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A NETWORK MODEL OF SYSTEMIC RISK: IDENTIFYING THE SOURCES OF DEPENDENCE ACROSS INSTITUTIONS

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Abstract

We design a financial network model that explicitly incorporates linkages across institutions through a direct contagion channel, as well as an indirect common exposure channel. In particular, common exposure is setup so as to link the financial to the real sector. The model is calibrated to balance sheet data on the colombian financial sector. Results indicate that commercial banks are the most systemically important financial institutions in the system. Whereas government owned institutions are the most vulnerable institutions in the system.

Keywords: systemic risk; network models; contagion; common exposure.
JEL Classification: C21,C58,G32.

1 Introduction

The 2007-2008 financial crisis has shifted the focus from the assessment of the resilience of individual financial institutions towards a more systemic approach. Hence, macro-prudential supervision and regulation will play a vital role in the new financial architecture, on Banking Supervison (2010). In particular, Basel 3 advocates financial regulation focused on limiting systemic risk. As illustrated by the crisis, an important aspect of systemic risk, which broadly speaking is the risk of a widespread crisis in the financial system, is the propagation of adverse shocks to a single institution through the rest of the system and the economy at large.

After the Great Depression, policymakers were faced with the daunting task of rethinking the way economic activity was measured. These innovations lead to the development of comprehensive set of national income accounts. Brunnermeier et al. (2011), argue that we now face a similar task, because during 2007-2008 financial crisis policymakers found themselves without the relevant information about the linkages within the financial sector as well as the linkages

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to the real economy.
The challenge of building this risk topography has received important methodological feedback from network and graph theory. This branch of mathematics has provided a set of definitions, tools, techniques that have been extensively applied to analyze financial networks(Newman (2010)). Within the field of financial network models, with applications to systemic risk, there is a branch of the literature preocupied with the more normative aspects. That is, identify the optimal topologies or mechanisms that mitigate the propagation of shocks (see Allen and Gale (2000); Allen and Babus (2008); Georg (2011); Amini et al. (2010); Becher et al. (2008)). One important result is that of Niera et al. (2007), they find that the degree of network connectivity is non-monotonic with respect to financial stability. In other words, even thought more connections across institutions increase the likelihood of contagion a suficient amount of connections can also increase the possibility of substitutability across existing institutions, this results is of course sensitive to the heterogeneity across the system with respect to the services that each institution provides.
A more applied branch of the same literature is focused on using data on counterparty financial exposures across and within countries to map existing networks, and in some cases analyze the robustness of financial systems using simulations (see Anand et al. (2009); Boss et al. (2003); Castrn and Kavonius (2009); Bastos and Cont (2010); Cajueiro and Tabak (2007); Degryse and Nguyen (2007); Chan-Lau (2010a,b); Espinosa-Vega and Sole (2010)). In a similar vein some financial network models have been developed in the Colombian Central Bank (see Estrada and Morales (2008); Machado et al. (2010); Leon et al. (2011)) but these have mainly focused on the interbank market liquidity and the topology of the large payments system that the Bank administers.
Although the literature has incorporated, in financial network models, different channels of contagion, including the use of market based data Elsinger et al. (2006), most of the applied literatures has focused, almost exclusively on the mapping the institutions that belong to the financial system. However, no attention has been payed to incorporate into such analysis, explicitly, the linkages to the real economy. In this paper we emphasize the importance of introducing the information of the real economy in financial network type models. The main reason to incorporate such information is to clarify the propagation mechanism of a shock that can have systemic consequences. The reason is that dependence across institutions can be a result of two underlying forces which unravel simultaneously: common exposure and a contagion channel. We build a financial network model that explicitly accounts for these two sources of dependence.
We design a financial network model based on balance sheet data from the Colombian financial system. Our model extends the financial network by explicitly accounting for linkages to sectors outside the financial system, such as the real and public sectors of the economy. This design allows us to incorporate two sources of dependence across financial institutions and systemic risk: common exposure and contagion.
The results indicate that commercial banks are the most systemically important financial institutions because they are heavily exposed to varios sectors,

and furthermore their failure leads to the failure of almost all other financial institutions. On the other hand, government owned institutions are overall the most vulnerable institutions in the system; vulnerable to shocks within the financial system and indirectly across the real sector. Whether this vulnerability is an intended part of their institutional design is a question that policymakers should ask themselves.
The remainder of the paper is organized as follows. Section 2 explains the nature of dependence that unravels during systemic events. In section 3 we explain the informational requirements through an example that looks at the case of the Colombian Financial sector. In section 4 we explain the mapping of counter parties and the common exposures and give a brief overview of descriptive statistics of the financial network for the Colombian financial system. Section 5 provides the network model of systemic risk and two exercises that simulate a pure credit shock and a credit and funding shock. Section 6 concludes.

2 Systemic risk: common exposure and contagion

Given the many sources and channels of systemic risk, there is no generally accepted definition for systemic risk or systemic importance. In some cases, a description of the "phenomenon" of systemic risk and its different dimensions rather than a succinct definition is given. More concise definitions of systemic risk can be found in e.g. Acharya and Richardson (2009) and IMF (2009), who define systemic risk as "the risk of a crisis in the financial sector and its spillovers to the economy at large" or "a risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy", respectively. Both of the definitions reflect the turmoil in financial markets during 2007 and 2008 and the consequence on the developed world economies. The definitions also emphasis the important relationship between the real and the financial sectors, although they implicitly focus mainly on the transmission of shock from the financial sector to the real sector, which is by many accounts a partial picture according to existing (pre-crises) literature.
The pre-crises literature is mainly focusses on the financial accelerator channel, which includes the borrower balance sheet channel and the bank balance sheet channel. The first of these channels, deals predominantly with how macroeconomic conditions affects the outlook of borrowers and the consequences of such shocks on the banks balance sheet. The second channel, looks at how do adverse shocks to the financial institutions balance sheets can entail sharp contractions in credit and its spill overs to the economy at large.[1] A further liquidity channel, has been identified recently (post-crises) literature, emphasizing the importance of the propagation of funding liquidity shock to financial sector and the market.

[1]For a recent survey on the transmission channels between the financial sector and the real sectors, see on Banking Supervison (2011).

The important gap in the pre-crises literature is the lack of feedback effects of a shock originating in the financial sector and its transmission to the real sector (or viceversa). As identified by the BIS working group "Macro stress testing considers the effects of real conditions on banks balance sheets but rarely accounts for the fact that such balance sheet developments have macroeconomic effects, which typically reinforce the effects of the initial macro shock".
Our contribution to the literature is closer to the bank balance sheet channel, since we are mainly interested in a model that is able to pick-up shocks originating in the real sector, using a financial network type model for systemic risk. We argue that the introduction of the real sector in a systemic risk model is not only important from the point view of the definition; but also for an adequate identification of the systemically important institutions or at least a clearer understanding of the propagation mechanism of a shock to the financial sector.
One important element that has emerged from the literature on measuring systemic risk and determining which are exactly the systemically important financial institutions is the challenge of properly accounting for the multiple interactions that seem to unravel in a systemic event. For example the ECB, has set its focus on three forms of systemic risk: a) contagion risk , b) the risk of macroeconomic shocks causing simultaneous problems at many financial institutions, and c) the risk of an abrupt unraveling of imbalances that have build up over time. We are interested mainly in the difficulties of observing simultaneously both a) and b). The dependence across the distress of multiple financial institutions is an important driver of both systemic risk and systemic importance, Castro and Ferrari (2010). This dependence structure is essentially determined by two underlying forces: common exposures and spillover channels.

The degree of **common exposures** of a financial institution determines to what extent the institutions' asset portfolios are vulnerable to similar risk factors. When financial institutions are to a large extent exposed to common risk factors, a systematic shock may adversely affect many institutions at the same time and pose a potentially large threat to the stability of the financial system. That is, common exposures increase the risk of a simultaneous failure or distress of a substantial number of financial institutions, and therefore the level of systemic risk. In addition common exposures may also strengthen the degree to which idiosyncratic shocks propagate to the system, for instance through the asset fire sales channel.
A second determinant of dependence of risk in the system is the presence of **spillover or contagion channels**; the larger the probability that (idiosyncratic) shocks propagate from one institution to the rest of the financial system, the larger is the potential level of systemic risk. That is, if adverse shocks easily spill over from one institution to the other institutions in the system, this again raises the probability that a substantial number of financial institutions fails at the same time. The literature has identified several direct and indirect channels through which spillover effects operate. The most obvious spillover channels are direct exposures between financial institutions through the interbank money market and counterparty relations (e.g. derivative markets, payment systems).

However, also indirect contagion channels exist, such as the adverse price effects on the asset portfolio of other financial institutions in the system in case of asset fire sales by a particular institution in distress. In addition, the failure of one institution may, due to imperfect and asymmetric information, lead to contagious bank runs in retail and wholesale (e.g. interbank) markets. Essentially, if there were no channels that directly or indirectly interconnect the different financial institutions in the system (and the financial institutions with the real economy), there would be no possibility for shocks to propagate through the system, thereby limiting the degree to which institutions can be expected to be systemically important.

Given these two main sources of dependence, unraveling simultaneously after some shock, it is vital that the measurement of a financial institution's systemic importance entails the separate identification of the spillover effect and the common exposure as drivers of the dependence in the system. In particular our identification strategy requires us to obtain information pertaining to these two channels. In order to control for common exposure we propose using the information of the portfolio of exposures on the extended financial sector, the public sector and the real sector, of the most important financial institutions to complement the balance sheet counterparty information that is regularly used to determine the direct linkages (our contaigion or spill over effect) in a financial network model. By introducing this new information into the setup, we expect to: a) Obtain a clearer picture of the propagation mechanism, and b) explicitly incorporate the relation between the financial and the real sector. Although, such relationship is important for identification purposes only, from the point of view of shocks originating from the real sector and its transmissions to the financial sector and not the other way around; which is more in line with the definition of systemic risk, that is shocks from the financial sector to real sector or feedback effects between both sectors.

3 The Colombian Financial sector

The colombian financial sector is mainly dominated by commercial banks and as such most firms (that have access to external finance) from the real sector get financed through the traditional channel of intermediation (the capital market is underdeveloped). Commercial financial corporations are mainly focused on leasing activities. Finally, participation of the public sector is still important in the financial system (although less than the period prior to the 1990), through government owned institutions which re-direct or subsidize credit to some economic sectors.

In the last three decades, Colombiaś financial sector has passed through different phases: of expansion and contraction, and, important changes in its business model.

The 1950s brought a strong state intervention in the financial system that lasted well into the end of the 1980s. The main purpose of state intervention was to direct resources to what the government deemed as key economic sectors. Dur-

Table 1: Financial institutions in the Colombian financial sector (December 2010)

Type of Institution	Number of Institutions	Assets ($ millions of pesos)
Commercial Banks	19	242.641.504
Investment Banks	3	7.165.373
Commercial Financial Corporation	23	16.873.147
Financial Cooperative	8	3.478.641
Government Owned	10	34.526.815
Pension Funds	14	99.163.670
Insurance	44	30.705.886
Fiduciaries and Mutual Funds	72	25.226.207

ing this period the central bank was the main agent of intervention through the use of different mechanisms: forced investment, percentage of credit directed to some sectors, obligation to hold liquid asset on central bank accounts, among others. The advent of the 1991 constitution brought first of all an independent central bank and a period of liberalization (a significant reduction of government lead financial repression) and a subsequent privatization of the financial sector. Although, government owned institutions (Table 1) are still today quite important. An important change in the business model of financial institutions, that took place thought the 90's, is that financial institution were no longer specialized. Early on the idea of directing credit to some sectors created the incentive to create specialized banks aligned with the needs of both the politically and economically important economic sectors (coffee growers association, cattle farmers, industrialist, mortgage banks, among others).

The recent financial history of Colombia is not free from boom-bust cycles. The early 80s (1982 − 1985) was a period of significant deterioration of the balance sheet of banks and commercial financial corporations as well as a sharp drop in earnings. The financial system instability during this period lead to the creation of a deposit insurance institution (FOGAFIN). Yet the most important financial crisis unwounded in 1998 and 1999, as a result of a deterioration of the housing market and its effect and subsequent collapse of the system of mortgage financing (UPAC) and the remnants of the specialized mortgage banks (CAV). The last decade has brought timid expansion in the financial sector[2]. Its important to note that most Latinamerican countries (including Colombia) financial depth (measured as the ratio of private sector credit to GDP) is on average 35%, far off from developed economies (150%) and even emerging countries in Asia (100%).

[2]For a complete account of the history of the financial sector in Colombia see Gonzalez and Garcia (2002).

4 Mapping conterparties and common exposures

4.1 The data

We will use counterparty data, from the institutions balance sheet (assets and liabilities), for the colombian financial sector for the year 2010. The unit of analysis is the type of institution not the institutions themselves. Although, not observing directly the information of the institution is a setback for this sort of exercise we can still map many bilateral exposures between institution types.

Table 2: Exposures by Institution Type (Example)

Type	Commercial Banks	Investment Banks
Commercial Banks	0	2
Investment Banks	4	0
Pension Funds	5	6
Manufacturing	10	2

Table 2 illustrates the type of data of the financial institutions. For example, if we are looking from the asset side of the balance sheet (investments, loans, derivative contracts) then Commercial Banks holds claims (loans) of 10 units on the Manufacturing sector. In this case, the sum of all elements in a given column correspond to the total assets held by Commercial Banks. On the other hand, if we are looking from the liability side of the balance sheet (deposits, outstanding debt, derivative contracts) then Investment Banks owes 6 units to Pension Funds. In this case, the sum of all elements in a given column correspond to the total liabilities held by Commercial Banks with other types of institutions (for simplicity we ignore assets and liabilities that are within the same type of institution). The full data (bi-directional positions or the columns in table 2) on the most relevant counterparty relationships between the institutions are only available for: commercial banks, investment banks, commercial financial corporations, financial cooperatives and government owned institutions (we denote this group the core system). Whereas for all other institutions (pension funds, insurance, fiduciaries, and mutual funds), we only observe those transactions generated on the asset or liability side of the first type of institutions (one directional positions). In particular, for the real sector we only have information on loans from the financial institutions to the different sectors. The information that we use to connect the financial counterparty data and the real sector, comes from a sample of the 5000 biggest private debtors of the financial system (organized by economic sector). This information gives us an idea of the distribution of the loan portfolio and the exposure of the most important financial institutions to the real sector.

4.2 The map of the Colombian Financial system

Once the bilateral and unilateral exposures have been calculated for the financial system, a network connecting all types of institutions can be constructed using the aggregate information on the asset side of the balance sheet (this includes investments, loans, derivative contracts, among others accounts). Its also posible to draw a similar graphs using the information on liabilities.

Definition

A network (graph) G consist of a non-empty set of elements V(G) called vertices, and a list of unordered pairs of these elements called edges E(G). The set of vertices (nodes) of the network is called a vertex set and the list of edges is called edge list.

If i and j are vertices of G, then an edge of the form (i,j) is said to joint or connect i and j.

In a our financial network the vertices represent the financial institution type and the edges represent the counterparty relationships on the asset side or the claims of each type of institution on the rest.

Figure 1 illustrate the network of counterparty exposures within the financial system. The circles indicate the institutions for which there is complete bilateral exposure information as mentioned in the previous section. The thickness of the edges (or links across the institutions) is determined in accordance to the importance of a particular exposure with respect to all other exposures of the institution in question. For example, the most significant exposure of the commercial financial corporations is, by far, to the private non-financial sector (in other words the real sector). On the other hand, investment banks have more or less a similar exposure to both the public and the private non-financial sectors. To get an idea or the order of magnitude (which is not evident in the graphs) of such exposures the Commercial Banks have an exposure to the private non-financial sector of \$163.173.566 millions of pesos. While government owned institutions have an exposure of \$5.374.483 millions of pesos to the same sector.

Figure 2 illustrate the network of portfolio holdings (asset side of the balance sheet: investments and loans) within the core of the financial system and the extended financial sector (circles), the public sector, the external sector (triangles) and the real sector (squares). Commercial banks are very important in the system because they seem to be the most connected type of institution, in contrast government owned institutions and investment banks are the least connected since they are exposed to few sectors of the real economy. Using the same metric as before, the thickness of the edges (or links across the institutions) is determined in accordance to the importance of a particular exposure with respect to all other exposures of the institution in question. For example, the most significant exposure of the commercial banks is to households, the manufacturing and the wholesale and retail sectors. The order of magnitude of such exposures is a follows: the Commercial Banks have an exposure to household of \$65.234.092 millions of pesos, the manufacturing sector of \$18.683.607 millions of pesos, while the exposure to wholesale, retail and repair is of $14.765.154 millions of pesos.

Figure 2 is very important for our identification strategy because it contains both the counter party information of the most relevant institutions in the financial system (inner ring) as well as the exposure of such institutions to the other sectors (outer ring). The common exposures (on the different sectors) that the financial institutions share will be the main channel of shocks coming, for example, from the real sector. Such shocks will propagate even to those institution that do not have a direct (loan portfolio) exposure to the real sector (such as investment banks) because of the counterparty relationships that the financial institutions have with their counterparts in the financial system.

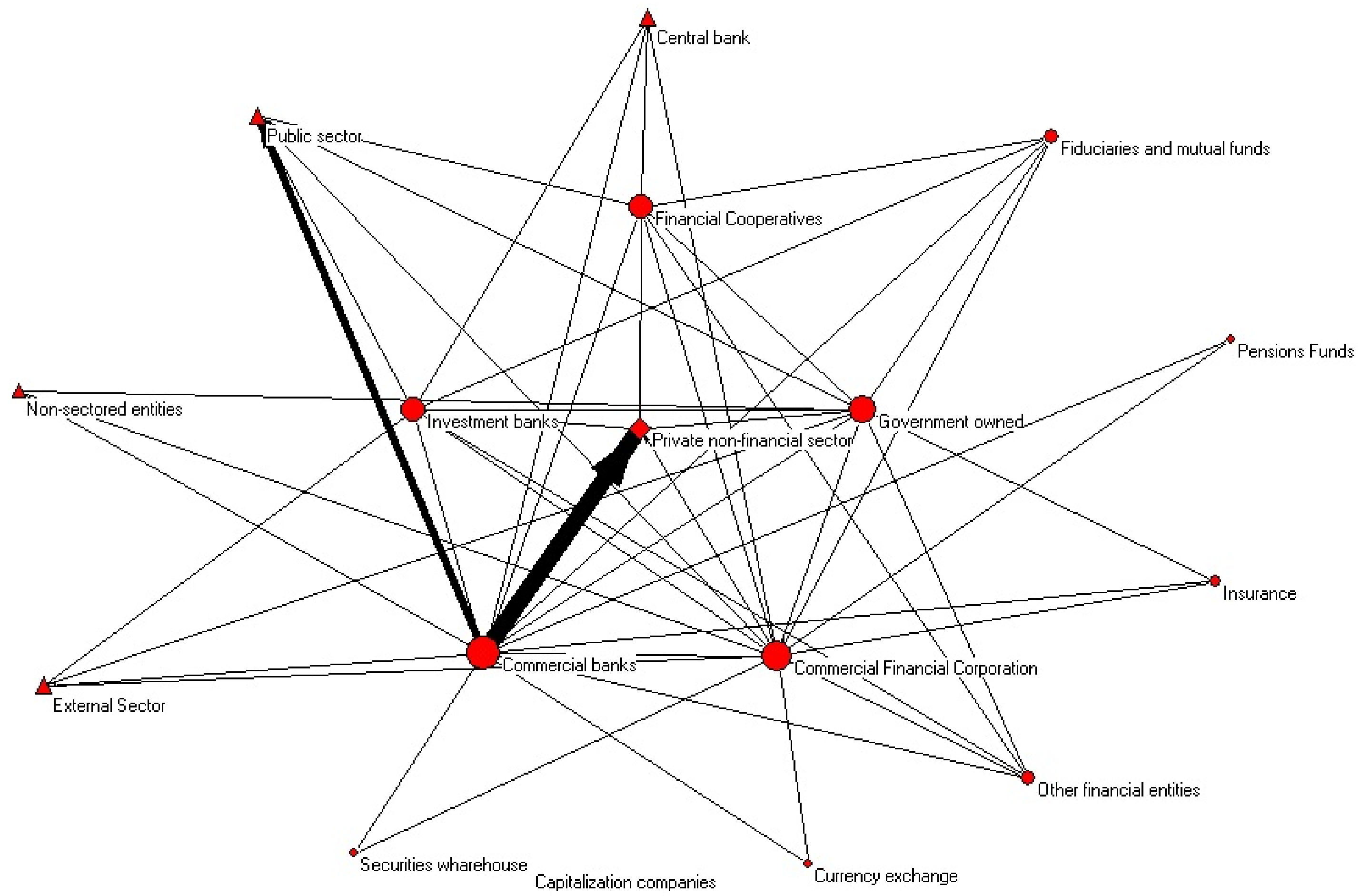

Figure 1: The Colombian financial network.

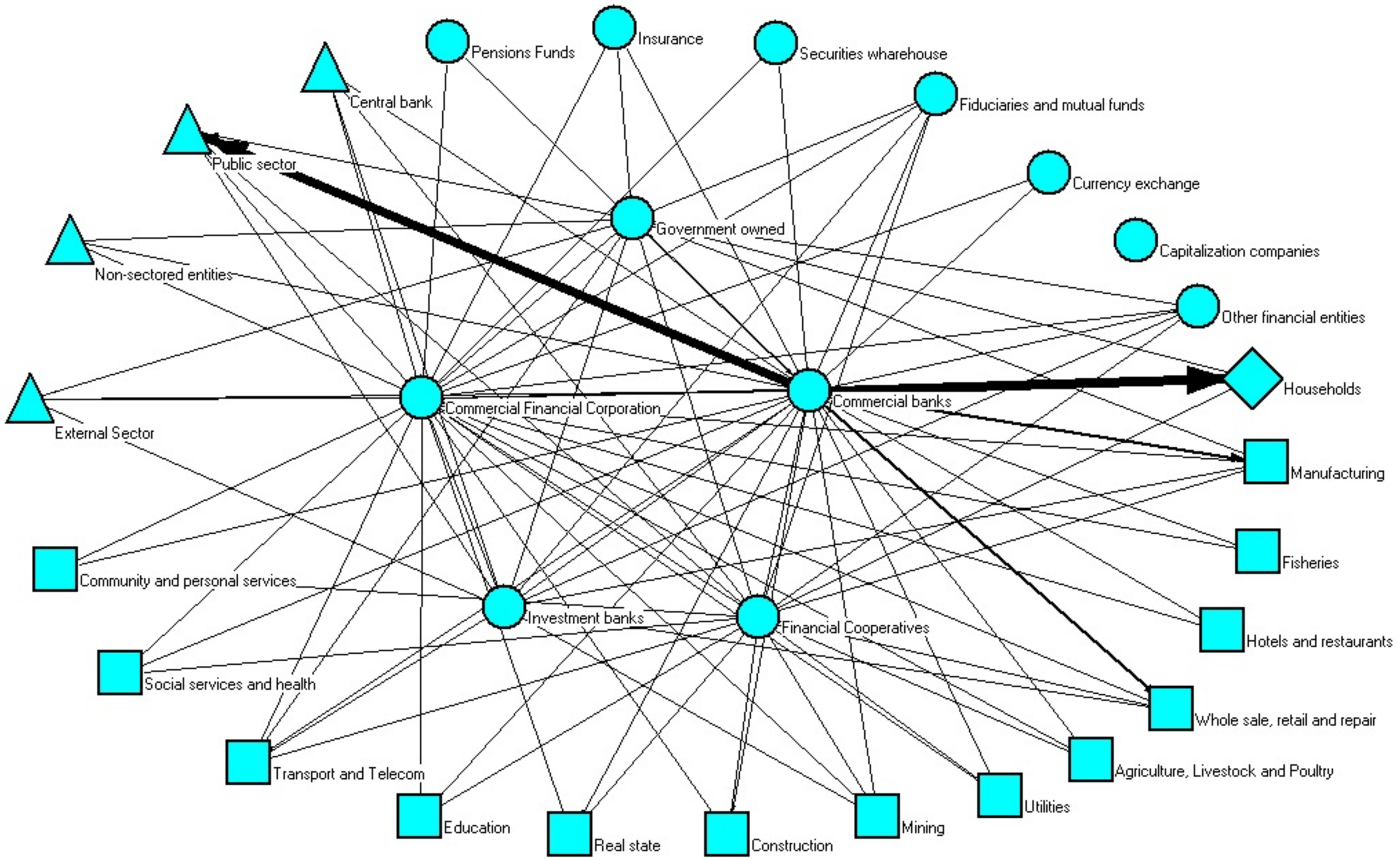

Figure 2: Exposure of the financial system to the economy.

4.3 Descriptive statistics of the network

A financial networks (or any network based data) can be described, analyzed and compared using a set of measures: density, degree, core among others. In the following paragraphs we explain such measures and calculated their respective values for the Colombian financial network that was introduced in the previous section (4.2). The advantage of network base measures is that they combine the properties of the own institutions with those of their counterparts. This approach is more in line with Basel 3.

In any sort of network (financial, social, biological) the amount of cohesion among the units of analysis (vertices) is captured by the density of the graph. The density consist of the quotient between the number of edges (counterparty relationships) that the network has over the possible edges that could exist. A network with density equal to one is called "complete network". In other words, in such network all of the vertices are connected to each other. In the context of systemic risk, the density of a financial network is a relevant measure because, as mentioned in some topological minded paper, Niera et al. (2007), when the number of edges increases the probability of a crisis decrease, in the sense that the affected institutions might be easily substituted by other financially healthier institutions. Nevertheless, in complete networks, the channel of contagion and spillover effects are greater, because of the of the increase of counterparty relationships.

$$\text{density} = \frac{\text{number of existing edges}}{\text{number of potential edges}}.$$

The financial networks presented in the previous section are not complete networks. The first network which represents the financial sector (Figure 1) has a density of 0.21. The network that involves the disaggregation of the other sectors (Figure 2) has a density of 0.11.

The density measure has a serious downside to it, since it depends on the network size, density and number of vertices (network size) have an inverse relation. When a new vertex is added to the network it has at most an edge with existing vertices, such that, at most the network with a new vertex has at most the same density. In this sense the measure does not allow comparisons across networks that do not have the same size. For this reason it is better to look at the number of edged in which each one of vertex is involved, the degree. When the edges of a network have a defined direction with respect to the relationships across the unit of analysis we can call the network a directed network. As mentioned previously the edges in the financial network (Figures 1 and 2) represents the claims of each type of institution on the rest.

For directed networks the measure of degree is divided into three groups: a) Indegree of a vertex is the number of edges that it receives (claims that other institutions have on the institution in question), Outdegree is the number of edges that it sends (claims on other institutions), and the Netdegree is the sum of the last two. When the network is an undirected network it means that the relation (edge) is bidirectional without value or any type of ponderable preference.

Table 3: Net-degree of financial network

Rank	Institution	Net-degree
1	Commercial banks	19
2	Commercial Financial Corporation	18
4	Government owned	15
5	Investment banks	12
3	Financial Cooperatives	10
6	Private non-financial sector	5
7	Public sector	5
8	Fiduciaries and mutual funds	5
9	Other financial entities	5
10	Central bank	4
11	External Sector	4
12	Insurance	3
13	Non-sectored entities	3
14	Pensions Funds	2
15	Currency exchange	2
16	Securities wharehouse	2
17	Capitalization companies	0

Table 3 shows the net-degree of each vertex (financial institutions) that exist in the Colombian financial sector and a network that examines the exposures (asset-side of the balance sheet) across sectors (Table 4). The first column shows the rank of each institution within the network, the second is the type of institution or economic sector, and in the final column we find net-degree of the each institution. We observe that in the financial sector the degree distribution reveals local concentrations of claims around some institutions (vertices) but it does not tell us whether vertices with a high degree are clustered or scattered all over the network. A k-core is the maximal sub network in which each vertex has at least degree k within the subnetwork, as with degree this measure is divided into three groups: incore, outcore and the netcore. This measure allows us to identify similarities in terms of degree between institutions (vertices). By looking at the outdegree or when trying to build the smallest outcore (2-core), we find that in the Colombian financial sector 5 out of 13 institutions hold claims on the real sector (private non-financial sector). This is of course due to regulatory restrictions.

Table 4: Net-degree of financial network and the real economy

Rank	Institution	Net-degree	Rank	Institution	Net-degree
1	Commercial banks	32	16	Insurance	3
2	Commercial Financial Corporation	31	17	Agriculture, Livestock and Poultry	3
3	Financial Cooperatives	21	18	Utilities	3
4	Government owned	17	19	Construction	3
5	Investment banks	15	20	Real state	3
6	Public sector	5	21	Education	3
7	Fiduciaries and mutual funds	5	22	Social services and health	3
8	Other financial entities	5	23	Community and personal services	3
9	Manufacturing	5	24	Non-sectored entities	3
10	Transport and Telecom	5	25	Pensions Funds	2
11	Central bank	4	26	Currency exchange	2
12	Households	4	27	Securities wharehouse	2
13	External Sector	4	28	Fisheries	2
14	Mining	4	29	Hotels and restaurants	2
15	Whole sale, retail and repair	4	30	Capitalization companies	0

5 Network model of systemic risk

Financial network models based on balance sheet data generally contain a mapping of the counter-party exposures across financial institutions (see Chan-Lau (2010a), Chan-Lau (2010b), Espinosa-Vega and Sole (2010)). Using the structure of the balance sheet of each institution of interest, and in particular the amount of capital that support the positions on the asset and liability-side of the balance sheet, it is posible to develop a series of simple exercises to asses the network externalities of the failure of one (or more) institutions. However, most applications only take into account the so call direct linkages across institutions, that is the bilateral exposures between institutions types that are explicitly mentioned in section 4.1. It is important to note that because of data limitations we only observed these bilateral exposures for a subset of the financial sector. This subset, that we denote the core, is composed of the following institutions: Commercial banks, investment banks, commercial financial corporations, financial cooperatives, government owned (institutions in the center, figure 1).

Another source of indirect linkages across institutions is common exposure. In our setup common exposure will arise primarily form the credit exposure on the real sector and from exposure (across the balance sheet) to other type of institutions (financial and non-financial) that because of data limitations we do not observe the bilateral exposures. In other words for these other institutions (or non-core institutions) we only observe the magnitude of exposure (on the asset and liability side of the balance sheet) of the core institutions. Off course there are other ways of introducing indirect linkages in a network model, Elsinger et al. (2006) use the market value of some elements in the asset side of the balance sheet as a way to incorporate common shocks and capture the stylized effects of an fire-sale of assets that can arise during a financial crisis.

To asses the potential systemic implications of the direct and indirect linkages of institutions we consider a network of N core financial institutions, M non-core institutions, and R real sectors. For our exercises we consider two types of scenarios: a credit shock scenario and a credit and funding shock escenario.

5.1 Credit shock scenario

Besides the counter-party and exposure data, another key element of financial network models is a series of reaction functions[3]. Since our model is based on the balance sheet of the core institutions we only requiere the following standard balance sheet identity for institution n.

$$\sum_{m=1}^{N+M} i_{n,m} + \sum_{m=1}^{N+M} x_{n,m} + \sum_{r=1}^{R} c_{n,r} + a_n = k_n + \sum_{m=1}^{N+M} d_{n,m} + \sum_{m=1}^{N+M} l_{n,m} + p_n \quad (1)$$

[3]Galbiati and Soramaki (2010) consider more complex reaction functions, for example based on agent based models. However, their models are based on transaction data from national payment systems, not balance sheet data.

where $i_{n,m}$ stands for investments of institutions n on institution m, $x_{n,m}$ loans of institutions n to institution m, $c_{n,m}$ loans of institutions n to economic sector k, a_n other asset of institution n. On the liability side, k_n stands for the capital of institution n, $d_{n,m}$ deposits of institution m on institution n, loans of institution m to institution n, p_n other liabilities of institution n.
In order to account for a credit shock the model asumes an initial shock of a failure of one or more exposures of institution n. These exposures are on any of the M non-core institutions or/and any of the R real sectors. The exercise then tracks the domino effects resulting from the subsequent failures of any of the N core institutions. For different assumptions on the loss given default λ it is assumed that the institutions capital k_n absorbs the loss of not only the initial shock but also the subsequent losses brought onto by the failure of the other core institutions.
Under the previous escenario the balance sheet identity (1) can be written as follows:

$$\sum_{m=1}^{N+M} i_{n,m} + \sum_{m=1}^{N+M} x_{n,m} - \lambda x_{n,h} + \sum_{r=1}^{R} c_{n,r} - \lambda c_{n,h} + a_n = (k_n - \lambda x_{n,h} - \lambda c_{n,h}) + \sum_{m=1}^{N+M} d_{n,m} + \sum_{m=1}^{N+M} l_{n,m} + p_n \quad (2)$$

The failure of institution n comes a results of its impossibility to cover its losses with the existing capital. In other words, when $(k_n - \lambda x_{n,h} - \lambda c_{n,h} < 0)$, at any point during which we observe the evolution of the network. The evolution of the network basically stand for the evolution of the balance sheet of the core institutions after the initial shock (coming form the non-core exposures) and the subsequent rounds that can be brought upon by the failure of the core institutions. The evolution of the network stops as soon as the number of failures does not change from one cycle to the other or when all core institutions have failed. Since we observe the colombian financial system by institution type and not by specific institutions we can only get a proper amount of interaction and therefore, evolution of the network if we consider large shocks, therefore we set $\lambda = 1$.

From analyzing the static exposure matrix we find the following: a) for commercial banks, commercial financial corporations and financial cooperatives, the only credit exposure large enough (in its own right) to cause a failure of one of these institutions is the exposure to households, b) for government owned institutions the largest significant exposure is to commercial banks and c) investment banks are not able to grant loan therefore they have no credit exposure. We consider an exposure as significant if the amount is larger than the existing capital. Even thought investment banks do not have credit exposure they are significantly exposed to public sector debt. These observations tell us that in order to get a shock of the magnitud needed to observed some dynamics across the network the shocks have to come from more than one of the exposures to the sectors (unless the shock comes from households).

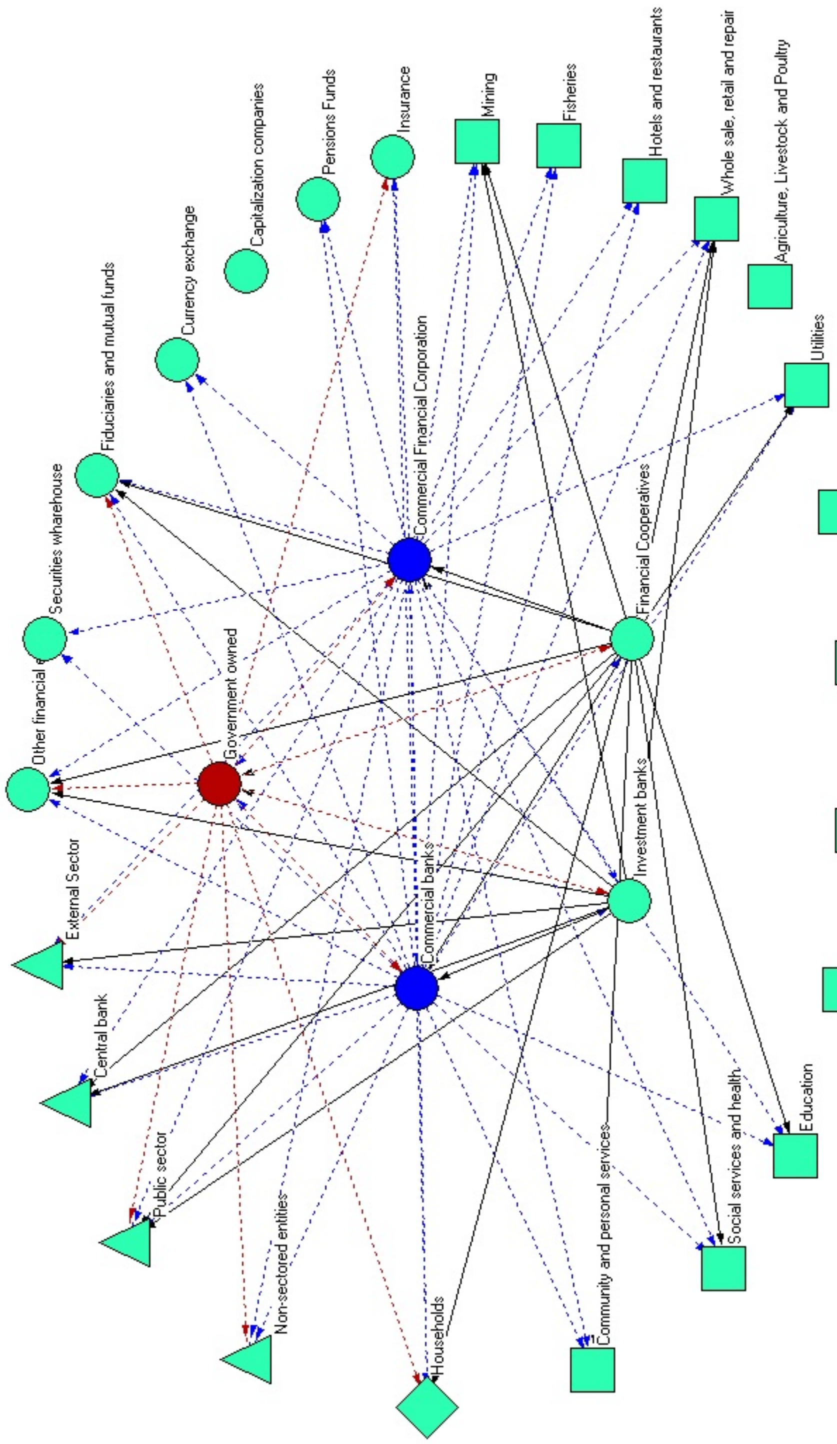

Figure 3: Credit Shock Scenario

Figure 3 maps the results of the simulation of a credit shock. The source of the credit shock is a common exposure shock on sectors: Agriculture, livestock and poultry, manufacturing, construction, transport and real state. The links to these sectors have been severed to indicate the initial shock to all of the institutions that had exposures to these sectors. Institutions that have failed in the first round, because of the shock, are labeled in blue as well as the links they had to other institutions in the system. Institutions that fail in the third round are labeled in red. The results indicate that the most affected institution are the government owned institutions, commercial banks and commercial financial corporations (in that order). After the commercial banks and commercial financial corporations have failed (in the first round) this leads to the failure of the government owned institution. The total losses, after depleting existing capital, for the institution types are as follows: commercial banks ($\$8,664,270$), commercial financial corporations ($\$2,192,625$), and government owned institutions ($\$20,569,344$), in millions of pesos.

5.2 Credit and funding shock scenario

In order to account for a credit and funding shock, in addition to the initial credit shock we account for two other sources of distress: First we asume that core institutions are unable to replace funding obtained from depositors and creditors, therefore the institution is forced to liquidate some of its investment in the market. Second these problems are aggravated since we assume that this is all going on during a time when we expect the market to be very volatile, a situation which generally triggers a fire sale of such assets (bonds, commercial paper, among other). Therefore, the institution is forced to sell these asset at a significant discount. The balance sheet effect that we should observe is an adjustment on the value of the investments, that ultimately will be reflected as unexpected loss absorbed by institutions n's capital. Under this scenario we incorporate three additional parameters: δ_d, fraction of loss funding from deposits, δ_o, fraction of loss funding from other institutions, and γ discount on the value of investments due to the fire sale of assets.

Under the previous escenario the balance sheet identity 1 can be written as follows:

$$\sum_{m=1}^{N+M}(1-\gamma)i_{n,m} + \sum_{m=1}^{N+M}x_{n,m} - \lambda x_{n,h} + \sum_{r=1}^{R}c_{n,r} - \lambda c_{n,h} + a_n =$$
$$(k_n - \lambda x_{n,h} - \lambda c_{n,h} - \gamma\sum_{m=1}^{N+M}i_{n,m}) + \sum_{m=1}^{N+M}(1-\delta_d)d_{n,m} + \sum_{m=1}^{N+M}(1-\delta_o)l_{n,m} + p_n \quad (3)$$

The failure of institution n comes a results of its impossibility to cover its losses, including the losses from the write down on the value of the investment that have been sold off to replace loss funding, with the existing capital, in other words when ($k_n - \lambda x_{n,h} - \lambda c_{n,h} - \gamma \sum_{m=1}^{N+M} i_{n,m} < 0$), at any point during which we observe the evolution of the network.

As mentioned previously, investment banks have a large and significant exposure on public sector bonds, but so do commercial banks, financial corporations and government owned institutions. For commercial banks deposits from households and the public sector is also significant. Whereas, commercial financial institution and financial cooperatives receive significant loans from commercial banks. We consider an exposure or a liability as significant if the amount is larger than the existing capital.

Figure 4 maps the results of the simulation of a credit and Funding shock ($\delta_d = 5\%$,$\delta_o = 12.5\%$,$\gamma = 35\%$). The source of the credit shock is a common exposure shock on sectors: Agriculture, livestock and poultry, manufacturing and construction. The links to these sectors have been severed to indicate the initial shock to all of the institutions that had exposures to these sectors. Institutions that have failed in the first round, because of the shock, are labeled in black as well as the links they had to other institutions in the system. Institutions that fail in the third round are labeled in blue. The results indicate that the most affected institution are the government owned institutions, commercial banks and investment banks (in that order). After the commercial banks and the government owned institutions have failed (in the first round) this leads to the failure of the investment banks. The total losses, after depleting existing capital, for the institution types are as follows: commercial banks ($\$2,344,542$), investment banks ($\$537,382$), and government owned institutions ($\$38,739,943$), in millions of pesos.

5.3 Estimation of the hazard rate for each core institution

The network model we have developed out of the data from the colombian financial sector, on the one hand, is entirely based on the balance sheet and any shocks (or distress) we incorporate on the elements of the balance sheet. On the other hand, the model is entirely deterministic in nature. In other words we choose where the shock comes from, the source of the exposure, and the dynamics that evolve after the shock are also deterministic. For this reason the scenarios that we generated before are deterministic simulations that take into account a particular path of the network given an specific shock. Network models are fast becoming an important tool for macro-prudential supervision. One often cited measure of vulnerability or fragility of a particular institution that can be obtained from such models is the hazard rate of the institution. The hazard rate for institution n is defined as the number of scenarios in which institution n failed (excluding those where the institution in question is the initial defaulting institution) over the total number of scenarios considered. Since our network model is deterministic and hence there is no random data generating process that drives default, we consider different combinations of sources and number of shocks as a mechanism to estimate the hazard rate for each of the N core institutions. Since we have in total 30 exposures we can consider all posible combinations of shocks caused by one through five exposures. This gives us a

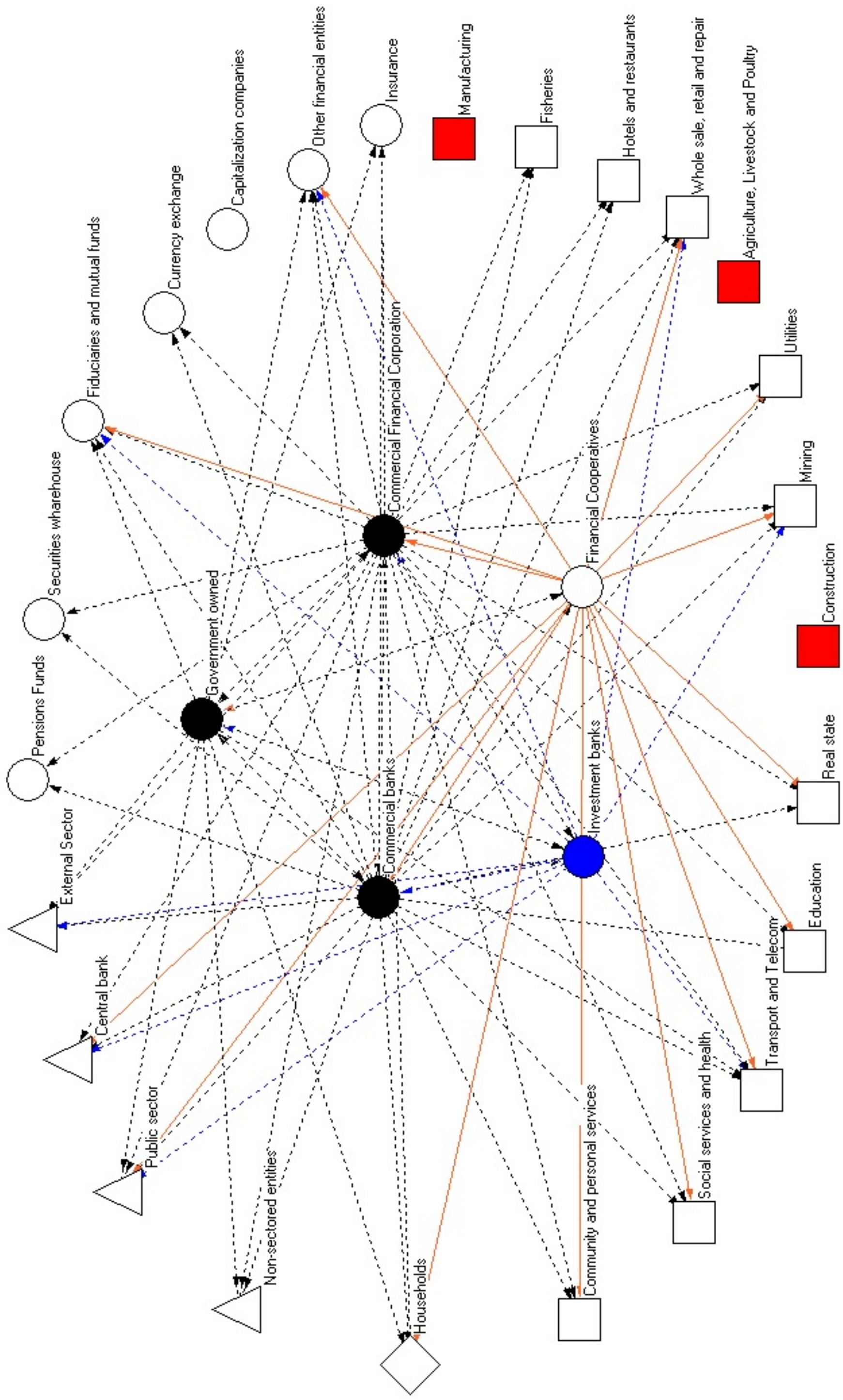

Figure 4: Credit-Funding Shock Scenario

total of $174,436$ scenarios. To obtain the hazard rate, for institution n we only have to determine the number of times that institution n has failed on average over the entire sample of scenarios.

Table 5: Hazard rates by Institution Type

Institution	Credit Shocks	Credit-Funding Shocks
Commercial Banks	0.2322	0.2124
Investment Banks	0	0.0344
Commercial Financial Corporation	0.3396	0
Financial Cooperative	0.4284	0
Government Owned	0.3742	0.4229

The results from table 5 indicates that although commercial banks are heavily exposed to different sectors and the most interconnected type of institutions, these conditions alone do not necessarily make them the most vulnerable institution in the system. However, since they provide important services and resources to the rest of the system they are quite systemically important. The most vulnerable institutions are the government owned, to both credit and funding shocks. This results is not surprising, because they are the second largest player in the core system by looking at assets under management, however the result is puzzling as to the role of government owned institutions. The reason is that if such institution provide a cushion for the shocks across the system then then their significant role is macro-prudentially sound. However, if the story is different, in that the large exposure across the system of government owned institutions is due to the large involvement of public institutions in financial intermediation, then we have to ask ourselves, what reason continue to justify this institutional arrangement? There is provably a combination of both effects, that because of data availability reasons for the moment we cannot properly address.

Commercial Financial Corporations and Financial Cooperatives are also very vulnerable to credit shocks, however this vulnerability vanishes when we include funding shocks. The main reason behind this result is that when funding is concern, institutions are able to liquidate investment positions (at a discount) to deal mainly with funding shocks, but they are also able to use these free-up resources to compensate any losses on the asset-side of the balance sheet. Therefore, their credit exposures can be in a way be covered by liquidating some of their marketable portafolio. Finally, investment banks by definition are not vulnerable to credit shock, only to funding shocks. They are specially sensitive to a forced liquidation (at a significant discount, close to 25% below current value) of their investment in public sector bonds.

6 Conclusions

Although the most recent financial crisis (2007, 2008) has not affected developing economies, the important lessons should not be overlooked by overconfident policy makers.

An important lesson that has come out of the crisis is the need to monitor more carefully the silent buildup of risk pockets and imbalances across the financial system. Network models provide a working horse to contribute to the required *"topography"* needed for macro-prudential supervision.

We contribute to the financial network literature by designing a network model that considers various sources of distress and spill-over mechanisms in the financial system, that are relevant to identify and manage systemic risk. Our results are in line with existing evidence on the Colombian financial system, Machado et al. (2010), to the extend that we find that commercial banks are the most systemically important financial institutions.

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