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Empirical evidence of jump behaviour in the Colombian intraday bond market *

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Abstract

Simulations and empirical studies suggest that incorporating a discontinuous jump process in asset pricing models improve volatility forecasting, pricing of instruments, and hedging positions in a portfolio. In this paper we analyze high frequency market data of Colombian sovereign bonds in order to study the presence or absence of discontinuities in the price generating process. We find that Colombian sovereign debt experiments jumps across all maturities but with different frequencies, in particular, we do not find that long term bonds jump less frequently than short term bonds. Furthermore, bonds with closer maturities cojump in greater magnitude than those with a greater distance between them. Finally, we find significant day-of-the-week effects, as well as an important increase in the jump frequency due to surprises in economic information related to US monetary policy and no effect due to direct monetary policy announcements in Colombia or the US.

Keywords: Jumps, Realized Variance, High Frequency, Preferred habitat theory, Monetary Policy Announcements.

JEL codes: G12, E43, C58.

1 Introduction

The mathematical modelling of financial assets is a key aspect of quantitative portfolio management. Stock market participants use it for pricing instruments, hedging positions, and forecasting uncertainty. Pricing models assume that an asset's log-price follows a time-continuous diffusion process, usually a geometric brownian motion. However, empirical studies and simulations suggest that incorporating pure jump processes is necessary for a correct specification of these models Johannes [2004]. Additionally, Johannes [2004] and Andersen et al. [2007] find that explicitly expressing discontinuities in price models improves volatility forecasting, while Piazzesi [2005] finds improvements in the pricing of US treasuries when incorporating FOMC news announcements as determinants of potential jump times.

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Recent literature extends the notion of price jumps to include cojumps, i.e., simultaneous jumps present in different assets; these cojump phenomena were first studied in Barndorff-Nielsen and Shephard [2004a]. Bollerslev et al. [2008] find strong evidence for modest-sized but highly significant cojumps in a panel of high-frequency stock return data. Additionally, Novotný and Urga [2017] find common discontinuities in stock prices within a portfolio. They prove these cojumps can be diversified by means of a correct combination of assets, though a method to find the combination which eliminates these jumps is left as a future endeavor.

Most of the work previously cited is focused on the equity market. Unlike stocks, fixed income instruments share many characteristics among themselves, and are usually only differentiated by maturity and coupon. Dungey et al. [2009] find "significant evidence of jumps and cojumps in the US term structure" in response to macroeconomic news announcements. Furthermore, around one fifth of cojump activity occurs independently of news. The authors look at this cojump activity and interpret their findings in the light of several theories about the formation and evolution of the term structure of the yield curve.

In order to test the presence of jumps, many of the previous literature uses the test statistic developed by Barndorff-Nielsen and Shephard [2004b] in which two measures of realized volatility are compared and contrasted: realized variance (RV) and bi-power variation (BV). By taking the difference between the former and the latter, we can obtain a notion of the size of a potential discontinuity (see Barndorff-Nielsen and Shephard [2004b], Andersen et al. [2003a], Huang and Tauchen [2005]). Intuitively, jumps are interpreted as the discrepancy between these two measures of realized volatility.

In this paper we test for the presence of jumps using high frequency Colombian sovereign bond data. Second, jump behaviour is described and characterized by analyzing the frequency and magnitude of its activity. Third, following the procedure presented in Dungey et al. [2009], cojumps across various assets are compared in the context of the two main theories of term structure formation: the expectation's theory of the term structure and the market segmentation/preferred habitat model. Finally, we look whether there are day-of-the-week effects or the relationship between jump frequency and economic announcements and surprises.

Results indicate that bonds distributed throughout the Colombian yield curve commonly experience jumps independently of maturity, this is different than what is found in the US data where long term bond show less jump activity than short term bonds. One possible explanation it that 15 year bonds in Colombia are less liquid than short term bond and hence there are important jumps in these types of assets. Furthermore, an average of 46.989% of jumps occur simultaneously across two assets. Most commonly, it is the bonds in the shorter end of the term structure which jump simultaneously, though illiquidity hinders a robust analysis for assets on the long end of the yield curve. Daily seasonalities are found in both univariate and multivariate jump activity, with both types of jumps being least likely to occur on Monday. Cojumps are most likely to occur on Wednesdays or Thursdays, depending on the sampling frequency. Furthermore, a panel logit model finds a persistent Thursday effect of an increase of 7% in the frequency of jumps for almost all sampling frequencies. In addition we find that investors in the Colombian sovereign bond market are more sensitive to external surprises that may impact a change in US monetary policy than local changes in monetary policy or any other economic

announcement. In particular, during 2017-2018 unexpected changes in CPI inflation in the US created a 37% increase on the probability of observing a jump, using 5 minute data.
This paper contributes to understanding the dynamics of bond markets in emerging economies and also provides empirical evidence regarding conflicting theories on the terms structure of interest rates (liquidity preference vs preferred habitat hypothesis). In particular, measuring the importance of co-jumps across different segments of the yield curve provides evidence regarding the behavior of investors along the curve.

The rest of the document is organized as follows. Section 2 discussed how different theories regarding the term structure of interest rate provide can lead to different hypothesis regarding the timing frequency of jumps in different maturities along the yield curve. Section 3 present the methodologies used to quantify and test for jumps using high-frequency transaction data. Section 4 contains an overview of the bond transaction database, along with considerations about sampling frequencies and methods. Section 5 presents an in-depth showcasing of results and the corresponding discussion. Finally, section 6 concludes.

2 Investor preference and the yield curve

Measuring jumps on bond has to consider the term structure of interest rates. Whereas jumps in specific stocks can be analyzed in isolation, jumps in bonds must have an important relationships among the different maturities. Term structure models are based on the idea that there exist a lower dimensional set of variables (factors) that capture most of the movements across the different maturities. So it is important to consider how much of these co-movement are related to discontinuities components of the data generating process. Although this is beyond the scope of the paper we empirically test for the relationship among jumps in different maturities.

The theories of liquidity preference and preferred habitat/market segmentation are two theories about how the term structure of the yield curve forms and evolves in time. Liquidity preference argues that yields of longer dated bonds are higher due to a liquidity risk premium. This liquidity risk premium arises from the greater possibility of capital loss in long term bonds in comparison to shorter term debt. Consequently, a greater risk of loss would imply that long dated bonds are more reactive to macroeconomic news announcements and external shocks than short bonds. Thus, we would expect to find greater jump activity in bonds of large maturities.

On the other hand, the preferred habitat hypothesis argues that individual investors operate in different segments of the term structure according to their own particular interest. Thus, movements in the yield curve respond to supply and demand pressures of investors who populate different sections of the market. For example, speculators who want to maximize profits may be more interested in trading short maturity bonds due to their liquidity. In contrast, pension funds or insurance companies may choose to trade long term bonds to fund future liabilities. Originally, this models assumes a rigid segmentation of markets. Modigliani and Sutch [1966] argue against this premise, proposing that investors may operate outside of their preferred segments if a risk premium compensates their aversion to reinvestment risk.

In this context, since prices respond to local behaviour of different investors, the short, medium, and long term yields would be independent of one another. Thus, it is reasonable to expect that if speculators and arbitrageurs tend to operate in the short end of the term structure, news and announcements may cause greater impact on short yields. At the same time, long bonds would be reactive to news relevant to the long term state of the economy. This qualitative overview of two theories of term structure behaviour will give us the guiding principles in our analysis of jump behaviour. In addition, under the preferred habitat hypothesis we would expect that bond with similar maturities would "jump together" more frequently than bond that are further apart. We explore this specific hypothesis in section 5.3.

3 Measuring and testing for jumps

Continuous time diffusion models are a vital tool in modelling the price evolution of financial instruments. Their analytic convenience makes them an extremely useful tool for drawing interpretations and simplifying hedging calculations on which modern financial derivatives are based on. These models commonly assume that the change of an asset's log-price p_t follows the stochastic differential equation:

$$dp_t = \mu_t dt + \sigma_t dW_t \tag{1}$$

where μ_t is the instantaneous drift given by a locally bounded variation process and σ_t is a strictly positive volatility process with well defined limits. W_t is a Brownian motion. Under the premise of equation (1) the j-th intraday log-return is defined as $r_{t,j} = p_{t,j} - p_{t,j-1}$. The associated quadratic variation of this model is given by:

$$\langle r, r \rangle_t = \int_0^t \sigma_s^2 ds \tag{2}$$

In what follows we assume that the data generating process for a bond's log-price is given by:

$$dp_t = \mu_t dt + \sigma_t dW_t + dL_J(t) \tag{3}$$

The new third term is a pure jump Levy process, where $L_J(t) - L_J(s) = \sum_{s \leq \tau \leq t} \kappa(\tau)$ is the jump size. We assume that this is a particular case of Levy process known as a Poisson compound process. Additionally, we also assume constant jump intensity λ and jump size $\kappa(\tau)$ as an identically distributed (i.i.d.) random variable. Now, the quadratic variation for this model is:

$$\langle r, r \rangle_t = \int_0^t \sigma_s^2 ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2 \tag{4}$$

In the more general process, expression (3), the quadratic variation includes the jump size.

Asymptotically, realized variance (RV) can give us a good approximation of the quadratic variation:

Definition 1 *Realized variance:*

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2$$

This means that, for our jump-diffusion model, the realized variance converges to expression (4) in the limit:

$$\lim_{M\to\infty} RV_t = \int_{t-1}^{t} \sigma_s^2 ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2 \tag{5}$$

Equation (5) gives us an estimate of daily volatility which captures the effect of the volatility process σ_t as well as the magnitude of variance attributed to discontinuous jumps, given by $\sum_{j=1}^{N_t} \kappa_{t,j}^2$

Barndorff-Nielsen and Shephard [2004a] and their following extensions in Barndorff-Nielsen and Shephard [2005a] and Barndorff-Nielsen and Shephard [2005b] suggest that, under reasonable assumptions, bi-power variation enables a consistent estimator of quadratic variation that is robust to jumps:

Definition 2 *Bi-power variation:*

$$BV_t = \mu_1^{-2} \frac{M}{M-1} \sum_{j=2}^{M} |r_{t,j}||r_{t,j-1}|$$

This definition of bipower variation (BV) is multiplied by a coefficient of standardization μ_k which allows for a direct comparison with RV. This coefficient is given by $\mu_k \equiv 2^{k/2}\Gamma\left[(k+1)/2\right]/\Gamma\left(1/2\right)$ for $k > 0$. Asymptotically, we have:

$$\lim_{M\to\infty} BV_t = \int_{t-1}^{t} \sigma_s^2 ds \tag{6}$$

We can use the fact that BV is robust to jumps, while RV is not, in order to obtain a notion of the size of a jump. By taking the difference between (5) and (6), asymptotically, we get:

$$RV_t - BV_t \to \sum_{t-1\leq\tau\leq t} \kappa_\tau^2 \tag{7}$$

Equation (7) implies that we can obtain a consistent estimate for the size of daily jumps. Despite this, for finite samples, the difference between RV and BV is not guaranteed to be positive. Nonetheless we can truncate its value at zero and consider only positive values.

Instead of analyzing the magnitude of jumps, it is more interesting to study the relative contribution of jumps to price variance. Thus, an initial expression the jump statistic (JS) in Barndorff-Nielsen and Shephard [2004a] is given by:

$$JS_t = \frac{RV_t - BV_t}{\sqrt{\left(\mu_1^{-4} + 2\mu_1^{-2} - 5\right)\int_{t-1}^{t} \sigma_s^4 ds}} \to \mathcal{N}(0,1)$$

Where the original difference in volatilities is now divided by a coefficient which standardises the statistic's distribution. This coefficient introduces the term $\int_{t-1}^{t} \sigma_s^4 ds$, which determines the scale of equation (7) in units of conditional standard deviation (see Huang and Tauchen [2005]). A jump-robust estimate of this term is given by tripower quarticity (TQ):

Definition 3 *Tripower quarticity*

$$TQ_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{j=3}^{M}|r_{t,j-2}|^{4/3}|r_{t,j-1}|^{4/3}|r_{t,j}|^{4/3} \to \int_{t-1}^{t}\sigma_s^4 ds$$

TQ is accompanied by the scale normalizing constant M since each absolute return is of the order $\sqrt{\Delta t}$. Since M is of order $\frac{1}{\Delta t}$, the whole expression approaches a well defined limit.

Even so, Huang and Tauchen [2005] find that simply using TQ tends to over-reject the null hypothesis of no jump. In its place, they propose the following modification:

$$JS_t = \frac{RV_t - BV_t}{\sqrt{\left(\mu_1^{-4} + 2\mu_1^{-2} - 5\right)\max\left(BV_t^2, TQ_t\right)}} \tag{8}$$

Several authors (Barndorff-Nielsen and Shephard [2005a], Andersen et al. [2001], Andersen et al. [2003b]) argue that finite sample performance may be improved by basing the jump test on the log-difference of the realized measures, i.e.:

$$JS_t = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{\left(\mu_1^{-4} + 2\mu_1^{-2} - 5\right)\max\left(BV_t^2, TQ_t\right)}} \tag{9}$$

This implies that the numerators of equations (8) and (9) have the same asymptotic distribution. According to Huang and Tauchen [2005] this is due to the fact that the first-order Taylor expansion term of both numerators, centered around the asymptotic mean of RV and BV (i.e. $\int_{t-1}^{t}\sigma_s^2 ds$), have the same distribution. Then, the difference of both realized (and log-realized) measures generate the same asymptotic distribution. Thus, equation (9) is the expression used to test the presence of jumps in our empirical application.

4 Data

Our database consist of intraday transactions on the Mercado Electrónico Colombiano (MEC) operated by the Bolsa de Valores de Colombia (BVC). Data entries span dates from January 2nd, 2017 to December 28th, 2018. This includes operations for a total of 485 trading days in these years. Colombian sovereign debt is issued in Colombian peso (COP) and Unidad de Valor Real (UVR)[1]. Despite having data for both types of assets, only COP issuances are considered since they are more liquid.

Mnemonic conventions for Colombian debt titles enconde information about the bond's coupon, year of issuance, and maturity. For example, TFIT16240724 is a fixed coupon treasury (TFIT) issued in 2016 (TFIT16) with expiration date 24th of July 2024 (TFIT16240724). For the sake of brevity we will denote bonds only by their expiration year in our discussions, i.e., we will refer to TFIT16240724 as T24.

[1]A Unit of Real Value (UVR) represents the acquisitive power of the Colombian peso, and is defined as the price of a predetermined bag of goods and services.

4.1 Bond selection criteria

Bonds are selected for analysis according to the following criteria: *i*) Liquidity: since the jump detection approach detailed in the theoretical framework is based on the asymptotic distributions of realized measures of variance, the most active assets will return the best results; *ii*) Maturity: the two theories of term structure formation discussed in section 2 give us preemptive expectations of the jump activity of bonds of different maturities (from 1 year up to 15 years). Thus, choosing bonds with maturities distributed along the term structure allows for an interesting comparison of jump behaviour in light of those hypothesis.

For the case study of Colombia, these two criteria present a serious challenge. The local market has few agents trading day to day, which means liquidity is generally low. Additionally, most of these agents trade mainly short and medium term bonds. This means long term debt is much more illiquid since market participants buy or sell long term bonds mostly to comply with regulations and to match long term liabilities. Consequently, analysis at the shorter end (less than 5 years) of the term structure will be much richer in comparison to the longer end (more than 10 years).

To aid in bond selection, table 1 displays daily descriptive statistics for all bond transactions. Maturity, total trading days, and average and median transactions are presented, as well as average and median inter-arrival times (IAT). IAT is defined as the time interval between transactions, thus, IAT is lower for more liquid assets and greater for illiquid ones. Values reported in this table help us quantify the daily liquidity of each title. For example, the T24 bond averages 189.971 transactions each day. Furthermore, IATs suggests that each transaction occurs every minute and 29 seconds on average. This means that this bond is much more liquid that the T20 title, which trades around 24 times each day, with each transaction occurring every 7 minutes and 18 seconds on average.

The bonds chosen for analysis are: TFIT06211118, TFIT06110919, TFIT15240720, TFIT-10040522, TFIT16240724 and TFIT16300632, hereafter T18, T19, T20, T22, T24, T32. In other words, if we take 2017 as a base year we are considering bonds with 1,2,3,5,7 and 15 years to maturity.

Even though these are the bonds which trade the most, illiquidity remains a real challenge. Only T18, T20 and T24 average more than 10 transactions per day, while the only long term bond (T32) averages 4.25 transactions per day. The most traded bond is T24 with 189.971 daily operations on average.

4.2 Data sampling and microstructure noise

In order to apply the jump test in equation (9), our trade data must be sampled at equal discrete time intervals Dungey et al. [2009]. Sampling high frequency data entails the following trade-off: choosing a high sampling frequency captures more information about the evolution of the real-time price formation process at the cost of greater microstructure noise. On the other hand, a lower sampling frequency minimizes noise, at the expense of masking information about the asset's instantaneous market price.

Even though optimal sampling frequency tests exist, their results differ for different bond maturities (Zhang et al. [2005], Bandi and Russell [2006]). Different sampling frequencies for different bonds makes comparisons across different assets

Mnemonic	Maturity	Trading days	Avg. trans.	Median trans.	Average IAT	Median IAT
TFIT16240724	7	485	189.971	198	1m 29.047s	13s
TFIT15240720	3	472	23.961	18	7m 18.138s	1m 16s
TFIT06211118	1	435	15.573	11	8m 32.520s	1m 40.5s
TFIT10040522	5	455	6.771	5	18m 42.474s	5m 56s
TFIT06110919	2	394	5.233	4	18m 54,712s	4m 16s
TFIT16300632	15	352	4.258	2	20m 04.405s	5m 35s
TFIT15260826	9	315	3.404	1	19m 42.871s	6m 35s
TFIT08261125	8	133	0.891	0	27m 06.659s	9m 30s
TFIT16180930	13	92	0.625	0	24m 55.545s	7m 36s
TFIT11241018	1	105	0.559	0	27m 40.795s	10m 25.5s
TFIT16280428	11	75	0.285	0	28m 51.778s	8m 00s

Table 1: Descriptive statistics of daily transactions throughout our sample; e.g., T24 averages 189.971 daily transactions in our database, with each trade happening almost every minute and a half on average. Maturities are in reference to 2017.

impossible. For this reason, instead of using optimal frequency tests, empirical literature cited so far applies several sampling frequencies for assets under consideration in order to compare and contrast the effects which sampling frequency has on the jump test. We will replicate this procedure, sampling at 5, 10, 15, and 30 minute intervals.

The optimal sampling method is also a source of debate among academics. Dungey et al Dungey et al. [2009] take the last price within a time bucket as representative of the market price within that interval. Sheppard [2006] argues that this approach may lead to scrambling problems[2] and could also bias the covariance of returns to zero for larger sampling frequencies.

On the other hand, Lee and Mykland Lee and Mykland [2012] propose a non-parametric approach which assumes that market noise has a zero-mean distribution. This way, taking local averages of prices within time intervals asymptotically removes noise from the underlying market price. Even though the authors assume that data is of ultra high frequency, we will adopt this method as our sampling procedure since scrambling problems are of greater magnitude for the more illiquid assets we are studying.

4.3 Additional statistics

This section presents additional information about daily bond transactions. Tables 2 and 3 present the same statistics as table 1 for each year in our sample. As previously mentioned, IAT for more liquid assets are smaller than for illiquid assets since the time between transactions is shorter, thus, their values would cluster near zero in the distribution. We have decided to crop IAT values at 3600 seconds since intervals larger than an hour are uncommon.

Figures 1 through 6 showcase the number of transactions of the selected bonds during all trading days of 2017-2018. Additionally inter-arrival time distributions for the selected bonds are included. This information on the trading activity in the bond market also show the impact on expected changes on the incentives on market makers in the bond market. At the end of 2018 the treasury reduced the financial

[2]Taking the last price in each time bucket could result in intervals of uneven length. Since we need equal length intervals, this problem is known as scrambling.

incentives for financial institutions that participate in the primary bond market. The incentive system in the Colombian bond market was established in the late nineties to foster the development of the bond market. However, recent studies indicated that the level of incentives was not necessary and created unnecessary trading activity from financial institutions in the secondary market in order to obtain the incentives in the primary market[3]. In particular in Figure 5 we observe a large drop in the number of transactions at the end of 2018 for the most actively traded bond, T24.

Mnemonic	Maturity	Trading days	Avg. trans.	Median trans.	Average IAT	Median IAT
TFIT16240724	7	242	233.636	226	1m 15.378s	12s
TFIT15240720	3	229	10.463	8	14m 47.167s	4m 17s
TFIT06211118	1	242	25.727	24	7m 9.708s	1m 24s
TFIT10040522	5	220	5.095	4	22m 4.609s	7m 39s
TFIT16300632	15	142	2.244	1	26m 43.930s	9m 17s
TFIT06110919	2	214	6.711	5.5	19m 19.944s	5m 51,5s
TFIT15260826	9	196	5.500	2	17m 23.907s	5m 33s
TFIT08261125	8	0	0	0	—	—
TFIT16180930	13	78	1.178	0	24m 21.121s	7m 8s
TFIT11241018	1	96	1.062	0	27m 38.963s	10m 27s
TFIT16280428	11	20	0.165	0	19m 9.950s	1m 23.5s

Table 2: Descriptive statistics of daily transactions during 2017.

Mnemonic	Maturity	Trading days	Avg. trans.	Median trans.	Average IAT	Median IAT
TFIT16240724	6	243	146.486	137	1m 50.813s	14s
TFIT15240720	2	243	37.403	34	5m 21.236s	58s
TFIT06211118	–	193	5.461	4	15m 49.509s	4m 44.5s
TFIT10040522	4	235	8.440	7	16m 49.719s	5m 7s
TFIT16300632	14	210	6.263	4	18m 2.294s	5m 16.5s
TFIT06110919	1	180	3.761	2	19m 32.676s	5m 34.5s
TFIT15260826	8	119	1.317	0	32m 47.567s	16m 57s
TFIT08261125	7	133	1.778	1	27m 6.659s	9m 30s
TFIT16180930	12	14	0.074	0	54m 37s	45m 56.5s
TFIT11241018	–	9	0.058	0	28m 39.800s	3m 42s
TFIT16280428	10	55	0.403	0	33m 22.395s	12m 16s

Table 3: Descriptive statistics of daily transactions during 2018.

[3] Here is a recent post (in spanish) that describe the regulatory changes.

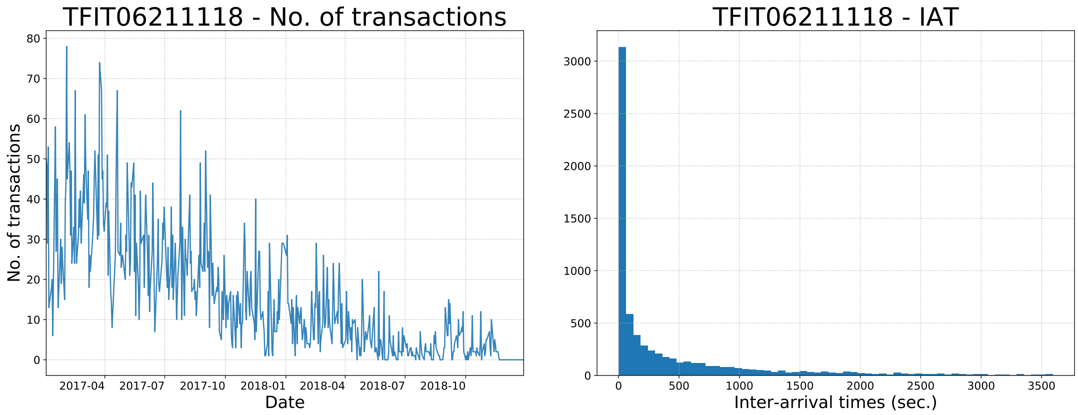

Figure 1: TFIT06211118: a) Daily transactions for 2017-2018; b) Inter arrival time distribution cropped at 3600 seconds

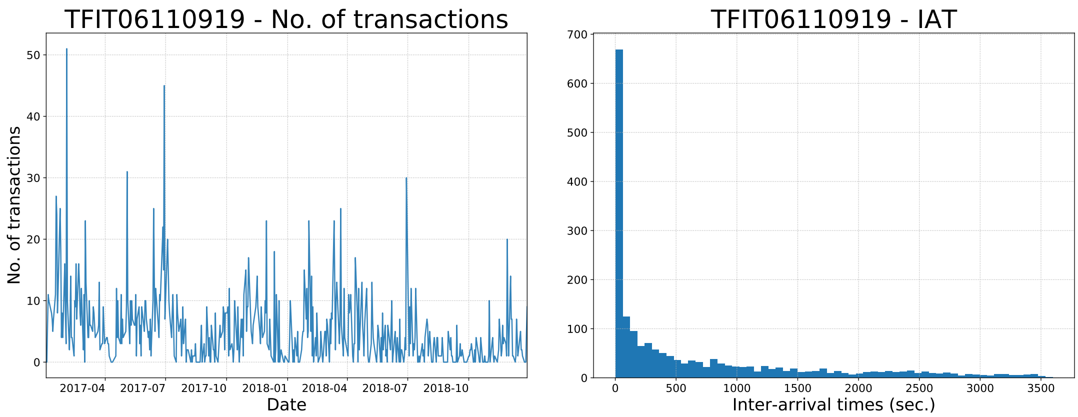

Figure 2: TFIT06110919: a) Daily transactions for 2017-2018; b) Inter arrival time distribution cropped at 3600 seconds

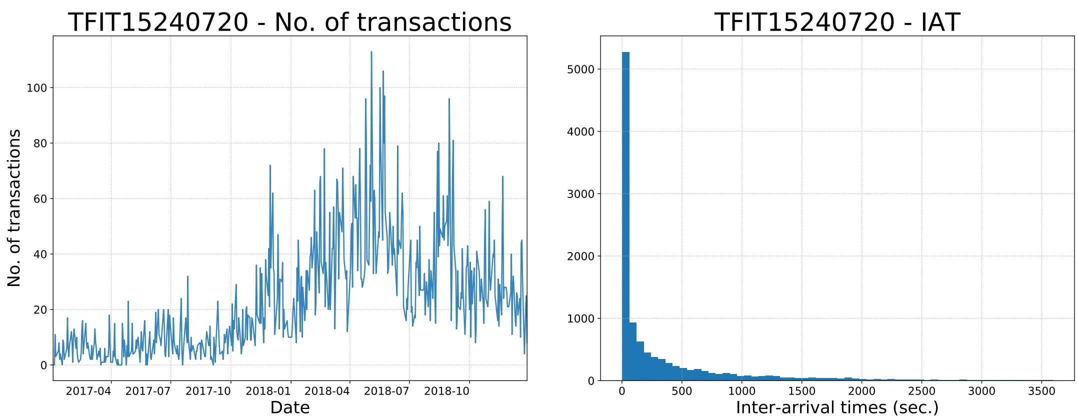

Figure 3: TFIT15240720: a) Daily transactions for 2017-2018; b) Inter arrival time distribution cropped at 3600 seconds

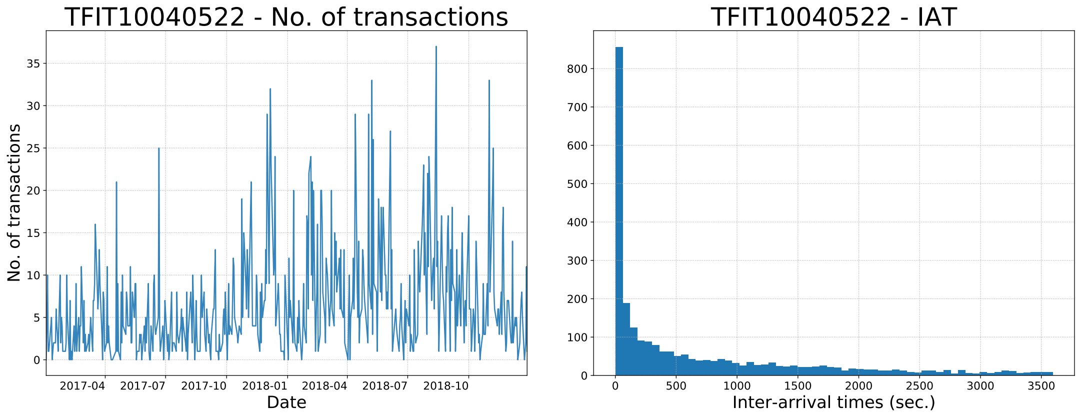

Figure 4: TFIT10040522: a) Daily transactions for 2017-2018; b) Inter arrival time distribution cropped at 3600 seconds

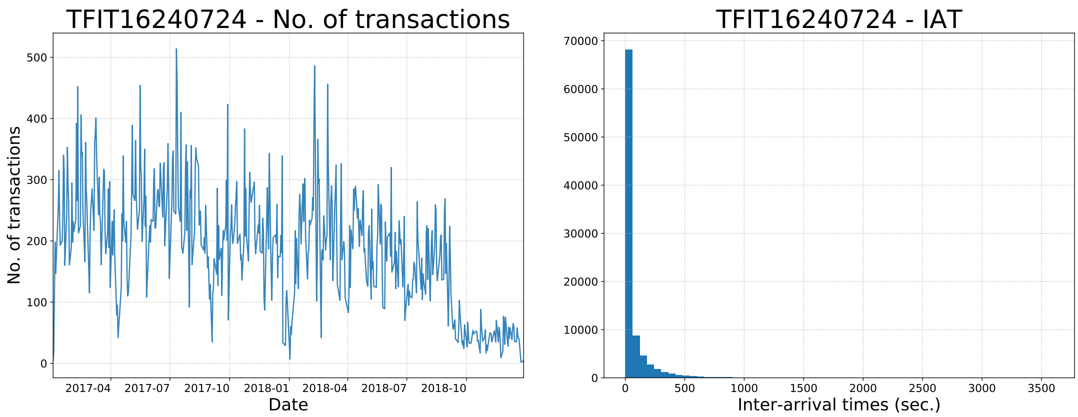

Figure 5: TFIT16240724: a) Daily transactions for 2017-2018; b) Inter arrival time distribution cropped at 3600 seconds

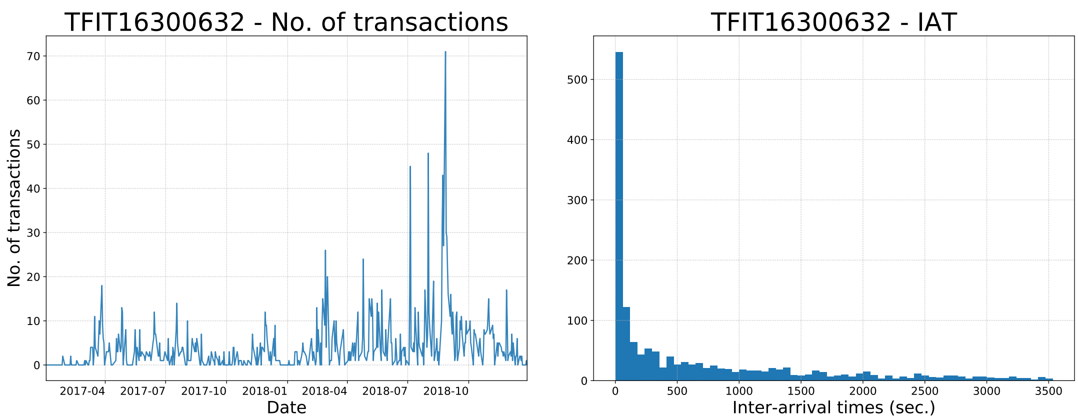

Figure 6: TFIT16300632: a) Daily transactions for 2017-2018; b) Inter arrival time distribution cropped at 3600 seconds

5 Empirical Results

5.1 Univariate jumps

Table 4 summarizes the results of applying equation (9) for 5, 10, 15, and 30 minute sampling frequencies at a 5% significance level. Despite trading for 472 out of the 485 total days, the T20 bond exhibits the most active jump behaviour at all frequencies except 30 minutes; jumping 68.4% of the time at 5 minute frequency and 55.2% on average. On the other hand, the T24 and T32 bills are among the least likely to jump. T24 jumps 55.7% of the time at the 5 minute sampling frequency, but this rejection rate quickly drops below 30% for all other frequencies. Meanwhile, the T32 rejection frequency increases for lower sampling frequencies.

This trend of lower sampling frequency resulting in higher rejection rates is unexpected, since the presence of noise in higher sampling frequencies should generate more rejections of the test statistic. Out of the six bonds studied, this inverse relationship between frequency and rejection is present in the more illiquid assets: T19, T22, and T32. Table 4 reveals that these assets increase the number of detected jump days when the sampling frequency decreases, which may indicate that the lower sampling frequency captures more information about transaction dynamics in illiquid assets. Thus, when the average of the time buckets is taken, the longer time intervals allow for a more representative average price.

On the other hand, more liquid bonds generate larger rejection rates as the sampling frequency increases. For example, the rejection rate for T24 grows from 0.14, 0.245, 0.272, and 0.557 as the sampling frequency increases from 30, 15, 10 to 5 minutes. This result is consistent with intuition that greater data granularity comes with greater noise, as well as with conclusions presented by Dungey et al. [2009] in their empirical study of US treasuries. Unlike their work, which finds that jumps are not as prevalent for longer term bonds in comparison with short term bonds, we find no relation between maturity and univariate jump rejection frequency.

Graphical representation of jump test results for the 30 minute sampling frequency are shown in figure 7. This plot shows the value of the jump statistic for each day in proportion to its critical value. It is clear by observation that jumps are a common occurrence for fixed income instruments, which suggests that simul-

Mnemonic	Avg. trans.	Rejection freq.	No. of jump days	Rejection freq.	No. of jump days
		5 minutes		10 minutes	
TFIT06211118	15.573	0.623	271	0.568	247
TFIT06110919	5.233	0.340	134	0.365	144
TFIT15240720	23.961	0.684	323	0.606	286
TFIT10040522	6.771	0.352	160	0.418	190
TFIT16240724	189.971	0.557	270	0.272	132
TFIT16300632	4.258	0.276	97	0.304	107
		15 minutes		30 minutes	
TFIT06211118	15.573	0.494	215	0.386	168
TFIT06110919	5.233	0.411	162	0.398	157
TFIT15240720	23.961	0.547	258	0.369	174
TFIT10040522	6.771	0.411	187	0.426	194
TFIT16240724	189.971	0.245	119	0.140	68
TFIT16300632	4.258	0.307	108	0.318	112

Table 4: Rejection frequency of the jump test for all sampling frequencies

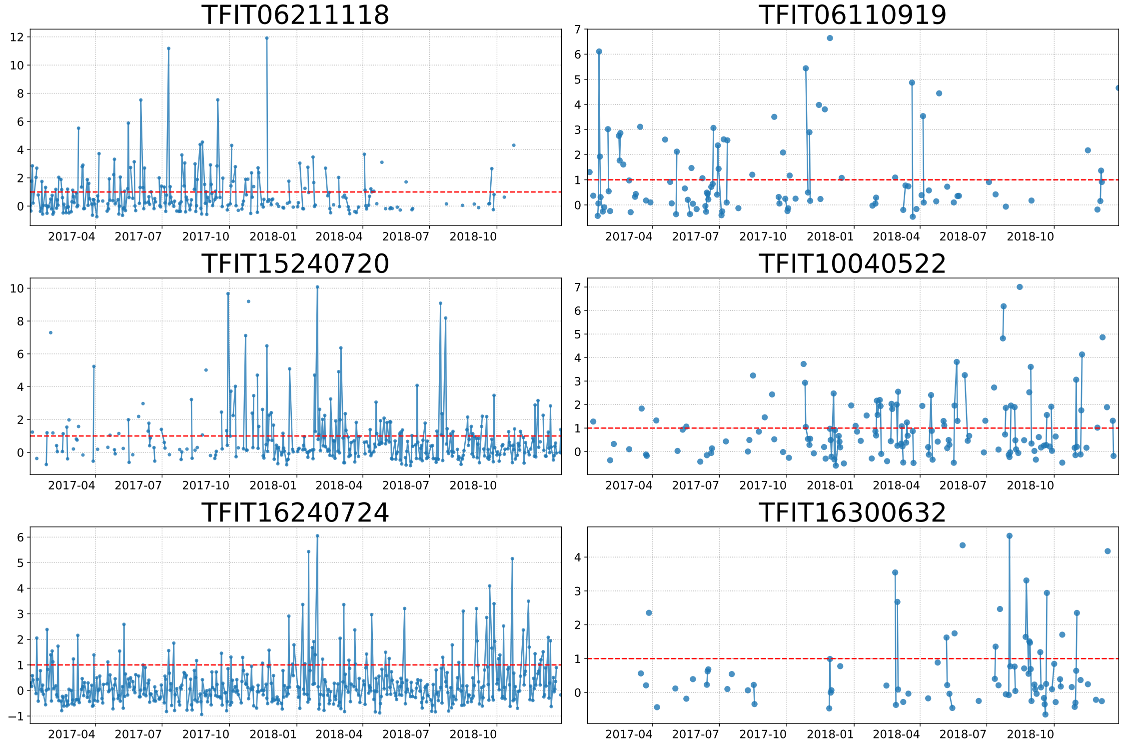

Figure 7: Time series of jump test results in proportion to the critical value at 30 minute sampling frequency and at a 95% confidence interval.

taneous jumps across different assets are a real possibility. We study the cojumping behaviour in detail in the next section. Univariate test results for different sampling frequencies are included in section 7.

5.2 Multivariate jumps

As a complement to the univariate jump test, we can also consider the case of multiple bonds of different maturities jumping on a given day. This cojump behaviour can be gauged by studying coexceedances, an approach developed by Bae et al. [2003] in the context of financial contagion and occurrence of extreme events. A coexeedance occurs when, on a particular day, a bond of maturity i exceeds the jump statistic's critical value *given* that a bond of maturity j has also surpassed the critical value for the same day. This would imply that the assets have jumped synchronically at the daily level.

More formally, the procedure is as follows. We begin by looking at the individual time series of $JS_{i,t}$ values for each bond. A dummy variable $d_{i,t}$ indicates if a bond of maturity i exceeds the statistic's critical value at day t:

$$d_{i,t} = \begin{cases} 1 & JS_{i,t} > JS_{critical} \\ 0 & \text{otherwise} \end{cases}$$

With the series of dummy values for each bond, the number of coexceedances will be given by the sum of all $d_{i,t}$ for $i \neq j$ given that $j = 1$:

$$E_{j,t|d_{j,t}=1} = \sum_{i=1,\ i\neq j}^{n} d_{i,t} \tag{10}$$

Mnemonic	Co-exceedances				Total jumps	Mnemonic	Co-exceedances				Total jumps
	0	1	2	3			0	1	2	3	
5 minute sampling						10 minute sampling					
TFIT06211118	47	132	76	16	271	TFIT06211118	69	120	51	7	247
TFIT15240720	39	166	102	16	323	TFIT15240720	78	141	60	7	286
TFIT16240724	27	129	98	16	270	TFIT16240724	22	66	37	7	132
TFIT16300632	4	29	48	16	97	TFIT16300632	12	47	41	7	108
15 minute sampling						30 minute sampling					
TFIT06211118	66	104	40	5	215	TFIT06211118	76	70	20	2	168
TFIT15240720	91	123	39	5	258	TFIT15240720	74	75	23	2	174
TFIT16240724	27	54	33	5	119	TFIT16240724	15	36	15	2	68
TFIT16300632	23	51	29	5	108	TFIT16300632	43	47	20	2	112

Table 5: Number of coexceedances for each bond at all sampling frequencies.

We have decided to limit the cojump analysis to the T18, T20, T24, and T32 emissions, since the first three are the most liquid and T32 is the longest dated bond in our database. This means that the number of coexceedances will range in values from 0 to 3, where 0 indicates the number of unique jumps and 3 the number of times when all bonds jump in a given day.

Table 5 presents the coexceedance results for all sampling frequencies as well as the total number of jumps. Interestingly, jumps of two assets are the most common event by a wide margin, followed by unique jumps. The least common occurrence is the simultaneous jump of all four bonds. Furthermore, these results persist across all maturities and sampling frequencies, which may point to an underlying dynamic which causes this behaviour in Colombian sovereign bond market.

Averaging the 2 asset cojump proportions across bonds and maturities (except for T32 at 5 minutes) accounts for 46.989% of all jump activity. This implies that when the term structure experiences a jump, it generally does so in tandem with another part of the curve. Identifying which ends move with which is difficult since all coexceedances of two assets are very similar in proportion, though, in magnitude, it is clear that T18 and T20 experience much more 2-asset co jumps at all frequencies. In section 5.3, the phenomenon of cojump pairs is described in more detail.

5.3 Cojump pairs

By limiting our view to coexceedances of only two assets, we can see how their cojump behaviour evolves in time. We do this by defining a counter which keeps track of every time a coexceedance occurs for a pair of bonds. Everytime $E_{j,t|d_{j,t}=1} = 1$, the counter goes up by 1. When graphing this counter's values as a time series, this procedure has a convenient interpretation, since the steepest curve indicates the most active pairing of cojumping bonds. Figure 8 shows the evolution of the cojump pairs for all sampling frequencies considered: T18-T20 as a green dashed and dotted line; T18-T24 as a solid orange line; and T20-T24 as a dashed blue line. Figure 9 graphs the same dynamic for the T24-T32 (dash and dot), T20-T32 (solid), and T18-T32 (dashed) pairs.

Our interest lies in comparing cojump behaviour of bonds distributed throughout the term structure. Thus, the analysis that follows is made more clear by referring to these pairs by the difference of their constituent's bond maturities. From smallest to largest difference, the pairs will be: T18-T20: 2Y pair; T20-T24: 4Y pair; T18-T24: 6Y pair. The second set would be T24-T32: 8Y pair; T20-T32: 12Y pair; T18-T32:

14Y pair.

Results at 5 minute sampling tend to align with the preferred habitat theory, since the two pairs of closest maturities, 4Y and 2Y, show the most (and second most) cojump activity. 4Y jumps 185 times, 2Y does so 161 times, while 6Y counts 129 coexceedances in our sample. Comparisons with the 10, 15 and 30 minute samplings show that 2Y is consistently the most active pair, with 4Y and 6Y being second and third. These results strengthen the case for cojump behaviour following the market segmentation hypothesis, which foresees bonds of nearer maturities reacting similarly to external shocks. Yet, for sampling frequencies of 10, 15, and 30 minutes, the 4Y and 6Y pairs tend to move more in tandem with one another. This low cojump number is explained by the low univariate activity of the T24 bond at those frequencies, since T24 only jumps on 132, 119, and 68 days respectively (see table 4). Thus, pairs which contain T24 will have fewer days on which a possible coexceedance may occur.

Meanwhile, casual observation of figure 9 tells us that pairs of dissimilar maturities are much less active than ones with similar maturities. Across all samplings, 12Y shows the most coexceedances, followed by 14Y (except at 5 minutes) and 8Y. Thus, we find no constructive evidence for either theory of the term structure of interest rates. Yet, we may replicate the argument that low univariate jump activity is responsible for the low cojump count for these pairs. In this case, it is the low activity of T32 which constrains the number of days for a coexceedance to occur. Since T20 is the most active bond, it has the most chance of cojumping with the T32 bond. By the same logic, T24 is the least active bond, making the T24-T32 pair the least likely to cojump. Our results for sampling frequencies other than 5 minutes reflect that this is indeed the case.

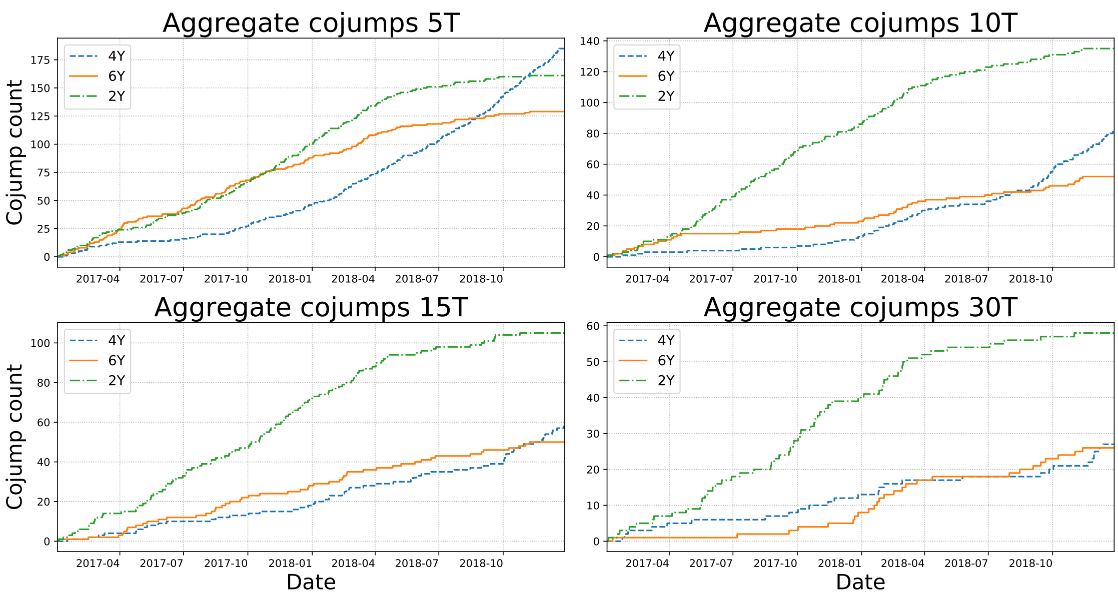

Figure 8: Time evolution of cojump pair activity at a) 5 minute sampling; b) 10 minute sampling; c) 15 minute sampling; d) 30 minute sampling frequency for pairs of 2Y, 4Y, and 6Y difference in maturity.

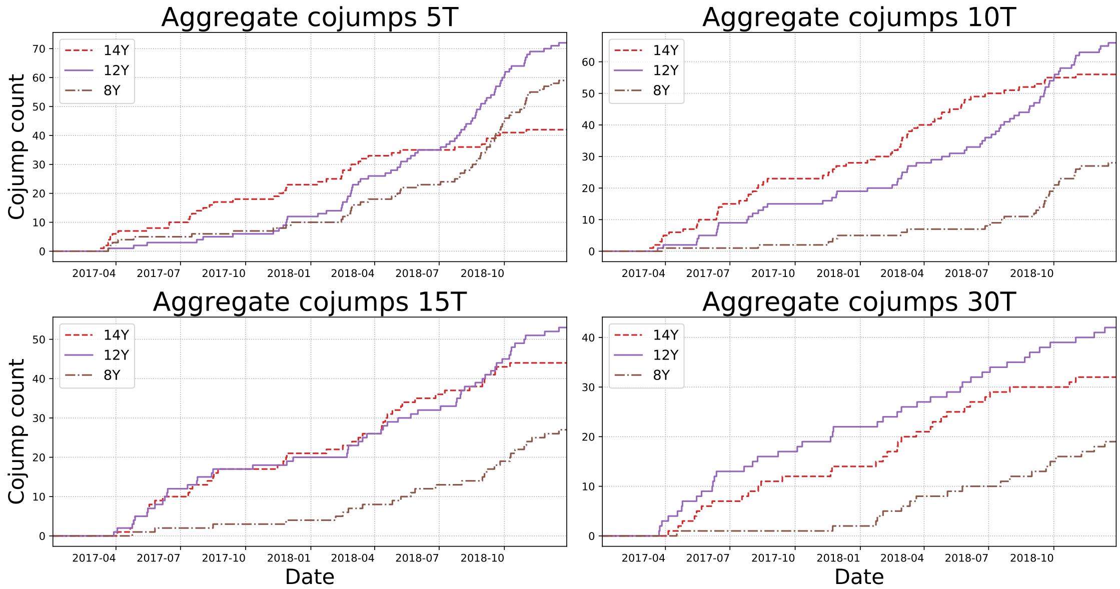

Figure 9: Time evolution of cojump pair activity at a) 5 minute sampling; b) 10 minute sampling; c) 15 minute sampling; d) 30 minute sampling frequency for pairs of 8Y, 12Y, and 14Y difference in maturity.

5.4 Stylized facts of the Colombian bond market

This section studies daily jump seasonalities in two ways: first, the daily distribution of the jump test results is studied in subsection 5.4.1. This allows us to more formally define a panel logistic regression model for a binary outcome of jump versus no jump. This approach lets us include central bank announcements. These results are presented in subsection 5.5.

5.4.1 Daily distribution of jumps

It is possible that both univariate and multivariate jumps exhibit daily seasonalities. For example, Das [2002] explicitly models day-of-the-week effects on jump behaviour and finds that jumps are more likely to jump on Wednesdays due to option expiry effects. Even though the procedure we have followed does not capture daily effects, we can observe the distribution of jumps and cojumps to check for daily patterns. Results of this analysis are presented in table 6.

For all sampling frequencies and almost all bonds, the least likely day for a jump to occur is Monday. Only T18, and T19 at 30 minute sampling, deviated from this behaviour. On the other hand, the assets studied did not reflect any particular seasonality for a most common jump day. On average, Wednesday was the most likely day for jumps at 10 and 30 minute sampling frequencies, with 22.1% and 22.4% of jumps happening on this day of the week on average. At 5 and 15 minute sampling frequencies, Thursday was the most common jump day, with 22.5% of jumps occurring that day for both frequencies.

Results for cojumps exhibit some similarity to univariate jumps. Analyzing only jumps of more than one asset (coexceedance > 0) no particular day at any sampling frequency stands out as one where a cojump is most likely to happen. As was the case for univariate jumps, least likely day for cojumps is once again Monday.

Mnemonic	Weekdays					Mnemonic	Weekdays				
	M	T	W	Th	F		M	T	W	Th	F
5 minute sampling						10 minute sampling					
TFIT06211118	0.193	0.226	0.181	0.189	0.211	TFIT06211118	0.235	0.189	0.205	0.182	0.189
TFIT06110919	0.155	0.238	0.195	0.214	0.198	TFIT06110919	0.140	0.241	0.185	0.227	0.206
TFIT15240720	0.148	0.214	0.218	0.196	0.225	TFIT15240720	0.145	0.178	0.255	0.206	0.215
TFIT10040522	0.106	0.206	0.275	0.250	0.163	TFIT10040522	0.116	0.221	0.221	0.237	0.205
TFIT16240724	0.104	0.201	0.179	0.276	0.239	TFIT16240724	0.104	0.201	0.215	0.250	0.229
TFIT16300632	0.072	0.237	0.278	0.227	0.186	TFIT16300632	0.112	0.178	0.243	0.215	0.252
Daily avg.	0.130	0.220	0.221	0.225	0.204	Daily avg.	0.142	0.201	0.221	0.220	0.216
Coexceedance	0.168	0.211	0.211	0.205	0.205	Coexceedance	0.160	0.208	0.219	0.205	0.208
15 minute sampling						30 minute sampling					
TFIT06211118	0.210	0.210	0.185	0.193	0.202	TFIT06211118	0.221	0.132	0.250	0.235	0.162
TFIT06110919	0.147	0.225	0.236	0.209	0.182	TFIT06110919	0.195	0.155	0.224	0.207	0.218
TFIT15240720	0.135	0.172	0.233	0.233	0.228	TFIT15240720	0.155	0.208	0.214	0.196	0.226
TFIT10040522	0.128	0.230	0.182	0.262	0.198	TFIT10040522	0.134	0.222	0.232	0.227	0.186
TFIT16240724	0.142	0.216	0.204	0.222	0.216	TFIT16240724	0.102	0.242	0.210	0.236	0.210
TFIT16300632	0.111	0.222	0.241	0.231	0.194	TFIT16300632	0.134	0.214	0.214	0.232	0.205
Daily avg.	0.146	0.213	0.214	0.225	0.203	Daily avg.	0.157	0.196	0.224	0.222	0.201
Coexceedance	0.151	0.202	0.221	0.214	0.212	Coexceedance	0.157	0.197	0.217	0.206	0.223

Table 6: This table shows the daily distribution of jump test results which exceeded the critical value at 5% significance for all sampling frequencies.

The apparent monday effect found in idiosyncratic jumps and cojumps contradicts findings for US treasuries presented by Dungey et al. [2009], where the authors do not find any evidence of daily seasonalities for neither jumps nor cojumps. Day of the week effects are studied more formally in the next subsection, as well as the effect of economic announcements on jump activity.

5.5 Economic announcements and jump activity

Having found daily seasonalities in both jump and cojump behaviour, we can now search for the impact of different economic announcements in jump activity. To do this, we define a panel logistic model which specifies the event of a jump occurring as a function of weekdays and economic announcements. The model is specified as follows:

$$\mathbb{I}(J^*_{i,t} \geq 1) = \beta_0 + \sum_{j=1}^{4} \beta_k D_k + \alpha \mathbb{D}_{\text{Announcement}} + \varepsilon_{i,t} \tag{11}$$

where $J^*_{i,t}$ is the result of the jump test applied to bond i at day t in proportion to the critical value. The identity function transforms the continuous values of the jump test into a binary outcome model which takes a value of 1 when the critical value is exceeded and zero otherwise. The D_k terms control for day of the week, from Tuesday through Friday. We expect the β_k coefficients to be positive since we found that Monday is the least likely day for a jump to occur. We estimate a random effects model,

$$\varepsilon_{i,t} = \tau_i + e_{i,t}$$

Where, $e_{i,t} \sim iid\mathcal{N}(0, \sigma_e^2)$ and $\tau_i \sim iid\mathcal{N}(0, \sigma_\tau^2)$, captures the unobserved heterogeneity across the propensity of different maturities to jump.
We consider different types of announcements and sources. First, we consider announcement as an indicator variable (i.e, $D_{\text{Announcement}}$ takes a value of 1 and 0 otherwise) on days that denotes the date of news releases or the day after if the release is after the market closes. The announcements are regarding macroeconomic

information from Colombia and the US: Monetary policy (interest rate announcements and FOMC meetings), CPI, Unemployment rate, Underemployment rate , GDP, Consumer confidence, trade balances, durable goods and rating changes on Colombian sovereign debt. Second, we also consider a different indicator variable that takes a value of 1 if the indicator that is released deviates from the expected value (from a survey of forecasters). This second definition provides a way to control for the content of the announcement and whether the surprise contained in the information is related to the jump rather than just the type of information that is being released to the public.[4]

Table 7 presents logistic regression results and the average marginal effects for the most relevant variables in terms of statistical significance.

Several day-of-the-week effects are found for 5, 10, and 15 minute samplings. We report positive Tuesday and Thursday effects, the former is specially important because it is consistently significant. At these frequencies, jumps are about 7.6% more likely to occur on Tuesdays and about 6% more likely to occur on Thursdays. The Thursday effect is robust to the introduction of different types of economic announcements. This results deviates from what is observed in Table 6 where we find a relatively similar distribution of jumps along weekdays, with a lower amount on Mondays and a larger amount on Wednesdays.

With respect to economic announcements and surprises we have mixed results. Overall, we find that very few variables have an impact on the jump frequency, in particular at the 15 minute sampling frequency we do not find any significant effects. Among the different sampling times we do not find common variables that increase the jump frequency, in particular we find that CPI inflation surprises regarding US data are more important for the 5 minute and 15 minute sampling frequencies. For the 10 minute sampling frequency surprises related to the Colombian trade Balance increase the probability of a jump by 8.7%. However, it is specific shocks rather than US (α_{NewsUS}) or Colombian shocks ($\alpha_{NewsCOL}$) that are relevant because when we aggregate all types of announcement or surprise's by country the effect is not statistically significant. During the sample we also observed two announcement regarding a stable and one negative outlook (by Fitch on the 22 of February of 2018) on Colombian sovereign rating, however, we find no statistically significant effect on the jump frequency and also there are mixed results regarding the sign across the different sampling frequencies.

Looking closely at the 5 minute sampling frequency and the 37% increase in the jump frequency due to the increase in the CPI inflation surprise in US, we find a possible explanation of the importance of external shocks to internal shocks. During the sampling period 2017-2018 and further on in 2019, there was a succession of US CPI inflation reports that have been significantly above expectations; these reports raised questions regarding the tightening of monetary policy[5]. On the other hand, during the same period 2017-2018 CPI inflation in Colombia was in line with the Central Bank's target. So it is not surprising that during the period investors in the Colombian sovereign bond market were more sensible to changes in the monetary policy in the US than any local shock.

[4]We obtain the dates of the announcement and the information regarding the observed and the expected macroeconomic indicator from Bloomberg.

[5]A discussion by Gregory Mankiw in The New York Times.

Panel	β_{TUE}	β_{THU}	α_{News}	$\alpha_{NewsCOL}$	α_{NewsUS}	α_{Rating}	Marginal effect
5 Minutes	0.303**	0.239*					0.076**
	(0.138)	(0.138)					(0.034)
	0.301**	0.220	1.768***				0.369***
	(0.139)	(0.138)	(0.657)				(0.094)
	0.308**	0.227		-0.0512	0.129		0.032
	(0.139)	(0.139)		(0.130)	(0.147)		(0.037)
	0.308**	0.244*				-0.438	-0.106
	(0.138)	(0.138)				(0.510)	(0.118)
10 Minutes	0.138	0.258*					0.063*
	(0.139)	(0.138)					(0.034)
	0.129	0.263*	0.378*				0.094*
	(0.139)	(0.138)	(0.222)				(0.055)
	0.131	0.254*		0.107	0.0736		0.017
	(0.139)	(0.139)		(0.128)	(0.146)		(0.035)
	0.137	0.257*				0.0630	0.015
	(0.139)	(0.138)				(0.493)	(0.121)
15 Minutes	0.143	0.250*					0.06*
	(0.138)	(0.137)					(0.033)
	0.146	0.250*	0.194				0.056
	(0.138)	(0.137)	(0.464)				(0.124)
	0.139	0.250*		0.0612	0.0127		0.003
	(0.138)	(0.138)		(0.128)	(0.146)		(0.035)
	0.142	0.248*				0.148	0.036
	(0.138)	(0.137)				(0.488)	(0.119)
30 Minutes	-0.0246	0.0993					0.022
	(0.143)	(0.142)					(0.031)
	-0.0191	0.106	0.377*				0.087*
	(0.143)	(0.142)	(0.218)				(0.052)
	-0.0221	0.0908		-0.0203	0.0993		0.022
	(0.144)	(0.142)		(0.134)	(0.150)		(0.033)
	-0.0244	0.0996				-0.0225	-0.005
	(0.143)	(0.142)				(0.513)	(0.112)

Table 7: Logistic regression results and Marginal effects for selected variables. The last column indicates the marginal effect. For the panel based on 5 and 15 minute sampling data the marginal effects reported are for: Tuesday, CPI inflation surprise in the US, US news and Credit rating announcements, respectively. For the panel based on 10 minute sampling data the marginal effects reported are for: Thursday, Trade Balance surprise in Colombia, US news and Credit rating announcements, respectively. For the panel based on 30 minute sampling data the marginal effects reported are for: Thursday, Underemployment rate announcements in the US, US news and Credit rating announcements, respectively.

We consider a broad range of announcements and surprises regarding economic conditions and consider both internal and external shocks (US) and find that the jump frequency is sensitive to specific shock that can have an incidence on monetary policy but not the policy announcement themselves. It is also important to note that external shocks seem to be more relevant than local shocks. Furthermore, we find systematic day-of-the-week effects that should be analyse further to determine whether they provide arbitrage opportunities.

6 Conclusions

In document we have found that price discontinuities are a common occurrence for Colombian sovereign bonds. Results presented in sections 5.1 and 5.2 show the extent of this activity, though no relation was found between maturity and univariate jump presence. Furthermore, no issuance was found to consistently be the one which jumps the most for the sampling frequencies studied, though T24 was the least active title for all sampling frequencies except 5 minutes.

By looking at the daily coexceedances, we can extend the notion of jumps to include simultaneous discontinuities across assets, which is interesting because of its effects on the yield curve. Analyzing results, almost half of all jump activity consists of the cojumping of two bonds. In particular, the assets which cojumped the most were the ones with shortest distance between maturities . This seems to suggest that the behaviour of the local market falls more in line with the market segmentation theory, as opposed to the liquidity risk premium hypothesis.

The widespread presence of jumps in bond prices allows for an interesting study of their weekly distribution. For both univariate and multivariate jumps, the least common jump day is Monday. For 10 and 30 minute sampling frequencies, the preferred cojumping day is Wednesday, while at 5 and 15 minute samplings the preferred cojumping day is Thursday. On the other hand, no particular day stood out as more prevalent for univariate bond jump activity.

A panel logit model for 6 bonds allows for a formal study of daily jump seasonalities and the effects of economic announcements and surprises. As we expected, there are multiple positive and significant day-of-the-week effects which diminish in number and significance with sampling frequency. In particular, a persistent Thursday effect was found for every sampling frequency except 30 minutes. We also find that jumps are determined by surprises and specific economic variables rather just the announcements. In addition we find that investors in the Colombian sovereign bond market are more sensitive to external surprises that may impact a change in US monetary policy than local changes in monetary policy or any other economic announcement.

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7 Appendix: Complementary results

This section includes the remaining results omitted in this chapter's previous discussions. Proportion of exceedance results are shown in figures 10, 11, and 12 for 5, 10, and 15 minute sampling frequencies. These results help highlight the interpretations given above, as well as illustrating the difficulty that liquidity imposes on our analysis.

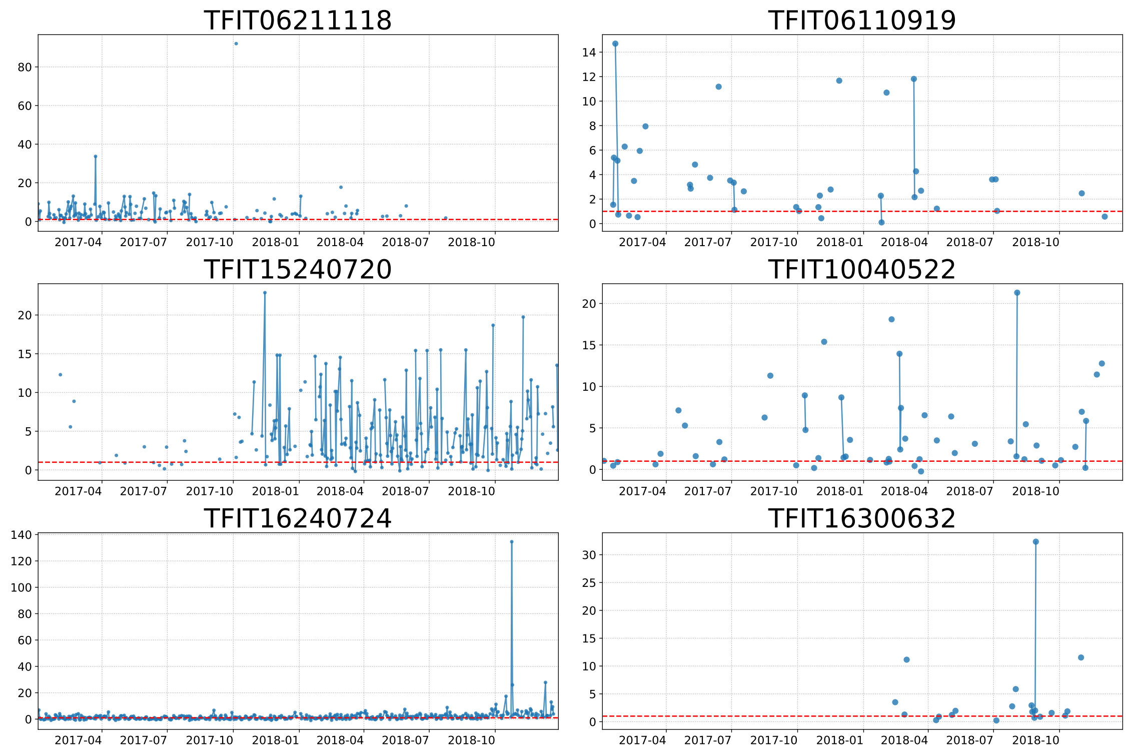

Figure 10: Jump statistic results in proportion to the critical value, 5 minute sampling

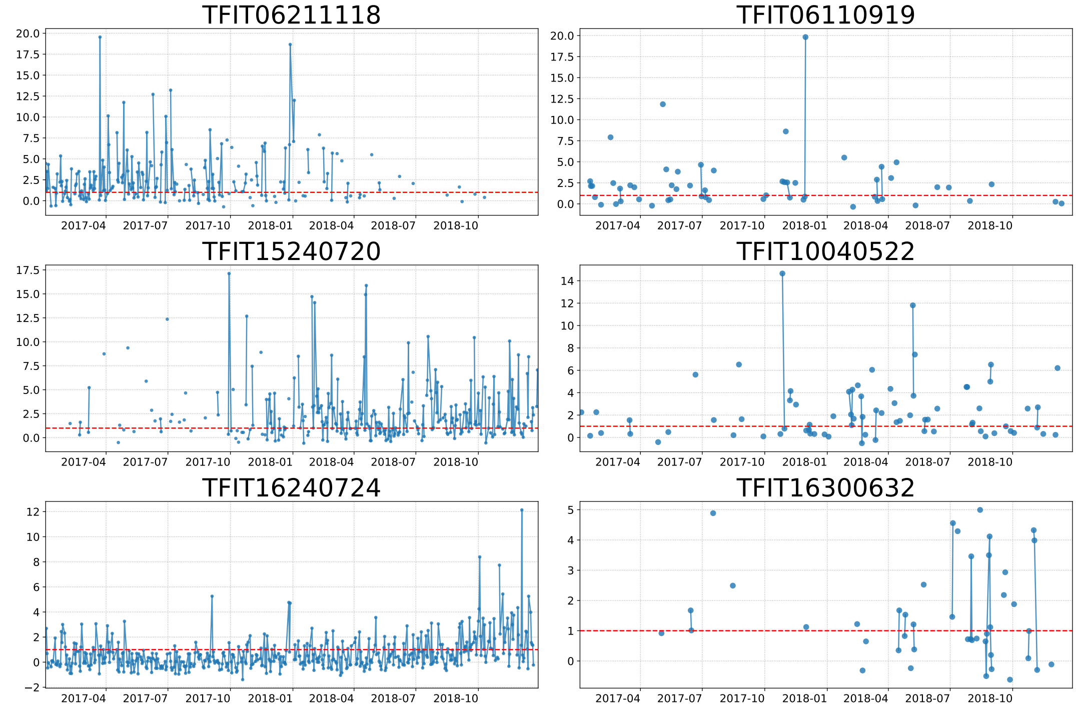

Figure 11: Jump statistic results in proportion to the critical value, 10 minute sampling

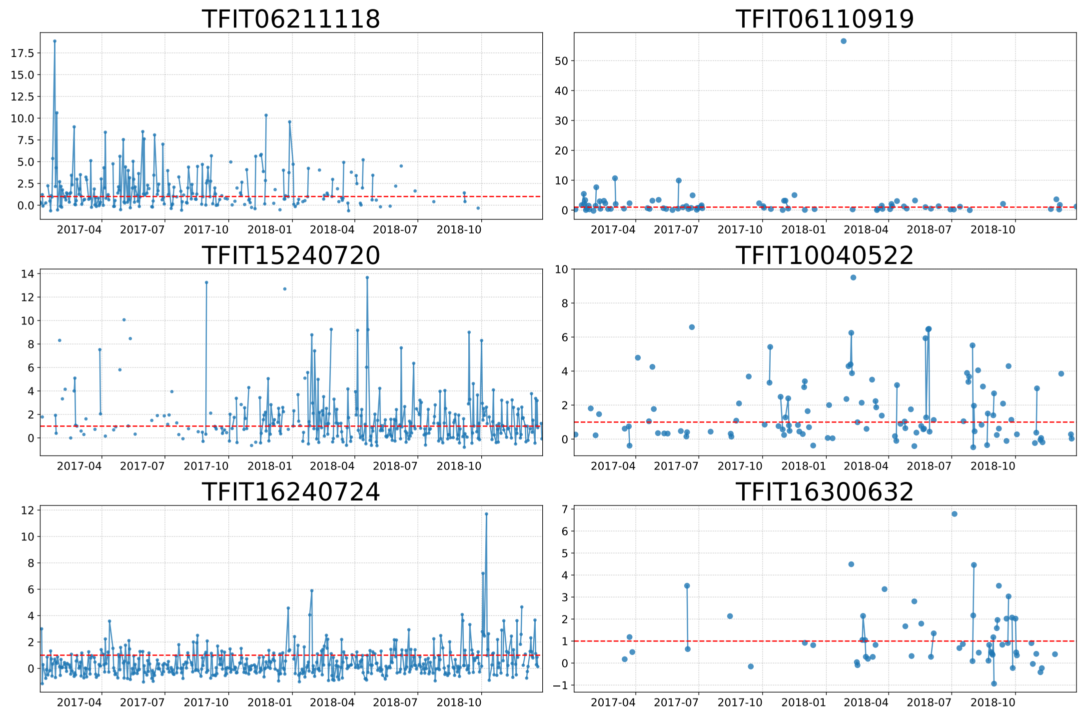

Figure 12: Jump statistic results in proportion to the critical value, 15 minute sampling