# Optimal Monetary policy with Informality: A Benchmark Framework

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# Introduction

- Developing countries are characterize by the presence of a large informal sector, where value-added activities avoid taxation and remain hardly registered by official statistics

-How does the presence of a large informal sector affects inflation dynamics and the optimal design of monetary policy?

- Surprisingly enough, very few papers have been devoted to the analysis of the monetary policy when the economy displays a so-called informal sector.

# Introduction

-In order to answer those questions we develop a simple New Keynesian model with two sectors. We extend the model in Galí (2008) by introducing an informal sector and variable taxes in the formal sector. The informal sector can avoid taxation

The size of the informal sector will be exogenously given by the proportion of the labor force in this sector.

Under this simple framework we are able to obtain analytical results about the effect of informality on monetary policy.

# Main results

- In economies with a large informal sector the cost-push on inflation is amplified .

- The sacrifice ratio increases with the weight of the informal sector.

- The inflation bias under discretion is increasing with informality

-The optimal trade-off between inflation and output gap increases with the size of the informal sector.

# Model

The analysis builds in a New-Keynesian closed economy framework with two sectors: formal and informal.

***The representative Household***

We assume a representative household that maximizes

$$U_t = E_0 \sum_{t=1}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - N_F \frac{{h_{F,t}}^{1+\eta}}{1+\eta} - (1 - N_F) \frac{{h_{I,t}}^{1+\eta}}{1+\eta} \right]$$

subject to the budget constraint

$$(1 - \tau_t^w) \, w_{F,t} N_F h_{F,t} + w_{I,t} (1 - N_F) \, h_{I,t} + \frac{(1 + i_{t-1})}{1 + \pi_t} b_{t-1} + T = c_t + b_t$$

# Model: Households

The first order conditions are given by:

$$(c_t)^{-\sigma} = \beta E_t \frac{(1+i_t)}{(1+\pi_{t+1})} (c_{t+1})^{-\sigma}$$

$$h_{F,t}^{\eta} = (c_t)^{-\sigma} (1-\tau_t^w) \, w_{F,t}$$

$$h_{I,t}^{\eta} = (c_t)^{-\sigma} \, w_{I,t}$$

# Model: Wholesale Firms

***Wholesale firms***

$$Y_{S,t} = N_S A_{S,t} h_{S,t} \quad S = F, I$$

where $A_{S,t}$ is an aggregate productivity shock common to all firms in sector $S$. $h_{S,t}$ are the working hours.

In a competitive environment, the maximization of profits implies prices equal to the marginal cost, this is:

$$P_t^Y = \frac{w_{S,t}}{A_{S,t}} = \phi_{S,t}$$

By the law of one price we have:

$$\phi_{F,t} = \phi_{I,t}$$

# Model: Retail Firms

***Retail firms*** purchase the wholesale output and transform it into differentiated final goods, $Y_t$,. We assume that each firm can reset its price with probability $(1-\omega)$ in any given period. A firm reoptimizing in period $t$ will solve the following problem

$$\max_{P_t^*} \quad E_t \sum_{j=0}^{\infty} \Gamma_{t,t+j} \omega^j \left[ P_t^* Y_{t+j/t} - P_{t+j} \phi_{t+j}^m Y_{t+j/t} \right]$$

subject to the sequence of demand constraints

$$Y_{t+j/t} = \left( \frac{P_t^*}{P_{t+j}} \right)^{-\theta} Y_{t+j}$$

The first order condition implies

$$\frac{P_t^*}{P_t} = \frac{\theta}{(\theta-1)} \frac{E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\sigma} \omega^j \left[ \phi_{t+j/t}^m \left( \frac{P_t}{P_{t+j}} \right)^{-\theta} \right]}{E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\sigma} \omega^j \left[ \left( \frac{P_t}{P_{t+j}} \right)^{1-\theta} \right]}$$

# Model: Government

Government always runs a balanced budget. Therefore, in each period Government budget constraint is as follows:

$$T + \tau^m \phi Y = \tau_t^w w_{F,t} N_F h_{F,t} + T^m$$

Tax rate varies over the business cycle in order to balance the constant subsidies and the fluctuating tax base.

$$\tau_t^w = \frac{T}{w_{F,t} N_F h_{F,t}}$$

It is certainly an extreme simplifying assumption, but it allows us to unveil some basic properties.

# Phillips curve, IS curve and sectoral integration condition

The standard New Keynesian model can be reduced to some log-linearized equations: the Phillips Curve, the IS relation, and the integration Condition.

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Upsilon \kappa_f \mathbb{X}_{F,t} + \Upsilon \kappa_I \mathbb{X}_{I,t} + \Upsilon c p_t$$

$$\mathbb{X}_t = -\frac{1}{\sigma}\left[\hat{i}_t - E_t\left[\hat{\pi}_{t+1}\right] - \hat{r}_t^e\right] + E_t\left[\mathbb{X}_{t+1}\right]$$

$$\mathbb{X}_{I,t} = \Omega_x \mathbb{X}_{F,t} - a_{x_I,t}$$

$$\hat{\mathbb{X}}_t = \Theta_m \hat{\mathbb{X}}_{f,t} + \left(1 - \Theta_m\right) \hat{\mathbb{X}}_{I,t} + a_{x,t}$$

Where $\mathbb{X}_{S,t} = \hat{Y}_{S,t} - \hat{Y}_{S,t}^e$ is the *welfare relevant* output gap in sector $S$, $\frac{\partial \Omega_x}{\partial N_f} > 0$, $\Theta_m = \frac{Y_F}{Y}$

# Phillips curve

In order to have a measure of the total sacrifice ratio we express the Phillips curve in terms the total welfare based output gap, $\hat{\mathbb{X}}_t$ .

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\Upsilon \left( \kappa_f + \kappa_I \Omega_x \right)}{\left( \Theta_m + \left( 1 - \Theta_m \right) \Omega_x \right)} \hat{\mathbb{X}}_t + \Upsilon CPT_t$$

The sacrifice ratio $SR = \left( \frac{\Upsilon(\kappa_f + \kappa_I \Omega_x)}{(\Theta_m + (1 - \Theta_m) \Omega_x)} \right)^{-1}$ and the cost push on inflation $CPT_t = \Gamma^{cp} \hat{A}_t$

$$\frac{\partial SR}{\partial N_I} > 0$$

$$\frac{\partial \Gamma_F^p}{\partial N_I} > 0$$

# Welfare Loss Function

Second-order Taylor approximation of the consumer's utility function around the steady state $(c,\ h_F,\ h_I)$ yields to the following welfare loss function $\mathbb{W}$

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{2} \left[ \frac{\theta}{\Upsilon} \pi_t^2 + (\eta + \sigma) \left( \frac{Y_f}{Y} \mathbb{X}_{F,t}^2 + \frac{Y_I}{Y} \mathbb{X}_{I,t}^2 \right) \right] - \frac{Y_f}{Y} \tau^w \mathbb{X}_{F,t} \right)$$

- Weight of inflation volatility is increasing on $\theta$, the elasticity of substitution among goods, and decreasing in the degree of price stickiness $\omega$

- The weight of the *welfare relevant* output gap volatility in the loss function is increasing in $\eta$ and $\sigma$ which determine the curvature of the utility function.

-The linear term $\frac{Y_f}{Y} \tau^w \mathbb{X}_{F,t}$ captures the fact that any marginal increase in output has a positive effect on welfare.

# Optimal monetary policy

We assume a Welfare-maximizing central bank. Under discretion, the central bank chooses $(\pi_t,\ \mathbb{X}_{F,t},\ \mathbb{X}_{I,t})$ in order to minimize the period losses:

$$\frac{1}{2}\left[\frac{\theta}{\Upsilon}\pi_t^2 + (\eta+\sigma)\left(\frac{Y_f}{Y}\mathbb{X}_{F,t}^2 + \frac{Y_I}{Y}\mathbb{X}_{I,t}^2\right)\right] - \frac{Y_f}{Y}\tau^w\mathbb{X}_{F,t}$$

Subject to the Phillips curve

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \Upsilon\kappa_f\mathbb{X}_{F,t} + \Upsilon\kappa_I\mathbb{X}_{I,t} + \Upsilon cp_t$$

and the sectoral integration condition

$$\mathbb{X}_{I,t} = \Omega_x\mathbb{X}_{F,t} - a_{x_I,t}$$

# Optimal monetary policy under discretion

-The first-order conditions implies:

$$(\eta+\sigma)\,\frac{\Theta_m+\Omega_x^2\,(1-\Theta_m)}{\Theta_m+\Omega_x\,(1-\Theta_m)}\hat{\mathbb{X}}_t=\mathit{\Delta}_t^{x\pi}-\theta\,(\kappa_f+\kappa_I\Omega_x)\,\hat{\pi}_t$$

- When the central bank puts some weight on stabilizing the output gap, it may have to accommodate adverse supply shocks through larger increases on inflation. How large will depend on the size of the sacrifice ratio and the weight of output gap in the welfare loss function.

- The optimal trade-off between inflation and output gap, $TO^{op}$, is given by:

$$TO^{op}=\frac{(\eta+\sigma)}{\theta\,(\kappa_f+\kappa_I\Omega_x)}\,\frac{\Theta_m+\Omega_x^2\,(1-\Theta_m)}{\Theta_m+\Omega_x\,(1-\Theta_m)}$$

$$\frac{\partial TO^{op}}{\partial N_I}>0$$

# Optimal Monetary Policy: inflation bias

$$(\eta+\sigma)\,\frac{\Theta_m+\Omega_x^2\left(1-\Theta_m\right)}{\Theta_m+\Omega_x\left(1-\Theta_m\right)}\hat{\mathbb{X}}_t=\mathit{\Delta}_t^{x\pi}-\theta\left(\kappa_f+\kappa_I\Omega_x\right)\hat{\pi}_t$$

the term $\mathit{\Delta}_t^{x\pi}$ represents the inflation bias that converge to $\Theta_m\tau^w$ in the steady state.

$$\mathit{\Delta}_t^{x\pi}=\Theta_m\tau^w+\Psi_{IB}\hat{A}_t$$

$$\frac{\partial\left(\Theta_m\tau^w\right)}{\partial N_I}>0$$

$$\frac{\partial\Psi_{IB}}{\partial N_I}>0$$

The presence of variable taxes generates an inflation bias that leads to a positive average inflation as a result of the central bank's incentive to drive output over its natural level.

# Conclusions

- Informality amplifies the cost-push shock on inflation

-The aggregate sacrifice ratio increases with the weight of the informal sector, what leads to recommend less inflation stability in economies with higher levels of informality

-When the central bank puts some weight on stabilizing the output gap, it may have to accommodate adverse supply shocks through larger increases on inflation. How large will depend on the size of the sacrifice ratio and the weight of output gap in the welfare loss function.

-The presence of variable taxes generates an inflation bias that leads to a positive average inflation as a result of the central bank's incentive to drive output over its natural level. That incentive increases with the size of the informal sector