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Abstract

I study a bilateral investment game where a buyer privately trades with several suppliers who compete by offering menus of non-exclusive contracts. When market trading is structured so that competition among suppliers is the most intense, the hold-up problem disappears for an extensive range of the investment costs. The investment of the supplier does not affect its bargaining position, and both the supplier and the buyer have the right incentives to invest. In any other equilibria, the efficient investment is not implemented: the reallocation of bargaining power as a result of investment distorts the incentives to invest efficiently. However, because under some parameters of the model investment decisions are strategic complements welfare is maximised for an intermediate level of competition.

Keywords: Bilateral Investment; Hold-up; Non-Exclusive Contracts; Competition.

JEL classification: D44; L11.

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1 Introduction

There are countless examples of trading situations in which trading partners undertake specific investments to increase the potential gains from trade. More often than not, investments are bilateral, and both parts devote resources to generate larger gains from their trading relationship. For instance, in 2013 Toyota spent in local manufacturing over $15 million in supplier development programs in Australia, and encouraged major tires' suppliers to invest in the design of specific tires for each of its vehicle programs, *(Toyota News, 2013)*. Also in the industry of information technology, Dell undertakes specific investment with its major suppliers, who in turn adapt their production process to Dell's particular needs, *(Kang et al., 2007)*.

In the existing literature, there is a wealth of works identifying the potential inefficiencies in industries where economic agents undertake ex-ante investments. Inefficiencies emerge due to the hold-up problem. This arises when a group of agents shares surplus from their trading interaction and when an agent investing is unable to receive all the benefits emerging from it. As a result, profitable investment decisions are not materialised. To reestablish efficiency, in addition to contractual solutions, the economic literature has currently considered the introduction of competition to tackle the problem of being held-up. Competition generates an available outside option that increases the bargaining position of an agent, leading to larger incentives to invest. Mailath, Postlewaite, and Samuelson (2011) and Felli and Roberts (2016) consider market situations where multiple suppliers and buyers match to create a surplus. Once the investment has been undertaken, agents decide on the trading partner and trade occurs exclusively between a single supplier and a buyer. Although these models are good representations of labor market relationships, where trade is exclusive, they do not consider situations when a single agent may transact with many partners.

This article revisits the approach of competition as a solution to the hold-up problem by relaxing the assumption of exclusive contracting. Situations where one buyer trades with several suppliers happens in many different markets. A useful example is the automobile industry. For instance, Toyota does not depend on a single source for most of their components and transacts with several suppliers. In these markets, a significant amount of trade materialises with a core supplier, who has invested in improving its ability to develop specific products and technologies. Liker and Choi (2014) found that Toyota asks core suppliers to send several of their design engineers to the manufacturers offices. Eventually, the suppliers engineers will understand the development process and come up with design ideas and a better production technology for the products that Toyota needs. In my model, I give a rationale for the incentives from a large buyer to transact with many suppliers even if a single supplier invests.

To explore the impact of competition on ex-ante investment decisions, I consider a common agency game of complete information in which a finite number of suppliers trade a homogenous input with a single buyer. Before the trade, I assume that only one of the suppliers can invest in a technology to reduce the cost of input production. The buyer invests in adapting its production process to increase its valuation for the input provided by the different suppliers. The assumption that only one supplier invests is a good representation of the Japanese car industry where the buyer has a preferred supplier who has typically superior knowledge relative to competing suppliers. My trading game is modelled as in Chiesa and Denicolò (2009) in which suppliers offer a menu of non-linear trading contracts to the buyer. In situations with investment, I need to distinguish between *ex-post* efficiency, which is restricted to the trading allocation of the common agency game, and ex-ante efficiency, which takes into account the investment decisions at the first stage of the game. I show that whereas the trading allocation is always efficient, given an investment profile the equilibrium trading allocation maximises the gains from trade, the investment profile will only be efficient if the competition in the trading game is the most intense.

The equilibrium in the trading game is fully characterised by the conditions of *bilateral efficiency* and *individual excludability*. With *bilateral efficiency*, each supplier's trading contract maximises the gains from trade between the buyer and himself. This explains why a supplier always offers the efficient trading allocation. By *individual excludability*, the buyer obtains the same equilibrium payoffs after excluding any supplier from trade. *Individual excludability* allows me to compute the equilibrium payoffs of the game. I find that the payoff of each supplier hinges on the outside option available to the buyer when the former is excluded from trade. For the construction of the outside option, I use the formulation of "latent" contracts. Those that in addition to the null contract and the trading contract that is accepted in equilibrium, are contracts constraining the payments of competitors for their prescribed equilibrium quantities. A "latent" contract entails larger trading quantities designed to give the best option to the buyer if it desires to exclude a supplier from trade. I then construct different equilibria depending on the number of suppliers who submit "latent" contracts. The more suppliers who offer those contracts, the easier it becomes to substitute the equilibrium trading allocation from an excluded supplier and the larger it is the outside option for the buyer. This translates, for a given investment profile, into a lower bargaining position for suppliers in favour of the buyer.

With the equilibrium of the trading game, I study the central theme of the article: how competition affects the bilateral investment decisions. My first finding asserts that when the market trading is structured so that competition among suppliers is the most intense, the hold-up problem disappears. Hence, the introduction of competition to one side of the market does not require the design of long-term contract as considered in the literature of contractual design (e.g., Aghion, Dewatripont and Rey, 1994; Chung, 1991 and Edlin and Reichelstein, 1996) to induce efficient investment. In my model, when competition is the most intense, the investment of the supplier does not affect its bargaining position, and it always appropriates its marginal contribution to the surplus. As a result, the supplier has the right incentive to invest. Then, when the buyer also invests efficiently, the equilibrium investment profile is efficient. This result does not extend to other equilibria of the trading game. Now, the investment profile affects the bargaining position of suppliers, and this allows the investing parties to behave strategically. The investment of the supplier reduces the equilibrium allocation for the competitive suppliers, which limits their ability to generate surplus with the buyer by submitting "latent" contracts. As a result, the supplier's bargaining position increase with investment: it appropriates more than its marginal contribution to the surplus and decides to over-invest. Moreover, the changes in the relative bargaining position of suppliers as a result of investment generates the investment decision of the buyer not to be monotone with the intensity of competition. Then, to come up with the investment decision of the buyer other elements such as changes in the trading allocation as a result of investment and the number of active suppliers need to be taken into account.

I finish my analysis by comparing the welfare generated by the different types of equilibria. To the best of my knowledge, this is the first work to consider welfare analysis for the different kind of equilibria that emerge in a common agency setting. The simplest situation to analyse is when the efficient investment profile is implemented in the most competitive equilibrium. In this case, welfare is maximised, and I show that due to the misallocation of the suppliers' investment welfare monotonically increase with the level of competition. For those situations wherein the most competitive equilibrium, ex-ante efficiency is not implemented, I can find cases where welfare may be largest in less competitive equilibria. This is because the over-investment of the supplier can generate the right incentives for the buyer to invest in those situations where investments are strategic complements. It is also the case that the efficient level of welfare can never be implemented.

Related literature

There is a sizeable literature studying the solutions to the hold-up problem. An early formulation of the hold-up problem appears in Klein, Crawford and Alchian (1978), and Williamson (1979, 1983). In those articles, the hold-up problem arises because parties are unable to bargain over specific investment as it is unverifiable. When contracts cannot be made contingent on investment, the literature of incomplete contracting can be, broadly speaking, categorised into two main groups. One group of articles (e.g., Grout, 1984; Grossman and Hart (1986) and Hart and Moore (1990)) takes an organisational approach, relating to the theory of the firm, which establishes the provisions for asset ownership and the allocation of residual rights of control needed to restore efficient investment. In the second group of articles, a long-term contract approach is considered. These works (e.g., Aghion, Dewatripont and Rey, 1994; Chung, 1991 and Edlin and Reichelstein, 1996) focus in the designs contractual arrangements aiming to relax potential conflicts of interests between trading parties.

More recently, and due to the impossibility to design and enforce ex-ante contracts, a new strand of the literature has considered the introduction of competition in settings affected by the hold-up problem. Then, Mailath, Postlewaite, and Samuelson (2011) and Felli and Roberts (2016), consider a matching market where once the investment has been undertaken, agents decide on the trading partner. The presence of market competition for matches provides incentives for investment but it may also generate inefficiencies arising from coordination problems in which matches cease to be efficient. In Cole, Mailath, and Postlewaite (2001a), and Cole, Mailath, and Postlewaite (2001b) the matching process is modelled as a cooperative assignment game. The authors show that although efficient matching characterises the equilibria in those articles, efficient investment is never implemented. Departing from these articles, my trading game does not consider a matching environment, and I use the notion of non-cooperative equilibrium.

The study of competition in a non-cooperative equilibrium and the hold-up problem together with elements of organisational design is considered in Cai (2003) and Chatterjee and Chiu (2006, 2007). In those articles, agents decide over the type of investment, general or specific, in a decentralised market. They find that more competition together with ownership of assets makes efficient and specific investment more likely. Otherwise, general investment and the ownership irrelevance phenomenon occur. Contrasting with these models, I do not consider asset ownership, and the level of competition and the subsequent investment of the supplier endogenously determines the level of buyer's investment specificity in my model.

By allowing the buyer to trade with several suppliers, my article is also related to the literature of second sourcing. Li and Debo (2009), and Lewis and Yildrim (2002, 2005) consider a multi-period setting, where a buyer may trade with different suppliers earlier or later in the game. Even if the trade happens exclusively in each period, a single buyer alternate suppliers to keep the competitive pressure active. In this second sourcing literature, in each period trading occurs only between one buyer and one supplier. I relax the assumption of exclusive contracting by allowing a buyer to trade with multiple suppliers at the same time. In this regard, my article builds on the literature of markets and contracts which considers the limits on the number of parties that can be part of the same contract. Bernheim and Whinston (1986), first thought of a contracting model between one agent and multiple principals. The authors take a group of principals aiming to provide incentives to a common agent and characterise necessary and sufficient conditions to achieve an efficient outcome. In a trading environment, Segal (1999b) demonstrates that in the absence of direct externalities, the equilibrium trading outcome is unique and efficient. Without externalities, the principals' payoffs depend only on their trade with the agent. Even in a bidding game, where multiple principals propose trading contracts to the common agent and inefficiencies may arise from the coexistence from the multiplicity of offers, efficiency remains.

Whereas in the absence of direct externalities, a unique and an efficient trading outcome exists, the literature has encountered multiplicity in the equilibrium payoffs. In Bernheim and Whinston (1986), and Laussel and Le Breton (2001), the authors address the analysis of the equilibrium payoffs by using a cooperative game characterisation. From the multiplicity of equilibrium payoffs, they concentrate on a subset of equilibria that call "truthful". Truthful equilibria include efficient actions that are focal and coalition-proof. In a non-cooperative game Chiesa and Denicolò (2009), show that the set of equilibrium payoffs is a semi-open hyper-rectangle and state that the maximum suppliers' payoff is not truthful. The maximum equilibrium payoffs are given by the "threat" of any principal to be unilaterally replaced by one of its competitors.[1] Similar to their work my model also explores the notion of the "threat" of being replaced by the competitors suppliers to be a determinant factor in the redistribution of the gains from trade.

However, contrary to Chiesa and Denicolò (2009), by using the notion of the number of "latent" contracts designed to compete for the equilibrium allocation of the excluded supplier, I can characterise a subset of the equilibrium payoffs belonging to their semi-open hyper-rectangle. I also

[1] Martimort and Stole (2009) show multiplicity of equilibria in a public common agency game and use asymmetric information as a tool for equilibrium refinement.

introduce a previous stage where both sides of the market undertake ex-ante investments. This investment stage allows me to compare equilibria concerning welfare. To the best of my knowledge, my article is the first to consider welfare analysis in a common agency game. I show how the re-distribution of the gains from trade has implications on investment decisions and the total surplus generated.

2 Model

I consider a bilateral investment game where a single buyer trades with many suppliers. The suppliers in my model are indexed by $i \in N = \{1, ..., N\}$ and produce a homogeneous input consumed for a single buyer indexed by 0. The game consists of two stages played sequentially. In stage one, investment takes place. Here, only supplier $i = 1$ invests in a cost-reducing technology, which allows the reduction of its production costs. Supplier's investment is a continuous variable $\sigma \geq 0$ with a convex cost $\psi_1(\sigma)$. At the same time, the buyer takes a binary investment decision $b \in \{0, 1\}$, incurring a fixed cost k. Buyer's investment is not rival and generates a larger valuation from the total input consumed. This investment can be interpreted as an adaptation of its production process to the homogeneous input provided by the suppliers.

Following Chiesa and Denicolò (2009), I model trade as a first-price auction in which suppliers simultaneously submit a menu of trading contracts and the buyer chooses the quantity it purchases from each supplier. The menu of trading contracts are denoted by $M_i \subset \Re^2_+$ for each supplier i. A trading contract consist of a pair $m_i = (x_i, T_i)$, where $x_i \geq 0$ represents the quantity of input supplied and $T_i \geq 0$ the transfer requested by each supplier i. The model belongs to private and delegated common agency. Private common agency means that a supplier cannot condition payments on the quantities others trade, and delegated implies that trade is voluntary and the buyer chooses the set of suppliers to trade with. In what follows, I state the model more formally.

Strategies and payoffs

A strategy for each supplier i is the set of menu of contracts $M_i \subset \Re^2_+$. With a menu profile of trading contract $\mathbf{M} = (M_1, M_2, ..., M_N) \in \Gamma^N$, a strategy for the buyer is a function $\mathcal{M}(\mathbf{M}) : \Gamma^N \to (\Re^+)^N$ such that $\mathcal{M}(\mathbf{M}) \in \times_{i=1}^N M_i$ for all $\mathbf{M} \in \Gamma^N$, and $\mathbf{m} = (m_1, m_2, ..., m_n)$ is the vector of contracts accepted by the buyer. I do not impose any restriction on the number and the form of trading contracts belonging to M_i, expect that each supplier i must offer the null contract $m_i^0 = (0, 0)$ - due to the voluntary of trade - and that the menu of trading contracts is a compact set Γ.[2]

For a given vector of contracts $\mathbf{m}$ accepted by the buyer and an investment profile (b, σ), the buyer's payoff is

$$\pi_0(\mathbf{m} \mid b) = U\left(X \mid b\right) - \sum_{i=1}^{N} T_i - k \times b, \tag{2.1}$$

where $X = \sum_{i=1}^N x_i$ represents the total input traded and the function $U(X \mid b) : \Re^+ \to \Re^+$ denotes, in monetary terms, the value to the buyer. The payoff for supplier 1 is

$$\pi_1(\mathbf{m} \mid \sigma) = \pi_1(m_1 \mid \sigma) = T_1 - C_1\left(x_1 \mid \sigma\right) - \psi_1(\sigma), \tag{2.2}$$

and

$$\pi_i(\mathbf{m}) = \pi_i(m_i) = T_i - C_i\left(x_i\right), \tag{2.3}$$

for $i \neq 1$, where $C_i(x_i \mid \cdot) : \Re^+ \to \Re^+$ is supplier i's cost function. Because the cost function is only dependent on own output, direct externalities are absent as the contracts that the buyer accepts from the other suppliers do not directly affect the payoffs of a given supplier.[3]

With the payoffs from the different players and an investment profile (b, σ), the maximum trading

[2] This last assumption is necessary to guarantee existence of an optimal choice for the buyer. The same assumption is considered in Chiesa and Denicolò (2009).

[3] Chiesa and Denicolò (2009) state that because the willingness to pay for the goods depends on the quantities traded with all principals, contractual externalities arise.

surplus gross of the investment costs is represented by

$$TS^*(b,\sigma) = \max_{x_1,\ldots,x_N} \left[U(x_1 + \ldots + x_n \mid b) - C_1(x_1 \mid \sigma) - \sum_{i \neq 1} C_i(x_i) \right], \tag{2.4}$$

where $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$ stands for the vector of quantities that solves the problem. For later use, I denote $X^* = \sum_{i=1}^N$ the sum of the efficient quantities, and by $X^*_{-H} = \sum_{i \notin H} x_i^*$, for $H \subset N$, the sum of the efficient quantities without taking the quantities of the subset of suppliers in set H.

To ensure an interior solution and that every supplier trades a strictly positive and finite quantity with the buyer, I introduce the following regularity assumptions, subscripts denote partial derivatives

Assumption 1. *(Regularity conditions)*

1. $U_x(\cdot) > 0,\ U_{xx}(\cdot) < 0,\ U(X \mid b = 1) > U(X \mid b = 0)$ *and* $U_x(X \mid b = 1) > U_x(X \mid b = 0)$.

2. $C_x(\cdot) > 0,\ C_{xx}(\cdot) > 0,\ C_\sigma(\cdot) < 0,\ C_{x\sigma}(\cdot) < 0,\ \psi_\sigma(\sigma) > 0\ C_{\sigma\sigma}(\cdot) > 0,$ *and* $\psi_{\sigma\sigma}(\sigma) > 0$.

3. $\lim_{X\to 0} U_x(\cdot) = +\infty,\ \lim_{X\to\infty} U_x(\cdot) = 0,\ \lim_{x_i\to 0} C_x(\cdot) = 0$ *and* $\lim_{x_i\to\infty} C_x(\cdot) = +\infty$.

Because the game is of complete information and played in different stages, I employ the solution concept of sub-game perfect Nash equilibrium.

Definition 1. ***(Equilibrium)****. A Subgame Perfect Nash equilibrium is a list of strategies* $\langle \hat{\mathcal{M}}(\mathbf{M}), \hat{M}_1, \hat{M}_2, ..., \hat{M}_N \rangle$ *such that:*

$$\hat{\mathcal{M}}(\mathbf{M}) \in \underset{\mathcal{M} \in \times_{i=1}^N M_i}{\arg\max} \quad \pi_0(\mathcal{M}) \quad \forall\, \mathbf{M} \in \Gamma^N,$$

$$\hat{M}_i \in \underset{M_i \in \Gamma}{\arg\max} \quad \pi_i(\hat{\mathcal{M}}(\hat{\mathbf{M}}_{-i}, M_i)) \quad \forall\, i \in N,$$

where $(\hat{\mathbf{M}}_{-i}, M_i) \equiv (\hat{M}_1, ...\hat{M}_{i-1}, M_i, \hat{M}_{i+1}, ..., \hat{M}_N)$*, and a pair of investments* $\langle \hat{b}, \hat{\sigma} \rangle$ *such that:*

$$\hat{b} \in \underset{b=\{0,1\}}{\arg\max} \quad \pi_0(\hat{\mathcal{M}}, b, \hat{\sigma}),$$

$$\hat{\sigma} \in \underset{\sigma \geq 0}{\arg\max} \quad \pi_1(\hat{\mathcal{M}}, \hat{b}, \sigma).$$

3 Trading game

The equilibrium in the trading game is fully characterised by the use of *bilateral efficiency* and *individual excludability*. *Bilateral efficiency* establishes the amount of input that each supplier will submit in any of its trading contracts. *Individual excludability* allows obtaining the equilibrium payoffs for each of the suppliers. After describing the properties of the equilibrium trading allocation (see Lemma 1), I characterise the equilibrium payoffs (see Proposition 1). Consistent with the common agent literature, whereas there exists a unique trading allocation the equilibrium payoffs are not unique. In my model, the transfer that a supplier obtains from its equilibrium allocation depends on the number of suppliers competing for its equilibrium allocation after it is excluded from trade.

Equilibrium allocation

The economic literature (e.g., Bernheim and Whinston (1986) and Segal (1999b)) has acknowledged that in a common agency game potential inefficiencies may arise due to externalities among suppliers. In my model, because the production cost of each supplier only depends on the amount of the input it produces, the trading contracts submitted by rivals do not affect its payoff.[4] Then, given a menu of trading contracts for the $N-1$ suppliers, who generate an amount of trade X_{-i}, each supplier effectively plays a bilateral trading game with the buyer in which the former has the whole bargaining power. As a result, when submitting a trading contract each supplier i maximises the potential gains from trade generated between the buyer and itself. This result is what the literature of markets and contracts have called "*bilateral efficiency*", and in equilibrium, there must exist a

[4] In the formulation of Segal, my model does not have externalities to non-traders.

trading contract where each supplier i offers an amount of input

$$x_i^*(b,\sigma) = \underset{x_i \geq 0}{\arg\max} \left[U\left(X_{-i} + x_i \mid b\right) - \sum_{j \neq i} T_j - C_i(x_i \mid \cdot) \right] = \pi_0(\mathbf{m} \mid b) + \pi_i(m_i \mid \cdot),$$

and $x_i^*(b,\sigma)$ is also the quantity that maximises the trading surplus in expression (2.4). Chiesa and Denicolò (2009) show (see their Lemma 2) that the contract that will be certainly accepted in equilibrium contains the efficient amount of trade. In my model, with a given investment profile, the efficient allocation is characterised by the system of equations

$$\begin{aligned} &U_x(X^* \mid b) = C_x(x_1^* \mid \sigma) && \text{for } i = 1, \\ &U_x(X^* \mid b) = C_x(x_i^*) && \text{for all } i \neq 1, \end{aligned} \tag{3.1}$$

where, given an investment profile (b, σ) the marginal production cost equals the marginal buyer's valuation.

An important element when eliciting the equilibrium investment profile would be the adjustment of the equilibrium allocation concerning changes in investment. I show that (see Proposition 6) changes in the equilibrium allocation due to investment redistributes the bargaining position of the supplier's bis a bis to the buyer which determine the incentives from both parts of the market to invest. Then, the changes in the equilibrium trading allocation are:

Lemma 1. *i) For a given investment of the buyer, an increase in the investment of supplier* 1 *raises the amount of trade between the buyer and itself, and decreases the amount of trade with the rest of the suppliers. However, the total amount traded increases, i.e.,*

$$\frac{dx_1^*}{d\sigma} > 0; \quad \frac{dx_j^*}{d\sigma} < 0 \ \textit{for all} \ j \neq 1; \quad \textit{and} \quad \frac{\partial}{\partial \sigma} X^* > 0.$$

ii) For a given investment of supplier 1*, an investment of the buyer increases the amount of trade* *with all suppliers, i.e.,*

$$x_i^*(1,\sigma) > x_i^*(0,\sigma) \qquad \forall i \in N.$$

The higher the investment of supplier 1, the more efficient it becomes with respect to the competing suppliers, and the buyer responds by substituting trading from them to supplier 1. The Lemma also states that this crowding-out effect is of second order: as the economy becomes more efficient as a result of the supplier's investment the total amount of trade increases. In the second part of the Lemma, for a given investment of supplier 1, the relative efficiency among suppliers does not change, and an investment of the buyer increases the amount of trade will all the suppliers. The buyer's investment rises its marginal utility from purchase and decides to buy a larger amount of input from all suppliers. Observe that Lemma 1 states only a partial equilibrium result. This is because, the investment of the buyer will have an effect on the investment decision of supplier 1, and this will change the equilibrium trading allocation. Nevertheless, (see Proposition 2) the crowding-out effect will be crucial for the analysis of the equilibrium investment profile.

Definition 2. ***(Allocative sensitivity)*** *The allocative sensitivity is the change in the equilibrium allocation $dx_j^*/d\sigma$ for $j \neq 1$ in response to an increase of investment by supplier 1.*

The allocative sensitivity depends on the fundamentals of the economy such as the concavity of buyer's utility, the convexity of cost functions and the number of suppliers active in the market. With the equilibrium allocation, I proceed with the analysis of the equilibrium transfers and payoffs of the trading game.

Equilibrium transfers and payoffs

The transfers that suppliers require as a payment for their efficient allocation $x_i^*(b,\sigma)$ is determined by the voluntarily of trade, which allows the buyer to decide on the subset of suppliers to trade with.[5] Then, the buyer can always decide to exclude a supplier from trade, if the latter asks a too

[5]In Martimort and Stole (2009), a common agent is forced to trade with all principals. In their setting two regulatory agencies interact with a single firm.

large a transfer for its efficient allocation. Indeed, the maximum transfer that a supplier can request depends on the outside option that the buyer will have available with the rest of the suppliers if it decides to exclude the former from trade. The larger this outside option, the more the supplier will face the "thread" to be excluded from trade and consequently, the lower will have to be its equilibrium transfer.

A distinguishing attribute in the literature on markets an contracts is that equilibrium transfers are not unique. This is because each supplier can offer many contracts in addition to the contract that will be accepted in equilibrium. The literature has called "latent" contracts, the set of trading contracts that are not accepted in equilibrium but that constraint the transfers that competitors can require for their equilibrium quantities. To better understand the role of those contracts, I show that the set of "latent" contracts determine the available outside option of the buyer when it excludes a supplier from trade, and therefore the transfers that suppliers obtain in equilibrium. Observe that because the buyer trades with all the suppliers in equilibrium (see Proposition 1), "latent" contracts will never be accepted. In my model, I construct different equilibria depending on the number of suppliers how submit "latent" contracts. Chiesa and Denicolò (2009), characterise the supplier's maximum payoff obtained when only a single supplier offers a "latent" contract.

To construct the equilibrium transfer for any supplier i, consider that a subset of suppliers $J_i \subset N$ and $i \notin J_i$, in addition to the null contract and the contract that will be accepted in equilibrium submit an extra contract. Consider also that the rest of suppliers $h \in N \setminus \{J_i, i\}$ only offer the null and the equilibrium contract. In principle, I do not impose any restriction on the form of those extra contracts submitted by suppliers $j \in J_i$, but to generate a constraint on the payoff of supplier i, the amount of input specified in those contracts need to maximise the gains from trade with the buyer when the latter does nit trade with supplier i. Then, by bilateral efficiency, given that the buyer does not trade with supplier i and the amount traded with the rest of the suppliers

$X_{-\{j,i\}}$, supplier j offers an amount equal to

$$\tilde{x}_j(b,\sigma) = \underset{x_j \geq 0}{\arg\max} \left[U\left(X_{-\{j,i\}} + x_j \;\middle|\; x_i = 0, b \right) - C_j(x_j \mid \cdot) \right] \quad \forall\, j \in J_i, \tag{3.2}$$

where the amount $\tilde{x}_j(b,\sigma)$ for each $j \in J_i$ constitutes the trading allocation that maximises the gains from trade when supplier i is excluded from trade. Then, when a supplier $j \in J_i$ aims at competing for the equilibrium allocation of the excluded supplier i, it must offer a "latent" contract with this trading allocation.

For later use in the article, it will be useful to compare the amount of input offered in the "latent" contracts with respect to the equilibrium allocation. Due to the convexity of the cost function, it is easy to show that the aggregate amount traded in equilibrium is always higher than the total amount traded when a supplier is excluded from trade. However, because "latent" contracts are designed to compete for the equilibrium allocation of the excluded supplier, the suppliers offer a larger amount of trade in its "latent" contract than in equilibrium. The next Lemma states this result.

Lemma 2. *For any investment profile (b,σ) and a set of suppliers in J_i, the aggregate trading quantity offered with the "latent" contracts is smaller than the aggregate equilibrium trade, i.e.,*

$$X^*(b,\sigma) > X^*_{-\{J_i,i\}}(b,\sigma) + \sum_{j \in J_i} \tilde{x}_j(b,\sigma \mid J_i),$$

but for each $j \in J_i$, the trade in the "latent" contracts is larger than equilibrium, $\tilde{x}_j(b,\sigma \mid J_i) > x^*_j(b,\sigma)$.

Because the input submitted in the "latent" contracts is larger than the equilibrium allocation, any supplier in $j \in J_i$ will also demand a larger transfer $\tilde{T}_j$. However, because in equilibrium a supplier must obtain the same payoff with the "latent" contract and its equilibrium contract, implies that the transfer demanded in the "latent" contract only pays for the increased cost of producing a larger input, i.e.,

$$\tilde{T}_j = T_j^* + \big(C_j(\tilde{x}_j \mid \cdot) - C_j(x_j^* \mid \cdot)\big) \quad \forall\, j \in J_i. \tag{3.3}$$

With the form of the "latent" contracts specified in expressions (3.2) and (3.3), I proceed to characterise the equilibrium transfer T_i^*. To this aim, I make use of a general result in the common agency literature, that of *individual excludability.* This means the buyer obtains the same equilibrium payoffs when transacting with supplier i than when it decides to exclude this supplier from trade. This result can be found in Bernheim and Whinston (1986) and in the fundamental equations of Laussel and LeBreton (2001). In my model, such fundamental equations are

$$U(X^* \mid b) - \sum_i T_i^* = U\left(X^*_{-\{J_i,i\}} + \sum_{j\in J_i} \tilde{x}_j(b,\sigma|J_i) \,\Big|\, b\right) - \sum_{j\in N\setminus\{J_i,i\}} T_j^* - \sum_{j\in J_i} \tilde{T}_j, \quad \forall i \in N. \tag{3.4}$$

The left-hand side is the equilibrium payoff of the buyer. The right-hand side stands for the payoff that the buyer obtains by excluding supplier i from trade. The buyer accepts the equilibrium contracts for those suppliers who do not offer "latent" contracts and the "latent" contracts from the set of suppliers in J_i. In an online Appendix, I show that there are no other combinations of contacts that gives a larger payoff to the buyer. The intuition of this result comes from the fact that the "latent" contracts are designed to optimally replace supplier i. Then, this right-hand side constitutes the available outside option of the buyer following the exclusion of supplier i.[6]

Summing up (3.3) with all the suppliers in J_i and introducing this result into expression (3.4) gives an equilibrium transfer for supplier i equal to:

$$\begin{aligned} T_i^*(b,\sigma\,|\,J_i) &= U(X^* \mid b) - U\left(X^*_{-\{J_i,i\}} + \sum_{j\in J_i}\tilde{x}_j(b,\sigma|J_i)\Big| b\right) + \sum_{j\in J_i}\big[C_j(\tilde{x}_j(b,\sigma|J_i)) - C_j(x_j^*(b,\sigma))\big] \\ &= TS^*(b,\sigma) - D_{J_i}\left(X^*_{-\{J_i,i\}} \mid b,\sigma\right) + C_i(x_i^*(b,\sigma) \mid J_i), \end{aligned} \tag{3.5}$$

[6] A similar equilibrium condition is shown in p.100 of Laussel and LeBreton (2001).

where $TS^*(b,\sigma)$ represents the maximum gains from trade as expressed in (2.4), and expression

$$D_{J_i}\left(X^*_{-\{J_i,i\}}\,|\,b,\sigma\right) = U\left(X^*_{-\{J_i,i\}}+\sum_{j\in J_i}\tilde{x}_j\Big|b\right)-\left(\sum_{j\in J_i}C_j(\tilde{x}_j(b,\sigma|J_i))+\sum_{j\in N\setminus\{J_i,i\}}C_j(x^*_j(b,\sigma|J_i))\right),$$

illustrates the gains from trade that can be generated with the rest of the suppliers when supplier i is excluded from trade. Notice that the equilibrium transfers always pays for the cost of production $C_i(x^*_i(b,\sigma)\mid J_i)$. This results from the suppliers' superior bargaining position form offering the trading contracts to the buyer. Hence, the equilibrium transfers can only be constrained by the surplus that the buyer can generate with the rest of suppliers. In some respect, the equilibrium transfer measures the degree of how indispensable the supplier is in the trading relationship.

In principle, there is a plethora of equilibria depending on both the number and the identity of the suppliers who offer "latent" contracts. In eliciting the identity of the suppliers who submit "latent" contracts, I introduce the following assumption

Assumption 2. *(No crossing) The marginal cost for producing an amount larger than efficiency is:*

$$C_x(x^*_1+\epsilon\mid\sigma) < C_x(x^*_2+\epsilon) \leq C_x(x^*_3+\epsilon) \leq \cdots \leq C_x(x^*_N+\epsilon),\;\;\forall\,\epsilon\geq 0.$$

The first strict inequality comes from the regularity condition $C_{x\sigma}(\cdot)<0$. The rest of inequalities are assumed for exposition simplicity.[7] I construct equilibria where the set of suppliers in J_i is made up by those suppliers who are more efficient in producing an input arbitrarily larger than their equilibrium allocation as presented in Assumption 2. This guarantees that the outside option available to the buyer is maximised given a cardinality of the set $\mid J_i \mid$. More formally:

$$J_i := \left\{j = \arg\min\left[C_j(x^*_j+\epsilon\mid\cdot)\setminus\{i\}\right]\;\mid\;\mid J_i\mid\;\text{ and }\epsilon>0\right\}.$$

To help clarification, the next example gives the identity of the suppliers submitting "latent"

[7] The same assumption is considered in Chiesa and Denicolò (2009) but without the investment from supplier 1.

contracts in an equilibrium with a cardinality for each of the sets equal to three.

Example 1. *Consider an equilibrium of the trading game where the payoff for each supplier i is constrained by three suppliers submitting "latent" contracts, i.e., $|J_i| = 3$ for all $i \in N$. With assumption 2, for any supplier $i = 4, ..., N$, the set of rival suppliers submitting "latent" contracts competing for the equilibrium allocation of supplier i is $J_i = \{1, 2, 3\}$. For supplier $i = 1, 2, 3$, the set of suppliers are $J_1 = \{2, 3, 4\}$, $J_2 = \{1, 3, 4\}$ and $J_3 = \{1, 2, 4\}$ respectively.*

Observe that, because supplier 1 is the most efficient supplier, due to its ability to invest, it always belongs to the set of suppliers offering "latent" contracts. Additionally, given that the rest of suppliers are identical, the same "latent" contract offered by supplier 1 is used to generate the equilibrium transfer for all the competing suppliers. For an equilibrium where the cardinality of the set is one, Chiesa and Denicolò (2009), show (in their Proposition 3) that only the first and the second most efficient suppliers submit a "latent" contract, i.e., $J_1 = \{2\}$ and $J_i = \{1\}$ for all $i \neq 1$.

Having indemnified the identity of suppliers offering "latent" contracts, it is important to analyse how the equilibrium transfers evolve with an increase in the number of suppliers who submit "latent" contracts. In my model, I construct different equilibria based on the number of suppliers offering "latent" contracts, i.e., the cardinality of the set J_i for all $i \in N$. The next Lemma identifies the changes in input allocation and the equilibrium transfers as a result of an increase in the number of suppliers submitting "latent" contracts.

Lemma 3. *For a given investment profile (b, σ), the trading allocation in the "latent" contracts and the equilibrium transfers are nonincreasing with the number of suppliers who offer "latent" contracts.*

The result stated in the Lemma is because the more suppliers offer "latent" contacts, the easier it becomes to substitute the equilibrium trading allocation from any excluded supplier. Then, each of the suppliers offers a lower trading allocation in their "latent" contracts. This result, together with the convexity of the production function, makes the outside option available to the buyer after excluding a supplier to be also increasing with the number of suppliers who offer "latent" contracts. This translates into lower equilibrium transfers for the suppliers.

With the recollection of the pervious results, the equilibrium payoffs of the trading game are:

Proposition 1. *For a given set of suppliers in J_i and an investment profile (b, σ):*

i) Suppliers' equilibrium payoffs are

$$\pi_1 (b, \sigma \mid J_1) = \underbrace{TS^*(b, \sigma) - \tilde{TS}_{-1}(b, \sigma \mid J_1)}_{\textit{Contribution to the surplus}} - \psi_1(\sigma); \quad \textit{for } i = 1, \tag{3.6}$$

$$\pi_i (b, \sigma \mid J_i) = TS^*(b, \sigma) - \tilde{TS}_{-i}(b, \sigma \mid J_i); \qquad \textit{for } i \neq 1. \tag{3.7}$$

The buyer obtains an equilibrium payoff

$$\pi_0 (b, \sigma \mid J_i) = TS^*(b, \sigma) - \sum_i \Big(TS^*(b, \sigma) - \tilde{TS}_{-i}(b, \sigma \mid J_i)\Big) - k \times b, \tag{3.8}$$

where $\tilde{TS}_{-i}(b, \sigma \mid J_i)$ is the trading surplus that can be generated without supplier i.

ii) The larger the number of suppliers in J_i redistribute rents from suppliers to the buyer.

In equilibrium each supplier obtains its contribution to the surplus which relates to the loss of the trading surplus originated from its exclusion to trade. The loss from exclusion is then determined by the buyer's outside option which depends on the set of suppliers who offer "latent" contracts. Because the trading surplus generated without supplier i increases with the number of suppliers in the set J_i, the rents of the suppliers are redistributed in favour of the buyer the more the number of suppliers offering "latent" contracts. In this regard, the model identifies a link between the number of suppliers offering "latent" contracts and the level of competition in the trading game. Competition is the most intense when all suppliers submit "latent" contracts. Here, my model reproduces the result in Laussel and LeBreton (2001) in which the solution of their fundamental equations generates the so-called "truthful equilibrium". In this equilibrium, each supplier obtains its marginal contribution to the surplus.[8] Conversely, the least competitive equilibrium emerges when the set of suppliers in J_i is a singleton. This generates the minimum rent equilibrium considered in Chiesa and Denicolò (2009) where the rent of the buyer is minimised. This discussion suggests that suppliers' bargaining position in the trading game is affected by the number of suppliers who submit "latent" contracts.

Definition 3. ***(Competition)***. *For a given investment profile, an outcome in the trading game is more competitive, generating a lower bargaining position of the suppliers, the larger the number of suppliers submitting "latent" contracts.*

The definition of the concept of competition and the subsequent distribution of the gains from trade are crucial to characterise the equilibrium investment profile. The next section tackles this question.

4 Investing game

I start the analysis with the form of the efficient investment profile. I later use this result to identify the type and magnitude of inefficiencies the equilibrium decisions may bring about. The equilibrium played in the trading game has is fundamental in eliciting the potential investment inefficiencies and show (see Proposition 2) that only when the market trading is structured so that competition among suppliers is the most intense in the can an efficient investment profile be implemented.

Efficient investment

Given a trading allocation, the efficient vector of investment (b^*, σ^*) is such that the trading surplus minus the investment costs are maximised:

$$\psi_\sigma(\sigma^*) = -C_\sigma\left(x_1^*(b, \sigma_b^*) \mid \sigma_b^*\right), \qquad \forall\, b; \tag{4.1}$$

[8] This result is stated in Proposition 3.3 in Laussel and LeBreton (2001) who argue that if the cooperative game is strongly sub-additive, then there is a unique solution of the fundamental equations such that each principal obtains its marginal contribution to the surplus.

$$k \begin{cases} \leq TS^*(1,\sigma_1^*) - TS^*(0,\sigma_0^*) - (\psi(\sigma_1^*) - \psi(\sigma_0^*)) \equiv K^*, & \text{then } b = 1; \\ > K^*, & \text{then } b = 0, \end{cases} \tag{4.2}$$

where under-scripts stand for partial derivatives, and the under-script on the supplier's investment state the investment of the buyer (e.g., σ_1^* is the efficient investment of the supplier when the buyer invests, i.e., $b = 1$). Expression (4.1) tells that the supplier invests until the marginal reduction on the production costs, in the right-hand side, equals its marginal cost of investment. Similarly, in expression (4.2) the buyer invests if the fixed cost of investment K is below the increase in welfare arising from its investment, represented with the threshold K^*. This threshold incorporates the gains in the trading surplus minus the increase in the cost of investment.

The regularity conditions guarantee that the vector (b^*, σ^*) exists and is unique. Moreover, a characteristic of the efficient investment profile is the strategic complementarity of investments: the more one party invests, the higher the incentives of the other party to increase investment. Investment complementarity comes from a variant of super-modularity, and the reason comes from the results stated in Lemma 1. The investment of one part increases the total amount of trade. Then, the value of investment from one party increases the marginal return of the other's party investment.

Equilibrium investment

For the analysis of the equilibrium investment profile, I first consider the investment decision of the supplier given the investment of the buyer and show that the supplier's investment crucially depends on the number of competing suppliers offering "latent" contracts (see Lemma 4). I later obtain the investment choice for the buyer and how this is affected by the intensity of competition (see Lemma 5 and Lemma 6).

In any investment game, investment decisions depend on the share of gains that the investor can appropriate. Lemma 4 below demonstrates that only in situations where the market trading is structured, so that competition among suppliers is the most intense, as considered in definition 3, is the investment from the supplier efficient. At any other equilibrium, the supplier over-invests. To understand the intuition of this finding, I analyse how the supplier's investment affects the gains from trade that the buyer generates with the rest of suppliers after exclusion of the former.

Then, when competition in the trading game is the most intense, i.e., $J_1 = N \setminus \{1\}$, the amount of trade offered in the "latent" contracts

$$\tilde{x}_j(b) = \arg\max_{x_j \geq 0} \left[U \left(X_{-\{j,1\}} + x_j \,\middle|\, x_1 = 0, b \right) - C_j(x_j) \right] \quad \forall\, j \in J_1, \tag{4.3}$$

does not depend on the supplier's investment, and the gains from trade generated after excluding the investing supplier, $TS^*_{-1}(b|J_1)$, are also independent on the supplier's investment. As a result, the bargaining position of the investing supplier stays constant, and it only appropriates its marginal contribution to the surplus

$$\pi_1(b, \sigma | J_1) = \underbrace{TS^*(b, \sigma) - TS^*_{-1}(b|J_1)}_{\text{Marginal contribution}} - \psi_1(\sigma)$$

Because the supplier appropriates only the benefits that its investment generates, it has the incentives to invest efficiently. However, notice that the supplier's investment decision is contained efficient as it depends on the investment from the buyer.

Efficiency in the supplier's investment decision does not apply when only a subset of suppliers $j \subset N \setminus \{1\}$ offer "latent" contracts. In this case, the trading amount offered in the "latent" contract

$$\tilde{x}_j(b, \sigma) = \arg\max_{x_j \geq 0} \left[U \left(X^*_{-\{J_1,1\}}(b, \sigma) + \sum_{j \in J_1} x_j \,\middle|\, x_1 = 0, b \right) - C_j(x_j) \right] \quad \forall\, j \in J_1, \tag{4.4}$$

now depends on the supplier's investment. To gain intuition of the result observe that the equilibrium allocation $X^*_{-\{J_1,1\}}(b, \sigma)$ of those suppliers not submitting "latent" contracts is a function of the investment profile (b, σ). The equilibrium allocation of rival suppliers decreases with the supplier's investment, (remember Lemma 1), and this generates a larger bargaining position for the investing supplier. The reduction in the equilibrium allocation pushes the set of suppliers in J_1 to offer a larger amount of trade in their "latent" contracts, and due to the convexity of the cost function, it now becomes more costly to substitute the equilibrium allocation of the excluded supplier. Therefore, the gains from trade generated without the infesting supplier shrink. This result is also verified by Proposition 1, showing that for any $J_1 \subset J_1'$, then $TS^*_{-1}(b,\sigma|J_1) < TS^*_{-1}(b,\sigma|J_1')$.

All these make the supplier appropriate more than its marginal contribution to the surplus

$$\pi_1(b,\sigma|J_1) = \underbrace{TS^*(b,\sigma) - TS^*_{-1}(b,\sigma|J_1)}_{\text{More than marginal contribution}} - \psi_1(\sigma),$$

and it has incentives to over-invest. This is because of the larger the investment, the more substantial its bargaining position becomes. Moreover, the degree of over-investment will depend own the sensitivity of investment in the equilibrium allocation of the competing suppliers. The next Lemma summaries the previous discussion and formally states the supplier's investment decision.

Lemma 4. *For a given investment of the buyer:*

i) The supplier invests efficiently in a market trading structured so that competition among suppliers is the most intense.

Otherwise, the supplier over-invests in a magnitude equal to

$$\gamma(J_1) = -\sum_{m \notin \{J_1,1\}} \left(\int_{X^*(b,\sigma)}^{X^*_{-\{J_1,1\}}(b,\sigma)+\sum_{j\in J_1}\tilde{x}_j(b,\sigma|J_1)} U_{xx}(\tau)d\tau \right) \frac{dx^*_m}{d\sigma}. \tag{4.5}$$

ii) The magnitude of over-investment decreases with competition, i.e., $\gamma(J_1) \geq \gamma(J_1')$ *for* $J_1 \subseteq J_1'$.

In addition to the effect that the allocative sensitivity has on the investment decision of the buyer as previously discussed, it is interesting to see also that for a given investment of the buyer, the magnitude of over-investment monotonically decreases with the level of competition in the market. This is because with more suppliers submitting "latent" contracts, the lower is the effect of the reduction in the equilibrium allocation to the trading surplus generated without a given supplier. Then, the investment of the suppliers experiences a lower effect in the determination of its bargaining position which gives less incentives to over-invest. It is also important to observe that the suppliers never suffers from the problem of being held-up. In equilibrium at least it appropriates its marginal contribution to the surplus. This result comes from the design of the game in which the suppliers submit trading contracts to the buyer.

With the unilateral investment decisions of the supplier, I proceed to analyse the buyer's investment. The buyer decides to invest if the fixed costs of investment k are below the gains

$$\hat{K}(J) \equiv TS^*(1, \sigma_1(J_1)) - TS^*(0, \sigma_0(J_1)) - \sum_{i \in N} (T_i^*(1, \sigma_1 \mid J_i) - T_i^*(0, \sigma_0 | J_i)) \tag{4.6}$$

it appropriates. The first part is the total gains from trade as a result of the buyer's investment. The second part represents the changes in the suppliers' equilibrium transfers. With a fixed investment of the supplier, the buyer never over-invests. In my model, suppliers offer the trading contracts to the buyer, and the latter is always forced to share with the suppliers a partition of the rents resulting from its investment. This generates the buyer to under-invests for some of its fixed investment costs. Also, when supplier's investment remains fixed, the more intense competition in the trading game becomes, the larger the incentives for the buyer to invest. It is precisely the competition between supplier that generate positive rents to the buyer, and these rents increase with the intensity of competition. The following lemma summarises the discussion and illustrates under which parameters of the fixed investment costs does the buyer under-investments.

Lemma 5. *For a given investment of the supplier, the buyer never over-invests, and under-invests for a cost* $k \in \left(\hat{K}(J), K^*\right]$*. The investment threshold* $\hat{K}(J)$ *is not decreasing with the intensity of competition.*

The Lemma states the problem of being held-up: the inability to appropriate the gains that originate from investment makes the buyer not to invest in situations when it will be efficient to do so. Also, for a given investment of the supplier, the problem of being held-up decreases with the intensity of competition in the trading game. More competition reduces the suppliers' bargaining position which gives to the buyer a larger proportion of the gains from trade and a larger return from its investment.

How the intensity of competition affects the equilibrium investment profile is more involved. To see this, consider a situation where competition becomes less intense. As just argued, less competition gives a larger partition of the gains from trade to suppliers, generating an increase (decrease) in the incentives for the supplier (buyer) to invest. However, a larger investment by the supplier translates also into changes in the relative bargaining position of the suppliers. More supplier's investment makes the investing supplier to becomes more efficient with respect to the other suppliers. As a result, it enjoys a larger market power relative to the competing suppliers, whose bargaining position shrinks. To see this, remember that a supplier's bargaining position depends on the available outside option of the buyer after its exclusion from trade. With more investment from the supplier, the gains from trade with this supplier and the buyer increase. This translates into a bigger outside option for the buyer, and a reduction of the bargaining position for the rest of the suppliers. The buyer has larger incentives to invest. On the other hand, the crowding-out effect that the investment of the supplier generates on the equilibrium allocation of the competing suppliers reduces the outside option of the buyer after excluding the investing supplier. The resulting increased bargaining position of the supplier reduces the incentives for the buyer to invest. The next Lemma states that the evolution of the buyer's investment threshold depends on the allocative sensitivity.

Lemma 6. *The change of the buyer's investment threshold $\hat{K}(J)$ with respect to competition depends on the magnitude of the allocative sensitivity.*

i) With a small enough allocative sensitivity, the investment threshold is not decreasing with the intensity of competition.

ii) Otherwise, buyer's investment threshold fails to be monotone with competition.

The second part of the Lemma illustrates the strategic complementarity of investment, and as I will later show in Proposition 3, precisely this strategic complementarity, that was always the case under the effect investment rule, may generate larger welfare in equilibria when the competition in the trading game is less intense. With all these results, the equilibrium investment profile is:

Proposition 2. *Only in the most competitive equilibrium can efficient investment be implemented. In any other equilibrium:*

i) When the buyer invest efficiently the supplier over-invests.

ii) When the buyer under-invest:

- The supplier over-invests for an allocative sensitivity

$$-\frac{dx_m^*}{d\sigma} > \frac{\int_{x_1^*(1,\sigma_1(J_1))}^{x_1^*(0,\sigma_0(J_1))} C_{x\sigma}(\tau)d\tau}{(N-(\mid J_1 \mid +1))\left(\int_{X^*(0,\sigma_0(J_1))}^{X^*_{-\{J_1,1\}}(0,\sigma_0(J_1))+\sum_{j\in J_1}\tilde{x}_j(0,\sigma_0(J_1)|J_1)} U_{xx}(\tau)d\tau\right)} = \lambda(J_1),$$

and under-invests otherwise.

The result shows that the introduction of competition to one side of the market per se is not a guarantee that the efficient investment profile will be implemented. The intensity of competition is also essential. Then, ex-ante efficiency can be achieved only in a situation where the competition in the trading game is the most intense. It this case, the investment of the supplier is constrained efficient as it appropriates its marginal contribution to the surplus. Then, if the buyer also invests efficiently, the efficient investment profile is implemented. The Proposition also states situations when both parties under-invest (see Figure 1). However, the origin of under-investment is very different from each of the investing parties. The under-investment of the buyer emerges as a result of the hold-up problem, i.e., the inability to appropriate all the gains arising from its investment reduces the incentives for the buyer to invest. Observe in the figure that as the intensity of competition decreases the lower are the incentives for the buyer to invest. The underinvestment of the supper it is never the result of being held-up. This is because in any equilibrium the supplier appropriates at least its marginal contribution of its investment. Under-investment emerges because efficiency

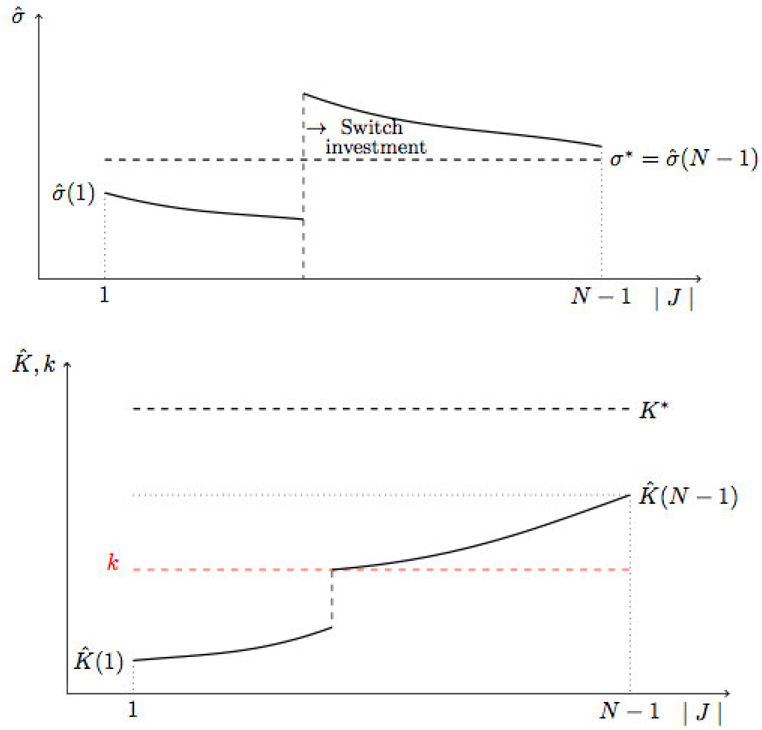

Figure 1: Equilibrium investment as a function of competition when the allocative sensitivity is small.

may require the buyer to invests, and the lower gains from trade generated due to the negative from the buyer to investors explain the little investment of the supplier. This is illustrated by the discreet jump in the supplier's investment in Figure 1. Observe that the equilibrium shown in the Figure only happens with a small allocation sensitivity, ensuring that the investment threshold of the buyer is always monotone with the intensity of competition.

A different equilibrium profile is depicted in Figure 2. The supplier always over-invests when the competition in the trading game is not the most intense. This occurs when the allocative sensitivity is high enough. Then, the substantial investment of the supplier and the subsequent reduction in the bargaining position for the competing suppliers makes the investment threshold of the buyer not monotone with the intensity of competition (see Lemma 6). In the Figure, the buyer does not invest in intermediate levels of competition and decides to invests when competition is very intense or very low. With large competition, the bargaining position of all supplier is small. With a little intensity of competition the gains from trade increase, due to a larger investment of the supplier,

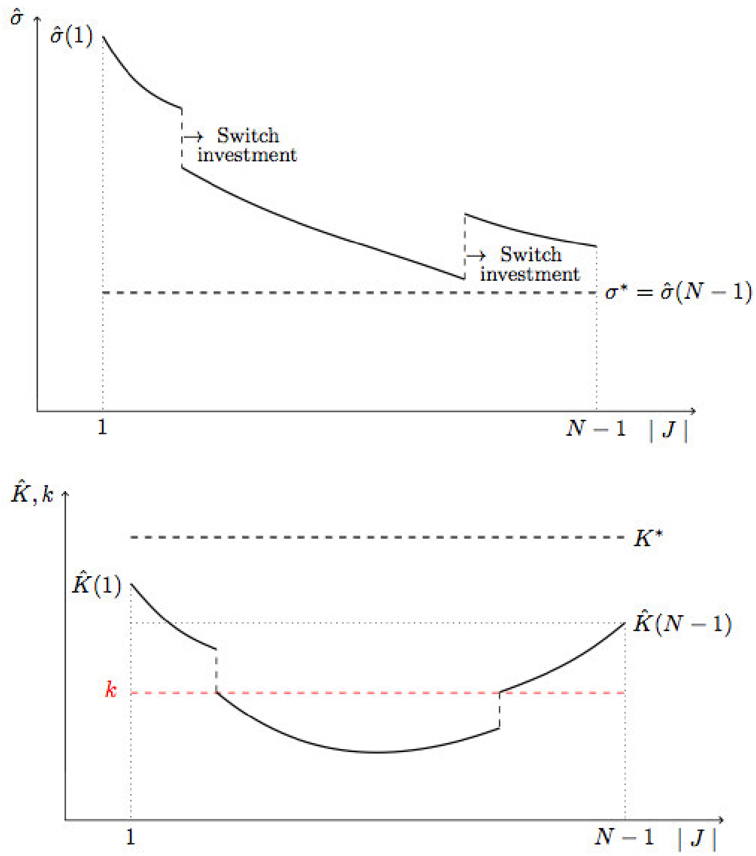

Figure 2: Equilibrium investment as a function of competition when the allocative sensitivity is large.

and the bargaining position of the non-investing suppliers goes in favour of the buyer.

5 Welfare analysis

The previous analysis has identified the equilibrium investment profile when a group of suppliers competes for a common buyer. The study suggests that the introduction of competition is a prerequisite to provide both parties with the right incentives to invest. However, the efficient investment profile only emerges when the trading game is structured so that competition is the most intense. An interesting question is the study of the welfare that can be generated in the market. Because my model abstracts from the existence of consumers who purchase the product manufactured the buyer, the measure of welfare that I take equals to the gains from trade minus the investment costs, i.e.,

$$\hat{W}(\hat{b}, \hat{\sigma}) = TS^*(\hat{b}, \hat{\sigma}) - k \times \hat{b} - \psi_1(\hat{\sigma}). \tag{5.1}$$

A clear result from Proposition 2 is that welfare is maximised in the most intense competition when the buyer invests efficiently. This is because $\hat{b} = b^*$ and the investment condition for the supplier is $\psi_\sigma(\hat{\sigma}) = -C_\sigma \left(x_1^*(b, \hat{\sigma}_{b^*}) \mid \sigma_{b^*}\right)$. In any other equilibria of the trading game, the supplier does not invest efficiently (see Lemma 4) and welfare is accordingly reduced. Nevertheless, in those situations when the buyer does not take the efficient investment decision in the most competitive equilibria, i.e., when the fixed cost of investment is $k \in \left(\hat{K}(N-1), K^*\right]$, an intermediate level of competition can maximise welfare. This is because the investment of the supplier reallocates the relative bargaining position of the competing suppliers, and the investment inefficiencies created to one side of the market may restore the efficient investment decision on the other side. The reduction of the bargaining position of the suppliers as a result of investment more than compensates the increased bargaining position of the investing supplier. Remember from Lemma 6 that the investment threshold of the buyer is not monotone with the intensity of competition when the allocative sensitivity is large. I then state the result:

Proposition 3. *Welfare is maximised with an intermediate level of competition when:*

i) Buyer's investment is inefficient in the most competitive equilibrium, and *ii) The investment threshold $\hat{K}(J)$ fails to be monotone with the intensity of competition.*

Otherwise, welfare is maximised with the most intense competition.

The proof of this Proposition is simple and emerges with the use of previous results. The first condition in point (i) states that the investment of the buyer need not be efficient in the most competitive equilibrium. If it was the case, and because the supplier invests efficiently, welfare will be maximised in the most competitive equilibria. Also, point (ii) states that the investment threshold from the supplier need not be monotone with competition. If it was monotone, then if the buyer did not invest in the most competitive equilibrium, it will never do so in equilibria that are less competitive. Again because of the over-investment of the supplier increase with less competition, welfare will be maximises in the most competitive equilibrium.

Figure 3 illustrates the result of the Proposition. Note first that the maximum level of welfare

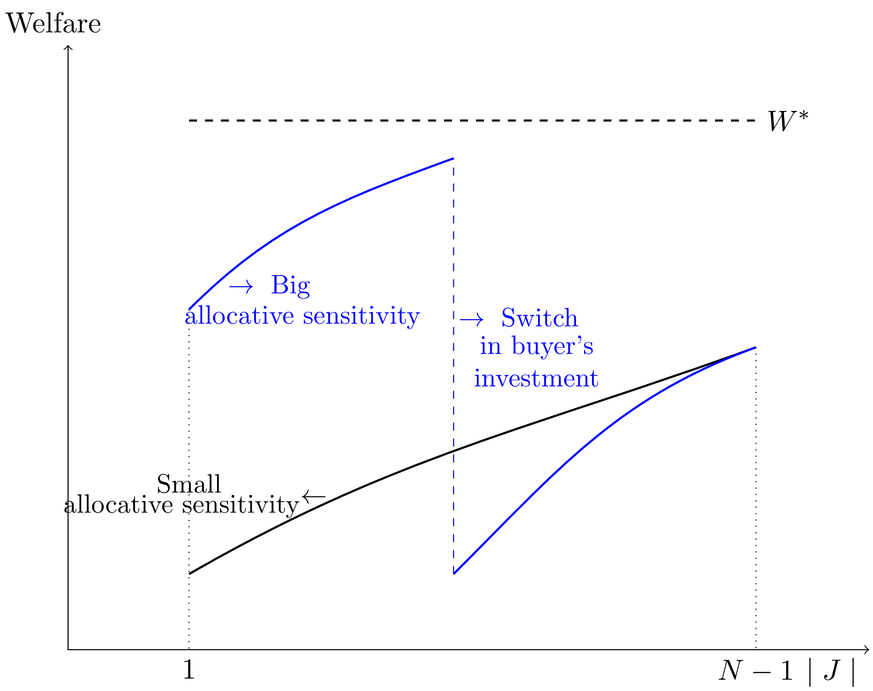

Figure 3: Welfare as a function of the intensity of competition

W^* is the same regardless the competition of the trading game: with an efficient investment profile (b^*, σ^*), the redistribution of the gains from trade does not affect welfare. This does not happen in equilibrium, where the level of competition and the subsequent partition of the gains from trade determine the investment profile. In Figure 3, I depict a situation where the efficient investment profile is not implemented in the most competitive equilibria. Then, two things can happen. When the allocative sensitivity is small, the investment threshold of the buyer is monotone with competition, and the less competitive the equilibrium becomes, the larger the inefficiency. In this case, the buyer never takes the efficient investment decision, and the over-investment from the supplier increases with less competition. The grey line illustrates this situation. The situation is different with a large enough allocative sensitivity. In this case, a more substantial investment of the suppliers significantly reduces the bargaining position of the competing suppliers, and as shown in point (ii) of Lemma 6 the investment threshold of the supplier fails to be monotone. Only in this situation, the over-investment from the supplier can restore the efficient investment of the buyer. When the buyer's investment generates enough gains from trade, is welfare maximised for an intermediate level of competition. This result is illustrated in the blue line where the discreet jumps stand for the buyer's investment decision.

6 Conclusion

This article has considered a bilateral investing game where suppliers compete by submitting trading contracts to a common buyer. I show that when the market trading is structured, so that competition among suppliers is the most intense, each supplier obtains its marginal contribution to the trading surplus. In this case, the investment of the supplier does not affect its bargaining position, and the buyer obtains a large partition of the gains from trade. This results in both parties to invest efficiently. This finding does not extend when the competition in the trading game is less intense. Now, the investment of the supplier redistributes the bargaining position and its dominant position increases with investment. This gives the supplier incentives to over-invest, and the investment decision of the buyer may fails to be monotone with the intensity of competition. Then, the introduction of competition to one side of the market alone is not sufficient to solve the hold-up problem. The study of the intensity of competitions is crucial. However, the article offers a solution of the hold-up problem in situations where ex-ante contracts cannot be designed or implemented.

The current model has studied the situation where only one of the suppliers invests in a cost-reducing technology. An extra layer of complexity emerges when all suppliers can reduce their production cost with investment. The investment decisions of the buyer and suppliers remain strategic complements, but the investment decisions among suppliers are strategic substitutes. I conjecture that strategic substitutability among suppliers' investment is of second order. Hence, in equilibrium investment complementarity persists and full efficiency can still be implementable in the most competitive equilibrium. However, the introduction of stability conditions guaranteeing that a supplier's response to a change in the investment from another supplier is below unity will be necessary. While the study of the introduction of extra suppliers is studied in a companion paper (see Roig, 2014), a market structure without a monopolistic buyer in which suppliers can sign multiple bilateral contracts with different buyers is harder to study. Despite the complexity of the equilibrium trading contracts that may emerge in this new environment, I expect the competitive advantage that the buyer obtains from investment to generate over-investment from the buyers. However, this formulation, whereas being in many respects more realistic, it is outside the scope of the current article where I limited my analysis in introducing competition to the side of the market. I leave the study with competition in both markets for further research.

Appendix

Proof. of Lemma 1.

I start by showing how the investment of supplier 1 affects the equilibrium allocation. I consider the case where the buyer decides not to invest, i.e., $b = 0$ but the proof is analogous for $b = 1$. For simplicity, I substitute $U(X^* \mid b = 0)$ for $U(X^*)$ in the analysis that follows.

Then, differentiating conditions (3.1) for x_j^* and $j \neq i$ with respect to σ gives:

$$U_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C_{xx}(x_j^*) \times \frac{dx_j^*}{d\sigma}. \tag{6.1}$$

Because the left hand side is independent of j all $dx_j^*/d\sigma$ have the same sign. Now suppose also that $dx_1^*/d\sigma$ has that same sign. Then also the sum has that same sign and because $U_{xx}(\cdot) < 0$ and $C_{xx}(\cdot) > 0$ this leads to a contradiction. Now suppose $dx_1^*/d\sigma < 0$. The other signs therefore have to be positive. By (6.1) I find that $\sum_{h=1}^N dx_h^*/d\sigma < 0$. But conditions (3.1) for x_1^*, differentiated with respect to σ gives

$$U_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C_{xx}(x_1^* \mid \sigma) \times \frac{dx_1^*}{d\sigma} + C_{x\sigma}(x_1^* \mid \sigma), \tag{6.2}$$

which would then have a positive left-hand side and a negative right-hand side due to $C_{x\sigma}(\cdot) < 0$ - a contradiction. Thus I have shown the first and the second part of the point (i) of the Lemma.

Again by (6.1) the last claim follows from $\partial X^*/\partial\sigma = \sum_{h=1}^N dx_h^*/d\sigma$. I proceed by analysing the effect that the investment of the buyer has on the equilibrium allocation. Again, I am going to make use of the conditions for the equilibrium allocation represented in equation (3.1), and for a fixed investment of the supplier, I obtain:

$$C_x(x_1^* \mid \sigma) = U_x(X^* \mid b = 1) > U_x(X^*) = C_x(x_1^* \mid \sigma) \qquad \text{for} \quad 1,$$

$$C_x(x_j^*) = U_x(X^* \mid b = 1) > U_x(X^*) = C_x(x_j^*) \qquad \text{for} \quad j \neq 1.$$

The strict inequality is by the assumptions of the model and by the convexity of the cost function.

Proof. of Lemma 2.

The first part of Lemma 2 claims that $X^*(b,\sigma) > X^*_{-\{J_i,i\}}(b,\sigma) + \sum_{j\in J_i} \tilde{x}_j(b,\sigma \mid J_i)$. To simplify notation, in what follows I eliminate the investment profile (b,σ) from the analysis.

Because $\sum_{h\neq J_i,i} x_h^* = X^*_{-\{J_i,i\}}$, the expression above is equivalent to $\sum_{j\in J_i} x_j^* + x_i^* > \sum_{j\in J_i} \tilde{x}_j(J_i)$. The regularity conditions assumed in the model imply that $x_i^* > 0$, and if $\sum_{j\in J_i}\left(x_j^* - \tilde{x}_j(J_i)\right) > 0$ the result is shown. For a given investment profile, if the previous is true, it has to be true for any supplier $j \in J_i$. If $x_j^* > \tilde{x}_j(J_i)$ the claim is proved. If the contrary occurs, $x_j^* < \tilde{x}_j(J_i)$, then from the efficient allocation, it has to be the case that

$$U_x\left(X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i)\right) = C_x(\tilde{x}_j(J_i)) > C_x(x_j^*) = U_x(X^*), \tag{6.3}$$

and by the concavity of $U(\cdot)$, the claim is true. Expression (6.3) also implies that for any $j \in J_i$, then $\tilde{x}_j(J_i) > x_j^*$, and this shows the second part of the Lemma.

Proof. of Lemma 3.

To show that the allocation in the trading contracts is non-increasing with the number of suppliers in the set J_i, I demonstrate that for $J_i \subseteq J_i'$ then $\tilde{x}_i(J_i) \geq \tilde{x}_i(J_i')$. Eliminating the profile of investments (b, σ), a similar argument as in the proof of Lemma 2 shows that if $X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i) \geq X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')$, then $\tilde{x}_i(J_i) \leq \tilde{x}_i(J_i')$. As a result, I obtain that:

$$\sum_{j \in J_i' \setminus J_i} \left(x_j^* - \tilde{x}_j(J_i')\right) + \sum_{j \in J_i} \left(\tilde{x}_j(J_i) - \tilde{x}_j(J_i')\right) \geq 0,$$

but from Lemma 2, I know that $x_j^* < \tilde{x}_j(J_i')$, and for the previous expression to be true I need $\tilde{x}_j(J_i) \geq \tilde{x}_j(J_i')$ - a contradiction. Then, the only possibility is that $X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i) \leq X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')$, which implies that $\tilde{x}_j(J_i) \geq \tilde{x}_j(J_i')$.

To show that the equilibrium transfers is also non-increasing with the number of suppliers submitting "latent" contract, i.e., for $J_i \subseteq J_i'$, then $T_i^*(J_i) \geq T_i^*(J_i')$ I make use of expression (3.5). Because for a given investment profile $TS^*(\cdot)$ and $c_i(x_i^*)$ is the same in both expressions, a necessary and sufficient condition for $T_i^*(J_i) \geq T_i^*(J_i')$ is that $D_{J_i'}\left(X^*_{-\{J_i',i\}}\right) \geq D_{J_i}\left(X^*_{-\{J_i,i\}}\right)$. Then:

$$
\begin{aligned}
D_{J_i'}\left(X^*_{-\{J_i',i\}}\right) &= \left[U\left(X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')\right) - \left(\sum_{j \in J_i'} C_j(\tilde{x}_j(J_i')) + \sum_{j \in N \setminus \{J_i',i\}} C_j(x_j^*(J_i'))\right)\right] \\
&> U\left(X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')\right) - U\left(X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i)\right) + D_{J_i}\left(X^*_{-\{J_i,i\}}\right) \\
\Longrightarrow D_{J_i'}\left(X^*_{-\{J_i',i\}}\right) - D_{J_i}\left(X^*_{-\{J_i,i\}}\right) &> U\left(X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')\right) - U\left(X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i)\right) \\
&> \int_{X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i)}^{X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')} U_x(\tau) d\tau > 0.
\end{aligned}
$$

The first strict inequality comes from the fact that it is more costly to replace the equilibrium amount from the excluded supplier the lower the number of suppliers submitting "latent" contracts. The transformation in the last line comes from then fundamental theorem of calculus and the last inequality is due to $X^*_{-\{J_i,i\}} + \sum_{j \in J_i} \tilde{x}_j(J_i) \leq X^*_{-\{J_i',i\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i')$, as shown in the first part of the proof, and the assumption $U_x(\cdot) > 0$.

Proof. of Proposition 1.

Without loss of generality, I eliminate the investment profile in the calculations that follow. Then, the payoff for each supplier i is equal to:

$$\pi_i(J_i) = T_i^*(J_i) - C_i(x_i^*),$$

and introducing the equilibrium transfers obtained in (3.5), I obtain

$$\pi_i(J_i) = TS^* - D_{J_i}\left(X^*_{-\{J_i,i\}}\right).$$

Noticing that expression $D_{J_i}\left(X^*_{-\{J_i,i\}}\right)$ stands for the maximum gains from trade generated given

$X^*_{-\{J_i,i\}}$ and without supplier i, i.e., $D_{J_i}\left(X^*_{-\{J_i,i\}}\right) = \tilde{TS}_{-i}(J_i)$, and introducing the investment profile gives the expressions (3.6) and (3.7). The equilibrium payoff for the buyer is

$$\begin{aligned}
\pi_0\left(J_i\right) &= U\left(X^*\right) - \sum_i T_i(J_i) - k \times b \\
&= U\left(X^*\right) - \left[\sum_i \left(TS^* - \tilde{TS}_{-i}(J_i) + C_i(x_i^*)\right)\right] - k \times b \\
&= U\left(X^*\right) - \sum_i C_i(x_i^*) - \left[\sum_i TS^* - \tilde{TS}_{-i}(J_i)\right] - k \times b \\
&= TS^* - \sum_i \left(TS^* - \tilde{TS}_{-i}(J_i)\right) - k \times b.
\end{aligned}$$

The proof of point (ii) is immediate from the equilibrium condition considered in Chiesa and Denicolò (2009) in their Proposition 1. They show that a vector of payoffs $(\pi_0, \pi_1, \pi_2, \cdots, \pi_N)$ is a vector of equilibrium payoffs if and only if it satisfies the condition that $\pi_0 + \pi_1 + \pi_2 + \cdots + \pi_N = TS^*$. Then, a reduction of the equilibrium transfers reduces the payoff for the supplies and increase the payoff of the buyer. In an online appendix I show that there is no profitable deviation from the equilibrium payoffs presented in the Proposition.[9]

Proof. of Lemma 4

When the market trading is structured so that competition among suppliers is the most intense, i.e., $|J_1| = N - 1$, from Proposition 1, the equilibrium payoff for supplier 1 is

$$\pi_1(b, \sigma \mid J_1) = TS^*(b, \sigma) - \tilde{TS}_{-1}(b \mid J_1) - \psi_1(\sigma),$$

where the gains from trade that can be generated without supplier 1

$$\tilde{TS}_{-1}(b \mid J_1) = U\left(\sum_{j \in J_1} \tilde{x}_j(b|J_1) \,\middle|\, b\right) - \sum_{j \in J_1} C_j(\tilde{x}_j(b)),$$

[9]The online Appendix is available upon request.

do not depend on the investment of the supplier σ. Then, by using the expression for $TS^*(b,\sigma)$ in (2.4), and by the envelope-theorem, the first-order condition of supplier 1 with respect to investment is

$$\psi_\sigma(\sigma) = -C_\sigma\left(x_1^*(b,\sigma_b)|\sigma_b\right), \quad \forall b,$$

which coincides with the efficient investment condition in (4.1). With the most intense competition, the supplier receives its marginal contribution of the trading surplus and becomes the residual claimant of its investment. As a result, it invests efficiently.

In any other equilibrium where competition is less intense, i.e., $|J_1| < N-1$, the gains from trade generated without supplier 1,

$$\tilde{TS}_{-1}(b,\sigma\,|\,J_1) = U\left(X^*_{-\{J_1,1\}}(b,\sigma) + \sum_{j\in J_1}\tilde{x}_j(b,\sigma|J_1)\,\Big|\,b\right) - \sum_{j\in N\setminus\{J_1,1\}} C_j(x_j^*(b,\sigma)) - \sum_{j\in J_1} C_j(\tilde{x}_j(b,\sigma)|J_1),$$

depend on the investment from the supplier, and the first order condition with respect to investment becomes

$$\psi_\sigma(\sigma) = -C_\sigma(x_1^*(b,\sigma_b)|\sigma_b) - \frac{\partial\left(\tilde{TS}_{-1}(b,\sigma\mid J_1)\right)}{\partial\sigma}. \tag{6.4}$$

The additional term $\partial\left(\tilde{TS}_{-1}(b,\sigma\mid J)\right)/\partial\sigma \neq 0$, creates a distortion of investment. To understand the origin of the distortion, the magnitude of the changes in the gains from trade with the exclusion of supplier 1 with respect to investment is

$$\frac{\partial\left(\tilde{TS}_{-1}(b,\sigma\mid J_1)\right)}{\partial\sigma} \equiv \sum_{m\neq J_1,1}\left(U_x\left(X^*_{-\{J_1,1\}}(b,\sigma) + \sum_{j\in J_1}\tilde{x}_j(b,\sigma|J_1)\mid b\right) - C_x(x_j^*(b,\sigma))\right) \times \frac{dx_m^*}{d\sigma} \tag{6.5}$$

From efficiency, I know that $U_x(X^*(b,\sigma)|b) = C_x(x_j^*(b,\sigma))$, $\forall j\in N$, and taking the sign from expression (6.4) gives

$$\gamma(J_1) \equiv -\sum_{m\neq J_1,1}\left(U_x\left(X^*_{-\{J_1,1\}}(b,\sigma) + \sum_{j\in J_1}\tilde{x}_j(b,\sigma|J_1)\mid b\right) - U_x(X^*(b,\sigma))\right) \times \frac{dx_m^*}{d\sigma}. \tag{6.6}$$

By applying the fundamental theorem of calculus, the amount of over-investment becomes

$$\gamma(J_1) \equiv -\sum_{m \neq J_1,1} \left(\int_{X^*(b,\sigma)}^{X^*_{-\{J_1,1\}}(b,\sigma)+\sum_{j\in J_1} \tilde{x}_j((b,\sigma)|J_1)} U_{xx}(\tau) d\tau \right) \times \frac{dx^*_m}{d\sigma} > 0, \tag{6.7}$$

and the whole expression is positive. By Lemma 2 and the concavity of the utility function the expression inside the brackets is positive. By Lemma 1, the amount traded with the rival suppliers decreases with σ, and the whole expression is positive.

To show point (ii) of the Lemma, I use a continuous approximation to show that the degree of over-investment decreases with the number of suppliers in J_1, i.e., $\partial\gamma(J_1)/\partial J_1 < 0$. Differentiating expression (6.7), and applying the Leibniz rule, I obtain

$$\begin{aligned} \frac{\partial \gamma(J_1)}{\partial J_1} = & \left(\int_{X^*}^{X^*_{-\{J_1,1\}}+\sum_{j\in J_1} \tilde{x}_j(J_1)} U_{xx}(\tau) d\tau \right) \times \frac{dx^*_m}{d\sigma} \\ & - U_{xx}\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1} \tilde{x}_j(J_1) \right) \times \underbrace{\frac{\partial \left(X^*_{-\{J_1,1\}} + \sum_{j\in J} \tilde{x}_j(J_1) \right)}{\partial J_1}}_{(+)} \times \frac{dx^*_m}{d\sigma} < 0, \end{aligned}$$

where I have erased the investment profile (b, σ) for ease of notation. The sign is due to Lemma 2 and the regularity conditions.

Proof. of Lemma 5

To demonstrate that the buyer does not over-invests, I compare the investing thresholds in equilibrium to the one under the efficient rule. Then, for a fixed investment of the supplier, the difference in the thresholds becomes

$$\hat{K}(J) - K^* = -\sum_{i\in N} \left(T^*_i(1, \sigma \mid J_i) - T^*_i(0, \sigma | J_i) \right). \tag{6.8}$$

Because the trade allocation increases with the investment of the buyer $x^*_i(1, \sigma) > x^*_i(0, \sigma)$ for all $i \in N$, as shown in lemma 1. The larger cost of production $C_i(x^*_i(1, \sigma) \mid \cdot) > C_i(x^*_i(0, \sigma) \mid \cdot)$, implies that $T_i^*(1,\sigma_1 \mid J_i) > T_i^*(0,\sigma_0|J_i)$ or all $i \in N$ for any $J_i \subset N$. All this makes $\hat{K}(J) - K^* < 0$, and the buyer under-investments when its cost of investment is $k \in \left(\hat{K}(J), K^*\right]$.

To show that the investment threshold is not decreasing with the intensity of competition I make use of the following claim:

Claim 1. *For a fixed supplier's investment and $J_i \subseteq J_i'$, then:*

$$D_{J_i'}(X^*_{-\{J_i',i\}} \mid 1,\sigma) - D_{J_i}(X^*_{-\{J_i,i\}} \mid 1,\sigma)$$
$$\geq D_{J_i'}(X^*_{-\{J_i',i\}} \mid 0,\sigma) - D_{J_i}(X^*_{-\{J_i,i\}} \mid 0,\sigma) \ \text{ for all } i \in N.$$

Proof. Operating I obtain

$$\begin{aligned}
&D_{J_i'}(X^*_{-\{J_i',i\}} \mid 1,\sigma) - D_{J_i}(X^*_{-\{J_i,i\}} \mid 1,\sigma) = \\
&\quad U\left(X^*_{-\{J_i',i\}}(1,\sigma) + \sum_{j\in J_i'} \tilde{x}_j(1,\sigma|J_i')\Big|1\right) - \left(\sum_{j\in J_i'} C_j(\tilde{x}_j(1,\sigma|J_i')) + \sum_{j\in N\backslash\{J_i',i\}} C_j(x_j^*(1,\sigma|J_i'))\right) \\
&\quad - \left[U\left(X^*_{-\{J_i,i\}}(1,\sigma) + \sum_{j\in J_i} \tilde{x}_j(1,\sigma|J_i)\Big|1\right) - \left(\sum_{j\in J_i} C_j(\tilde{x}_j(1,\sigma|J_i)) + \sum_{j\in N\backslash\{J_i,i\}} C_j(x_j^*(1,\sigma|J_i))\right)\right] \\
&\quad > U\left(X^*_{-\{J_i',i\}}(0,\sigma) + \sum_{j\in J_i'} \tilde{x}_j(0,\sigma|J_i')\Big|1\right) - U\left(X^*_{-\{J_i,i\}}(0,\sigma) + \sum_{j\in J_i} \tilde{x}_j(0,\sigma|J_i)\Big|1\right) \\
&\quad - \left[U\left(X^*_{-\{J_i',i\}}(0,\sigma) + \sum_{j\in J_i'} \tilde{x}_j(0,\sigma|J_i')\Big|0\right) - U\left(X^*_{-\{J_i,i\}}(0,\sigma) + \sum_{j\in J_i} \tilde{x}_j(0,\sigma|J_i)\Big|0\right)\right] \\
&\quad + D_{J_i'}(X^*_{-\{J_i'\}} \mid 0,\sigma) - D_{J_i}(X^*_{-\{J_i\}} \mid 0,\sigma) \ \text{ for all } i \in N.
\end{aligned}$$

The inequality comes from the inefficient allocation generated when the trading allocation is substituted by the allocation when the buyer does not invest. With the use of lemma 3, the degree of inefficiency is larger in equilibrium with the set J_i' than with J_i. Because the investment from the supplier does not change, this condition applies to all of the suppliers. Finally, by using the fundamental theorem of calculus, the previous expression is equivalent to

$$\begin{aligned} D_{J_i'}(X^*_{-\{J_i',i\}} \mid 1,\sigma) - D_{J_i}(X^*_{-\{J_i,i\}} \mid 1,\sigma) &> D_{J_i'}(X^*_{-\{J_i',i\}} \mid 0,\sigma) - D_{J_i}(X^*_{-\{J_i,i\}} \mid 0,\sigma) \\ &> \int_{X^*_{-\{J_i,i\}}(0,\sigma)+\sum_{j\in J_i}\tilde{x}_j(0,\sigma|J_i)}^{X^*_{-\{J_i',i\}}(0,\sigma)+\sum_{j\in J_i'}\tilde{x}_j(0,\sigma|J_i')} (U_x(\tau \mid 1) - U_x(\tau \mid 0))\, d\tau > 0, \end{aligned}$$

where the last inequality is due to lemma 3 and the regularity conditions assumed in the model.

□

Then, for any $J_i \subseteq J_i'$ and a fixed investment of the supplier, the difference in the buyer's investment threshold is

$$\begin{aligned} \hat{K}(J') - \hat{K}(J) &= \sum_{i\in N} (T_i^*(1,\sigma \mid J_i) - T_i^*(0,\sigma|J_i)) - \sum_{i\in N} \left(T_i^*(1,\sigma \mid J_i') - T_i^*(0,\sigma|J_i')\right) \\ &= N \times [TS^*(1,\sigma) - TS^*(0,\sigma)] - \sum_{i\in N} \left[D_{J_i}\left(X^*_{-\{J_i,i\}} \mid 1,\sigma\right) - D_{J_i}\left(X^*_{-\{J_i,i\}} \mid 0,\sigma\right)\right] \\ &\quad - N \times [TS^*(1,\sigma) - TS^*(0,\sigma)] + \sum_{i\in N} \left[D_{J_i'}\left(X^*_{-\{J_i',i\}} \mid 1,\sigma\right) - D_{J_i'}\left(X^*_{-\{J_i',i\}} \mid 0,\sigma\right)\right] \\ &= \sum_{i\in N} \left[D_{J_i'}\left(X^*_{-\{J_i',i\}} \mid 1,\sigma\right) - D_{J_i'}\left(X^*_{-\{J_i',i\}} \mid 0,\sigma\right) - \left(D_{J_i}\left(X^*_{-\{J_i,i\}} \mid 1,\sigma\right) - D_{J_i}\left(X^*_{-\{J_i,i\}} \mid 0,\sigma\right)\right)\right] > 0 \end{aligned}$$

where the last inequality is due to claim 1. Because the investment from the supplier does not change, the production cost for supplier i can be disregarded.

Proof. of Lemma 6

When the allocative sensitivity is very small, for any $J_1 \subset J_1'$, then $\sigma_b(J_1) \approx \sigma_b(J_1')$, and $TS^*(1,\sigma_1(J_1')) - TS^*(0,\sigma_0(J_1')) \approx TS^*(1,\sigma_1(J_1)) - TS^*(0,\sigma_0(J_1))$. Hence, for $J_i \subset J_i'$, the difference in the investment thresholds becomes

$$\begin{aligned} \hat{K}(J') - \hat{K}(J) = &\sum_{i\in N} \left[D_{J_i'}\left(X^*_{-\{J_i',i\}} \mid 1,\sigma_1\right) - D_{J_i'}\left(X^*_{-\{J_i',i\}} \mid 0,\sigma_0\right)\right] \\ &- \sum_{i\in N} \left[\left(D_{J_i}\left(X^*_{-\{J_i,i\}} \mid 1,\sigma_1\right) - D_{J_i}\left(X^*_{-\{J_i,i\}} \mid 0,\sigma_0\right)\right)\right] \end{aligned} \tag{6.9}$$

Because $\sigma_b(J_1) \approx \sigma_b(J_1')$, then, expression (6.9) can be expressed as in claim 1. This proves that the investing threshold of the buyer is not decreasing with the intensity of competition, i.e., $\hat{K}(J') > \hat{K}(J)$. The results are different when the allocative sensitivity is such that the invests of the supplier changes significantly with the level of competition. Lemma 4 shows that $\sigma_b(J_1) > \sigma_b(J_1')$, and the difference in the investment threshold becomes

$$\begin{aligned}
&\hat{K}(J') - \hat{K}(J) = \\
&\quad -(N-1) \times \left[TS^*(1, \sigma_1(J_1)) - TS^*(0, \sigma_0(J_1)) - \left(TS^*(1, \sigma_1(J_1')) - TS^*(0, \sigma_0(J_1'))\right)\right] \\
&+ \sum_{i \in N} \left[D_{J_i'}\left(X^*_{-\{J_i', i\}} \mid 1, \sigma_1(J_1')\right) - C_i(x_i^*(1, \sigma_1(J_1'))) - \left(D_{J_i'}\left(X^*_{-\{J_i', i\}} \mid 0, \sigma_0(J_1')\right) - C_i(x_i^*(0, \sigma_0(J_1')))\right)\right] \\
&- \sum_{i \in N} \left[D_{J_i}\left(X^*_{-\{J_i, i\}} \mid 1, \sigma_1(J_1)\right) - C_i(x_i^*(1, \sigma_1(J_1))) - \left(D_{J_i}\left(X^*_{-\{J_i, i\}} \mid 0, \sigma_0(J_1)\right) - C_i(x_i^*(0, \sigma_0(J_1)))\right)\right].
\end{aligned}$$

Defining:

$$\gamma_0(\Delta J) := TS^*(1, \sigma_1(J_1)) - TS^*(0, \sigma_0(J_1)) - \left[TS^*(1, \sigma_1(J_1')) - TS^*(0, \sigma_0(J_1'))\right];$$

$$\begin{aligned}
\gamma_1(\Delta J_1) := {}& D_{J_1'}\left(X^*_{-\{J_1', 1\}} \mid 1, \sigma_1(J_1')\right) - C_1(x_1^*(1, \sigma_1(J_1')) \mid \sigma_1(J_1')) \\
&- D_{J_1'}\left(X^*_{-\{J_1', 1\}} \mid 0, \sigma_0(J_1')\right) + C_1(x_1^*(0, \sigma_0(J_1')) \mid \sigma_0(J_1')) \\
&- D_{J_1}\left(X^*_{-\{J_1, 1\}} \mid 1, \sigma_1(J_1)\right) + C_1(x_1^*(1, \sigma_1(J_1)) \mid \sigma_1(J_1)) \\
&+ D_{J_1}\left(X^*_{-\{J_1, 1\}} \mid 0, \sigma_0(J_1)\right) - C_1(x_i^*(0, \sigma_0(J_1)) \mid \sigma_0(J_1));
\end{aligned}$$

and

$$\begin{aligned}
\gamma_i(\Delta J_i) := {}& D_{J_i'}\left(X^*_{-\{J_i', i\}} \mid 1, \sigma_1(J_1')\right) - C_i(x_i^*(1, \sigma_1(J_1'))) \\
&- D_{J_i'}\left(X^*_{-\{J_i', i\}} \mid 0, \sigma_0(J_1')\right) + C_i(x_i^*(0, \sigma_0(J_1'))) \\
&- D_{J_i}\left(X^*_{-\{J_i, i\}} \mid 1, \sigma_1(J_1)\right) + C_i(x_i^*(1, \sigma_1(J_1))) \\
&+ D_{J_i}\left(X^*_{-\{J_i, i\}} \mid 0, \sigma_0(J_1)\right) - C_i(x_i^*(0, \sigma_0(J_1))),
\end{aligned}$$

the threshold becomes

$$\hat{K}(J') - \hat{K}(J) := -(N-1)\gamma_0(\Delta J) + \gamma_1(\Delta J_1) + \sum_{i\neq 1} \gamma_i(\Delta J_i). \tag{6.10}$$

The element $\gamma_0(\Delta J)$ stands for the gains from trade as a result of the buyer's investment, and $\gamma_1(\Delta J_1)$ and $\gamma_i(\Delta J_i)$ represent the change in the suppliers' bargaining position as a result of the buyer's investment. With a small allocative sensitivity, we had $\gamma_0(\Delta J) = 0$ and both $\gamma_1(\Delta J_1)$ and $\gamma_i(\Delta J_i$ for $i \neq 1$ moved in the same direction. This implied that $\hat{K}(J') > \hat{K}(J)$. Now, to show that the threshold may not be monotone with respect to the intensity of competition I proceed by calculating a lower bound of $\gamma_0(\Delta J)$ and an upper bound for $\gamma_1(\Delta J_1)$ and $\gamma_i(\Delta J_i)$. To determine the non-monotonicity of $\hat{K}(J)$ it is crucial to study the change of the bounds with respect to the allocative sensitivity.

For the lower bound of $\gamma_0(J)$, observe that

$$\begin{aligned}
&TS^*(1,\sigma_1(J_1)) - TS^*(0,\sigma_0(J_1)) = \\
&\qquad U(X^*(1,\sigma_1(J_1)) \mid 1) - C_1(x_1^*(1,\sigma_1(J_1) \mid \sigma_1(J_1)) - \sum_{i\neq 1} C_i(x_i^*(1,\sigma_1(J_1))) \\
&\qquad - \left(U(X^*(0,\sigma_0(J_1)) \mid 0) - C_1(x_1^*(0,\sigma_0(J_1) \mid \sigma_0(J_1)) - \sum_{i\neq 1} C_i(x_i^*(0,\sigma_0(J_1))) \right) \\
&\quad > U(X^*(1,\sigma_1(J_1')) \mid 1) - C_1(x_1^*(1,\sigma_1(J_1') \mid \sigma_1(J_1)) - \sum_{i\neq 1} C_i(x_i^*(1,\sigma_1(J_1'))) \\
&\qquad - \left(U(X^*(0,\sigma_0(J_1')) \mid 0) - C_1(x_1^*(0,\sigma_0(J_1') \mid \sigma_0(J_1)) - \sum_{i\neq 1} C_i(x_i^*(0,\sigma_0(J_1'))) \right) \\
&\quad = TS^*(1,\sigma_1(J_1')) - TS^*(0,\sigma_0(J_1')) - C_1(x_1^*(1,\sigma_1(J_1') \mid \sigma_1(J_1)) + C_1(x_1^*(1,\sigma_1(J_1') \mid \sigma_1(J_1')) \\
&\qquad + C_1(x_1^*(0,\sigma_0(J_1') \mid \sigma_0(J_1)) - C_1(x_1^*(0,\sigma_0(\Delta J_1') \mid \sigma_0(J_1')) \\
&\qquad \Longrightarrow \gamma_0(J) > \int_{x_1^*(0,\sigma_0(J_1'))}^{x_1^*(1,\sigma_1(J_1'))} -C_{x\sigma}(\tau|J_1')d\tau = \underline{\gamma_0}(\Delta J).
\end{aligned}$$

The first inequality comes from an inefficient allocation of trade and the use of lemma 3. The upper bound for $\gamma_1(\Delta J_1)$ and $\gamma_i(\Delta J_i)$ are obtained by using a similar argument as in Claim 1. Then:

$$\gamma_1(\Delta J_1) < \int_{X^*_{-\{J_1,1\}}(1,\sigma_1(J_1))+\sum_{j\in J_1}\tilde{x}_j(1,\sigma_1(J_1)|J_1)}^{X^*_{-\{J_1',1\}}(1,\sigma_1(J_1'))+\sum_{j\in J_1'}\tilde{x}_j(1,\sigma_1(J_1')|J_1')} (U_x(\tau \mid 1) - U_x(\tau \mid 0))\, d\tau = \bar{\gamma}_1(\Delta J_1),$$

and

$$\gamma_i(\Delta J_i) < \int_{X^*_{-\{J_i,i\}}(1,\sigma_1(J_i))+\sum_{j\in J_i}\tilde{x}_j(1,\sigma_1(J_i)|J_i)}^{X^*_{-\{J_i',i\}}(1,\sigma_1(J_i'))+\sum_{j\in J_i'}\tilde{x}_j(1,\sigma_1(J_i')|J_i')} (U_x(\tau \mid 1) - U_x(\tau \mid 0))\, d\tau = \bar{\gamma}_i(\Delta J_i).$$

With the bound of the integral, it can be shown that $d(\gamma_0(\Delta J))/(dx_i^*/d\sigma) > 0$, $d(\gamma_1(\Delta J_1))/(dx_i^*/d\sigma) > 0$ and $d(\gamma_i(\Delta J_i))/(dx_i^*/d\sigma) < 0$. Therefore, for some values of the utility and cost function $\hat{K}(J') < \hat{K}(J)$.

Proof. of Proposition 2

From lemma 4, in the most competitive equilibrium the supplier appropriates its marginal contribution from investment and invests efficiently. With a fixed investment cost for the buyer $k \geq K^*$ and $k \leq \hat{K}(N-1)$ it invests efficiently, and ex-ante efficiency is implemented. The proof of point (i) comes directly from lemma 4. For the proof of point (ii), for any $J_1 \subset \{N \setminus \{1\}\}$, the buyer under-invests when $k \in \left(\hat{K}(J_1), K^*\right]$. Then, because it would be optimal for the buyer to invest, the efficient investment buy the supplier is given by

$$\psi_\sigma(\sigma) = -C_\sigma\left(x_1^*(1,\sigma)|\sigma\right).$$

From lemma 4, when the buyer does not invest, the investment decision of the supplier is given by

$$\psi_\sigma(\sigma) = -C_\sigma\left(x_1^*(0,\sigma)|\sigma\right) - \sum_{m\neq J_1,1}\left(\int_{X^*(0,\sigma)}^{X^*_{-\{J_1,1\}}(0,\sigma)+\sum_{j\in J_1}\tilde{x}_j((0,\sigma)|J_1)} U_{xx}(\tau)d\tau\right) \times \frac{dx_m^*}{d\sigma},$$

and over-investment occurs when

$$
\begin{aligned}
&- C_\sigma \left(x_1^*(0,\sigma)|\sigma\right) - \sum_{m\neq J_1,1}\left(\int_{X^*(0,\sigma)}^{X^*_{-\{J_1,1\}}(0,\sigma)+\sum_{j\in J_1}\tilde{x}_j((0,\sigma)|J_1)} U_{xx}(\tau)d\tau\right)\times\frac{dx_m^*}{d\sigma} > -C_\sigma\left(x_1^*(1,\sigma)|\sigma\right) \\
&\Longrightarrow -\frac{dx_m^*}{d\sigma} > \frac{\int_{x_1^*(1,\sigma_1(J_1))}^{x_1^*(0,\sigma_0(J_1))} C_{x\sigma}(\tau)d\tau}{(N-(\mid J_1\mid+1))\left(\int_{X^*(0,\sigma_0(J_1))}^{X^*_{-\{J_1,1\}}(0,\sigma_0(J_1))+\sum_{j\in J_1}\tilde{x}_j(0,\sigma_0(J_1)|J_1)} U_{xx}(\tau)d\tau\right)} = \lambda(J_1).
\end{aligned}
$$

Otherwise, the supplier will under-invest.

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