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Credit Spreads and Monetary Policy in a Small Open Economy: an Analysis of the Colombian Economy

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ABSTRACT

This paper estimates Bejarano and Charry (2014)'s small open economy with financial frictions model for the Colombian economy using Bayesian estimation techniques. Additionally, I compute the welfare gains of implementing an optimal response to credit spreads into an augmented Taylor rule. The main result is that a reaction to credit spreads does not imply significant welfare gains unless the economic disturbances increases its volatility, like the disruption implied by a financial crisis. Otherwise its impact over the macroeconomic variables is null.

1 INTRODUCTION

The 2008 financial crisis renewed economists' interest over the debate about the role of central banks on the achievement financial stability. In this debate, whether monetary authority should target a financial measure besides inflation and output to prevent a negative impact over the economy has gained importance.

Motivated by Taylor (2008) and Mishkin (2008), which proposed that the Federal Reserve should lower the interest rate to increases of credit spreads as a way to mitigate the adverse effect of the financial crisis over the real economy, Curdia and Woodford (2010) analyzed the effects of including the spread between deposits and loans interest rate into a Taylor Rule in a DSGE framework calibrated to reflect US economy dynamics . They found that this reaction is recommended for financial and non-financial disturbances, but the optimal size of the response would depend on the source and the persistence of the disturbance. Additionally, empirical studies such as Belke and Klose (2010), Castro (2011), Martin and Milas (2013), and Huang (2015) have concluded that the Federal Reserve, the European Central Bank or the Bank of England have responded to credit spreads during or before the 2008 financial crisis.

These studies were focused mainly on developed economies. However, analysis of the relation between monetary policy and credit spreads are scarce in countries like Colombia. Since Colombian monetary policy is conducted similarly to US, it is possible to apply Curdia and Woodford

(2010) analysis to the Colombian framework. However, US and Colombian economies differ in several aspects; mainly that Colombia is a small open economy.

In this sense, Bejarano and Charry (2014) extended Curdia and Woodford (2010) model to a small open economy framework. Their conclusions differ substantially from Curdia and Woodford (2010) model since they found that including spreads on the Taylor rule is just recommended if the source of the disturbance is the foreign interest rate. An important remark about Bejarano and Charry (2014) model is that most of the parameters used were the same as Curdia and Woodford (2010) analysis in order to focus mainly on the effect of opening the economy.

In order to apply this analysis to the Colombian economy, this paper estimates Bejarano and Charry (2014) model's structural parameters through Bayesian estimation techniques using Colombian data on main macroeconomic variables, as on interest rates. The main reason to estimate the structural parameters using Colombian data is for the model to reflect Colombian aggregate variables dynamics. Both, Curdia and Woodford (2010) and Bejarano and Charry (2014) conclusions are based on a calibration that reflects US macroeconomics variable dynamics, but these conclusions might differ in the context of the Colombian economy.

Once the parameters are estimated, this paper computes the optimal response to credit spreads in a Taylor rule by maximizing an aggregate welfare function approximated up to second order. Then, to asses the gains of implementing the optimal response described above, we compute the welfare loss as the percentage of consumption sacrificed from not reacting at all to credit spreads instead of implementing the optimal reaction. Finally, we perform counterfactual analysis comparing historic data to simulated data where monetary authority implements the optimal response. This is done to to determine the impact on macroeconomic variables, such as output or debt, of implementing the optimal reaction.

The main result of this research is that implementing an optimal reaction to credit spreads do not imply significant welfare gains because credit spreads volatility is low compared to other macroeconomic variables. The Credit spread is rather a stable measure that does not have great impact over the economy. If the the central bank implemented an optimal response to credit spread, aggregate macroeconomic variables would display almost exactly the same dynamics as if it did not respond at all. However, similar to the 2008 financial crisis, during financial turmoil credit spreads volatility increases significantly and impacts adversely debt and output. In this scenario, it is highly recommended to react to credit spreads as it would stabilize output faster and prevent it from falling that much compared to not reacting at all.

The structure of this paper is as follows: Section 2 describes the structure of the model, emphasizing in the introduction of the credit spread and the main characteristics of the small open economy. In Section 3 estimates the model's structural parameters through Bayesian estimation techniques. Section 4 computes the optimal response to credit spreads and the welfare gains implied, explaining under which circumstances such a reaction is recommended. Finally, in Section 5 the main conclusions are summarize.

2 A SMALL OPEN ECONOMY WITH FINANCIAL FRICTIONS

This is a basic New Keynesian model with the following additional assumptions: i) the existence of two different types of households: savers and borrowers, ii) the economy can only channel funds from savers to borrowers through a financial intermediary, iii) the financial intermediary can be funded by foreign liabilities, besides savers funds, and iv) households can consume three different baskets: domestic traded goods, domestic non-traded goods, and imported goods. The first two assumptions account for the introduction of a financial friction that creates a wedge between the deposit and the credit interest rate. The last two account for setting the small open economy framework.

2.1 *Financial Friction: the Credit Spread*

2.1.1 *Heterogeneous Households: Intertemporal Optimization Problem*

Heterogeneity is introduced by assuming that households differ in their preferences. Hence, at period t they can be either a saver or a borrower household according to the following discounted intertemporal objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u^{\tau_t(i)}(c_t(i); \varepsilon_t) - \int_0^1 v^{\tau_t(i)}(h_t(j; i); \varepsilon_t) dj \right] \tag{1}$$

where $\tau_t(i) \in \{b, s\}$ indicates the household's type in period t. Period utility $u^{\tau_t(i)}(c_t(i); \varepsilon_t)$ is function of a Dixit-Stiglitz aggregator of differentiated consumption goods $c_t(i)$. Disutility of work $v^{\tau_t(i)}(h_t(j; i); \varepsilon_t)$ is function of a continuum of different types of specialized labor, indexed by j, that households supply to domestic non-traded good firms. Finally, both $u^{\tau_t(i)}$ and $v^{\tau_t(i)}$ can be shifted by a vector of aggregate taste shocks ε_t.

For this analysis we assume that utility of consumption and disutility of work are of the form:

$$u^{\tau_t(i)}(c_t(i); \varepsilon_t) = \frac{c_t(i)^{1-\frac{1}{\sigma_\tau}} \bar{C}^{\tau \frac{1}{\sigma_\tau}}}{1 - \frac{1}{\sigma_\tau}} \tag{2}$$

and

$$v^{\tau_t(i)}(h_t(j; i); \varepsilon_t) = \frac{\psi_\tau}{1+\nu} h_t(i, j)^{1+\nu} \bar{H_t}^{-\nu} \tag{3}$$

where σ_τ is the intertemporal elasticity of substitution of type τ; $\bar{C}_t^\tau$ is an aggregate preference shock to consumption of type τ; ψ_τ is a scalar factor for type τ, ν is the inverse Frisch elasticity of labor supply, and $\bar{H}_t$ is an aggregate preference shock to labor.

Households may change type according to an independent two-state Markov chain. Each period, with probability $1 - \delta$ each household may drawn a new type; otherwise it remains the same. After a new type is drawn, it becomes a borrower with probability π_b and a saver with probability π_s. It is assumed risk-sharing households that can sign state contingent contracts with one another to insure against aggregate and idiosyncratic risk from the household random drawn of its type.[1] This insurance is intermittent, and households

1 This facilitates aggregation by avoiding that financial wealth depend on household idiosyncratic risk.

are only able to sign these contracts whenever they face a new type drawn. In this case, household will receive a net transfer $T_t(i)$ and learns its type. Then, it makes its spending, saving and borrowing decisions, taking into account its new type.

Household's beginning-of-period nominal net financial wealth is given by:

$$A_t(i) = [B_{t-1}(i)]^+(1 + i^d_{t-1}) + [B_{t-1}(i)]^-(1 + i^b_{t-1}) + D^{int}_t + T_t(i) \quad (4)$$

where B_{t-1} is the household net financial wealth for period $t-1$; $[B]^+ \equiv max(B, 0)$ is the financial wealth household i has on deposits to the financial intermediaries or government debt; $[B]^- \equiv min(B, 0)$ if the financial wealth from credit owed to the financial intermediaries; i^d_t is the riskless one-period deposits interest rate, i^b_t is the borrowing interest rate; and D^{int}_t is the distributed profits from the financial intermediary.

Households end-of-period nominal net financial wealth is given by:

$$B_t(i) = A_t(i) - P_t c_t(i) + \int_0^j W_t(j) h_t(i, j) dj + D_{n,t} + D_{x,t} + D_{m,t} + T^g_t \quad (5)$$

where P_t is the Dixit-Stiglitz price index for period t; $W_t(j)$ is the wage of labor type j; $D_{n,t}$ are the profits from the non-traded good producing firms; $D_{x,t}$ are the profits from the traded good producing firms; $D_{m,t}$ are the profits from the imported good producing firms; and T^g_t is the lump-sum transfer received from the government.

If it is assumed that ψ_s and ψ_b are calibrated so that both types choose to work the same hours in steady state, the existence of heterogeneous households is guaranteed, at least around the equilibrium, if $u^b_c(c; \varepsilon) > u^s_c(c; \varepsilon)$ (so borrowers have a higher impatience to consume) and $i^b_t > i^d_t$. The fist condition is assured by calibrating $\sigma_b > \sigma_s$, the second will be shown in the next section. These two conditions imply that the set of borrowers B will always choose $B_t(i) < 0$, the set of savers S will choose $B_t(i) > 0$, and $i = [B, S]$.[2]

By maximizing (1) with respect to $c_t(i)$, $h_t(i, j)$, and $B_t(i)/P_t$ subject to (4) and (5) and, then, aggregating over B and S we have the following fist order conditions:

$$\lambda^\tau_t = \left(\frac{\bar{C}^\tau_t}{c^\tau_t}\right)^{\frac{1}{\sigma_\tau}} \quad (6)$$

$$\mu^w_t \psi_\tau \left(\frac{h_t(j)^\tau}{\bar{H}_t}\right)^\nu = \lambda^\tau_t \frac{W_t(j)}{P_t} \quad (7)$$

$$\lambda^s_t = \beta E_t \left\{ \frac{(1 + i^d_t)}{\Pi_{t+1}} [\delta \lambda^s_t + (1 - \delta)\Lambda_t] \right\} \quad (8)$$

$$\lambda^b_t = \beta E_t \left\{ \frac{(1 + i^b_t)}{\Pi_{t+1}} \left[\delta \lambda^b_t + (1 - \delta)\Lambda_t\right] \right\} \quad (9)$$

$$\Lambda_t = \pi_b \lambda^s_t + \pi_s \lambda^b_t \quad (10)$$

where λ^τ_t is the marginal utility of consumption for type τ, Λ_t is the marginal utility aggregated over i , μ^w_t is a wage mark-up shock which is assumed exogenous, and $\Pi_t = P_t/P_{t-1}$ is period t inflation.

2 For more details see section 1.1 from Curdia and Woodford (2009) on-line appendix.

2.1.2 *Financial Intermediaries*

As it was stated above, when households cannot sign state-contingent contracts, we assume that funds from savers to borrowers can only be channeled through a financial intermediary. Additionally, we assume that the financial intermediary can also channel funds to borrowers from international financial markets. [3] Then, expected real profits from the financial intermediary sector can be expressed as [4]

$$E_t \frac{D^{int}_{t+1}}{P_{t+1}} = E_t \left\{ \frac{b_t(1+i^b_t)}{\Pi_{t+1}} - \frac{d_t(1+i^d_t)}{\Pi_{t+1}} - q_{t+1} b^*_t (1+i^{d*}_t)(1+\varphi_t) \right\} \tag{11}$$

where b_t are the aggregate loans, d_t are the aggregate deposits, q_t is the real exchange rate, b^*_t are international financial markets funds, i^{d*}_t is the foreign interest rate, which is assumed to be exogenous, and φ_t is the risk premium with the form suggested by Schmitt-Grohe and Uribe (2003) where it increases to deviations of international financial markets funds to GDP ratio from its steady state value:

$$\varphi_t = \psi_1 \left(exp(\frac{q_t b_t}{gdp_t} - \frac{q_{ss} b_{ss}}{gdp_{ss}}) - 1 \right) \tag{12}$$

The financial intermediary can only channel these funds assuming a cost $\Xi(b_t)$ and faces an aggregate default probability χ_t on every loan, which is assumed exogenous. We assume that the costs associated to channeling funds come from consuming non-traded goods. In order to repay savers and international financial markets we have the following real resource constraint for financial intermediaries:

$$b_t + \widetilde{P_{n,t}} \Xi(b_t) + \chi_t b_t = d_t + q_t b^*_t \tag{13}$$

where $\widetilde{P_{n,t}} = P_{n,t}/P_t$ is the non-traded good price relative to the Dixit-Stiglitz price index. We assume that $\Xi(b_t) = \widetilde{\Xi}_t b^{\eta}_t$, with $\eta > 1$ in order to have non-decreasing and convex cost over aggregate loans, and we assume $\widetilde{\Xi}_t$ to be an exogenous shock. By maximizing (11) with respect to b_t, d_t, and b^*_t subject to (13) we have that:

$$(1+i^b_t) = (1+i^d_t)(1+\omega_t) \tag{14}$$

$$(1+i^d_t) = E_t \left\{ \frac{q_{t+1}(1+i^{d*}_t)(1+\varphi_t)\Pi_{t+1}}{q_t} \right\} \tag{15}$$

where ω_t is the credit spread from deposits and loans interest rate of the form:

$$\omega_t = \widetilde{P_{n,t}} \widetilde{\Xi}_t \eta b^{\eta-1}_t + \chi_t \tag{16}$$

As it was discussed above, one condition to guarantee the existence of heterogeneous households was that $i^b_t > i^d_t$, which we make sure since from the definition $\omega_t > 0$.

3 We calibrate the model in order that financial intermediaries always choose to borrow from international markets instead of lending them; hence $b^*_t > 0$.

4 Most international funds for the Colombian economy come through Foreign Direct Investment. However, since is a model without investment, we just choose this structure to reflect the fact that these international funds finance domestic debt.

2.2 *The Small Open Economy*

2.2.1 *Intratemporal Optimization Problem*

Another important characteristic of a Small Open Economy is for the possibility to import a foreign traded-good and to export a domestic traded good. In this sense, it is assumed that the Dixit-Stiglitz aggregator of differentiated consumption goods $c_t(i)$ is of the form:

$$c_t(i) = \left[\gamma^{\frac{1}{\rho_h}} c_{h,t}(i)^{\frac{\rho_h - 1}{\rho_h}} + (1-\gamma)^{\frac{1}{\rho_h}} c_{f,t}(i)^{\frac{\rho_h - 1}{\rho_h}} \right]^{\frac{\rho_h}{\rho_h - 1}} \tag{17}$$

where γ is the parameter controlling the participation of households expenditure in domestic goods, ρ_h is the elasticity of substitution between imported and domestic goods consumption, $c_{h,t}(i)$ is household i consumption of the domestic good, and $c_{f,t}(i)$ is households i consumption of the imported good.

Additionally, the consumption of the domestic good can be differentiated between the consumption of a non-traded good and a traded good by

$$c_{h,t}(i) = \left[\gamma_n^{\frac{1}{\rho_n}} c_{n,t}(i)^{\frac{\rho_n - 1}{\rho_n}} + (1-\gamma_n)^{\frac{1}{\rho_n}} c_{x,t}(i)^{\frac{\rho_n - 1}{\rho_n}} \right]^{\frac{\rho_n}{\rho_n - 1}} \tag{18}$$

where, similarly as above, γ_n controls the participation of household consumption in the domestic non-traded good, ρ_n is the elasticity of substitution between the domestic non-traded good and the domestic traded good, $c_{n,t}(i)$ is household i consumption of the domestic non-traded good, and $c_{x,t}(i)$ is households i consumption of the domestic traded good.

We assume that the domestic non-traded good and the imported good are baskets of differentiated goods indexed by j and m, respectively. Then it follows that:

$$c_{n,t}(i) = \left[\int_0^1 c_{n,t}(j)^{\frac{\theta_n - 1}{\theta_n}} dj \right]^{\frac{\theta_n}{\theta_n - 1}} \tag{19}$$

$$c_{f,t}(i) = \left[\int_0^1 c_{f,t}(m)^{\frac{\theta_f - 1}{\theta_f}} dm \right]^{\frac{\theta_f}{\theta_f - 1}} \tag{20}$$

Since household heterogeneity has no impact over the optimal composition of consumption baskets, we can aggregate over i and solve households intra-temporal consumption decision problems. This will provide us with the following equilibrium conditions:

- For the optimal allocation between domestic and imported goods

$$c_{h,t} = \gamma c_t \widetilde{P_{h,t}}^{-\rho_h} \tag{21}$$

$$c_{f,t} = (1-\gamma) c_t \widetilde{P_{f,t}}^{-\rho_h} \tag{22}$$

$$P_t = \left[\gamma P_{h,t}^{1-\rho_h} + (1-\gamma) P_{f,t}^{1-\rho_h} \right]^{\frac{1}{1-\rho_h}} \tag{23}$$

- For the optimal allocation between domestic non-traded and traded goods

$$c_{n,t} = \gamma_n c_{h,t} \overline{P_{n,t}}^{-\rho_n} \tag{24}$$

$$c_{x,t} = (1-\gamma_n) c_{h,t} \overline{P_{x,t}}^{-\rho_n} \tag{25}$$

$$P_{h,t} = \left[\gamma_n P_{n,t}^{1-\rho_n} + (1-\gamma_n) P_{x,t}^{1-\rho_n}\right]^{\frac{1}{1-\rho_n}} \tag{26}$$

Where $\overline{P_{n,t}} = P_{n,t}/P_{h,t}$ and $\overline{P_{x,t}} = P_{x,t}/P_{h,t}$.

- For the optimal allocation between differentiated domestic non-traded goods

$$c_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\theta_n} c_{n,t} \tag{27}$$

$$P_{n,t} = \left[\int_0^1 P_{n,t}(j)^{\theta_n - 1} dj\right]^{\frac{1}{\theta_n - 1}} \tag{28}$$

- For the optimal allocation between differentiated imported goods

$$c_{f,t}(m) = \left(\frac{P_{f,t}(m)}{P_{f,t}}\right)^{-\theta_f} c_{f,t} \tag{29}$$

$$P_{f,t} = \left[\int_0^1 P_{f,t}(m)^{\theta_f - 1} dm\right]^{\frac{1}{\theta_f - 1}} \tag{30}$$

2.2.2 *Firms' Pricing Decision*

Domestic Non-Traded Good

We assume that households supply labor to each j non traded good producing firms. From equation (7) we can solve for h_t^τ and aggregate over i to obtain labor supply to firm j

$$h_t(j) = \bar{H}_t \left[\frac{W_t(j)}{P_t} \frac{\widetilde{\lambda}_t}{\mu_t^w \psi}\right]^{\frac{1}{\nu}} \tag{31}$$

where

$$\left(\frac{\widetilde{\lambda}_t}{\psi}\right)^{\frac{1}{\nu}} = \pi_b \left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}} + \pi_s \left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}} \tag{32}$$

Additionally, we assume that non-traded good producing firms have a production technology of the form:

$$y_{n,t}(j) = z_t \left(h_t(j)\right)^{\frac{1}{\phi}} \tag{33}$$

where z_t is a productivity shock common to all non-traded good producing firms, and ϕ is assumed to be grater than 1. We define labor real expenditure of firm j as:

$$linc_t(j) = \frac{W_t(j)}{P_t} h_t(j) \tag{34}$$

Given that $y_{n,t}(j) = c_{n,t}(j)$, and replacing equation (27), we can determine $linc_t(j)$ by solving for $h_t(j)$ in equation (33) and $W_t(j)/P_t$ in equation (31). Then, we have that:

$$linc_t(j) = \left(\frac{\mu_t^w \psi}{\widetilde{\lambda}_t \bar{H}_t^\nu}\right)\left(\frac{y_{n,t}}{z_t}\right)^{1+\omega_y}\left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\theta_n(1+\omega_y)} \tag{35}$$

where $\omega_y = \phi(1+\nu)$. We can define firm j nominal profits as

$$D_{n,t}(j) = [P_{n,t}(j)y_{n,t}(j)(1-\tau_{n,t}) - P_t linc_t(j)] \tag{36}$$

where $\tau_{n,t}$ is the tax payed for each sold non-traded good. Firm j can only update its price $P_{n,t}(j)$ with probability α each period. Then, replacing equations (27), (33), and (35), firm j maximizing problem becomes:

$$\begin{aligned} P_{n,t}(j)^* =& argmax \left[E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ P_{n,t}(j)\left(\frac{P_{n,t}(j)}{P_{n,T}}\right)^{-\theta_n} y_{n,T}(1-\tau_{n,T}) - \right.\right. \\ & \left.\left. P_T \left(\frac{\mu_T^w \psi}{\widetilde{\lambda}_T \bar{H}_T^\nu}\right)\left(\frac{y_{n,T}}{z_T}\right)^{1+\omega_y}\left(\frac{P_{n,t}(j)}{P_{n,T}}\right)^{-\theta_n(1+\omega_y)} \right\}\right] \end{aligned} \tag{37}$$

Where $Q_{t,T}$ is the stochastic discount factor given by

$$Q_{t,T} = \beta^{T-t}\frac{\Lambda_T P_t}{\Lambda_t P_T} \tag{38}$$

From the maximization problem stated above we get the following fist order condition:

$$\left(\frac{P_{n,t}(j)^*}{P_{n,t}}\right)^{\theta_n\omega_y+1} = \frac{K_{n,t}}{F_{n,t}} \tag{39}$$

Where

$$K_{n,t} = \Lambda_t \mu_n (1+\omega_y)\left(\frac{\mu_t^w \psi}{\widetilde{\lambda}_t \bar{H}_t^\nu}\right)\left(\frac{y_{n,t}}{z_t}\right)^{1+\omega_y} + \alpha\beta E_t[K_{n,t+1}\Pi_{n,t+1}^{\theta_n(1+\omega_y)}] \tag{40}$$

$$F_{n,t} = \Lambda_t \widetilde{P_{n,t}} y_{n,t}(1-\tau_{n,t}) + \alpha\beta E_t[F_{n,t+1}\Pi_{n,t+1}^{\theta_n-1}] \tag{41}$$

Here $K_{n,t}$ is the discounted marginal cost, $\mu_n = \theta_n/(\theta_n-1)$ is the firm's mark up, $\Pi_{n,t}$ is the non-traded good inflation, and $F_{n,t}$ is the discounted marginal revenue. Because only a share α of all non-traded good producing firms can update their prices, we replace equation (39) in (28), aggregate over j , and, after some algebra, get

$$\Pi_{n,t} = \left[\frac{1-(1-\alpha)\left(\frac{K_{n,t}}{F_{n,t}}\right)^{\frac{1-\theta_n}{1+\theta_n\omega_y}}}{\alpha}\right]^{\frac{1}{\theta_n-1}} \tag{42}$$

Imported Good

We assume that an importing firm can differentiate an imported good at no cost and then sell it to the households. Then, firm m nominal profits are given by

$$D_{f,t}(m) = \left[P_{f,t}(m)y_{f,t}(m)(1-\tau_{f,t}) - S_t P^*_{f,t}y_{f,t}(m)\right] \tag{43}$$

where S_t is the nominal exchange rate, $\tau_{f,t}$ is a tax for each imported good sold, and $P^*_{f,t}$ is the imported good international price, which is assumed exogenous. Similarly to the non-traded good producing firms, they can only update its price with probability α each period. Since $c_{f,t}(m) = y_{f,t}(m)$, replacing equation (29) into equation (43), firm m maximizing problem becomes:

$$\begin{aligned} P_{f,t}(m)^* =& argmax \left[E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ P_{f,t}(m) \left(\frac{P_{f,t}(m)}{P_{f,T}}\right)^{-\theta_f} y_{f,T}(1-\tau_{f,T}) - \right.\right. \\ & \left.\left. S_t P^*_{f,t} \left(\frac{P_{f,t}(m)}{P_{f,t}}\right)^{-\theta_f} c_{f,t} \right\}\right] \end{aligned} \tag{44}$$

From this maximization problem we get the following fist order condition:

$$\left(\frac{P_{f,t}(m)^*}{P_{f,t}}\right) = \frac{K_{f,t}}{F_{f,t}} \tag{45}$$

Where

$$K_{f,t} = \Lambda_t \mu_f q_t P^*_{f,t} y_{f,t} + \alpha\beta E_t[K_{f,t+1}\Pi^{\theta_f}_{f,t+1}] \tag{46}$$

$$F_{f,t} = \Lambda_t \widetilde{P_{f,t}} y_{f,t}(1-\tau_{f,t}) + \alpha\beta E_t[F_{f,t+1}\Pi^{\theta_f - 1}_{f,t+1}] \tag{47}$$

Here $K_{f,t}$ is the discounted marginal cost, $\mu_f = \theta_f/(\theta_f - 1)$ is the firm's mark up, $\Pi_{f,t}$ is the imported good inflation, and $F_{f,t}$ is the discounted marginal revenue. Similarly to the non-traded good case, by replacing equation (45) into (30), and aggregating over m we get

$$\Pi_{f,t} = \left[\frac{1-(1-\alpha)\left(\frac{K_{f,t}}{F_{f,t}}\right)^{1-\theta_f}}{\alpha}\right]^{\frac{1}{\theta_f - 1}} \tag{48}$$

Domestic Traded Good

Finally, we assume that the domestic traded good price is set abroad. Hence, the price that households pay for the domestic traded good can be described by

$$\widetilde{P_{x,t}} = q_t P^*_{x,t} \tag{49}$$

where $\widetilde{P_{x,t}} = P_{x,t}/P_t$ and $P^*_{x,t}$ is the domestic traded price payed at international markets, which is assumed exogenous. At the same time, we assume that the domestic economy is endowed with $y_{x,t}$ units of the domestic traded good, then domestic traded good firm profits is described by:

$$D_{x,t} = \widetilde{P_{x,t}} y_{x,t}(1 - \tau_{x,t}) \tag{50}$$

where $\tau_{x,t}$ is a tax charged by the government for every unit of domestic traded good produced.

2.2.3 *Aggregate Supply*

Given that the domestic non traded good is used by households, government expenditure and the financial intermediary in order to channel funds to borrowers, aggregate domestic non-traded good can be expressed as

$$y_{n,t} = c_{n,t} + g_t + \Xi_t b_t^{\eta} \tag{51}$$

At the same time, the domestic traded good is exported, so the the firms produce this to satisfy domestic and foreign demand given by

$$y_{x,t} = c_{x,t} + c_{x,t}^{*} \tag{52}$$

where $c_{x,t}^{*}$ corresponds to exports of the domestic traded good, which is assumed exogenous. Finally, firms import the foreign good just in order to satisfy domestic demand. This implies

$$y_{f,t} = c_{f,t} \tag{53}$$

Using equations (51), (52), and (53) we can characterize Gross Domestic Product as

$$gdp_t = \widetilde{P_{n,t}} y_{n,t} + \widetilde{P_{x,t}} y_{x,t} + \widetilde{P_{f,t}} y_{f,t} - q_t P_{f,t}^{*} y_{f,t} \tag{54}$$

2.3 *Public Sector*

Fiscal Policy

The government is assumed to purchase quantity g_t of the domestic non-traded good. This purchase is financed by taxes to each unit sold of the domestic traded good $\tau_{x,t}$, the domestic non-traded good $\tau_{n,t}$, the imported good $\tau_{f,t}$, and additionally, lump-sum taxes to households T_t^g and borrowing from households at $(1 + i_t^d)$. Hence, government's real budget constraint is given by

$$\widetilde{P_{n,t}} g_t = \widetilde{P_{x,t}} y_{x,t} \tau_{x,t} + \widetilde{P_{n,t}} y_{n,t} \tau_{n,t} + \widetilde{P_{f,t}} y_{f,t} \tau_{f,t} + b_t^g + \frac{T_t^g}{P_t} - b_{t-1}^g \frac{(1 + i_t^d)}{\Pi_t} \tag{55}$$

We assume that $\tau_{x,t}$, $\tau_{n,t}$, $\tau_{f,t}$, g_t, and b_t^g are exogenous shocks.

Monetary Policy

Following Curdia and Woodford (2010) we assume that monetary policy is represented by an augmented Taylor Rule that account for deviations of the credit spread from its steady state value besides deviations of inflation from its target value and gdp_t from its natural value. This means that the deposits interest rate is represented as

$$i_t^d = i_t^{n,d} + \phi_\pi \pi_t + \phi_y (gdp_t - gdp_t^n) - \phi_\omega (\omega_t - \omega_{ss}) \tag{56}$$

Here $i_t^{n,d}$ and gdp_t^n represents the natural deposit interest rate and gross domestic product in a economy without financial frictions and flexible price setting for the domestic non-traded and imported good. [5]

2.4 *Aggregate Private Indebtedness Dynamics*

We can see from equation (16) that the credit spread is a function of aggregate debt and, differently from the basic New Keynesian model, we not longer assume that in the aggregate this equates to zero. Then, it is important to characterize its dynamics in order to close the model. For this purpose, we should aggregate equation (5) over B. Then we have:

$$P_t b_t = \int_B B_t(i) di = -\int_B \{A_t(i) + R_t(i)\} di \tag{57}$$

The fist term is the beginning of period net financial wealth held by borrowers, and the second is their excess of consumption for period t given by:

$$R_t(i) = \int_0^1 W_t(j) h_t(i,j) dj + D_{n,t} + D_{x,t} + D_{f,t} + T_t^g - P_t c_t(i) \tag{58}$$

Borrowers Beginning of Period Net Financial Wealth is characterize by

$$\int_B A_t(i) di = \delta \left\{ -P_{t-1} b_{t-1}(1 + i_{t-1}^b) + \pi_b D_t^{int} \right\} + (1-\delta)\pi_b A_t \tag{59}$$

The first term relates the beginning of period financial wealth for those who did not change type and, hence, did not receive any insurance transfer. The second term correspond to those who did receive the transfer. Aggregating equation (4) over i we have that:

$$A_t = P_{t-1}(b_{t-1} + b_{t-1}^g)(1 + i_{t-1}^d) - P_{t-1} b_{t-1}(1 + i_{t-1}^b) + D_t^{int} \tag{60}$$

Using the definition of D_t^{int} from equation (11), and the uncovered interest parity condition from equation (11), and expressing in real terms, we have

$$\frac{A_t}{P_t} = \left(b_{t-1}^g - b_{t-1}^* q_{t-1} \right) \frac{(1 + i_t^d)}{\Pi_t} \tag{61}$$

Then, the beginning of period financial wealth is the debt held by the government less the debt from international financial markets. Aggregating the second term in equation (57) we have

$$\int_B R_t(i) di = \pi_b R_t^b = \pi_b \left\{ W_t^b + D_{n,t} + D_{x,t} + D_{f,t} + T_t^g - P_t c_t^b \right\} \tag{62}$$

where $W_t^b = \int_B \int_0^1 W_t(j) h_t(j) dj di$ is the nominal labor income for borrowers. For both types, we have that aggregating equation (35) over j and over each type, real labor income for each one is:

$$\int_\tau \int_0^1 \frac{W_t(j)}{P_t} h_t(j) dj di = w_t^\tau = \left(\frac{\mu_t^w \psi_\tau}{\lambda_t^\tau \bar{H}_t^\nu} \right) \left(\frac{y_{n,t}}{z_t} \right)^{1+\omega_y} \Delta_{n,t} \tag{63}$$

5 Following Curdia and Woodford (2009) financial frictions are taken out by setting Ξ_t and χ_t to their steady state values.

where $\Delta_{n,t}$ is the domestic traded good price dispersion. Following Curdia and Woodford (2009) appendix, $\Delta_{n,t}$ is expressed as

$$\begin{aligned}\Delta_{n,t} &= \int_0^1 \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\theta_n(1+\omega_y)} dj \\ &= \alpha\Delta_{n,t-1}\Pi_{n,t}^{\phi(1+\omega_y)} + (1-\alpha)\left(\frac{K_{n,t}}{F_{n,t}}\right)^{\frac{-\phi(1+\omega_y)}{1+\omega_y\theta}}\end{aligned} \tag{64}$$

Aggregating profit equations (36) and (43), using government budget constraint in equation (54), and expressing in real terms, we get from equation (62):

$$\begin{aligned}\frac{\pi_b R_t^b}{P_t} =& \pi_b \left\{ c_t^b + \pi_s(w_t^b - w_t^s) + gdp_t \right.\\ & \left. - \widetilde{P_{n,t}} g_t + b_t^g - b_{t-1}^g \frac{(1+i_t^d)}{\Pi_t} \right\}\end{aligned} \tag{65}$$

where w_t^s is the real labor income that corresponds to savers. Replacing equations (51), (52), and (53), that characterize aggregate supply, equations (22), (24), and (25) that determine the optimal allocation for the different consumption baskets, into equation (54) we can reexpress gdp_t as

$$gdp_t = c_t + \widetilde{P_{n,t}}(\Xi_t b_t^\eta + g_t) + \widetilde{P_{x,t}} c_{x,t}^* - q_t P_{f,t}^* y_{f,t} \tag{66}$$

where $c_t = \pi_b c_t^b + \pi_s c_t^s$. Finally, replacing equation (66) into (65), we get:

$$\begin{aligned}\frac{\pi_b R_t^b}{P_t} =& \pi_b\pi_s \left((c_t^b - c_t^s) - (w_t^b - w_t^s) \right) \\ & + \pi_b \left(b_t^g - b_{t-1}^g \frac{(1+i_{t-1}^d)}{\Pi_t} + Tb_t + \widetilde{P_{n,t}}\Xi_t b_t^\eta \right)\end{aligned} \tag{67}$$

Where,

$$Tb_t = \widetilde{P_{x,t}} c_{x,t}^* - q_t P_{f,t}^* y_{f,t} \tag{68}$$

We can further reexpress equation (67) by finding a relation between the trade balance Tb_t, and the dynamics of foreign debt. By defining aggregate saving similar to equation (57) we can express financial intermediaries resource constraint in equation (13) as:

$$q_t b_t^* = b_t^g - \frac{A_t}{P_t} - \pi_b \frac{R_t^b}{P_t} - \pi_s \frac{R_t^s}{P_t} + \widetilde{P_{n,t}}\Xi(b_t) + \chi_t b_t \tag{69}$$

Through a similar process to reach equation (67) we have that

$$\pi_b \frac{R_t^b}{P_t} + \pi_s \frac{R_t^s}{P_t} = b_t^g + Tb_t + \widetilde{P_{n,t}}\Xi_t b_t^\eta - b_{t-1}\frac{(1-i_{t-1}^d)}{\Pi_t} \tag{70}$$

Replacing (66) into (65) we have that

$$q_t b_t^* = q_{t-1} b_{t-1}^* \frac{(1-i_{t-1}^d)}{\Pi_t} + \chi_t b_t - Tb_t \tag{71}$$

Replacing equations (59), (61), (67), and (71) into (57) we have that the dynamics of private indebtedness are characterized by the following expression

$$
\begin{aligned}
b_t(1+\pi_b\omega_t) =& \delta\left[b_{t-1}\frac{(1-i^b_{t-1})}{\Pi_t} - \pi_b\frac{D^{int}_t}{P_t}\right] \\
&+\pi_b\left[\left(q_t b^*_t - \delta q_{t-1}b^*_{t-1}\frac{(1+i^d_{t-1})}{\Pi_t}\right) - \left(b^g_t - \delta b^g_{t-1}\frac{(1+i^d_{t-1})}{\Pi_t}\right)\right] \\
&\pi_b\pi_s\left[(c^b_t - c^s_t) - (w^b_t - w^s_t)\right]
\end{aligned}
\tag{72}
$$

I can be seen, that aggregate debt depends on the beginning-of period financial wealth held by borrowers, the change of the aggregate beginning-of period financial wealth (we can see from equation (61) that the middle term is proportional to the change of A_t/P_t from period $t-1$ to t), and the consumption-labor income difference from borrowers to savers.

3 BAYESIAN ESTIMATION

Since the purpose of this article is to determine the existence on any benefits to the Colombian economy from targeting credit spreads in the Taylor Rule described in equation (56), we need to set the model's parameters to reflect Colombian aggregate variable dynamics. In order to do this we use Bayesian estimation techniques.

3.1 *The Structural Parameters*

First, it is useful to list the model's parameters. This is is a model with 15 exogenous shock that, as described above, come from the following variables: $\bar{C}^b_t, \bar{C}^s_t, \bar{H}_t, \mu^w_t, i^{b*}_t, \widetilde{\Xi}_t, \chi_t, z_t, P^*_{f,t}, P^*_{x,t}, c^*_t, \tau_{n,t}, \tau_{f,t}, g_t, b^g_t$.[6] We assume that each one follows an independent AR(1) process; so this means that we would have 30 parameters related to the exogenous processes accounting for the autoregressive and the standard error parameters. Additionally, the model has 23 structural parameters which are listed below:

Table 1: Structural Parameters

1.	σ_b	2.	σ_s	3.	ν	4.	ψ_b	5.	ψ_s
6.	ψ	7.	δ	8.	π_b	9.	π_s	10 .	ψ_1
11.	η	12.	γ	13.	ρ_h	14.	γ_n	15 .	ρ_n
16.	θ_n	17.	θ_f	18.	ϕ	19.	α	20 .	β
		21.	ϕ_Π	22.	ϕ_y	23.	ϕ_ω		

Differently from most Real Business Cycle models, the steady state deposits interest rate no longer reflect the inverse of β. Now it has to account

6 $\tau_{x,t}$ was state as an exogenous shock. However, this has no aggregate effect because it acts as a government lump-sum tax. This can be seen from equations (5) and (50).

for the both types of interest rates. In this sense, we can solve β from equations (8) and (9), which would be calibrated to

$$\beta = \frac{\delta + 1 + \omega_{ss}(\delta + (1-\delta)\pi_b) - \sqrt{(\delta + 1 + \omega_{ss}(\delta + (1-\delta)\pi_b))^2 - 4\delta(1+\omega_{ss})}}{2 i^d_{ss}\delta(1+\omega_{ss})} \tag{73}$$

As it was stated above, ψ_b and ψ_s are calibrated in order to equate worked hours for both types in steady state. Since wages are the same for both types of households, we can see from equation (72) and (63) that equating worked hours is the same as equating real labor income for both types. This implies that

$$\frac{\psi_b}{\psi_s} = \frac{\lambda^b_{ss}}{\lambda^s_{ss}} = \Omega \tag{74}$$

If equation (8) is divided by equation by λ^s_{ss} we have that

$$\Omega = \frac{1 - i^d_{ss}\beta(\delta + (1-\delta)\pi_s)}{i^d_{ss}\beta(1-\delta)\pi_b} \tag{75}$$

Additionally, we can express ψ_s from equation (32) as

$$\psi_s = \psi[\pi_b \Omega^{\frac{1}{\nu}} + \pi_s]^{\nu} \tag{76}$$

If we set $\psi = 1$, by replacing (75) and (76) in equation (74) we wil have the calibration for ψ_b. Given that the probabilities for the drawn of the new type must add to one, we set $\pi_s = 1 - \pi_b$. Following Curdia and Woodford (2010) we assume that the average of the intertemporal elasticities of substitution for both types must equate the intertemporal elasticity of substitution of a conventional New Keynesian model, this means $\sigma = \pi_b\sigma_b + \pi_s\sigma_s$; it is also assumed that $\sigma_b/\sigma_s = 5$. We set $\sigma = 5$ following Hamann et al (2006). This implies that each parameter is calibrated as follows:

$$\sigma_s = \frac{\sigma}{1 + 4\pi_b}; \tag{77}$$

$$\sigma_b = 5\sigma_s; \tag{78}$$

Similar to Lopez et al (2009), the risk premium parameter ψ_1 is set to 0.000003. The reason is that incorporation of a risk premium is for technical reasons in order to avoid non stationary dynamics, and is suggested by Gertler et al (2007) to keep this parameter close to zero so it does not affect high-frequency dynamics. We set $\theta_n = \theta_f = 6$, and $\rho_n = \rho_h = 1$ as Lopez et al (2009) given these parameters weak identification. For the same reason we follow Hamann et al (2006) and set $\phi = 1.6$, and $\eta = 5$ following Curdia and Woodford (2010).

The parameter δ is set equal to 0.9; this value is lower than Curdia and Woodford (2010), but it does not conflict with credit spread steady state set at 1.0174 at quarterly frequency. We set $gdp_{ss} = 1$ in order to express the variables a ratios to Gross Domestic Product. It is assumed that purchase power parity holds in steady state, hence $q_{ss} = 1$. It is assumed that stead state gross inflation Π_{ss} is 1. We set $c_{ss} = 0.65$, $g_{ss} = 0.165$, $b_{ss} = 1.07$, $b^*_{ss} = 1.07$, and $i^d_{ss} = 1.01$ to be consistent Colombian aggregate quarterly frequency data and calibrate other variables steady states to be consistent with these values. [7] We set the parameter for the credit spread in the

7 For more details see Bejarano and Charry (2014) calibration appendix

Taylor Rule $\phi_\omega = 0$ for two reasons: first, since the establishment of inflation targeting Colombian Central bank objective focuses by law in stabilizing inflation and output. Second, assuming the central bank does not react to credit spreads serves us as a benchmark to study the effects over aggregate variables of changing the value of ϕ_ω. Finally, we assume that $\chi_{ss} = 0$ so the aggregate default probability does not affect the steady state of the model.

3.2 *Priors and Posteriors*

So far we have described that calibrations of some parameters in order to fulfill some assumptions or reflect Colombian economy steady state relationships. The remaining parameters have to be estimated in order to reflect Colombian aggregate data dynamics. Using Bayesian estimation techniques allows us to identify these parameters by making more stable a high non-linear optimization algorithm such as a likelihood estimation over the parameters of a DSGE model. The procedure, based on Bayes' law, involves updating our beliefs about the distribution of the parameters using the likelihood function of the data that comes from the log-linear state representation of the model through a Kalman Filter. This updating becomes the posterior distribution of the parameters. By Bayes' Law have that:

$$P\left(\theta|Y^T, A\right) = \frac{P\left(Y^T|\theta, A\right) P\left(\theta|A\right)}{P\left(Y^T|A\right)} \tag{79}$$

Where A stands for the model, θ stands for the model parameters, and Y^T stands for the data sample considered. Here $P\left(\theta|A\right)$ is the prior distribution of the structural parameters, $P\left(Y^T|\theta, A\right)$ is the likelihood function of the data conditional on the parameters, $P\left(\theta|Y^T, A\right)$ is the posterior distribution of the structural parameters given the data and the model, and $P\left(Y^T|A\right)$ is the marginal density of the data conditional on the model. Since this last expression is a constant for any parameter value, we can express the posterior Kernel (or un-normalized posterior distribution) as:

$$P\left(\theta|Y^T, A\right) \propto P\left(Y^T|\theta, A\right) P\left(\theta|A\right) \equiv K\left(\theta|Y^T, A\right) \tag{80}$$

Since this expression may not have a known distribution, computational techniques are required to find the mode and use a sampling algorithm around the mode to get an empirical distribution. The idea is to use a sampling algorithm called Metropolis-Hastings algorithm which involves the following steps:

1. Use a numerical optimization routine to find the posterior mode of the logarithm of the posterior distribution and compute the inverse of the Hessian Σ at the mode.

2. Draw a proposal θ^* from a jumping distribution

$$J\left(\theta^*|\theta^{t-1}\right) = N\left(\theta^{t-1}, c\Sigma\right) \tag{81}$$

3. Compute the acceptance ratio:

$$r = \frac{P\left(\theta^*|Y^T, A\right)}{P\left(\theta^{t-1}|Y^T, A\right)} \tag{82}$$

4. Accept or discard the proposal θ^* according to the following rule:

$$\theta^t = \begin{cases} \theta^* & \text{with probability } min(r,1), \\ \theta^{t-1} & \text{with probability } r-1. \end{cases} \quad (83)$$

Following this procedure we can obtain a sample of the posterior distribution which can be used to construct the first and second moments of the structural parameters distribution. Then, the mean of the posterior distribution would be used as the parameter values in the this model, so optimal monetary policy analysis can be done in the section below.

The data used for this estimation consists on quarterly data on Gross Domestic Product, consumption, inflation, deposit interest rate, deposits and credits interest rate spread, debt, and wages for the period 2001:1-2014:2. The sources of these datasets are Banco de la Republica and Departamento Administrativo Nacional de Estadistica (DANE). The inflation is the percentage change on CPI index; the credit spread is the ratio from average gross interest rate from fixed term deposits (DTF) and the average gross interest rate on loans; debt are the loans held by the financial institutions; wages are calculated by DANE for the manufacturing industry. GDP, consumption, and debt are divided by the labor force to express them in per capita terms. All series are deseasonalized using a X12 arima algorithm, then expressed in logarithms and detrended using a one-sided HP-filter as Stock and Watson(1999).

The priors and posterior distributions are described in the table 2. For all the autoregressive parameters a Beta distribution prior was chosen with mean 0.5; for all the standard errors a Inverse Gamma distribution was chosen with mean 0.02 and infinite standard deviation. The Calvo parameter α has a Beta prior with range $(0, 1)$ with mean 0.5; the Taylor Rule coefficients for inflation and Gross Domestic Product have a Gaussian prior with mean 1.5 and 0.125, respectively, following Hamann et al (2006). The prior for the inverse Frisch elasticity ν is a Gaussian with mean 0.5, according to Gonzalez et a(2013). The share prior of borrowers π_b was set as a Beta distribution with mean 0.5 and range $(0, 1)$ following Cuardia and Woodford (2010). The parameters controlling for the share of consumption on the domestic goods and the domestic non-traded good, γ and γ_n, are set with beta distribution priors, mean 0.75, and range $(0.5, 1)$. [8] Finally, we set a measurement error to the wage series following Pfeifer (2014). Its prior distribution is a Inverse Gamma with mean set to 10% of the original series standard deviation.

The estimated parameters are based on two Metropolis-Hastings samples of 100,000, burning the first 50,000 on each one. It seems that the data set is informative over most parameters since the estimated standard deviation is lower to the one assumed in the priors. Similarly to Lopez et al (2009) the Calvo parameter mean is estimated around 0.4. The Taylor Rule coefficients were estimated at 1.45 and 0.4, respectively, which implies a stronger response to output from what was first assume. ν is estimated to 0.34 which is in line with Prada and Rojas (2009). The share of borrowers was estimated at 0.35. This means that to the dataset it is not longer consistent an economy where half the population has negative financial wealth. Estimated values for γ and γ_n are over 0.75. This shows a strong inclination for consumption of domestic and domestic non-traded good for the Colombian economy. Finally, most exogenous shocks parameters do not seem to be strong since the autoregressive parameters are not estimated far over 0.5 and many estimated standard deviations go below the initial 0.02.

8 This range was chosen due that values below 0.5 drive indeterminacy

Table 2: Priors and Posteriors

		Prior			Posterior			
Parameter	Distribution	Mean	S.D	Range	Mean	S.D	5%	95%
α	Beta	0.500	0.100	$(0,1)$	0.4288	0.0774	0.3066	0.5487
ϕ_Π	Gaussian	1.500	0.100	(∞,∞)	1.4493	0.0880	1.3115	1.5914
ϕ_y	Gaussian	0.125	0.100	(∞,∞)	0.3994	0.0668	0.2831	0.5123
ν	Gaussian	0.500	0.100	(∞,∞)	0.3447	0.0622	0.2390	0.4506
π_b	Beta	0.500	0.100	$(0,1)$	0.3534	0.0506	0.2732	0.4382
γ_n	Beta	0.750	0.100	$(0.5,1)$	0.7756	0.0613	0.6756	0.8806
γ	Beta	0.750	0.100	$(0.5,1)$	0.7829	0.0543	0.7033	0.8643
$\rho_{i^{b*}}$	Beta	0.500	0.100	$(0,1)$	0.5274	0.0773	0.3848	0.6712
ρ_χ	Beta	0.500	0.100	$(0,1)$	0.7600	0.0461	0.6747	0.8431
ρ_Ξ	Beta	0.500	0.100	$(0,1)$	0.5044	0.1066	0.3374	0.6596
$\rho_{P_x^*}$	Beta	0.500	0.100	$(0,1)$	0.5710	0.1075	0.4109	0.7287
$\rho_{P_f^*}$	Beta	0.500	0.100	$(0,1)$	0.5349	0.1092	0.3701	0.7060
$\rho_{c_x^*}$	Beta	0.500	0.100	$(0,1)$	0.5123	0.0716	0.3962	0.6367
ρ_{μ^w}	Beta	0.500	0.100	$(0,1)$	0.5565	0.0920	0.4127	0.7101
$\rho_{\bar{H}}$	Beta	0.500	0.100	$(0,1)$	0.5182	0.1034	0.3461	0.6778
ρ_z	Beta	0.500	0.100	$(0,1)$	0.5359	0.0512	0.4453	0.6272
$\rho_{\bar{C}^b}$	Beta	0.500	0.100	$(0,1)$	0.7683	0.0417	0.6988	0.8383
$\rho_{\bar{C}^s}$	Beta	0.500	0.100	$(0,1)$	0.5673	0.0591	0.4638	0.6654
ρ_{b^g}	Beta	0.500	0.100	$(0,1)$	0.5207	0.1063	0.3572	0.6883
ρ_g	Beta	0.500	0.100	$(0,1)$	0.5145	0.0977	0.3662	0.6607
ρ_{τ_n}	Beta	0.500	0.100	$(0,1)$	0.4954	0.1060	0.3321	0.6568
ρ_{τ_f}	Beta	0.500	0.100	$(0,1)$	0.5280	0.1042	0.3731	0.6879
$\sigma_{i^{b*}}$	Inv.Gamma	0.500	∞	$(0,\infty)$	0.0049	0.0008	0.0032	0.0063
σ_χ	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0028	0.0003	0.0024	0.0032
σ_Ξ	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0150	0.0035	0.0047	0.0268
$\sigma_{P_x^*}$	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0096	0.0021	0.0041	0.0150
$\sigma_{P_f^*}$	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0101	0.0026	0.0052	0.0152
$\sigma_{c_x^*}$	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0741	0.0144	0.0475	0.1026
σ_{μ^w}	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0064	0.0009	0.0047	0.0080
$\sigma_{\bar{H}}$	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0097	0.0025	0.0048	0.0140
σ_z	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0129	0.0016	0.0101	0.0154
$\sigma_{\bar{C}^b}$	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0835	0.0119	0.0609	0.1067
$\sigma_{\bar{C}^s}$	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0284	0.0041	0.0216	0.0350
σ_{b^g}	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0437	0.0074	0.0301	0.0569
σ_g	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0081	0.0018	0.0048	0.0114
σ_{τ_n}	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0130	0.0036	0.0054	0.0217
σ_{τ_f}	Inv.Gamma	0.020	∞	$(0,\infty)$	0.0116	0.0029	0.0049	0.0191
σ_{wages}	Inv.Gamma	0.001	∞	$(0,\infty)$	0.0008	0.0001	0.0003	0.0015

4 OPTIMAL MONETARY POLICY

Given the estimated parameters of the previews section, now we can identify the optimal response from the central bank to credit spreads movements and the benefits within. In order to do this, we follow Bejarano and Charry (2014) and Schmitt-Grohé and Uribe (2004a) were a Welfare function approximated up to second order, following Schmitt-Grohé and Uribe (2004b), is maximized over the parameter ϕ_ω in the Taylor Rule given by equation (56). Then, we compute the welfare loss associated on using the reaction to credit spreads in the benchmark model, $\phi_\omega = 0$, instead of the one that maximizes the welfare function.

4.1 *The Aggregate Welfare Function*

Considering the model described in 2th section, Optimal Monetary Policy problem consists in maximizing the following Welfare function over ϕ^*_ω:

$$\begin{aligned}\phi^*_\omega = & argmax \Bigg\{ W_t = \pi_b u(c^b_t; \bar{C}_t^{\,b}) + \pi_s u(c^s_t; \bar{C}_t^{\,s}) \\ & - \frac{\phi}{1+\nu}\left(\frac{y_{n,t}}{z_t}\right)^{1+\omega_y}\left(\frac{\widetilde{\Lambda}_t}{\widetilde{\lambda}_t}\right)^{\frac{1+\nu}{\nu}}\frac{\Delta_t}{\bar{H}_t^{\,\nu}} + \beta E_t W_{t+1}\Bigg\}\end{aligned} \tag{84}$$

Where

$$\widetilde{\Lambda}_t^{\frac{1+\nu}{\nu}} = \psi^{\frac{1}{\nu}}\left(\pi_b \psi_b^{-\frac{1}{\nu}} \lambda_t^{b\;\frac{1+\nu}{\nu}} + \pi_s \psi_s^{-\frac{1}{\nu}} \lambda_t^{s\;\frac{1+\nu}{\nu}}\right) \tag{85}$$

The welfare function comes from aggregating equation (1) over i. Since we are maximizing the welfare equation approximated to second order, it is not possible to find a reduced form solution. However, it is possible to set a discrete grid for ϕ_ω and evaluate equation (84) in each grid point, choosing the grid point that gives the highest value. For this exercise we choose grid points with length of 0.05.

Additionally, we need to compute welfare loss associated with using an different reaction to credit spreads in the Taylor rule instead of ϕ^*_ω. To do this, let W^*_t be the welfare level implied by ϕ^*_ω and W^a_t the welfare implied by any other alternative $\phi^a_\omega \neq \phi^*_\omega$. Welfare loss is measured as the percentage of consumption, $\Gamma_{\phi^{*a}_\omega}$, sacrificed by consumers associated to ϕ^*_ω in order to obtain W^a_t. This implies that

$$W^*_t[c^{b*}_t, c^{s*}_t] \geq W^{*a}_t[(1-\Gamma_{\phi^{*a}_\omega})c^{b*}_t, (1-\Gamma_{\phi^{*a}_\omega})c^{s*}_t] = W^a_t \tag{86}$$

Once the welfare values W^*_t and W^a_t are computed, $\Gamma_{\phi^{*a}_\omega}$ can be solved from equation (86).

4.2 *The Optimal Response to Credit Spread and Welfare Loss*

We compute the optimal response to credit spreads ϕ^*_ω taking into account all the exogenous shocks in order to reflect all possible disturbances Colombian economy can face according to our model. We can see from table 3 that for this case $\phi^*_\omega = -0.6$. This can be interpreted as the average optimal response to all financial and non financial disturbances. This result was not computed by Curdia and Woodford (2010) or Bejarano and Charry (2014).

Table 3: Optimal Response and Welfare Loss on Individual Shocks

Shock	ϕ_ω^*	$100x\Gamma_{\phi_\omega=0.6}$	$100x\Gamma_{\phi_\omega=0}$	$S.D_\omega$
ALL	-0.60	0%	28.787e-3%	4.2938e-3
χ_t	-2.15	61.581e-3%	102.07e-3%	3.6955e-3
$\bar{C}_t^b$	0.40	44.202e-3%	6.0920e-3%	1.5930e-3
b_t^g	-4.15	8.2472e-3%	9.5010e-3%	996.22e-6
z_t	-1.20	3.4382e-3%	12.326e-3%	716.29e-6
$\bar{C}_t^s$	-0.80	263.97e-6%	3.9026e-3%	659.36e-6
i_t^{b*}	-1.95	6.3032e-3%	12.070e-3%	635.29e-6
$P_{x,t}^*$	6.50	7.4369e-3%	6.3444e-3%	325.95e-6
$\tau_{f,t}$	-0.15	107.29e-6%	9.7918e-6%	307.69e-6
c_t^*	-2.40	10.052e-3%	17.438e-3%	285.38e-6
$\tilde{\Xi}_t$	-4.80	698.54e-6%	776.15e-6%	256.20e-6
$P_{f,t}^*$	5.20	960.42e-6%	779.10e-6%	207.22e-6
g_t	-1.30	158.77e-6%	505.62e-6%	137.30e-6
$\bar{H}_t$	-1.10	51.903e-6%	240.42e-6%	112.81e-6
μ_t^w	-1.25	24.380e-6%	82.161e-6%	56.939e-6

However, this result is important as it reflects a general recommendation for monetary policy that can be tractable and easy to understand; the same way that it is usually recommended that $\phi_\Pi > 1$ in order to assure inflation stability. For this recommendation to be implementable, we could expect, at least, that the welfare gains associated to this recommendation are higher than those implied by $\phi_\omega = 0$ for each shock.

Table 3 shows the optimal response to credit spreads to each individual shock, as the welfare loss with respect to the recommended reaction $\phi_\omega = -0.6$ and benchmark reaction $\phi_\omega = 0$. Almost all optimal reaction to credit spreads are negative, except for international prices for imports and exports, and borrowers preference shock. It is immediately clear that for shocks that imply $\phi_\omega^* \geq 0$, the economy is better off with the benchmark reaction $\phi_\omega = 0$, than with the recommended reaction $\phi_\omega = -0.6$, since $\Gamma_{\phi_\omega=0.6} \geq \Gamma_{\phi_\omega=0}$.

If, for example, we assume a shock to borrowers preferences, $\bar{C}_t^b$, we can see from the impulse response in figure 1 that borrowers are inclined to take more loans from financial intermediaries and, hence, face a higher credit spread. The optimal response for this shock would imply a positive reaction to the increase of the credit spread in order to rise borrowers interest rate i_t^b, so that output and inflation do not fall far from target values. However, the recommended reaction $\phi_\omega = -0.6$ would lower borrowers interest rate and increase the deviation of output and inflation from target values, implying a higher deposits interest rate to control these, after all. Finally, we can see that the benchmark reaction $\phi_\omega = 0$ falls somewhere in the middle; justifying less welfare loss in this scenario.

We can see, also, from table 3, that the welfare loss from implementing $\phi_\omega = 0$, instead of the recommended value is 0.028%. This means that households would sacrifice this consumption percentage in order to obtain the same level of welfare as the benchmark scenario. In order to see how this welfare loss is reflected in aggregate variable dynamics, we conduct a counterfactual analysis, where several historic data series are compared to simulated data where monetary policy is conducted following the recommended $\phi_\omega^* = -0.6$.

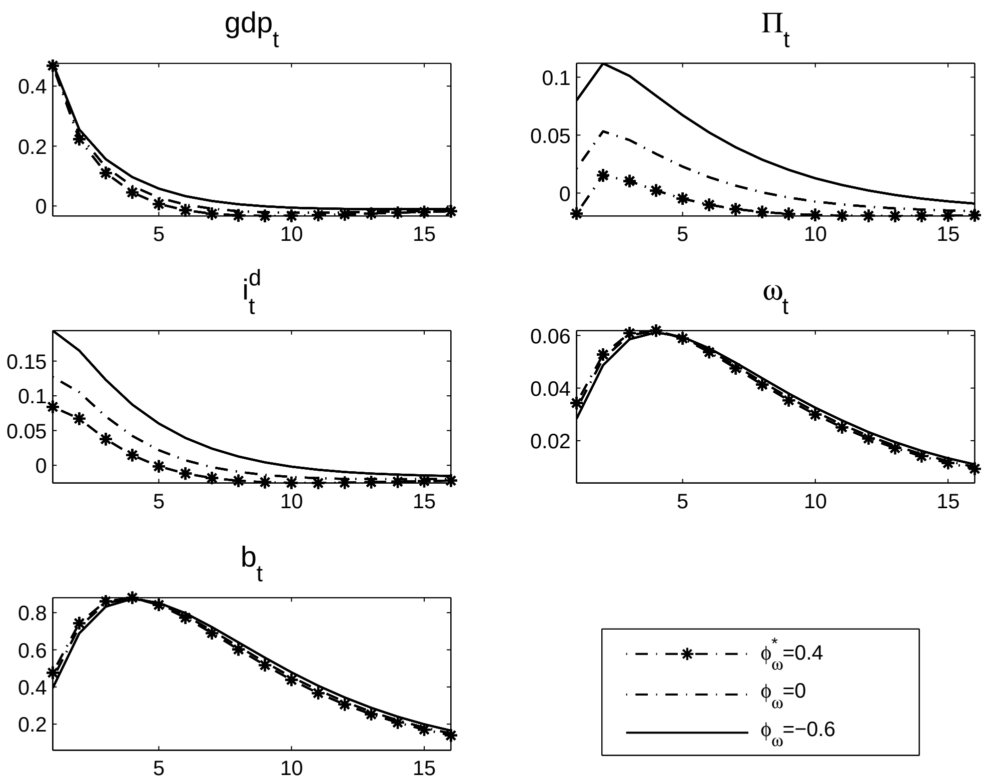

Figure 1: Impulse Response to shock $\bar{C}_t{}^b$ using estimated parameters $\rho_{\bar{C}_t{}^b}$ and $\sigma_{\bar{C}_t{}^b}$. Result expressed as percentage points deviations from steady state.

Figure 2 shows us that implementing the recommended reaction to credit spreads does not imply significant changes to aggregate variables dynamics. The only simulated variables that show a different dynamic to its historic counterpart are the deposit interest rate and inflation (although it is less pronounced).The deposit interest rate shows a positive comovement with the credit spread, which at first glance might seem counterintuitive. As Curdia and Woodford (2010) explains, although the recommended reaction implies a negative reaction to credit spread, it does not mean that the interest rate will actually fall to increases of the credit spread. Near 2010 credit spreads fell around 0.03% below trend, the recommended reaction would imply a rise in the deposit interest rate to discourage a rise of loans; however, this would result in less pressure on price and a lower inflation, which would imply a drop of the deposit interest rate.

So far we have learned that a general recommendation does not imply considerable welfare gains to all shocks compared to the benchmark reaction $\phi_\omega = 0$; as is the case for a borrowers preferences shock. Additionally, the fact that the implementation of the recommended reaction to credit spreads does not translate into considerable changes in macroeconomics variables might explain why welfare loss is not that significant between $\phi_\omega = -0.6$ and $\phi_\omega = 0$. These are an important result as they imply a weak benefit from implementing a general response to credit spreads. It is possible that considerable benefits exist not to a global reaction to all shocks, but to an individual shock once the source of the disturbance is identified.

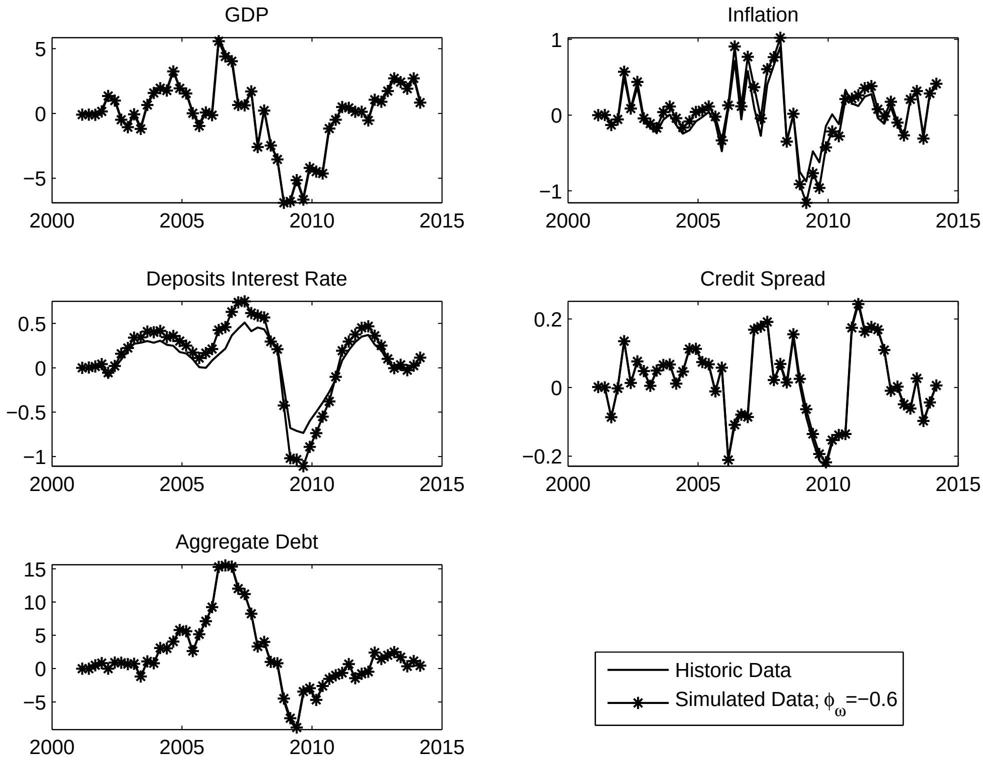

Figure 2: Counterfactual analysis between historic data and simulated data assuming $\phi_\omega = -0.6$. Result expressed as percentage points deviations from trend.

We see from table 3, that must of the optimal reactions are strong compared to the recommended $\phi^*_\omega = -0.6$; only the optimal reaction to credit spreads for both types preference shocks and imports taxes shocks are in the $(-1, 1)$ range. However, these differences in the strength of the reaction to the credit spread do not translate to significant welfare loss from setting $\phi_\omega = 0$. The biggest welfare loss occurs to the aggregate default probability shock, χ_t. In this case, household would sacrifice 0.102% of their consumption if $\phi_\omega = 0$ was set instead of $\phi^*_\omega = -2.15$. A similar logic follows to $\phi_\omega = -0.6$, the biggest welfare loss is associated to the aggregate default probability χ_t, being 0.061%. This means that households would sacrifice less that 0.1% of their consumption from a reaction to spreads of $\phi_\omega = -0.6$, instead of $\phi^*_\omega = -2.15$, when an aggregate default probability shock occurs.

The reason why most optimal responses to individual shocks are outside the $(-1, 1)$ range can be found in Curdia and Woodford(2010). They found that the strength of the response tends to be grater the lower the persistence of the credit spread is. Curdia and Woodford(2010) found an optimal response of $\phi^*_\omega = -0.66$ for the aggregate default probability shock, χ_t, with an autoregressive parameter $\rho_\chi = 0.9$ and standard error $\sigma_\chi = 0.04$, while our estimated parameters for this shock were 0.76 and 0.0028, respectively. If we calibrate the same shock to our model, the optimal response implied would be of -1.15, which is almost half the response to the estimated shock. However, if we keep the persistence the same and only change the standard error the optimal response would be the same.

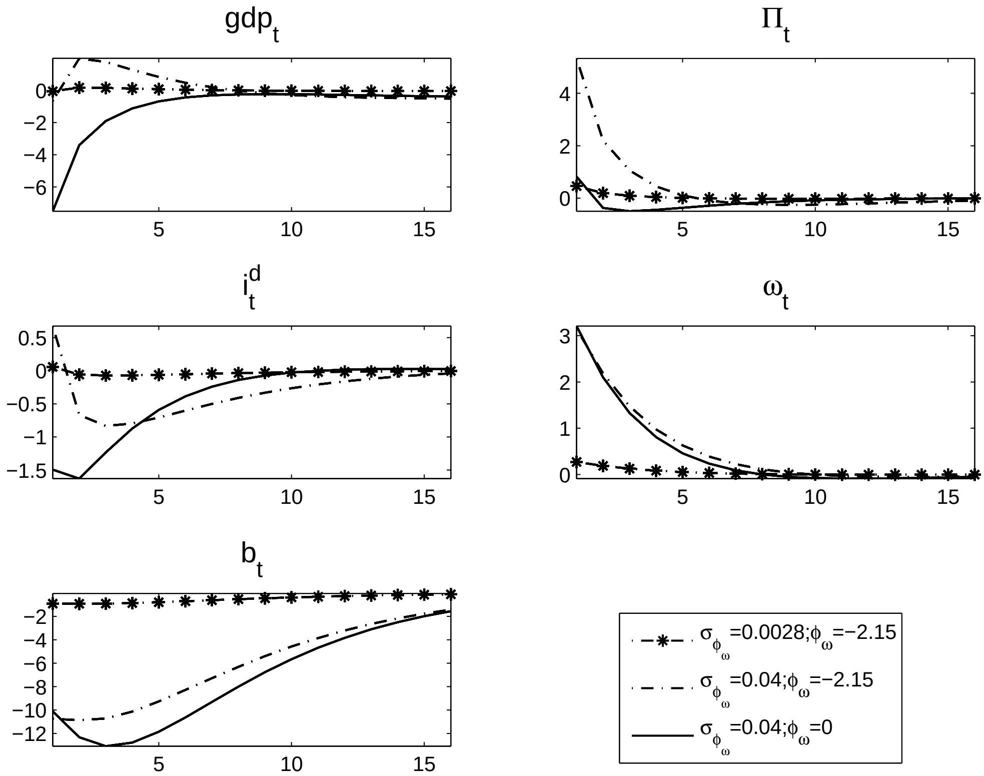

Figure 3: Impulse Response to shock χ_t comparing estimated parameters and Curdia and Woodford's (2010) σ_χ. Result expressed as percentage points deviations from steady state.

Table 3 also provides an interesting fact about this model, the fifth column shows the standard deviation of the credit spread if $\phi_\omega = 0$ was assumed. There is a positive relation between the standard deviation and the welfare loss compared to both, $\phi_\omega = -0.6$ and $\phi_\omega = 0$. These standard deviations are not far from the standard deviation of credit spreads implied from historic data on figure 2, which is 1.1384e-3. The sample period selected for the estimation was chosen to reflect a monetary policy compatible with the Taylor rule, since for the new millennium Target inflation criteria was adopted, but is not known to have a financial crisis. For times without financial turmoil credit spreads do not tend to be volatile, but periods for with financial stress this spread rises significantly. Chari et al (2008) show evidence of this fact for US economy during the 2008 financial crisis. They analyzed several measures of credit spreads and showed that these were a stable measure until Lehman Brothers bankruptcy. Specially for the Tbill and Libor rate spread; this measure was a steady measure of 0.5%, but the following month after Lehman Brother collapsed it increased up to 4.5%.

Figure 3 compares the impulse response to a shock to , χ_t, changing its standard deviation from 0.0028 to 0.4 as Curdia and Woodford (2010), but keeping the autoregressive parameter fixed at 0.76. It is clear that credit spreads reacts significantly more in the latter case; as well as the other variables. This figure also compares the optimal reaction to credit spreads to the benchmark. If monetary authority reacted optimally at $\phi^*_\omega = -2.15$ output would rise instead of falling, it would imply a higher inflation, deposit interest rate and debt. It seems that, at least for the standard deviation of this

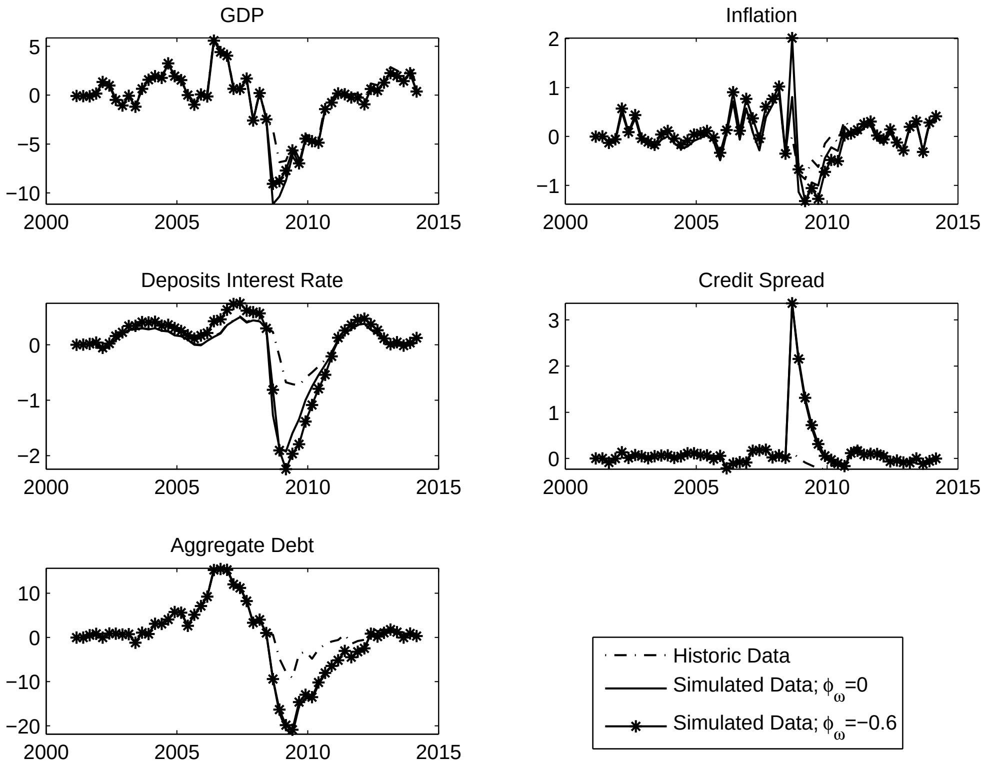

Figure 4: Counterfactual analysis between historic data and simulated data assuming $\sigma_\chi = 0.4$ in 2008:II. Result expressed as percentage points deviations from trend.

shock, the optimal reaction would imply a significant change to macroeconomic variables. Additionally,it also implies an increase in welfare loss. This scenario would imply welfare loss of $\Gamma_{\phi_{\omega=0}} = 11.9\%$ and $\Gamma_{\phi_{\omega=0.6}} = 7.38\%$, which are 1,166 and 1,209 times bigger, respectively, compared to the estimated standard deviation.

Finally, historic data is compared to simulated data assuming that in September 2008 the aggregate default probability standard deviation increased to 0.04, as was set on figure 3. Is an example were is simulated a scenario where the 2008 financial crisis were transmitted to the Colombian economy. The results are shown in figure 4. This figure also compares the general recommendation $\phi_\omega = -0.6$ to $\phi_\omega = 0$. As in figure 3, now the increase of credit spreads has a significant impact over the economy with respect to historic data. The implementation of an optimal response would avoid output from falling beyond 10% below trend, with a less significant effect over inflation. Additionally, It would not affect significantly debt dynamics. This figure also shows that the fall of the deposit interest rate would be similar to both reactions, but if $\phi_\omega = 0$ was assumed, it would rise earlier. The overall conclusion from these last two figures is that a reaction is recommended whenever credit spreads increase its volatility, as it would indicate a financial disruption.

5 CONCLUSIONS

After estimating Bejarano and Charry (2014) model's structural parameters the optimal response to credit spreads to all disturbances were computed. First, it was analyzed if a general reaction to all disturbances interacting simultaneously could improve monetary policy with respect to a non responsive scenario. Unfortunately, for some disturbances the Colombian economy is better off if the central bank does not react at all. Additionally, this general response does not imply significant welfare gains, since the impact over macroeconomic variables is null.

Then, each shock was analyzed individually. Similar to Curdia and Woodford (2010) the optimal response depended on the source of the disturbance and on its persistence. Although many of these optimal reactions were outside the $(-1, 1)$ range, similarly to the general recommendation, they did not imply significant welfare gains or responses from output and inflation.

The main explanation to the lack of responsiveness is that the credit spread is not volatile for the sample considered for the estimation. The sample from 2001:I-2014:II is not associated to a financial crisis period, although it serves as a good approximation of monetary policy conducted by a Taylor rule, since Inflation targeting was implemented after 2000. However, a counterfactual analysis, that compared the historic data to a simulated data where the 2008 financial crisis spreads to the Colombian economy, showed that after a increase in credit spreads volatility the Colombian economy could obtain considerable gains and stabilized output from reacting to this variable.

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