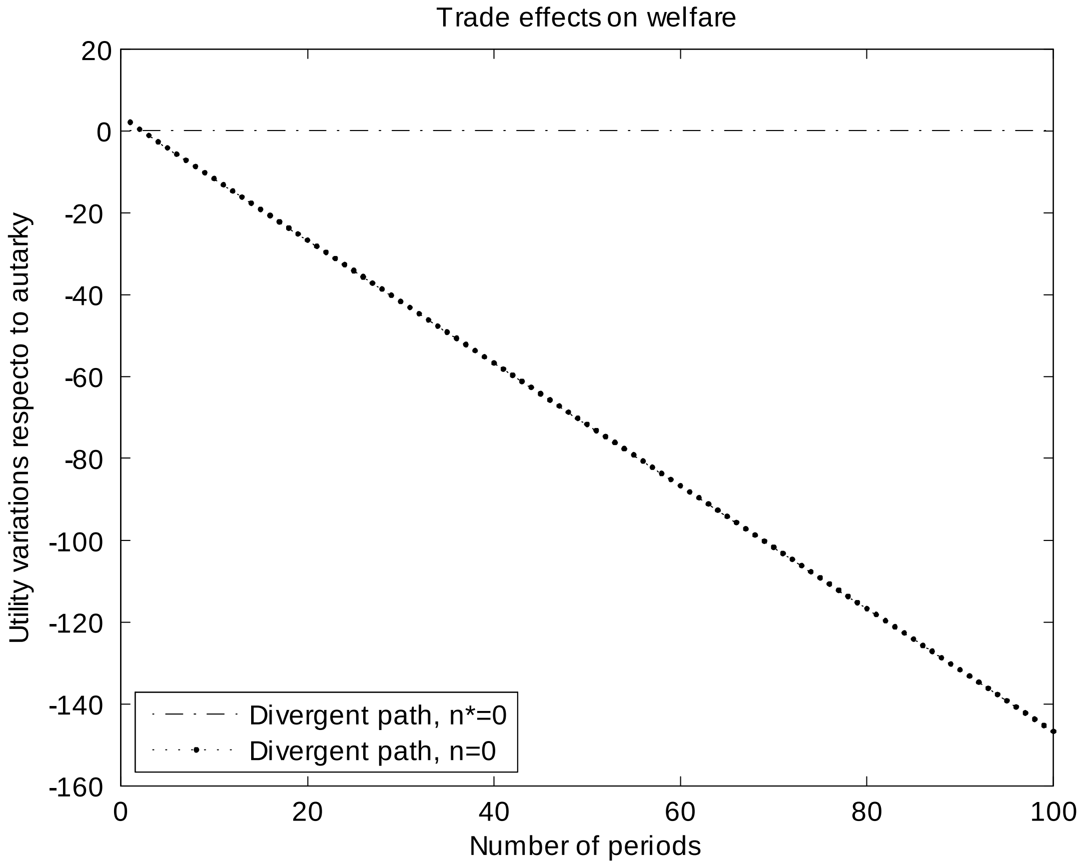

Figure 3: Welfare levels in relation to autarky without positive knowledge spillovers

In short, the effects of trade liberalization on economic growth and welfare are only positive for domestic markets in the following cases. Trade with a similar country in the presence of high levels of knowledge spillover, or trade with a country with a smaller market size, which allows it to take a divergence path with complete specialization in heterogeneous goods in the domestic market through HME.

Trade with a large country generates negative implications for domestic growth and welfare. Under positive knowledge spillovers there exists income divergence and less welfare than under autarky in the short term. Without knowledge spillovers, divergence in growth rate and income occurs and, furthermore, welfare levels are at their worst, even when compared with the conditions under autarky. According to this section, autarky is strictly preferred to free trade in this last case.

In contrast to the mainstream theory of the effects of international trade, the model presented in this paper shows that the trade effects on economic growth and welfare may be positive or negative. The final result of international trade depends on the types of countries that are trading and the economic policy that regulates any trade. The best results of the trade are presented when similar countries trade, or different countries trade under positive knowledge spillover.

3.4 Conclusions

In this paper, I analyze the conditions under which trade has positive and negative effects on growth. The results show some elements that contribute to this paradigmatic discussion.

The existence of different productivity expansion paths shows the diversity of scenarios that might be generated according to the countries that trade. The divergence or convergence of these paths reveals the impact of international trade on economic growth. In particular, the results show that productivity levels, the dependency ratio, knowledge spillovers and population size are determinants of the effects of international trade on economic growth.

Trade between similar countries in the presence of knowledge spillover generates positive effects for the countries. The countries converge to a steady state with equal long-term growth rates and better welfare than in autarky. When there are low levels of knowledge spillover or the countries' sizes are very asymmetrical, international trade causes different effects on growth. The divergent scenarios show that the country with a greater market size improves its productivity and supernumerary income after trade, while the smaller country would have positive or negative results depending on the existence of knowledge spillovers. The net productivity gap between countries expands over time, so that income and productivity levels are more distant every time, until a new steady state in a new regime of complete specialization is established. Under the complete specialization scenario with knowledge spillover the long-term growth rate is equal between the countries, but the levels of productivity and income are divergent. However, the complete specialization scenario without knowledge spillover produces divergence in both the long-term growth rate and income and productivity levels.

The effects on welfare go in the same direction as the effects on growth. In cases in which countries converge, welfare is greater in relation to the welfare levels under autarky for both countries. For divergent cases, welfare after trade is better for both countries in the long run only when technology transfer is present (positive knowledge spillover). Without technology transfer, only the country that keeps growing after trade sees an increase in its welfare levels with respect to autarky, while its counterpart reduces these utility levels permanently. The last dynamic effect occurs despite the fact that initially both countries increase their welfare, which is reflected in the positive static effect widely presented in static international trade models.

The discussion surrounding this relationship continues and these results are nothing more than a contribution that aims to direct the discussion towards the search for more scenarios and determinants that clarify the different implications of economic globalization on country development.

3.5 Appendices

3.5.1 Appendix 1: Properties of the dynamic function of the net productivity gap (propositions 1 and 2)

This appendix shows a particular analysis of the characteristics of the dynamic function of the productivity gap, which supports the results presented in the article. In the first instance, it can be said that the part outside the square bracket of the next equation is a positive constant that only modifies the speed rate in the variable H. Therefore, the functional form is determined by the equation within the square brackets.

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\left(\frac{N\gamma^*}{N^*\gamma} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t} \right)}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}} H_t \frac{N\gamma^*}{N^*\gamma} \right)} (1 - \delta H_t) - \left(1 - \frac{\delta}{H_t} \right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{3.56}$$

The assumptions about transportation costs and productivity in the model establish $\theta = \tau^{\frac{\sigma}{1-\sigma}}$, and a new variable can be defined $\phi = \frac{N\gamma^*}{N^*\gamma} = \frac{L}{L^*}$.

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\left(\phi - \frac{\theta}{H_t} \right)}{(1 - \theta H_t \phi)} (1 - \delta H_t) - \left(1 - \frac{\delta}{H_t} \right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{3.57}$$

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\left(\phi - \delta\phi H_t - \frac{\theta}{H_t} + \delta\theta \right)}{(1 - \theta H_t \phi)} - \left(1 - \frac{\delta}{H_t} \right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{3.58}$$

The functional form is determined by the function inside the square brackets, thus redefining the part of function to be analyzed as $Z(H_t)$:

$$Z_t = \left[\frac{\left(\phi - \delta\phi H_t - \frac{\theta}{H_t} + \delta\theta \right)}{(1 - \theta H_t \phi)} - \left(1 - \frac{\delta}{H_t} \right) \right] \tag{3.59}$$

First, it is possible to determine the zeros of the function to identify its functional form and its possible equilibriums. Reorganizing the function Z_t.

$$Z_t = \frac{\phi - \delta\phi H_t - \frac{\theta}{H_t} + \delta\theta - 1 + \frac{\delta}{H_t} + \theta\phi H_t - \delta\theta\phi}{(1 - \theta H_t \phi)}$$

The numerator determines the zeros of the function. Equalizing the numerator to zero and multiplying by H finds the following quadratic expression with its respective solution:

$$H_t^2 + \frac{(\phi + \delta\theta - 1 - \delta\theta\phi)}{(\theta - \delta)\phi} H_t - \frac{1}{\phi} = 0$$

$$H_{1,2} = \frac{-\left(\frac{\phi + \delta\theta - 1 - \delta\theta\phi}{(\theta-\delta)\phi} \right) \pm \sqrt{\left(\frac{\phi + \delta\theta - 1 - \delta\theta\phi}{(\theta-\delta)\phi} \right)^2 + \frac{4}{\phi}}}{2} \tag{3.60}$$

After solving the quadratic expression one could conclude that the function has two roots, one positive and one negative, the values of which depend, in particular, on the parameter values. The negative segment of this function is of no interest within the context of this model, since it does not have any valid economic interpretation (negative productivity). Similarly, a vertical asymptote is verified when H_t takes the value $\frac{1}{\theta\phi}$ which adds to the information used to define the form that takes different expansion paths. The information obtained about the function's characteristics proves the existence of a stationary equilibrium (*proposition 1*) in the positive root of the function, and the existence of a vertical asymptote which denotes a regime change towards a complete specialization scenario.

After obtaining the above information, one should determine the shape of the expansion paths that cross the equilibrium. Z_t is differentiated in order to determine these paths:

$$\frac{\partial Z_t}{\partial H_t} = \frac{\theta - \delta\phi H_t^2 - 2\theta^2\phi H_t + \theta\phi^2 H_t^2 + \delta\theta^2\phi H_t^2 - \delta + 2\delta\theta\phi H_t - \delta\theta^2\phi^2 H_t^2}{\left(1-\theta H_t\phi\right)^2 H_t^2}$$

Organizing the expression, the following is found:

$$\frac{\partial Z_t}{\partial H_t} = \frac{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) H_t^2 + \left(2\delta\theta\phi - 2\theta^2\phi\right) H_t + (\theta - \delta)}{\left(1-\theta H_t\phi\right)^2 H_t^2}$$

The denominator is always positive. The differentiation sign will depend on the values that take the numerator. In particular, for the following calculations we will call $b(H_t)$ the numerator, which fundamentally defines the values of the differentiation.

$$b(H_t) = \left[\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) H_t^2 - 2\theta\phi\left(\theta - \delta\right) H_t + (\theta - \delta)\right]$$

A particular analysis shows:

$$\begin{aligned} b(0) &= (\theta - \delta) \\ \frac{\partial b}{\partial H_t} &= 2\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) H_t - 2\theta\phi\left(\theta - \delta\right) \\ b'(0) &= -2\theta\phi\left(\theta - \delta\right) \\ \frac{\partial^2 b}{\partial H_t^2} &= 2\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) \end{aligned} \tag{3.61}$$

These criteria determine intervals in which the function is concave or convex and, therefore, if it has a minimum or a maximum in the critical value where the first differentiation is zero:

$$\frac{\partial b}{\partial H_t} = 0 \Leftrightarrow H_t^{critical} = \frac{\theta\phi\left(\theta - \delta\right)}{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right)}$$

Entering this critical value $H_t^{critical}$ in the function $b(H_t)$:

$$b(H_t^{critical}) = \frac{-\theta^2\phi^2\left(\theta - \delta\right)}{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right)} + 1$$

If the parameter values generate a $b(H_t)$ function with positive concavity and the function evaluated at the critical value is positive, it means that the slope of the function Z_t is always positive. If the parameter values generate a $b(H_t)$ function with negative concavity and the function evaluated at the critical value is negative, it means that the slope of the function Z_t is always negative. In other cases, a change of slope that modifies the expansion paths is presented. After evaluating $b(H_t^{critical})$ it has the following inequality:

$$b(H_t^{critical}) > 0 \Leftrightarrow \frac{\theta^2\phi^2\left(\theta-\delta\right)}{\left(\theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2\right)} < 1 \tag{3.62}$$

The inequality results depend in the first instance on the value that takes the denominator. This value is determined by the values of the parameter ϕ, which is related to the labor force size of the countries trading.

$$\begin{aligned} \text{Denominator} &> 0 \Leftrightarrow \theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2 > 0 \\ &\Leftrightarrow \theta\left(1-\delta\theta\right)\phi^2+\delta\left(\theta^2-1\right)\phi > 0 \\ \phi &> \frac{\delta\left(1-\theta^2\right)}{\theta\left(1-\delta\theta\right)} = \frac{\delta-\delta\theta^2}{\theta-\delta\theta^2} \end{aligned} \tag{3.63}$$

Solving for ϕ the polynomial in the denominator, the root of the same is obtained $\phi = \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ which establishes the threshold where the inequality changes direction. This value is the same critical point that determines the sign of the second differentiation, that is, the concavity or convexity of the function. These results define the function's form, given the values that take different parameters. Two cases may occur in the resolution of the inequality:

When $\phi > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ the denominator is positive and the function is convex. If in addition the next condition on the parameter ϕ is accomplished then together these two criteria guarantee the existence of a positive minimum.

$$\begin{aligned} \frac{\theta^2\phi^2\left(\theta-\delta\right)}{\left(\theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2\right)} &< 1 \Leftrightarrow \theta^2\phi^2\left(\theta-\delta\right) < \theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2 \\ \theta^3\phi^2 &< \theta\phi^2+\delta\theta^2\phi-\delta\phi \\ 0 &< \theta\left(1-\theta^2\right)\phi^2+\delta\left(\theta^2-1\right)\phi \\ \frac{\delta}{\theta} &< \phi \end{aligned} \tag{3.64}$$

When $\phi < \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ the denominator is negative and the function is concave. If in addition the next condition on the parameter ϕ is accomplished then together these two criteria guarantee the existence of a maximum negative.

$$\begin{aligned} \frac{\theta^2\phi^2\left(\theta-\delta\right)}{\left(\theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2\right)} &< 1 \Leftrightarrow \theta^2\phi^2\left(\theta-\delta\right) > \theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2 \\ \theta^3\phi^2 &> \theta\phi^2+\delta\theta^2\phi-\delta\phi \\ 0 &> \theta\left(1-\theta^2\right)\phi^2+\delta\left(\theta^2-1\right)\phi \\ \frac{\delta}{\theta} &> \phi \end{aligned} \tag{3.65}$$

These findings for the characteristics of the differentiation of function Z_t generate six different scenarios according to the parameter values and the interval in which the parameter ϕ occurs.

Condition 1: If $\theta > \delta$ there are three different criteria according to the interval in which the parameter ϕ is, and these determine the function's form.

Case 1: If $\phi > \frac{\delta}{\theta} > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ the two conditions of ϕ (see (3.64)) are reached, the minimum value is positive, therefore the function $b(H_t)$ is always positive. The function $Z(H_t)$ always increases and describes a divergent expansion path of the productivity gap between countries after trade.

Case 2: If $\frac{\delta}{\theta} > \phi > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ only one of the two conditions of the parameter ϕ (see (3.64)) is reached. This implies that the minimum in the function is not always positive and that there is a slope change in the $Z\left(H_t\right)$ function. However, appendix 2 indicates that in this case the equilibrium $H^{Equilibrium}$ is always subsequent to the asymptote, which means that the path is still divergent since it occurs in a complete specialization regime.

Case 3: If $\frac{\delta}{\theta} > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \phi$ the two conditions on the parameter ϕ (see (3.65)) are reached, but given the condition $\theta > \delta$ the maximum is positive (see abscissa intercept (3.61)) and the function decreases in the interval (see second derivative criterion). This result presents the existence of a slope change in the $Z\left(H_t\right)$ function, so this variation also involves a change in the direction of the expansion path, meaning that it decreases. However, as in the previous criterion the equilibrium point is always more to the right than the asymptote of the function, which implies a divergence in the expansion path despite its change of direction because it occurs in a complete specialization regime (appendix 2).

Condition 2: If $\theta < \delta$, as in the previous condition, there are three different criteria determined by ϕ values that describe the function's shape.

Case 1: If $\frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \frac{\delta}{\theta} > \phi$ the two conditions of ϕ (see (3.65)) are reached, the maximum value is always negative, so the function $b(H_t)$ is always negative. This demonstrates that the $Z(H_t)$ function always decreases and describes a convergent expansion path for the productivity gap between countries after trade (*proposition 2*).

Case 2: If $\frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \phi > \frac{\delta}{\theta}$ only one of two conditions of the parameter ϕ (see (3.65)) is reached. This implies that the maximum in the function is not always negative and that there is a slope change in the $Z(H_t)$ function. With appendix 2 in mind, this may show that this case involves a divergent complete specialization scenario.

Case 3: If $\phi > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \frac{\delta}{\theta}$ the two conditions of the parameter ϕ (see (3.64)) are reached, but given the condition $\delta > \theta$ the minimum is negative (see intercept abscissa (3.61)) and the function increases. This implies a slope change in the $Z(H_t)$ function and therefore in the path direction. As in previous atypical cases, appendix 2 shows that this case represents a divergent complete specialization scenario.

The analysis of the different possible cases in the trade between different countries shows that there is only one equilibrium $H^{Equilibrium}$ (*proposition 1*) and one convergent path achieved when $\delta > \theta \wedge \frac{\delta}{\theta} > \phi$ (*proposition 2*). The other cases generate complete specialization for the production of countries after trade and a divergent expansion path in H_t.

After knowing the behaviour of function $Z(H_t)$ via its properties and the properties of its differentiations $b(H_t)$, we can return to the function (3.57) to determine the trajectories of the state variable H_t, as defined above. The dynamic productivity function will be:

$$\frac{\dot{H_t}}{H_t} = Z(H_t)\widehat{n_t^*}\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{3.66}$$

Accordingly, the different expansion paths are determined by the Z function multiplied by a positive constant (in intervals with economic interpretation), which does not alter its functional form.

3.5.2 Appendix 2: The relation between equilibrium and asymptote

Cases 2 and 3 from the two conditions in the previous appendix can be solved by determining whether the equilibrium comes before the asymptotic behavior of the function where economic interpretation is present under the incomplete specialization of the countries, or, outside of this, where a complete specialization scenario exists and the function that determines the expansion path is different. In particular, it is necessary to determine whether:

$$\frac{-\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)+\sqrt{\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)^2+\frac{4}{\phi}}}{2} > \frac{1}{\theta\phi}$$

It can be shown that this inequality is always reached for the criteria in question. Therefore, regardless of the slope change in the expansion path for criteria 2 and 3 of both conditions, these are always divergent, since they occur in complete specialization scenarios.

Bibliography

[1] Acemoglu, D. (2009). Introduction to Modern Economic Growth. New Jersey: Princeton.

[2] Acemoglu, D., Johnson, S., & Robinson, J. (2005). The Rise of Europe: Atlantic trade, institutional change and economic growth. American Economic Review, 546-579.

[3] Aghion, P., & Howitt, P. (2005). Growth with Quality-improving Innovations: An integrated framework. Handbook of Economic Growth.

[4] Baldwin, R., Martin, P., & Ottaviano, G. (2001). Global Income Divergence, Trade and Industrialization: The geography of growth take-offs. Journal of Economic Growth, 5-37.

[5] Barro, R., & Sala-i-Martin, X. (1992). Convergence. Journal of Political Economy, 223-251.

[6] Barro, R., & Sala-i-Martin, X. (2004). Economic Growth. Cambridge: MIT press.

[7] Ben-David, D. (1993). Equalizing Exchange: Trade liberalization and income convergence. Quarterly Journal of Economics, 653-679.

[8] Chang, H.-J. (2001). Intellectual Property Rights and Economic Development: Historical lessons and emerging issues. Journal of Human Development: A Multi-disciplinary journal for people-centred development, 287-309.

[9] Chang, R., Kaltani, L., & Loayza, N. (2009). Openness can be Good for Growth: The role of policy complementarities. Journal of Development Economics, 33-49.

[10] Chung, C. (2006). Nonhomothetic Preferences and the Home Market Effect: Does Relative Market Size Matter? Georgia Institute of Technology.

[11] Dollar, D. (1992). Outward-oriented developed economies really do grow more rapidly: Evidence from 95 LDC's. Economic Development and Cultural Change, 523-544.

[12] Dollar, D. (2005). Globalization, poverty, and inequality since 1980. Oxford University Press, 145-175.

[13] Edwards, S. (1998). Openness, prosuctivity and growth: What do we really know? Economic Journal, 383-398.

[14] Frankel, J. A., & Romer, D. (1999). Does Trade Cause Growth? The American Economic Review, 379-399.

[15] Galor, O., & Mountford, A. (2008). Trading Population for Productivity: Theory and Evidence. The Review fo Economic Studies, 1149-1179.

[16] Gancia, G., & Zilibotti, F. (2005). Horizontal Innovation in the Theory of Growth and Development. Handbook of Economic Growth.

[17] Giraldo Salazar, I. (2014). Home Market Effect and Relative Market Size.

[18] Grossman, G. M., & Helpman, E. (1997). Innovation and Growth in the global economy. Cambridge: The MIT Press.

[19] Kind, H. (2002). Endogenous Growth and Trade Liberalization Between Small and Large Countries. Review of International Economics, 151-165.

[20] Krugman, P. (1987). The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Tatcher. Journal of Development Economics, 41-55.

[21] Lucas, R. (1988). On the Mechanics of Economics Develpment. Journal of Monetary Economics, 3-42.

[22] Maddison, A. (1983). A Comparison of Levels of GDP Per Capita in Developed and Developing Countries, 1700-1980. The Journal of Economic History, 27-41.

[23] Martin, P., & Ottaviano, G. (1999). Growing locations: Industry location in a model of endogenous growth. European Economic Review, 281-302.

[24] Rivera-Batiz, L., & Romer, P. (1991). Internationaltrade with endogenous technological change. European Economic Review, 971-1001.

[25] Rodriguez, F., & Rodrik, D. (2001). Trade Policy and Economic Growth: A Skeptic's Guide to the Cross-National Evidence. NBER Macroeconomics Annual 2000, 261-338.

[26] Romer, P. (1990). Endogenous technological change. Journal of Political Economy, 71-102.

[27] Sachs, J., & Warner, A. (1995). Economic reform and the process of global integration. Brookings Papers on Economic Activity, 647-668.

[28] Sala-i-Martin, X. (1996). The Classical Approach to Convergence Analysis. The Economic Journal, 1019-1036.

[29] Singh, T. (2010). Does International Trade Cause Economic Growth? The World Economy, 1517-1563.

[30] Thoenig, M., & Verdier, T. (2003). A Theory of Defensive Skill-Biased Innovation and Globalization. The American Economic Review, 709-728.

[31] Ventura, J. (1997). Growth and Interdependence. The Quaterly Journal fo Economics, 57-84.

[32] Williamson, J. (1990). Latin American Adjustment: How Much Has It Happened? Washington: Institute for International Economics.

[33] Williamson, J. (2000). What Should the World Bank Think about the Washington Consensus? The World Bank Research Observer, 251-264.

[34] Williamson, J. (2002). Winners and Losers Over Two Centuries of Globalization. NBER working papers.

[35] Young, A. (1993). Substitution and Complementarity in Endogenous Innovation. The Quaterly Journal of Economics, 775-807.

Chapter 4

Catching up with the "Joneses", the Home Market Effect and Economic Growth.

4.1 Introduction

International trade not only involves an exchange of goods, but also implies a constant flow of goods, information, values, behaviors and more, all of which affect agent preferences. Besides economic issues, international trade has direct implications for the preferences and tastes of agents. However, extant literature about the effects of international trade focuses on welfare and economic growth, leaving aside the impacts of international trade on the preferences of agents. Indeed, it assumes exogenous preferences.[1]

The objective of the present paper is to analyse the implications of international trade on the evolution of agent preferences. To this end, I focus on the following questions. What are the impacts of international trade on the preferences of agents? Will preferences or cultural values converge towards homogeneous patterns? These questions are important when analyzing the discussion surrounding the implications of trade on the preferences of agents and, at the same time, the consequences for economic performance.

The new perspective that I propose is based on the existence of external habit formation in the consumption of heterogeneous goods,[2] which depends on the historical consumption of these kinds of good

[1] In spite of scarce economic research into this topic, some social sciences such as sociology and anthropology have studied this subject, providing some elements of analysis for this new field in economic research.

[2] Heterogeneous goods refers to manufactured goods or modern products with high levels of added value. Homogeneous

in the trading countries. The levels of consumption of other individuals affect agent utility. Therefore, under autarky agents take as their reference the consumption patterns of domestic individuals. After trade, each agent compares his own consumption with both national and foreign consumption. The reference points of consumption for the agents are dynamic and are determined by the levels of consumption for heterogeneous goods in both countries; this represents a clear interaction between taste channels and the preferences of agents after trade. Meanwhile, the levels of consumption for heterogeneous goods also determine trade effects on economic growth and, consequently, future levels of consumption. The effects of international trade on preferences and economic growth are jointly determined.

The present article shows that consumption converges towards a homogeneous pattern between countries after trade. Nevertheless, productivity and per capita income might either converge or diverge. The combination of convergence in consumption habits and convergence or divergence in countries' incomes after trade produces different scenarios for welfare levels, some of which exhibit autarky as strictly preferred to trade. Welfare performs better in scenarios with convergence in countries' incomes and performs at its worst under trade rather than under autarky in some divergent income scenarios. This last point is so because agents in the poorer country exhibit the same consumption patterns as the richer trade partner, but they do not have the income needed to satisfy their consumption preferences.

Emerging economic literature about the implications of international trade on the preferences of agents has been developed around the relationship between globalization and cultural diversity. The implications of trade for cultural values, consumption traditions and preferences are studied through different models that show the interactions that occur between cultures after trade and the implications for the native cultures of each country. A recent survey of this literature can be found in Bisin and Verdier (2014).

In spite of the findings of the existing literature about the topic, none of the current papers use the habit-formation mechanism to evaluate the impact of international trade on agent preferences. For example, Van Ypersele and Francois (2002), Bala and Van Long (2005), Janeba (2004) and Rauch and Trindade (2009) use cultural diversity as an exogenous static process. Other papers develop an endogenous dynamic process of cultural diversity similar to the process analyzed in the present paper, but with other specific mechanisms. Olivier et al. (2008) model cultural identity as a positive externality under perfect competition. Finally, Maystre et al. (2014) present a model in which trade integration leads to cultural convergence through the cultural socialization of parents under a monopolistic competition scenario.

The empirical evidence for trade impact on agent preferences comes from Maystre et al. (2014),

goods refers to agricultural goods, that is, goods free from any manufacturing process.

in which the authors show the link between trade and convergence values. They use the World Value Survey dataset to build an index of cultural distance and derive two stylized facts: "bilateral cultural distances exhibit significant time variation and that time variation in bilateral cultural distances is correlated with time variation in trade in (differentiated) goods" Maystre et al. (2014).

The models for habit formation in consumption have largely been developed from the literature of asset pricing, structural change, and growth and distribution wealth. The seminal papers of Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) show how the models for habit formation fit the data better than the models for fixed preferences. However, this specification has not yet been used in the study of the impact of international trade on consumer behavior.

The idea of using the habit formation channel in the present paper comes from evidence that utility or happiness depends on relative income, as shown by Clark and Oswald (1996) and Alesina et al. (2003). "The accepted standard of expenditure in the community largely determines what a person's standard of living will be" Pigou (1903). External habit formation in consumption suggests that agents not only appreciate consumption *per se*, but also the relative position it affords them with respect to the consumption of other agents. In an international trade context, agents not only compare their consumption with that of domestic agents but also with the agents of the trading partner country. This is so because trade is more than an exchange of goods, it is also an exchange of behaviors, tastes, traditions, information, publicity, and so on. Bisin and Verdier (2014).

International trade opens the door to cultural interchange, which affects agent behaviors. In relation to the literature concerning habit formation, international trade generates a new reference consumption level to be reached and, consequently, the utility levels of agents depend on the possibility of achieving the reference consumption level and finding a special status inside society which provides high levels of utility.

The results of the model presented in this paper are coherent with evidence concerning the correlation between trade and convergence in cultural values. Moreover, the results show that the impacts of international trade upon preferences and economic variables are codetermined and that the welfare impact of trade might be positive or negative in relation to autarky.

The article consists of this introduction and six additional sections. The second section exposes the fundamentals of the model, the third presents the habit specification in a closed economy, while the fourth presents the specification in an open economy. The fifth section exposes a simulation of the model with all dynamic effects, the sixth section presents the implications of trade on welfare and the seventh section concludes.

4.2 The Model

The general model is based on three elements. The first of these is that of interdependent preferences among countries and among agents (catching up with the "Joneses"). The second is a trade model based on the Home Market Effect (HME) with non-homothetic preferences, as found in Giraldo (2015), which includes demand and supply side elements in the determination of trade effects. The third element constitutes endogenous economic growth through a "learning by doing" process in the heterogeneous sector, as found in Krugman (1987) and Lucas (1988).

I assume two regions, domestic and foreign $(*)$.[3] There are two types of good: homogeneous (X), which represents agricultural goods, presents constant returns to scale in production and does not generate transportation costs in its trade; and heterogeneous (Y), which represents manufactured goods, exhibits economies of scale in production and demands transportation costs for its international trade. The varieties of heterogeneous good are horizontally differentiated *à la* Dixit-Stiglitz and the firms in this sector maximize the potential benefits under monopolistic competition. Labor (L), is the only existing factor of production and is mobile among sectors but immobile among countries.

With the idea of modeling the effects of demand composition on the internal market in a simple way, I use Chung's (2006) strategy. This assumes that the number of people who consume is different from the number of people who produce. Countries have the same amount of labor $(L = L^*)$, but their populations (N and N^*) may differ. It is thus supposed that domestic households offer $(\frac{1}{\gamma})$ units of work for each resident $(N = \gamma L = L)$, while foreign households offer $(\frac{1}{\gamma^*})$, meaning $(N^* = \gamma^* L^*)$.

Intuitively, γ captures the demographic and redistribution factors that affect relative demand for diverse goods in comparison to homogeneous goods. Using this modification it is possible to interpret γ as the population proportion that earns an income.

The consumption side supposes that all households demand goods and demand each variety of heterogeneous goods symmetrically (Y). Households in both countries have the same utility function with interdependent preferences (external habit formation). Catching up with the "Joneses" shows that the utility level of agents depends on their own consumption and average consumption in the market (autarky or global market). The intertemporal utility function is:

$$U_t = \int_0^\infty e^{-\rho t} u_t dt \tag{4.1}$$

The respective intratemporal utility function depends on the consumption of homogeneous goods X_t, the consumption of heterogeneous goods Y_t and the reference consumption level (Joneses) J_t:

$$u_t = u_t\left(X_t, Y_t, J_t\right) \tag{4.2}$$

[3] Hereafter, the variables corresponding to the foreign region have the superscript $*$.

The varieties of heterogeneous goods are horizontally differentiated *à la* Dixit-Stiglitz. Y_t is the aggregate consumption of all n varieties of heterogeneous good at time t and y_{it} is the consumption of the i-th variety at every moment.

$$Y_t = \left(\int_{i=1}^{n_t} y_i^{\sigma} di \right)^{\frac{1}{\sigma}} \tag{4.3}$$

The dynamic equation for the reference consumption level J_t (catching up with foreign and domestic "Joneses") is defined by the following equation:

$$J_{t+1} = \nu \overline{Y_t} + (1-\nu) J_t \tag{4.4}$$

Where the actual reference consumption level is a linear combination of the previous reference consumption level J_t and the previous average of consumption $\overline{Y_t}$, weighted by parameter ν.

$$\overline{Y_t} = \left\{ \begin{array}{c} \eta_t \overline{Y_t} \quad \text{in autarky} \\ \varphi \eta_t \overline{Y_t} + (1-\varphi) \eta_t^* \overline{Y_t}^* \ \ with\ 0 < \varphi < 1 \ \ \text{in trade} \end{array} \right\} \tag{4.5}$$

External habits or the reference consumption level for heterogeneous goods depends on the average consumption for this type of good ($\overline{Y_t}$). It is assumed that the consumption of each agent is negligible in relation to the aggregate consumption. In trade, the level of habit depends on the domestic and foreign average for the consumption of heterogeneous goods, each weighted by parameter φ. The average is normalized by variable η, which suppresses the preference of the agents for variety.[4] It takes the value $\left(\eta_t = n_t^{\frac{\sigma-1}{\sigma}}\right)$ in autarky and $\left(\eta_t = \left(n_t + \frac{n_t^*}{\tau}\right)^{\frac{\sigma-1}{\sigma}}\right)$ in trade.

Both goods use the same factor of production, namely labor. The production of each good is determined by the amount of labor used and its productivity. The production of homogeneous goods and all varieties of the heterogeneous goods sector use the same production function in both countries. The homogeneous goods sector has the following production function:

$$NX_t = D_{Xt} = L_{Xt} A_t \tag{4.6}$$

Where D_x is the aggregate demand of the homogeneous good, L_X is the amount of labor used in the production of these goods, and A_t is the productivity. The costs function in the heterogeneous goods sector is given by:

$$l_{it} = \frac{\mu}{A_t} + \frac{\beta D_{it}}{A_t} \quad i = 1, 2 \ldots n \quad \text{donde } D_{it} = N y_{it} \tag{4.7}$$

[4] This is a convenient normalization that has been used in other specifications, see for example, Blanchard and Kiyotaki (1987). This normalization avoids some of the analytical problems that would arise for the presence of a heterogeneous goods index price in agent demands.

Where D_{it} is the aggregate demand of the i-th variety, l_{it} is the amount of labor used in the production of each variety and A_t is the productivity level at time t. Moreover, β and μ are the parameters of the fixed and variable costs, respectively.

Technological progress only occurs in heterogeneous goods production through a specific learning process for each country, namely "learning by doing". The evolution of productivity depends on both domestic production in the manufacturing sector and the productivity of this sector abroad, which represents knowledge spillover from the outside into the domestic economy. For the case of autarky, this last factor is equal to zero ($\delta = 0$).

$$A_t = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right) \, ds \tag{4.8}$$

K_s and K_s^* are the levels of knowledge for each economy. These accumulate with the production evolution of heterogeneous goods. Thus:

$$K_s = \int_{1}^{n_t} y_{it} di \quad \text{and} \quad K_s^* = \int_{1}^{n_t^*} y_{jt} dj \tag{4.9}$$

Finally, the full-employment condition is assumed:

$$L = L_{Xt} + L_{Yt} = \frac{D_{Xt}}{A_t} + \sum_{i=1}^{n} \left(\frac{\mu}{A_t} + \frac{\beta D_{it}}{A_t} \right) \tag{4.10}$$

4.3 Habit Specification

In this section, I study the implications of catching up with the "Joneses" under a closed economy in order to determine the dynamic consequences of interdependent preferences after trade. The intratemporal utility function is:

$$u_t = \alpha \ln \left(X_t\right) + (1 - \alpha) \ln \left(\eta_t Y_t - \xi J_t\right) \tag{4.11}$$

Where ξ is an elasticity parameter that defines the importance of reference consumption levels for the welfare of the agents ($0 < \xi < 1$).

The intratemporal optimization problem of the agents is conventional, maximizing (4.11), subject to its budget restriction. Here I use the methodology presented by Chung (2006) to enter per capita income differences for domestic and foreign agents. This model differentiates between the members of the household who work and those who only consume. In more formal terms, each worker in the home has (γ) additional agents under its responsibility that only consume. $N = \gamma L$.

$$\max u_t = \alpha \ln X_t + (1 - \alpha) \ln \left(\eta_t Y_t - \xi J_t\right) \quad \text{s.a. } P_{Xt} X_t + P_{Yt} Y_t = \frac{w_t}{\gamma} \tag{4.12}$$

The equation for habit formation is:

$$\overline{Y_t} = \varphi \eta_t Y_t + (1-\varphi)\, \eta_t^* Y_t^{*1-\varphi} \text{ with } \varphi = 1 \text{ in autarky} \tag{4.13}$$

Optimal demand for agricultural goods X_t and aggregate manufacturing goods Y_t result from the optimization program by every agent in every moment in time:

$$X_t = \frac{\alpha}{P_{Xt}} \left(\frac{w_t}{\gamma} - \frac{P_{Yt}\xi J_t}{\eta_t} \right) \tag{4.14}$$

$$Y_t = \frac{(1-\alpha)}{P_{Yt}} \left(\frac{w_t}{\gamma} - \frac{P_{Yt}\xi J_t}{\eta_t} \right) + \frac{\xi J_t}{\eta_t} = \frac{(1-\alpha)}{P_{Yt}} \frac{w_t}{\gamma} + \frac{\alpha \xi J_t}{\eta_t} \tag{4.15}$$

The optimal demands at every moment in time depend on supernumerary income, each weighted by price, which in the case of manufactured goods is an index price that is established in reference to the price of each of the existing varieties. The demand for each of the varieties of heterogeneous good at every moment depends on the aggregate spending on such goods, weighted by the added prices of all varieties.

$$y_{it} = \frac{p_{it}^{\frac{1}{\sigma-1}} \left((1-\alpha) \frac{w_t}{\gamma} + \frac{\alpha P_{Yt} \xi J_t}{\eta_t} \right)}{P_{Yt}^{\frac{\sigma}{\sigma-1}}}$$

With the index price of heterogeneous goods $P_{Yt} = \left(\int_1^{n_t} p_{it}^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} = n_t^{\frac{\sigma-1}{\sigma}} p_{it}$ under autarky and $P_{Yt} = \left(n_t + \frac{n_t^*}{\tau^{\frac{\sigma}{\sigma-1}}} \right)^{\frac{\sigma-1}{\sigma}} p_{it}$ under trade.

4.3.1 Producer

Perfect competition is assumed in the production of homogeneous goods, which means that after cost minimization one may have an equilibrium price determined by the level of wages and productivity. The price of this good is established as a numeraire, so productivity determines salary levels in this economy.

$$P_{Xt} = 1 = \frac{w_t}{A_t} \tag{4.16}$$

Since the production of every variety of heterogeneous good uses the same technology of production, and monopolisitic competition exists in the sector, the price of each of the varieties is the same, and is determined by the markup $\left(\frac{1}{\sigma}\right)$ and the marginal costs $\left(\frac{\beta w_t}{A_t}\right)$.

$$p_{it} = \frac{\beta w_t}{\sigma A_t} \tag{4.17}$$

Inserting the prices into the zero-benefits condition, determined by the free entry and exit of firms in the manufacturing sector, the aggregate production of each variety is:

$$D_{it} = \frac{\mu \sigma}{(1-\sigma)\,\beta} \tag{4.18}$$

Finally, from the full-employment condition (4.10), it is possible to obtain the number of varieties of heterogeneous good produced in this economy for each time t.

$$n_t = \frac{L_{yt}\,(1-\sigma)\,A_t}{\mu} \tag{4.19}$$

The full-employment condition determines the amount of labor used in the heterogeneous goods sector L_{yt}, which is equal to the total available workforce (L), minus the quantity used in the production of homogeneous goods $\left(L_{xt} = \frac{N\alpha}{A_t}\left(\frac{A_t}{\gamma} - \frac{P_Y \xi J_t}{\eta}\right)\right)$. In this way, the number of varieties produced in every moment can be rewritten as:

$$n_t = \left(A_t - \alpha\left(A_t - \frac{\gamma P_Y \xi J_t}{\eta}\right)\right)(1-\sigma)\frac{N}{\mu\gamma} \tag{4.20}$$

This expression shows how the number of varieties of heterogeneous good produced in the domestic market, which we assume to be the level of industrialization of a country, is determined by demand and supply elements. The dependence factor (γ), productivity levels (A_t) and the reference consumption level (J_t) determine supernumerary income, which represents the purchasing power of the agents in each country and is a direct consequence of interdependent preferences. Population size (N) determines the market size in the standard way.

At the same time, the dynamic is imposed by the learning process generated in the manufacturing output, which develops according to production in this sector. Accordingly, the productivity variation rate is proportional to the amount of labor used in this sector.

$$\dot{A}_t = K_t = n_t y_i = n_t\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) = \left(A_t - \alpha\left(A_t - \frac{\gamma P_Y \xi J_t}{\eta}\right)\right) L\frac{\sigma}{\beta} \tag{4.21}$$

The productivity growth rate is obtained from the last equation, and from this, the growth rate for other variables:

$$\frac{\dot{A}_t}{A_t} = \frac{n_t}{A_t}\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{4.22}$$

The productivity growth rate is determined by the number of varieties of heterogeneous goods produced in the county which, at the same time, is determined by the dependence factor, the consumption reference level and the population size, as shown in equation (4.20).

Shortly, I will present the long-term equilibrium under autarky. I will define $\widehat{n_t}$ and $\widehat{J_t}$ as the number of varieties of heterogeneous good and the reference level of consumption normalized by the productivity level:

$$\widehat{n_t} = \frac{n_t}{A_t} \tag{4.23}$$

$$\widehat{J_t} = \frac{J_t}{A_t} \tag{4.24}$$

From the index price of heterogeneous goods and the equations (4.15), (4.4) and (4.5), it is possible to find the dynamic equation for consumption habits:

$$\dot{J}_t = \nu \left(\frac{(1-\alpha)\,\sigma A_t}{\beta\gamma} - (1-\alpha\xi)\, J_t \right) \tag{4.25}$$

From the equation of the prices of manufacturing and the number of varieties produced at each moment, it is possible to define:

$$\widehat{n_t} = \left(\frac{(1-\alpha)}{\gamma} + \frac{\alpha\beta\xi J_t}{\sigma A_t} \right) (1-\sigma)\, \frac{N}{\mu} \tag{4.26}$$

Following this, the productivity growth rate is defined as:

$$\frac{\dot{A}_t}{A_t} = \widehat{n_t} \left(\frac{\mu\sigma}{(1-\sigma)\,\beta} \right) = \left(\frac{(1-\alpha)\,\sigma N}{\gamma\beta} + N\alpha\xi \widehat{J_t} \right) \tag{4.27}$$

The dynamic of the habits of consumption is determined by:

$$\dot{\widehat{J_t}} = \nu \frac{(1-\alpha)\,\sigma}{\beta\gamma} - \left(\nu\,(1-\alpha\xi) + \frac{\sigma N\,(1-\alpha)}{\beta\gamma} \right) \widehat{J_t} - N\alpha\xi \left(\widehat{J_t} \right)^2 \tag{4.28}$$

From the last equation, it is possible to show that there exists a long-term equilibrium for the reference level of consumption and that this equilibrium is stable. The value of the long-term equilibrium depends, in particular, on the weight parameter of habits (ν) and the importance of the reference consumption level for the welfare of the agents (ξ). The dynamic of the other state variable, productivity, is determined by the next growth rate:

$$\frac{\dot{A}_t}{A_t} = \frac{1}{2} \left(\frac{\sigma N\,(1-\alpha)}{\beta\gamma} - (1-\alpha\xi)\,\nu + \sqrt{\left((1-\alpha\xi)\,\nu + \frac{\sigma N\,(1-\alpha)}{\beta\gamma} \right)^2 + 4N\alpha\xi\nu \frac{(1-\alpha)\,\sigma}{\beta\gamma}} \right) \tag{4.29}$$

Therefore, long-term economic growth in a closed economy directly depends on the amount of labor in the country and the importance of the reference consumption level for agent utility. The greater the amount of labor and the more important the stock of habits for the utility of the agents, the greater the long-term economic growth rate.

4.4 Open Economy

In this section, I show the implications of international trade on economic growth in a scenario with interdependent agent preferences. As mentioned before, the international trade model is based on the HME model with non-homothetic preferences presented in Giraldo (2015). However, in this case the preferences are dynamic and the reference consumption level moves according to the average domestic and foreign consumption of heterogeneous goods. HME determines production for each trade partner after trade.

Assuming costless international trade for homogenous good (X), its price equalizes in the two countries. This price is taken as numeraire ($P_x = P_x^* = 1$), and so productivity determines the salary levels for each economy in the same way that it does in a closed economy.

The international trading of heterogeneous goods generates positive transportation costs, which are modeled as iceberg costs.[5] According to the HME model, in the presence of positive transportation costs for the heterogeneous goods trade, the economy with the greater market size gathers the majority production of the varieties of heterogeneous good. In accordance with the international trade model, aggregate demand of heterogeneous goods in each country is the sum of domestic and foreign demand for this type of good:

$$n_t p_t D_t = \tfrac{n_t}{n_t+\theta n_t^*}\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha P_{Yt}\xi J_t}{\eta}\right)N + \tfrac{\theta^* n_t}{\theta^* n_t + n_t^*}\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha P_{Y^*t}^*\xi J_t^*}{\eta^*}\right)N^* \quad (4.30)$$

$$n_t^* p_t^* D_t = \tfrac{\theta n_t^*}{n_t+\theta n_t^*}\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha P_{Yt}\xi J_t}{\eta}\right)N + \tfrac{n_t^*}{\theta^* n_t + n_t^*}\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha P_{Y^*t}^*\xi J_t^*}{\eta^*}\right)N^* \quad (4.31)$$

With

$$\theta = \left(\frac{p_{it}}{p_{it}^*}\right)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \text{ y } \theta^* = \left(\frac{p_{it}}{p_{it}^*}\right)^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \quad (4.32)$$

After computations, the previous equations generate the next HME dynamic equation (4.33),

$$\frac{n_t}{n_t^*} = \frac{\frac{\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N}{\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha\beta\xi^* J_t^*}{\sigma}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}\frac{\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N}{\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha\beta\xi^* J_t^*}{\sigma}\right)N^*}} \quad (4.33)$$

This equation shows the interaction between supply and demand factors for the dynamic effects of trade. In addition, this equation presents the reference consumption level for heterogeneous goods as a new determinant of the number of varieties produced in each country.

After trade, heterogeneous goods production basically depends on the demand size for this kind of good in each country, which is determined by productivity levels, the dependence factor, population size and, in particular, the stock of habits in the consumption of heterogeneous goods. This last of these variables is a new determinant of trade effects in HME theory.

From the HME equation (4.33), it is possible to show that consumption habits share a direct relation with HME and therefore the effects of trade on the production of heterogeneous goods are determined by consumption habits. At the same time, these habits influence economic growth through the number of varieties produced in each country.[6] The relationship between number of varieties of heterogeneous good produced and the stock of consumption habits for this kind of good is direct.

$$\frac{\partial\left(\frac{n_t}{n_t^*}\right)}{\partial J_t} > 0 \quad (4.34)$$

[5] The "iceberg cost" supposes that a τ portion of transported good arrives, and that $(1-\tau)$ is lost in transit.

[6] Giraldo (2015) presents a detailed analysis of Home Market Effect determinants.

Proposition 8 *: HME is determined by productivity, income, population size and the stock of habits. The effect of the stock of habits in post-trade production is similar to the effects of productivity, income and population size. Ceteris paribus, the country with a higher stock of habits in heterogeneous goods consumption will produce a greater number of varieties in this sector.*

Proposition 1 shows the importance of habits of consumption in determining the dynamic effects of trade. For each country, the history of patterns of consumption before trade is an important determinant of the consequences of trade. So the country with the highest consumption path in heterogeneous goods will exhibit a greater demand for these goods, a larger market size and, as a result, a tendency towards the agglomeration of production for most varieties of heterogeneous good after trade.

Countries with a long history of heterogeneous goods consumption have a large stock of habits for such goods. Indeed, where there exists a large stock of habits and high levels of current consumption for heterogeneous goods, there exists a huge market size for this type of good. The market size of heterogeneous goods in these countries encourages the establishment of firms within this sector in order to take advantage of economies of scale. The history of consumption for heterogeneous goods increases demand for this type of good and the production of varieties in the country with a greater demand for manufacturing.

On the other side, the productivity dynamic is established through the learning by doing function in the production of heterogeneous goods

$$A_t = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right)\, ds \tag{4.35}$$

Differentiating over time, it is possible to obtain the rate of adjustment for productivity in each country.

$$\dot{A}_t = \frac{\partial A_t}{\partial t} = K_t + \delta K_t^* = \left(n_t + \delta n_t^*\right)\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{4.36}$$

$$\dot{A}_t^* = \frac{\partial A_t^*}{\partial t} = K_t^* + \delta K_t = \left(n_t^* + \delta n_t\right)\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{4.37}$$

From the above equations, it is possible to derive the dynamics of productivity after trade. The rates of adjustment for the productivity variable will be determined by the trade scenario generated through HME (complete or incomplete specialization) and the knowledge spillover between economies. On the one hand, HME determines the number of varieties produced in each country after trade, which at the same time determines the learning by doing process and then the economic growth rate. On the other hand, the level of spillover determines technology transfer among economies, which is an element that generates convergence or divergence in the economic growth rate of the trading countries.

$$\frac{\partial \dot{A}_t}{\partial n_t} > 0 \quad \text{and} \quad \frac{\partial \dot{A}_t}{\partial \delta} > 0 \tag{4.38}$$

The growth rate of productivity after trade is determined by the number of varieties produced in each country (which are defined by HME) and knowledge spillover. In cases where complete specialization is present, the country specializing in the production of homogeneous goods grows according to the levels of knowledge spillover and productivity growth of its trading partner. The country specializing in the production of heterogeneous goods grows in relation to the number of varieties produced. In cases of incomplete specialization, knowledge spillover determines the speed of the convergence or divergence of growth rates between the economies.[7]

Equations (4.36) and (4.37) summarise the results for the dynamic of productivity presented in Giraldo (2015). The long-term effects of trade on economic growth depend on the number of varieties of heterogeneous good produced in each country and the knowledge spillover between the economies. Consequently, the stock of habits is a determinant of the dynamic effects of trade on economic growth because it determines HME, which at the same time determines the number of varieties produced in each country after trade.

Proposition 9 *: The interdependence of the preferences of agents and habit formation in the consumption of heterogeneous goods determines the effects of trade on production, economic growth and the welfare levels of the agents of the trading partners.*[8]

According to the above proposition, the stock of habits determines the trade effects on economic growth as well as welfare levels. This is so because it is a dynamic determinant of HME, the number of varieties and, ultimately, the levels of productivity for each country.

The catching up with the "Joneses" affects the welfare levels of the agents because they try to reach a reference level of consumption imposed by the external and domestic habits of consumption for heterogeneous goods. Simultaneously, the consumption of heterogeneous goods increases consumption habits over time and expands the demand size for this type of good. This last effect has direct implications for trade performance on economic growth. The greater the consumption of heterogeneous goods, the larger the number of varieties produced of this kind of good, and the better the long-term effects of trade on economic growth.

4.4.1 The Dynamic Implications of Habits

In the present section, I assume constant productivity levels in order to explain in an analytical way the dynamic behavior of the economy under habit formation in consumption. This section assumes that the dynamic is only generated by the evolution of the habits variable, while productivity is assumed

[7] See Giraldo (2015).

[8] It is easy to show that $\frac{\partial u_t}{\partial J_t}$ is negative, and that given $\frac{\partial J_t}{\partial t} > 0$ the greater the habits the lower the welfare in the expansion path towards equilibrium.

to be exogenous. As a consequence, a learning by doing process remains absent. This last assumption allows for an analytical solution to the dynamic of the model.

In this case, the model is represented by four equations that result from the imposition of the above assumptions on equations (4.33) and (4.4) for the domestic and foreign countries:

$$\frac{n_t}{n_t^*} = \frac{\frac{\left((1-\alpha)\frac{A}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N}{\left((1-\alpha)\frac{A^*}{\gamma^*}+\frac{\alpha\beta\xi^* J_t^*}{\sigma}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}\frac{\left((1-\alpha)\frac{A}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N}{\left((1-\alpha)\frac{A^*}{\gamma^*}+\frac{\alpha\beta\xi^* J_t^*}{\sigma}\right)N^*}} \tag{4.39}$$

$$n_t^* = \left[\frac{\theta}{\frac{n_t}{n_t^*}+\theta}\left((1-\alpha)\frac{A}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N + \frac{1}{\theta\frac{n_t}{n_t^*}+1}\left((1-\alpha)\frac{A^*}{\gamma^*}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N^*\right]\frac{1-\sigma}{\mu} \tag{4.40}$$

$$\dot{J}_t = \nu\left(\overline{Y_t}-J_t\right) \tag{4.41}$$

$$\dot{J}_t^* = \nu\left(\overline{Y_t}-J_t^*\right) \tag{4.42}$$

The equations (4.39) and (4.40) determine the effects of trade on production and the optimal demands of the agents of each country. The optimal demands for the heterogeneous goods are inserted into the dynamic equations for habits in order to determine the dynamic of the economy.

$$\dot{J}_t = \nu\varphi\left(\frac{(1-\alpha)A\sigma}{\beta\gamma}\right)+\nu(1-\varphi)\left(\frac{(1-\alpha)A^*\sigma}{\beta\gamma^*}\right)+\nu(\varphi\alpha\xi-1)J_t+\nu(1-\varphi)\alpha\xi J_t^* \tag{4.43}$$

$$\dot{J}_t^* = \nu\varphi\left(\frac{(1-\alpha)A\sigma}{\beta\gamma}\right)+\nu(1-\varphi)\left(\frac{(1-\alpha)A^*\sigma}{\beta\gamma^*}\right)+\nu\varphi\alpha\xi J_t+\nu((1-\varphi)\alpha\xi-1)J_t^* \tag{4.44}$$

The dynamic of the model is determined by the evolution of the habit variables, which define the path of growth, possible equilibriums and stability. From the equations (4.43) and (4.44), the evolution of habits can be graphically represented by two equilibrium locus with a positive slope (see figure 1). Nevertheless, the slope of the locus linked to the dynamic of foreign habits is greater than that of the locus linked to domestic habits. This guarantees convergence towards a long-term stationary equilibrium independently of the initial conditions,[9] which is equal to:

$$J_\infty = J_\infty^* = \frac{\Psi}{\nu(1-\alpha\xi)} = \frac{(1-\alpha)\sigma}{(1-\alpha\xi)\beta}\left(\varphi\left(\frac{A}{\gamma}\right)+(1-\varphi)\left(\frac{A^*}{\gamma^*}\right)\right) \tag{4.45}$$

The long-term equilibrium in the level of habits variable is determined by the levels of productivity and the dependence factor in both countries.[10] As a result, there exists a trajectory of equilibria according to the values of the parameters, which corresponds to a forty-five degree line. In the long-term equilibrium, the level of habits is equal between the countries.

Proposition 10 *: There is one unique equilibrium for the consumption habits of heterogeneous goods where $J_t = J_t^*$. This equilibrium is stable and is determined by the levels of productivity for each*

[9] See proof in appendix 1.

[10] The other parameters are the same for both countries.

country, the dependence factor and the parameters of habits function.[11]

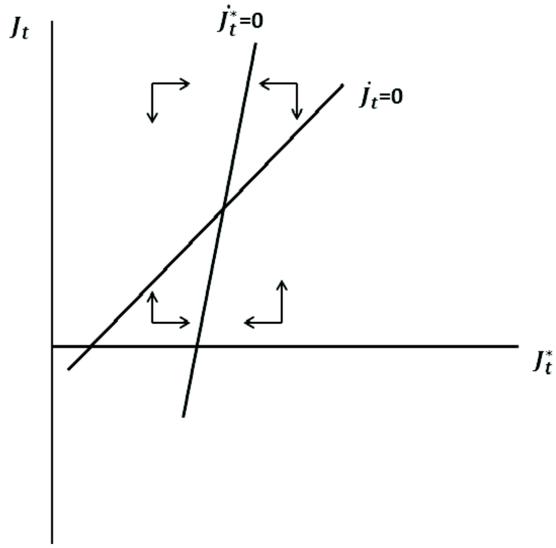

Figure 1: Habits phase diagram

The proposition means that independently of the initial levels of consumption for heterogeneous goods in each country, after trade the consumption habits for heterogeneous goods tend towards the same level. The phase diagram shows how the dynamic of consumption after trade reaches an unique long-term equilibrium, where the reference level of consumption for heterogeneous goods is the same between the countries (see figure 1).

The trade produces homogenization in the reference consumption level. Independently of the initial conditions of consumption, the trade partners tend to have the same patterns of consumption, in particular, the pattern of consumption for the country with the greater income level, which means greater consumption. This is known as McDonaldization, a term coined by Ritzer (2004).

Information flow is greater after trade and the preferences of the agents are affected by this new information. Agents try to reach the reference consumption level, which is driven by the country with the highest income. The agents know the average preferences of consumption of the people in the world economy and try to achieve these, yet are restricted by budget constraints.

In other words, the agents have knowledge of the last-generation goods used by the average person in the world economy. People in the poorer country know that there are last-generation cell phones, new car models, typical fast-food restaurants, and many other goods that are consumed in the higher income country and they too want to consume the same goods. Aside from information flow about consumption preferences in the higher income country, trade also generates a flow of goods. Consequently, agents

[11] The analytical solution (see appendix 2) for the dynamic system shows that the system has two real and negative eigenvalues. This proves that the system is stable and convergent.

know which goods that they want in relation to the reference consumption level and they can find these in local stores due to trading. Therefore, accomplishing the desired consumption level depends on the income levels of the agents, which are directly determined by productivity levels and economic growth.

Replacing (4.45) in equation (4.39), we find the long-term relative production of heterogeneous goods between countries:

$$\frac{n_t}{n_t^*} = \frac{\frac{\left((N+(1-\alpha\xi)N^*)\frac{A}{\gamma}+\alpha\xi N^*\frac{A^*}{\gamma^*}\right)N}{\left((N^*+(1-\alpha\xi)N)\frac{A^*}{\gamma^*}+\alpha\xi N^*\frac{A}{\gamma}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}\frac{\left((N+(1-\alpha\xi)N^*)\frac{A}{\gamma}+\alpha\xi N^*\frac{A^*}{\gamma^*}\right)N}{\left((N^*+(1-\alpha\xi)N)\frac{A^*}{\gamma^*}+\alpha\xi N^*\frac{A}{\gamma}\right)N^*}} \tag{4.46}$$

In the long run, the reference level of consumption for heterogeneous goods is equal between the trading countries, but their production differs and is determined by HME. The long-term relative production of heterogeneous goods depends on population size, the dependence factor and productivity levels, as explained in Giraldo (2015). However, interdependence among the preferences of the agents shows that the historical composition of consumption determines the production of heterogeneous goods. The greater the consumption of heterogeneous goods, the greater the long-term production of such goods.

4.4.2 The Dynamic Implications of Productivity

A convergence in consumption habits implies that the long-term convergence or divergence between economies after trade depends on the evolution of productivity levels in each country. Now, I assume exogenous preferences (constant habits of consumption) and focus on the dynamic of productivity. In this case, J_t is exogenous and constant and the productivity of the countries is the dynamic variable. This allows for the finding of an analytical solution with two state variables.

Assume the next normalization for productivity variables

$$\widehat{A_t} = (1-\alpha)\frac{A_t}{\gamma} + \frac{\alpha\beta\xi J}{\sigma} \tag{4.47}$$

Consequently, the growth rate of this new variable in each country is:

$$\frac{\dot{\widehat{A_t}}}{\widehat{A_t}} = \frac{(n_t+\delta n_t^*)}{\widehat{A_t}}\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{4.48}$$

$$\frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = \frac{(n_t^*+\delta n_t)}{\widehat{A_t^*}}\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{4.49}$$

The results for productivity growth rates allow for a determination of the relative change in productivity of the two countries as the difference between their growth rates. This new variable is defined as the net productivity gap $H_t = \frac{\widehat{A_t}}{\widehat{A_t^*}}$, then, the growth rate of this variable is the subtraction of productivity growth rates between domestic and foreign productivity.

$$\dot{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = \left[(1-\delta H_t) \frac{\widehat{n_t}}{\widehat{n_t^*}} - \left(1 - \frac{\delta}{H_t}\right) \right] \frac{\widehat{n_t^*} \mu \sigma}{(1-\sigma)\beta} \tag{4.50}$$

Where $\widehat{n_t} = \frac{n_t}{\widehat{A_t}}$ and $\widehat{n_t^*} = \frac{n_t^*}{\widehat{A_t^*}}$

A system with three equations and three unknowns is thus obtained:

$$\frac{\widehat{n_t}}{\widehat{n_t^*}} = \frac{\frac{L}{L^*} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t}}{1 - \tau^{\frac{\sigma}{1-\sigma}} \frac{H_t L}{L^*}} \tag{4.51}$$

$$\widehat{n_t^*} = \left(\frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + \tau^{\frac{\sigma}{1-\sigma}}} H_t L + \frac{L^*}{\tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + 1} \right) \frac{(1-\sigma)}{\mu} \tag{4.52}$$

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\widehat{n_t}}{\widehat{n_t^*}} (1-\delta H_t) - \left(1 - \frac{\delta}{H_t}\right) \right] \frac{\widehat{n_t^*} \mu \sigma}{(1-\sigma)\beta} \tag{4.53}$$

The first two equations of the system, (4.51) and (4.52), are entered into the dynamic equation of productivity differences between countries (4.53), and the dynamic equation of the net productivity gap between countries is found:

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\frac{L}{L^*} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t}}{1 - \tau^{\frac{\sigma}{1-\sigma}} \frac{H_t L}{L^*}} (1-\delta H_t) - \left(1 - \frac{\delta}{H_t}\right) \right] \frac{\widehat{n_t^*} \mu \sigma}{(1-\sigma)\beta} \tag{4.54}$$

The productivity gap and the parameters value determine the long-term equilibriums in the same way as in Giraldo (2015). From equation (4.53), we find one unique equilibrium and one unique convergent expansion path for the variable of net productivity gap between countries under the incomplete specialization scenario. A convergence in productivity between countries after trade could be achieved for similar countries in terms of market size and productivity, whenever there is a high level of knowledge spillover between economies. The other scenarios present divergent paths with complete specialization in production between countries.[12]

Habit formation in consumption has a direct effect on the economy in a way that it predicts HME and consequently determines the effects of international trade on economic growth. However, the consumption habits variable is convergent by itself after trade, so the convergence or divergence among countries after trade depends on the convergence or divergence present in the productivity levels.

Under external habit formation, the patterns of consumption in heterogeneous goods tend to be equal between economies after trade. Nevertheless, the productivities and income levels after trade depend on a convergence or divergence in the net productivity gap. This added result is part of the analysis presented in the fifth section, where I simulate a dynamic model with the four state variables.

[12] For more details, see Giraldo (2015).

4.5 The Dynamic in Productivity and Habits

In this section, I simulate the model in the presence of the four state variables, two from the dynamic of habit formation and two from the evolution of productivity. The idea is to show the consistency of the results where both dynamics exist and understand the effects of trade on economic growth and agent consumption preferences without the assumptions established in the past sections.

The model is the same as that used in the previous sections, but here both external habit formation and productivity levels are dynamic. A system with six equations and six unknowns is now present, four of which are dynamic. This added model allows for an understanding of the evolution of each of the variables in each country and their interrelations, and therefore the effects of international trade on both economic growth and agent preferences. The system of equations is as follows:

$$\frac{n_t}{n_t^*} = \frac{\frac{\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N}{\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha\beta\xi^* J_t^*}{\sigma}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}\frac{\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N}{\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha\beta\xi^* J_t^*}{\sigma}\right)N^*}} \tag{4.55}$$

$$n_t^* = \left[\frac{\theta}{\frac{n_t}{n_t^*}+\theta}\left((1-\alpha)\frac{A_t}{\gamma}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N + \frac{1}{\theta\frac{n_t}{n_t^*}+1}\left((1-\alpha)\frac{A_t^*}{\gamma^*}+\frac{\alpha\beta\xi J_t}{\sigma}\right)N^*\right]\frac{1-\sigma}{\mu} \tag{4.56}$$

$$\dot{A}_t = \left(n_t+\delta n_t^*\right)\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{4.57}$$

$$\dot{A}_t^* = \left(n_t^*+\delta n_t\right)\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{4.58}$$

$$\dot{J}_t = \nu\varphi\left(\frac{(1-\alpha)A_t\sigma}{\beta\gamma}\right)+\nu(1-\varphi)\left(\frac{(1-\alpha)A_t^*\sigma}{\beta\gamma^*}\right)+\nu(\varphi\alpha\xi-1)J_t+\nu(1-\varphi)\alpha\xi J_t^* \tag{4.59}$$

$$\dot{J}_t^* = \nu\varphi\left(\frac{(1-\alpha)A_t\sigma}{\beta\gamma}\right)+\nu(1-\varphi)\left(\frac{(1-\alpha)A_t^*\sigma}{\beta\gamma^*}\right)+\nu\varphi\alpha\xi J_t+\nu((1-\varphi)\alpha\xi-1)J_t^* \tag{4.60}$$

The effects of trade on the production of heterogeneous goods is determined by levels of productivity and pre-trade habits. The history of production and the consumption of heterogeneous goods under autarky determines HME through the ratio of varieties of heterogeneous goods produced after trade. However, the production of heterogeneous goods determines the evolution of productivity levels, and the levels of productivity plus the stock of consumption habits determines the evolution of external habits in the consumption of heterogeneous goods. There is a clear interrelation among the variables.

Feedback between the dynamic variables determines the growth path of the variables in the model. The productivity level of each country determines the evolution of consumption habits for heterogeneous goods, that is, the levels of demand for such goods. At the same time, this demand determines the production of heterogeneous goods, and the number of varieties produced determines the productivity growth path.

The analytical solution of the model shows that the stock of consumption habits in heterogeneous goods tends to be equal between countries after trade and that this is so independently of the countries that trade. In contrast, the levels of productivity might converge or diverge depending on the characteristics of the trading countries. The dependence factor, supernumerary income and knowledge spillover are the determinants of the effects of trade on productivity growth. These analytical results are maintained in the simulation, which shows their robustness.

In order to summarize the simulation results, a new variable is defined that helps to show the results of trade for the dynamic variables in a clearer way. $Jr_t = \frac{J_t}{J_t^*}$ represents the relative level in the stock of habits for heterogeneous goods between countries (relative Joneses) and, as before, $H_t = \frac{\widehat{A_t}}{\widehat{A_t^*}}$ represents the net productivity gap between countries. These two variables reveal the convergence or divergence in productivity and consumption preferences after trade in the different scenarios.

Figure 2 shows the simulation of trade effects for three different scenarios in the presence of knowledge spillover. The first row represents the convergent scenario for productivity levels. The second row represents the divergent scenario for productivity levels with complete specialization in the production of heterogeneous goods in the domestic country. Finally, the third row represents the divergent scenario in productivity levels with complete specialization in the production of heterogeneous goods in the foreign country. The last two scenarios show the transition from an incomplete specialization regime to a complete specialization regime. The divergence in the incomplete specialization regime displaces the variable of the net productivity gap to the complete specialization regime, where the variable converges in the growth rates but at different levels.

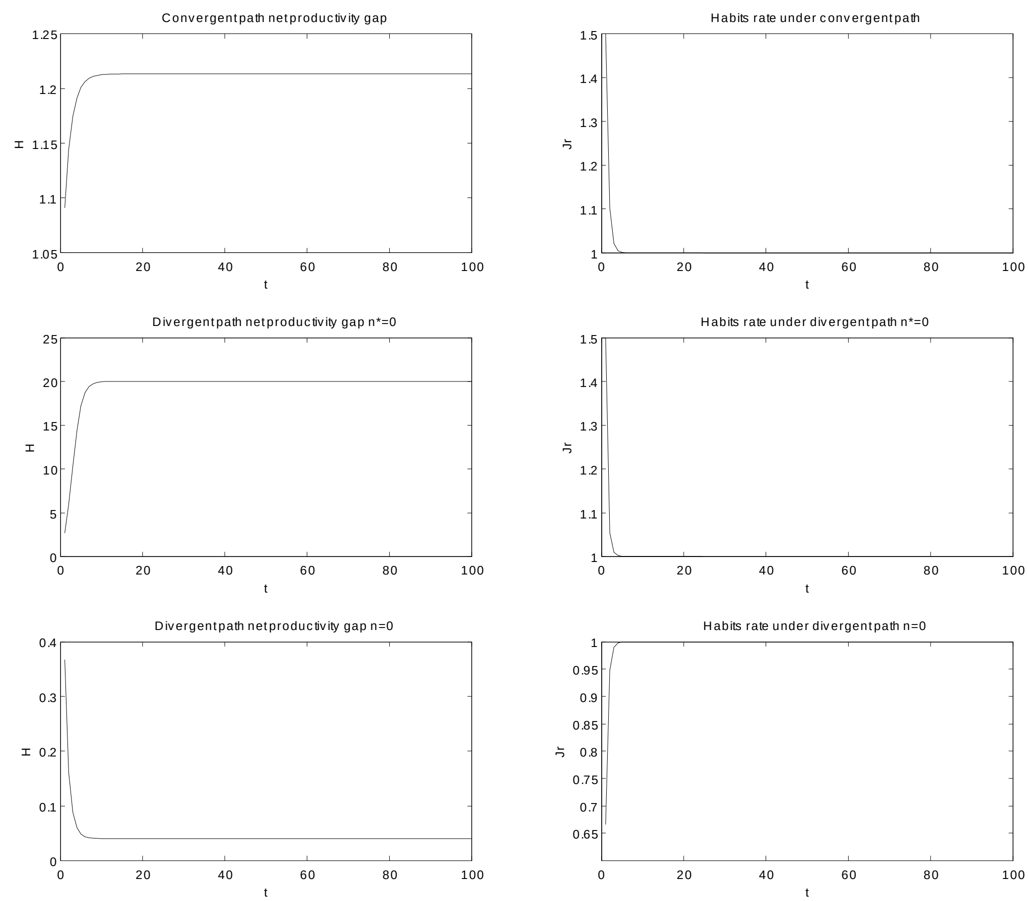

Figure 2: Trade scenarios under positive knowledge spillover

The results from the simulation are consistent with the analytical results. The first column shows how productivity levels converge to a steady state in the first case, or diverge towards a new regime of complete specialization in production where productivity grows at the same rate but in different levels according to the level of knowledge spillover (δ) in the other two scenarios. In the long run, $H^{ss} = \frac{1}{\delta}$ for the second row and $H^{ss} = \delta$ for the third row. The country specializing in homogeneous goods production only grows according to technology transfer.

The second column shows how the effects of trade on the stock of habits for heterogeneous goods is independent of the results regarding productivity, which at the same time determines income. After trade, the external habits of consumption for heterogeneous goods converge towards the same level in the countries that trade. This means that trade generates a homogenization in the consumption preferences of agents in the trading countries.

Figure 3 shows the simulation of trade effects for divergent scenarios without the presence of knowledge spillover. The first row represents the divergent scenario for productivity levels with complete specialization in heterogeneous goods production in the domestic country. The second row represents the divergent scenario in productivity levels with complete specialization in heterogeneous goods production for the foreign country. In both scenarios, the productivity gap rises each time and the income differences between the countries increase.

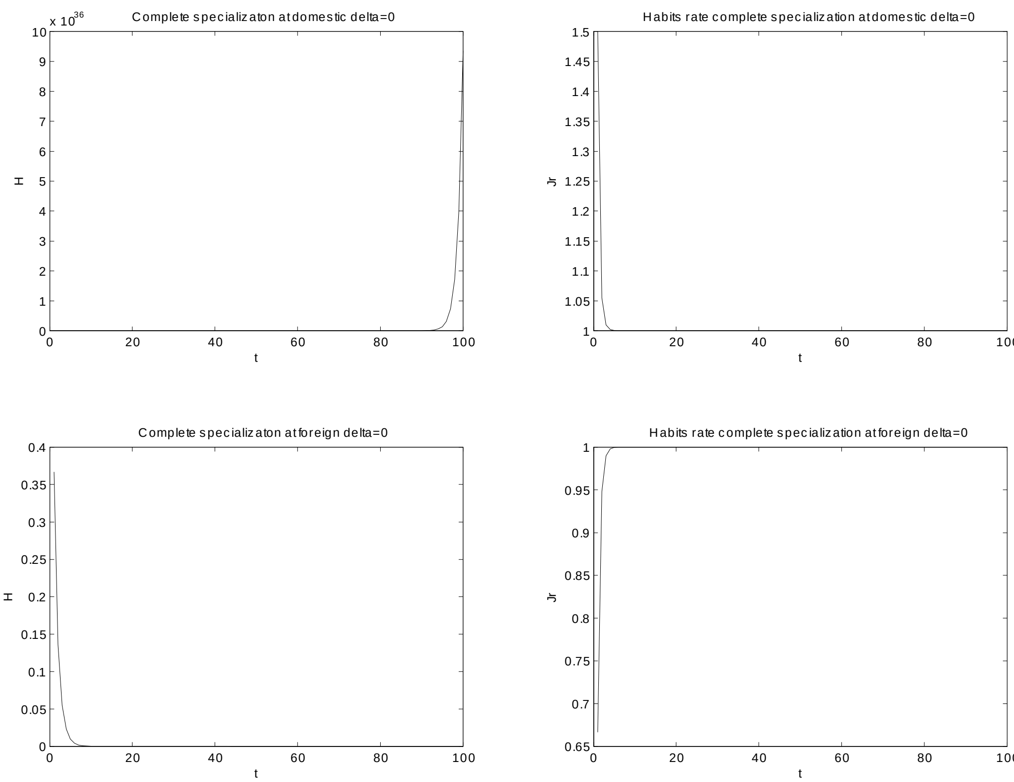

Figure 3: Trade scenarios without knowledge spillover

Similar to the scenarios presented in figure 2, convergence or divergence in productivity levels does not have implications for the convergence of habits in the consumption of heterogeneous goods. After trade, consumption preferences become equal between the countries, in spite of the divergence that is evident for productivity and income. Countries with different levels of income wish for the same level of consumption in relation to heterogeneous goods, as a result, welfare is affected in different ways after trade according to the convergence or divergence scenario for productivity levels.

4.6 Welfare

In spite of the long-term convergence in the external habits of heterogeneous goods consumption, the levels of productivity might converge or diverge between the countries that trade. In the long-term, this differentiation affects the dynamic variables and has direct implications for the effects of trade on welfare.

The countries that trade will have the same preference for heterogeneous goods in the long run. However, the convergence or divergence in productivity levels determines the income differences between these countries. Consequently, the relation between income and stock of habits for heterogeneous goods determines welfare levels after trade.

In order to establish the effects of trade on welfare, I simulate the utility path under trade and under autarky for the different possible scenarios. Following this, I compare the utility paths in order to identify the impact of trade on welfare. The utility function for both scenarios is:[13]

$$U_t = \int_0^\infty e^{-\rho t} u_t = \int_0^\infty e^{-\rho t} \left[\alpha \ln \left(\widehat{A_t} - \frac{A_t}{\gamma} \right) + (1-\alpha) \ln \left(\left(\widehat{A_t} - \frac{\xi \beta J_t}{\sigma} \right) \frac{\sigma}{\beta} \right) \right]$$

The subtraction between intertemporal utility under trade and under autarky provides a comparative framework for the effects of international trade on welfare:

$$\begin{aligned} U_t - U_t^A &= \int_0^\infty e^{-\rho t} \left(u_t - u_t^A \right) \\ U_t - U_t^A &= \int_0^\infty e^{-\rho t} \left[\alpha \ln \left(\frac{\widehat{A_t} - \frac{A_t}{\gamma}}{\widehat{A_t^A} - \frac{A_t^A}{\gamma}} \right) + (1-\alpha) \ln \left(\frac{\widehat{A_t} - \frac{\xi \beta J_t}{\sigma}}{\widehat{A_t^A} - \frac{\xi \beta J_t^A}{\sigma}} \right) \right] \end{aligned}$$

The effects of trade on welfare depend, in essence, on the relationship between productivity and stock of habits. Under autarky, the stock of habits grows according to the demand for heterogeneous goods, which is, at the same time, determined by productivity levels. On the other hand, under trade the stock of habits grows according to the demand for heterogeneous goods in both countries and converges towards the same level, while productivity might converge or diverge depending on the characteristics of the trading countries and knowledge spillover.

With the purpose of showing the intertemporal effects on welfare in a clearer way, I simulate a utility comparison between autarky and trade under the five different scenarios of the productivity path for the domestic country. Figure 4 shows the three first scenarios in the up figure, that is, convergence and divergence in the presence of knowledge spillover, and the other two scenarios in the down figure, namely divergence without knowledge spillover.

[13] Under autarky this is the same function as that with superscript A in the variables.

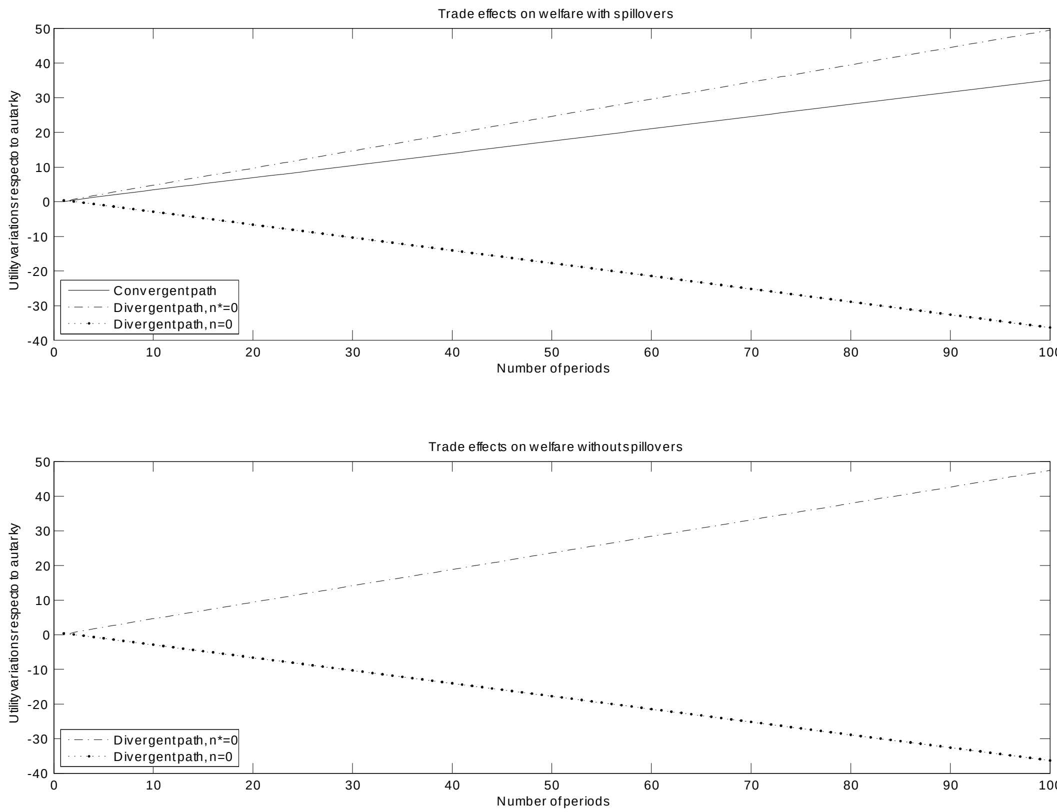

Figure 4: Trade effects on welfare

The simulation clearly shows how the effects on welfare depend on the productivity growth path, given the convergence for habits. The convergence scenarios and the divergence scenarios with complete specialization in heterogeneous goods production in the domestic country are scenarios that present better welfare after trade in comparison with autarky for domestic agents. This is so because the level of consumption for heterogeneous goods increases along with the levels of productivity, which allows for a greater level of income in order to satisfy higher levels of consumption.

The divergent cases with complete specialization in heterogeneous goods for the foreign country are scenarios with worse welfare after trade for domestic agents in comparison with autarky. Autarky is strictly preferred to trade in these scenarios. Intuition appears contrary to that of the other scenarios. After trade, the habits of consumption for heterogeneous goods converge towards a higher level imposed by the highest income country, but productivity levels are divergent and the country specializing in homogeneous good production does not have a large enough income to satisfy the longed for levels of demand for heterogeneous goods. Accordingly, the actual consumption of heterogeneous goods is less than that wished for by the agents and consequently the utility levels decrease with higher habits.

The convergent scenario exposes the trade scenario between similar countries, where these share a pattern of consumption for heterogeneous goods after trade and have the purchasing power to fulfil new consumption preferences. Divergent scenarios with complete specialization in the production of heterogeneous goods in the domestic country present trade scenarios in which the domestic country is the partner with the greatest productivity and income. In these cases, habits are addressed by the domestic country because it experiences a higher demand for heterogeneous goods and higher levels of productivity. Clearly, in this case the levels of consumption for heterogeneous goods are satisfied.

On the other hand, divergent scenarios with complete specialization in the production of heterogeneous goods in the foreign country present trade scenarios in which the foreign country is the partner with the greatest productivity and income. In these cases, habits are addressed by the foreign country. The foreign country establishes a high level of preference for the consumption of heterogeneous goods, a level that the domestic country cannot reach due to its low productivity and low income level.

The dynamic trade effects for welfare are not always positive. Indeed, there exist certain scenarios in which welfare is worse after trade in relation to autarky, in spite of a positive static effect during the early phases. Trade between similar countries generates convergence in both dimensions, that is to say, productivity and habits of consumption, and therefore generates better levels of welfare. However, trade between very asymmetrical countries only generates convergence in the habits of consumption. As a result, a preference for high consumption combined with a low level of income produces reduced welfare for the poorer country.

Such a loss in welfare level might be understood in light of a change in the preferences of the agents. Trade implies a high information transfer (through publicity, tourism, marketing, and so on), which modifies the preferences of the agents according to consumption wishes. However, it fails to modify agent income. Agents from the poorer country wish to achieve the level of consumption of the richer country.[14] However, they do not have the income level to achieve this and so welfare is lower than under autarky, where the reference level is accomplished. After trade, everybody wants the last-generation cellphone, but it is easier to buy for agents in a high-income country than for agents in a low-income country. Welfare is worse in the low-income country because the consumption wish remains unfulfilled.

[14] In the literature concerning habits, this behavior is explained by envy, greed, agent ambitions and other ideas, because the agents try to achieve any goal, identity or status that might generate greater levels of utility.

4.7 Conclusions

In this paper, I analyze trade implications for the preferences of agents and economic growth. I find three main results that add to the discussion about trade effects in relation to economic and cultural issues. The first of these addresses the importance of the historical composition of consumption for countries in the determination of international trade effects on industrialization. The second is unconditional convergence in the preferences of the agents after trade. The third comprises the different implications of trade for welfare, which includes various scenarios in which autarky is strictly preferred to trade.

The results show how the effects of international trade depend on the trading countries. The industrialization of the countries, the growth rate of productivity and income are all determined by the characteristics of the countries that trade, while the reference level of consumption that brings the greatest levels of utility is determined by the historical composition of consumption for these countries. Consequently, the effects of international trade are different and are, to reiterate, determined by the characteristics of the trading countries. There are scenarios in which trade improves the welfare of countries, but others in which it reduces it and autarky is strictly preferred to trade.

The core of this analysis exposes the effects of international trade on the preferences of agents, which provides a different dimension in relation to the standard economic literature. The inclusion of external habits formation in the utility function is a novel methodology in emerging literature about the cultural implications of trade. However, more empirical estimations are necessary in order to bolster the emergent hypothesis currently surrounding this new dimension in international trade literature.

4.8 Appendices

4.8.1 Appendix 1

If I assume the constant term in equations (4.43) and (4.44) to be $\Psi = \nu\varphi\left(\frac{(1-\alpha)A\sigma}{\beta\gamma}\right) + \nu(1-\varphi)\left(\frac{(1-\alpha)A^{*}\sigma}{\beta\gamma^{*}}\right)$, then the dynamic equations of habits are:

$$\dot{J}_t = \Psi + \nu(\varphi\alpha\xi - 1)J_t + \nu(1-\varphi)\alpha\xi J_t^{*} \tag{4.61}$$

$$\dot{J}_t^{*} = \Psi + \nu\varphi\alpha\xi J_t + \nu((1-\varphi)\alpha\xi - 1)J_t^{*} \tag{4.62}$$

Equating the last equations to zero, I obtain the dynamic equations of habits for each country:

$$J_t = \frac{\Psi}{\nu(1-\varphi\alpha\xi)} + \frac{(1-\varphi)\alpha\xi J_t^{*}}{(1-\varphi\alpha\xi)} \tag{4.63}$$

$$J_t^{*} = \frac{\Psi}{\nu(1-(1-\varphi)\alpha\xi)} + \frac{\varphi\alpha\xi J_t}{(1-(1-\varphi)\alpha\xi)} \Leftrightarrow J_t = -\frac{\Psi}{\nu\varphi\alpha\xi} + \frac{(1-(1-\varphi)\alpha\xi)J_t^{*}}{\varphi\alpha\xi} \tag{4.64}$$

The slope of the dynamic function of domestic habits is therefore greater than the dynamic function of foreign habits if and only if:

$$\begin{aligned} \frac{(1-\varphi)\,\alpha\xi}{(1-\varphi\alpha\xi)} &< \frac{(1-(1-\varphi)\,\alpha\xi)}{\varphi\alpha\xi} \\ 0 &< 1-\varphi\alpha\xi-(1-\varphi)\,\alpha\xi \\ 0 &< 1-\alpha\xi \end{aligned} \tag{4.65}$$

It is thus shown that the slope of the equilibrium locus of the dynamic equation for foreign habits is always greater than its dynamic counterpart. Solving systems (4.63) and (4.64), the stationary equilibrium is:

$$J_\infty = J_\infty^* = \frac{\Psi}{\nu\,(1-\alpha\xi)} = \frac{(1-\alpha)\,\sigma}{(1-\alpha\xi)\,\beta}\left(\varphi\left(\frac{A}{\gamma}\right)+(1-\varphi)\left(\frac{A^*}{\gamma^*}\right)\right) \tag{4.66}$$

4.8.2 Appendix 2

The dynamic system of habit formation can be written as a matrix, for example:

$$\begin{bmatrix} \dot{J}_t \\ \dot{J}_t^* \end{bmatrix} = \begin{bmatrix} \nu\,(\varphi\alpha\xi-1) & \nu\,(1-\varphi)\,\alpha\xi \\ \nu\varphi\alpha\xi J_t & \nu\,((1-\varphi)\,\alpha\xi-1) \end{bmatrix} \begin{bmatrix} J_t \\ J_t^* \end{bmatrix} + \mathbf{\Psi} \tag{4.67}$$

Where $\mathbf{\Psi}$ is a vector with a constant term that is equal between countries. The polynomial characteristic of the system is found and is:

$$\lambda^2 + (2-\alpha\xi)\,\nu\lambda + (1-\alpha\xi)\,\nu^2 = 0 \tag{4.68}$$

The solution to the polynom gives:

$$\lambda_{1,2} = \frac{-(2-\alpha\xi)\,\nu \pm \sqrt{(2-\alpha\xi)^2\,\nu^2 - 4\,(1-\alpha\xi)\,\nu^2}}{2} \tag{4.69}$$

The first step is to determine whether the roots are real:

$$\begin{aligned} (2-\alpha\xi)^2\,\nu^2 - 4\,(1-\alpha\xi)\,\nu^2 &> 0 \\ (2-\alpha\xi)^2\,\nu^2 &> 4\,(1-\alpha\xi)\,\nu^2 \\ \alpha^2\xi^2 &> 0 \end{aligned} \tag{4.70}$$

The two roots are real. The second step is to determine if both the roots are negative. When the numerator in (4.69) is subtracted, the result is evidently negative, but when it is added it is necessary to show that:

$$\begin{aligned} (2-\alpha\xi)\,\nu &> \sqrt{(2-\alpha\xi)^2\,\nu^2 - 4\,(1-\alpha\xi)\,\nu^2} \\ (2-\alpha\xi)^2\,\nu^2 &> (2-\alpha\xi)^2\,\nu^2 - 4\,(1-\alpha\xi)\,\nu^2 \\ 4\,(1-\alpha\xi) &> 0 \quad \text{Always} \end{aligned} \tag{4.71}$$

So both eigenvalues are negative and the system is convergent and stable.

Bibliography

[1] Abel, A. (1990). Asset Prices under Habit Formation and Catching Up with the Joneses. American Economic Review, 38-42.

[2] Alesina, A., Di Tella, R., & MacCulloch, R. (2003). Inequality and Happiness: Are Europeans and Americans Different? NBER Working Papers.

[3] Bala, V., & Van Long, N. (2005). International Trade and Cultural Diversity with Preference Selection. European Journal of Political Economy, 143-162.

[4] Bisin, A., & Verdier, T. (2014). Trade and Cultural Diversity. In V. Ginsburgh, & D. Throsby, Handbook of the Economics of Art and Culture (págs. 440-482).

[5] Campbell, J., & Cochrane, J. (1999). By Force of Habit: A consumption-based explanation of aggregate stock market behavior. Journal of Political Economy, 205-251.

[6] Chung, C. (2006). Non-homothetic Preferences and the Home Market Effect: Does relative market size matter? Georgia Institute of Technology.

[7] Clark, A., & Oswald, A. (1996). Satisfaction and Comparison Income. Journal of Public Economics, 359-381.

[8] Constantinides, G. (1990). Habit Formation: A resolution of the equity premium puzzle. Journal of Political Economy, 519-543.

[9] Francois, P., & van Ypersele, T. (2002). On the Protection of Cultural Goods. Journal of International Economics, 359-369.

[10] Giraldo Salazar, I. (2015). Implications of International Trade on Economic Growth: An approach from the Home Market Effect and Non-homothetic Preferences.

[11] Giraldo Salazar, I. (2015). Productivity, Demand and Home Market Effect.

[12] Janeba, E. (2004). International Trade and Cultural Indentity. NBER Working Paper Series.

[13] Krugman, P. (1987). The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Tatcher. Journal of Development Economics, 41-55.

[14] Lucas, R. (1988). On the Mechanics of Economics Development. Journal of Monetary Economics, 3-42.

[15] Maystre, N., Olivier, J., Thoenig, M., & Verdier, T. (2014). Product-based Cultural Change: Is the village global? Journal of International Economics, 212-230.

[16] Olivier, J., Thoenig, M., & Verdier, T. (2008). Globalization and the Dynamics of Cultural Identity. Journal of International Economics, 356-370.

[17] Pigou, A. (1903). Remarks on Utility. Economic Journal, 58-68.

[18] Rauch, J., & Trindade, V. (2009). Neckties in the Tropics: A model of international trade and cultural diversity. Canadian Journal of Economics, 809-843.

[19] Ritzer, G. (2004). The McDonaldization of Society. New Century Editors.

Chapter 5

Conclusions

The aim of this work is to deepen the analysis of the effects of international trade on economic growth, the preferences of agents and welfare. The study is based on the identification of new channels that determine the static and dynamic implications of international trade. Income differences between countries, demand composition and habits of consumption are novel variables in the theoretical approach to this topic. Interactions among these variables determine the effects of international trade in terms of industrialization, economic growth rate, the consumption habits of individuals, and the welfare of the countries' partners.

The Home Market Effect defines the industrialization of countries after trade by way of three main variables: supernumerary income, productivity levels and population size. The greater the supernumerary income, productivity and population size, the greater the level of industrialization. The effects of international trade on countries' production are determined by both supply and demand side variables. However, demand plays an important role in the dynamics of the model through supernumerary income, which is the main determinant of any long-terms effects. Contrary to the typical method used to model the Home Market Effect, it is shown that its determinant is the economic size of a country more than population size.

The effects of international trade on economic growth differ according to the countries that trade. The economic growth rate after trade depends directly on the trade effects on industrialization and knowledge spillovers between countries. The long-term stable equilibrium under incomplete specialization is unique and is only possible for trade between similar countries and high levels of tech transfer. The other equilibriums are presented under the complete specialization of production for the countries; these present convergences on the long-term growth rate but divergence for the income levels of countries. Without knowledge spillovers between countries, countries will diverge in both growth rates and income levels.

The interrelation of the preferences of the agents shows that international trade produces homogenization in the consumption preferences of agents. The agents of countries that trade will have the same habits of consumption in the long run, independent of trade effects on industrialization and economic growth rate. This effect shows another dynamic dimension within trade effects, which entails direct implications for the traditions and cultural values of the countries.

Finally, the trade effects on welfare can be divided into two groups: first, the usual static effect in the international trade literature, which is always positive in relation to the autarky; second, the dynamic effect, which depends directly on trade implications in terms of industrialization and economic growth rate. The different scenarios that are present, according to the distinct countries that can trade, show that trade effects on welfare are not always positive. Indeed, some scenarios indicate that, in terms of welfare, autarky is strictly preferred to trade.

All the results of the present work contribute to the discussion about international trade effects, revealing other dimensions in which these affect the dynamics within and among economies. The effects of international trade are not ever equal; on the contrary, they are singular and depend on the characteristics of the countries that trade. These conclusions present a framework of analysis for countries when determining better trade partners. Trade policy makers should analyze the trade-off between the static and dynamic gains of trade to determine the lineaments of trade policy and the best partners for each particular case.