# Tax policies, informality, and real wage rigidities

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# Tax policies, informality, and real wage rigidities [*]

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**Abstract**

Developing countries have a vast informal sector generally associated with low productivity levels. The informal employment persistence could respond to rigidities in the labor market and the lack of a combination of labor policies and state capacity. This paper proposes a theoretical framework to understand the role of tax policies that discourage informality, such as lower payroll taxes in the formal sector or increased enforcement expenditure, in an economy with real wage rigidities. I develop a search and matching model with a shirking mechanism for formal and informal workers. The analytical results show that decreasing payroll taxes increases formal employment demand, and enforcement expenditure decreases informal employment offers. The model suggests that a tax policy combination leads to a significant impact on informality reduction. Furthermore, the flexibility of real wage rigidities could significantly decrease informal employment. On the other hand, the tax policies' magnitude effect depends on real wage rigidities. When the economy is in front of high real wage rigidities, the tax policies have a higher effect on informality reduction than those with low real wage rigidities. However, with fewer real wage rigidities, there is an increase in tax revenue through tax policies, affecting less formal employment.

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# 1 Introduction

The informal sector, defined as a labor force that does not comply with government regulations, has a remarkable size that persists over time. Specifically, developing countries have a vast informal sector generally related to low productivity levels (Perry, 2010). Informality is related to less pension and health coverage, workers without unemployment compensation, and less tax revenue via widespread tax evasion (Schneider & Enste, 2000). Also, the persistence of informality could be a response to rigidities in the labor market, with a combination of high non-wages costs and high minimum wages (Maloney, 2004). Some authors as Mondragón-Vélez et al. (2010) and Santa María et al. (2010) highlight the relevance of the labor market rigidities to understand the persistence of the informality.

Accordingly, this paper sheds light on understanding the tax policy's effect that seeks to reduce the informality in a case with real wage rigidities and restrictions associated with the search and matching process in the labor market. The model seeks to understand how the combination of labor policies and the state capacity could decrease informal workers, increase formal workers, and increase tax revenues in an economy with real wage rigidities in the formal sector. In specific, the model proposed in this paper is similar in spirit to Martin & Wang (2020) which defines a model with a search and matching frictions in the labor market that includes a shirking mechanism. However, the present paper includes a government in a dual labor market with informal and formal workers.

The model has formal and informal firms hiring formal and informal workers, respectively. The formal firm can hire formal workers by a matching process as Diamond (1982), Mortensen & Pissarides (1994) and Pissarides (2000). In contrast, the informal firm does not have any friction and hires informal workers given the marginal labor productivity. Unemployed in the economy, search for a job in the formal or informal sector. If the unemployed are employed in the formal firm, they could be shirking, or non-shirking workers as in Shapiro & Stiglitz (1984)

The real wage rigidities in the formal sector comes from shirking mechanism. That is, the workers have an additional disutility to become non-shirking workers in terms of the real wage. The real wage increase as a consequence of the incentive from the firm to hire non-shirking workers leading to real wage rigidities.

The government's main objective is to increase its revenues through two tax policies: to reduce payroll tax in the formal sector and increase the expenditure on law enforcement in the informal sector. The first policy decreases the cost of hiring in the formal sector, but the other increases the cost of working in the informal sector. In this sense, combined mechanisms reduce informality and increase the formal labor in the economy. Given the above, the increase in the formal worker leads to an increase in the tax base and an increase in the government income.

The main model results suggest that tax policies, such as variations in payroll taxes and law enforcement expenditures, are relevant instruments to reduce informal employment. The magnitude of their effect depends on the economy's real wage rigidities. The results show that a decrease in payroll taxes increases formal worker demand, while the increase in law enforcement expenditure reduces the size of the informal sector.

Therefore, both tax policies combination would have a significant impact on reducing informality. These results hold under scenarios of real wage rigidities. In this sense, the model suggests that complementing state capacity and labor policies is essential to reduce informality and encourage formal employment. Also, the simulation results show the existence of a policy with constant government expenditure, where it is possible to reduce payroll taxes with increases in law enforcement expenditure.

Similarly, there is a scenario in which it is possible to decrease the payroll taxes to increase the public expenditure and the formal employment. In addition, I observe a case where increases in enforcement spending lead to a peak in public expenditure with low levels of informal employment. The model also suggests a more significant effect of tax policies in the informal employment reduction when the economy presents high real wage rigidities.

This paper contributes to the literature that seeks to understand the effect of tax policies, such as variations in tax policies and the enforcement expenditure on labor informality. Related to the reduction of payroll taxes, findings of D'Erasmo & Boedo (2012) and Haanwinckel & Soares (2016) suggest that reductions in payroll tax seem to generate an increase in the formalization. Also, in line with the previous results Pratap & Quintin (2006); Santa María et al. (2010) and Osorio-Copete (2016) show that reductions in payroll tax has a significant effect on reducing the informality.

Furthermore, Rocha et al. (2018) studied the causal effect of reducing the tax burden on firms. The authors found a positive effect in the reduction of informality given the reduction in tax burden. Hence, the literature highlights the relevance of tax policies in reducing informality. Accordingly, this research contributes to the evidence favoring reducing tax burdens and payroll taxes to decrease informality.

In terms of law enforcement, the paper is related to Meghir et al. (2015), Ulyssea (2018), Bardey & Mejía (2019), and Acosta-Henao (2020), who find strong evidence in favor of informality reduction as a consequence of increasing in law enforcement expenditure. However, this research differs from the conclusion proposed by Ulyssea (2010), and Charlot et al. (2015) in which an increase in enforcement expenditure could lead to an increase in unemployment. In specific Charlot et al. (2015) defines the formal and informal sectors with matching frictions and points out that fiscal policies such as reducing taxes and increasing enforcement can generate a trade-off, reduce informality and increase unemployment.

In contrast to the above, the present paper assumes an informal sector close to De Soto

(1989) subsistence view without search and matching frictions; furthermore, the results show that an increase in enforcement can lead to decreases in unemployment. This last result is more in line with the contribution of Meghir et al. (2015) and Dix-Carneiro et al. (2021), who find no effect on unemployment with increases in enforcement.

Hence, the research contributes to the debate on the possible enforcement effects on unemployment and informality. Moreover, the paper explores the possible dynamics of unemployment given changes in law enforcement expenditure when the labor market has rigidities.

Regarding research on the reduction of informality with tax policies in Colombia, the results of the present paper have a similar conclusion with, Antón (2014), Kugler & Kuler (2015), Fernández & Villar (2017) and Garlati-Bertoldi (2020), who find a reduction in the informality given the Colombian labor reform of 2012, which decreased the payroll tax. On the other hand, the results of Posada & Mejía (2012) and Acosta-Henao (2020) on increased spending on law enforcement decrease informality are in agreement with the results in the present paper.

Finally, the model of the paper is similar to the approximation of Albrecht et al. (2009), Flórez (2015), Bosch & Esteban-Pretel (2015) and García-Suaza et al. (2021) who modeled a labor market with search and matching frictions and an informal sector. Nevertheless, to the best of my knowledge, there is no research linking the effect of tax policies on informality in a labor market with micro-founded real wage rigidities, including search and matching frictions with unemployment.

The above is in line with Ulyssea (2020), which argues the importance of understanding the informality determinants that build decisions from the micro-level to the macro-level. In this regard, a relevant contribution of this paper is including micro-founded real wage rigidities to understanding how these rigidities can affect the optimal workers' decisions to enter the formal or informal sector and how these workers' decisions affect the aggregate employment variables in the economy.

The following section presents the informality context in Colombia. Section 3 presents the model, while section 4 provides the calibration. Section 5 presents the analytical and simulations results, and section 6 concludes.

# 2 Informality context in Colombia

Despite the different informality definitions, Colombia has presented high levels of informality over time. The most recent data from National Statistics Department (DANE, for its acronym in Spanish) shows that the share of employment of the informal sector is around 43.9%. High levels of unemployment and informality have characterized the labor market behavior in Colombia, even during economic boom periods. The most recent unemployment figures, according to DANE, are 14.6%[1]. DANE (2022)

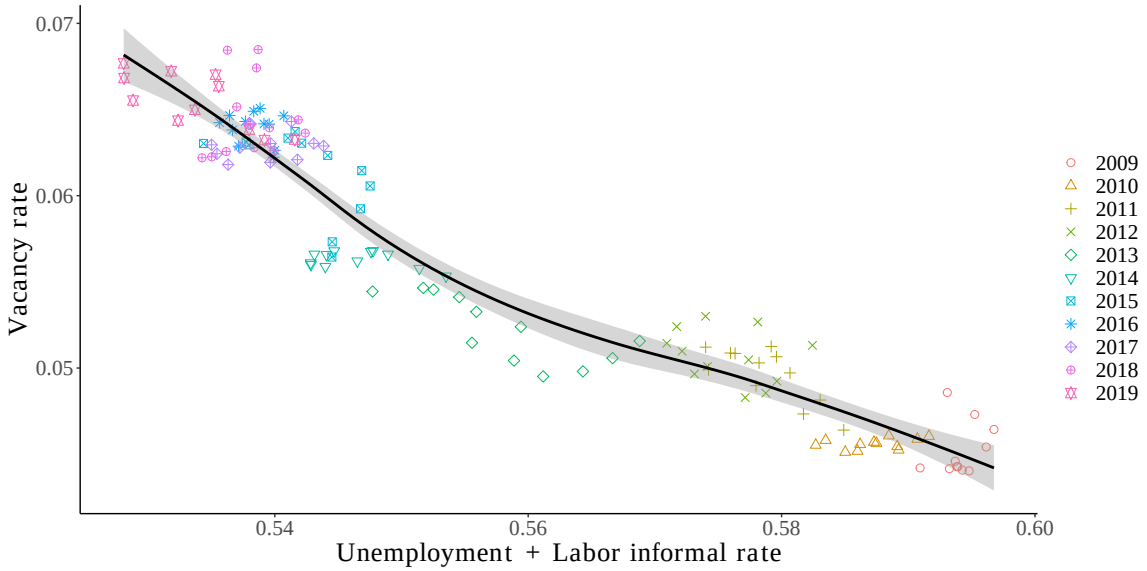


Figure 1: Beveridge curve with non-formal sector (each point is a month). Source: (Morales & Lobo, 2021) and Colombian Household Surveys (GEIH for its acronym in Spanish) published by the DANE for the 23 main metropolitan areas.

Also, based on the vacancy data provided by Morales & Lobo (2021), it is possible to observe an inverse and non-linear relationship between the vacancy rate and the unemployment rate. This relationship is known as the Beveridge curve, which suggests that unemployment levels will be lower with increases in the number of vacancies. In this sense, the Beveridge curve allows us to understand unemployment's possible inflows and outflows.

However, the existence of high levels of informality in the country persistent over time highlights the relevance of taking into account the informal sector when analyzing the role of vacancies. Thus, this paper shows a convex and negative relationship between formal vacancies and the non-formal sector defined as the unemployment and informal-

[1]Official data for January 2022

ity rate, as shown in Figure (1). The Beveridge curve with the non-formal sector allows us to understand the relationship between exit and entry to the formal sector.

Thus, the model presented below can replicate the empirical relationship of the Beveridge curve with the non-formal sector observed in the data. This relationship is a crucial element of the model and allows us to understand the ability of the formal sector to absorb the labor force in the face of tax policies that affect the employment decisions of workers.

# 3 Model

I develop a dynamic general equilibrium model to understand the role of tax policies in an economy with real wage rigidities through shirking mechanisms and search and matching processes. The model is based on Martin & Wang (2020), who modified the search and matching model summarized in Pissarides (2000), incorporating a shirking mechanism by Shapiro & Stiglitz (1984). Nevertheless, I include an informal sector with no labor market frictions.

The model has three agents: households, firms, and government. The representative household supplies labor, owns firms in the economy, and chooses consumption to maximize an intertemporal function. The workers in the economy belong to one of three states: informal workers, formal workers, or unemployed. If the worker is employed in the formal sector, she could be a shirking or non-shirking worker.

The firms are formal or informal. Formal firms can hire up to one worker, and these workers could be shirking or non-shirking. Also, there is a fixed vacancy cost; in this sector, the formal wage is determined by shirking mechanisms. On the other hand, informal firms are labor-intensive and have marginal decreasing returns; wages in this sector correspond to workers' marginal productivity.

Finally, the government's primary balance depends on revenue and expenditure. The government revenue comprises formal firms' taxes, households' taxes, and informal firms' fines. In contrast, the government expenditure depends on the transfers to households, unemployment transfer, enforcement expenditure to find informal firms, and unproductive expenditure.

Moreover, the government has two principal tax policies to increase the formal sector and government revenue: reduce the formal firms' taxes, and change the enforcement expenditure to increase the probability of auditing and finding an informal firm. The above policies create incentives for the transition from the informal to the formal sector, increasing government revenue by increasing the taxable base.

## 3.1 Households

The representative households derives utility from consumption according to the intertemporal utility function.

$$\max_{c_t,b_t} E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(\mathcal{U}(c_t^{wi}-\chi)l_t^i + \mathcal{U}(c_t^{wns}-\chi-\zeta)l_t^{wns} + \mathcal{U}(c_t^{ws}-\chi)l_t^{ws} + \mathcal{U}(c_t^u)u_t\right) \tag{1}$$

Where $\rho$ is the time discount rate. $c_t^{wi}$, $c_t^{wns}$, $c_t^{ws}$, and $c_t^u$ are per capita consumption for informal workers, non-shirking workers, shirking workers, and unemployed respectively. For simplicity, I assume the functions $\mathcal{U}(\cdot)$ are strictly increasing and strictly concave. Households own firms and workers in the economy, and there is risk-sharing. In each period, the household earns (and consumes) real wage from the labor supply in the formal sector and informal sector defined as $w_t^f$ and $w_t^i$ respectively.

Formal and informal workers face a disutility $\chi$. Also, employed in the formal can be shirking or non-shirking. Hence, workers who do not shirk incur a disutility $\zeta$. Furthermore, in each period, the household receives formal firms dividends $Div_t$, and an unemployment transfer for the government given by $s_t$. By the above, the household's problem consists of maximizing the intertemporal utility function (1) subject to the following budget constraint:

$$c_t + b_t = (1 + (1-\tau_t^{\pi})r)b_{t-1} + w_t^f l_t^f + w_t^i l_t^i + s_t u_t + (1-\tau_t^{\pi})Div_t \tag{2}$$

In equation (2), the households bonds are represented by $b_t$, and $r$ is the return rate of the bonds. The total household consumption $c_t$ is defined as $c_t = c_t^{wi}l_t^i + c_t^{wns}l_t^{wns} + c_t^{ws}l_t^{ws} + c_t^u u_t$. Additionally, the government taxes the formal profits of the firms with $\tau_t^{\pi}$. Given the labor market, the total labor force in the economy is divided by non-shirking workers $l_t^{ns}$, shirking workers $l_t^s$, informal workers $l_t^i$ and unemployed $u_t$ so in per-capita terms $1 = l_t^f + l_t^i + u_t$, where $l_t^f = l_t^{ns} + l_t^s$. Given the above, the first-order conditions from the household maximization problem is the following:

$$\frac{\partial \mathcal{U}(c_t^{wi}-\chi)}{\partial c_t^{wi}} = \frac{\partial \mathcal{U}(c_t^{wns}-\chi-\zeta)}{\partial c_t^{wns}} = \frac{\partial \mathcal{U}(c_t^{ws}-\chi)}{\partial c_t^{ws}} = \frac{\partial \mathcal{U}(c_t^u)}{\partial c_t^u} = \lambda_t \tag{3}$$

$$\lambda_t = \left(\frac{1}{1+\rho}\right) E_t \lambda_{t+1}(1 + (1-\tau_{t+1}^{\pi})r_{t+1}) \tag{4}$$

From equation (3) is possible to define the following consumption behavior $c_t^u = c_t^{wi} - \chi = c_t^{wns} - \chi - \zeta = c_t^{ws} - \chi$. Combining the equation (3) and (4) is possible get the

following expression, in which $\Omega_{t+1}$ is defined as the ratio of the first-order conditions $\frac{\lambda_{t+1}}{\lambda_t}$

$$\frac{1}{E_t(1+(1-\tau^{\pi}_{t+1})r_{t+1})} = \left(\frac{1}{1+\rho}\right)\frac{\lambda_{t+1}}{\lambda_t} = \left(\frac{1}{1+\rho}\right)\Omega_{t+1} \tag{5}$$

## 3.2 Firms

The firms are formal or informal. Formal firms hire up to one formal worker, and informal firms hire up to one informal worker. The formal and informal firms are homogeneous. In the formal sector, the firms maximize profits. The firms post a vacancy and unemployed search for a job. The formal wage is given by the shirking mechanism, creating the real wage rigidities. In contrast, the informal firms do not face any friction, only hire non-shirking workers and maximize profit. The marginal productivity gives the worker's wage.

### Matching Process

The matching function that determines the aggregate hiring in the formal sector is the following.

$$m(u_t, v_t) = k u_t^{\phi} v_t^{1-\phi} \tag{6}$$

Where $m(u_t, v_t)$ is the number of workers hired in the formal sector, $u_t$ is the unemployment rate, and $v_t$ is the formal vacancy rate. As equal to the standard literature of search and matching models, $\phi$ and $k$ are parameters characterizing the constant returns of the matching function. Moreover, I define the labor market tightness $\theta_t$ as the ratio of vacancy rate to the unemployment rate.

$$\theta_t = \frac{v_t}{u_t} \tag{7}$$

when $\theta_t$ is higher, the labor market is tighter from an entrepreneur perspective. It is possible to define the probability of filling a vacancy in the formal sector as $q(\theta_t)$ equal to the ratio of the number of workers hired in the formal sector to the vacancy rate.

$$q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = k\left(\frac{1}{\theta_t}\right)^{\phi} \tag{8}$$

While the probability that an unemployed finds a job in the formal sector $\alpha(\theta_t)$ is defined by the ratio of matching workers in the formal sector to the unemployment rate.

$$\alpha(\theta_t) = \frac{m(u_t, v_t)}{u_t} = k\,(\theta_t)^{1-\phi} \tag{9}$$

The formal labor force evolves according to equation (10). In the period $t$ there is a pool of unemployed workers that find a formal work with probability $\alpha(\theta_t)$, and there is a fraction of non-shirking workers who is fired with an exogenous separation rate $\mu \in (0,1)$. Also, the shirking workers has an additional exogenous probability of being unemployed defined by $d$

$$l^f_{t+1} = (1-\mu)l^{ns}_t + (1-\mu-d)l^s_t + \alpha(\theta_t)u_t \tag{10}$$

The production function of the formal firm depends on the worker. For simplicity, if the worker is non-shirking the production is given by $y^f_t = \psi^f$, where $\psi^f$ is the formal firm productivity. But, the production is zero if the worker shirks. In addition, the formal firm has a fixed cost $\eta$ to create a vacancy. Also, in each period, the formal firm pays profit tax defined by $\tau^\pi_t$ and pays a payroll tax $\tau^w$ to hire a worker. Then, the formal firms' net profits for each period $t$ are given by:

$$\Pi^f_t = Div_t\,(1-\tau^\pi_t) = \left(y^f_t l^{ns}_t - (1+\tau^w_t)w^f_t l^f_t - \eta v_t\right)(1-\tau^\pi_t) \tag{11}$$

Given the above, it is possible to get the three different states of the formal firms' profits along the time. The first possible state is represented by the value function of the formal firm to create a vacancy $V_t$ is defined as follow:

$$V_t = -(1-\tau^\pi_t)\eta + \left(\frac{1}{1+\rho}\right) E_t\left\{\Omega_{t+1}\left[q(\theta_t)H_{t+1} + (1-q(\theta_t))V_{t+1}\right]\right\} \tag{12}$$

Where $H_{t+1} = J^{ns}_{t+1}$ if the worker chooses not to shirk, $J^{ns}_t$ is the formal firm value function of filled job with a non-shirking worker. In contrast, $H_{t+1} = J^s_{t+1}$ in other case. That is, $J^s_t$ is the formal firm value function of filled job with a shirking worker. The equation (12) represents the net cost of a vacancy will be $(1-\tau^\pi_t)\eta$, also the value function have an expected probability $q(\theta_t)$ in which the vacancy is filled by a worker, but with probability $1-q(\theta_t)$ the vacancy is still open.

The value function of filling the vacancy with a non-shirking worker is $J^{ns}_t$. Hence, at the beginning of the period $t$ the firm have a net profit given by $(1-\tau^\pi_t)(y^f_t - w^f_t(1+\tau^w_t))$, and with an exogenous probability $\mu$ the worker is fired and the vacancy is open. In contrast, with $1-\mu$, the vacancy is still filled by a shirking or non-shirking worker.

$$J_t^{ns} = (1-\tau_t^{\pi})(y_t^f - w_t^f(1+\tau_t^w)) + \left(\frac{1}{1+\rho}\right) E_t\left\{\Omega_{t+1}\left[\mu V_{t+1} + (1-\mu)H_{t+1}\right]\right\} \quad (13)$$

Differently, if the vacancy is filled with a shirking worker, the net profit of the firms does not have production and is defined by $-(1-\tau_t^{\pi})(1+\tau_t^w)w_t^f$. However, the probability of firing a worker and open a vacancy increase by $d$. In consequence, with probability $\mu + d$, the vacancy is open.

$$J_t^{s} = -(1-\tau_t^{\pi})(1+\tau_t^w)w_t^f + \left(\frac{1}{1+\rho}\right) E_t\left\{\Omega_{t+1}\left[(\mu+d)V_{t+1} + (1-\mu-d)H_{t+1}\right]\right\} \quad (14)$$

In general terms, equations (13) and (14) show the different states of the formal firm profits along the time that depends on the worker state. If the worker is non-shirking, the production of the firm is given by $y_t^f$, and the probability of firing a worker, and therefore open a vacancy is $\mu$ and open the vacancy. On the other side, if the worker is shirking, the firm's production is zero, but the probability of firing a worker is $\mu + d$ and, consequently, open the vacancy.

**Informal Firms**

Following Ulyssea (2018), the informal firm has a profit function defined in the equation (15). For simplicity, I assume that the production of the informal firm is given by $y_t^i = \psi^i$, where $\psi^i$ is the productivity of the informal firm. Also, I suppose that the formal productivity is greater than the informal productivity $\psi^f > \psi^i$.

$$\max \Pi^i = y_t^i l_t^i [1 - A(e_t)] - w_t^i l_t^i \quad (15)$$

In this case, similar to Posada & Mejía (2012), Bardey & Mejía (2019) and Acosta-Henao (2020), the informal sector does not have any taxes, but is subject to an probability of auditing $A(e_t)$ that depends on enforcement expenditure by the government $e_t$.

$$A(e_t) = 1 - exp\left\{-\gamma e_t\right\} \quad (16)$$

I assume that $A(e_t)$ have a exponential distribution probability. Where $A_e(\cdot) > 0$. It means that an increase in the enforcement expenditure in the economy increases the probability of auditing an informal firm and in this case, it loss whole production.

Furthermore, $l_t^i$ is the informal labor force in the period $t$ with a law of motion given by:

$$l_{t+1}^i = (1-\sigma)l_t^i + \xi u_t \tag{17}$$

The probability that an unemployed was employed in the informal sector is exogenous and defined by $\xi$. While with probability $\sigma$, the informal worker is fired. Lastly, the expected informal net income is given by the marginal productivity in the informal firm and depends on the audited probability.

$$w_t^i = y_t^i(1 - A(e_t)) \tag{18}$$

## 3.3 Workers

Workers in the model are homogeneous. In the period $t$, a worker is in one of the three states: employed in the formal sector, employed in the informal sector, or unemployed. However, workers in the formal sector could be employed as shirking workers or non-shirking workers. As a result, the workers' utility has four possible states summarized in the following value functions.

$$W_t^{ns} = w_t^f - \chi - \zeta + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1}\left[\mu U_{t+1} + (1-\mu)M_{t+1}\right]\right\} \tag{19}$$

The equation (19) shows the utility along the time if the worker is employed in the formal sector and is non-shirking, where $M_{t+1} = \text{Max}\{W_{t+1}^{ns}, W_{t+1}^{s}\}$. In this case, at the beginning of the period $t$ the worker has a real wage given by $w_t^f$ but suffer a disutility in term of the real wage given by $\zeta$ to decide to be non-shirking, and there is another disutility to work $\chi$. Also, with probability $\mu$, the worker is fired from the formal sector and becomes unemployed.

In contrast, if the worker is shirking does not has a disutility $\zeta$, but have the disutility to work given by $\chi$, so utility at the beginning of the period $t$ is equal to $w_t^f - \chi$. Nevertheless, the probability of being fired and become unemployed increases and is equal to $\mu + d$. The value function of being employed in the formal sector as a shirking worker is given by:

$$W_t^{s} = w_t^f - \chi + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1}\left[(\mu+d) U_{t+1} + (1-\mu-d)M_{t+1}\right]\right\} \tag{20}$$

The value function of a worker who is employed in the informal sector is defined in

equation (21). At the beginning of the period $t$, the worker has a utility of $w_t^i - \chi$ and has an exogenous probability of being fired from the informal work given by $\sigma$.

$$W_t^i = w_t^i - \chi + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1}\left[\sigma U_{t+1} + (1-\sigma)W_{t+1}^i\right]\right\} \tag{21}$$

Similarly, the value function of being unemployed is defined as:

$$U_t = s_t + \left(\frac{1}{1+\rho}\right) E_t \left\{\Omega_{t+1}\left[\alpha(\theta_{t+1})M_{t+1} + \xi W_{t+1}^i + (1-\alpha(\theta_{t+1})-\xi)U_{t+1}\right]\right\} \tag{22}$$

In this case, if the worker is unemployed, have a government transfer of $s_t$, and the endogenous probability of being employed in the formal sector is $\alpha(\theta_{t+1})$ while the probability of being employed in the informal sector is exogenous and equal to $\xi$. On the other hand, the worker holds a probability of being unemployed of $1 - \alpha(\theta_{t+1}) - \xi$.

## 3.4 Government

In each period, the government is represented by a balanced budget rule as follows:

$$g_t + e_t + s_t u_t = \tau_t^w w_t^f l_t^f + y_t^i l_t^i A(e_t) + \tau_t^\pi Div_t \tag{23}$$

The government expenditures is given by the unproductive expenditure $g_t$, the enforcement expenditure $e_t$, and the unemployment transfers $s_t u_t$. The government income is defined by payroll taxes from the formal sector $\tau_t^w w_t^f l_t^f$, profit taxes from formal firms $\tau_t^\pi Div_t$, and income from the informal sector if informal firm is audited $y_t^i l_t^i A(e_t)$.

Also, from the equation (23) it is possible to understand the public expenditure (or unproductive expenditure) $g$ as a proxy of government income net to the expenditures associated with the enforcement and unemployment transfers.

In baseline case of the model, I assume that the unproductive expenditure $g_t$ is endogenous, and the tax policies $\tau_t^w$ and $e_t$ are exogenous. The above implies that any change in one of the tax policies remains the other constant lead to a change in the unproductive expenditure.

$$\begin{aligned} \Delta\tau_t^w \quad &\rightarrow \quad \Delta g_t,\ \overline{e}_t \\ \Delta e_t \quad &\rightarrow \quad \Delta g_t,\ \overline{\tau}_t^w \end{aligned}$$

Nevertheless, further simulations assume that enforcement expenditure $e_t$ is endogenous. Hence, any change in one of the tax policies remains the unproductive expenditure constant, leading to a change in the endogenous tax policy.

$$\Delta\tau_t^w \quad \rightarrow \quad \Delta e,\ \overline{g}$$

## 3.5 Steady State

Given the above, the model's analysis focuses on the steady-state to understand the role of the real wage rigidities given the tax policies that encourage the formality in the long run. For this purpose, I compare different scenarios in the long run for the interest variables under changes in the tax policies.

### Firms

In the steady-state, the value function of the formal firms described in the equations (12) - (18) is presented as follow:

$$\rho V = -(1-\tau^{\pi})\eta(1+\rho) + q(\theta)\left(J^{ns} - V\right) \tag{24}$$

Given the steady-state in the economy, the vacancy is filled with probability $q(\theta)$ with a non-shirking worker, and the cost of creating a vacancy is affected by a profit tax. As shown below, there are not shirking workers in equilibrium because the firm defines a formal wage level at which the worker is indifferent between being a shirking or non-shirking worker

In the same way, the formal firm value function of filling a vacancy with non-shirking and shirking workers is defined in the equation (25) and (26) respectively.

$$\rho J^{ns} = (1-\tau^{\pi})(y^f - w^f(1+\tau^w))(1+\rho) + \mu\left(V - J^{ns}\right) \tag{25}$$

$$\rho J^{s} = -(1-\tau^{\pi})(1+\tau^w)w^f(1+\rho) + (\mu+d)\left(V - J^{s}\right) \tag{26}$$

From the side of the informal firm the expected informal net income in the steady-state is given by:

$$w^i = y^i(1 - A(e)) \tag{27}$$

**Workers**

The value functions of the workers summarized in the equations (19) – (22) in the steady-state are the following:

$$\rho W^{ns} = (w^f - \chi - \zeta)(1+\rho) + \mu(U - W^{ns}) \tag{28}$$

If the worker is formal, then the value functions could be of the non-shirking or shirking worker.

$$\rho W^{s} = (w^f - \chi)(1+\rho) + (\mu + d)(U - W^{s}) \tag{29}$$

When the worker is informal, the value functions in steady-state is the following:

$$\rho W^{i} = (w^i - \chi)(1+\rho) + \sigma(U - W^{i}) \tag{30}$$

Finally, when the worker is unemployed, the value functions are presented below. In this case, I assume that in a steady-state the unemployed become a formal non-shirking worker with probability $\alpha(\theta)$

$$\rho U = s(1+\rho) + \alpha(\theta)(W^{ns} - U) + \xi(W^{i} - U) \tag{31}$$

It is essential to highlight that there is no job-to-job transition in the setup of the model. To change the state, the worker must first be unemployed and become a formal or informal worker. The worker does not have a direct transition from formal to informal or informal to formal state.

**Real Wage Rigidities**

The shirking mechanism determines the formal wage in the model. That is, non-shirking workers suffer a disutility $\zeta$ when is employed. If this disutility increase, the workers will have an incentive to shirk the work. However, formal firms do not want to hire shirking workers. Given the above, the formal firm has the incentive to increase the formal wage $w^f$, and this increase is above the Walrasian equilibrium wage (equilibrium wage with any distortion in the market). Hence, the formal wage has real wage rigidities that are persistent along the time.

To determine the real wage and follow the standard literature of the shirking models, I assume the free entry condition where the value of creating a vacancy is equal to zero $V = 0$. From equation (24) is possible to get:

$$J^{ns} = \frac{(1-\tau^{\pi})\eta(1+\rho)}{q(\theta)} \tag{32}$$

Replacing the equation (32) in equation (25) is possible to obtain the job creation curve:

$$w^f = \left(y^f - (\mu+\rho)\left(\frac{\eta}{q(\theta)}\right)\right)\frac{1}{1+\tau^w} \tag{33}$$

Given the formal firm's limited ability to control workers, the firm seeks to generate incentives to induce workers' effort. In this sense, the non-shirking condition ensures that the formal wage paid by the firm is high enough to encourage workers to be non-shirking, and consequently, the firm ensures an output different from zero. Hence, using the non-shirking condition $W^{ns} = W^s$ it is possible find the following eqution: $W^{ns} - U = \frac{\zeta}{d}$, which implies that formal firms set wage high enough that worker strictly prefer formal employment to unemployment. With the above condition and using the equations (28) – (31) is possible to obtain the wage curve.

$$w^f = \left(\rho + \mu + \frac{(\rho+\sigma)\alpha(\theta)}{\rho+\sigma+\xi}\right)\frac{\zeta}{d} + \Gamma \tag{34}$$

$$\Gamma = \chi + \zeta + s + \frac{\xi}{\rho+\sigma+\xi}\left(y^i(1-A(e)) - \chi - s\right)$$

Equation (34) shows that the informal wage affects the formal wage and depends on the probability that an unemployed is employed in the informal sector $\xi$. Also, the rigidities in the formal wage are given by the disutility of being a non-shirking worker $\zeta$. The formal wage in the model with informality is a generalized version of the Shapiro - Stiglitz wage. Consider the case in which there is not informal sector, and lets assume that the probability of being an informal worker is equal to zero $\xi = 0$ the formal wage become:

$$w^f = (\rho + \mu + \alpha(\theta))\frac{\zeta}{d} + \chi + \zeta + s \tag{35}$$

Hence, equation (35) represents the classic Shapiro - Stiglitz wage curve that represents the wage needed to induce effort.

**Beveridge Curve**

The classic search and matching model makes it possible to obtain the Beveridge curve that shows the relationship between the vacancy rate and the unemployment rate. For

the above, consider the equations of law motion of labor force in the formal and informal sector (10), (17), and the equation of the total labor force in the steady-state.

$$
\begin{aligned}
1 &= l^f + l^i + u && (36)\\
\mu l^f &= \alpha(\theta) u && (37)\\
\sigma l^i &= \xi u && (38)
\end{aligned}
$$

Whit the above equations is possible to get the classic Beveridge curve defined as follow:

$$u = \frac{\mu}{\alpha(\theta) + \frac{\xi+\sigma}{\sigma}\mu} \tag{39}$$

The equation (39) is a generalized version of the Beveridge curve. Let us assume that there is not informal sector $\xi = 0$ is possible to return the Beveridge curve of the simple search and matching model.

$$u = \frac{\mu}{\alpha(\theta) + \mu} \tag{40}$$

Additionally, the informal sector and the equations (36) – (38) is possible to get a new version of the Beveridge curve that relates the formal vacancy with the non-formal sector. The above is a replication of the stylized fact presented in Figure (1) in which it is possible to see a non-linear relationship between formal vacancy and non-formal sector.

$$u + l^i = \frac{\mu}{\mu + \left(1 - \left(\frac{\xi}{\sigma+\xi}\right)\right)\alpha(\vartheta)} \tag{41}$$

Where $\alpha(\vartheta) = k\left(\frac{v}{u+l^i}\left(\frac{\sigma+\xi}{\sigma}\right)\right)^{1-\phi}$ with $\vartheta = \frac{v}{u+l^i}$. To get the equilibrium from Beveridge curve side I use the equation (41) and the job creation line.

$$v = \theta\frac{\sigma}{\sigma+\xi}(u + l^i) \tag{42}$$

Hence to understand who the formality could be change given different tax policies, in the following sections, I use the equations (33), (34), (41), and (42) to get the equilibrium.

**Equilibrium Condition**

Given the workers and firms optimization summarized in the steady states value functions, the labor market equilibrium of the economy is characterized by the equations system (33), (34), (41), and (42). Consequently, it is possible to get the real formal wage of equilibrium $w^{f*}$ and the labor market tightness of equilibrium $\theta^*$.

Hence, it is possible to obtain the formal and informal workers in equilibrium and the unemployed workers. Also, given that the economy does not have capital, the bonds in equilibrium are equal to zero $b = 0$. Replacing the informal firm's optimal condition from the equation (27) and the steady-state government balance on the steady-state budget balance of household is possible to find the equilibrium balance defined as follow:

$$GDP = y^f l^f + y^i l^i - \eta v = c + g + e \tag{43}$$

Figure (2) represents the equilibrium in the economy in which the intersection of curves (33) and (34) returns the formal wage of equilibrium $w^{f*}$ and the labor market tightness of equilibrium $\theta^*$. Also, with the equations (41), (42), and the labor market tightness of equilibrium, it is possible to obtain the formal vacancy rate of equilibrium $v^*$ and the unemployment plus the informal rate of equilibrium $(u + l^i)^*$

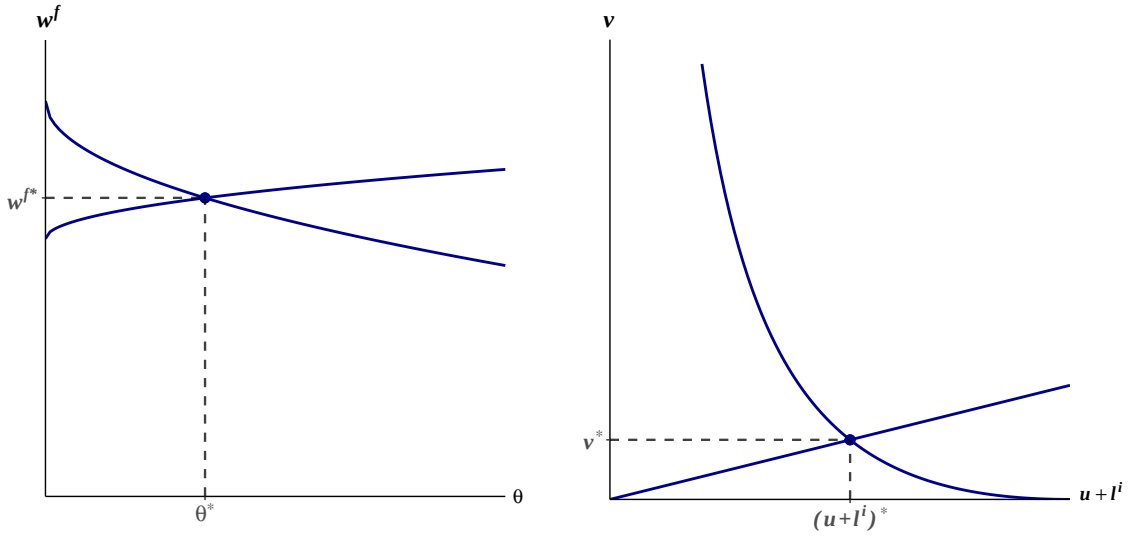


Figure 2: Equilibrium in the economy

The left panel of Figure (2) shows the behavior of the wage curve (positively sloped curve) and the job creation condition curve (negatively sloped curve). The wage curve

shows the wage that the formal firm must pay so that workers are non-shirking and have incentives to work in the formal sector.

The above implies that, given increases in the labor market tightness and consequently in the probability of finding formal job, it is easier for the unemployed to find a formal employment. Therefore, the cost of being fired as a shirking worker decreases, increasing the wage that the formal firm must pay to discourage shirking workers and compensate for the profits in other sectors, such as being unemployed or informal.

While the job creation curve corresponds to the marginal condition of labor demand, a high level of formal wages makes job creation less profitable, which generates a lower demand for workers by formal firms. The wage curve and the job creation curve replace the Walrasian economy supply and demand curves, respectively; therefore, the two curves' intersection generates a unique formal wage and labor market tightness of equilibrium ($w^{f*}$, $\theta^*$)

The right panel of Figure (2) shows a new version of the Beveridge curve comparing vacancies in the formal sector with the workers in the non-formal sector (unemployment and informality). The convex curve with a negative slope corresponds to the non-formal Beveridge curve, which replicates the behavior observed in the data and is presented in Figure (1).

When there are more formal vacancies, unemployment and informality are lower because of the probability of finding employment in the formal sector increases. On the other hand, the curve with a positive slope is the job creation line, which shows the ratio between vacancies and the non-formal sector. The intersection of the two curves allows us to find the formal vacancies and unemployment more informality of equilibrium ($v^*$, $(u + l^i)^*$).

# 4 Calibration

The calibration of the model seeks to adjust to the main characteristics of the Colombian labor market, and the period is set to one month. For this reason, I divide the parameters into two groups. In the first group are parameters that are difficult to identify with the data. Consequently, I use typical values from search and matching models that incorporate an informal sector in the Colombian economy or parameters from the international literature.

| Parameter | Description | Source | Value |
|---|---|---|---|
| $\rho$ | Discount rate | Granda Carvajal & Hamann (2020) | 0.009 |
| $k$ | Matching process efficiency | Albrecht et al. (2009) | 0.25 |
| $\phi$ | Matching elasticity | Albrecht et al. (2009) | 0.5 |
| $s$ | Unemployment transfer | Albrecht et al. (2009) | 0 |
| $\eta$ | Vacancy cost | Albrecht et al. (2009) | 0.5 |
| $\mu$ | Formal separation rate | García-Suaza et al. (2021) | 0.0244 |
| $\sigma$ | Informal separation rate | García-Suaza et al. (2021) | 0.0235 |
| $\tau^w$ | Payroll tax | Rincón-Castro (2021) | 0.33 |
| $\tau^\pi$ | Income tax | Rincón-Castro (2021) | 0.13 |
| $e$ | Enforcement expenditure | Posada & Mejía (2012) | 0.007 |
| $A(e)$ | Auditing probability | Posada & Mejía (2012) | 0.27 |
| $\chi$ | Disutility of working | Martin & Wang (2020) | 0.62 |
| $d$ | Detection rate | Calibrated | 0.96 |
| $\zeta$ | Disutility of effort | Calibrated | 0.29 |
| $\xi$ | Prob of working informally | Calibrated | 0.08 |
| $\gamma$ | Institutional efficiency | Calibrated | 42.44 |
| $\theta$ | Labor market tightness | Calibrated | 0.17 |
| $y^i$ | Informal production | Calibrated | 1 |
| $y^f$ | Formal production | Calibrated | 1.37 |

Table 1: Parameter values

Following Granda Carvajal & Hamann (2020), I assign the value of the discount rate $\rho$ equal to $\left(\frac{1}{0.972^{1/12}} - 1\right)$. For the parameters describing matching and frictions, I use the standard values in the literature, $\phi = 0.5$, $\eta = 0.5$, and $k = 0.25$ used in Albrecht et al. (2009). Regarding the parameters of labor market dynamics are set as follow: for simplicity, I assume the unemployment transfer $s = 0$; the formal and informal separation rate are $\mu = 0.0244$ and $\sigma = 0.0235$ given estimations of García-Suaza et al. (2021). Also, due to the difficulty of estimating the disutility of working, I use the Martin & Wang (2020) value $\chi = 0.62$.

As for the tax fiscal policy parameters benchmark, I set the value of $\tau^w$ and $\tau^\pi$ as 0.33 and 0.13, respectively, following Rincón-Castro (2021). The enforcement expenditure and

the probability of being audited are taken from Posada & Mejía (2012), who develop a model with informal sector and enforcement policies, hence $e = 0.007$ and $A(e) = 0.27$

The second set of parameters is calibrated to match the average unemployment, formal and informal labor rates observed in the data from 2008 to 2019 and normalizing the informal production $y^i = 1$. Using the equilibrium equations described in the previous section, I select the value of $\zeta$ and $d$ to match the unemployment rate $u = 0.112$, the formal rate $l^f = 0.478$, and the informal rate $l^i = 0.410$[2]; the ratio between formal and informal wage observed in the data $\frac{w^f}{w^i} = 1.39$[3]

Finally, the probability of working in the informal sector $\xi$ and labor market tightness $\theta$ is derived from the labor market equations (36) – (38). The institutional efficiency $\gamma$ is obtained using the compliance probability equation (16), and the formal production is derived using the equation (43). The result of the calibrated parameters is presented in the Table (1).

[2]The average rates are calculated based on information from the GEIH published by the DANE for the 23 main metropolitan areas.

[3]The relationship between formal and informal wages was estimated using Mincer equations with GEIH data from 2008 to 2019. Section A presents the estimation results.

# 5 Policies

This section explores the long-run effect of tax policies, defined as the change in payroll taxes and law enforcement expenditures. In the first part of the analysis, I perform comparative statics to get the formal wage, vacancy rate, and non-formal sector results given changes in the tax policies and a decrease in the real wage rigidities. In the second part, I simulated the model under the tax policies effect in two scenarios: one assuming an endogenous public expenditure and the other considering endogenous law enforcement expenditure.

## 5.1 Analytical Results

Using the model's equilibrium, I assume the case of a decrease in the payroll taxes $\tau^w$. Figure (3) shows the policy effect. A decrease in payroll taxes generates an outward shift of the job creation curve. The above is a consequence of reducing hiring costs, which increases worker demand given the additional profit to hire a worker.

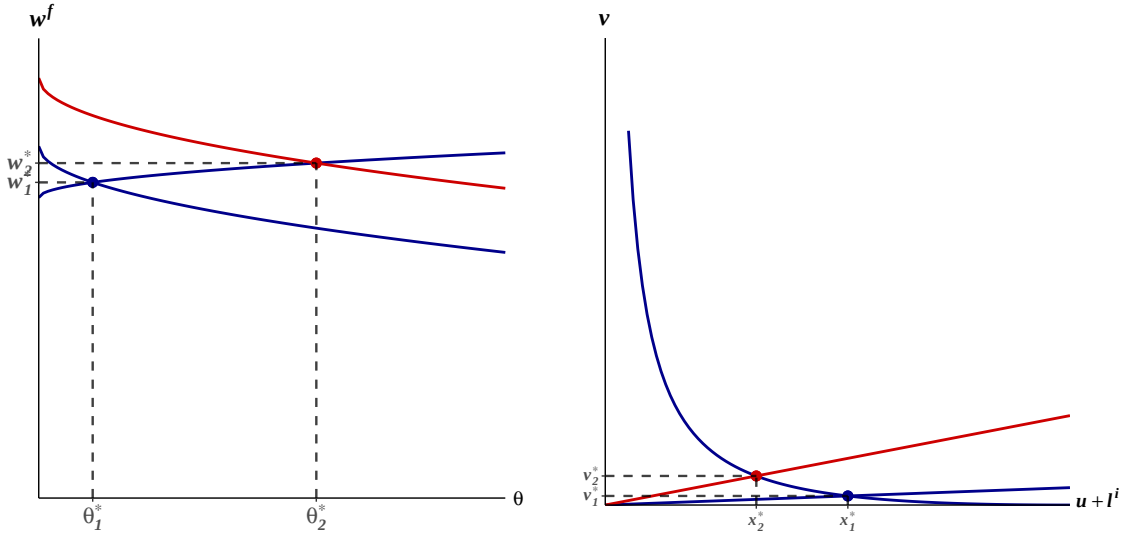


Figure 3: The effects of a decrease in payroll taxes $\tau^w$. Note: $x_j^*$ refers to $(u + l^i)_j^*$

The decrease in payroll taxes leads to an increase in the formal wages at which a firm would be willing to hire a formal worker at all productivity levels, leading to an increase in the formal equilibrium wages from $w_1^{f*}$ to $w_2^{f*}$. Likewise, the shift of the job creation curve generates an increase in labor market tightness from $\theta_1^*$ to $\theta_2^*$. Consequently, there

is an increase in the slope of the job creation line and given the Beveridge curve the vacancy rate increase to $v_2^*$ and the equilibrium unemployment plus informality rate decrease to $(u + l^i)_2^*$

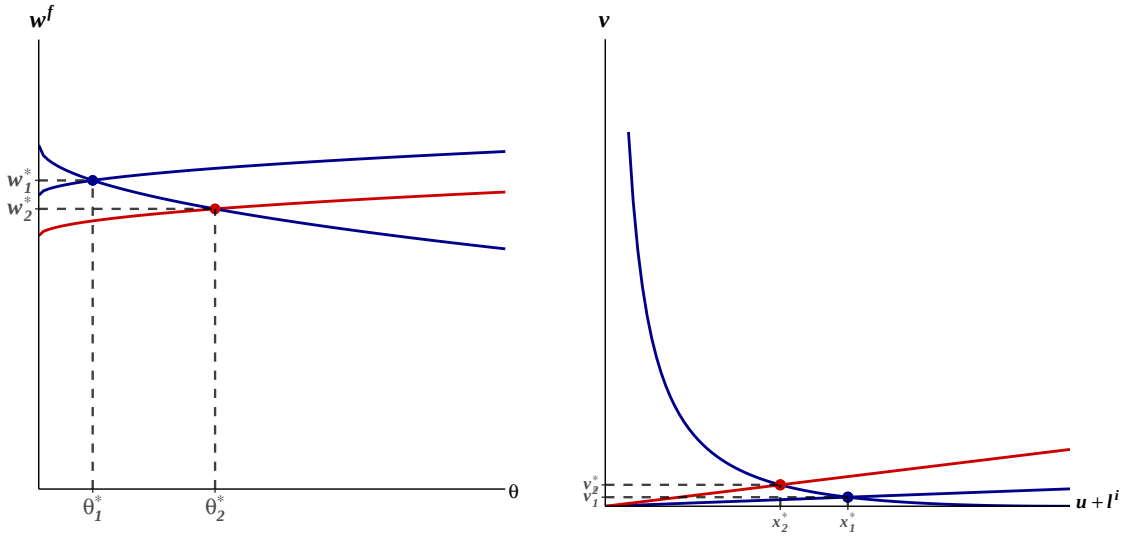


Figure 4: The effects of an increase in law enforcement expenditure $e$. Note: $x_j^*$ refers to $(u + l^i)_j^*$

On the other hand, Figure (4) shows an increase in the law enforcement expenditure $e$. This policy shifts the wage curve downward due to an increase in the auditing probability and consequently a decrease in expected informal net income. The wage curve represents the formal wage that induces workers' effort and compensates for other scenarios such as working in the informal sector or being unemployed. Therefore, the formal sector's compensation to induce formal work decreases, given the expected informal net income reduction. The above makes workers willing to enter the formal sector with a lower wage.

The shift of the curve leads to a decrease in the formal equilibrium wage to $w_2^{f*}$ and an increase in the labor market tightness to $\theta_2^*$. Accordingly, the increase in $\theta$ leads to an increase in the slope of the job creation line and a decrease in the unemployment and informal rate from $(u + l^i)_1^*$ to $(u + l^i)_2^*$ and an increase in the vacancy rate from $v_1^*$ to $v_2^*$

Figure (5) shows the reduction in real wage rigidities by decreasing the disutility of effort $\zeta$ shifts the wage curve downward. The reduction in the disutility of effort causes a decrease in the formal wage needed to induce effort. Therefore, formal wages are close to Walrasian equilibrium wages. The policy effect leads to a decrease in the formal equilibrium wage $w^f$ and a rise in the labor market tightness $\theta$. As a result, there is a

reduction in unemployment and informality $u + l^i$ and increased vacancies $v$, as shown in the Beveridge curve.

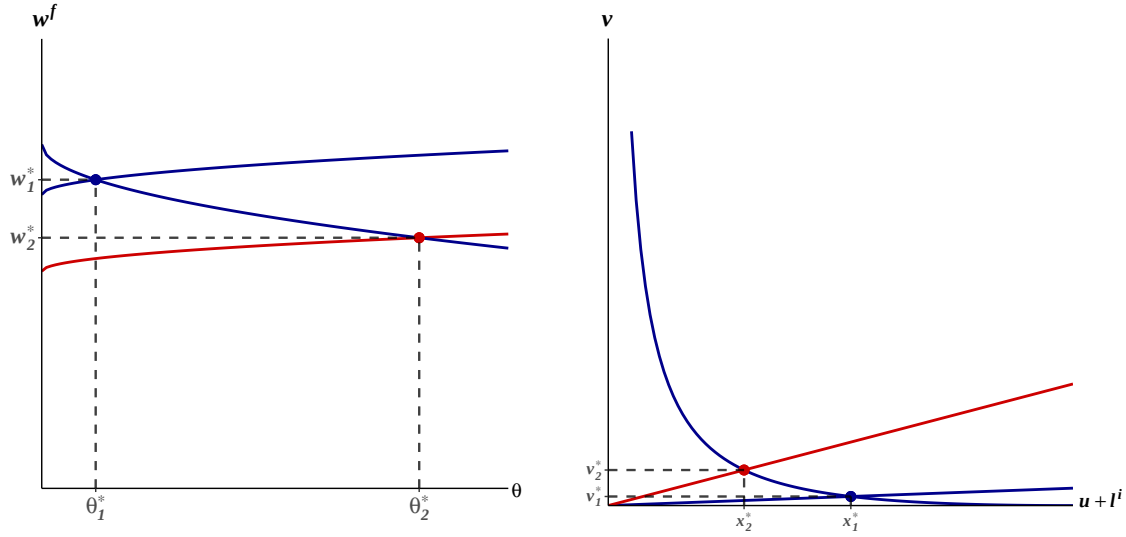


Figure 5: The effects of a decrease in real wage rigidities $\zeta$. Note: $x_j^*$ refers to $(u + l^i)_j^*$

The analytical results suggest that tax policies such as decreasing payroll taxes and increasing law enforcement expenditure reduce informality and unemployment in the long run. The first policy encourages the formal workers' demand and increases the formal equilibrium wage. The second policy reduces the informal worker offer by increasing the formal worker offer leading to a reduction of the formal equilibrium wage. The above suggests that an effective policy to reduce informality is the combination of labor policies and state capacity. Therefore, combining tax policies that increase formal worker demand and reduce the informal worker offer is an effective way to reduce informality and unemployment.

Figure (6) shows the combined effect of tax policies. The effect of combining both tax policies makes an increase in the labor market tightness from $\theta_1^*$ to $\theta_2^*$. Hence, the increase is more than the increase in the labor market tightness with a decrease in the payroll taxes, which effect is an increase from $\theta_1^*$ to $\theta^{\tau^w}$. Also, the effect is higher than the increase in the law enforcement expenditure, which increases the labor market tightness from $\theta_1^*$ to $\theta^e$. However, the combination tax policies impact over real formal wage is uncertain.

Based on the above, the combination of tax policies has a more significant impact on reducing unemployment and informality than reducing only the payroll tax or increasing law enforcement expenditure. The unemployment and informality decrease from $(u +$

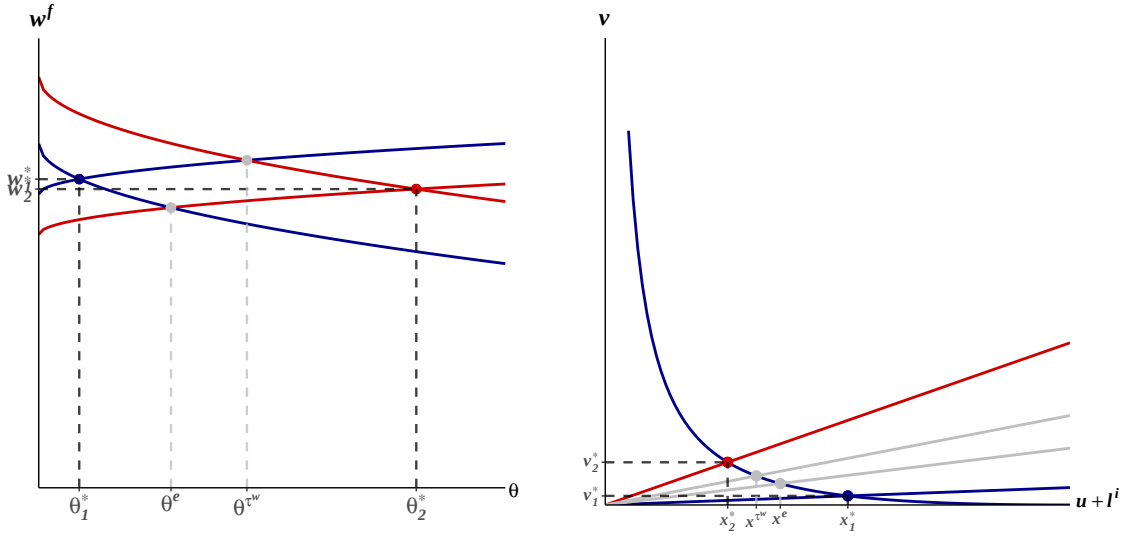


Figure 6: The effects of a decrease in payroll taxes $\tau^w$ and increase in law enforcement expenditure $e$. Note: $x_j^*$ refers to $(u + l^i)_j^*$

$l^i)_1^*$ to $(u + l^i)_2^*$, which is a higher reduction than the effect of reducing uniquely $\tau^w$ or increase $e$ that reduce unemployment and informality until $(u + l^i)^{\tau^w}$ and $(u + l^i)^{\theta}$, respectively.

On the other hand, the analytical results show that the decrease in the rigidity of real wages generates a significant decrease in informality and unemployment and a decrease in formal equilibrium wages who suggest that policiy that make more flexible the labor market has a greater impact in the informality and unemployment reduction. The following section shows the simulation results of the model's long run effects on the economy's main variables in case of changes in payroll taxes and law enforcement spending. Also, I compared two scenarios sensitive to rigidities in real wages.

## 5.2 Simulations

The simulation of the model presents the primary outcome variables in the long run, given the change of the tax policies. Figures (7) and (8) estimate the model with endogenous public expenditure. Hence, the government expenditure adjusts for changes in the payroll tax and law enforcement expenditure. In contrast, Figure (8) shows the simulation results when the public expenditure in the economy is constant, and the enforcement expenditure is endogenous. In both simulations, I present the case when the economy is in front of high real wage rigidities and low real wage rigidities.

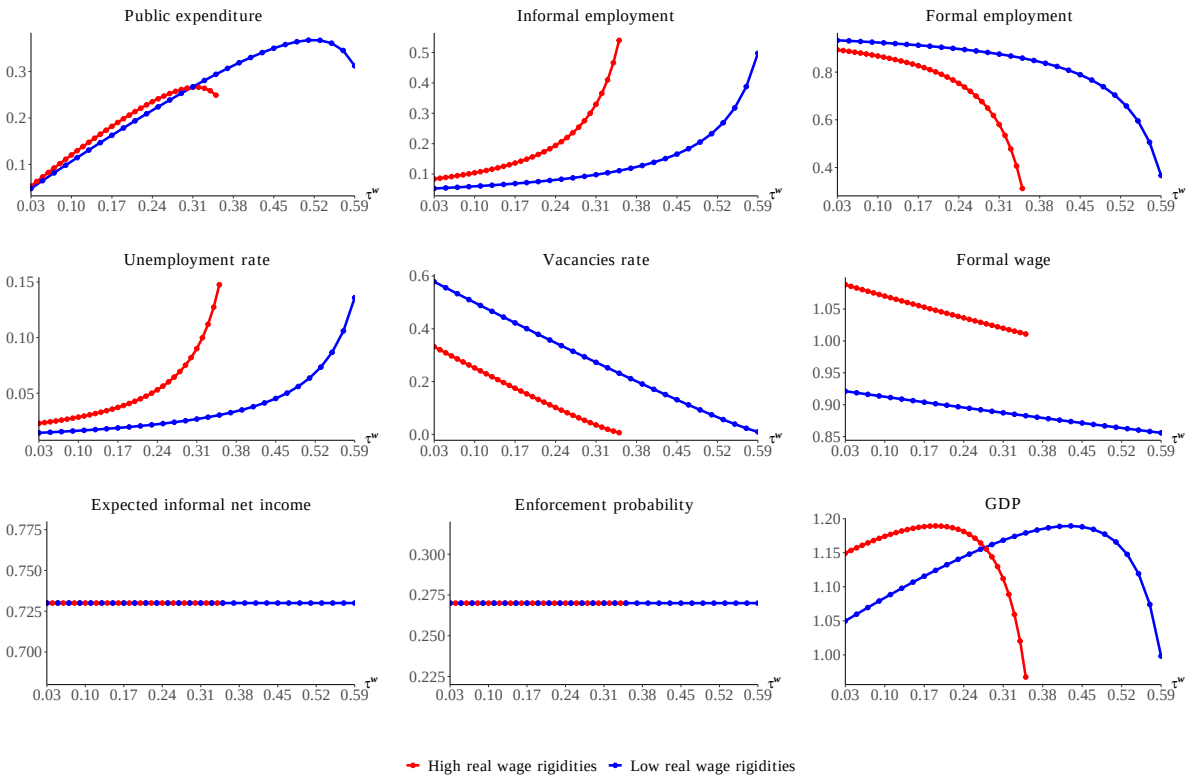


Figure 7: Multiple payroll tax policies with constant enforcement expenditure, each point is a steady-state value of the variable given the value of $\tau^w$. High wage rigidities: $\zeta = 0.29$; Low wage rigidities: $\zeta = 0.14$

Figure (7) show the simulations results for multiple payroll tax policies. The increase in the payroll taxes lead to an increase in the public expenditure, with a decrease in the formal employment and an increase in the unemployment and informality. The results suggest that when the economy has low real wage rigidities, increasing payroll taxes increases informal employment and decreases formal employment more slowly relative to the scenario with higher real wage rigidities. The increase payroll taxes effect on unemployment is similar with the informality behavior: the unemployment increase is slower with low rigidities than with high rigidities.

Additionally, the laffer curve of public expenditure suggest that it exists a policy in which is possible decrease the payroll taxes, increase the public expenditure and the formal employment, and reduce the informality and unemployment rate. However with low real wage rigidities is possible increase the public expenditure with the increase in payroll taxes whit low increase in the unemployment and informal rate, and also with low decrease in the formal employment.

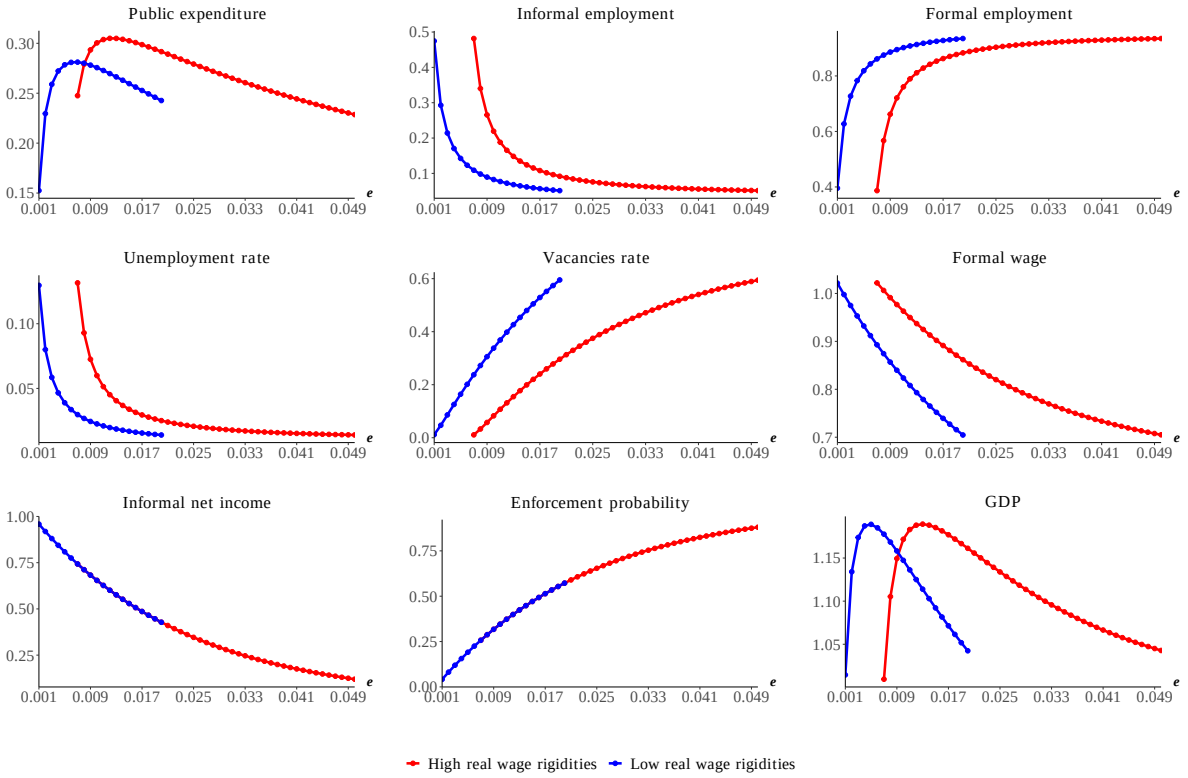


Figure 8: Multiple enforcement expenditure policies with constant enforcement expenditure, each point is a steady-state value of the variable given the value of $e$. High wage rigidities: $\zeta = 0.29$; Low wage rigidities: $\zeta = 0.14$

Concerning the law enforcement expenditure policy, Figure (8) shows the results of the simulation. An increase in the law enforcement expenditure generates increases in public expenditure, with a decrease in the unemployment and informal rate and increased formal employment. The results suggest that with high real wage rigidities, there must be a more significant increase in law enforcement expenditure for there to be a significant effect on the reduction of informal employment and unemployment. In contrast, when the economy has greater flexibility in real wages, a minor increase in law enforcement expenditure is required to reduce informal employment and unemployment.

Also, the results present a public expenditure Laffer's behavior in both scenarios, with high and low real wage rigidities. In both scenarios, it is possible to find a level of law enforcement expenditure that leads to a peak in public expenditure with a relevant decrease in unemployment and informality rates and increases in formal employment. With low real wage rigidities, the economy reaches the public expenditure peak with less law enforcement expenditure. In contrast, the economy with high real wage rigidities reaches the public expenditure peak with a considerable increase in law enforcement expenditure.

After a certain level of enforcement, the public expenditure present a permanent decrease. However, when the economy is in front of higher real wage rigidities, increases in enforcement spending tend to increase public spending by a more significant proportion than the economy with lower real wage rigidities.

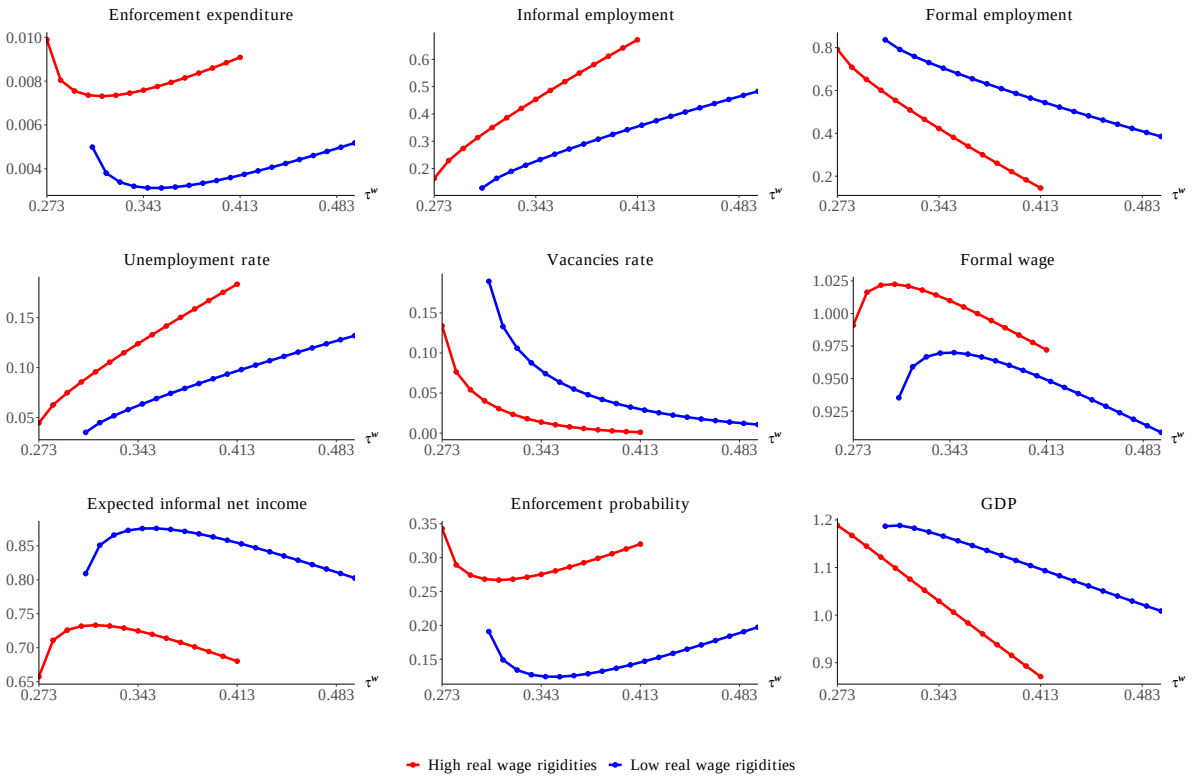


Figure 9: Multiple payroll tax policies with constant public expenditure, each point is a steady-state value of the variable given the value of $e$. High wage rigidities: $\zeta = 0.29$; Low wage rigidities: $\zeta = 0.14$

Finally, Figure (9) shows the simulation results when the economy has a constant public expenditure and the law enforcement expenditure is adjusted for changes in the payroll taxes. These simulations illustrates the combination of tax policies' effect on the labor market. The law enforcement expenditure has a convex behavior with a minimum point. Hence the increase in the payroll taxes leads to a decrease in the enforcement expenditure. However, there is a point where the increase in payroll taxes increases law enforcement expenditure.

The results show that the increase in the payroll taxes induces an increase in the unemployment and informality rates, no matter the level of law enforcement expenditure. The increase in payroll taxes in the economy with high real wage rigidities hurts formal

employment more than the economy with low real wage rigidities. Also, the law enforcement expenditure is higher with high real wage rigidities economy than low wage rigidities economy. Nevertheless, the simulation could suggest a combination of tax policies where it is possible to reduce the payroll taxes and increase the law enforcement expenditure to reduce informality significantly.

In this sense, the simulation results agree with the analytical results, where the combination of tax policies, such as decreasing payroll taxes and increasing law enforcement expenditure, generate significant effects on reducing unemployment and informality. However, when the economy has high real wage rigidities, law enforcement expenditure will be higher before payroll taxes decrease compared to an economy with low real rigidities.

The simulations results show that labor policies and state capacity, defined as tax policies such as reduction in payroll taxes and increase in enforcement expenditure, generate relevant effects on the increase the formal employment and decrease the informality and unemployment. Likewise, a real wage flexibility policy could facilitate an increase in tax revenues, increasing the ability of tax policies to achieve objectives related to informality and unemployment reduction and increases in government revenues.

An economy with lower real wage rigidities can increase its fiscal revenues with payroll taxes increases affecting formal employment to a lesser extent. An economy with low real wage rigidities can increase its fiscal revenues and decrease unemployment and informality, with fewer increases in law enforcement expenditure compared to an economy with higher wage rigidities.

When the economy has high real wage rigidities, the results show that tax policies such as reducing payroll taxes or increasing law enforcement expenditure have a relevant effect on informality and unemployment reduction. First, the reduction of payroll taxes can increase tax revenues with lower rates of informality and unemployment. Second, increases in law enforcement expenditure can generate greater increases in government revenue while reducing unemployment and informality in the economy. Also, the tax policy combination can lead to significant effects in reducing informality and unemployment.

On the other hand, Tables (2) through (4) show the changes in the main variables of interest given a reduction in payroll taxes and an increase in law enforcement expenditure. The simulations were performed assuming an initial value of $\tau^w = 0.33$ and decreasing it up to 25 percentage points (pp) and an initial value of $e = 0.007$ increasing it up to 2.5 pp. Tables (2) and (3) present the impact of tax policies when the economy has endogenous government expenses, while Table (4) has the results with a payroll tax change with endogenous law enforcement expenditure.

Table (2) shows the change in the main variables with a decrease in the payroll taxes,

| Rigidities | Variables | 5pp | 10pp | 15pp | 20pp | 25pp |
|---|---|---|---|---|---|---|
| High real wage rigidities | Public expenditure | -0.01 | -0.04 | -0.07 | -0.12 | -0.16 |
| | Informal employment | -0.16 | -0.23 | -0.27 | -0.29 | -0.31 |
| | Formal employment | 0.20 | 0.29 | 0.34 | 0.37 | 0.40 |
| | Unemployment rate | -0.04 | -0.06 | -0.07 | -0.08 | -0.09 |
| Low real wage rigidities | Public expenditure | -0.03 | -0.07 | -0.12 | -0.15 | -0.18 |
| | Informal employment | -0.01 | -0.03 | -0.04 | -0.04 | -0.05 |
| | Formal employment | 0.01 | 0.03 | 0.04 | 0.05 | 0.06 |
| | Unemployment rate | -0.00 | -0.01 | -0.01 | -0.01 | -0.01 |

Table 2: Decrease in payroll taxes assuming $\tau^w = 0.33$ as the initial value with endogenous government expenditure. The decrease in $\tau^w$ is from 5 pp to 25 pp. Baseline rates. HWR: Public expenditure = 0.26, Informal employment = 0.41, Formal employment = 0.47, Unemployment rate = 0.11. LWR: Public expenditure = 0.28, Informal employment = 0.10, Formal employment = 0.86, Unemployment rate = 0.02.

assuming an initial payroll tax value of 0.33. When the economy has high real wage rigidities, a decrease of 5 pp leads to a decrease in the informality of 0.16 pp. In contrast, the decrease in informality is 0.01 pp when the economy has lower real wage rigidities. If the payroll taxes decreases 25 pp, the decrease in the informality is around 0.31 pp and 0.05 pp for high and low real wage rigidities, respectively. On the other hand, when a decrease of 5 pp, the increase in formal employment is around 0.20 pp for an economy with high real wage rigidities and 0.01 pp for an economy with low real wage rigidities.

Also, formal employment increased by 0.40 pp and 0.06 pp with a decrease in payroll taxes of 25 pp in an economy with high and low real wage rigidities. With decreases in payroll taxes, the unemployment has a reduction but with a low magnitude than the reduction of the informality. Given the above, the economy with high real wage rigidities has a higher reduction in informality and unemployment and relevant increases in the formality with a decrease in payroll taxes than the economy with low real wage rigidities.

| Rigidities | Variables | 0.5pp | 1pp | 1.5pp | 2pp | 2.5pp |
|---|---|---|---|---|---|---|
| High real wage rigidities | Public expenditure | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 |
| | Informal employment | -0.32 | -0.37 | -0.40 | -0.41 | -0.42 |
| | Formal employment | 0.40 | 0.48 | 0.51 | 0.52 | 0.53 |
| | Unemployment rate | -0.09 | -0.10 | -0.11 | -0.11 | -0.11 |
| Low real wage rigidities | Public expenditure | -0.01 | -0.03 | -0.04 | | |
| | Informal employment | -0.04 | -0.05 | -0.06 | | |
| | Formal employment | 0.05 | 0.07 | 0.07 | | |
| | Unemployment rate | -0.01 | -0.01 | -0.02 | | |

Table 3: Increase in law enforcement expenditure assuming $e = 0.007$ as the initial value with endogenous government expenditure. The increase in $e$ is from 0.5 pp to 2.5 pp. Baseline rates. HWR: Public expenditure = 0.24, Informal employment = 0.48, Formal employment = 0.38, Unemployment rate = 0.13. LWR: Public expenditure = 0.28, Informal employment = 0.10, Formal employment = 0.86, Unemployment rate = 0.02.

Table (3) shows the results related to the increase in law enforcement expenditure with endogenous government expenditure for both scenarios of real wage rigidities. First, with high real wage rigidities, the increase of 0.5 pp in the enforcement expenditure reduces the informal employment by 0.32 pp and the unemployment by 0.09 pp, and increases the formal employment by 0.40 pp. Second, the low real wage rigidities case shows that the increase in enforcement expenditure of 0.5 pp causes a reduction in the informality close to 0.04 pp and the unemployment close to 0.01 pp. In comparison, the increase in formal employment is 0.05 pp. Based on the above, the increase in law enforcement expenditure is a policy with relevant results in the informal employment reduction, which effect is greater when the economy is in front of high real wage rigidities than in the economy with low real wage rigidities.

| Rigidities | Variables | 5pp | 10pp | 15pp | 20pp | 25pp |
|---|---|---|---|---|---|---|
| High real wage rigidities | Enforcement expenditure | -0.00004 | 0.000005 | 0.0001 | 0.0006 | 0.002 |
| | Informal employment | -0.03 | -0.07 | -0.11 | -0.15 | -0.22 |
| | Formal employment | 0.04 | 0.09 | 0.14 | 0.19 | 0.28 |
| | Unemployment rate | -0.009 | -0.01 | -0.03 | -0.04 | -0.06 |
| Low real wage rigidities | Enforcement expenditure | 0.0001 | 0.0005 | 0.001 | | |
| | Informal employment | -0.02 | -0.04 | -0.08 | | |
| | Formal employment | 0.02 | 0.06 | 0.10 | | |
| | Unemployment rate | -0.006 | -0.01 | -0.02 | | |

Table 4: Decrease in payroll taxes assuming $\tau^w = 0.33$ as the initial value with endogenous enforcement expenditure. The decrease in $\tau^w$ is from 5 pp to 25 pp. Baseline rates. HWR: Enforcement expenditure = 0.007, Informal employment = 0.38, Formal employment = 0.50, Unemployment rate = 0.10. LWR: Enforcement expenditure = 0.003, Informal employment = 0.21, Formal employment = 0.73, Unemployment rate = 0.05.

Finally, Table (4) shows the results of a reduction in payroll taxes when the economy has endogenous enforcement expenditure and constant government expenditure. In the scenario with high real wage rigidities, a decrease in 5 pp returns a decrease in the informal employment around 0.03 pp, a decrease in the unemployment of 0.009 pp, and an increase in the formal employment around 0.04 pp. With low real wage rigidities, the reduced payroll tax of 5 pp reduces informal employment and unemployment by 0.02 pp and 0.006 pp, respectively. At the same time, the increase in formal employment is close to 0.02 pp. Also, the effect on enforcement expenditure is relatively lower in both rigidities scenarios.

The simulations results note the relevance of the tax policies change that affects the principal labor market variables in an economy. Also, highlight the role of the real wage rigidities in the magnitude effect of these policies. The results pretend to understand the effect of tax policies and the combination of these policies as an alternative to reduce informal employment and unemployment and increase formal employment.

When the economy has high real wage rigidities, the tax policies have a more significant impact on reducing informal employment and the increase in formal employment. Also, a fixed government expenditure with an endogenous law enforcement expenditure highlights the case in which it is impossible to modify the government spending, and there is a combination of tax policies that reduce the informality. In addition, the economy with low real wage rigidities has lower levels of informality and a wider margin of action in the tax policies that do not negatively affect formal employment and increase the government income. Hence, a relevant policy to reduce the informality is a flexibilization of real wage rigidities. Besides the above, the results highlight that the tax policies are effective in the case of low real wage rigidities to encourage formal employment.

# 6 Conclusion

This article has developed a dynamic general equilibrium model with search and matching frictions and rigidities of real wages through shirking mechanisms, based on Martin & Wang (2020). It includes a government and an informal labor market. The simulation results suggest that the rigidities of the real wage in the economy are a relevant determinant of the magnitude of the tax policies that seek to reduce informality.

The model highlights the tax policies combination as a relevant instrument to reduce informal employment and increase formal employment. In this respect, the paper shows the importance of labor policies and state capacity in reducing informality and the government income increase. First, the analytical results show that a decrease in payroll taxes increases the formal employment demand, and an increase in enforcement expenditure decreases the informal employment offer.

Hence, both policies significantly impact the reduction of informality and unemployment and the increase in formal employment. Based on the above, the tax policies combination has a most significant impact on the reduction of informality and unemployment because it affects the demand and supply side of the economy. Also, a policy associated with a flexibilization in the real wage rigidities decreases informal employment and unemployment.

The above is coherent with the simulations results. The decrease in payroll taxes and increase in law enforcement impact the informal employment reduction for high and low real wage rigidities. The model shows that for both wage rigidities scenarios, there is the possibility that it is possible to reduce the informality with an increase in the government income with a decrease in the payroll taxes. Also, there is a case in which the increase in law enforcement expenditure leads to a peak in public spending with low levels of informal employment. However, the simulations present the relevance of the wage flexibility as a policy that could impact the reduction of the informality and an increase the fiscal revenues with the tax policies.

Additionally, with an endogenous law enforcement expenditure and a constant government expenditure, there is a case in which it is possible to reduce informal employment and unemployment with a decrease in payroll taxes and an increase in enforcement expenditure. The above is coherent with the comparative statics in which the tax policies combination has a greater impact on the informality reduction.

In addition to the above, the impact of the policy on the informal employment reduction depends on the initial value of the tax policies. Although, the results suggest that tax policies could have a greater impact on the reduction of the informality when the economy has a high real wage rigidities relative to the economy with low real wage rigidities. Also, the tax policy effect has a bigger impact on informality reduction than the unemployment reduction. Notwithstanding the previous, the tax policies have a

greater effect on reducing informal employment in an economy with high and low real wage rigidities.

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# 7 Appendix

## A Mincer Regressions

I estimate the wage gap between the formal and informal sectors using a Mincerian regression model, the Table (A1) present the results. In this way, I used the information available in the Gran Encuesta Integrada de Hogares (GEIH) from 2008 to 2019 for the 23 leading main metropolitan. The dependent variable is the logarithm of the monthly hourly income ($w_h$) reported by workers in the economy.

Among the explanatory variables used in the estimation, the informality dummy variable takes the value of 1 if the person is informal and 0 if he/she is formal; the variable was constructed based on the DANE informality definition, which classifies a worker as informal if he/she meets at least one of the following criteria: employees of a private

company with five or fewer workers, domestic employees, self-employed workers who work in establishments with up to five people, family workers, unpaid workers, day laborers, or employer in a company with five or fewer workers.

The model considers variables such as age, years of schooling, and sex defined as a dichotomous variable that takes the value of 1 if the worker is a man and 0 if the worker is a woman. In addition to the above, the estimations were made, including fixed effects by metropolitan areas and by year. On the other hand, I exclude the three percent observations from the tail distributions to clean the data.

| Variables | $\log(w_h)$ | $\log(w_h)$ | $\log(w_h)$ | $\log(w_h)$ |
|---|---|---|---|---|
| Informal DANE | -0.625*** | -0.354*** | -0.342*** | -0.339*** |
| | (0.000765) | (0.000760) | (0.000769) | (0.000748) |
| Age | | 0.0331*** | 0.0336*** | 0.0350*** |
| | | (0.000158) | (0.000157) | (0.000154) |
| $Age^2$ | | -0.000310*** | -0.000314*** | -0.000339*** |
| | | (1.93e-06) | (1.93e-06) | (1.90e-06) |
| Schooling | | -0.0145*** | -0.0157*** | -0.0228*** |
| | | (0.000350) | (0.000353) | (0.000348) |
| $Schooling^2$ | | 0.00442*** | 0.00451*** | 0.00476*** |
| | | (1.78e-05) | (1.80e-05) | (1.78e-05) |
| Man | | 0.175*** | 0.178*** | 0.180*** |
| | | (0.000699) | (0.000695) | (0.000678) |
| Constant | 8.386*** | 6.996*** | 7.048*** | 6.870*** |
| | (0.000543) | (0.00356) | (0.00374) | (0.00397) |
| 23 main metropolitan areas FE | No | No | Yes | Yes |
| Time FE per year | No | No | No | Yes |
| N. of obs | 2,766,183 | 2,654,843 | 2,654,843 | 2,654,843 |
| $R^2$ | 0.191 | 0.376 | 0.386 | 0.416 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A1: Mincerian regression models. The wage gap is the exponential of the Informal DANE coefficient with both fixed effects.

# B Sensitivity Analysis

Given the relevance of the parameter $d$ on the effect of real wage rigidities, Figures A1 – A3 show the results of the simulations presented earlier in the paper, assuming values of the detection rate between 0.2 and 0.95.

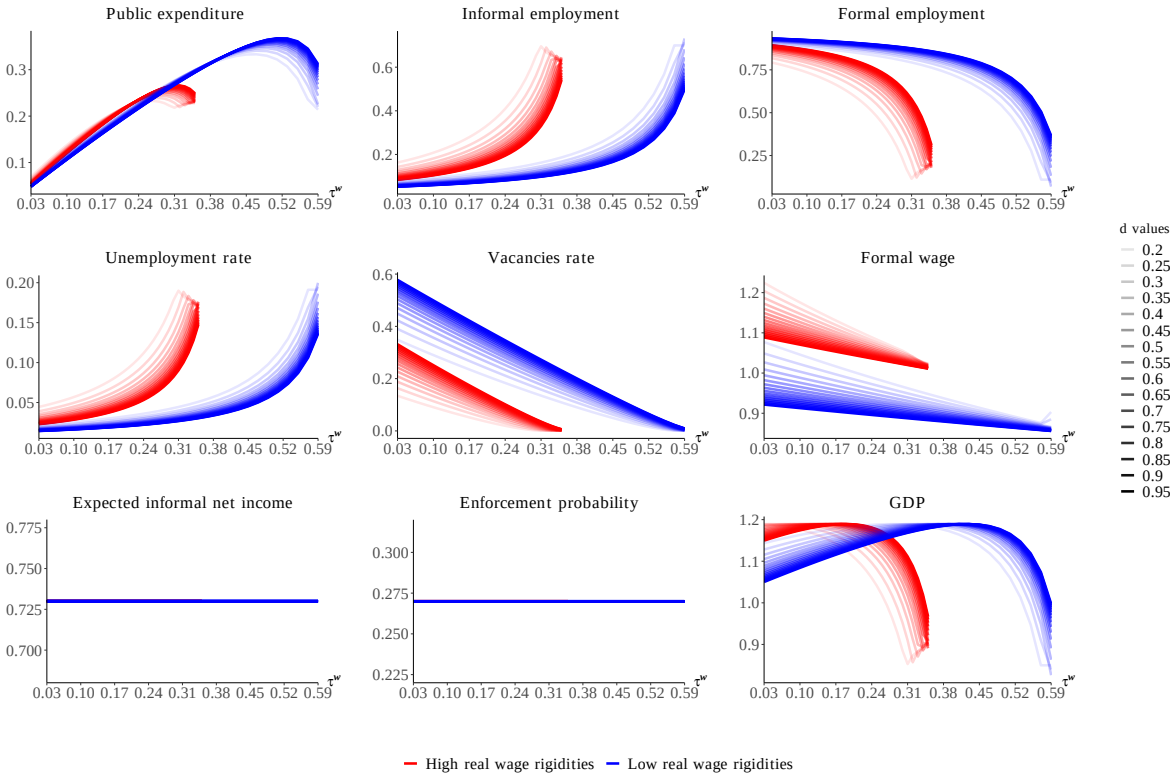


Figure A1: Multiple payroll tax policies with constant enforcement expenditure, for different detection rate values $d$. Each point is a steady-state value of the variable given the value of $\tau^w$.

Figure A1 shows the simulations results for multiple payroll tax policies, given different detection rate values. The main results are the same presented previously in the paper; the increase in payroll tax can lead to an increase in public expenditure until a given level of $\tau^w$, with a formal employment decrease and an increase in the informality and unemployment.

However, the principal difference in the results is the magnitude. Whit low values in the detection rate, the increase in the informal employment given increases in payroll taxes is higher than the scenario with high values of detection rate for both real wage rigidities cases. The above pattern is equal for formal and unemployment. Also, the increase in

public expenditure whit low values of detection rates tends to be lower than the scenario with high values of detection rate for high and low real wage rigidities.

For the above, under increases in probability detection rate, the results in the reduction of informality and unemployment due to decreases in payroll taxes tend to be a higher magnitude for both real wage rigidities scenarios. Also, the public expenditure laffer curve suggests that the decrease in payroll taxes could increase the public expenditure with a high detection rate compared to the case of a low detection rate.

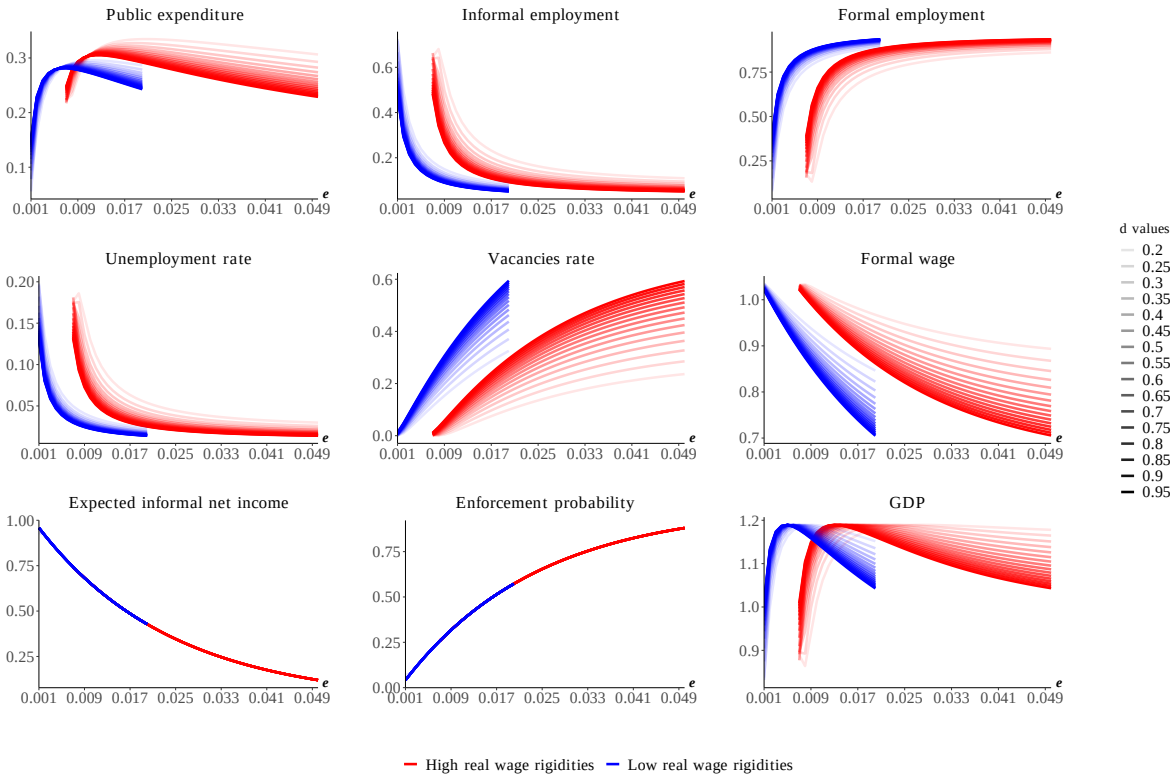


Figure A2: Multiple enforcement expenditure policies with constant enforcement expenditure, for different detection rate values $d$. Each point is a steady-state value of the variable given the value of $e$.

Similarly, Figure A2 shows the simulations results given the increases in the enforcement expenditure for the principal economic variables. With increases in law enforcement expenditure, the simulations suggest that an economy with high detection rate values tends to lower informality and unemployment levels more than an economy with low detection rate values for high and low real wage rigidities.

Regarding public expenditure, the results differ from the simulation in the Figure A1. There are scenarios in which increases in enforcement expenditure with low values of

detection rate could lead to a high level of public expenditure for low and high real wage rigidities. The above is a consequence of the levels of informal employment with low levels of detection rates. The probability of auditing increases given the increase in law enforcement expenditure. Consequently, there is an increase in the informal sector fines that increase public expenditure.

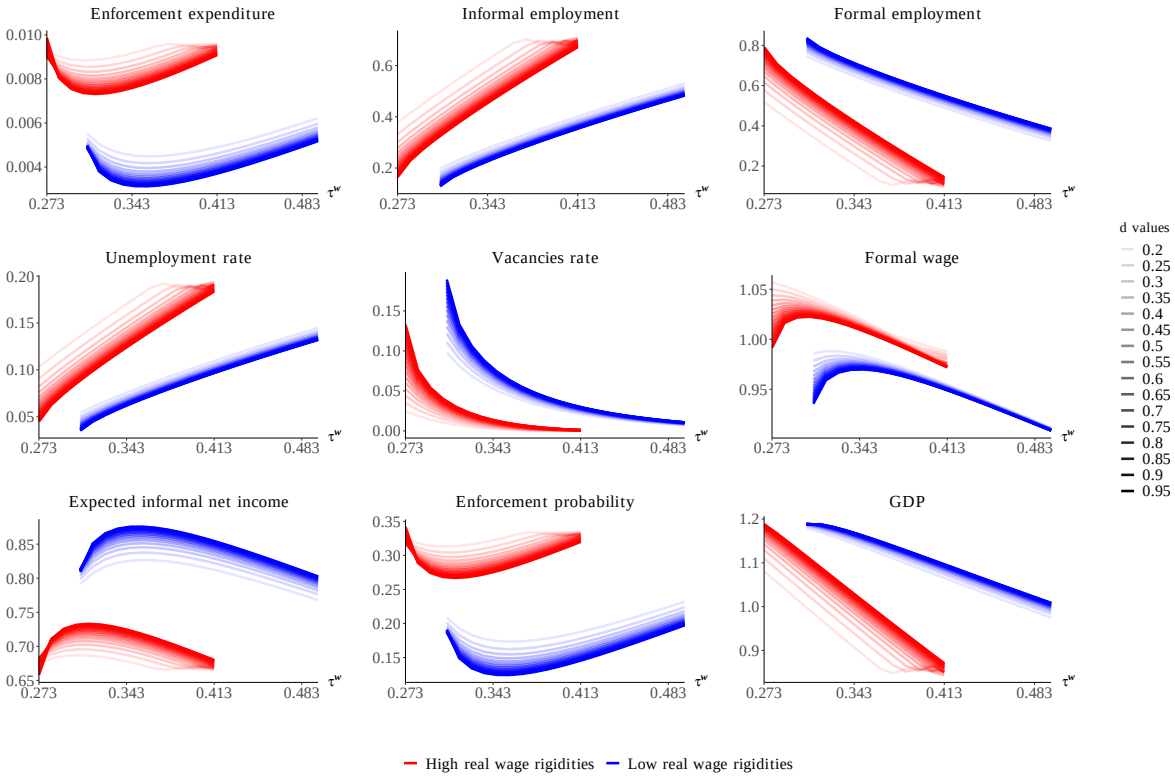


Figure A3: Multiple payroll tax policies with constant public expenditure, for different detection rate values $d$. Each point is a steady-state value of the variable given the value of $e$.

Finally, Figure A3 shows the simulation results given the change in payroll taxes with endogeneous enforcement expenditure and different values of the detection rate probability. Similarly, the simulations are consistent with the main results in the paper. There is a scenario in which a decrease in payroll tax could increase enforcement expenditure and consequently reduce the levels of informality and unemployment significantly and increase formal employment.

Nevertheless, the payroll tax magnitude effect change given the detection rate values. With high detection rate values, the reduction in the level of informality and unemployment is higher with the decrease in payroll taxes than in the scenario with low values of detection rate. The above is satisfied for both cases of real wage rigidities. In addition,

with low values of detection rate, the levels of enforcement expenditure tend to be higher when the $d$ value is lower. The above suggests that the enforcement expenditure effort is inversely related to the detection rate probability for high and low wage rigidities.