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Abstract

The causality between international trade and industrialization is still ambiguous. We consider a model of international trade with the Home Market Effect - with differences in income and productivity between sectors and between countries - in order to identify additional channels for determining the effects of international trade on industrialization. Introducing non-homothetic preferences and differences in productivity aids in the interpretation of any apparent paradoxes within international trade, such as the commercial relations between more populated countries like China and India and large economies such as the U.S. Population size, demand composition and productivity levels constitute the three main channels for determining the effects of international trade. Interactions among these channels define the results obtained in terms of industrialization, while welfare levels are always higher in relation to autarky.

Keywords: *International Trade, Non-homothetic Preferences, Home Market Effect, Monopolistic Competition.*

JEL Classification: F10, F12, F17.

1 Introduction

The causality between international trade and industrialization is still ambiguous. From one side the classic international trade literature presents trade as the mechanism to generate economic growth. On the other side, the new literature like Young (1991) and Melitz (2005) suggests that the infant industry should be protected to generate industrialization previous to trade. The present discussion and the diverse performance of countries newly liberalized for world trade suggest the importance of determine what are the main variables that determine whether trade effects on industrialization are positive or negative.

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In contrast to the standard literature of Home Market Effect[1] (Krugman 1980,1991, etc.), we consider the aforesaid question through a general equilibrium model of bilateral trade based on the existence of non-homothetic preferences of agents and differences in income and productivity between sectors and between countries. This allows us to introduce additional mechanisms by which trade increases or decreases Gross Domestic Product (GDP) and well-being, and identify the most suitable trade partners for an economy. Indeed, where non-homothetic preferences are at play, income differences between countries and differences in productivity between sectors and between countries contribute to the structure of demand inside each country. At the same time, in a model with HME, demand acts over the effects of trade liberalization on welfare and the industrialization of countries.

The model proposed herein analyses interactions between supply-side variables, like productivity, and demand-side factors, like the size and the composition of internal demand. It identifies three mechanisms through which HME acts: population size, demand composition and levels of productivity. The consequences of international trade in terms of industrialization - as evident under positive transportation costs - can be analyzed through the interplay of these three mechanisms. In fact, this article shows that population size, demand structure and productivity levels determine the level of industrialization generated after international trade. In addition, we discuss the effects of international trade on welfare, which are positive whenever the global market of manufacturing increases after trade.

Traditional models of international trade focus on the supply side. In contrast, the new theory of international trade, particularly that of Krugman (1980), takes into account the effects of demand on trade. The Home Market Effect establishes that the market size of a closed economy determines its trade patterns and industrial development. This is an important factor that is first mentioned by Linder (1961). This approach has allowed for the identification of agglomeration and dispersion effects generated by trade, which show the positive and negative effects of international trade on economic performance, depending on the size of the market of each economy. Helpman and Krugman (1985).

The HME was first proposed by Corden (1970) and then extended by way of formal changes in a seminal article by Krugman (1980). Further modifications have mainly been carried out by the same author and presented in Helpman and Krugman (1985). The literature surrounding HME focuses on population size as a demand element in determining patterns of trade and the industrial distribution of countries, showing transportation costs as the crucial variable. Although this literature does not exclude the possibility of additional mechanisms, it does not give sufficient importance to these and assumes the size of the country as being the only channel through which demand determines trade patterns.

The traditional HME suggests that the most densely populated countries concentrate the production of manufacturing internationally. However, in contrast with reality, China and India would have

[1] Hereafter HME

benefited the most from trade relations at the international level. In particular, China and India have populations 4.32 and 3.95 times larger than that of the United States respectively.[2] However, trade between these countries has not allowed full specialization in the manufacturing industry for the former examples (as predicted by the traditional HME), and much less any specialization in the other sectors for a commercial counterpart.

In the presence of HME, the number of agents in an economy is a fundamental variable in determining trade patterns and the distribution of industrial production among countries that trade. However, there are additional variables that complement this, and which therefore contribute to the final effects of international trade on GDP. Income and competitiveness differentials determine the composition of demand for countries, but these variables are shelved in HME standard modeling because this theory assumes homothetic preferences. With the inclusion of the variables analyzed in our model, the size of the market for industrial goods is limited by both the number of agents that comprise it and the ability to generate demand in relation to their commercial counterpart. The countries with low levels of income must have a structure of demand mainly concentrated in vital consumer goods. In contrast, countries with higher levels of income principally demand manufactured goods. Mitra and Trindade (2005); Bohman and Nilsson (2007); Dalgin, Mitra and Trindade (2008).

The empirical evidence proves the importance of the composition of demand in patterns of trade and specific specializations within countries (Markusen 1986; Dalgin, Mitra and Trindade 2008). This evidence shows the presence of HME between different partner countries and the interactions among supply and demand elements (Davis and Weinstein 1996; Davis, Hanson and Weinstein 2003; Xiang 2004). Indeed, Yu (2005) shows the differentiated effects of HME after including symmetrical transportation costs for both goods and differences in the elasticity of substitution. Chung (2006) shows the importance of demand composition in the determination of HME. Crozet and Trionfeti (2008) show the non-linearity of HME. Huang and Huang (2011) demonstrate the possibility of reversing HME with a technological advantage in production, based on a sample of six types of industry. Indeed, the evidence supports the importance of building a good indicator of HME.

Less conclusive estimates about the presence of HME, such as Davis (1998) and Antweile and Trefler (2002), reveal the presence of additional channels that are not taken into account by the traditional model. Even the lack of a robust HME effect may be due to the omission of key channels in the determination of the structure of demand and thus due to the omission of key explanatory variables.

The effects of trade between symmetric countries (North-North or South-South), as well as asymmetric countries (North-South), can be studied by the model proposed herein. In addition, studying HME through different channels gives robustness to empirical exercises that seek to establish the presence of HME in international trade. Indeed, when it is only the population size that is included in an

[2] World Bank data 2013.

econometric exercise, there exists a bias due to omitted variables. The model shows that the inclusion of additional variables that determine the structure of demand, such as productivity among countries and among sectors, differences in income and the composition of the population, allow for a more robust analysis of the effects of HME in relation to international trade.

This article consists of six sections, including the introduction. The second section presents the characteristics of the model, the third shows the effects of an open economy, the fourth exposes the alternative HME in the model, the fifth presents comparative statics, and the sixth section concludes.

2 The Model

We start from the basic structure of the theory of the Home Market Effect, as presented by Krugman (1980), but we break the homothetic preferences assumption and add the Stone-Geary utility function. In addition, differences in productivity between sectors and between countries are used.

We assume the presence of two regions, domestic and foreign $(*)$,[3] independent of size. There are two types of good: homogeneous (X), which represents agricultural goods and presents constant returns to scale in production, and heterogeneous (Y), which represents manufactured goods and exhibits increasing returns to scale in production. The varieties of heterogeneous good are horizontally differentiated *à la* Dixit-Stiglitz, and the firms in this sector maximize their benefits under monopolistic competition. Labor (L) is the only existing factor of production and is mobile among sectors but immobile among countries.

With the idea of modeling the effects of demand composition on the internal market in a simple way, we use the Chung (2006) strategy. This assumes that the number of people consuming differs from the number of people producing; countries have the same amount of labor $(L = L^*)$, but their populations (N and N^*) may be different. So it is supposed that domestic households offer one unit of work for each resident $(N = \gamma L = L)$, while foreign households offer $(\frac{1}{\gamma^*})$, meaning $(N^* = \gamma^* L^*)$.[4]

Intuitively, γ captures the demographic and redistribution factors that affect the relative demand for diversity goods in comparison to homogeneous goods. According to this modification, it is possible to interpret γ as the number of dependents under the economic responsability of each worker. This factor let us find the per capita incomo in an easy way.

The consumption side assumes that all households demand both goods and that they symmetrically demand each variety of heterogeneous good (Y). Households in both countries have the same non-homothetic utility function.

$$U = (X - \overline{X})^{\alpha} Y^{1-\alpha} \tag{1}$$

[3] Hereafter the variables corresponding to foreign have the superscript *.

[4] $\gamma^* > 1$

$$\text{With } Y = \left(\sum_{i=1}^{n} y_i^{\sigma}\right)^{\frac{1}{\sigma}}, \; 0 < \sigma < 1, \quad n = \text{ number of varieties consumed} \tag{2}$$

Where $\overline{X}$ is the minimum consumption (of survival) of the homogeneous good,[5] and X is the consumption of this same good beyond the threshold of survival. Y is the aggregate consumption of all n varieties of heterogeneous good and y_i is the consumption of the ith variety.

Both goods use the same factor of production, namely labor. The production of homogeneous goods, and all varieties of the heterogeneous sector, is performed with the same function of production in both countries. The homogeneous goods sector has the following production function:

$$Q_x = L_x A_x \tag{3}$$

In equilibrium it should be equal to added demand for this good.

$$NX = D_x = Q_x = L_x A_x \tag{4}$$

Where Q_x is the aggregate production of a homogeneous good, D_x is the aggregate demand for the homogeneous good, L_x is the amount of labor used in the production of this good, and A_x is the productivity in this sector. The cost function for the heterogeneous goods sector is given by:

$$l_i = \frac{\mu}{A_y} + \frac{\beta Q_i}{A_y} = \frac{\mu}{A_y} + \frac{\beta D_i}{A_y} \qquad i = 1, 2...n \quad \text{where } D_i = N y_i \tag{5}$$

Where Q_i and D_i are, respectively, the aggregate supply and demand of the i-th variety, l_i is the amount of labor used in the production of each variety, and A_y is the productivity in this sector. Moreover, μ and β are the parameters of fixed costs and variable costs, respectively.

Finally, the full-employment condition is assumed, meaning that:

$$L = L_X + L_Y = \frac{D_X}{A_X} + \sum_{i=1}^{n} \left(\frac{\mu}{A_y} + \frac{\beta D_i}{A_y}\right) \tag{6}$$

2.1 Closed Economy

2.1.1 Consumer

Agents maximize their utility function (1) subject to the budget restriction. With the aim of introducing differences in the incomes of agents, as differentiated by Chung (2006), this model differentiates between members of the household who work and those who only consume. It is assumed that in one of the countries each worker supports γ additional agents that only consume; they are part of the total

[5] This consumption is equal for all countries, indicating that everybody needs the same minimum consumption of food to survive.

population but not of the employed population, and they do not receive any income. The population is proportional to the number of employees, $N = \gamma L$.[6].

$$MaxU = (X - \overline{X})^{\alpha} Y^{1-\alpha} \text{ s.t. } P_x X + P_y Y = \frac{w}{\gamma} \tag{7}$$

After having been normalized by the total population (N), the consumer maximization program defines the optimal quantities demanded of good X, and the aggregate demand of all varieties of heterogeneous good Y.

$$Y = \frac{1-\alpha}{P_y}\left(\frac{w}{\gamma} - P_x\overline{X}\right) \tag{8}$$

$$X = \frac{\alpha}{Px}\left(\frac{w}{\gamma} - P_x\overline{X}\right) + \overline{X} \tag{9}$$

Where P_x is the price of a homogeneous good, P_y is the price index of heterogeneous goods, which is an aggregate price of each variety's price. The optimization process defines the demand of each variety of heterogeneous good, which is determined by aggregate spending on such goods, the price of each variety i, and the sum of the price of all the n varieties.

$$y_i = \frac{p_i^{\frac{1}{\sigma-1}}(1-\alpha)\left(\frac{w}{\gamma} - P_x\overline{X}\right)}{P_y^{\frac{\sigma}{\sigma-1}}} \tag{10}$$

$$\text{Where } P_y = \left(\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}} \tag{11}$$

P_y is the index price for the heterogeneous good that is found from y_i and its implications on the aggregate demand for heterogeneous goods (2). This index is established as an aggregate of prices of different varieties, weighted by the degree of substitutability between them.

2.1.2 Producer

In the production of homogeneous goods there exists a competitive environment, thus implying an equilibrium with zero profit. At the same time, the price of the homogeneous good has been established as a numeraire. As a result, the wages (w) in the homogeneous goods sector are exogenous and equal to productivity:

$$P_x = 1 = \frac{w}{A_x} \tag{12}$$

A direct consequence of the last equation is that the per capita income, in terms of homogeneous goods, is completely determined for productivity in this sector and the dependence factor γ.

$$\frac{wL}{N} = \frac{A_x L}{N} = \frac{A_x}{\gamma} \tag{13}$$

[6] γ represents the relationship between the population and labor. The dependence factor.

For the heterogeneous goods sector, the monopolistic competition scenario for which the production of such goods is inscribed, there is an explicit relationship between price elasticity and marginal cost, which maximizes benefits for the firms that produce some of the varieties of heterogeneous good.

$$p_i\left(1-\frac{1}{\epsilon_{y,p}}\right) = Cmg \tag{14}$$

$$p = p_i = \frac{\beta w}{\sigma A_y} \tag{15}$$

From (15), the price of each variety is defined by parameters, being constant for all varieties. Inserting the prices into the zero benefits condition, determined by the free entry and exit of firms, it is possible to find the production of each variety of heterogeneous good, which is equal to the total demand for each variety.

$$\pi_i = pD_i - \left(\frac{\mu}{A_y}+\frac{\beta D_i}{A_y}\right) w = 0 \tag{16}$$

$$D = D_i = \frac{\mu\sigma}{(1-\sigma)\,\beta} \tag{17}$$

In the last equation, it is possible observe that the quantity demanded of and produced for each variety is independent of the productivity rate and country size. As is typical in models with monopolistic competition and preferences *á la* Dixit-Stiglitz, increases in productivity are reflected in a rise in the number of varieties in demand but not in the amount demanded of each, Romer (1990).

Finally, from the full-employment condition (6) one can obtain the number of varieties of heterogeneous good present in this economy.

$$L_y = \sum_{i=1}^{n}\left(\frac{\mu}{A_y}+\frac{\beta D_i}{A_y}\right) \tag{18}$$

$$n = \frac{L_y\,(1-\sigma)\,A_y}{\mu} \tag{19}$$

The amount of labor used in the heterogeneous sector is the total available workforce minus the quantity used in the production of homogeneous goods.

$$L_y = L - L_x \tag{20}$$

$$L_y = L - N\left(\frac{\alpha}{\gamma}+\frac{(1-\alpha)}{A_x}\overline{X}\right) \tag{21}$$

Replacing (21) and (19) it is possible to find the number of varieties as a function of the parameters of the model, the amount of the available productivity factor, and the productivity within each sector of the economy.

$$n = \frac{\left(L-N\left(\frac{\alpha}{\gamma}+\left(\frac{1-\alpha}{A_x}\right)\overline{X}\right)\right)(1-\sigma)\,A_y}{\mu} \tag{22}$$

The Equation (22) can be rewritten in the following way:

$$n^A = \left(\frac{A_x}{\gamma} - \overline{X}\right) \frac{(1-\sigma)(1-\alpha)}{\mu} \frac{A_y}{A_x} N \tag{23}$$

$$n^A = (A_x - \gamma\overline{X})(1-\sigma)(1-\alpha)\frac{A_y}{A_x}\frac{L}{\mu} \tag{24}$$

For the last equation (23), it can be noted that the number of varieties (n) produced in a country corresponds to its level of industrialization.[7] Basically, this depends on three particular elements. First, the comparative advantages ($\frac{A_y}{A_x}$), defined by the ratio between the productivity of the two sectors of the economy. This ratio determines the competitiveness of a country, according to its relative production advantages in one of the two sectors. Second, the population size of the country, in the sense of the standard theory of HME. Third, the supernumerary income ($A_x - \gamma\overline{X}$), which determines the purchasing power of workers in terms of heterogeneous goods. These elements persist in the open economy and determine the effects of international trade.

Additionally, the degree of industrialization that is determined by the number of varieties produced in the manufacturing sector also determines welfare levels.

$$U^A = \left(\alpha(\frac{A_x}{\gamma} - \overline{X})\right)^{\alpha} \left(\frac{(1-\alpha)}{n^{\frac{\sigma-1}{\sigma}}\frac{\beta A_x}{\sigma A_y}}\left(\frac{A_x}{\gamma} - \overline{X}\right)\right)^{1-\alpha} \tag{25}$$

$$U^A = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\sigma}{\beta}\right)^{1-\alpha}\left(n^A\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}}\left(\frac{A_y}{A_x}\right)^{1-\alpha}\left(\frac{A_x}{\gamma} - \overline{X}\right) \tag{26}$$

The greater the degree of country industrialization, the greater the welfare. Similarly, the same variables that determine the level of industrialization of a country affect welfare levels. In summary, the levels of productivity in the sector of manufactured goods, the population size and the supernumerary income define both the degree of industrialization of a country and its welfare.

3 Open Economy

In this section, we extend the model to an open economy scenario, which establishes the basis for the HME model. There are two trading countries that differ in population size (N) and productivity in each of the sectors (A_x y A_y).

Assuming costless international trade in the homogenous good (X),[8] its price is equalized in the two countries. This price will be taken as a numeraire ($P_x = P_x^* = 1$). There are transportation costs associated with the heterogeneous goods trade, which are modeled as "iceberg" transportation costs.

[7] The level of industrialization can also be defined by the amount of labor available in the manufactured goods sector (Desdoigts & Jaramillo, 2009).

[8] This is a simplified assumption that is widely used (Helpman & Krugman 1985; Krugman 1991, etc.) and which does not affect the essential argument of the model.

In particular, it is supposed that a τ portion of the transported goods arrives, while $(1-\tau)$ is lost in transit. Including this relationship of costs to prices in the international market, the prices of each variety of heterogeneous good are as follows:

$$\text{Domestic}\begin{cases} p = p \\ \widehat{p^*} = \frac{p^*}{\tau} \end{cases}, \quad \text{Foreigner}\begin{cases} p^* = p^* \\ \widehat{p} = \frac{p}{\tau} \end{cases} \tag{27}$$

Therefore, the consumption of national varieties differs from the varieties imported due to price differences. The representative home maximizing program is then modified in relation to the varieties of heterogeneous domestic and foreign goods.[9] The aggregate consumption of varieties of heterogeneous good is no longer represented by (2), but it becomes an aggregate of both domestic and foreign varieties that differ in price $Y' = \left(\sum_{i=1}^{n} y_i^{\sigma} + \sum_{j=1}^{n^*} y_j^{*\sigma}\right)^{\frac{1}{\sigma}}$. Therefore, the budget constraint is now defined by:

$$\sum_{i=1}^{n} p_i y_i + \sum_{i=1}^{n} \widehat{p_j^*} y_j^* - (1-\alpha)\left(\frac{w}{\gamma} - P_x \overline{X}\right) \tag{28}$$

Where y_i is the demand for each domestic variety and y_j^* is the demand for each foreign variety. The budget restriction at the foreign level is symmetric to this.

As a result of the maximization, we find the ratio between the demand for domestic and foreign varieties to be a function of the price ratio of these,

$$\frac{y_j^*}{y_i} = \left(\frac{p_i}{\widehat{p_j^*}}\right)^{\frac{1}{1-\sigma}} \tag{29}$$

The local demand for each variety of heterogeneous domestic and foreign good (y_i and y_j^*), resulting from the maximization program of the domestic agent, is defined by the proportion of revenue earmarked for heterogeneous goods demand and the price of each variety weighted by the addition of the prices of all available varieties around the world.

$$y_i = \frac{p_i^{\frac{1}{\sigma-1}}(1-\alpha)\left(\frac{w}{\gamma} - P_x\overline{X}\right)}{\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} + \sum_{j=1}^{n^*} \widehat{p}_j^{*\frac{\sigma}{\sigma-1}}} \tag{30}$$

$$y_j^* = \frac{\hat{p}_j^{*\frac{1}{\sigma-1}}(1-\alpha)\left(\frac{w^*}{\gamma} - P_x\overline{X}\right)}{\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} + \sum_{j=1}^{n^*} \widehat{p}_j^{*\frac{\sigma}{\sigma-1}}} \tag{31}$$

The new basket of varieties available worldwide that enters into the aggregation of heterogeneous goods Y', modifies its index price, similarly affecting the proportion of income available for the consumption of such goods. Performing the same procedure as that used with the price index in a closed

[9]
$$\mathcal{L} = \left(\sum_{i=1}^{n} y_i^{\sigma} + \sum_{j=1}^{n^*} y_j^{*\sigma}\right)^{\frac{1}{\sigma}} - \lambda\left(\sum_{i=1}^{n} p_i y_i + \sum_{i=1}^{n} \widehat{p_j^*} y_j^* - (1-\alpha)\left(\frac{w}{\gamma} - P_x\overline{X}\right)\right)$$

economy, the free-trade index, depends on the price of existing domestic and foreign varieties of heterogeneous goods:

$$P_{Y'} = \left[\sum_{i=1}^{n} p_i^{\frac{\sigma}{\sigma-1}} + \sum_{j=1}^{n^*} \widehat{p}_j^{*\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}} \tag{32}$$

Equation (29) is the ratio between the consumption of domestic and foreign varieties in terms of the ratio between prices. In order to determine the world equilibrium we need to add the quantities of the goods used for the transportation of products. The demand rate for foreign heterogeneous goods, in terms of the domestic (θ) and corresponding rates for foreign goods (θ^*) is equal to:

$$\theta = \frac{y_j^*}{y_i} = \left(\frac{p_i}{p_i^*}\right)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \tag{33}$$

$$\theta^* = \frac{y_i}{y_j^*} = \left(\frac{p_i^*}{p_i}\right)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} = \left(\frac{p_i}{p_i^*}\right)^{-\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \tag{34}$$

After determining the ratio of demand for varieties among foreign and domestic varieties, one can define individual demand patterns for heterogeneous goods in each country, which are restricted by the proportion of expenditure for manufacturing consumption. The national demands for domestic and foreign heterogeneous goods are:

$$y_i = \left(\frac{p_i}{P_{Y'}}\right)^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{\frac{w}{\gamma} - P_x\overline{X}}{P_{Y'}}\right) \tag{35}$$

$$y_j^* = \left(\frac{\widehat{p_j^*}}{P_{Y'}}\right)^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{\frac{w}{\gamma} - P_x\overline{X}}{P_{Y'}}\right) \tag{36}$$

3.1 Producer

Using equations (35) and (36) it is easy to show that the elasticity of demand for exports is the same as in a closed economy for heterogeneous goods ($\frac{1}{1-\sigma}$). Therefore, transportation costs have no effect on the pricing policy of the firm. This result shows that the domestic and foreign prices of each variety of heterogeneous good remain the same as under autarky, in their respective local markets.

$$p = \frac{\beta w}{\sigma A_y} \quad \wedge \quad p^* = \frac{\beta w^*}{\sigma A_y^*} \tag{37}$$

Given the characterization of monopolistic competition in the market of heterogeneous goods, every variety of this type of good is only produced by one firm.[10] The number of varieties produced in each region is determined in the first instance by productivity in this sector, the amount of labor force used in the production of these goods (L_y and L_y^*) and the model parameters.

$$n = \frac{L_y\,(1-\sigma)\,A_y}{\mu} \quad \wedge \quad n^* = \frac{L_y^*\,(1-\sigma)\,A_y^*}{\mu} \tag{38}$$

[10] The only way in which the results are modified in relation to the closed economy is if the wages between countries differ, a central element in the section below.

With regard to the homogeneous good, the equalization of prices at the international level sets a relationship of proportionality between the wages and agricultural productivity of both countries.

$$w = A_x \quad \text{and} \quad w^* = A_x^* \tag{39}$$

In accordance with the equation (39), the per capita incomes, in terms of the homogeneous good, are the same as in a closed economy.

$$\frac{wL}{N} = \frac{A_x}{\gamma} \text{ and } \frac{w^* L^*}{N^*} = \frac{A_x^*}{\gamma^*}$$

4 The Home Market Effect

The presence of increasing returns to scale in the production of heterogeneous goods, and transportation costs generated for its trade at the international level, create an incentive to produce such goods in the "biggest market", thus taking advantage of economies of scale and minimizing transportation costs (Krugman 1980, 1991, etc.). In this sense, and according to the purposes of this article, the "largest market" is not only determined by the number of agents in a country, but also by their productivity and per capita supernumerary incomes. In other words, the demand effect, through the purchasing power and the level of competitiveness of the agents, constitutes a market.

Starting with two countries that possess the established features, aggregate demand for heterogeneous goods in each country is the sum of the domestic and foreign demand for this type of good, that is, the domestic consumption of heterogeneous goods plus exports of these kinds of good (40 and 41).[11]

$$npD = \frac{n}{n + \left(\frac{p^*}{p}\right)\theta n^*}(1-\alpha)\left(\frac{w}{\gamma} - \overline{X}\right)N + \frac{\theta^* n}{\theta^* n + \left(\frac{p^*}{p}\right)n^*}(1-\alpha)\left(\frac{w^*}{\gamma^*} - \overline{X}\right)N^* \tag{40}$$

$$n^*p^*D = \frac{\theta n^*}{\left(\frac{p}{p^*}\right)n + \theta n^*}(1-\alpha)\left(\frac{w}{\gamma} - \overline{X}\right)N + \frac{n^*}{\left(\frac{p}{p^*}\right)\theta^* n + n^*}(1-\alpha)\left(\frac{w^*}{\gamma^*} - \overline{X}\right)N^* \tag{41}$$

Aggregate demand for heterogeneous goods in a closed economy, which is only determined by the proportion of domestic spending dedicated to this type of good, is now determined by a combination of variables regarding the economies that are trading. In particular: a) the proportion of spending on such goods $(1-\alpha)$; (b) the demand rate among domestic and foreign varieties, that is, ultimately, a price ratio (fractions depending on (n and θ)); (c) the supernumerary income of the agents; and (d) the total population. Additionally, the productivity in each of the two sectors plays a fundamental role in the demand for such goods through the real income of workers. On the one hand, the productivity of the homogeneous good sector determines wages, demarcating agents' revenues and the costs of firms, while the productivity of the heterogeneous sector determines the price of each variety.

[11] D is determined by the zero profit condition $D_i = D = \frac{\mu\sigma}{(1-\sigma)\beta}$

Solving (40) and (41) obtains the relationship between the number of varieties produced domestically against those produced overseas as a measure of HME, which is determined by the interactions between supply and demand elements that are additional to those presented in the traditional approach.

$$\frac{n}{n^*} = \frac{\left[\frac{\left(\frac{A_x}{\gamma}-\overline{X}\right)N(1-\theta)}{\left(\frac{A_x^*}{\gamma^*}-\overline{X}\right)N^*(1-\theta^*)}\right]-\theta}{1-\theta^*\left[\frac{(1-\theta)\left(\frac{A_x}{\gamma}-\overline{X}\right)N}{(1-\theta^*)\left(\frac{A_x^*}{\gamma^*}-\overline{X}\right)N^*}\right]}\left(\frac{p^*}{p}\right) \tag{42}$$

Equation (42) is a novel result in the theory of international trade with increasing returns to scale. In the first instance, it is evident that the HME presented through the varieties rate of the heterogeneous goods produced in each country depends on the same elements as its traditional version, the parameter (θ), which mainly includes the effects of the trade frictions, particularly transportation costs. However, in equation (42) we identify other channels by which the ratio ($\frac{n}{n^*}$), and therefore HME, can be changed.

The term between the brackets of equation (42) collects most of the different effects evident in this relationship. The first fraction of this term $\left(\frac{\frac{A_x}{\gamma}-\overline{X}}{\frac{A_x^*}{\gamma^*}-\overline{X}}\right)$, corresponds to the relative supernumerary income, which is a direct consequence of the non-homothetic preferences assumption and relates to the purchasing power of the agents. The second term corresponds to the relationship with population sizes $\left(\frac{N}{N^*}\right)$, which shows the effects of the ratio among the sizes of the markets, in the standard form of HME. Finally, the last expressions in parentheses convey the degree of competitiveness of the markets according to their productive advantages, weighted by existing trade frictions $\left(\frac{1-\theta}{1-\theta^*}\right)$. In global terms, the expression reflects the relationship between the relative sizes of the demands of the two countries, which is determined by population size, the purchasing power of agents and the degree of competitiveness of these.

The disparity between ($\gamma \neq \gamma^*$) reflects the difference in per capita income ($\frac{A_x}{\gamma}$), which will modify the demand for heterogeneous goods in each country, affecting the number of varieties of heterogeneous goods produced in each region. This channel identifies differences in the purchasing power of the residents of a market. It is hoped that countries with greater purchasing power demand a higher proportion of heterogeneous goods, creating an incentive for the establishment of firms in this market, which will increase the number of varieties produced.

Variation in the productivity of both sectors is another channel through which international trade can affect the degree of industrialization of a country. Variations in productivity in the homogeneous good sector alters wages, generating two contrary effects within the economy that result in a differentiated aggregate effect. The first is an expenditure effect, which alters the level of revenue dedicated to the purchase of heterogeneous goods. The second is a cost effect, which changes the prices for each variety of these goods. These two effects act in opposite directions, and so the result of an increase of A_x, in terms of the number of varieties produced, depends on the magnitude of each one of these effects. Furthermore, variations in the productivity of heterogeneous goods modify the prices of such goods and thus the number of varieties in demand, leading to alterations in the number of varieties produced in each country. Productivity within both sectors figures strongly in the case of comparative static, which is developed in the next section.

In order to simplify the initial analysis, equal productivity among the sectors of each country is assumed ($A_x = A_y = A$), but it differs between countries ($A \neq A^*$). In this way, there exists a scenario in which relative income and productivity vary between countries. The final effects of trade will therefore depend on the three fundamental channels described in this article, which produce HME: population size, relative income differences and differences in productivity. The implications of the assumptions presented for the general result (42) are verified in the following equation:

$$\frac{n}{n^*} = \frac{\frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}\frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*}} = \frac{Z-\tau^{\frac{\sigma}{1-\sigma}}}{1-\tau^{\frac{\sigma}{1-\sigma}}Z} \tag{43}$$

where

$$Z = \frac{\left(\frac{A}{\gamma}-\overline{X}\right)N}{\left(\frac{A^*}{\gamma^*}-\overline{X}\right)N^*} \tag{44}$$

This equation presents the effects of the different channels on the HME through the number of produced varieties of heterogeneous good. The Z variable in the equation (43) corresponds with the supernumerary income ratio (the centerpiece of this result, since it depends on the three channels in question) and the ratio of population size. The Z variable collects the different channels in the model, so that supernumerary income is affected by the country's productivity levels, the dependence factor (γ) and population size. Increased productivity or a reduced dependence factor increases per capita income levels, creating a demand effect. This in turn stimulates the production of more varieties of heterogeneous good in the country with a higher income. This is so because it boosts the size of demand, which allows it to exploit economies of scale. Countries with a greater supernumerary income, as caused by any of the channels presented in this case, will then have a higher real income, which increases the economic market size, directly affecting the number of varieties of heterogeneous good produced in the economy.[12]

In HME function (43), the interval of incomplete specialization, where both countries produce two types of good, occurs when the (Z) variable belongs to the interval $\left(\tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}}\right)$. The greater the transportation costs and the lower the economies of scale, the greater the range of incomplete specialization. Outside this interval of the variable, the full specialization of the partners takes place.

[12] In the annex we show that, even when international trade reduces the degree of industrialisation in the countries, the welfare of the representative agent improves because the international market offers a greater number of varieties.

A trade balance in the heterogeneous goods of a domestic country is obtained from the demands of these goods from domestic and foreign countries.

$$TB_Y = \frac{\theta^* n}{\theta^* n + n^*}(1-\alpha)\left(\frac{A^*}{\gamma^*} - \overline{X}\right)N^* - \frac{\theta n^*}{n + \theta n^*}(1-\alpha)\left(\frac{A}{\gamma} - \overline{X}\right)N \tag{45}$$

The behavior of the trade balance in the range of incomplete specialization depends on exogenous variables (productivity, dependence factors, proportion of income devoted to spending on heterogeneous goods and transportation costs), and the number of varieties of heterogeneous good produced in each country. When the countries have the same supernumerary income and population size $\left(Z = \frac{\left(\frac{A}{\gamma} - \overline{X}\right)N}{\left(\frac{A^*}{\gamma^*} - \overline{X}\right)N^*} = 1\right)$, both produce the same number of varieties of heterogeneous good, presenting equilibrium in the trade balance of manufacturing. Out of equilibrium, the performance of the trade balance depends on the number of varieties produced in each country (46), which is directly defined by the relationship between the supernumerary incomes of the countries and population size (43). The country with a higher per capita income will produce more varieties than the other, and will experience a trade balance surplus in heterogeneous goods at this interval.

$$\begin{aligned} TB_Y &= \frac{\left(A^* - \gamma^* \overline{X}\right)(1-\alpha)L\tau^{\frac{\sigma}{1-\sigma}}}{\tau^{\frac{\sigma}{1-\sigma}} n + n^*}(n - n^*) && (46) \\ TB_Y &> 0 \Leftrightarrow n > n^* && (47) \end{aligned}$$

If $Z > \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}}$, the trade between the two countries will involve a full specialization in heterogeneous goods domestically, and in homogeneous goods overseas. On the other hand, if $Z < \tau^{\frac{\sigma}{1-\sigma}}$, the trade between the two countries will take on a full specialization in heterogeneous goods overseas, and in homogeneous goods domestically. The more similar (different) the traded countries are ($Z \approx 1$) in terms of population size, income and productivity, the higher (lower) the probability of incomplete specialization, intra-industry trade (inter-industry trade).

The effects of (Z) on the number of varieties produced in each country can be determined analytically and show a positive relation. The country with a higher income will be the largest producer of heterogeneous goods within a bilateral trade relationship, with positive transportation costs:

$$\frac{d\left(\frac{n}{n^*}\right)}{dZ} = \frac{1 - \tau^{\frac{2\sigma}{1-\sigma}}}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}}\left(\frac{\left(\frac{A}{\gamma} - \overline{X}\right)N}{\left(\frac{A^*}{\gamma^*} - \overline{X}\right)N^*}\right)\right)^2} > 0 \quad \text{given } \tau < 1 \tag{48}$$

Given the characteristics of the HME function (43), this can be represented in a graph, as shown below.

Figure 1: Population size, supernumerary income and HME

The graph is similar to the traditional HME. It shows the variable Z and the channels involved, while defining the number of varieties produced by each country. Located on the right of the asymptote are the cases of complete specialization in heterogeneous goods on domestic production after trade implementation. Similarly, the points to the left of the intercept demarcate the overseas cases of complete specialization for such goods. The interval between the intercept and the asymptote is the area of incomplete specialization and illustrates the case in which both countries are equal ($Z = 1$) and produce the same number of varieties.

Is important to highlight that the HME is determined by the Z variable and not only by population size, as in traditional models. It shows the importance of the demand composition in the results of the trade. More than country size, economic market size is key in the sense of the purchasing power of agents, which determines the consequences of international trade on the degree of industrialisation for countries that trade. International trade increases industrial production in relation to autarky levels if the relative supernumerary income is sufficently greater in relation to the transportation cost.

On the other hand, via this same model it is possible to determine the trade implications for welfare. The relationship between utility under an open economy and autarky is such that it will only depend on the number of varieties to which the country has access after and before trade, *ceteris paribus*.

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \tau^{\frac{\sigma}{1-\sigma}}}{n_A} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \tag{49}$$

The outcome in terms of welfare depends on two effects: the first is the number of varieties produced domestically after trade, in relation to the number produced under autarky, and the second is the number of additional varieties that are accessed after trade in relation to those available under autarky. These two effects can go in the same direction or in opposite directions, depending on whether specialization exists or not, and the impact of trade on the production of heterogeneous goods. However, using the definitions about the number of varieties produced under autarky and under trade, we find that welfare is better under trade than under autarky.[13] This means that in the static scenario exposed in this model, trade is strictly preferred to autarky.

$$\frac{U}{U_A} = \left(1 + \tau^{\frac{\sigma}{1-\sigma}}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} > 1 \tag{50}$$

5 Comparative Statics

From (42), it is possible to generate the different comparative statics that enable the identification of the different channels through which HME may occur after trade liberalization, and they demonstrate that the outcome of international trade depends on the specific characteristics of the partner countries.

[13] See Appendix 1.

5.1 Variations in Population Size

Assume the absence of a homogeneous good, that the population is equal to the number of workers, that productivity is equal between the countries, and that transportation costs are positive ($\alpha = 0$, $L = N$, $\overline{X} = 0$ and $\theta > 0$). With these assumptions we achieve the classic results obtained by Krugman (1980), who presents the relationship between population sizes as determinant of the number of varieties of heterogeneous good produced in each country. HME is determined by the population size of each country ($Z = \frac{N}{N^*}$). The graphic representation of this scenario is illustrated in figure 1, where HME is determined by the values that take the variable Z.

$$\frac{n}{n^*} = \frac{Z - \theta}{1 - \theta Z} = \frac{\frac{N}{N^*} - \theta}{1 - \theta\left(\frac{N}{N^*}\right)} \tag{51}$$

Given $\left(\frac{dZ}{dN} > 0\right)$, the larger a country in terms of population, the greater the number of varieties of heterogeneous good being produced. It is clear that the size of the population, as Krugman (1980) states, is an important channel in the determination of HME, but it is not the only element that comes into this determination because, as we shall see later on, both the purchasing power and level of productivity in each of the sectors complement the channels through which the size of a market becomes a determinant of the type of product that one country trades (HME).

5.2 Variations in Relative Income

Assume the existence of a homogeneous good with a minimum level of consumption $\left(\alpha \neq 0 \text{ and } \overline{X} > 0\right)$, the population differs from the number of workers in each economy ($L \neq N$, with $N = \gamma L$), and the other variables are equal between countries, while HME is obtained from the relative demand of the market. Given the above assumptions, per capita income differentials determine the structure of the demand and so delimit the heterogeneous good varieties produced in each country after trade liberalization.

According to the hypothesis, the prices of every variety are the same in both countries, therefore $\theta = \theta^* = \tau^{\frac{\sigma}{1-\sigma}}$. By definition $N = \gamma L$, assuming $\gamma = 1$ for domestic and $\gamma^* > 1$ for foreign, the relationship presented in (42) is defined in the following way:

$$\frac{n}{n^*} = \frac{Z - \tau^{\frac{\sigma}{1-\sigma}}}{1 - Z\tau^{\frac{\sigma}{1-\sigma}}} \quad \text{with} \quad Z = \frac{\left(A - \overline{X}\right) N}{\left(\frac{A}{\gamma^*} - \overline{X}\right) N^*} \tag{52}$$

(52) has the same functional form of the standard HME, which in this case is presented through other channels and is represented in figure 1. The Z variable determines the complete or incomplete specialization of countries at the same intervals set out in the general case. There is a point where the income is equal for both countries ($\gamma = \gamma^*$), and they therefore produce the same number of varieties while maintaining a balanced trade for such goods. However, outside of this point the trade balance

in terms of heterogeneous goods exhibits a particular behavior that is determined by the number of varieties produced in each country (53), which in turn is determined by the level of income of each region. Therefore, the country with higher levels of income (fewer γ) will have a trade balance surplus in manufacturing.

$$TB_Y = \frac{(A-\gamma^* X)(1-\alpha)L}{\tau^{\frac{\sigma}{1-\sigma}} n + n^*}(n-n^*) \tag{53}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \ \wedge \ n > n^* \Leftrightarrow \gamma^* > \gamma \tag{54}$$

In equation (48) we show that $\frac{d(\frac{n}{n^*})}{dZ} > 0$, and so the effect of the increases in the variable γ^*, which contains the differentials of per capita income against a foreign partner, is positive.[14] Accordingly, the higher the per capita income of countries, the greater the number of varieties produced.

$$\frac{dz}{d\gamma^*} = \left(\frac{\left(A_x - \overline{X}\right)\overline{X}}{\left(A_x - \gamma^* \overline{X}\right)^2} \right) > 0 \ \ if \ A_x > \overline{X} \tag{55}$$

Proposition 1 *: In a world characterized by the presence of homogeneous and heterogeneous goods, where productivity is equal among countries and among sectors, and each country has a different level of supernumerary income, after trade the country with the higher relative income (less γ) will produce a greater number of varieties in the heterogeneous goods sector in relation to its trade partner and its autarky production. At the same time, the country with a higher level of income will have a trade surplus in this sector and its industrial production will be greater than under autarky.*

The result presented in proposition 1 goes in the same direction as the issues raised in the introduction and as that of the argumentation of the overall result. Countries with higher levels of supernumerary income spend the bulk of their income on elaborate items, which increases market size for this type of good, and so it becomes attractive to establish firms in this sector of production in order to take advantage of economies of scale. For this case, the countries with higher per capita income will have greater production of varieties of heterogeneous good due to the greater demand for these goods. The higher the per capita income of the countrty the higher the industrialization level after trade.

In terms of welfare, the results can be obtained by comparing levels of utility under autarky and in an open economy (50). The welfare levels under an open economy are superior to those under autarky because of the greater number of varieties available to agents. International trade increases the varieties available around the world, which allows for an increase in the levels of utility for both countries in relation to their situation under autarky.

[14] This is true so long as worker remuneration is higher than the survival consumption of the agricultural good; this is one of the assumptions made in the present paper.

5.3 Productivity

5.3.1 Total productivity

Equalizing the labor-force sizes of the countries, and assuming equality between the population and the number of employees ($N = L$), productivity differences among countries are entered. Productivity differs between countries but productivity among sectors is equal for each country ($A_x = A_y = A$ and $A \neq A^*$). Incorporating the assumptions above into the general equation (42), the following expression is reached, which is also represented in figure 1:

$$\frac{n}{n^*} = \frac{Z - \tau^{\frac{\sigma}{1-\sigma}}}{1 - \tau^{\frac{\sigma}{1-\sigma}} Z} \quad \text{con} \quad Z = \frac{\left(A - \overline{X}\right)}{\left(A^* - \overline{X}\right)} \tag{56}$$

Differences in productivity, as they are shown, allow for the inclusion of supply and demand effects within the relationship of the varieties of heterogeneous goods. The demand effect dominates through differences in income, and can even generate a complete specialization via productivity differentials. The external position of each economy in the range of incomplete specialization behaves similarly to the previous case. When the productivity factor is equal between countries, they produce the same amount of varieties and achieve a trade balance for differentiated goods. However, when productivity differs, that trade balance depends on the number of varieties produced (57), as directly defined by the productivity factor. A more productive country will have a positive trade balance in heterogeneous goods.

$$TB_Y = \frac{\left(A - \overline{X}\right)(1-\alpha) L \tau^{\frac{\sigma}{1-\sigma}}}{\tau^{\frac{\sigma}{1-\sigma}} n + n^*} (n - n^*) \tag{57}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \ \wedge \ n > n^* \Leftrightarrow A > A^* \tag{58}$$

Given $\frac{d(\frac{n}{n^*})}{dZ} > 0$, the effect of variations in the productivity factor on the number of varieties of heterogeneous goods produced in each country can be determined analytically, showing their direct relationship. In this way, the most productive country within the trade relationship will be the largest producer of heterogeneous goods.[15]

$$\frac{d(z)}{dA} = \left(\frac{1}{A^* - \overline{X}}\right) > 0 \ \ si \ A^* > \overline{X} \tag{59}$$

Proposition 2 *: Bilateral trade in countries that only differ in their productivity factors, these being equal between sectors, means that the country with higher productivity produces a superior number of varieties of heterogeneous good in relation to its trade partner and its autarky production. At the same time, the country with higher productivity will have a trade surplus in the manufacturing sector.*

Proposition 2 goes in the same direction mentioned above. The productivity channel, such as it arises in this case, raises the supernumerary income in the more productive country, creating a demand

[15] It is always true that productivity is higher than the minimum level of consumption of homogeneous goods.

effect that leads to an increase in the market of heterogeneous goods, making the establishment of firms within this sector in the said country attractive. Countries with high levels of productivity will have high levels of income, which increases the demand for heterogeneous goods and the number of varieties produced.

The current result exposes agglomeration and dispersion effects posed by the standard theory of HME, but via changes in productivity. However, the demand effect is much higher, resulting in the agglomeration effect dominating the dispersion. More productive countries will generate better remunerations for workers (income effect), while the cost effect is cancelled due to a reduction in this via productivity in equal magnitude to the increase in wages. Therefore, a more productive country will have a greater weight of heterogeneous goods in the composition of the individual demand, leading the producers of such goods to becoming established in this market in order to exploit economies of scale.

In terms of welfare, the result compared with autarky is determined in the same way by (50). The increase in global demand for heterogeneous goods generates greater varieties, which raises the levels of welfare for countries with respect to their situation in a closed economy.

5.3.2 Comparative advantage in heterogeneous goods

Detailing a little more regarding the implications of changes in productivity, this is a singular case in which there are variations between regions and sectors. Initially, the effects of productivity variations in the heterogeneous goods sector are examined, when these differ between countries and within the homogeneous goods sector, *ceteris paribus*. In a formal way $A_y \neq A_y^* \neq A_x$; $A_x = A_x^*$, then $z = 1$. Defining $\eta_y = \frac{A_y}{A_y^*}$ as the relationship pertaining to productivity in the manufacturing sector. The inclusion of these assumptions in (42) generates the following output:

$$\frac{n}{n^*} = \frac{\eta_y \left(H - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} \right)}{1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}} H} \tag{60}$$

where

$$H = \frac{1 - \eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}}{1 - \eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \tag{61}$$

As in the previous cases, the functional form is maintained, although the variables involved are clearly different. H defines the positive range of the function, which presents the possibility of incomplete specialization $\left(\eta_y^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\eta_y^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right)$, as figure 2 shows.

Figure 2: HME and the competitiveness factor

The trade balance of the heterogeneous goods in this interval is again determined by the number of varieties produced in each country, and also by the relationship with productivity in this sector (62). Similarly, at the point where both economies have the same productivity, they produce the same number of varieties and present an external equilibrium in this sector.

$$TB_Y = (\eta_y^{\frac{1}{1-\sigma}} - \eta_y^{\frac{-1}{1-\sigma}}) \frac{\tau^{\frac{\sigma}{1-\sigma}}(1-\alpha)\left(\frac{A_x^*}{\gamma^*} - \overline{X}\right) N^*}{\left(1-\eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)\left(1-\eta_y^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)} \tag{62}$$

$$TB_Y > 0 \Leftrightarrow n > n^* \Leftrightarrow \eta_y > 1 \tag{63}$$

Equation (60), presents the competitiveness factor H and the relationship between productivities (η_y, which represents the price ratio) as determinants of HME. The way in which these variables relate can be verified analytically using the following expressions.

$$\frac{\partial \frac{n}{n^*}}{\partial H} = \frac{\eta_y\left(1+\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}H\right)^2} > 0 \tag{64}$$

$$\frac{\partial\left(\frac{n}{n^*}\right)}{\partial \eta_y} = \frac{\left(H - \eta_y^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)}{1-\eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}H} + \frac{\left(\frac{1}{1-\sigma}\right)\left(\eta_y^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}H^2 - 2\tau^{\frac{2\sigma}{1-\sigma}}H\right)}{\left(1-\eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}H\right)^2} > 0 \tag{65}$$

Competitiveness factor H and the relationship between productivities of heterogeneous goods among countries η_y are directly related to the quotient of varieties produced between countries. The effects of variations in the productivity of heterogeneous goods are channeled via prices of this type of good. In this way, both supply and demand effects are presented. The first reduces costs for firms in the more productive country, after producing the same with less labor, increasing the number of varieties offered by the added open market. The second, the price effect, increases the number of varieties in demand, given the lower price of each. Finally, the differentiation in the factor of competitiveness with respect to the relationship for productivity is:

$$\frac{dH}{d\eta_y} = \frac{\left(\frac{1}{1-\sigma}\right)\left(\eta_y^{\frac{-2+\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_y^{\frac{\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} - 2\tau^{\frac{2\sigma}{1-\sigma}}\eta_y^{-1}\right)}{\left(1-\eta_y^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)^2} > 0 \tag{66}$$

The result of (66), related to (64), determines the aggregate effect of costs, which is presented as positive due to increased efficiency in manufacturing output. At the same time, the effect of the relationship between productivities is direct (65).[16] Effects in the same direction create a positive aggregate effect. The more productive the country in the manufacturing sector, the greater the number

[16] These results are always true under the interest interval.

of varieties that it produces.

$$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_y} > 0 \Leftrightarrow H \in \left(\tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}}\right) \tag{67}$$

Proposition 3 *: Trade between countries that only differ in productivity regarding the heterogeneous goods sector means that the country with greater productivity in this sector produces a higher number of varieties of heterogeneous good in relation to its trade partner and its autarky production. The greater the productivity in the heterogeneous goods sector, the greater the number of varieties produced. The effect on the trade balance of the more productive country is also positive.*

This proposition contributes to a delimiting of the effects of productivity as the channel of competitiveness among economies that trade. Thus productivity in the heterogeneous goods sector directly affects the size of the market and may strengthen HME. The result is clear that in the interest interval, where there exists incomplete specialization, the agglomeration effect dominates, so a direct relationship between comparative advantage in the heterogeneous goods sector and the number of varieties produced in the country is present. The higher the productivity in the manufacturing sector the higher the industrialization level after trade.

Similar to the above cases, it is possible to determine the effects on welfare by comparing the levels of utility between an open economy and autarky. In this case, the expression that determines these effects takes into account the implications of differences in productivity on the relative prices of the domestic varieties with respect to the foreigner.

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^*\left(\frac{\tau}{\eta_y}\right)^{\frac{\sigma}{1-\sigma}}}{n_A}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} = \left(\frac{1-\tau^{\frac{2\sigma}{1-\sigma}}}{1-\eta_y^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} > 1 \tag{68}$$

The welfare levels under an open economy are superior to those under autarky because of the greater number of varieties available to agents.

5.3.3 Comparative advantage in homogeneous goods

The other way to see particular differences in productivity between countries and between regions is through productivity in the homogeneous goods sector. Maintaining all other variables equal, countries only differ in productivity in relation to homogeneous goods, which in turn differs from productivity in relation to heterogeneous goods $\left(A_x \neq A_x^* \neq A_y \text{ and } A_y = A_y^*\right)$. Defining $\eta_x = \frac{A_x}{A_x^*}$ as the relationship between productivity for homogeneous goods in the two countries obtains the following result:

$$\frac{n}{n^*} = \frac{\eta_x^{-1}\left[\left(\frac{A_x-\overline{X}}{\frac{A_x}{\eta_x}-\overline{X}}\right)\left(\frac{1-\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}}{1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}}\right)-\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right]}{1-\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}\left(\frac{A_x-\overline{X}}{\frac{A_x}{\eta_x}-\overline{X}}\right)\left(\frac{1-\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}}{1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}}\right)} = \frac{\eta_x^{-1}\left(ZH-\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)}{1-\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}ZH} \tag{69}$$

The functional form persists and ZH defines the interval of incomplete specialization $\left(\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}, \frac{1}{\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}}\right)$ through different channels. This result is presented in figure 3, which shows a graphic representation similar to the previous cases but with different implications.

Figure 3: HME supernumerary income and competitiveness factor

This time, the behavior of the trade balance in this interval is given by the number of varieties produced in each country and the relationship between productivities for homogenous goods (wages) (70). Similarly, there is a case of balanced trade in this sector when the countries have the same level of productivity in the sector in question.

$$TB_Y = \frac{\tau^{\frac{\sigma}{1-\sigma}}\left(A_x^* - \overline{X}\right)(1-\alpha)L}{\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}n + n^*\eta_x^{-1}}\left(\eta_x^{\frac{-1}{1-\sigma}}n - \eta_x^{\frac{1}{1-\sigma}}n^*\right) \tag{70}$$

$$TB_Y > 0 \Leftrightarrow \frac{n}{n^*} > \eta_x^{\frac{2}{1-\sigma}} \tag{71}$$

The last term in (69) incorporates the factor of competitiveness among countries H, the supernumerary income ratio Z, and the ratio of productivity between countries in the homogeneous goods sector $\eta_x = \frac{A_x}{A_x^*}$, as determinants of HME. This expression allows for an analytical verification of how these determinants relate to the ratio of varieties between countries.

$$\frac{\partial \frac{n}{n^*}}{\partial z} = \frac{\eta_x^{-1}H\left(1-\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}ZH\right)^2} > 0 \tag{72}$$

$$\frac{\partial \frac{n}{n^*}}{\partial H} = \frac{\eta_x^{-1}Z\left(1-\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}ZH\right)^2} > 0 \tag{73}$$

$$\frac{\partial\left(\frac{n}{n^*}\right)}{\partial \eta_x} = \frac{-\eta_x^{-2}\left(ZH - \eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)}{1-\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}ZH} - \frac{\eta_x^{-1}}{1-\sigma}\left(\frac{\eta_x^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_x^{\frac{-2+\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}Z^2H^2 - 2\eta_x^{-1}\tau^{\frac{2\sigma}{1-\sigma}}ZH}{\left(1-\tau^{\frac{\sigma}{1-\sigma}}\eta_x^{\frac{-1}{1-\sigma}}ZH\right)^2}\right) < 0 \tag{74}$$

Competitiveness factor H and relative supernumerary income Z relate directly to the ratio of varieties produced between countries, while the relationship of productivity in the homogeneous goods sector η_x relates indirectly. However, ending the differentiation:

$$\frac{dz}{d\eta_x} = \left(\frac{\frac{A_x}{\eta_x^2}\left(A_x - \overline{X}\right)}{\left(\frac{A_x}{\eta_x} - \overline{X}\right)^2}\right) > 0 \quad if\ A_x > \overline{X} \tag{75}$$

$$\frac{dH}{d\eta_x} = \frac{\frac{-1}{1-\sigma}\left(\eta_x^{\frac{\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} + \eta_x^{\frac{-2+\sigma}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}} - 2\eta_x^{-1}\tau^{\frac{2\sigma}{1-\sigma}}\right)}{\left(1-\eta_x^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}\right)^2} < 0 \tag{76}$$

The result presented in (75), combined with (72), allows for the identification of the aggregate effect of demand, which is positive and is the same as that presented in case three, variations in the productivity factor. Similarly, the result (76), related to (73), determines the aggregate effect of costs, which is negative by the increase in the remuneration of labor.

The results in this case are ambiguous, since two contrary effects coexist. On the one side there is an income effect through the increase in wages that increases the demand, and therefore raises the proportion of the heterogeneous goods demanded. This effect from the demand side increases the number of varieties produced in the more productive country. On the other side is the cost effect, which arises from increases in wages after increases in productivity, and rises one to one the remuneration of labor, thus reducing the number of varieties produced in the more productive country because of the high costs of production and the tendency to specialize in the production of homogeneous goods. This ambiguity in the relationship shows the presence of the agglomeration and dispersion effects of the HME referenced above, and presents the existence of a trade-off between them, allowing any final result depending on the magnitudes of each.

$$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_x} = \underset{(-)}{\frac{\partial\left(\frac{n}{n^*}\right)}{\partial\eta_x}} + \underset{(+)}{\frac{\partial\left(\frac{n}{n^*}\right)}{\partial Z}}\frac{dZ}{d\eta_x} + \underset{(-)}{\frac{\partial\left(\frac{n}{n^*}\right)}{\partial H}}\frac{dH}{d\eta_x}$$

Proposition 4 *: The trade between countries that only differ in their productivity in the homogeneous goods sector generates opposite effects on the number of varieties of heterogeneous good produced in each country, the aggregate outcome being dependent on the magnitude of the effects presented. On the one side, the demand effect stimulates the production of more varieties given increases in wages, but on the other side, the cost effect reduces the number of varieties produced because the cost of production is higher.*

Proposition 4 contributes, from an alternative angle, to the delimitation of a competitiveness channel as a determinant of the market size of the economies that trade. The presence of contrary effects adds ambiguity to the aggregate result, presenting the result that productivity in this sector reinforces the HME when the agglomeration effect dominates; or, on the contrary, it weakens when the dispersion effect is predominant.

To reduce ambiguity in the results, a simulation was executed in order to determine the values of the parameters within which the dispersion or agglomeration effect dominates. Table 1 shows the values that the parameters must take for the agglomeration effect to dominate the dispersion effect. The first row presents the values that should take the transportation cost parameter if we use the standard values of substitution elasticity among varieties. Similarly, the second row presents the values that should take the substitution elasticity parameter among varieties if we use the standard values of transportation costs.[17]

Table 1: Parameter values, agglomeration dominates dispersion effect

These results suggest that the dispersion effect dominates the agglomeration effect in situations with economic sense in the value of the parameters. The agglomeration effect would only dominate in cases in which the transportation costs or the elasticity of substitution between varieties were extraordinarily high, cases in which international trade is possibly not established. This result justifies the fact that some industries will migrate to developing countries with low levels of productivity and wages. This is the case for industries in China or India, where an increase in the number of firms is due more to low-wage labor than to a large and effective demand for heterogeneous goods. When the dispersion effect dominates, it is possible to produce far from the larger markets and assume the costs of international trade, due to the low production costs of less productive countries.

As in other cases, the effect on welfare in relation to levels of utility could be set under autarky. The following expression exhibits this relationship, including differentials in productivity in the homogeneous goods sector.

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \left(\tau \eta_x\right)^{\frac{\sigma}{1-\sigma}}}{n_A} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} = \left(\frac{1 - \tau^{\frac{2\sigma}{1-\sigma}}}{1 - \eta_x^{\frac{-1}{1-\sigma}} \tau^{\frac{\sigma}{1-\sigma}}} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} > 1 \tag{77}$$

Similar to the previous cases, the effect on welfare is always positive in relation to autarky when the size of the global market for heterogeneous goods increases after trade. Regardless of the outcome of the trade in relation to the industrialization of countries, welfare increases along with the number of varieties to which people have access.

5.3.4 Comparative statics summary

The comparative statics show how the different variables affect the number of varieties produced in each country. This is evidence of the existence of different channels in HME determination. Interactions among channels from the demand and supply side determine HME in a distinct way. The next table summarizes the effects of each determinant on HME. The first column shows the relationship between each determinant and the quotient of varieties produced for each country (the sign of the first differentiation). The second column presents what happens to the HME when two countries start

[17] These standard values are used in papers as: Krugman (1991), Martin and Ottaviano (1999), Corsetti, Martin and Pesenti (2007), etc.

to trade and each variable becomes greater at the domestic rather than foreign level. Finally, the third column presents the trade effect on industrialization (number of varieties produced) in relation to autarky.

Table 2: Variable effects on HME[18]

The demand size directly affects the number of varieties produced. Population size, the dependence factor and total productivity,[19] define the supernumerary income, which determines the purchasing power of the agents and, indeed, the size of the market for each country. The interplay of these variables determines the demand size in each country, and so, the larger the demand size of the country, the larger the number of varieties of heterogeneous good that it produces.

The supply side affects the number of varieties produced in different ways. Total productivity and the comparative advantage in heterogeneous goods directly impact the number of varieties produced; these allow for higher production in the heterogeneous goods sector and raise the real income of the agents. On the other hand, the comparative advantage in homogeneous goods reduces the number of varieties produced because it increases production costs and promotes specialization in agricultural production.

HME is generated through the interaction of supply and demand side variables. These variables determine the comparative advantages and the demand size of the countries. The results of international trade in terms of the industrialization of countries depends on the different determinants of HME and their interplay. The effects of international trade on welfare are always positive, relative to levels of autarky (*ceteris paribus*).

5.4 Variations in Productivity and Supernumerary Income

With the intention of illustrating the empirical implications of the model, we present a simple exercise which varies population size, relative income and total productivity. For this purpose, the United States is taken as the domestic economy and six other countries/regions as the foreign economies. The results of a hypothetical bilateral trade agreement between the domestic economy and each of the foreign regions is analyzed under the exposed model (equation (43), which combines the first three cases of comparative statics). There is evidence of the importance of these three channels (population size, purchasing power and productivity level) for trade outcomes and the presence of HME. In addition, the exercise allows us to highlight the apparent paradoxes of international trade, as in the case of commercial relations between large countries in terms of population size, such as China and India, and

[18] The comparative advantage variable in the homogeneous good sectors is assumed under economic interpretation in the value of the parameters.

[19] When productivity is equal among sectors, total productivity determines the wages of the workers.

economically large countries like the United States. Similarly, it would explain why some regressions in search of the HME are not so robust.

Tabla 3: Results of hypotetical trade between the U.S. and six countries/regions.

The exercise involves three key variables: population size, number of people employed and level of productivity. The first two determine the value of γ, which is 2.17 for the case of the United States. Foreigners are presented in the third column of table 3.[20] Similarly, the level of labor productivity per worker is taken as the productivity variable; the value for the United States is 93.3 and for other countries/regions is shown in the fifth column of the same table.

The estimation of trade effects on these scenarios through traditional HME derives from situations of complete specialization,[21] however, the results obtained from the exposed HME (sixth column) show that beyond the physical size of the market, HME is determined by economic size. The number of people that constitute a market is important in determinating the effects of trade, but equally important is their purchasing power, which for the case study is determined by the level of productivity and the supernumerary income. In a more graphic way, countries like China and India, with a population almost four times larger than that of the United States, would not achieve concentration in the manufacturing industry after trading with the latter, as traditional HME puts it. In contrast, the United States would present a high concentration of the industry of manufactured goods, due to their large purchasing power being fundamentally related to high levels of productivity.

More generally, it may be seen that trade results are less asymmetrical as the gap in purchasing power among the regions closes, that is, the more similar their productivity and their γ. The results of the trade between the United States and Europe or Australia do not have a concentration of this type of industry, as is the case with other countries, including the case of Nepal, where the gaps are big enough that the heterogeneous goods sector ends up being suppressed.

Parallel to the previous exercise, an additional simulation is performed by modifying the definition of (γ), not as a relationship between population and employees, but, as the ratio between the population and the people above the poverty line.[22] This is done in order to present the robustness of the findings when income distribution is entered. The results are presented in the columns accompanied by (p), and demonstrate consistency with those presented previously. The effects are virtually identical and underline the importance that the levels of productivity play in the determination of the demand structure in each market.

[20] Data from the World Bank and APO Productivity Data Book 2012.

[21] Bilateral trade would result in a complete specialization in heterogeneous goods in Europe, China and India; while in the other cases, the United States would concentrate the production of such goods.

[22] We take the criterion of the population living on less than 2 US dollars a day from the World Bank.

The last columns of table 3 show the effects of bilateral trade on the welfare of different regions. The result is found from (49) and shows the variation of the utility under free trade in relation to autarky. The welfare in all regions increases considerably, regardless of the effects on the domestic industry of heterogeneous goods. That is, while the heterogeneous United States goods industry increases in all cases, the utility of all the countries/regions is much better after the trade agreement due to the increased number of varieties available, which immediately raises welfare levels. The country that increases in welfare the most is Nepal, which in turn is the most different from the United States in terms of the variables in question; this means that the increase in the number of varieties will be much more representative than in other economies.

In the case of the United States, it also benefits from trade in terms of welfare, mainly with Europe, with which its welfare increases by 25.8%; given the access to a greater number of varieties. With other countries the increase in welfare is in the order of 8 to 20%. With Nepal, the variation is only 5% due to the reduced number of additional varieties that this country can offer. In the way that the model predicts, the outcome of international trade is a positive-sum game in which everyone wins. However, it should be clarified that these results have been generated in a static environment, and that the conclusions may be modified in a dynamic setting.[23]

6 Conclusions

The traditional literature on HME has focused primarily on the number of agents that make up a market as a determinant of demand for trade between countries, and despite acknowledging the existence of additional channels it has not focused enough on these. This article contributes to the exploration of some of these additional channels in the determination of the effects of international trade, in terms of the industrialization and welfare of participating countries.

The alternative way in which HME has been modeled, with non-homothetic preferences and differences in productivity, shows the importance of three channels in the determination of international trade effects and HME: population size, relative income and productivity levels. Thus the effects of international trade on the industrialization of countries depends on the way in which these three channels interact, depending on the particularities of the countries participating in the trade.

The greater the population size, relative income, total productivity levels and productivity in the heterogeneous goods sector, the greater the number of varieties produced by a country after trade in relation to its trade partner and autarky production. On the other side, the greater the productivity in the homogeneous goods sector, the fewer the number of varieties produced by a country after trade

[23] The application of this open economy model to an endogenous economic growth model is part of the subsequent article on the research agenda.

in relation to its trade partner and autarky production. In terms of the industrialization of countries, international trade can be positive or negative according to each particular scenario. However, the effects on welfare are always positive, relative to levels of autarky (*ceteris paribus*).

Such results have become one of the dimensions of international trade that can be analyzed by different countries when determining the nations with which it should trade. However, the static nature of the results opens up research into the dynamic effects of trade and to future extensions of this model.

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7 Appendix 1

In this section we demonstrate that welfare levels are always better after trade in relation to autarky levels. The next equation relates the utility levels under trade and under autarky:

$$\frac{U}{U_A} = \left(\frac{n}{n_A} + \frac{n^* \tau^{\frac{\sigma}{1-\sigma}}}{n_A} \left(\frac{p}{p^*} \right)^{\frac{\sigma}{1-\sigma}} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \tag{78}$$

Using the equations (23) and (42)

$$\frac{n}{n^A} = \left(\frac{\frac{n}{n^*}}{\frac{n}{n^*} + \theta \frac{p^*}{p}} \right) + \left(\frac{\theta^* \frac{n}{n^*}}{\frac{p^*}{p} + \theta^* \frac{n}{n^*}} \right) \frac{1}{Z} = \frac{1}{1-\theta^*} - \frac{\theta}{1-\theta} \left(\frac{1}{Z} \right) \tag{79}$$

$$\frac{n^*}{n^A} = \left[\left(\frac{\theta}{\frac{n}{n^*} \frac{p}{p^*} + \theta} \right) + \left(\frac{1}{1 + \theta^* \frac{n}{n^*} \frac{p}{p^*}} \right) \frac{1}{Z} \right] \frac{p}{p^*} = \theta \left(\frac{1}{(1-\theta) Z} - \frac{\theta^*}{1-\theta^*} \right) \tag{80}$$

Entering the final equations in (78) we will have:

$$\frac{U}{U_A} = \left(\frac{1-\theta\theta^*}{1-\theta^*} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \tag{81}$$

Given $\theta < 1 \Rightarrow U > U^A$ welfare is always better after trade in relation to autarky.

$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_x} > 0$	$\sigma = 0.4 \wedge \tau < 0.025$	$\sigma = 0.8 \wedge \tau < 0.38$	$\sigma = 0.9 \wedge \tau < 0.59$	Standard σ Values
$\frac{d\left(\frac{n}{n^*}\right)}{d\eta_x} > 0$	$\tau = 0.7 \wedge \sigma > 0.95$	$\tau = 0.8 \wedge \sigma > 0.973$	$\tau = 0.9 \wedge \sigma > 0.9998$	Standard τ values

Table 1: Parameter values, agglomeration dominates dispersion effect

Variable	$\partial\left(\frac{n}{n^*}\right)$	**HME**	**Industrialization**
Population Size N	> 0	$\uparrow$	$\uparrow$
Dependence Factor γ	< 0	$\downarrow$	$\downarrow$
Total Productivity A	> 0	$\uparrow$	$\uparrow$
Comparative Advantage Heterogeneous Goods A_y	> 0	$\uparrow$	$\uparrow$
Comparative Advantage Homogeneous Goods A_x	< 0	$\downarrow$	$\downarrow$

Table 2: Variable effects on HME

Country/Region ($*$)	N^*/N	γ	$\gamma(p)$	Productivity	n/n^*	$n/n^*(p)$	$\triangle U^*$	$\triangle U$
Europe	1.62	2.28	1	69.7	1.95	1.94	23.3%	25.8%
Australia	0.07	1.99	1	74.3	1.76	1.76	56.1%	8.2%
Japan	0.41	2.01	1	63.9	2.18	2.17	36.8%	15.1%
China	4.32	1.74	1.43	12.4	4.24	4.25	28.3%	20.7%
India	3.95	2.88	3.03	8.3	4.42	4.42	39.2%	12.6
Nepal	0.09	3.34	2.34	3	4.64	4.63	114.3%	5%

Tabla 3: Results of hypotetical trade between the U.S. and six countries/regions.

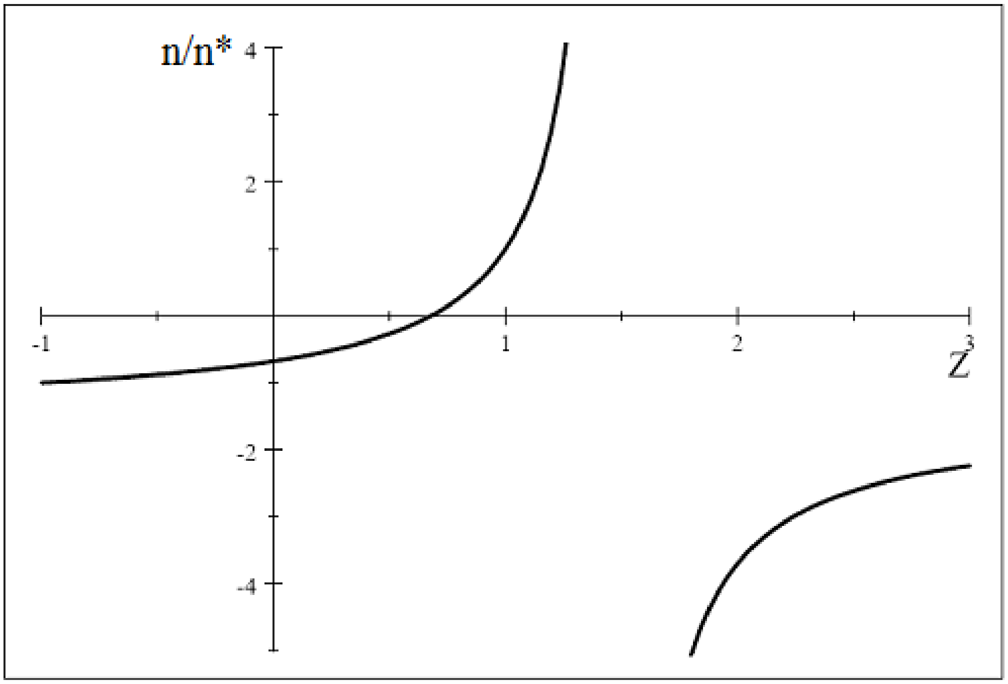

Figure 1: Population size, supernumerary income and HME

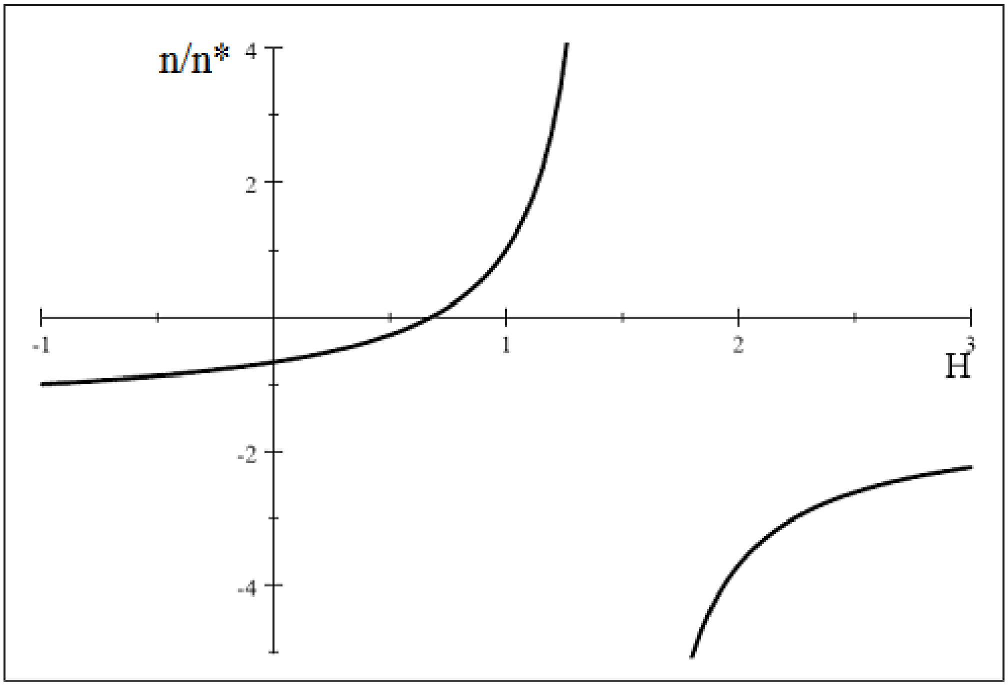

Figure 2: HME and the competitiveness factor

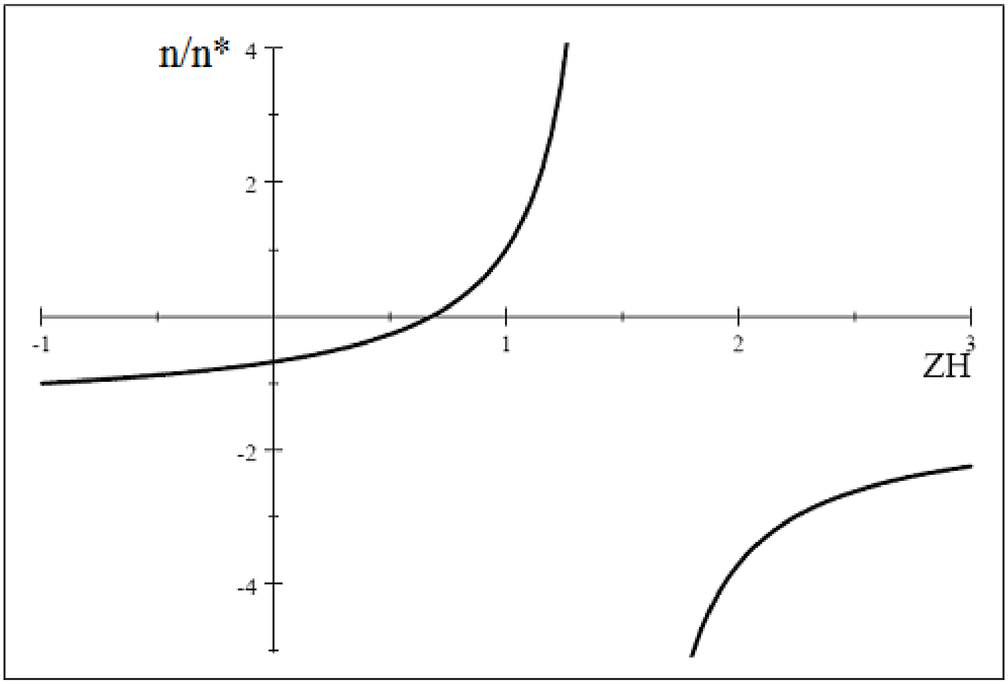

Figure 3: HME supernumerary income and competitiveness factor