Essays on Telecommunications Regulation and Innovation Economics

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Dedication

To my beloved wife Marisol, for unconditionally and lovingly supporting me in this endeavor.

To my parents, Ramiro and Clara, for encouraging me from the very beginning to do my best.

To my sister Mónica, for teaching me the value of self-confidence.

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1. Introduction

Along my engineering profesional life in telecommunications, I had the opportunity to get in touch with regulatory issues in the industry and related businesses. The Industrial Organization literature in this field has provided answers to the classical market failures problems that stem from network externalities and call termination monopoly. Nonetheless, other questions still drive interest. In addition to this, the enrollment with innovation models captured my attention about innovation races and incentives to innovate.

The spread of mobile telecommunications networks enables access to new technological products. Mobile banking services such as e-wallets and mobile money provide drive new businesses for network operators and third party players, and serve a channel for financial inclusion. The cost of transactions between consumers is a matter of interest, since these tariffs can be used by network operators to behave anticompetitively. In Chapter 2, I analyze the implication of regulating the retail price of a dominant firm in a market where consumers subscribe to any of two platforms to send and recieve transactions, like money transfers. The work is motivated by observing that in some banking markets, the retail prices are regulated but in others not. The policy implication of my findings state that that this type of regulation triggers retail price increase.

In industries such as pharmaceuticals and technology, Research and Development is a key driver for innovation. In the video game market, Nintendo and Sega engaged in an innovation race for 16-bit console. Sega, playing as the challenger in the market, released a 16-bit console right ahead of Nintendo. It turns out that Nintendo had already a 16-bit console available by the time Sega released its product. In Chapter 3, written together with professor Guillem Roig Ph.D., we analyze the incentives of an incumbent to delay the introduction into the market of a discovered product. When the discovered innovation is highly profitable, if the incumbent discover the innovation first, it has an incentive to delay the product release into the market. This strategy seeks to capture monopoly profits by discouraging the entrant to continue on the race. Our results imply that public policy should aim to foster research joint ventures.

Another relevant question in telecommunications markets relates to switching costs. Firms can use switching costs as a lock-in device to discourage consumers from looking better deals from other suppliers. In response, regulators pursue the elimination of switching costs to encourage competition. In Colombia, the telecommunications market regulator banned the use of Early Termination Fees -ETF- in mobile telecommunications services. Firms argued against the ban, stating that ETFs are the vehicle to encourage new technology penetration.

This would be so, since consumers could buy mobile handsets at a lower price by just accepting the ETF. The effects in the handsets market has been proven beneficial. However, there is no evidence to date about the outcome of this regulation in the mobile voice and mobile Internet markets. In the Chapter 4 I tackle this question. I find that this regulation was beneficial since prices decreased in the provision of mobile voice. There is no any effect in the provision of mobile Internet access.

2. Receiving Money Matters: Regulation of Transaction Tariffs

Renzo Clavijo[1]

2.1. Introduction

Firms that offer services subject to network effects such as traditional and mobile banking pursue discriminatory practices at the retail level. Indeed, some other financial services such as e-wallets do not even allow transactions between platforms.[2] In general, making transactions outside the platform is more costly for consumers than transactions with other consumers of the same platform, such as mobile money services in Indonesia (Bourreau and Valetti, 2015) or Uganda (Paelo and Roberts, 2022)[3,4]. Transactions' costs can even discourage service adoption.[5] Conversely, price discrimination for mobile money services is banned in Kenya since April 2018, (CBK, 2018). Therefore, regulating retail prices in this industry raises the question about the suitability of this decision.

The outcomes of retail uniform pricing regulation in network industries were analyzed by Hoernig (2008). In his model, consumers transact evenly with any other consumer in the market. However I go a step further to analyze how the strategic decisions of a network regulated at the retail level change when consumers make transactions more intensely with friends, family, colleagues (circles) and consumers derive utility from receiving transactions as well (what I call transaction externality). To answer these inquiries, I develop a theoretical model of network competition and discuss price discrimination for the case of unregulated firms. Then, I extend my model to introduce retail price regulation. The outcomes of my

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[2]Consider, for example, Daviplata and Nequi services in Colombia.

[3]Paelo and Roberts (2022) show that price differential can be as high as 200% for transactions in Uganda.

[4]In Congo, SMS off-net prices can be as high as 100% on-net tariffs. This is a concern since SMS is widely used in Africa for surveys (Lau et al., 2019), election monitoring, health information campaigns, disaster relief operations, and mobile banking (Jack and Suri, 2014, Aker et al., 2017).

[5]According to financial products demand survey in Colombia, 42% of respondents report not to have any savings or payment product (such as e-wallet). From this percentage, 35% considers products transactions as being very costly and 23% do not even know what savings or payment products are. Among the respondents holding a savings or payment product, almost 70% cast a bad perception regarding the costs of using these products.

model convey a message against retail regulation.
This paper also talks about features of markets where the traditional price differential rule, i.e. off-net price higher than on-net, is not met. Hence, my approach unveils the relevant features of markets where different than usual on/off-net pricing discrimination behavior is observed. My setting is a model of network competition with consumers located on a Salop circle and two networks competing for consumers. The networks strategically set retail prices for outgoing transactions. Consumers do not bear any cost for incoming transactions. The firms have different market shares, which is the only source of asymmetry between networks. Besides, I assume that access charges are set by a regulator at the symmetric cost level.[6]
Consumers decide consumption of outgoing transactions, and derive utility from transactions made to destinations either inside their network or in the rival network. Transactions are made to destinations in a non-uniform pattern such that more transactions are sent to certain destinations than to others. The equilibrium prices I obtain are such that the corresponding on-net Lerner indices are dampened the higher the transaction externality, while off-net Lerner indices increase when transaction externality is accounted for.
My model extends previous results in the literature of network competition. I show that if consumers transact heavily with destinations in their circle, the large firm sets a higher on-net price compared to the on-net price set by the small network. This outcome still holds if circles are weak but the market is sufficiently concentrated. This result constitutes theoretical support to regulators' concerns about the inconvenience of a highly concentrated market. For off-net prices, the large firm sets a higher price than the small network if circles are not strongly concentrated. This condition depends only on transaction externality and price elasticity of demand.
Regarding on/off-net price differential, my model explains both traditional and non-traditional outcomes. I find a series of conditions on structural parameters such that traditional lower on-net than off-net prices and non-traditional on-net higher than off-net prices can be explained within the same theoretical framework. Hoernig et al. (2014) find a condition for which on-net prices will be lower than off-net prices without distinction of price differentials for each network. Their result holds for a profit maximizing access charge. In my setting I find conditions when access charge is set at the cost level. Therefore, my contribution fills this gap considering the effect that the interaction of circles and transaction externalities have on price differentials on a per firm basis when access charges are regulated at cost.
The equilibrium off-net price in a market where circles are present is lower compared to a market where consumers transact evenly to any destination. If consumers make a significant amount of transactions to destinations in their circle, the demand for transactions outside the circle is weak. Since off-net destinations are mostly located outside the circle, the previous rationale implies that demand for off-net transactions is also weak.
The low off-net pricing behavior just described is reinforced if consumers display a low trans-

[6]By the same argument as in López and Rey (2016), asymmetrical access charges are fading away as a regulatory tool to enhance competition.

action externality. When consumers value incoming transactions much less than outgoing ones, the amount of surplus derived by consumers for incoming transactions is very small. Hence, outgoing off-net transactions yield a small utility for consumers in the rival network making the firm less willing to discourage off-net operations through a higher price. In summary, either strong circles or low transaction externalities draw off-net equilibrium prices closer to zero.

My model is extended to account for retail price differential ban of the large network. The regulated price is always higher than on-net discriminating price. In markets with consumers who make evenly distributed transactions among destinations and derive high utility from incoming transactions, the regulated price is lower than off-net discriminating price. If transactions are strongly concentrated in the circles and consumers value incoming transactions much less than outgoing ones, regulated price shifts to lower values, even below off-net discriminating price. Lastly, the larger the market share of the large network the larger the space for which the regulated price is lower than off-net discriminating price.

The theoretical literature about network competition is motivated in telecommunications industry. It has shifted from models that consider consumers transacting uniformly to every destination, towards settings in which consumers make transactions to certain destinations more heavily than to others, also known as circles. Armstrong (1998), Laffont et al. (1998a and 1998b) constitute the groundwork in network competition. In these models, two firms compete in prices for consumers located along a Hotelling line. The networks set either a linear or a two-part tariff while consumers make calls uniformly to any other destination in the market. Further contributions, Gabrielsen and Vagstad (2008) and Calzada and Valletti (2008), account for calling circles by considering each consumer makes a proportion of calls to certain destinations while the remaining calls are uniformly distributed among any destination in the market. My work is closer to the framework of Hoernig et al. (2014), who model calling circles as a probability distribution centered around each consumer. However, they do not incorporate in their setting the transaction externality to establish how pricing strategies of the networks are shaped either in the discrimination case nor in the retail regulated scenario.

Implications on competition when consumers derive utility for receiving calls were first discussed in the literature related to competition under receiver party pays -RPP- principle since the relevance of call externality was clearer in that context.[7] For the case of calling party pays -CPP-, Berger (2005) takes call externality into consideration to analyse the problem of access pricing, while Hoernig (2007) analyses retail price behavior of firms when consumers derive utility from incoming calls (transaction externality in my model). I take transaction externality into account and depart from the access charge price problem by considering the compensation for the usage of the rival network is set at cost.

Retail price regulation has been subject of analysis aimed to establish implications on welfare. From a theoretical perspective, Hoernig (2008) proposes a model of network competition that

[7] Under RPP, the callee party pays a fee when answering the call.

predicts a strategic reaction of the regulated firm increasing on-net prices and reducing off-net prices. Hoernig's model takes into consideration call externalities but not calling circles effects. Unlike his work, my model is able to predict simultaneous price increase (on-net and off-net) when the regulator bans the retail price differential. Empirical analysis by Rojas (2015) finds how consumer surplus and profits change according to different retail price regulation rules in Chile. He finds that consumers might be harmed while firms are better-off with uniform prices. Similarly, Agostini et al. (2017) carry out the analysis for the Chilean market aiming to identify if retail differentials are a vehicle of predatory behavior. These empirical contributions set their framework on theoretical models considering call externalities but disregarding calling circles.

The rest of this paper is organized as follows. Section 2.2 contains the detailed description of the model. Discriminatory price outcomes are discussed in section 2.3. Section 2.4 contains price differential results. I discuss retail price regulation in section 2.5. Finally, section 2.6 concludes.

2.2. Model set-up

This work develops a competiton model between two networks whose sizes are exogenously given by the amount of consumers subscribed to each. Figure **2-1** depicts concepts related to market shares, on-net transactions, off-net transactions and corresponding prices.

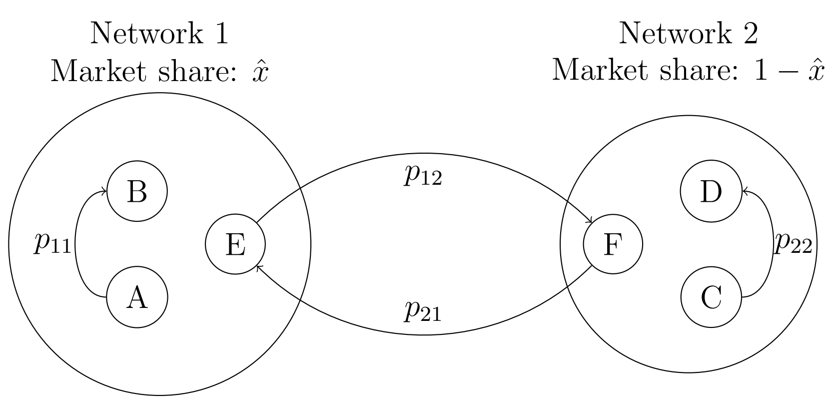


Figure **2-1**.: Basic elements for the model.

The networks ($i = 1, 2$) behave like profit maximizing firms by competing in the market and facing marginal costs for originating (c_O) and terminating transactions (c_T) which are the same for both firms, i.e., firms are considered equally efficient when providing on-net and off-net transactions. Furthermore, each firm holds a portion of consumers that constitutes its market share. These market shares are given as follows: $\hat{x} > 1/2$ corresponds to the market share of network 1 and network 2 holds a market share $1 - \hat{x}$. $\hat{x}$ represents the location of

the consumer that is indifferent between offers of network 1 and network 2. This location is considered exogenous in the model (see Figure **2-2**).
Consumers can either make on-net or off-net transactions. On-net are those transactions terminated in the same network they were initiated (i.e., those originated in network i whose destination is network i) which are charged by the originating firm at a price p_{ii}. Off-net transactions are those initiated on any network and terminated on the rival network (i.e., transactions originated in network i whose destination is network j), which are charged at a price p_{ij} by the originating network.

2.2.1. Consumers

Consumers in my model locate uniformly on a salop cirle in the interval $[-1, 1]$ while firms are located in $x = 0$ and $x = 1$, as displayed in Figure **2-2**:

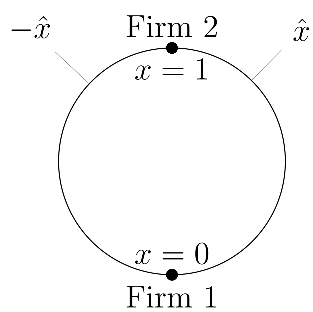


Figure **2-2**.: Consumers' location.

Regarding consumers location and subscription to networks, consumers can belong to one of two firms present in the market: consumers belonging to network 1 are located in the interval $[-\hat{x}, \hat{x}]$ of the circle, while consumers belonging to network 2 are located in the interval $[-1, -\hat{x}] \cup [\hat{x}, 1]$. One of the key features of the model is the consumer behavior when making transactions: each consumer initiates transactions with a higher probability to consumers located close to her, representing a higher probability of transact with consumers with similar preferences. This behavior is known as circles, where the circle means a set of destinations to which the consumer transacts more frequently with. [8]
Circles are modeled using a function $G(y|x)$, where $G(\cdot)$ is a cumulative distribution function -CDF-, indicating the likelihood a consumer located at x transacts with every other subscriber located below point y (consumers located at $y' < y$). Consumers y' can belong either to network 1 or network 2, which means that transactions pattern for each subscriber

[8] According to Hoernig et al. (2014), this behavior can be explained by the affinity consumers feel with a specific brand thanks to market effort developed by the firms, or just the coverage level offered in a given area. Authors like Birke and Swann (2010), even point the possibility that consumers take into consideration subscribing the network where their relatives or acquaintances belong to.

is a CDF that averages the probability of making transactions uniformly to every subscriber in the market (uniform CDF described by $U(\cdot)$) and the probability of making transactions to subscribers in the circle (described by $H(\cdot)$), as follows:

$$G(y|x) = (1-\lambda)U(y) + \lambda H(y-x), \tag{2-1}$$

where $0 \leq \lambda \leq 1$ is the circle weight in the transaction pattern of the consumer, and $H(z)$ is a uniform CDF with density $h(\cdot)$ with the functional form:

$$h(z) = \frac{1}{\varepsilon}, 0 \leq z \leq \varepsilon. \tag{2-2}$$

This function describes the circle centered around each consumer where ε is the circle size for each consumer. The circle size in my model is considered to be small enough compared to the market size of each network, as stated in Assumption 1.

Assumption 1. *Circle size is small enough compared to the market shares of the firms:* $\varepsilon < 2\hat{x}(1-\hat{x})$.

Given on-net and off-net prices, consumers demand transactions with money amount $q_{ii} = q(p_{ii})$ and $q_{ij} = q(p_{ij})$ deriving an indirect utility given by:

$$\upsilon(p) = \max_{q} \left\{ u(q) - pq \right\},$$

where,

$$u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}},$$

and parameter η stands for the price elasticity of demand for both on-net and off-net transactions.
Given this framework, I allow for the possibility for the consumers to derive utility not only by making transactions but also by receiving transactions: the utility function for every consumer must include the corresponding terms allowing this novel conjoint approach; therefore, the utility a consumer located at x derives follows the next functional form:

$$U_1 = G(\hat{x}|x)\left[\upsilon(p_{11}) + \gamma u(q_{11})\right] + \left[1 - G(\hat{x}|x)\right]\left[\upsilon(p_{12}) + \gamma u(q_{21})\right] + u_0 - F_1 - \tau x, \tag{2-3}$$

for the case of a consumer belonging to network 1, and

$$U_2 = \left[1 - G(\hat{x}|x)\right]\left[\upsilon(p_{22}) + \gamma u(q_{22})\right] + G(\hat{x}|x)\left[\upsilon(p_{21}) + \gamma u(q_{12})\right] + u_0 - F_2 - \tau(1-x), \tag{2-4}$$

for the case of a consumer in network 2. These expressions account for the intrinsic utility of subscribing to the network (u_0) which I consider high enough to have a fully covered market, utility for making transactions ($v(p)$) and utility for receiving transactions ($\gamma u(q)$)[9]. Note that the indirect utility is used for the case of transactions made by the agent since it reflects the reaction to any decision of the firms on prices, while direct utility is used when accounting for utility of received transactions since the agent does not make any decision on the amount of received transactions.[10] F_1 and F_2 correspond to the fixed fee of the two-part tariff set by the firms, and τ represents the dis-utility the consumer bears due to the distance between his location and the one of the network he is subscribed to.

2.2.2. Firms

There are two firms in the market which are located at points $x = 0$ and $x = 1$ on the Salop circle and behave as profit maximizing agents competing *à la Bertrand* by making decisions on fixed fees and prices offered to consumers,

$$T_i = F_i + q_{ii}p_{ii} + q_{ij}p_{ij},$$

meaning that for on-net transactions inside network i firm charges a price p_{ii}, while for off-net transactions originated at network i and terminated at network j firm i charges a price p_{ij} to consumers, besides the fixed fee F_i.

Since my model considers consumers located along the Salop circle, competitive dynamics in the interval $[0, 1]$ are mirrored in the interval $[-1, 0]$, in consequence the analysis in the interval $[0, 1]$ explains the behavior of the whole market. In this work the large network will also be regarded as network 1, big network or large firm. Likewise, the small network will be regarded as network 2 or small firm.

Total benefits of firm 1 from any consumer x are

$$\Pi_1(x, \hat{x}) = \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f,$$

where π_1 are the payoffs generated by consumer x to the firm 1:

$$\pi_1(x, \hat{x}) = G(\hat{x}|x)(p_{11} - c_{11})q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c_{12})q(p_{12}),$$

F_1 is the fixed fee of the two-part tariff offered to consumers subscribed to network 1, R_{12} are the perceived benefits of the firm due to access charges received for terminating transactions originated in network 2:[11]

$$R_{12}(x, \hat{x}) = [1 - G(\hat{x}|x)]\,(a - c_T)q_{p21},$$

[9] The intrinsic utility can be thought of as the utility a consumer derives to be able to use basic services such as keeping money in the account, before making transactions to other consumers in the market. Jeon et al. (2004) provide an interpretation from the telecom markets perspective.

[10] Call externalities are considered in the literature as the ratio of incoming call utility to outgoing call, see Jeon et al. (2004) and Sobolewski and Czajkowski (2018).

[11] Access charges correspond to money paid by network 2 to network 1 by using the latter's infrastructure to terminate transactions originated in the former.

and f is the fixed cost of serving a subscriber. Recalling that analysis for $x \in [0,1]$ explains the competitive dynamics in the whole market and integrating for all consumers, total benefits for network 1 are:

$$\bar{\Pi}_1(\hat{x}) = 2\int_0^{\hat{x}} \Pi_1(x,\hat{x})\,dx. \tag{2-5}$$

A similar definition applies for network 2 benefits, considering its market share are the consumers located in the interval $\hat{x} \leq x \leq 1$ and $-1 \leq x \leq -\hat{x}$ on the Salop circle:

$$\bar{\Pi}_2(\hat{x}) = 2\int_{\hat{x}}^{1} \left[\pi_2(x,\hat{x}) + F_2 + R_{21}(x,\hat{x}) - f_2\right]\,dx \tag{2-6}$$

Networks maximize profits given by equations (2-5) and (2-6), a process that requires the definition of some terms that will be useful later on. Along section 2.2.1, circles were defined by means of a CDF -denoted $G(\hat{x}|x)$- that considers the likelihood for each consumer to transact in her circle as well as the likelihood to make transactions towards destinations outside her circle.
Following Hoernig et al. (2014), the amount of on-net transactions generated in network 1 can be computed by summing up for every consumer x belonging to this network:

$$L_{11}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)\frac{1}{2}dx.$$

Given that I am interested in the case when there is a big network, the aggregate number of on-net transactions in network 1 when $\hat{x} > 1/2$ is given by:

$$L_{11}(\hat{x}) = \int_0^{2-2\hat{x}} H(z)dz + 2\hat{x} - 1.$$

In a similar fashion, the aggregate number of off-net transactions originated in network 1 when $\hat{x} > 1/2$ is given by the expression that follows:

$$L_{12}(\hat{x}) = \hat{x} - L_{11}(\hat{x}).$$

For the case of network 2, a similar approach can be followed to define both aggregate number of on-net transactions (L_{22}) and aggregate number of off-net transactions originated by consumers of this network (L_{21}).
Lastly, the case of the circle for the marginal subscriber $G(\hat{x}|\hat{x})$ is considered. If every inframarginal consumer had the same circle pattern of the marginal consumer, the on-net amount of transactions of network 1 are given by the next expression:

$$\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x}).$$

Consequently, off-net transactions originating in network 1 if every infra-marginal consumer holds the same circles pattern as marginal consumer would be given by

$$\hat{L}_{12}(\hat{x}) = \hat{x}\left[1 - G(\hat{x}|\hat{x})\right].$$

Similar definitions apply for $\hat{L}_{22}(\hat{x})$ and $\hat{L}_{21}(\hat{x})$.

2.2.3. Timing and Equilibrium Concept

Regarding the timing of the game, at $t = 1$ firms set fixed fees given marginal consumer $\hat{x}$; in $t = 2$ firms decide prices, consumers make transactions and payoffs are realized.
The game I develop consists of complete information, therefore the strategic decisions of the firms will lead to a Nash equilibrium in pure strategies. The equilibrium consists of a strategy profile that formulates the on-net price and the off-net price set by the network.

2.3. Market outcomes

2.3.1. Equilibrium on-net prices

Given the framework along section 2.2 for the game between firms and consumers, equilibrium on-net prices set by the firms can be obtained after maximizing the profits for each network (equations (2-5) and (2-6)), as stated in Proposition 1.

Proposition 1. *Equilibrium on-net Lerner indices are given by the next expressions:*

$$\frac{p_{11} - c_{11}}{p_{11}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{11}}{L_{11}}\right] \underbrace{-\gamma\frac{\hat{L}_{11}}{L_{11}}}_{C.E.\ effect}, \tag{2-7}$$

and

$$\frac{p_{22} - c_{22}}{p_{22}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{22}}{L_{22}}\right] \underbrace{-\gamma\frac{\hat{L}_{22}}{L_{22}}}_{C.E.\ effect}. \tag{2-8}$$

Proof. See section A.1 in the appendix. □

From the Lerner indices given in (2-7) and (2-8), some interesting features about the optimal on-net prices can be discussed. Regarding price elasticity of demand (η) it can be seen in the last term of each expression that this is not shaping the behavior of the firms due to the presence of transaction externalities, resembling the results obtained by Hoernig (2007). This reflects how transaction externalities affect consumer surplus through utility derived due to received transactions but not as a result of consumer decisions given prices in the market. In other words, the term $1/\eta$ explains the reaction of the firm in response to consumers' behavior when deciding the amount of transactions to place by virtue of price set by the network. Meanwhile, the transaction externality in the second term of the expressions explains how the firm takes into account utility derived by consumer x as a result of other consumers transacting with him/her, a utility that is not a result of consumers' own decision.

Absent transaction externalities, my results resemble the Lerner indices obtained by Hoernig et al. (2014); furthermore, absent circles ($\lambda = 0$), lerner indices of Hoernig (2007) are recoverd when accounting for two-part tariff equilibrium. My result states that as long as transaction externality is greater than zero, consumers will derive benefits not only for making transactions but also from receiving transactions, and this will trigger the incentives of firms to encourage a higher volume of transactions inside their networks since more transactions mean more consumer surplus, leading to more benefits that firms can extract. On-net transactions consumption can be fostered by reducing the on-net price, which is the mechanism at work explained by the negative term preceding the transaction externality in on-net Lerner indices.

The ratio $\hat{L}_{ii}/L_{ii}$ mandates how strong is the effect of transaction externality for consumers in network i. $\hat{L}_{ii}$ represents the utility of the marginal consumer for incoming transactions when on-net price decreases, being this consumer the one that benefits the least among the consumers in the network. Thereby, a price decrease translates into an additional surplus for marginal consumer that can be extracted through fixed fee. For inframarginal consumers, a price decrease will set them better-off than marginal consumer, however, lower on-net price could lead to a reduction of profits despite the growth in on-net transaction volume, then firms will face a limitation to exploit transaction externality effects on consumer surplus. That is why L_{ii} appears in the denominator of the ratio being discussed. Taken together, absent circles, quotient equals to one and decreases in magnitude as circles become stronger. This means that the effect of the circle becomes weaker and firms set higher on/net prices. These insights are stated at Corollary 1.1 as follows.

Corollary 1.1. *Transaction externality effect on on-net Lerner index is weaker the larger the circle weight (λ).*

Proof. See section A.2 in the appendix. □

2.3.2. Equilibrium off-net prices

Equilibrium off-net prices set by the firms can be obtained after maximizing the profits for each network (equations (2-5) and (2-6)), as stated in Proposition 2.

Proposition 2. *Equilibrium off-net lerner indices are given by the next expressions:*

$$\frac{p_{12} - c_{12}}{p_{12}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{12}}{L_{12}}\right] \underbrace{+\gamma\frac{\hat{L}_{11}}{L_{12}}}_{C.E.\ effect}, \tag{2-9}$$

and

$$\frac{p_{21}-c_{21}}{p_{21}}=\frac{1}{\eta}\left[1-\frac{\hat{L}_{21}}{L_{21}}\right]\underbrace{+\gamma\frac{\hat{L}_{22}}{L_{21}}}_{C.E.\ effect}, \tag{2-10}$$

Proof. See section A.3 in the appendix. □

In a similar fashion as for on-net Lerner indices, price elasticity of demand is not shaping the effects of transaction externality on the behavior of the firm when setting off-net prices. This conduct is implied by a mechanism akin to the one described for on-net prices. Consumers derive utility for making off-net transactions such that the amount of these transactions is decided by consumers according to prices set by the firms, which is reflected by the expression including $1/\eta$; meanwhile, utility due to outgoing transactions that consumers of network i generate to reach consumers in network j is explained by the term containing the transaction externality.

From the Lerner indices given in (2-9) and (2-10), absent transaction externalities, I obtain the same results as Proposition 1 in Hoernig et al. (2014); furthermore, for the case of no circles, I recover the results of Hoernig (2007) when accounting for two-part tariffs. Off-net Lerner indices are shaped by transaction externality in the opposite direction to on-net ones, stemming from firms' reaction to the utility that outgoing transactions generate for consumers in the rival network. If transaction externality is greater than zero, consumers in network i derive utility by making off-net transactions, but will also allow subscribers receiving this type of transactions in the rival network to derive utility; this effect is internalized by firm i which reacts by refraining consumers to make off-net transactions through an increase in p_{ij}. This mechanism becomes evident by the positive sign of the factor containing the transaction externality in the off-net Lerner indices.

The magnitude of $\hat{L}_{ii}/L_{ij}$ ratio can be understood as follows. $\hat{L}_{ii}$ represents the benefit derived by marginal subscriber of network j due to received transactions generated by consumers in network i. This consumer is the one that derives the highest benefits from receiving transactions generated by consumers in network i. Hence, raising off-net price will shift marginal consumer utility downwards and this will be reflected as a smaller dis-utility for network i that becomes a higher fixed fee. This is the reason why the term multiplying transaction externality in off-net lerner index is positive. However, the firm also internalizes the reduction of outgoing transactions generated by their own subscribers if price is raised, which is accounted by the denominator L_{ij}.

For a circle weight increase, marginal consumer reduces off-net consumption less strongly than infra-marginal subscribers which translates into a higher off-net transactions aggregate reduction if infra-marginal consumers have a different circles pattern than marginal consumer. Thus, as circle weight rises, the proportion of off-net transactions generated by marginal consumer is higher than for infra-marginal, such that infra-marginal consumers drive aggregate reduction demand of off-net transactions leading to a price decrease. The described reduction in off-net transactions has two effects due to utility derived by consumers of the network and utility generated for consumers in the rival network. When circle weight is higher, more on-net transactions are placed and demand for off-net transactions decreases. Firms react by setting lower off-net prices to foster consumption (elasticity effect). Fostered consumption, however, will also raise benefits derived by rival network's consumers, an effect that is internalized by originating network by means of price increase because of the increased amount of outgoing transactions (transaction externality effect). Then, there is a trade-off between reducing off-net price due to the elasticity effect and increasing the off-net price because of transaction externality effect. These two forces act against each other and one will overcome the opposite depending on transaction externality magnitude and elasticity strength. For inelastic markets, consumers react mildly to price changes and elasticity effect dominates giving the firms more power to extract higher benefits from consumers by an overall price reduction. On the other hand, if transaction externality is very strong, utility provided to rival network is large enough so that firms react by increasing prices to deplet off-net consumption. This analysis is stated at Corollary 2.1.

Corollary 2.1. *The transaction externality effect on off-net Lerner index is stronger the larger the circle weight* (λ)*.*

Proof. See section A.4 in the appendix. □

2.4. Price differentials

In this section I study the relationship between on-net prices, as well as on-net/off-net price differential.

2.4.1. On-net prices ratio (p_{11}/p_{22})

Depending upon consumer attributes, on-net prices will vary and the mechanism behind those decisions is of interest to further establish the strategies under retail regulation. The condition under which $p_{11} \geq p_{22}$ is stated in Proposition 3.

Proposition 3. *On-net price set by the large network is greater than or equal to the on-net price of the small network if the next inequality on circle weight holds:*

$$\lambda \geq \frac{\hat{x}(1-\hat{x}) - \frac{\varepsilon}{2}}{\hat{x}(1-\hat{x}) - \frac{\varepsilon}{4}} = \hat{\lambda}_{on}(\hat{x}, \varepsilon). \tag{2-11}$$

Proof. See section A.5 in the appendix. □

Corollary 3.1. $\hat{\lambda}_{on}(\hat{x}, \varepsilon)$ *is always positive and less than one.*

Proof. Numerator and denominator of $\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon)$ are positive from Assumption 1, while numerator is smaller than the denominator, then this ratio is positive and less than 1. □

For the particular case of $\lambda = 1$ and using the result given in Corollary 3.1, the on-net prices relationship turns out to be: $\lambda = 1 \Rightarrow p_{11} > p_{22}$. This condition means that if consumers transact to destinations in the circle only, then the largest firm finds incentives to charge a higher on-net price compared to on-net price set by network 2. In this scenario, consumers place no transactions outside their circle leading to a greater amount of on-net transactions and less off-net transactions. Then, competition is weakened and more benefits can be extracted from consumers by means of on-net pricing strategy.

In addition, it can be seen that the interval $[\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon), 1]$ expands when either $\hat{x}$ or ε increases. The larger the market share of network 1 allows for a wider range of vales of λ for which the on-net price of network 1 will be higher than on-net price of network 2. A similar behavior for $\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon)$ holds when circle size increases. This result supports the claim of regulators about the concern that a highly concentrated market can have on the price outcomes of the large firm. Larger firm 1 market share or larger circle size translates into a higher probability of making on-net transactions for subscribers in network 1, which is a market force -stronger demand- that raises incentives for this firm to set higher on-net prices. The relationship between on-net prices does not depend upon price elasticity of demand nor the magnitude of transaction externality, only market shares and calling circle size are involved. This is explained because both firms are internalizing the utility consumers derive for making and receiving transactions inside their corresponding networks.

If the circle size approaches to zero in the limit and consumers only make transactions to their circle ($\lambda = 1$), there will only be on-net transactions and both firms will set the same on-net price. This resembles monopoly level prices, as only on-net transactions will flow in the networks. In this case, there will be two separate markets, one for each network, and the firms behave as monopolies on each market. Departing far enough from $\lambda = 1$, on-net price of bigger network will be less than on-net price of small network. In this case the larger network loses a greater amount of on-net transactions than its rival, which translates into a stronger on-net price reduction than on-net price reduction of network 2. These insights are summarized in Corollary 3.2.

Corollary 3.2. *On-net prices approach the monopoly level if circle size approaches zero and consumers make transactions to destinations in their circle only.*

2.4.2. Off-net prices ratio (p_{12}/p_{21})

Now, I analyze the off-net prices differential of the firms present in the market, i.e., the differential between p_{12} and p_{21}. The condition under which $p_{12} \geq p_{21}$ holds is stated in Proposition 4 as follows:

Proposition 4. *Off-net price set by the large network is greater than or equal to the off-net price set by the small network if the next inequality on circle weight holds:*

$$\lambda \leq \frac{2}{1 + \frac{1}{\gamma\eta}} = \hat{\lambda}_{off}(\gamma, \eta). \tag{2-12}$$

Proof. See section A.6 in the appendix. □

Inequality (2-12) states that p_{12} will be greater than or equal to p_{21} as long as $\lambda \leq \hat{\lambda}_{\text{off}}(\gamma, \eta)$. In the limit, when transaction externality approaches zero, p_{12} is less than p_{21}. Transaction externality creates an incentive for the large firm to increase off-net price to dampen the amount of benefits rival consumers can derive. Network 2 faces the same incentive. If circles are weak, the incentive to increase prices is stronger for the large network since its consumer base has the potential to generate more benefits to rival consumers than conversely.

For a given level of transaction externalities, the incentives to modify off-net prices in response to circle weight changes are also asymmetrical among firms. Consider the case of non-existing circles. Off-net price of the large firm is higher than the off-net price of the small network due to the amount of benefits created to rival consumers for receiving transactions. If the transactions pattern changes to a more concentrated shape, the demand for off-net transactions decreases, an incentive for both networks to drop off-net prices. In this scenario, the forgone benefits of consumers in the small network due to less incoming transactions is higher than the foregone benefits of consumers in the large network due to transactions coming from rival consumers. This asymmetry makes the large network to drop its off-net price at a faster pace than the small network if the circles become even stronger. Eventually, the large network sets an off-net price below the off-net price of the small network if circles are sufficiently strong.

Hoernig (2007) describes a similar mechanism in terms of the Lerner indices for on-net and off-net prices. When transaction externality is taken into consideration, the incentives to raise off-net prices come into play due to additional utility derived by the consumers of the rival network therefore enhancing its competitive position. To the best of my knowledge, the existing literature has only highlighted the role of transaction externality (see Hoernig (2007) and Hoernig (2008)), but not the circles counterforce lessening the strength of transaction externality effects. This dampening response is evident from the behavior of $\hat{\lambda}_{\text{off}}(\gamma, \eta)$, which states that circle weight can surpass the relevance of transaction externality explaining the differential between off-net prices. Thus, inequality (2-12) is stating that transaction externality encourages a competitive behavior in the market that becomes apparent by off-net price raising, however this incentive is dampened by the existence of circles, which means that consumer transaction preferences is the mechanism through which firms moderate the described conduct.

Regarding the price elasticity of demand, the effect is similar as for the case of transaction externality. Higher price elasticity leads to a higher threshold $\hat{\lambda}_{\text{off}}(\gamma, \eta)$ which means that a wider range of circle weight values (i.e. interval $[0, \hat{\lambda}_{\text{off}}(\gamma, \eta)]$) is allowed for the network 1 to set a higher off-net price than network 2. In this case, a high price elasticity of demand means that consumers are highly sensitive to price changes, hence a small decrease in off-net price of either firm will turn into a strongly boosted consumption for this type of transactions. In this case, the large network drops its off-net cautiously accounting for the benefits the rival consumers derive. The small network faces similar incentives. The benefits that consumers of the large network trigger for consumers of the small network are larger than conversely, thus the large network is more cautious the stronger the elasticity. This makes the off-net price of firm 1 to remain higher for a wider range of circle weights. The effect is strong the higher the transaction externalities.

2.4.3. On/off-net prices differential (p_{ii}/p_{ij})

Unlike thresholds $\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon)$ and $\hat{\lambda}_{\text{off}}(\gamma, \eta)$, the differentials for prices set by each network depend on every structural parameter of the model, namely, transaction externality, price elasticity of demand, market shares and circle size. For the case of transaction externality and price elasticity of demand, dependence is algebraically the same as for $\hat{\lambda}_{\text{off}}(\gamma, \eta)$ but this effect is now shaped by market shares and circle size. The condition for the relationship $p_{ii} \geq p_{ij}$ to hold is given in Proposition 5.

Proposition 5. *On-net price set by network i will be greater than or equal to its own off-net price if the next inequalities on circle weight hold.*
For network 1:

$$\lambda \geq \frac{\hat{x}^2}{\hat{x}\left(\hat{x}-\frac{1}{2}\right)+\frac{1}{2\gamma\eta}\left(\hat{x}-\frac{\varepsilon}{2}\right)} = \hat{\lambda}^1_{dif}(\hat{x}, \gamma, \eta, \varepsilon). \tag{2-13}$$

For network 2:

$$\lambda \geq \frac{(1-\hat{x})^2}{(1-\hat{x})\left(\frac{1}{2}-\hat{x}\right)+\frac{1}{2\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)} = \hat{\lambda}^2_{dif}(\hat{x}, \gamma, \eta, \varepsilon). \tag{2-14}$$

Proof. See section A.7 in the appendix. □

Proposition 5 states that when transaction externality approaches zero $p_{ii} \geq p_{ij}$, resembling the case in which networks behave according to Hoernig et al. (2014). Subject to these conditions, networks do not face incentives to extract benefits from consumers due to incoming on-net transactions and do not face incentives to discourage transactions whose destination are consumers in the rival network. Each firm is taking advantage of rent extraction due to generated transactions: transactions inside its network because of calling circles strength -p_{ii} grows- and promoted transactions to destinations in the rival network by decreasing p_{ij}. As transaction externality increases, consumers derive utility from incoming transactions but also trigger additional utility for receivers in both the own and the rival network. This induces the firms to set strategies to exploit benefits of own consumers (p_{ii} drops) and lessen derived utility by consumers of the rival (p_{ij} rises). However, this force might be outweighed by circles if these are strong enough so that transaction volume displays the behavior that follows. Originated transactions in network 1 consolidate inside the network (which correspondingly holds in network 2), leading outgoing off-net transaction volume to decline and off-net prices to drop. Since circles pattern strenghtening yields an increase of on-net transaction volume, which can be understood as an on-net demand increase, on-net prices rise. Thus, if λ is high enough, the price outcomes for high transaction externality are reversed.

When large firm market share increases, interval $\left[\hat{\lambda}^1_{\text{dif}}, 1\right]$ shrinks while interval $\left[\hat{\lambda}^2_{\text{dif}}, 1\right]$ widens. As market share of the big network becomes even larger, the proportion of on-net transactions is higher and the incentives to exploit transaction externalities (by means of a lower on-net price) from its consumers are more relevant than the incentives to discourage off-net transactions (by means of higher off-net price) due to the same externality. In consequence, on-net price can be lower than off-net. These outcomes can only be offset if calling circle weight is high enough, i.e. $\lambda > \hat{\lambda}^1_{\text{dif}}$.

For the case of network 2 the effect is the opposite. Shrinking this network market share leads to a lower proportion of on-net transactions and the incentives to exploit own consumers' transaction externality is weaker. Therefore, the network exploits circles more heavily (by means of a higher on-net price), and becomes more aware about transaction externality effects on consumers of the rival (increase off-net price). In this case it is not required to have very high values of circle weight to observe $p_{22} > p_{21}$, i.e. $\hat{\lambda}^2_{\text{dif}}$ decreases. A graphical interpretation is provided in Figure **2-3**.

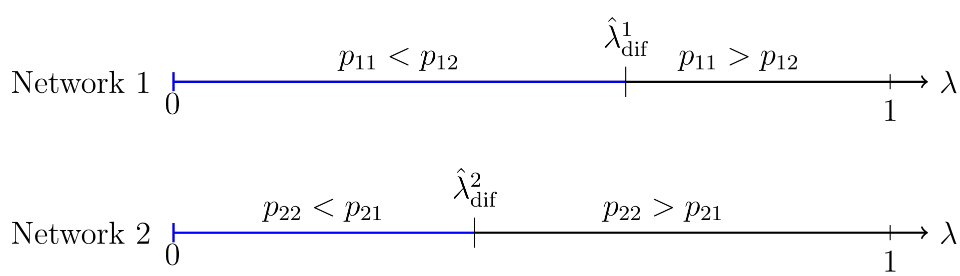


Figure **2-3**.: Threshold illustration for $\hat{\lambda}^i_{\text{dif}}$.

2.4.4. Price differentials discussion

To provide some insights on the purpose of my model, I recall two cases of transaction pricing in banking. The off-net transactions in Colombian banks is costly for consumers. However, on may 2019 BBVA bank reduced the off-net transaction costs to zero.[12] A similar decision

[12]See press release here.

was made by bank Caja Social on april 2021.[13] These two banks, fourth and fifth larger players in the country account for 15% of the active savings accounts.[14] Therefore most of the banks, the largest ones in particular, keep charging for off-net transactions higher fees than on-net transactions, while a few do not follow that strategy. This is evidence of the different outcomes discussed along this work.

Regarding off-net prices, the largest banks charge higher fees. Bancolombia charges COP 7.090, Banco de Bogotá COP 6.800 and Davivienda charges COP 6.450. Small banks charge lower fees, such as Banco Popular, whose off-net tariff amounts to COP 2.450.This outcome casts insights related to Proposition 4: circles are not very strong or the transaction externality is high. Weak circles means transacting more evenly with every consumer in the market, but this is not a likely scenario given the size of the market. Instead, high circle weight and strong transaction externalities can explain the off-net outcomes.

The results of my model can also be seen in other markets. Mobile prepaid price figures for Colombian and Congolian markets reflect diverse pricing strategies.[15,16] The largest operator in Colombia sets on-net price always below off-net (Figure **2-4**a) which is a classic prediction in the literature, while for the rest of the competitors (Figure **2-4**b) on-net tariffs are set above off-net ones. An additional finding from these data is the difference in on-net prices among operators (as well as off-net differential): on-net price set by the large network is always less than the on-net price of its competitors and off-net price set by large operator is always greater than off-net tariffs of its rivals. For smaller firms, the differential off-net prices are set above on-net prices (Figures **A-1** through **A-3** and **A-4** in the appendix). In average, off-net price differential is reversed for Comcel competitors (as shown in Figure **2-4**) mainly due to small networks in the market.

Looking at the mobile prepaid market from The Republic of Congo, three salient features of the retail prices are highlighted. First, on-net prices are lower than off-net. Second, the on-net price set by the large network (MTN) is lower than the corresponding price set by the small network (check left hand side panel of Figure **A-6** in the Appendix). Finally, off-net price of the big network is lower than that of the small network.

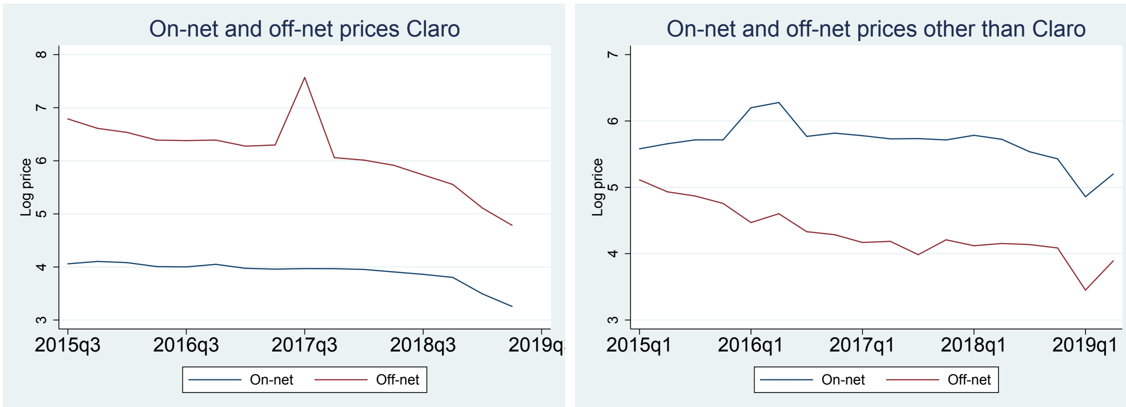
With this information, my model casts insights about the markets. On/off-net differential in The Republic of Congo allows to say that calling circle weight for consumers is low enough so as to make off-net prices of both large and small firm to be higher than on-net prices, a prediction from Proposition 5. For the case of Colombia, given the differential of average prices is reversed for smallest firms, casts evidence to conclude that consumers have a circle weight not as low compared to the consumers in The Republic of Congo. This means that a market where off-net prices are lower than on-net, becomes evidence of lower circle weight than markets where the opposite holds. This can also be the case for small banking firms

[13]See press release here.

[14]See press release here.

[15]Data for Colombia can be found at https://colombiatic.mintic.gov.co/.

[16]Data for The Republic of Congo can be found at http://www.arpce.cg/telecharger-observatoires.



(a) Claro. (b) Other operators than Claro.

Figure **2-4**.: Prepaid prices in the Colombian mobile market.
Source: MINTIC web portal and author's calculations.

that reduce off-net transactions fee to the on-net price level.
Nonetheless, the transaction externality is playing a key role, since the presence of on-net prices below off-net for the large network but a reverse outcome for the small one implies the existence of the blue line regions in Figure **2-3**. This means that in The Republic of Congo transaction externality effects are not negligible and also, that market is highly concentrated, which is an equivalence to the Colombian case.

2.5. Regulating price differential

I model the regulation of the on/off-net price differential as in Hoernig (2008), by assuming that off-net price equals on-net price plus a difference Δ: $p_{ij} = p_{ii} + \Delta$. I am interested in the setting for differential regulation on the prices for network 1, while network 2 can still price discriminate on-net and off-net transactions.

Proposition 6. *When price differential is regulated, retail uniform price set by network 1 will be as follows:*

$$p_{11} = \frac{c_{11} + \frac{L_{12}}{L_{11}}\frac{q'_{12}}{q'_{11}}\left\{c_{12} + \frac{\hat{L}_{12}}{L_{11}} - \Delta\left[1 - \frac{1}{\eta} - \frac{\hat{L}_{11}}{L_{12}}\gamma\right]\right\}}{1 - \frac{1}{\eta} + \frac{\hat{L}_{11}}{L_{11}}\frac{1+\gamma\eta}{\eta} + \frac{L_{12}}{L_{11}}\frac{q'_{12}}{q'_{11}}\left[1 - \frac{1}{\eta} - \frac{\hat{L}_{11}}{L_{12}}\gamma\right]}. \tag{2-15}$$

Proof. See section A.8 in the appendix. □

Results from Proposition 6 represent an implicit solution for the uniform price that network 1 would set if price differential is regulated. I am particularly interested in the case when the regulator bans the price discrimination for the larger network. In such a scenario, the uniform price set by the regulated network is stated in Corollary 6.1.

Corollary 6.1. *When price differential is regulated to zero* ($\Delta = 0$)*, uniform price set by the large network will be as follows:*

$$p_1^{\Delta} = \frac{\eta c_{11} L_{11}^2 + \eta c_{12} L_{11} L_{12} + \eta L_{12} \hat{L}_{12}}{L_{11} \left\{ (\eta - 1) \left[L_{11} + L_{12} \right] + \hat{L}_{11} \right\}}. \tag{2-16}$$

Proof. See section A.9 in the appendix. □

Price elasticity of demand shapes the behavior of uniform price in the expected way: highly elastic markets foster competition strength leading to overall lower prices, which can be easily checked in Equation 2-16. Regarding transaction externality my results show that, under a price discrimination ban, this parameter becomes irrelevant in the price setting strategy for the regulated firm. This resembles a previous result by Hoernig (2008). This is so because the marginal utility of the off-net transactions received by consumers in the small network -regarded as a cost- when the differential approaches zero, is the same as the marginal utility derived by consumers in the large network due to received transactions originated by consumers belonging to this same firm.
Meanwhile, circles involve an interesting implication for the regulated firm. Previous literature such as Hoernig et al. (2014) shows how the regulation under discussion could be beneficial for the market, but under some circumstances this could not be the case. In what follows I state a couple of results from my model that allow me to state how uniform price is set and how it compares to on/off-net discrimination prices.

Corollary 6.2. *When price differential is regulated to zero* ($\Delta = 0$)*, uniform price set by the large network is greater than on-net price set under price discrimination*

Proof. See section A.10 in the appendix. □

Corollary 6.3. *In markets with equally efficient firms and access charges set at cost, equilibrium uniform price of the large network is set above the off-net discrimination price when call externality is weak enough:*

$$\gamma < \gamma_{\Delta}(\hat{x}, \eta, \lambda, \varepsilon) = \frac{1}{\eta \hat{L}_{11}} \left[(\eta - 1) L_{12} + \hat{L}_{12} - \frac{c_{11} L_{11} L_{12} \left(\eta \hat{x} - \hat{L}_{12} \right)}{c_{11} \hat{x} L_{11} + L_{12} \hat{L}_{12}} \right] \tag{2-17}$$

Proof. See section A.11 in the appendix. □

The equilibrium uniform price set by the bigger network is higher than the on-net discriminatory price. This result means that under regulation consumers in this network would pay a higher fee for transactions whose destination belongs to the same network.
Conversely, the uniform price is set above the off-net discriminatory price if conditions on transaction externality and circle weight are met. Leader network will set a uniform price

above off-net price in markets where consumers display a low valuation for incoming transactions. For this type of markets, the firm is not highly concerned about utility derived by consumers of the rival network, which is a more evident position when the market also displays a consumer behavior towards making transactions to circles in a high proportion.

In the discriminating scenario, utility derived by consumers in the rival network turns more relevant and the off-net pricing will reflect this by a higher price level if transaction externality is high. In the regulated scenario the firm is not concerned about this, which means a uniform price can be set below discriminating off-net one. Under these conditions, circles turn out to be more relevant to consider: if consumers heavily place transactions to destinations in their circle, uniform price will reflect the firm's strategy to exploit transactions behavior of its own consumers.[17] This strategy is not longer in place if consumers interact weakly with other consumers in their circle, so that the firm adopts a different strategy by encouraging its consumers to make more transactions without concerning about derived utility of recipients in the rival network. This is the case when uniform price is set below off-net discriminating one.

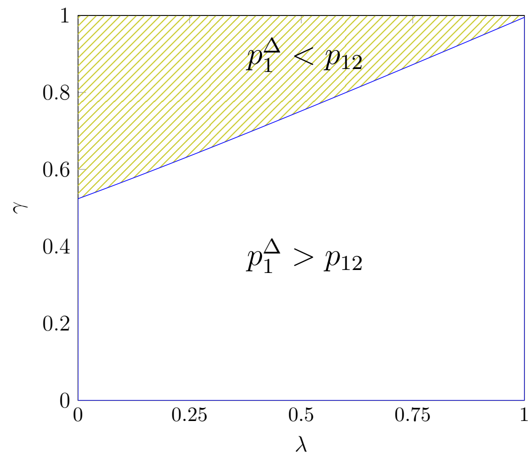
There is still a third force to consider. A concentrated market implies that consumers in the large network will more likely place transactions to destinations in the same network, allowing the firm to extract rents by fostering these transactions with lower prices. The more concentrated the market is, the lower will be the uniform price as long as circles are weak. In such a case, uniform price decrease in concentrated markets leads to a wider range of transaction externality such that this price is lower.

Therefore, in non-concentrated markets where consumers display a low valuation for incoming transactions and transactions are made towards destinations with a heavy weight to circles, regulation would lead consumers in the large network to face a higher price for making transactions towards destinations located in the small network. This is indeed an undesirable outcome, given that consumers are also facing a higher on-net price in any case. Regulating price differentials without considering important features in the market such as circles and transaction externalities can lead to outcomes such as the ones described. This means that in order to lay such a regulatory rule, accurate measurements about transaction externalities and circles strength should be available at hand. This is at odds to regulatory decision in Kenya to ban price discrimination for mobile money transactions or the policy recommendations raised to the CRC related to keep the price differential regulation active.[18]

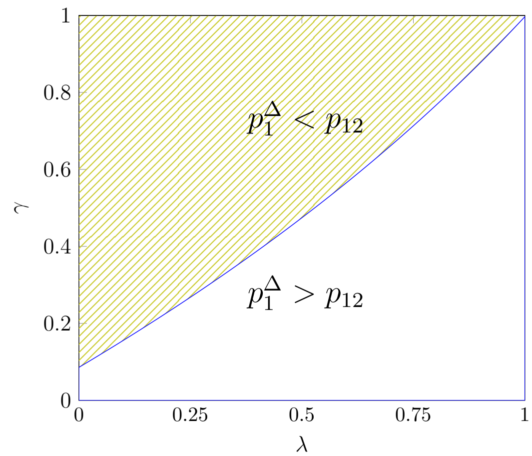
Figure **2-5** summarizes different regulatory scenarios depending on the structure of the market and the fundamental behavior of consumers. As stated formerly, more concentrated markets and consumers with lower valuation for incoming transactions as well as lighter circles will lead to regions where regulated uniform price will be lower than off-net discriminatory price.

[17] It can be proved that (2-16) approaches discriminating on-net pricing under these conditions and highly concentrated markets.

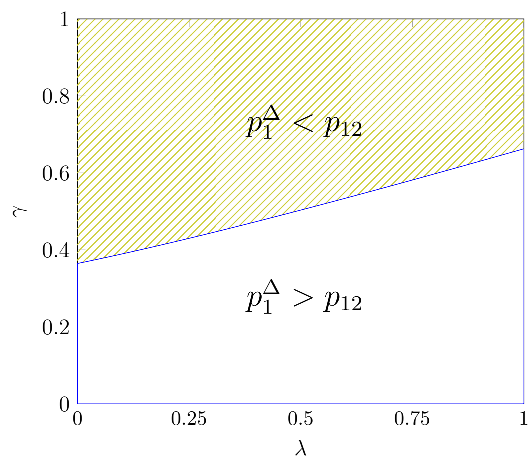
[18] See TMG (2016).



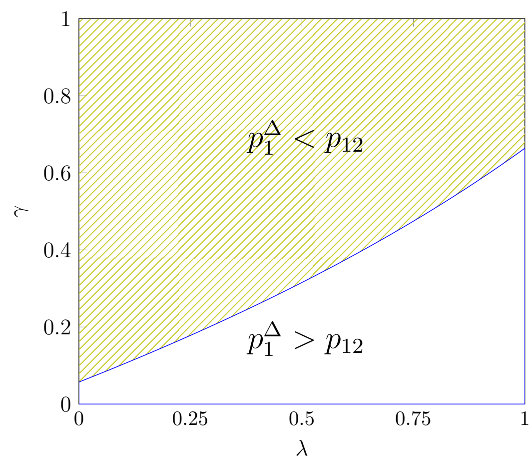
(a) $\hat{x} = 1/2$, $\eta = 1$.



(b) $\hat{x} = 3/4$, $\eta = 1$.



(c) $\hat{x} = 1/2$, $\eta = 1.5$.



(d) $\hat{x} = 3/4$, $\eta = 1.5$.

Figure **2-5**.: Regions for uniform price higher than discriminating off-net price level.

2.6. Conclusion

This paper analyzed how pricing behavior is shaped under a setting of network competition between two firms when both circles and transaction externality are considered. Traditional outcomes state that equilibrium off-net prices are set above on-net prices, however I propose a different approach to establish the reasons why that might be otherwise.

Transaction externality plays a role that counteracts circle effects for the on/off-net price decisions the firms must deal with. Existence of circles drives transaction demand towards on-net destinations in a more relevant way than just uniformly distributed demand, providing firms incentives to exert their power by increasing on-net prices as circle weight rises. On-net prices differential is driven by market shares and circle size. Transaction externality and price elasticity of demand do not play a role in this differential since on-net consumers derive their utility of transactions originated and terminated in the same network they belong to.

The transaction externality effect is relevant for off-net price differentials. Comparing off-net prices set by the firms, I found that transaction externality is shaping the incentives of the firms to extract surplus derived by consumers from received transactions and these incentives are strengthened as the transaction externality increases. Looking at on/off-net differentials, absent transaction externality, off-net prices would be set above on-net prices for any value of circle weight. However, high values of transaction externality could allow to obtain a reversed price differential.

These results are meaningful since they allow to explain how consumers' attributes -circles and transaction externality- can lead to non-traditional outcomes where the small firm in the market sets on-net price below off-net, while its rival acts the opposite. This type of outcome is expected to be observed in concentrated markets where consumers have a non zero valuation for receiving transactions and the proportion of transactions to circles is moderated. Besides, this model also allows outcomes in which both firms set on-net price above off-net, when consumers heavily place transactions to destinations in their cicles.

Finally, the model is extended to analyze how retail price regulation for the large network shapes the pricing stretegic behavior of the firm. The regulated price is always higher than the discriminating on-net price, while it can be higher or lower than discriminating off-net price. When markets are more concentrated and consumers display a moderate to high valuation for incoming transactions together with moderate probability of making transactions to destinations in their circle, the uniform price will be lower than off-net price set under discrimination. This means that price regulation should be avoided in markets that do not display these features, since consumers will always face higher prices for transactions to any destination.

3. Introduction Delay as a Pre-emptive Strategy in Product Innovation

Guillem Roig [1]
Renzo Clavijo[2]

3.1. Introduction

In innovation races, firms delay the introduction of finished new products into the market. For instance, in the middle 1980s, the innovation race in game consoles was centered around the consoles' performance, encouraging the firms to improve the image and sound quality of the devices.[3] In 1989, Sega released the Genesis system -a 16-bit video processor- to beat Nintendo's dominant position. By Genesis's introduction, Nintendo had already undertaken R&D efforts to create a 16-bit console but delayed the release (Schilling, 2003). Another example is the Bank of America (BoA) decision in 1988 to abandon, after 5 years of work, the efforts to create a new technological platform from trust accounts. The company Premier Systems (PS) was in charge of developing the platform for BoA. PS was an entrant competitor challenging the incumbent SEI Corp. After the exit of PS, SEI began to handle trust accounts formerly belonging to BoA.[4] SEI delayed the introduction of his product for BoA, waiting for PS to abandon the efforts on software development, Frantz (1988).

This article develops a model that explicitly studies the firm's decision to delay the introduction of a new product in the market. Some interesting questions arise from the previous examples in which firms delayed the introduction of their innovations. Why would firms strategically delay the release of a profitable product during an innovation race? Does such delay depend on the characteristics of the firms involved in the innovation race? From the public policy perspective, what are the inefficiencies, if any, from keeping finished new products unmarketed?

To answer these questions, we consider an innovation race between an incumbent and a potential entrant. In the model, firms face uncertainty regarding the market profitability

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[3]Pereira (1990) describes how 16-bit consoles provided ten times more colors than Nintendo's 8-bit console.

[4]The abandonment decision of BoA was followed by a transfer of managed funds related to trust accounts to other companies such as Wells Fargo and Seattle First National Bank.

of the new product. When a firm completes the innovation process, it privately learns the profitability of the market. However, a firm does not observe the research strategy from the opponent, if it has completed or abandoned research, nor the private outcomes, the profitability of the market that resulted from research. A firm only observes when the opponent has introduced the new product in the market.

In our model, product introduction decisions may convey information about market profitability to the opponent. To see this, consider a situation in which a firm will only introduce the product if the market for the product innovation is highly profitable. Therefore, the introduction of the new product indicates the rival the market is highly profitable. Similarly, the opponent learns from the lack of product introduction. No product introduction could represent two exclusive events: i) the firm did not finish research, or ii) the firm finished research and learned a low profitability market. Over time, the opponent becomes more pessimistic about the prospects of the market. This learning by the opponent explains why a firm, when it finishes research and discovers the market to be highly profitable, delays product introduction. Waiting is beneficial because as long as no product is introduced, the opponent becomes more pessimistic and eventually drops out of the race.

The strategy to delay product introduction does not apply to all firms in the market. A potential entrant introduces the new product innovation immediately regardless of market profitability. As a result, the incumbent never learns from the entrant's product introduction or the lack of it. The incumbent, however, obtains profits from a pre-existing market and decides not to introduce the new product if the market profitability is small. The potential entrant becomes pessimistic over time, which incentivizes the incumbent to delay introduction.

In the model, the product introduction strategy of the incumbent is characterized as follows. If the incumbent finishes research early enough, it introduces the product immediately. Waiting for product introduction until the entrant abandons would generate a loss in foregone profits that would dominate the expected reduction of profits from future competition. If discovery occurs later, the incumbent has incentives to wait for product introduction as the entrant is closer to abandoning research. The expected profits of becoming a monopoly in the market innovation outweigh the loss of foregone profits from waiting. The waiting of the incumbent means that it introduces the product with positive probability. In equilibrium, the probability to introduce the product is defined such that the incumbent is indifferent between introducing the product immediately or waiting until the entrant abandons research. Finally, in a late discovery (once the entrant has abandoned research), the incumbent introduces the product immediately. These results emerge whenever the players sufficiently differ in their research technology.

When research technologies become similar, the potential entrant is expected to introduce the new product nearly as fast as the incumbent. In this case, the strategy to delay product introduction becomes very costly, as it becomes unlikely that the entrant does not complete innovation before its exit time. Then, the incumbent decides to introduce the new product once it completes research. The incumbent prefers to replace itself than be replaced by a potential entrant.

When strategic delays occurs in equilibrium, the market outcome presents three types of inefficiencies: duplication of R&D costs, dampened product introduction pace, and early exit from research. The duplication of R&D costs takes place because the entrant devotes resources to achieve a discovery, while the incumbent is just waiting with its own discovery at hand. This unmarketed product involves a slower rate of innovations available. Even if the invention is completed, the innovator and the consumers do not benefit from this new product. Early exit results from becoming pessimistic about market profitability which implies less market competition.

Those market inefficiencies raise interesting policy interventions. A competition authority should incentivize technology adoption by a potential entrant. If this is not possible, other approaches may contemplate the creation of a research joint venture (RJV) in which information from research outcomes is shared, since inefficiency arises from the lack of information transmission about market profitability. We explore this possibility and show that strategic delay does not take place. Meanwhile, the incumbent may still have incentives to either over-invest or under-invest in product innovation.

The differences in innovation decisions between an incumbent and a potential entrant relate our work to an old research question that has explored the relationship between incumbency and the incentives to innovate. In the seminal articles of Dasgupta and Stiglitz (1980) and Gilbert and Newbery (1982), an incumbent has incentives to undertake R&D activities to engage in preemptive patenting and, consequently, keep its monopolized position. In those models, the patent system creates opportunities for firms to maintain their monopoly power by making "sleeping patents". Opposite to this result, Reinganum (1983) finds that an incumbent researches less for product innovations than a potential entrant. In her model, an incumbent derives a stream of profits from an existing product, and the introduction of a product innovation jeopardizes such profits. The Schumpeterian "replacement effect" gives the incumbent lower incentives to develop and commercialize new products. We extend Reinganum's model by adding market uncertainty and private research outcomes. When a firm completes the innovation, it privately learns the market profitability; hence, its decision to introduce the product may provide information about market profitability to the rival. In our model, the incumbent may have a lower incentive to innovate due to such involuntary information transmission.

The involuntary information transmission relates our work to the literature in which information from research spills over to the rival. In Choi (1991) and Malueg and Tsutsui (1997), when information arrives, firms learn the feasibility of the innovation process. The good news of project feasibility encourages the opponent to continue with research. Contrary to this result, Dasgupta and Stiglitz (1980), Fudenberg et al. (1983), Harris and Vickers (1985), and Lippman and McCardle (1988) information always hurts the rival. News from the competitor reveals that it is ahead in the race, and the lagging firms decide to exit. A similar result is found in Gill (2008) and Jansen (2010). Adding to these papers, involuntary information transmission through product introduction has two countervailing effects in our model. First, I may indicate to the rival that the market for the new product is highly profitable (good news), but it also indicates that if the rival continues with research, the most it can hope for are duopoly profits (bad news). If the good news dominates, the rival continues with research, lowering the expected profits from the firm that introduced the new product. If the opposite happens, the leading firm obtains monopoly profits in the new market due to the exit from the rival.

Our work also relates to the literature considering an innovation race with learning. In Choi (1991), firms are uncertain about the arrival rate of innovation, and as time passes without success, they become more pessimistic about future innovation and decide to abandon the race. More closely related to ours is how firms' actions change the competitor learning process. Bag and Dasgupta (1995) extend Choi's work by allowing firms to strategically announce the information they receive after completion of the first research stage. In their model, the decision to disclose depends on the time of discovery, which affects firms' learning process. In Erkal and Roig (2019), firms' learning about project feasibility is affected by the equilibrium decision to either disclose or keep intermediate research outcomes secret. Although the evolution of beliefs in our setting is similar to the previous works, our work departs from them because information is transmitted through rival's product release, rather than the firms' position in the race or by strategic information disclosure.

In a more recent paper, Akcigit and Liu (2015) consider a model with two research projects (one risky and one safe). They show that the firm's decision not to disclose bad results makes firms become pessimistic about project feasibility as long as no information arrives. Over time, if no information arrives, firms become pessimistic and abandon research. A similar result is found in Erkal and Roig (2019), but the depression of beliefs about project feasibility accelerates when firms disclose intermediate research outcomes rather than when they keep it secret. In our model, firms become pessimistic when the rival does not introduce the product innovation, as it indicates that the new product's market may be of low profitability. As a result, the incumbent may strategically delay product introduction to induce pessimism about market profitability and pre-empt a potential entrant.[5]

Finally, the firm's product introduction delay contributes to the literature that considers firms' strategy to delay their actions to change a competitor's behavior. Michael L. and Carl (1987) study two firms competing in an innovation race in which firms benefit from competitors' innovation by imitating the product. Imitation gives incentives to firms to delay research, turning the innovation race into a waiting game. In Weeds (2002), firms delay investment in R&D to avoid a potential innovation race. Closer to our setting is Banerjee and Sarvary (2009). They develop a model in which an incumbent and an entrant engage in an innovation race and decide their investment levels and product introduction

[5]Our pre-emptive strategy differs from Guéron and Lee (2022), where the incumbent might deter competition by pursuing more R&D efforts to improve the product.

time. In their model, firms delay product introduction due to the potential diffusion growth for their existing product. Different from theirs, we do not consider technological spillovers or product imitation. In our model, product introduction delay emerges to avoid the involuntary information transmission about market profitability and make the rival more pessimistic about market profitability and, eventually, abandon the race.
The outline of the article is as follows. Section 3.2 presents the model. We start with a benchmark in Section 3.3, in which only the incumbent can research for product innovation. Section 3.4 considers the full-fledged innovation race between an incumbent and a potential entrant. Later in Section 3.5, we discuss the effects of making information about research outcomes public, and Section 3.6 concludes. For ease of exposition, all the proofs are in the Appendix.

3.2. Model set-up

We consider an innovation environment where two firms, an incumbent (I) and a potential entrant (E), compete to develop a product innovation. Research comprises a single stage, after which a firm can decide to commercialize the product resulting from it. We consider an environment in which there is uncertainty in the market profitability for the product innovation. Firms share a common prior p_0 that the new product will generate large profits, and with probability $(1-p_0)$ the new product generates low profits. Over time, firms may update their beliefs about market profitability, and we assume that a firm privately learns market profitability after research is completed but before the product is finally released. This timing may occur because the innovator may still have to perform cross-checks validations regarding expected benefits once the innovation process is completed. The product may be given to consumer focus groups for feedback, and the firm learns the future profitability of the new product.[6]
The innovation for the new product follows a Poisson discovery process. Time is continuous, and firms have a common discount rate r. The firms competing for product innovation are asymmetric. We assume that the incumbent has better research technology than the entrant and expects to finish research for the new product earlier. For example, an incumbent may be better suited to research than a potential entrant because it can exploit its customer knowledge, customer franchise, and market power to lead innovative processes that create product innovation (see Bauer, 1960, Folkes, 1988, and Gregan-Paxton and John, 1997). We model this by a larger hazard rate of the rival with respect to the entrant, $\lambda^I > \lambda^E$. Following Lee and Wilde (1980), we consider that firm i incurs a flow cost of research and assume that better research technology is also associated with higher flow costs from research $c\lambda^I > c\lambda^E$; a better research team may have higher salary costs.

[6]See Cooper (2011) for further insights about product release cycles and simulated tests market to predict expected market sales for product innovation.

At each point in time, a firm decides whether or not to research for product innovation. When it completes research, the firm can choose whether to launch the product or when to do it. A firm starts to earn profits from the innovation only after commercializing the product. For a firm that has not completed research for the product innovation, a decision not to pay the flow cost is assumed to be irreversible and equivalent to dropping out of the game. Also, a firm that has abandoned research can not resume it in the future.

3.2.1. Information structure

We consider a research environment with private learning (i.e., research outcomes are not observable) and private actions (i.e., firms' investment or exit decisions are not observable). Research outcomes and the firm's research activity are private information. Specifically, after a firm completes the innovation process, the market profitability from such innovation is private information. Also, a firm does not observe when the rival completes or abandons research; it only observes the rival's product introduction in the market.

The assumption of private learning about market profitability raises the question of whether a firm would like to introduce the product in the market. The introduction of the product or the lack of it may provide information about the product's profitability. Then, we denote by p_t^i the firm i's belief at time t that the product innovation will generate high profits.

3.2.2. Product market competition

We assume competition in the product market. Hence, the first innovator for the new product can not exclude the opponent from commercializing its version of the product. The impossibility to exclude means that even if the first innovator patents the new product, the laggard has no cost to invent around the patent.

Once the innovation is completed and the firm introduces the product, it begins to obtain flow profits. The flow profit depends on market profitability and on how many firms are active in the market. If the profitability for the new product is high, a firm obtains flow profits $\bar{R}$ if it is the only one commercializing the product. The flow profits are $\bar{D} \in \left(0, \bar{R}/2\right)$ if two firms commercialize it. In the case that the new product generates low market profitability, a single firm commercializing the product obtains $\underline{R} > 0$. No profits are generated with two active firms in the market, $\underline{D} = 0$.

In addition to the market for the new product, the incumbent is active in an already existing market. The incumbent obtains flow profits R in this existing market as long as the new product is not introduced. We assume that the existing and the potential new market are related. We model this relation by considering that when the new product is introduced, the incumbent loses a proportion of consumers $\gamma \in (0, 1]$ in its existing market. We assume this proportion to be invariant of the firm's identity introducing the new product and the market profitability for the new product. As a result, once the new product is introduced,

the flow profits for the incumbent in its existing market become $(1-\gamma)R$.[7]
To avoid a taxonomy of uninteresting equilibria, we make the following assumptions.

Assumption 2. *i) Monopoly flow profits are such that:* $R > c \geq \underline{R}/r > 0$, $\bar{R} - R \geq c$, *and* $\bar{R}\underline{R} > R^2$.
ii) The prior of high profitability is large enough $p_0 > rc/(\bar{R} - R)$, *and the firms' discount factor is small enough,* $r < (\bar{R} - R)/(\bar{R} - \underline{R})$.
iii) The entrant's arrival rate is $\lambda^E \in \big(0, (r(\bar{R} - R) + R)/R\big]$.

Point (i) in the assumption states that the research in the new product is not worth undertaking if market profitability is low. Point (ii) ensures that if the incumbent is the only firm undertaking product innovation, it always innovates. This assumption is important, because we want to study if with competition, the incumbent changes its decisions to innovate. Point (iii) delimits the entrant's research efficiency such that we do not have undesirable equilibria.[8]

3.2.3. Equilibrium concept

We solve for a Perfect Bayesian Nash equilibrium. An equilibrium is defined by a strategy profile and a belief system for each firm such that the expected payoff for each firm is maximized given the strategy profile and belief system of the rival. The strategy profile specifies at each $t > 0$ a research activity and a decision to introduce the product. Several options define the strategy of a firm: the firm can either continue to invest or exit the game with zero payoffs. When research is completed, it can decide whether to introduce the product or not. The belief system specifies at each $t > 0$ the probability of the market for the new product being highly profitable and the probability that the rival introduces the product for each market state.
We assume that when a firm is indifferent between introducing the new product or not, it decides not to introduce the product due to some small costs associated to it.

3.3. Only the incumbent innovates in a new product

To understand the effects of competition in product innovation, we first study the incumbent's incentives to research for product innovation without the threat of a potential entrant. We start by eliciting the incumbent's beliefs about the market profitability for the product innovation. The incumbent learns the profitability of the new product when it completes the innovation and gives the product to a focus group of consumers to test it. During the

[7] The parameter γ can be interpreted as a measure of how radical the new product innovation is. More radical innovations are more disruptive and are associated with a larger γ.
[8] For larger values of the entrant arrival rate, there may be equilibria in which firms introduce the product when the market profitability is small, but not when it is high.

innovation process, the incumbent does not update its beliefs about market profitability for the new product. No belief updating is because the arrival of information does not depend on the profitability of the new product; hence, the incumbent always believes the new product to be highly profitable with probability p_0.

With the incumbents beliefs about market profitability, when deciding whether or not to innovate, it must take into account the profits it gets in its existing market. The incumbent obtains flow profits of R in its existing product from which it loses a proportion of $\gamma \in (0,1]$ when it introduces the product innovation. In the market for the product innovation, the incumbent obtains flow profits $\bar{R}$, if highly profitable, and flow profits $\underline{R}$ if the profitability is low.

Conditional on having completed the innovation, the incumbent always introduces the product innovation when the market is highly profitable. Introducing the product gives the incumbent flow profits $\bar{R} + (1-\gamma)R$. If the product is not introduced the incumbent gets the flow profits of its existing market, R. For any loss of consumers in the existing market, represented by γ, we get $\bar{R}+(1-\gamma)R > R \iff \bar{R} > \gamma R$. If the market for the new product turns out to be of low profitability, the incumbent's product introduction decisions depends on loss of consumers in its existing market. Because the flow profits in the new market are below those in the existing market, $\underline{R} < R$, the incumbent introduces the product when it can still obtain significant profits in its existing market. If the loss in the existing market is large, (high γ), the incumbent prefers to keep all the consumers of its existing market and does not introduce the product innovation. Then, the incumbent introduces the new product when

$$\underline{R} + (1-\gamma)R > R \iff \gamma < \underline{R}/R := \gamma^M. \tag{3-1}$$

The ratio between the flow profits in the new market with that of the existing market establishes the proportion of consumers the incumbent can afford to lose in its existing market and still finds it profitable to introduce the new profit in a market with low profitability.

With the incumbent's product introduction decision, the incumbent's expected profits from undertaking product innovation can be expressed with the recursive continuous-time Bellman equation

$$W^M = -c\lambda^I dt + e^{-rdt}\left[\lambda^I dt\left(p_0\bar{R}/r + (1-p_0)\underline{R}/r + (1-\gamma)R/r\right)\right.$$
$$\left. +(1-\lambda^I dt)\left(Rdt + W^M\right)\right], \tag{3-2}$$

whenever the incumbent introduces the product innovation regardless of market profitability, ($\gamma < \gamma^M$), and by equation

$$W^M = -c\lambda^I dt + e^{-rdt}\left[\lambda^I dt\left(p_0\left(\bar{R}/r + (1-\lambda)R/r\right) + (1-p_0)R/r\right)\right.$$
$$\left. +(1-\lambda^I dt)\left(Rdt + W^M\right)\right], \tag{3-3}$$

when the incumbent only introduces the new product in a highly profitable market ($\gamma \geq \gamma^M$). In both equations, the first term on the right-hand side is the flow costs associated with research. The second term is the discounted expected return from arrival in dt. The third term is the discounted expected continuation payoff.
Proposition 1 states the incumbent's research decision and its product introduction choice.

Proposition 1. *Suppose the incumbent is the only firm undertaking product innovation. The incumbent always researches for product innovation. Consider γ^M as defined in* (3-1). *For $\gamma < \gamma^M$, the incumbent introduces the product innovation regardless of market profitability. For $\gamma \geq \gamma^M$, the incumbent introduces the product innovation only when the market is highly profitable.*

The proposition states that the incumbent researches for a product innovation even if it decides not to commercialize due to the loss generated to its existing market. In our model, the incumbent chooses to innovate to learn the market profitability for the new product. Hence, it can select after it learns market profitability, whether to introduce the product. This increased flexibility explains why our result contrasts with other models in which the replacement effect makes the incumbent unwilling to innovate (see Gilbert and Newbery, 1982; Reinganum, 1983).

3.4. Innovation with a potential entrant

This section studies a product innovation race between an incumbent and a potential entrant. We will see that the firms' asymmetric position in the market and the differences in research technology will be crucial to determine the equilibrium research, product introduction decisions, and firms' learning process about market profitability. We assume that market profitability for the new product is private information, but a firm can still infer market profitability by the introduction decision of the rival. We relax our assumption about private information of market profitability in Section 3.5.
We start by characterizing the firms' beliefs about market profitability since their equilibrium research strategy depends on their beliefs about market profitability and the rival's product introduction decisions.

3.4.1. Learning with private information about market profitability

In the model, research outcomes are private information. A firm does not observe market profitability when the rival finishes the innovation. Yet, a firm may learn about market profitability from the rival's product introduction decision. We are interested in characterizing the evolution of firms' beliefs about a highly profitable market for the new product. We will show that firms' beliefs depend on the firms' product introduction decisions after completing the innovation process.

Consider the belief p_t^i that firm i holds at t that the product innovation will generate high profits given no information has arrived up to time t. Take first an equilibrium in which firms introduce the new product regardless of market profitability. Then, if at any time $t=\tau$ the rival introduces the product, the firm does not learn market profitability, $p_\tau^{i,IN}=p_0$. We use the superscript IN to denote that firms introduce the new product regardless of market profitability. No learning occurs because product introduction is not indicative of market profitability. Now, consider that no firm has introduced the new product up to time t, and suppose further that the new product is not introduced from time t to $t+dt$. Then, firms do not update their beliefs about market profitability, as illustrated by Bayes' rule:

$$p_{t+dt}^{i,IN}=\frac{p_t^{i,IN}(1-\lambda^i dt)(1-\lambda^{-i}dt)}{p_t^{i,IN}(1-\lambda^i dt)(1-\lambda^{-i}dt)+(1-p_t^{i,IN})(1-\lambda^i dt)(1-\lambda^{-i}dt)}=p_t^{i,IN}=p_0.$$

In the numerator, conditional on the market for the new product being highly profitable, no firm introduces the product. The denominator represents the probability of no product introduction. The absence of product introduction means that none of the firms have completed the innovation because firms would have introduced the product if they had. This occurs with instantaneous probability $(1-\lambda^i dt)(1-\lambda^{-i}dt)$. As we have obtained in the monopoly case, the innovation process alone does not provide information about the profitability of the new product.

Now consider an equilibrium in which firms only introduce the product innovation when the market is highly profitable. As we will show, firms' beliefs about a highly profitable market become more pessimistic as time passes without any product introduction. This is so because two events may have occurred if no product had been introduced. Either the rival did not finish the innovation, or the rival completed the innovation but realized low profitability and decided not to introduce the new product in the market. Then, assume that as firms get more pessimistic, they abandon the innovation at time T^{INH} if no firm has introduced the new product by then. We use the superscript INH to denote that firms only introduce the product in a highly profitable market. Now suppose that the rival completes research at time $\tau<T^{INH}$ and introduces the product. After the rival introduces the product, the firm learns that the market is highly profitable, $p_\tau^{i,INH}=1$. Now consider that no product has been introduced up to time t and for an additional time dt no product is introduced. Then, firm i's belief about a highly profitable market is:

$$\begin{aligned}p_{t+dt}^{i,INH}&=\frac{p_t^{i,INH}(1-\lambda^i dt)(1-\lambda^{-i}dt)}{p_t^{i,INH}(1-\lambda^i dt)(1-\lambda^{-i}dt)+(1-p_t^{i,INH})(1-\lambda^i dt)}\\&=\frac{p_t^{i,INH}(1-\lambda^{-i}dt)}{p_t^{i,INH}(1-\lambda^{-i}dt)+(1-p_t^{i,INH})}.\end{aligned}$$

The numerator represents the probability that, conditional on the market being highly profitable, none of the firms completes the innovation. The denominator represents the probability of no product introduction. Again, firms' i learning about market profitability does

not depend on its innovation process, but now, it depends on the arrival rate of the rival λ^{-i}.
To see this, subtracting $p_t^{i,INH}$ from both sides and dividing by $dt \approx 0$ gives the law of motion

$$\dot{p}_t^{i,INH} = -p_t^{i,INH}(1 - p_t^{i,INH})\lambda^{-i}, \tag{3-4}$$

and the larger the arrival rate of the rival λ^{-i} becomes, the faster firm i learns about the market profitably absent product introduction.
Finally, consider a candidate equilibrium in which firms do not introduce the new product innovation in a market with low profitability. Still, they do in a high profitability market with probability $\alpha \in (0,1)$. Similar to the case in which the product is introduced in a highly profitable market with certainty, we will show that firms become pessimistic about high market profitability and abandon research at time $T^{INH\alpha}$ if no product has been introduced. We use the superscript $INH\alpha$ to denote that firms only introduce the product in a highly profitable market with probability α. Suppose the rival completes innovation and introduces the new product at time $\tau < T^{INH\alpha}$. As before, the firm learns the market is highly profitable, $p_\tau^{i,INH\alpha} = 1$. Now, consider the case in which no product has been introduced up to time t, and it is not introduced for an extra time dt. In this candidate equilibrium, conditional on the market for the new product being highly profitable, the rival does not introduce the product if i) it has not yet completed innovation, which happens with instantaneous probability $(1 - \lambda^{-i}dt)$; or ii) it completed innovation but did not introduce the product, which occurs with instantaneous probability $(1 - \alpha^{-i})\lambda^{-i}dt$. The sum of both probabilities is then the probability of not observing a product introduction of the rival, i.e., $(1 - \alpha^{-i}\lambda^{-i}dt)$, and applying Bayes' rule, we obtain:

$$\begin{aligned} p_{t+dt}^{i,INH\alpha} &= \frac{p_t^{i,INH\alpha}(1 - \lambda^i dt)(1 - \alpha^{-i}\lambda^{-i}dt)}{p_t^{i,INH\alpha}(1 - \lambda^i dt)(1 - \alpha^{-i}\lambda^{-i}dt) + (1 - p_t^{i,INH\alpha})(1 - \lambda^i dt)} \\ &= \frac{p_t^{i,INH\alpha}(1 - \alpha^{-i}\lambda^{-i}dt)}{p_t^{i,INH\alpha}(1 - \alpha^{-i}\lambda^{-i}dt) + (1 - p_t^{i,INH\alpha})}. \end{aligned}$$

From the previous expression, the law of motion becomes:

$$\dot{p}_t^{i,INH\alpha} = -p_t^{i,INH\alpha}(1 - p_t^{i,INH\alpha})\alpha^{-i}\lambda^{-i}. \tag{3-5}$$

Comparing this expression with the law of motion when firms introduce the product with centrality in (3-4), we observe that the parameter λ^{-i} is multiplied by the probability α^{-i}. Because a firm receives information from the rival only when the rival introduces the product, the firm's learning depends on the probability that the rival introduces the product.
Lemma 1 summarizes the previous results and introduces the explicit form of the beliefs.

Lemma 1. *With two competing firms, if no product has been introduced up to time t, a firm's beliefs can be characterized as follows:*

i) Suppose the rival always introduces the product. Then, at any time t, $p_t^{i,IN} = p_0$.
ii) Suppose the rival only introduces the product in a high profitable market. Then, at any time t,

$$p_t^{i,INH} = \frac{p_0}{p_0 + (1-p_0)e^{\lambda^{-i}t}}. \tag{3-6}$$

iii) Suppose the rival only introduces the product in a high profitable market with probability $\alpha^{-i} \in (0,1)$. *Then, at any time t,*

$$p_t^{i,INH\alpha} = \frac{p_0}{p_0 + (1-p_0)e^{\alpha^{-i}\lambda^{-i}t}}. \tag{3-7}$$

iv) Firms become pessimistic about project feasibility: $\partial p_t^{i,j}/\partial t < 0$ *and* $\partial p_t^{i,j}/\partial \lambda^{-i} < 0$ *for* $j = INH, INH\alpha$. *Also,* $\partial p_t^{i,INH\alpha}/\partial \alpha^{-i} < 0$.

Some remarks in Lemma 1 are in order. When the rival only introduces the product in a highly profitable market, as time passes without any product introduction, the firm becomes pessimistic about market profitability. Also, the rate at which the firm becomes pessimistic increases with the arrival rate of the rival λ^{-i}. As time passes with no product introduction, the firm gives more weight to the event that the rival has completed the innovation but realized that the market for the new product is of low profitability, and hence, it decided not to introduce the product. The firm is convinced faster of this event when the rival's arrival of information is large. Learning from the rival's experimentation is also obtained in Akcigit and Liu (2015). In their model, the learning results from the equilibrium information disclosure rule, while in our model, learning comes from observing the rival's product introduction decisions.

Finally, in the equilibrium where the product is introduced with probability α, the rate at which the firm becomes more pessimistic increases with α. When the rival introduces the product with a larger probability, the lack of product introduction is more indicative that the market for the new product may be of low profitability. Moreover, if the probability in which the rival introduces the product tends to 0, we obtain $lim_{\alpha \to 0}[p_t^{i,INH\alpha}] = p_t^{i,IN} = p_0$, and the firm does not update its beliefs about market profitability. When the probability of introducing the product tends to 1, we get $lim_{\alpha \to 1}[p_t^{i,INH\alpha}] = p_t^{i,INH}$.

3.4.2. Research strategy with and without product introduction

In our model, when a firm introduces the product innovation, it provides two pieces of information to the rival. First, the introduction of the new product may indicate that the market is highly profitable (good news for the rival). Second, it also shows that the firm is already commercializing the product, and the most that the rival can hope for are duopoly profits (bad news for the rival). Then, one question that emerges is whether the rival should continue with research or abandon it. The decision will depend on the trade-off between the good and the bad news and the expected profits under duopoly.

We begin by shutting down the good news and consider an equilibrium in which firms do not learn a highly profitable market after product introduction. Remember that this will be the case when firms introduce the new product regardless of the market profitability. Then, the expected profit from continuing with research is

$$V_F^i = -c\lambda^i dt + e^{-rdt}\left[\lambda^i dt\left(p_0\bar{D}/r\right) + (1-\lambda^i dt)V_F^i\right]. \tag{3-8}$$

The under-script F stands for the follower as the new product has been introduced by the other firm. The first term in the right-hand side is the flow with research. The second term is the discounted expected return form completing the innovation in dt. The third term is the discounted expected continuation profit. By taking the limit $dt \to 0$ and using $e^{-rdt} = (1 - rdt)$, the solution of (3-8) is $V_F^i = \lambda^i\left(p_0\bar{D}/r - c\right)/(r+\lambda^i)$, and the firm continues with research if

$$V_F^i = \lambda^i\left(p_0\bar{D}/r - c\right)/\left(r+\lambda^i\right) > 0 \iff \bar{D} > rc/p_0 := \hat{D}. \tag{3-9}$$

Otherwise, it abandons.

Now consider an equilibrium in which the follower learns that the market is highly profitable after product introduction. Suppose further that the product is introduced before any firms abandon research, $\tau < T$. Then, by continuing with research, the follower gets $V_F^i = \lambda^i\left(\bar{D}/r - c\right)/(r+\lambda^i)$. The difference from the previous case, we have $p_\tau^i = 1$ instead of p_0. Then, the follower continues with research when

$$V_F^i = \lambda^i\left(\bar{D}/r - c\right)/\left(r+\lambda^i\right) > 0 \iff \bar{D} > rc := \tilde{D}. \tag{3-10}$$

Otherwise it abandons.

Lemma 2, summarizes the firms' research decisions after the rival has introduced the new product.

Lemma 2. *Suppose a firm has introduced the the new product. Then:*
i) In an equilibrium in which product introduction does not give information about market profitability. For $\bar{D} > \hat{D}$, where $\hat{D}$ as defined in (3-9)*, the follower continues with research. For $\bar{D} \leq \hat{D}$, the follower exists.*
ii) In an equilibrium in which product introduction indicates that the market is highly profitable. For $\bar{D} > \tilde{D}$, where $\tilde{D}$ as defined in (3-10)*, the follower continues with research. For $\bar{D} \leq \tilde{D}$, the follower exists.*

Note that the decision to either abandon or continue with research after the rival has introduced the product is the same for each type of firm. The difference between the incumbent and the entrant is the former's flow profit in its existing market. However, when the entrant has introduced the new product, the incumbent obtains flow profits of $(1-\gamma)R$ in its existing market regardless of its decisions to either continue or abandon research for the new product. Then, those profits do not affect the incumbents' research decisions.

Now consider a situation in which no firm has yet introduced the product. In an equilibrium in which firms' beliefs about market profitability do not evolve absent product introduction, i.e., when firms introduce the product regardless of market profitability (see Lemma 1), no firm ever abandons research before the product is introduced. Once the product is introduced, the research strategy is represented in Lemma 2.

Now consider an equilibrium in which firms' beliefs about market profitability evolve as long as the new product has not been introduced. Remember that this will happen whenever firms only introduce the new product when the market is highly profitable. We have shown in Lemma 1 that firms become pessimistic about market profitability absent product introduction, and consequently, may abandon research. The exit will occur when the cost to continue is above the expected profit, and a firm will abandon when it becomes sufficiently convinced that the market is of low profitability.

For a firm to abandon research at exactly t, it must be indifferent between leaving at time t or researching for an extra time dt and then abandon. A firms' payoff for "exploring for another time dt and then abandon" is:

$$(1-rdt)\left[\lambda^i dt\left(p_t^{i,INH}V_{LH}^i+(1-p_t^{i,INH})V_{LL}^i\right)\lambda^{-i}dt\left(p_t^{i,INH}V_F^i\right)\right]-c\lambda^i dt. \tag{3-11}$$

With instantaneous probability $\lambda^i dt$, the firm completes research. When the market is highly profitable, with probability $p_t^{i,INH}$, it obtains profits V_{LH}^i - the under-script LH stands for being the first firm to finish research and when and the market is highly profitable. The profit V_{LH}^i, depends on the identity of the firm $I = I, E$, and whether the rival either continues or abandons research, after the new product is introduced as stated in Lemma 2. In a low profitability market, with probability $(1-p_t^{i,INH})$, the firm obtains profits V_{LL}^i- the under-script LL stands for being the first firm to finish research, and the market is of low profitability. If the rival finishes research (with instantaneous probability $\lambda^{-i}dt$), the firm that is still researching will decide as stated in Lemma 2 giving an expected profit of V_F^i. Finally, if no firm completes the innovation (with probability $(1-\lambda^i dt)(1-\lambda^{-i}dt)$, the firm abandons in the next period and obtains no profits; the firm would have become more pessimistic by then and abandoning research will be the optimal choice. The instantaneous cost of research is $c\lambda^i dt$.

Taking the limit $dt \to 0$, the indifference condition at t between researching for an extra time dt or abandoning becomes $\lambda^i\left(p_t^{i,INH}V_{LH}^i+(1-p_t^{i,INH})V_{LL}^i\right)+\lambda^{-i}\left(p_t^{i,INH}V_F^i\right)=c\lambda^i$, and the abandoning time $T^{i,INH}$ is characterized by

$$\lambda^i\left(p_{T^{i,INH}}^{i,INH}V_{LH}^i+(1-p_{T^{i,INH}}^{i,INH})V_{LL}^i\right)+\lambda^{-i}\left(p_{T^{i,INH}}^{i,INH}V_F^i\right)=\lambda^i c. \tag{3-12}$$

A similar condition is obtained when firms introduce the product only in a highly profitable market with probability $\alpha \in (0,1)$. But substituting $p^{i,INH}$ by $p^{i,INH\alpha}$ in expression (3-12). Lemma 3 characterizes the firms' research strategies if no product has been introduced.

Lemma 3. *Suppose no firm has introduced the new product in the market. Then:*
i) In an equilibrium in which product introduction does not indicate market profitability, no firm ever abandons the research before a product has been introduced.
ii) In an equilibrium in which product introduction indicates that the market is highly profitable. If no product has been introduced, firm i abandons research at time:

$$T^{i,INH} = \frac{1}{\lambda^{-i}} ln \left[\frac{p_0}{1-p_0} \left(\frac{\lambda^i (V^i_{LH} - c) + \lambda^{-i} V^i_F}{\lambda^i (c - V^i_{LL})} \right) \right], \tag{3-13}$$

if firms always introduce the product in a highly profitable market, and at time

$$T^{i,INH\alpha} = \frac{1}{\alpha^{-i}\lambda^{-i}} ln \left[\frac{p_0}{1-p_0} \left(\frac{\lambda^i (V^i_{LH} - c) + \lambda^{-i} V^i_F}{\lambda^i (c - V^i_{LL})} \right) \right], \tag{3-14}$$

if firms introduce the product in a highly profitable market with probability $\alpha \in (0,1)$.

Several elements are worth mentioning from the lemma. First, firms exit the research before a product gets introduced in the market in those equilibria where firms only introduce the new product in a highly profitable market. Absent product introduction, firms become more convinced that the rival firm found evidence of the market being of low profitability. A firms' pessimism about market profitability will make it eventually exit the innovation race. We show in the next section that an eventual exit from the rival will provide incentives to delay product introduction.

In addition, the lemma states that the abandoning times are firm-specific. Different exit times may come from differences in the expected profits from research, but the main difference is firms' learning process. In particular, a firms' learning process and beliefs about market profitability is completely governed by the arrival rate of the rival, λ^{-i}. When the rival's arrival rate is large, a firm gets convinced faster that the market is of small profitability if no production is introduced. Because the incumbent is more efficient with research than the potential entrant, $\lambda^I > \lambda^E$ "ceteris paribus," the entrant abandons research first absent product introduction.

Finally, a firm abandons research earlier, the larger the probability that the rival introduces the product in a highly profitable market, i.e., $\partial T^{i,INH\alpha} / \partial \alpha^{-i} < 0$. When the probability of introducing the product in a highly profitable market becomes large, the faster a firm becomes pessimistic about market profitability if no product is introduced. Similar to the discussion that we introduced after Lemma 1, it is easy to verify that $lim_{\alpha^{-i} \to 0} [T^{i,INH\alpha}] = \infty$, and $lim_{\alpha^{-i} \to 1} [T^{i,INH\alpha}] = T^{i,INH}$. The firm either never abandons before the new product is introduced, or the abandoning time coincides with the time firms abandon absent product introduction in an equilibrium in which the product is always introduced in a highly profitable market.

3.4.3. Product introduction and strategic delay

In this section, we characterize the firms' equilibrium product introduction decisions. In our model, firms start to gain profits from the innovation once they introduce the product in the marketplace. The profits depend on the profitability of the market and the opponent's strategy once the product is introduced.

Consider a situation in which the entrant has been the first firm to complete the innovation race, and it does it before any of the firms abandons research. Suppose also that the market for the new product is highly profitable and that after the product is introduced, profits under duopoly are such that the incumbent continues with research (see Lemma 2). Then, by introducing the new product, the entrant gets

$$V_I^E = e^{-rdt}\left[\lambda^I dt\bar{D}/r + (1-\lambda^I dt)\left(\bar{R}dt + V_I^E\right)\right]. \tag{3-15}$$

With instantaneous probability $\lambda^I dt$, the incumbent completes the innovation and introduces the product. This generates present discounted duopoly profits $\bar{D}/r$. With probability $(1-\lambda^I dt)$, the incumbent does not make a discovery, and the entrant obtains flow monopoly profits $\bar{R}$ together with the continuation profit of V_I^E.

Now consider the case that the duopoly profits are such that after the entrant introduces the product, the incumbent abandons research (see Lemma 2). In this case, the entrant becomes the monopolist for the new product and obtains present discounted profits of $\bar{R}/r$.

Now suppose that the entrant is the first to complete research, but the market for the new product is of small profitability. Since a low profitability market does not generate duopoly profits, if the entrant introduces the new product, it becomes the monopolist to obtain $\underline{R}/r$. If the entrant does not introduce the new product, it gets no profits. The next proposition states that introducing the new product is a dominant strategy for the entrant regardless of the market profitability for the new product.

Proposition 2. *Suppose the potential entrant is the first firm to complete the innovation process. Introducing the new product for any state of the market is the entrant's dominant strategy.*

Introducing the product always generates positive profits to the entrant, while not introducing the product gives no earnings. Also, delaying product introduction is not profitable for the entrant. In equilibrium, the entrant cannot commit not to introduce the product in a small profitability market. Not introducing the product will make it lose sure monopoly profits. Then, a delay of product introduction can only happen if the market is highly profitable. But then, in the equilibrium path, the incumbent will become more optimistic over time about market profitability absent product introduction. Consequently, the incumbent will never abandon research, and hence, the entrant has no incentive to delay product introduction.

A significant result is that the incumbent never learn about market profitability when the entrant introduces the new product. Also, the incumbent does not update its beliefs about the market profitably without product introduction. The incumbent only learns the profitability of the market when it completes the innovation process. The non-learning result will be later used when determining the incumbent's incentives to engage in product innovation. We proceed to characterize the incumbent's product introduction decisions. Suppose then that the incumbent is the first to complete the innovation process. Suppose further that the market for the new product is of small profitability. When the market is of small profitability, duopoly profits are zero. Then, if the incumbent introduces the product, it gets

$$V_I^I = \underline{R}/r + (1-\gamma)R/r. \tag{3-16}$$

The incumbent abandons present discounted profits $\underline{R}/r$ but loses the mass of consumers γ in its existing market. If the incumbent decides not to introduce the new product and assuming that the entrant always continues with research until a product is introduced, the incumbent obtains

$$V_{NI}^I = e^{-rdt}\left[\lambda^E dt\left((1-\gamma)R/r\right) + (1-\lambda^E dt)\left(Rdt + V_{NI}^I\right)\right]. \tag{3-17}$$

The incumbent loses the mass of consumers γ in its existing market when the entrant finishes the research project, an event that occurs with probability $\lambda^I dt$. If the entrant does not finish research, with probability $(1-\lambda^E dt)$, the incumbent keeps all the profits in its existing market and obtains the continuation payoff of V_{NI}^I. By taking the limit $dt \to 0$ and using $e^{-rdt} = (1-rdt)$, the solution of (3-17) is $V_{NI}^I = \left(R + \lambda^E(1-\gamma)R/r\right)/\left(r+\lambda^E\right)$. Then, comparing both profits, the incumbent introduces the new product in the small profitability market if

$$V_I^I = \frac{\underline{R} + (1-\gamma)R}{r} \geq \frac{R + \lambda^E(1-\gamma)R/r}{r+\lambda^E} = V_{NI}^I \iff rR \leq (r+\lambda^E)\underline{R} + r(1-\gamma)R$$
$$\iff \gamma \leq \frac{\underline{R}(r+\lambda^E)}{rR} := \underline{\gamma}(\lambda^E). \tag{3-18}$$

Otherwise, the incumbent does not introduce the product. The incumbent does not introduce the product because of the loss it creates in its existing market. The threat of a potential entrant explains why the incumbent introduces the product innovation more often than when it researches alone, $\underline{\gamma}(\lambda^E) > \gamma^M$. Also, a better entrant's research technology (an increase in λ^E) makes the cut-off level $\underline{\gamma}(\lambda^E)$, below which the incumbent introduces the product in a low profitability market, to increase, $\partial\underline{\gamma}(\lambda^E)/\partial\lambda^E > 0$. With a more efficient entrant, the incumbent expects early product introduction by the entrant and its expected profits from not introducing the product decrease. Then, the incumbent introduces the product even when it loses more in its existing market. For an efficient-enough entrant's, i.e., $\lambda^E \geq (r(R-\underline{R}))/\underline{R}$, then, $\underline{\gamma}(\lambda^E) \geq 1$; the incumbent introduces the product even if it entails losing all its existing market.

Consider a situation in which $\gamma \leq \underline{\gamma}(\lambda^E)$. Assume further that now the market is highly profitable. Then, if the incumbent introduces the new product, the entrant does not learn market profitability. Suppose also that the entrant continues with research after product introduction (i.e.. $\bar{D} \geq \hat{D}$, see Lemma 2). Then, by introducing the product, the incumbent gets:

$$V_I^I = e^{-rdt}\left[(1-\gamma)R/r + \lambda^E dt(\bar{D}/r) + (1-\lambda^E dt)\left(\bar{R}dt + V_I^I\right)\right]. \tag{3-19}$$

When the product is introduced, the incumbent loses the present discounted profits $\gamma R/r$ from its existing market. With instantaneous probability $\lambda^E dt$, the entrant finishes research, and the incumbent obtains present discounted duopoly profits of $\bar{D}/r$. With probability $(1-\lambda^E dt)$, the entrant does not complete research, and the incumbent obtains monopoly profits $\bar{R}$ and the continuation profit of V_I^I.
Now consider a situation in which, after the incumbent introduces the new product, the potential entrant abandons research, i.e., $D < \hat{D}$. In this case, introducing the product gives the incumbent

$$V_I^I = \bar{R}/r + (1-\gamma)R/r. \tag{3-20}$$

Not consider that the incumbent decides not to induce the new product in the market. In this case, the incumbent obtains

$$V_{NI}^I = e^{-rdt}\left[\lambda^E dt\left(\bar{D}/r + (1-\gamma)R/r\right) + (1-\lambda^E dt)\left(Rdt + V_{NI}^I\right)\right]. \tag{3-21}$$

By not introducing the product, the loss γ in the existing market only occurs when the entrant introduces the product. This happens with instantaneous probability $\lambda^E dt$. Note that after the entrant. Note that when the entrant introduces the product, the incumbent also does and gets duopoly discounted profits $\bar{D}/r$. The incumbent keeps all the profits in its existing market if the entrant does not finish research.
By taking the limit $dt \to 0$ and using $r^{-rdt} = (1-rdt)$, the solution of expressions (3-19) and (3-21) are $V_I^I = \frac{\bar{R}+(1-\gamma)R/r+\lambda^E \bar{D}/r}{r+\lambda^E}$ and $V_{NI}^I = \frac{R+\lambda^E(\bar{D}/r+(1-\gamma)R/r)}{r+\lambda^E}$, respectively. Then, the incumbent always introduces the product in a highly profitable market because $V_I^I > V_{NI}^I \iff \frac{(\bar{R}-R)+(1-\lambda^E)(1-\gamma)R/r}{r+\lambda^E} > 0$.[9] This is also the case, when the entrant abandons research after the incumbent has introduced the new product. By introducing the new product, the incumbent becomes the monopolists in the research stage, while by not introducing the product, the incumbent will eventually face competition in the product market.
Suppose now a situation in which $\gamma > \underline{\gamma}(\lambda^E)$. Remember that the incumbent does not introduce the product in this case if the market is of low profitability. Then, if the market is highly profitable and the incumbent introduces the product, the entrant learns about a highly profitable market after the new product is introduced. If the good news of learning

[9] The LHS decreases with λ^E. Substituting for the largest possible $\lambda^E = (r(\bar{R}-R)+R)/(R)$, we get $LHS = 0$.

a highly profitable market is dominated by the bad news of only expecting duopoly profits, i.e., $\bar{D} < \tilde{D}$, where $\tilde{D}$ as defined in (3-10), the potential entrant abandons research after product introduction. In this situation, it is easy to see that the incumbent will introduce the new product in a highly profitable market since this secures monopoly profits.

Consider a situation in which the entrant will continue with research if the incumbent introduces the product, i.e, $\bar{D} \geq \tilde{D}$. In this case, introducing the product right away may not be the best option for the incumbent. To see this remember from Lemma 1 that when the incumbent introduces the product only in a high profitability market, the entrant becomes pessimistic about the market profitability and eventually abandons research absent product introduction. Anticipating the exit of the entrant, the incumbent may strategically delay product introduction in a high profitability market. The incentive to delay comes from the fact that the entrant learns that the market is highly profitable if the incumbent introduces the product. Then, the entrant continues with research, and the incumbent faces competition in the product market once the entrant finishes research. By delaying product introduction, the entrant becomes pessimistic about market profitability and drops out of the race if it does not make a discovery.

The delaying strategy of the incumbent depends on the time of discovery. If the incumbent makes an early discovery, it introduces the product immediately. Waiting for product introduction until the entrant abandons research would generate a loss in foregone profits that would dominate the expected reduction of profits from the eventual competition in the product market. If the incumbent makes a later discovery, it has incentives to wait for product introduction because the entrant is closer to dropping out of the race. The expected profits of becoming the monopoly in the market innovation once the entrant has abandoned research outweigh the loss of foregone profits from waiting.

In the model, waiting is modeled as a probability of introducing the product in a highly profitable market, such that, the incumbent is indifferent between introducing the product immediately or waiting until the entrant abandons research. The incumbent's equilibrium decision is in mixed strategies. In choosing the probability to introduce the product in a highly profitable market, the incumbent trades off the rival's speed in the learning process against a lower continuation profit after the product is introduced. A higher introduction probability makes the potential entrant pessimistic about market profitability faster and hence abandons research earlier. Suppose the entrant believes in the equilibrium path that the incumbent introduces the new product with a large probability, and no product is introduced. In that case, the entrant becomes more convinced that the market is of small profitability, a situation in which the incumbent never introduces the product. If the incumbent introduces the product, it obtains lower continuation profits. The entrant learns the market to be highly profitable and researches until completion. After this, the incumbent faces competition in the product market.

Interestingly, the introduction probability is not constant. It monotonically decreases over time. At the start, the incumbent gives more weight to the entrant's learning speed and

introduces the product with a high probability to depress the entrant's beliefs about market profitability. When time advances and the entrant is closer to abandoning the race, introducing the product will have been in vain as the entrant will continue researching until it introduces the product. The incumbent then gives more weight to the decrease in continuation payoffs and responds by lowering the product's introduction probability. Note that once the entrant has abandoned research, the incumbent introduces the product immediately.
The incumbent's product introduction decision is summarized in the following proposition.

Proposition 3. *Suppose the incumbent is the first firm to complete the innovation process. Define $\underline{\gamma}(\lambda^E)$ as in* (3-18)*. Then, the incumbent's product introduction decisions are:*
i) For $\gamma < \underline{\gamma}(\lambda^E)$, the incumbent introduces the product in any state of the market.
ii) For $\gamma \geq \underline{\gamma}(\lambda^E)$, the incumbent does not introduce the product in a market of low profitability. With $\bar{D} \leq \tilde{D}$, where $\tilde{D}$ is defined as in (3-10)*, the incumbent introduces the product in a high profitable market. With $\bar{D} > \tilde{D}$, the incumbent's product introduction decision in a highly profitable market depends on the time of discovery, such that:*
a) The incumbent introduces the product immediately when discovery occurs at $\tau^I \in \left(0, \underline{T}(T^{E,INH})\right)$;
b) The incumbent introduces the product with probability $\alpha_t \in (0,1)$ whenever the discovery happens at $\tau^I \in \left[\underline{T}(T^{E,INH}), T^{E,INH\alpha}(\alpha_t)\right]$ with $\partial\alpha_t/\partial t < 0$; and
c) The incumbent introduces the product immediately if discovery takes place after $T^{E,INH\alpha}(\alpha_t)$.

Figure **3-1** illustrates the incumbent's product introduction decisions for $\bar{D} \geq \tilde{D}$. If compe-

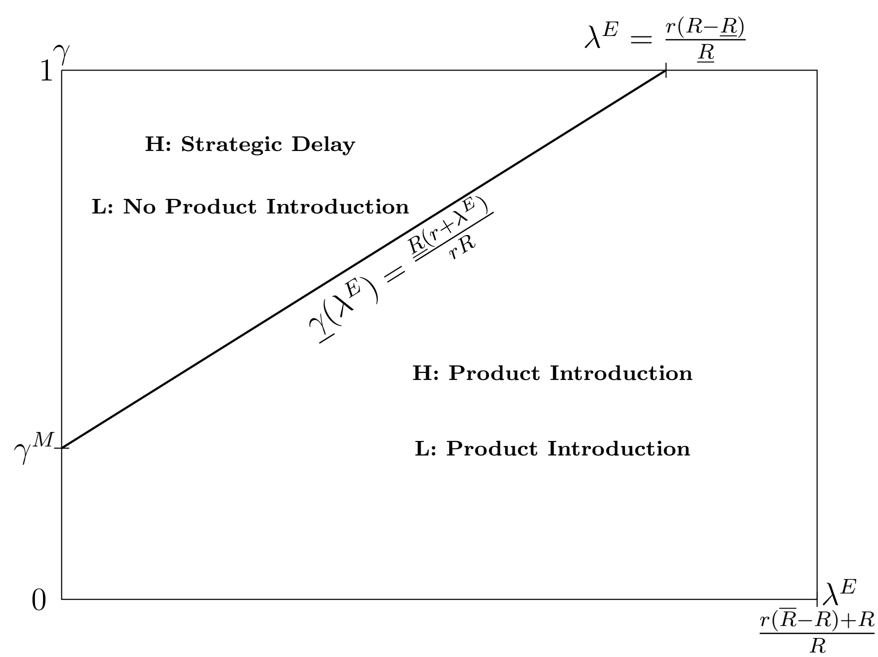

Figure **3-1**.: Incumbent's product introduction decision with $\bar{D} \geq \tilde{D}$.

tition in the product market becomes severe, i.e., $\bar{D} < \tilde{D}$, the incumbent does not undertake strategic delay, and it always introduces the product in a highly profitable market. The picture illustrates that the region in which the incumbent undertakes product introduction delay shrinks with a better entrant's research technology (larger λ^E). When the entrant becomes more efficient in research, the incumbent expects an early product introduction by the rival. The latter responds by introducing the new product in a small profitability market. The incumbent prefers to replace itself than be replaced by the potential entrant. When the incumbent introduces the product in a small profitability market, the rival does not become pessimistic absent product introduction. Then, there is no profit from delaying product introduction, and the incumbent immediately introduces the product in any market profitability when it completes research.

3.4.4. Research decisions

With the firms' equilibrium product introduction decisions and the subsequent firms' beliefs about market profitability, we elicit the firms' decisions to research in product innovation. In Proposition 1, we have stated that the incumbent always decides to research in the new product if it is the only active firm in the market. This section considers the innovation race between the incumbent and a potential entrant. Firms decide to research as long as the expected profits from innovating in a new product outweigh the profits of no innovating. We assume that firms take research decisions at the start of the game and do not allow for strategic waiting. If a firm does not pay the flow costs of research at the beginning of the game, this is equivalent to dropping out.

Consider first an equilibrium in which the incumbent introduces the new product regardless of market profitability, i.e., $\gamma < \underline{\gamma}(\lambda^E)$ (see Proposition 3). Remember that the entrant always introduces the new product. In this equilibrium, no firm learns market profitability. Also, firms research until a firm introduces the product in the market. Then, by undertaking research in product innovation, the entrant obtains

$$W^E = -c\lambda^E dt + e^{-rdt}\left[\lambda^E dt\left(p_0 V_L^E + (1-p_0)\underline{R}/r\right) + \lambda^I dt V_F^E + (1-\lambda^E dt - \lambda^I dt)W^E\right]. \tag{3-22}$$

Inside the bracket, with probability $\lambda^E dt$, the entrant is the first to complete research. It obtains the expected profits of leading the new market if the market is highly profitable or obtains present discounted monopoly profits $\underline{R}/r$ if the market is of small profitability. With probability $\lambda^I dt$, the incumbent is the first to complete the innovation process, and the entrant obtains the expected profits of being the follower in the product market. With probability $(1-\lambda^E dt - \lambda^I dt)$, no firm completes innovation and the entrant gets its continuation profits W^E.

By undertaking product innovation, the incumbent obtains

$$W^I = -c\lambda^I dt + e^{-rdt}\left\{\lambda^I dt\left[p_0(V_L^I) + (1-p_0)(\underline{R}/r) + (1-\gamma)R/r\right]\right.$$
$$\left. +\lambda^E dt\left[V_F^I + (1-\gamma)R/r\right] + (1-\lambda^I dt - \lambda^E dt)\left(Rdt + W^I\right)\right\}. \quad (3\text{-}23)$$

Inside the bracket, with probability $\lambda^I dt$, the entrant is the first to complete research. If the market is highly profitable, it obtains the expected profits of leading the new market. If the market is of small profitability, it obtains present discounted monopoly profits $\underline{R}/r$. Also, when the product is introduced, the incumbent loses the present discounted profits $\gamma R/r$ in its existing market. With probability $\lambda^E dt$, the entrant is the first to complete the innovation process, and the incumbent obtains the expected profits of being the follower in the product market. Also, the incumbent loses the present discounted profits of $\gamma R/r$. With probability $(1-\lambda^E dt - \lambda^I dt)$, no firm completes innovation and the incumbent gets the flow profits in its existing market and the continuation profits W^I.

Observe that the expected profits from research depend on the degree of competition in the product market. Then for $\bar{D} < \hat{D}$, where $\hat{D}$ is defined as in (3-9), firms abandon research after the rival has introduced the new product. Then $V_F^i = 0$ and $V_L^i = \bar{R}/r$ for $i = I, E$. When market competition becomes less intense, $\bar{D} \geq \hat{D}$, the leader in the market, expects to lose the monopoly profits when the opponent completes the innovation, after which it only gets duopoly profits. The follower gets duopoly profits when it completes innovation.

Now consider an equilibrium in which the incumbent does not introduce the new product in a small profitability market, i.e., $\gamma \geq \underline{\gamma}(\lambda^E)$. Suppose further that the incumbent always introduces the product in a highly profitable market, i.e., $\bar{D} \leq \tilde{D}$ (see Proposition 3). In this equilibrium, if the incumbent introduces the product, the entrant learns the market to be highly profitable. Also, absent product introduction, the entrant's belief about market profitability evolves. Particularly, the entrant becomes pessimistic about the market being of high profitability. Then, absent product introduction, the entrant abandons research at time $T^{E,INH}$ as characterized in Expression (3-13) in Lemma 3. Then, researching for a product innovation gives to the entrant

$$W^E = \int_0^{T^{E,INH}} e^{-(r+\lambda^E+\lambda^I)t}\lambda^E\left[p_t^{E,INH}\bar{R}/r + \left(1 - p_t^{E,INH}\right)\left(1+\lambda^I\right)\underline{R}/r - c\right]dt. \quad (3\text{-}24)$$

The expected profits of the entrant take into account that its beliefs about market profitability evolve absent product introduction. Then, as the entrant becomes pessimistic, it will only be active in research until time $T^{E,INH}$. After this time, it abandons research if no product has been introduced. Also, if the entrant is the first to complete research, it becomes the monopolist. For $\bar{D} < \tilde{D}$, the incumbent abandons research after product introduction. The same occurs if the incumbent is the first to complete research, except when the market is of small profitability because the incumbent does not introduce the product. This is illustrated in the expected profit of $\lambda^E\lambda^I\underline{R}/r$.

Knowing that the entrant may eventually abandon research if no product is introduced before $T^{E,INH}$, researching for a product innovation gives the incumbent

$$W^I = \int_0^{T^{E,INH}} e^{-(r+\lambda^E+\lambda^I)t} \left\{\lambda_I \left[p_0 \left(\bar{R}/r + (1-\gamma)R/r\right) + (1-p_0)\left(\frac{\lambda^E(1-\gamma)R/r+R}{r+\lambda^E}\right) - c\right]\right.$$
$$\left. + \lambda^E \left[(1-\gamma)R/r\right] + R\right\} dt + e^{-(r+\lambda^E+\lambda^I)T^{E,INH}} \left(\int_0^\infty e^{-(r+\lambda^I)t} \left\{\lambda^I \left[p_0 \left(\bar{R}/r + (1-\gamma)R/r\right)\right.\right.\right.$$
$$\left.\left.\left. - c\right] + R\right\}\right) dt. \tag{3-25}$$

The first line represents the expected profits of the incumbent when the entrant is active in research. Different from the expected profits of the incumbent in Expression (3-23), if the incumbent is the first to complete research but realizes that the market is of small profitability, it decides not to introduce the product. Then, the incumbent obtains the flow profits R in its existing market until the entrant completes research and introduces the product. This is illustrated in the expected profit of $\lambda^I(1-p_0)((\lambda^E(1-\gamma)R/r+R)/r+\lambda^E)$. Also, if the entrant is the first to complete research, the incumbent abandons the race. The second line is the incumbent's expected profits after the entrant abandons research. Note that the incumbent becomes the monopolist after $T^{E,INH}$, if none of the firms have made a discovery. This occurs with probability $e^{-(r+\lambda^E+\lambda^I)T^{E,INH}}$.

Finally, it is left to consider the case in which the incumbent engages in strategic delay. This occurs whenever $\gamma > \underline{\gamma}(\lambda^E)$ and $\bar{D} > \tilde{D}$ (see Proposition 3). In this case, the entrant's expected profit from researching is

$$W^E = \overbrace{\int_0^{\underline{T}(T^{E,INH})} e^{-(r+\lambda^E+\lambda^I)t} \left\{\lambda^E \left[p_t^{E,INH} V_L^E + \left(1-p_t^{E,INH}\right)\underline{R}/r - c\right] + \lambda^I V_F^E\right\} dt}^{\text{Immediate incumbent's product introduction}}$$
$$+ \underbrace{\int_{\underline{T}(T^{E,INH})}^{T^{E,INH\alpha}(\alpha_t)} e^{-(r+\lambda^E+\lambda^I)t} \left\{\lambda^E\left[p_t^{E,INH\alpha}(\alpha_t) V_L^E + \left(1-p_t^{E,INH\alpha}(\alpha_t)\right)\frac{\underline{R}}{r} - c\right] + \lambda^I \alpha_t V_F^E\right\} dt}_{\text{Incumbent's strategic delay}}. \tag{3-26}$$

The first line coincides with Expression (3-24), but with the difference in the continuation payoff when the incumbent is the first to complete research. Now, when the incumbent introduces the new product, the entrant continues with research, generating expected profits of V_F^E. The second line is the entrant's expected profits when the incumbent undertakes strategic delay. Note first that the entrant will undertake research up to time $T^{E,INH\alpha}(\alpha_t)$. Also, the evolution of beliefs absent product introduction depends on the introduction probability in a highly profitable market of the incumbent α_t. Also, the continuation payoff V_F^E depends on the introduction probability.

With strategic delay, the incumbent obtains

$$\begin{aligned}
W^I = & \overbrace{\int_0^{\underline{T}(T^{E,INH})} e^{-(r+\lambda^E+\lambda^I)t} \{\lambda_I[p_0(V_L^I+(1-\gamma)R/r)+(1-p_0)A-c]+\lambda^E[V_F^I+(1-\gamma)R/r]+R\}\, dt}^{\text{Competition with the potential entrant}} \\
& + \int_{\underline{T}(T^{E,INH})}^{T^{E,INH\alpha}(\alpha_t)} e^{-(r+\lambda^E+\lambda^I)t} \left\{\lambda^I \left[p_0 \left(\alpha_t \left(V_L^I + (1-\gamma)R/r\right) + (1-\alpha_t)R\right)\right.\right. \\
& \left.\left. +(1-p_0)A - c\right] + \lambda^E \left[V_F^I + (1-\gamma)R/r\right] + R\right\} dt \\
& \underbrace{+ e^{-(r+\lambda^E+\lambda^I)T^{E,INH\alpha}(\alpha_t)}\left(p_0\lambda_I T^{E,INH\alpha}(\alpha_t)B + \int_0^{\infty} e^{-(r+\lambda^I)t}\{\lambda^I[p_0B-c]+R\}\right)dt,}_{\text{Monopolistic incumbent}}
\end{aligned} \tag{3-27}$$

where:

$$A = \frac{\lambda^E(1-\gamma)R/r + R}{r+\lambda^E},$$

$$B = \frac{\bar{R}}{r} + (1-\gamma)\frac{R}{r}.$$

The first three lines represent the incumbent's expected profit when the entrant is active in research. The first line is the same as Expression (3-25) but with the extra expected profit V_F^I that the incumbent obtains when the entrant is the first to complete research. In the second and third lines, note that the expected profits depend on the probability α_t in which the incumbent introduces the new product if the market is highly profitable. The last line represents the expected profit after the entrant has abandoned research. There, the first part illustrates the probability that by time $T^{E,INH\alpha}(\alpha_t)$, the incumbent have finished research, $p_0\lambda_I T^{E,INH\alpha}(\alpha_t)e^{-(r+\lambda^E+\lambda^I)T^{E,INH\alpha}(\alpha_t)}$, and introduces the product right away.Recall that the probability of only the incumbent finishing research by $T^{E,INH\alpha}(\alpha_t)$ is given by:

$$\int_0^{T^{E,INH\alpha}(\alpha_t)} e^{-(\lambda^I+\lambda^E)s}\lambda^I e^{-(\lambda^I+\lambda^E)(T^{E,INH\alpha}(\alpha_t)-s)}ds = \lambda^I T^{E,INH\alpha}(\alpha_t)e^{-(\lambda^I+\lambda^E)(T^{E,INH\alpha}(\alpha_t)}$$

The second part of the third line in expression (3-27) states the expected profits if the incumbent has not finished research by $T^{E,INH\alpha}(\alpha_t)$.

A firm that decides not to undertake research in product innovation does not need to pay the flow cost of research; however, it does not profit from the new product. If the potential entrant decides not to innovate, it gets zero profits. The incumbent's expected profits depend on the potential entrant's research decision. Suppose the entrant decides not to innovate. Then, by not innovating, the incumbent guarantees the present discounted profit of R/r from its existing market. The incumbent does not face any threat from a potential competitor. Suppose that the potential entrant researches for product innovation. Then, if the incumbent decides not to innovate, it gets

$$W_{NR}^I = e^{-rdt}\left[\lambda^E dt\,((1-\gamma)R/r) + (1-\lambda^E dt)\left(Rdt + W_{NR}^I\right)\right] dt. \tag{3-28}$$

The incumbent keeps the whole profits in its existing market as long as the entrant does not completes research. With probability $\lambda^E dt$, the entrant finishes research and the incumbent loses a proportion of profits $\gamma(R/r)$ from its market.

To establish the firms' research decisions, note that because the potential entrant obtains no profits if it does not innovate, it always researches for product innovation in equilibrium. Proposition (4) below shows that researching in product innovation is a dominant strategy for the potential entrant.

Knowing that the entrant always researches, if the incumbent decides not to research, it knows the entrant will eventually replace it. The threat of being replaced provides the incumbent with more incentives to research. A more efficient entrant (higher λ^E), achieves the innovation earlier, and this reduces the incumbent's outside option from not researching - the profits from the incumbent in its existing market absent product innovation. Consequently, the incumbent introduces the product regardless of market profitability. As a result, the entrant's learning from the incumbent's product introduction disappears, decreasing the entrant's willingness to continue with research after the incumbent has introduced the product. The lack of learning from the entrant eliminates the good news effect of the incumbent's product introduction. Expecting only duopoly profits and being unsure about market profitability makes the entrant exit research.

When the entrant becomes less efficient in undertaking research, the incumbent research in product innovation is not guaranteed. Fewer incentives to innovate result from the incumbent's difficulty to pre-empt the entrant. To see this, note that with a less efficient entrant, the incumbent does not perceive the threat of being replaced by the entrant as important and decides not to introduce the product if the market is of small profitability. As a result, when the incumbent introduces the product, it signals a market with high profitability, and the entrant responds by continuing to research until it also introduces the product. To mitigate the good news, the incumbent engages in strategic product delay to depress the entrant's beliefs and make it eventually exit research. However, if the entrant abandons research late, either due to a low incumbent's research efficiency or a small cost of research, the delaying strategy becomes less effective. With an inefficient incumbent (small λ^I), the learning of the entrant becomes sluggish (see Lemma 1 and Lemma 3), and it takes longer to drop out of the race. Abandoning research late also occurs when the cost of research is small because the entrant does not lose much by carrying on the innovation. Hence, the incumbent anticipation of competition in the product innovation due to the impossibility to pre-empt the entrant lowers its incentives to innovate. This result complements Reinganum (1983), who shows that the incumbent's replacement effect gives an equilibrium with less innovation.

With the characterization of firms' expected profits from research, the following proposition contains a complete description of the equilibrium.

Proposition 4. *Suppose there are two firms competing for product innovation, an incumbent and a potential entrant. Then, the PBE of the game is defined as follows.*

i) The entrant always researches for product innovation.

a) For $\gamma < \underline{\gamma}(\lambda^E)$ the entrant never exits research absent product introduction. For $D \leq \hat{D}$, where $\hat{D}$ is defined as in (3-9)*, when the incumbent introduces the new product, it abandons. For $D > \hat{D}$, the entrant continues with research after the incumbent introduces the new product.*
b) For $\gamma \geq \underline{\gamma}(\lambda^E)$, if no product is introduced the entrant abandons research. For $\bar{D} \leq \tilde{D}$ where $\tilde{D}$ is defined as in (3-10)*, the entrant exists at time $T^{E,INH}$. For $\bar{D} > \tilde{D}$, the entrant exists at $T^{E,INH\alpha}(\alpha_t)$, with $T^{E,INH} < T^{E,INH\alpha}(\alpha_t)$. Also, when the incumbent introduces the new product, for $D \leq \tilde{D}$, the entrant abandons, for $D > \tilde{D}$, it continues.*
ii) When the incumbent researches, it never abandons research absent product introduction. When the entrant introduces the product, for $D \leq \hat{D}$, the incumbent abandons, for $D > \hat{D}$, it continues. However, the incumbent not always decides research in the new product.
a) For $\gamma < \underline{\gamma}(\lambda^E)$, the incumbent researches in product innovation.
b) For $\gamma \geq \underline{\gamma}(\lambda^E)$, with $\bar{D} > \hat{D}$ and $\bar{D} \leq \tilde{D}$, the incumbent researches in product innovation. With $\bar{D} \in \left(\tilde{D}, \hat{D}\right]$, for $p_0 \leq \tilde{p}(\gamma, c, \lambda^E, \lambda^I, \bar{D})$ the incumbent does not research in product innovation.

3.4.5. Market inefficiencies

The market equilibrium of our research game presented in Proposition 4 is prompt to inefficiencies that arise due to duplication research costs, early exit, and inefficient delay in product introduction. Those inefficiencies mainly arise from the firms' private learning of market profitability when they complete research. Section 3.5 below shows how most of those inefficiencies may be solved by the creation of a Research Joint Venture in which firms share the information they obtain from research.
To characterize the duplication research costs, consider a situation in which the opponent introduces the product in an equilibrium where a firm does not learn market profitability from the opponent's product introduction. Suppose further that the firm, with the hope that the market is highly profitable, decides to continue with research. However, when the firm finishes research and learns the market to be of low profitability, it does not introduce the product. In this case, the expected duplication costs equal to

$$DC^i = \int_0^\infty e^{-(r+\lambda^i+\lambda^{-i})t}\left[(1-p_0)\lambda^{-i}\left(\lambda^i c\right)\right]dt = \left((1-p_0)\lambda^{-i}\lambda^i c\right) / \left(r+\lambda^i+\lambda^{-i}\right). \quad (3\text{-}29)$$

Those costs are an upper-bound for the entrant because for $\gamma > \underline{\gamma}(\lambda^E)$ it drops out research at $T^{E,INH\alpha}(\alpha_t)$.
With probability $(1-p_0)\lambda^{-i}$, the rival completes research, and the market has small profitability. Note that the firm will pay flow costs $\lambda^i c$ until it completes research to realize that the market is of small profitability. Note that the flow costs would not have been paid if the firm had known that the market was of small profitability after the rival had introduced the product. The firm would have abandoned research, then, and flow costs of research would have been saved.

Another market inefficiency emerges from an early exit from research. Early exit may occur in an equilibrium in which the entrant becomes pessimistic about market profitability and decides to exit research before the new product has been introduced to the market (see Proposition 3 for $\gamma \geq \underline{\gamma}(\lambda^E)$). The early exit creates a loss in expected profits that may have emerged from the firm continuing with research which equals to

$$EE^E = \int_{T^{E,INH}}^{\infty} e^{-(r+\lambda^E+\lambda^I)t} \left\{ \lambda^E \left[p_0 \left(\bar{R} + \lambda^I \bar{D}/r \right) + (1-p_0)\underline{R}/r \right] + \lambda^I \left[p_0 \lambda^E \left(\bar{D}/r \right) \right] - \lambda^E c \right\} dt \tag{3-30}$$

With probability $\lambda^E dt$, the entrant would have made a discovery if it was still present in research, from which it will get expected profits $p_0(\bar{R} + \lambda^I \bar{D}/r)$ if the market is of high profitability, and profits $\underline{R}/r$ if the market is of small profitability. With probability, λ^I, the incumbent would have introduced the product. Then, the entrant would have made profits $\bar{D}/r$, after it has completed the innovation, with probability λ^E, and the market is highly profitable, with probability p_0.

Finally, In an equilibrium where the incumbent engages in strategic delay, there is a loss equal to

$$ID = \int_{\underline{T}(T^{E,INH})}^{T^{E,INH\alpha}(\alpha_t)} e^{-(r+\lambda^E+\lambda^I)t} \left\{ \lambda^I \left[(1-\alpha_t) p_0 \bar{R} \right] \right\} dt \tag{3-31}$$

For the time in which the incumbent delays product introduction, with probability $(1-\alpha_t)$ there are forgone profits equal to $\bar{R}$.

Because market inefficiencies are more severe, in the form of delayed product introduction and early exit, when the firms' research strategies differ, from a policy perspective, a competition authority would like to promote efficiency technology adoption by a potential entrant. If this is not possible, the following section proposes another solution that considers the creation of a research joint venture in which firms share their private outcomes from research.

3.5. Research Joint Venture with public research outcomes

In the model, we have studied the incentives to innovate when firms privately learn the state of the market once they finish research. In this section, we consider the case in which the results from research become public information. Ways in which firms may share research information could be creating a research joint venture or through a contract in which firms agree to make outcomes from their research public.

With public research outcomes, firms learn the state of the market when a product is introduced, but there is no learning about market profitability, absent product introduction. We show that such learning of market uncertainty will not generate duplication research costs, early exit from research, and inefficient introduction delays that will emerge when firms privately learn the state of the market.

To elicit possible inefficiencies in the incumbent's research strategy, we characterize its expected profits under public research outcomes. With $\gamma < \underline{\gamma}(\lambda^E)$, the incumbent gets:

$$W^I_{PI} = \int_0^\infty e^{-(r+\lambda^E+\lambda^I)t} \left\{\lambda^I \left[p_0 \left(V^I_{L,PI} + (1-\gamma)R/r\right) + (1-p_0)\left(\underline{R}/r + (1-\gamma)R/r\right) - c\right] + \lambda^E \left[V^I_{F,PI} + (1-\gamma)R/r\right] + R\right\} dt, \tag{3-32}$$

Under public information, by learning the market being of high profitability, the incumbent obtains lower profits than private outcomes if it is the first to introduce the product $V^I_{L,PI} = (\bar{R} + \lambda^E \bar{D}/r)/(r+\lambda^E) < V^I_L = \bar{R}/r$, and larger profits when the entrant introduces the product first $V^I_{F,PI} = \lambda^I(\bar{D}/r - c)/(r+\lambda^I) > V^I_F = 0$.
With $\gamma \geq \underline{\gamma}(\lambda^E)$, the incumbent gets:

$$W^I_{PI} = \int_0^\infty e^{-(r+\lambda^E+\lambda^I)t} \left\{\lambda^I \left[p_0 \left(V^I_{L,PI} + (1-\gamma)R/r\right) + (1-p_0)\left(\frac{R+\lambda^E(1-\gamma)R/r}{r+\lambda^E}\right) - c\right] + \lambda^E \left[V^I_{F,PI} + (1-\gamma)R/r\right] + R\right\} dt, \tag{3-33}$$

where $V^I_{L,PI} = V^I_L$, and $V^I_{F,PI} = \lambda^I(\bar{D}/r - c)/(r+\lambda_I) > V^I_F = 0$. When the entrant is the first to introduce the product, the incumbent drops out, whereas, with public information, it keeps on researching as it learns a high profitability market.
Proposition 5 establishes the differences in the firms' research strategies between private and public research outcomes.

Proposition 5. *With public research outcomes:*
i) The entrant always researches and never abandons research before product introduction.
ii) The incumbent research with private information may not efficient with $\bar{D} \in (rc, rc/p_0]$:

iia) For $\gamma < \underline{\gamma}(\lambda^E)$, the incumbent over-invests when $r < \underline{r}(\lambda^E, c)$.

iib) For $\gamma \geq \underline{\gamma}(\lambda^E)$, there exists a level of competition $\breve{D}$ such that: when $\bar{D} \in \left(\tilde{D}, \breve{D}\right)$, the incumbent over-invests, and when $\bar{D} \in \left(\breve{D}, \tilde{p}_0^{-1}\left((\cdot, \tilde{D}) = rc/(\bar{R} - R)\right)\right)$ it under-invests.

The proposition states that the incumbent may over-invests with private outcomes for $\gamma < \underline{\gamma}(\lambda^E)$. With private outcomes, the entrant does not learn market profitability after the incumbent introduces the product, and abandons research due to the fear of a small market. The entrant's exit gives the incumbent monopolistic profits. With public information, when the entrant learns a highly profitable market, it continues with research. Consequently, the incumbent obtains duopoly profits under public information instead of monopoly profits. This explains the over-investment with private outcomes when the discount rate r is sufficiently small. For $\gamma \geq \underline{\gamma}(\lambda^E)$, the proposition states that the incumbent may over or under-invest with private outcomes. The incumbent researches too much resulting from the entrant's learning. The incumbent expects monopolistic profits after $T^{E,INH\alpha}(\alpha_t)$. Under public information, the entrant never drops out the race before product introduction. The incumbent may also under-invest with private outcomes. The incumbent abandons research for an intermediate level of competition, but with public information, it continues with research providing more incentives to innovate.

3.6. Conclusion

This work has studied the incentives to research product innovation between an incumbent and a potential entrant with market uncertainty and private research outcomes. Product introduction by an incumbent indicates a market with high profitability. The entrant's belief update encourages to continue research until it introduces the product innovation, hence, generating competition in the product innovation market. To mitigate the entrant's response to persist researching, the incumbent decides to wait for product introduction. Because the product is not introduced, the entrant becomes convinced that it is a market with low profitability and eventually abandons research.

The incumbent's strategic delay is less effective by holding an inefficient research technology and/or research cost are small. An ineffective research technology makes the entrant's learning absent product introduction sluggish, and it will take a longer time for the entrant to drop out of the race. With a small research cost, exit also happens late because the entrant does not lose much from continuing research. Eventual competition in the product market becomes more likely, which reduces the incumbent's incentives to innovate. This confirms the so-called "incumbent's curse," supporting the view that large incumbent firms rarely introduce product innovations, and potential entrants play an essential role in the research and development of new products.

An interesting feature of the model is that an efficient entrant disciplines the incumbent's research decisions. A capable entrant is expected to complete innovation and introduce the product in a short time. With the fear of being replaced by the entrant, the incumbent has more incentives to innovate and launch the product innovation without delay. This result suggests that competition policies should incentivise early technology adoption by potential entrants.

In the model, we have considered a single research stage that privately reveals the state of the market. The main results of the model will be maintained by considering, for instance, that the research completion provides an imperfect signal about market profitability. Also, our model assumes that research outcomes are not verifiable, implying that information cannot be credibly disclosed. The analysis with verifiable information would allow the study of strategic information disclosure, which may explain strategies such as sleeping patents or pre-emptive announcements. Extensions could also introduce extra dynamic complexities: for example, by allowing firms to decide on the intensity of research, which will enable us to study how these changes when the opponent introduces the product. These interesting research questions are left for future research.

4. The Effects of Banning Permanence Clauses in Mobile Phone Markets

Renzo Clavijo[1]

4.1. Introduction

Consumers in mobile telecommunications markets face barriers to switch from one network provider to another. There exist technical barriers such as non-portability of the mobile number and handset locking. There are also barriers due to conditions agreed upon in the service contract signed between the network provider and the consumer, in particular early termination fees -also known as permanence clauses (PC). There are opposing views about the effects of PC. From one side, operators argue PC are a tool to recover subscription costs in order to maintain the pace of investment. From the other, regulators are concerned about the implications of these switching costs on welfare.

In some countries operators can use PC while in others can not. In Canada the regulator allows PC claiming that, absent early termination charges, service providers would not compete by offering plans with better prices to consumers willing to sign fixed-term contracts.[2] In the UK, the contract can have a condition on early termination fees as a reimbursement for the damages caused to the service provider if the consumer drops out.[3] For the case of US, the Civil Code section 1671 allows the operators to charge consumers a fee for contract termination in similar terms to the UK.[4] In short, the consumer faces costs when she winds up the contractual relationship.

Despite of the existing approaches supporting PC, other countries ban the use of this arrangement. In Colombia, the Comisión de Regulación de Comunicaciones (CRC) enacted a rule mandating mobile operators to avoid the use of PC.[5] The CRC supported this decision arguing that banning PC is a tool that enhances competition in the provision of services. This is so since operators stop looking for rents by means of handsets trade, CRC (2013). The National Congress of Ecuador –2010–, and the regulators in Chile –2012– and Peru –2015–,

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[2]See Telecom Regulatory Policy CRTC 2013-271, paragraph 227.

[3]Currently under revision, here.

[4]See a case here.

[5]See Resolution CRC 4444 of 2014.

also banned the minimum forced duration in mobile service contracts.[6] The regulators in these countries seek to strengthen competition in the supply of services.

The opposing avenues described for contract termination, opens the question about the consequences of restricting the use of PC. To the best of my knowledge there is only one contribution in this direction. Serna et al. (2019) measure the effect of the PC ban on the Colombian mobile handsets market. The authors estimate an structural model to establish that the banning has improved the welfare levels in the market under analysis. There is, however, no evidence of the effect that banning PC brought to the services market.

This paper explores the effects of the PC ban on the Colombian mobile voice telecommunications market. I construct a theoretical framework to model a market in which consumers are subject to switching costs and network externalities. The theoretical model casts several predictions related to switching costs that I test in the data.

I gather evidence using a difference in differences approach to establish a causal relationship between the PC ban and the ARPU in the market under discussion. I use the fixed voice and fixed Internet ARPU figures as counterfactuals to the mobile services markets, given the former remained exposed to permanence clauses while the latter were subject to an exogenous change on this regard. I find that the quarterly ARPU in the mobile voice market had a statistically significant average decrease of USD 3.7 as a result of the PC ban enactment, while I find no evidence of any significant effect for the mobile Internet market. This means that removing PC forced networks to offer lower voice tariffs to their subscribers.

There are several contributions in the literature to estimate the effects of reducing switching costs in telecommunications markets. Some of these contributions use structural modeling while others use reduced form evidence. Grzybowski and Pereira (2011) use a panel data from Portugal to estimate a mixed multinomial logit demand model. The authors perform a policy experiment by setting switching costs to zero. The experiment shows that the market shares reaccommodate leading to remarkable improvements in welfare. García-Mariñoso and Suárez (2019) analyze bundled offers using information from Spain[7]. Authors find that, in addition to PC, bundling mobile and fixed services gives rise to a switching cost. Consumers' awareness of PC conditions better explains switching decisions, i.e., consumers not aware of the contract details are less prone to switch.

The only work closely related to the estimation of PC banning effects is Serna et al. (2019). They use individual purchase data for handsets in Colombia spanning three months before the ban and two years after. Authors estimate a supply/demand model and find that PC banning is welfare improving in the mobile handsets market.

The reduced form evidence contributions focus on Number Portability (NP). Buehler et al. (2006), and Lyons (2010) show that prices in the market drop after the implementation of

[6] For the Ecuador case, see Ley 21 de 2000, Artículo 44.
For Perú, see Resolución 138-2014-CD/OSIPTEL.
For Chile, , see the decision here.

[7] Endogeneity issues are tackled with an IV approach as the one employed by Lee (2017).

NP. However, switching is worth as long as the time required for the porting process is short enough. Estimation of effects relies on country panels data. Researchers exploit variation among countries with and without NP to elicit the effects.
Xavier and Ypsilanti (2008) provide survey evidence about switching. In the UK roughly 62 % of interviewees in 2006 switched their plan, however, a similar amount of people avoids changing their current provider due to PC. This figure is even stronger in the prepaid segment, amounting to 82% of the consumers. For the case of Portugal, around one third of switching consumers declared to have switched because their contacts belong to other network. A similar proportion assert that better tariffs were the reason to change their operator.
The rest of the document is organized as follows. Section 4.2 lays the theoretical framework. Section 4.3 describes the data and the variables. Section 4.4 describes the colombian mobile telecommunications market including insightful evidence about switching indicators. In section 4.5, I set the empirical setting and discuss the identification strategy to isolate the causal effect of the ban. Section 4.6 shows the results of the empirical model. Finally, section 4.7 concludes.

4.2. Theoretical Framework

The theoretical literature has predicted the price outcomes due to the existence of consumers' switching costs. In two-period models, firms fiercely compete in prices in the first stage of the game to capture as many consumers as possible, and then ripoff the locked in costumers by means of high prices, Klemperer (1987a, 1987b). Multiperiod models have also predicted that prices increase in the presence of switching costs as the result of a strong effect to exploit loyal customers in the current period, instead of decreasing prices to attract new consumers, Farrell and Shapiro (1988), Villas-Boas (2006). Either two-period or infinite period models reach similar conclusions.
A further strand of literature studies the price outcomes in the market when switching occurs in equilibrium. In a dynamic setting, firms with larger market shares set higher prices in the short-term in the presence of switching costs; however, this outcome reverses in an evenly splitted market, Fabra and García (2015a,b). Therefore, accounting for switching in equilibrium leads to outcomes that depend on the market shares of the firms competing in the market. However, this strand does not account for network externalities.
I build on Klemperer (1987a) to propose a model with network externalities. The setting considers consumers derive utility in direct proportion to the market size of the firm. This framework supports the fact that mobile markets display network externalities. I look to understand consumers' and firms' reaction to changes in switching costs when externalities are taken into account.
The game has two stages $t \in \{1, 2\}$. In stage 1, firms decide first period prices and consumers make purchase decisions. In second period, firms set prices and consumers purchase. The

market is fully covered every period and both firms and consumers use discount factor λ. There are two firms $i \in \{A, B\}$ that compete *à la Bertrand*. They sell an undifferentiated product. Each firm maximizes expected profits. The market size is $\rho = 1$. Firm $i = A$ locates at position 0 of the Hotelling line, and firm $i = B$ sits at position 1 of the market. Consumers are located on a Hotelling line. Each consumer buys one unit of the product each period. A consumer located at x purchasing from firm $i = A$ at period t derives utility,

$$U_t^A = r - p_t^A - x + \alpha\sigma_t^A,$$

where α accounts for network externality strength and σ_t^i is the number of units sold by the firm from which consumer x purchases. A similar utility definition applies for consumers purchasing the product of firm $i = B$. I model consumers as boundedly rational since they can not fully internalize the reaction of firms in the second period. Consumers expect utility due to network externalities in second period is the same as in the first period, i.e., consumers expect market shares of networks remain unchanged for this purpose.

The consumers' preferences can change. A proportion ν of consumers leave the market after purchasing in period 1. A proportion μ of consumers have changing preferences at second period. The remaining proportion $(1 - \mu - \nu)$ of consumers have the same preferences in both periods.

I solve the model using backward induction. In the second period, I set the indifference conditions for each consumer type. From these conditions I can establish the expressions for demand. Using the demand, I solve the maximization problem of the firms and find the equilibrium prices of second period. Then, I solve the first period setting the indifference condition of consumers and solving the maximization problem of the firms to find first period prices. Please find the details at Appendix A.15.

Equilibrium prices and market shares follow:

$$p_1^A = p_1^B = c+1+\frac{2\lambda\mu s(\mu s+1)}{3(\mu+\nu)}+\frac{1}{3}\left\{(1-\mu-\nu)\lambda - 4\lambda\mu s - \alpha\left[3(1-\mu-\nu)\lambda + 3\lambda\mu s + 3 + 2\lambda\right]\right\},$$

$$p_2^A = p_2^B = c + \frac{1}{\mu+\nu}.$$

The results show that in a symmetric equilibrium, only first period price reacts to switching costs. If network externality is strong, switching costs play a secondary role. Firms try to poach consumers by decreasing prices, knowing consumers expect the same utility due to network externality in the second stage. If network externality is not strong, firms prefer to exploit consumers in the first place because consumers are more prone to switch since they derive less value from the size of the firm and become more sensitive to price. Consumers with changing preferences are eager to pay a higher price even if switching cost are high because changing of operator provides more utility.

I extend this model by considering fully rational consumers, in a similar fashion to Doganoglu and Grzybowski (2004). In this model, consumers are fully aware of firms' strategy in second

period. They form an accurate expectation about market shares, such that utility due to network externality becomes relevant in the profit maximization problem of firms. Absent network externalites, first period equilibrium prices decrease on switching costs, an invest strategy to exploit profits in second period. If the model accounts for network externalities, firms compete more fiercely in second period, because consumers are eager to move to the biggest firm. Price change due to switching costs follow a similar argument as for the boundedly rational consumers model. See full model details in Appendix A.16.
The telecommunications market I analyse is subject to network externalites. Network operators offer lower tariffs for calls to destination in the same network. Therefore, consumers are more attracted to networks where potential call destinations (friends and relatives) are subscribed to. Both boundedly rational and fully rational models predict prices increase in switching costs.

4.3. Data and Descriptive Statistics

The data I use in this work includes quarterly fixed and mobile operator-specific revenues and users. Firms report this information to the CRC, spaning 28 quarters from 2012-Q1 through 2018-Q4.[8] There are 6 mobile and 24 fixed operators in the market at the begining of the time frame, turning into 10 mobile and 17 fixed operators by the end. I also collect yearly data for regulated access charge tariffs and regulated national roaming tariffs.
The revenues and users for voice and internet services show a contrasting behavior during the time frame under analysis. Fixed voice revenues decreased from 0.65 trillion to 0.54 trillion Colombian pesos. Similary, revenues for mobile services went down from 2.09 trillion to 1.04 trillion Colombian pesos. Meanwhile, fixed internet revenues increased from 0.43 trillion to 0.95 trillion Colombian pesos, and mobile internet revenues went up from 0.26 trillion to 1.38 trillion Colombian pesos. Fixed voice users slightly decreased from 7.09 to 6.97 million and mobile voice users increased from 47.17 million to 62.22 million. Fixed Internet subscribers rised from 2.85 to 5.37 million and mobile Internet consumers went up from 4.68 to 24.97 million.
Voice revenue sources for fixed operators comprise calls classified as local, extended local, national long distance, and international long distance.[9,10] In particular, the local revenue for Telefonica jumps around 700% from 2015-Q2 to 2015-Q3. I inspect an alternative data source, provided by MINTIC, that discriminates local and extended local revenue to explain the reason of this behavior.[11] Comparing CRC and MINTIC data allows me to conclude that the jump is explained due to the fact that local revenues for Telefonica before 2015-Q3

[8]Information available at https://postdata.gov.co/.
[9]Extended local call means a call whose destination is a number located in any municipality located in the same department of the origin municipality of the call.
[10]Local calls revenue information includes revenue from extended local calls.
[11]See https://colombiatic.mintic.gov.co/.

only include local extended calls earnings.[12] I fix this issue by merging CRC and MINTIC data sources to consolidate fixed voice revenues of Telefonica.

I use revenue and subscribers information to build the quarterly average revenue per user (ARPU) for each network. ARPU reflects the spending of each consumer independent of the amount of minutes the consumer talks. I take this approach to account for the willingness consumers have to hold a fixed or mobile service. For example, the usage (amount of minutes) of fixed voice services displays a downward trending behavior, but the number of users do not decline.[13] This means that consumers are willing to hold a fixed line even if the usage is low. For the case of mobile services, both the number of consumers as well as the amount of minutes increase during the time period of interest, however I focus on the information about consumers to emphasize the analysis on the PC ban on switching decisions rather than on consumption choices.

Previous data sheds light on revenue figures in the market at the retail level, however, additional data is used to take the wholesale level into account. When voice calls are originated in one network and terminated in other network, the originating firm has to pay a fee to the receiving one for terminating the call; this payment is known as access charge. Since access charges are regulated, the CRC updates and publishes these tariffs on a yearly basis.[14] The CRC uses a mathematical framework modeling a theoretical firm that carries voice traffic. Thereby, the purpose of this methodology is to compute the termination price set by an efficient firm for terminating calls. This information serves the purpose of controlling how the revenue per user can drop due to declining costs per minute in the market.

One additional source of revenue at the wholesale level comes from national roaming access. This arrangement comes into play when any network intends to offer its services in an area not covered by its own antennas: The interested operator can ask the incumbent in the area to rent the radio access infrastructure. The renting fees are regulated and updated on a yearly basis by the CRC. The regulator uses a similar methodology to compute and update national roaming rates as for access charges.

In addition to information about revenues and subscribers, I also collect information about the amount of granted radio spectrum. Radio spectrum availability for each network is obtained from technical reports issued by the regulator, ANE and CRC (2015), and CRC (2019). Spectrum is a scarce resource that operators can use only under authorization of the central government. This authorization is granted through an auction process in which the winner firms are given the permission to use one portion of the spectrum. To account for the availability of this resource, I build a database with the amount of spectrum per band (in MHz) each operator has been granted with. Spectrum is costly, but it also provides network with an essential input to widen their service coverage. More spectrum allows operators to

[12] The difference between CRC and MINTIC data for local revenues of Telefonica is less than 1.5% for the period 2015-Q3 through 2016-Q4.

[13] The amount of fixed voice minutes went from 9.10 billion to 4.01 billion minutes in the time lapse available.

[14] Information available at https://www.crcom.gov.co/es/pagina/valores-regulados.

serve more consumers, which could explain decreasing ARPU.

Variable Name	Description
ARPU	Quarterly average revenue per user. It is computed separately for voice and Internet services.
Ban	Dummy variable that represents whether PC are active or not. It is set to 1 before 2014-q3, to convey the idea of active PC before the ban.
Treated	Dummy variable that identifies the treated companies. I set to 1 for mobile operators.
Access Charge	Regulated fee paid by mobile operators for termination of calls in rival networks.
Exchange Rate	Quarterly average COP/USD exchange rate.
RAN Data	Regulated fee paid by mobile operators for use of access infrastructure of rival networks to allow Internet (data) traffic.

Table **4-1**.: Data description for selected variables.

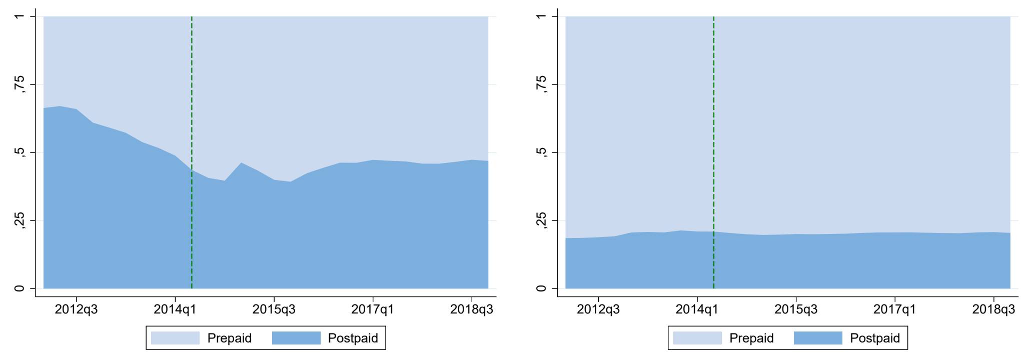
Variable Name	Voice		Internet	
	Mean	Std. Err.	Mean	Std. Err.
ARPU	69621.5	135698.4	1323227	4940984
Ban	0.65	0.48	0.75	0.44
Treated	0.3	0.46	0.24	0.43
Access Charge	45.52	32.14	-	-
Exchange Rate	2475.03	535.84	2552.73	518.99
RAN Data	-	-	3.44	7.83

Table **4-2**.: Descriptive statistics.

4.4. The Colombian Telecommunications Market

The telecommunications industry –fixed and mobile– in Colombia is a relevant economic activity in the country. The share of telecommunications in the domestic GDP amounts to 2.3% by 2012-Q1, a similar figure of other salient sectors such as energy (2.2%) and coal and metals mining (2.0%).

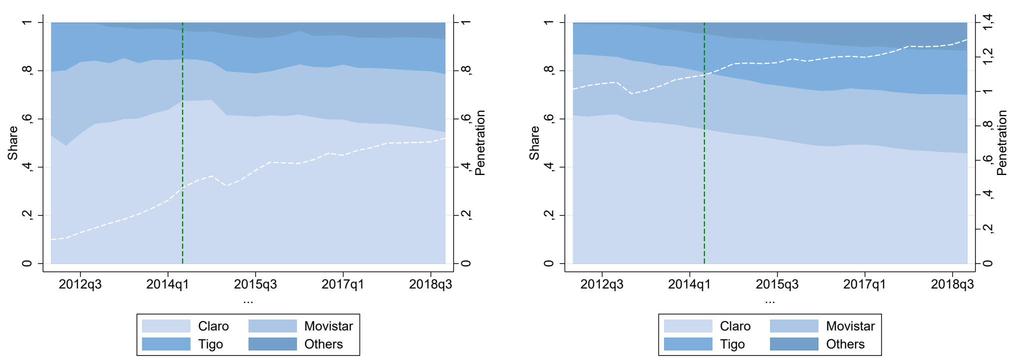
The mobile market is an oligopoly with three large firms holding more than 97% of market share and several small networks. For voice services, 80% of consumers belong to the prepaid segment, while the remaining 20% are postpaid consumers. This figure has remained roughly the same since 2012-Q1 (see Figure **4-1**b). Meanwhile, mobile Internet access share for prepaid subscribers increased from 10% in 2012-Q1 to more than than 60% in 2014-Q4 and remains stable around 55% ever since (see Figure **4-1**a). Claro is the network with the largest share of consumers in both Internet access and voice services, followed by Telefonica, Tigo, and others (see Figure **4-2**). There is however, a sustained increase of the market share for the group of other operators, going from less than 2% in 2012-Q1 to 10% by 2017-Q4 for both Internet and voice services. Service penetration rates are also a key indicator of the market: voice service has already surpassed 100% rate, while Internet penetration has been steadily increasing from 10% in 2012-Q1 to 50% in 2017-Q4.



(a) Internet access. (b) Voice.

Vertical green dashed lines represent the time at which permanence ban was officially announced by the regulator.

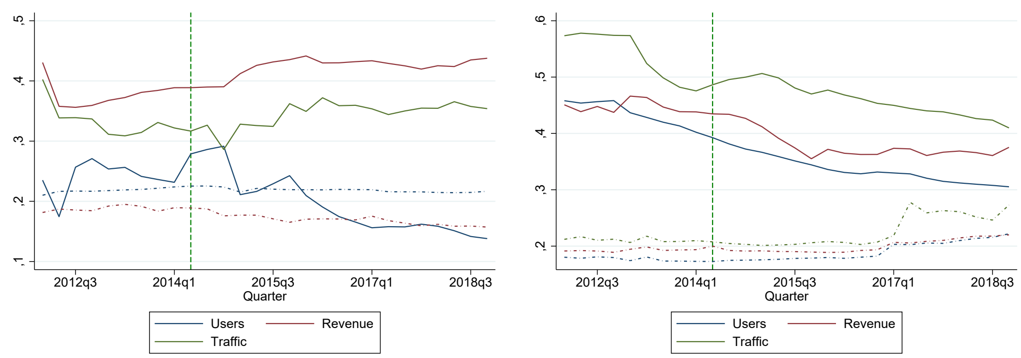
Figure **4-1**.: Mobile prepaid/postpaid users shares.
Source: MINTIC and author's calculations.



(a) Internet access. (b) Voice.

Figure **4-2**.: Mobile users market shares (left axis) and penetration (right axis).
Source: MINTIC and author's calculations.

The Herfindahl-Hirschman Index (HHI) is an important indicator of the behavior of the market. HHI trends are tracked in terms of consumers, traffic and revenue. The evolution of this indicator shows that the concentration for Internet access has remained high along the period under analysis, while the evidence for voice service states that market concentration has decreased for all the indicators considered, particularly traffic shares, as displayed with solid lines in Figure **4-3**.[15]



(a) Internet access. (b) Voice.

Solid lines represent mobile market and dashed lines represent fixed market.

Figure **4-3**.: HHI evolution of telecommunications services in Colombia.
Source: MINTIC and author's calculations.

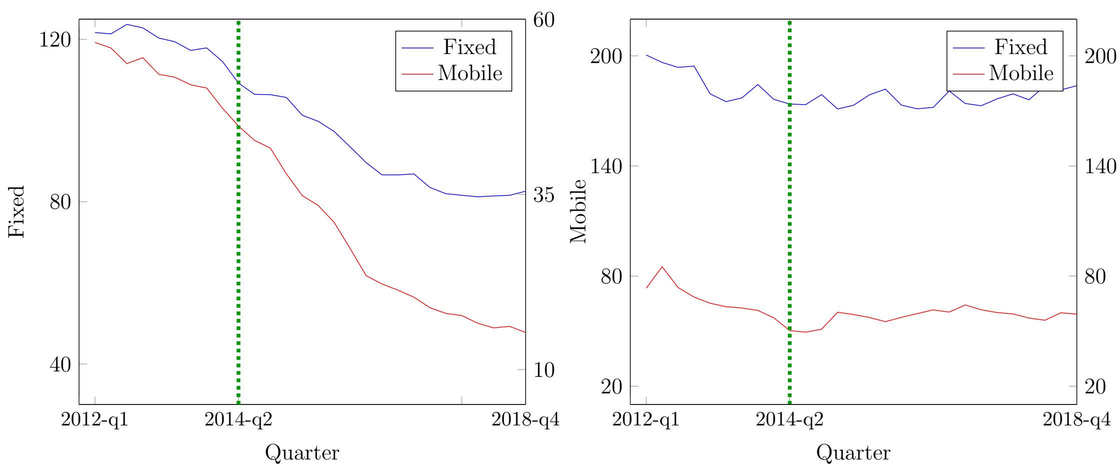
Given the high concentration of consumers in the market, the CRC has made decisions to encourage competition. Effective since january 2013, the regulator set a limit on the retail price differential between on-net and off-net calls offered by the largest operator. This decision mandates Claro to set off-net prices lower to or equal than on-net prices. After this regulation, the amount of quarterly voice traffic generated by this network droped 34.8% from 2012-Q4 to 2013-Q4, improving the traffic HHI indicator, as displayed in the solid green plot of Figure **4-3**b. One additional regulation mandated Claro to offer lower access charges to its competitors, starting on 2012-Q1 going through 2014-Q4. This decision was motivated to encourage the off-net price reduction of Claro's rivals and prevent the strengthening of tariff-mediated network externalities.[16] Further regulation to foster competition relates to switching barriers.

There are three main decisions made by the CRC to reduce switching barriers in the market. First, since august 2011, the regulator mandated all the mobile operators to implement NP aiming to allow consumers to switch among different operators keeping their phone number. Moving across networks with the same number is an advantage for consumers only if they can also use the same handset, CRC (2011). It is, however, not the case at the time when NP was mandated in Colombia. For this reason, the regulator made another decision to reduce switching barriers: effective on may 2011, prohibit the use of any kind of handset blocking. This means that any mobile phone can be used to make calls on any network across the country. The third decision, effective on the first quarter of 2014, is the prohibition of PC in postpaid contracts for mobile telecommunications services.

[15] For example, antitrust authorities in the United States consider a market highly concentrated when HHI > 2500. See the criterion here.

[16] See Resolution CRC 4002, page 75.

The time frames before and after the PC ban, highlighted with the vertical dashed green line in figures **4-1** to **4-3**, do not display remarkable changes. However, the ARPU trends for mobile voice show a steep decline after 2014-Q1, Figure **4-4**a. In the two years period 2012-Q1 to 2013-Q4, mobile ARPU decreased 8.47%. Meanwhile, for the two years period 2014-Q1 to 2015-Q4, the ARPU went down 28.70%. In the case of fixed voice, ARPU displays a persistent downward sloping trend, with bi-anual modest price drops around 6%. For mobile and fixed Internet access, it is evident how the ARPU figures follow a similar trend both before and after the PC ban, Figure **4-4**b.



(a) Voice. (b) Internet access.

Vertical green dashed lines represent the time at which permanence ban was officially announced by the regulator. ARPU displayed in constant thousand Colombian pesos of 2012-Q1.

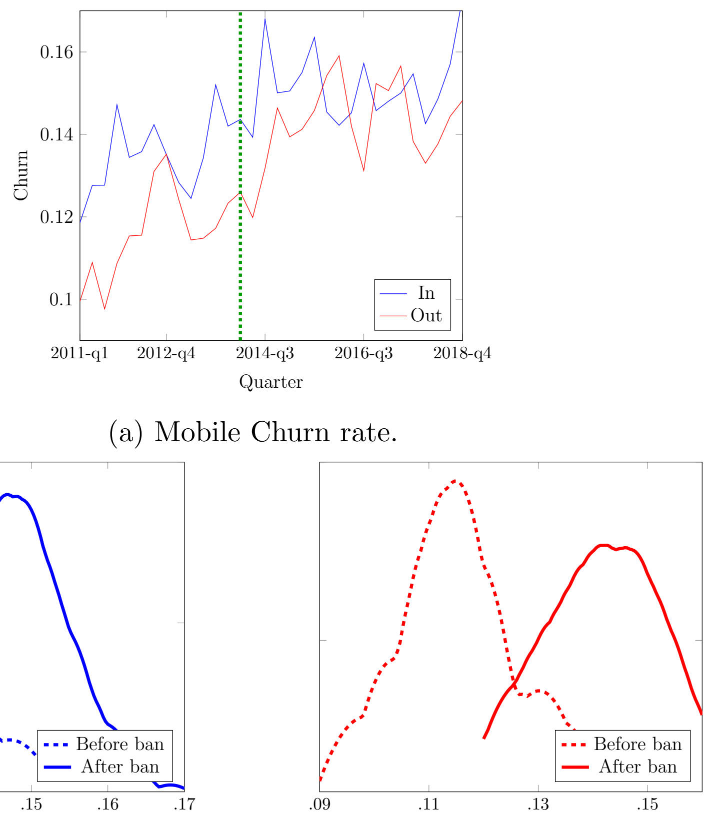
Figure **4-4**.: Quarterly ARPU (thousands COP) for telecommunications services in Colombia.

Source: MINTIC and author's calculations.

Preventing networks from using permanence clauses translates into the reduction of a switching cost that allows consumers to move more freely among different operators. The Figure **4-5** shows evidence about this insight. The figure displays how subscription rate and de-subscription rate (churn) behaves along time. These rates show that mobile consumers were switching more heavily among operators the time period after the PC ban. Figure **4-5**a shows the evolution over time for churn rates, where it is evident that churn in was located around 12% and churn out was located around 11% before the ban.[17] After the ban, Both rates settled after two quarters around 15%. These figures are more easily displayed in Figure **4-5**b and Figure **4-5**c: dashed plots correspond to churn rate distributions before the ban, while dotted plots reflect the distribution for the rates after the ban.[18]

[17] For 2011-q4 and 2013-q1, Comcel performed a couple of consumer database debugs, therefore the churn is adjusted to be the average between the previous and the next quarter for each period. Further details here.

[18] Performing a Kolmogorov-Smirnov test on each pair of distributions, it can be said that the distributions



(a) Mobile Churn rate.

(b) Churn in rate density.

(c) Churn out rate density.

Figure **4-5**.: Quarterly and density behavior for churn in and churn out rates for mobile services.

Source: MINTIC and author's calculations.

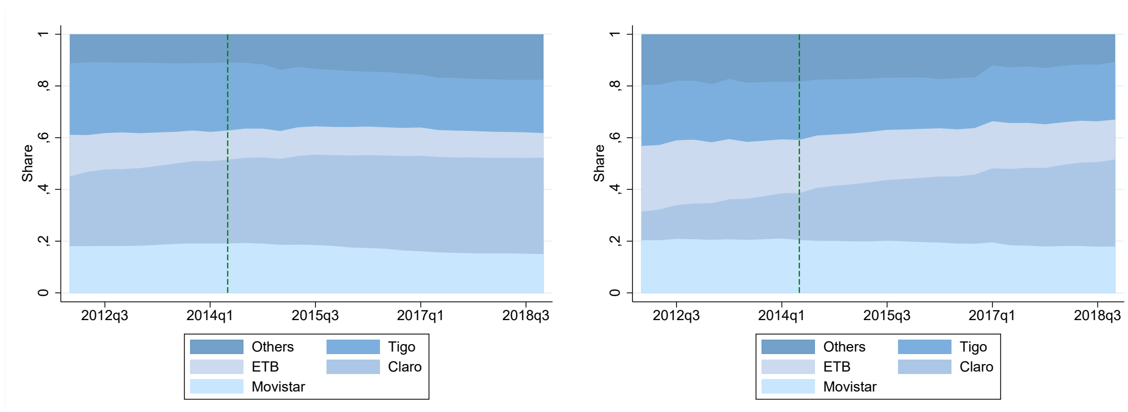
Turning into the fixed telecommunications services, this market resembles the structure of the mobile market. Four major operators shape the market, accounting for more than 80% of the users. These networks deliver fixed services across the country and also offer mobile services. The market shares for the largest operators remain stable for the period under analysis, while the market share for others locates around 18% both for Internet (Figure **4-6**a) and voice services (Figure **4-6**b). HHI levels are lower than the case of mobile markets (see dashed lines in Figure **4-3**a and Figure **4-3**b). This is explained by the presence of several small fixed operators that deliver services within a bounded geographic area. In addition to this, the evolution of HHI indicators display less fluctuations than the mobile market. This market structure has motivated some regulatory decisions.

The relevant regulations in the fixed market are related to wholesale tariffs and switching barriers. The interchange of traffic between networks is a concern for the regulator, since the receiver network holds a dominant position for call termination. Therefore, termination tariffs in the wholesale market are regulated and updated on a yearly basis following a similar

of churn rates after the ban are statistically significant (p-value less than 0.04%) higher than before the prohibition.

methodology as for mobile networks.

In addition to wholesale regulation, the CRC has attempted to enact obligations aiming to decrease switching barriers. The implementation of NP did not take place because the cost of deployment is higher than the expected benefits, CRC (2017). The regulator also conducted the analysis to remove the PC enshrined in fixed services contracts. After a thorough analysis, the CRC decided to avoid the prohibition of PC, CRC (2015). This decision is motivated on the argument that operators need to recover the expenses they incur to install and activate a new subscriber. This way, consumers in the fixed telecommunications market sign contracts with PC involved. In summary, the consumers of fixed services in Colombia remain subject to switching barriers due to absence of NP and the presence of PC in the service contracts.



(a) Internet. (b) Voice.

Figure **4-6**.: Market shares for fixed services.

4.5. Empirical Strategy

I employ a difference-in-differences approach to identify the effect of the PC ban regulation in the Colombian mobile telecommunications market. I compare the revenue per user for each network in the mobile market before and after the prohibition to the same indicator for the case of fixed telecommunications services in the country.

I propose a difference-in-differences model to estimate the causal effect of PC banning on the mobile market:

$$y_{it} = \alpha T_i + \beta D_t + \gamma T_i D_t + \boldsymbol{x}_{it}\boldsymbol{\psi} + \delta_i + \mu_t + \varepsilon_{it}. \tag{4-1}$$

The dependent variable, y_{it}, represents ARPU for the service offered by network i at quarter t. T_i is a dummy variable representing the treated group, it means, whether firm i belongs to the group of networks subject to PC ban. D_t is a dummy variable that indicates when the PC ban is active, $\boldsymbol{x}_{it}$ is a vector of firm attributes, δ_i accounts for network fixed effects constant through time, while μ_t accounts for time fixed effects constant across firms and ε_{it} is the error term. I use fixed effects to account for the unobservable heterogeneity at the firm level. Likewise, time fixed effects account for unobservable heterogeneity across firms. In this specification, γ is the coefficient of interest, since it describes the magnitude and direction of the effect of the permanence clauses on the ARPU for mobile services. This empirical approach needs the definition of the treatment and control groups. PC ban regulation is targeted to mobile networks, therefore I use firms offering mobile services as treated subjects. The counterfactual is a set of firms the resemble the strategic behavior of mobile networks. To assess the effect of the prohibition, I use the firms in the fixed market as control subjects. In this setting, it is relevant to note that I consider a firm as a single player supplying either mobile of fixed services. Therefore, if any commercial brand offers both mobile and fixed services, this brand is actually regarded as a different firm in each market.

4.5.1. Identification

The PC regulation is an obligation to every mobile network in the market, which implies a challenge to look for a causal effect. Some contributions that analyze the effects of switching costs reduction, such as number portability, have resorted to panel data from different countries as counterfactuals. I look for a Colombian counterfactual market to exploit the panel data at hand.

I use the fixed telecommunications market as a counterfactual for the reasons that follow. The market structure for fixed services is similar to mobile services, since few firms hold the largest share of consumers. This structure behaves similarly over time for both markets, with the main movements in the group of other operators. In addition to the market structure, the contracts for services in the fixed market include PC along the whole time period of analysis. This is a key feature given it is precisely the service attribute whose change I analyze in this paper.

Regarding switching costs other than permanence clauses, NP and terminal compatibility are not relevant for consumers in the fixed market. In section 4.4, I show that fixed services consumers do not value to keep their phone number when switching among networks. Terminal compatibility is given since any fixed telephone can be used to make calls over any fixed network, therefore the terminal device is not a barrier to switch operator. This means that fixed services share technical attributes with mobile services in the sense that consumers are not attached to their fixed operator due to phone number or technical compatibility.

One additional reason to consider the fixed market as a counterfactual, relates to investment drivers. The infrastructure that supports fixed and mobile services (access appliances, optical fiber, among others) is mostly manufactured overseas, which implies that local companies import equipment to upgrade and sustain their business. For this reason, fixed services face the same dependency on imports as mobile. This is relevant since in the time period of analysis the colombian currency depreciated 65%, against US dollar, putting a higher cost burden to every network operator. I gather evidence from this claim in the data, by analysing if exchange rate has any relevance on the results of the empirical strategy.

I now claim the ground for the exogeneity of the prohibition of PC, given the counterfactual I use for the analysis. The discussion prior to final regulation started on June 2013 and extended through December the same year. The CRC anounced on November that the decision would take place on March 26, 2014, a commited deadline sice the enactment was indeed published in March 25, 2014.[19] This commitment is a serious signal to the market about the upcoming regulation. The regulation is effective if consumers are aware of it, not only firms. The press announcement about PC ban encourages consumers to look for better deals.[20] These press announcements spread the date at which regulation takes place, july 01, 2014.

Under the stated claims, I look for causal evidence using the available information on ARPU since this figure represents the upfront tariffs that consumers take into account to decide switching from one operator (or plan) to another. This isolates the switching behavior and allows me to avoid any confounders related to consumption behavior. This is an important feature of the specification I propose since for the case of fixed voice service, the available data displays an steady decline in the total amount of minutes consumed in the market, which reflects a consumption trend different to mobile voice service.

The substitutability between voice services and Internet access services threatens the identification strategy. Different regulators across the globe have addressed this question, with mixed results.[21] Particularly, for the case of Colombia, usability of OTT platforms for the purpose of voice calls is not considered to substitute mobile voice services. This conclusion is supported on the limited coverage of fourth generation networks, limited affordability of smartphones for consumers and the suparssing quality features of traditional voice services over OTT voice services, CRC (2016). Therefore, the analysis of the PC ban on fixed and Internet access services does not take into account any substitution between them.

There is an additional threat to identification. PC ban makes mobile services more attractive to consumers. It can be the case that consumers of fixed services substitute their subscriptions to mobile services, looking for reduced switching costs. If this mechanism is at work, there should be evidence of consumers dropping fixed services towards mobile services after the ban. If this is the case, the effect can be underestimated or even turn out to be non significant.

I claim there is no substitution between fixed and mobile services due to the ban. I support

[19] See the anouncement of the CRC here.

[20] Semana magazine: (link), El Tiempo (link), El Espectador (link1, link2), RCN (link), Caracol TV (link).

[21] In countries such as Korea, Oman and Sweeden, mobile voice and Internet access services are considered to belong to the same relevant market, while for countries such as Jordania these services are considered to consitute separate relevant markets.

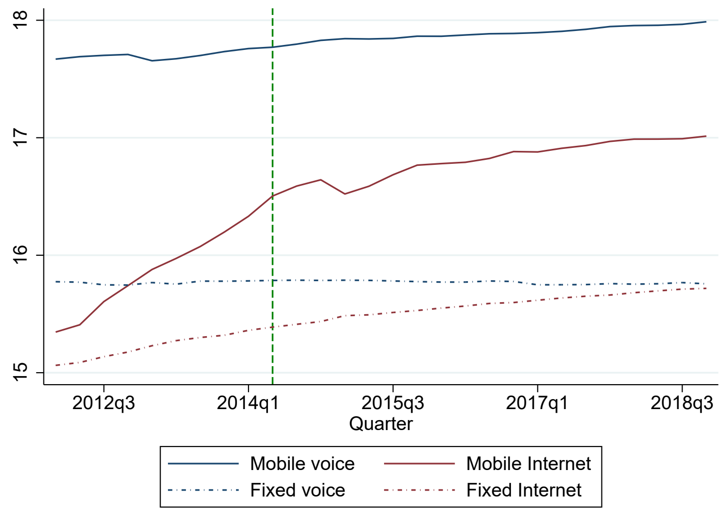


Figure **4-7**.: Mobile and fixed users trends.
Source: MINTIC and author's calculations.

this exploring the trends of consumers subscribed to fixed and mobile services around the time of the ban. The results are displayed in Figure **4-7**. The number of subscribers of mobile voice services is steadily growing in the period of interest: there is no any jump or sudden change in this trend. For the case of fixed voice subscriptions, the series does not display any sudden change after the ban. In the case of Internet access services, both fixed and mobile subscribers series are growing and do not display any trend change. The mobile Internet access subscribers series drops in the first quarter of 2015, an event that remains unexplained by the operator according.[22] Therefore, there is no evidence to state that consumers are unsubscribing fixed services to subscribe mobile services instead.

Firms providing both fixed and mobile services pose an additional identification threat. Given the regulation in the mobile services market, these firms might be willing to use the fixed services to seek for more rents in that market if mobile competition becomes fiercer. To test for such a scenario, I run the following specifications: estimate the model removing mobile and fixed firms that provide services in both markets and, to make these specifications more stringent, set the regulation dummy variable in different quarters both before and after the regulation date. I then check for the significance for the relevant coefficient in the model.

4.6. Results

The regression results for equation (4-1) are displayed in Table **4-3**. The column (1) provides evidence about the causal effect of the PC prohibition on the ARPU for the mobile postpaid service when the fixed voice service is used as the counterfactual. Additional specifications

[22] According to information provided by CRC.

(columns (2) - (3) in Table **4-3**) show that the magnitude and significance of the coefficient γ (Treated*Ban in the table) persists among different specifications. This allows to conclude that ARPU for mobile voice services was higher before the ban rule. Including additional variables that could potentially have an effect on the average revenue for both fixed and mobile services such as (asymmetrical) access charge rates, and excahnge rate, does not significantly shift the magnitude nor statistical significance of the coefficient for the PC ban effect in the mobile postpaid voice market. In average, the quarterly ARPU for mobile voice service decreases USD 8.61 due to the PC ban regulation.[23]

Table **4-3**.: Differences in differences specifications for voice and Internet access service.

	Dependent variable: arpu					
	Voice service			Internet access		
	(1)	(2)	(3)	(4)	(5)	(6)
Treated*Ban	19749,8*	18990,1*	18990,1*	70334,6	70334,6	231564,5
	(11338,8)	(11280,1)	(11280,1)	(219020,0)	(219020,0)	(234528,0)
Ban	17153,5	-25629,2	-25519,8	-1362408,4**	-1446240,8**	-1597179,4**
	(14232,4)	(21651,5)	(20793,6)	(653641,4)	(688804,8)	(709537,2)
Treated	-43267,9***	-45882,3***	-45882,3***	243705,7	243705,7	473216,9**
	(13053,0)	(13910,8)	(13910,8)	(193252,9)	(193252,9)	(224274,6)
Access Charge		367,3	367,3			
		(258,7)	(258,7)			
Exchange Rate			1,01		-776,6*	-806,2*
			(16,3)		(425,0)	(426,4)
RAN Data						-29785,0***
						(8781,1)
Constant	64723,2***	59558,4***	57624,2	1241634,7**	2724262,3**	2846319,6**
	(14906,0)	(16687,0)	(42780,3)	(524790,1)	(1309756,0)	(1317775,8)
Operator FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	757	757	757	891	891	891
R^2	0,329	0,329	0,329	0,814	0,814	0,815

Standard errors in parentheses.

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Columns (1)-(3) display results for regressions using voice data. Columns (4)-(6) display results for regressions using Internet access data.

I give a stronger support to the causal effect of the PC ban by constructing a placebo test to evaluate the effect of the ban had it been enacted one, two and three quarters before the

[23]I use the quarterly average exchange rate applicable on 2014-Q1.

actual date of the starting point of the ban. The results for this placebo test are displayed in columns (1)-(3) in Table **A-1**. The magnitude and the statistical significance for the coefficient of interest are not significant . Therefore, this robustness check supports the claim about the effect of the prohibition of PC on the mobile market.
For the case of Internet access, there is no visual evidence of a change in the bahavior of the ARPU for the mobile market, when considering the fixed Internet access market as the counterfactual (Figure **4-4**b). I use equation (4-1) to look for causal evidence about the likely effect of PC ban in the mobile Internet access market. The basic regression in the column (4) of the Table **4-3**, shows that the coefficient for PC Ban is not significant, a first clue about the non-existent effect of the prohibition on the mobile postpaid Internet access market. Furthermore, additional variables that could potentially affect the behavior of mobile and fixed Internet ARPU are included, however the coefficient for the causal effect remains in a non-significant level (columns (5)-(6) of Table **4-3**). Therefore, I do not find any evidence of an effect of PC banning in the Internet access market. The same placebo tests as for voice service were conducted (see columns (4)-(6) of Table **A-1**), leading to expected results, i.e., no significant effect.
Removing mobile and fixed firms that offer services in both markets, leads to similar results. For voice services, the effect becomes stronger one quarter after regulation, as displayed in Table **A-4**. For mobile Internet services, I do not find any evidence of price effects after the regulation, see Table **A-6** and Table **A-7**.

4.7. Conclusion

This work has addressed the question about the effect of the prohibition of using permanence clauses in contracts for mobile services after the enactment of a regulation on the Colombian mobile telecommunications market. Using a difference in differences methodology, the results show that banning the use of permanence clauses strengthens the competitive environment in the market reflected in the decrease of the ARPU for postpaid voice service. For the case of the Internet services, there is no evidence of any effect of the ban.
The findings for the mobile postpaid voice service are robust to different specifications including asymmetric regulation of access charges and wholesale national roaming fees, as well as to spectrum allocation for voice services. Similar specifications were used for the case of mobile Internet access services, but still no robust evidence is found.
Although finding evidence of an improved competitive environment due to the decrease in the average revenue per user could be understood as a beneficial outcome for consumers, the methodology does not allow to make any statement about the overall effect in the market in terms of welfare. Thereby, further research is required to establish the welfare impact of this regulatory decision.

A. Appendix

A.1. Proof of Proposition 1

To find optimal p_{ii} both firms perform the following maximization program: $\max_{p_{ii}} \bar{\Pi}_i(\hat{x})$, which can be written as:

$$\max_{p_{ii}} \bar{\Pi}_i(\hat{x}) = \max_{p_{ii}} \int \Pi_i(x, \hat{x})\, dx = \frac{\partial}{\partial p_{ii}} \left\{ \int \Pi_i(x, \hat{x})\, dx \right\}.$$

A.1.1. Network 1 problem to decide on-net price

Solving the problem of firm 1 to set prices, network decides on-net price p_{11} performing the maximization program:

$$\begin{aligned}
\frac{\partial}{\partial p_{11}} \left\{ \bar{\Pi}_1(\hat{x}) \right\} &= \frac{\partial}{\partial p_{11}} \left\{ \int_{-\hat{x}}^{\hat{x}} \Pi_1(x, \hat{x})\, dx \right\} \\
&= \frac{\partial}{\partial p_{11}} \left\{ \int_{-\hat{x}}^{\hat{x}} \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f\, dx \right\} = 0 \\
&= \frac{\partial}{\partial p_{11}} \left\{ \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)(p_{11} - c_{11})q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c_{12})q(p_{12}) \right. \\
&\left. + F_1 + [1 - G(\hat{x}|x)]\,(a - c_T)q_{p21} - f_1\, dx \right\} = 0 \\
&= \frac{\partial}{\partial p_{11}} \left\{ \hat{x}(F_1 - f_1) + L_{11}(\hat{x})(p_{11} - c_{11})q(p_{11}) + \right. \\
&\left. L_{12}(\hat{x})(p_{12} - c_{12})q(p_{12}) + L_{21}(\hat{x})(a - c_0)q(p_{21}) \right\} = 0
\end{aligned} \tag{A-1}$$

where I have defined:

$$L_{11}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)\, dx \tag{A-2}$$

$$L_{12}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} [1 - G(\hat{x}|x)]\, dx \tag{A-3}$$

$$L_{21}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} [1 - G(\hat{x}|x)]\, dx. \tag{A-4}$$

Performing the derivative stated at equation (A-1), it is possible to establish the condition from which p_{11} can be found:

$$\frac{\partial \overline{\Pi}_1}{\partial p_{11}} = \hat{x}\frac{\partial F_1}{\partial p_{11}} + L_{11}(\hat{x})q(p_{11}) + L_{11}(\hat{x})(p_{11} - c_{11})q'(p_{11}) = 0 \tag{A-5}$$

$$\begin{aligned}\frac{\partial F_1}{\partial p_{11}} &= \frac{\partial}{\partial p_{11}}\left\{F_2 + u_1(\hat{x},\hat{x}) - u_2(\hat{x},\hat{x}) - \tau\hat{x}\right\}\\ &= \frac{\partial u_1(\hat{x},\hat{x})}{\partial p_{11}}\\ &= G(\hat{x}|\hat{x})\left[v'(p_{11}) + \gamma u'(q_{11})q'(p_{11})\right].\end{aligned}$$

Substituting $\partial u_1(\hat{x},\hat{x})/\partial p_{11}$ into (A-5):

$$\frac{\partial \overline{\Pi}_1}{\partial p_{11}} = \hat{x}G(\hat{x}|\hat{x})\left[-q(p_{11}) + \gamma p_{11}q'(p_{11})\right] + L_{11}(\hat{x})q(p_{11}) + L_{11}(\hat{x})(p_{11} - c_{11})q'(p_{11}) = 0$$

and defining

$$\hat{L}_{11} = \hat{x}G(\hat{x}|\hat{x}) \tag{A-6}$$

first order condition follows:

$$q(p_{11})\left[-\hat{L}_{11} + L_{11}(\hat{x})\right] + q'(p_{11})\left[\hat{x}G(\hat{x}|\hat{x})\gamma p_{11} + L_{11}(\hat{x})(p_{11} - c_{11})\right] = 0$$

$$\begin{aligned}q'(p_{11})\left[\hat{L}_{11}(\hat{x})\gamma p_{11} + L_{11}(\hat{x})(p_{11} - c_{11})\right] &= q(p_{11})\left[\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})\right]\\ q'(p_{11})\left[\hat{L}_{11}(\hat{x})\gamma + \frac{L_{11}(\hat{x})(p_{11} - c_{11})}{p_{11}}\right] &= \frac{q(p_{11})}{p_{11}}\left[\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})\right]\\ \frac{L_{11}(\hat{x})(p_{11} - c_{11})}{p_{11}} &= \frac{q(p_{11})}{q'(p_{11})\,p_{11}}\left[\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})\right] - \gamma\hat{L}_{11}(\hat{x})\\ \frac{p_{11} - c_{11}}{p_{11}} &= \frac{q(p_{11})}{q'(p_{11})\,p_{11}}\left[\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})} - 1\right] - \gamma\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\\ \frac{p_{11} - c_{11}}{p_{11}} &= \frac{1}{\eta}\left[1 - \frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\right] - \gamma\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}.\end{aligned} \tag{A-7}$$

The latest expression is the equation (2-7) stated in Proposition 1 and can be easily written as follows:

$$p_{11} = \frac{\eta c_{11}L_{11}(\hat{x})}{\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})(1-\eta) + \eta\gamma\hat{L}_{11}(\hat{x})}, \tag{A-8}$$

or in a closed form:

$$p_{11} = \frac{\eta\, c_{11}\,\left((1-\lambda)\hat{x}^2 + \lambda\left(\hat{x} - \frac{\epsilon}{4}\right)\right)}{(\eta - 1)\left[(1-\lambda)\hat{x}^2 + \lambda\left(\hat{x} - \frac{\epsilon}{4}\right)\right] + (1+\gamma\eta)\left[(1-\lambda)\hat{x}^2 + \frac{\lambda\hat{x}}{2}\right]}. \tag{A-9}$$

A.1.2. Network 2 problem to decide on-net price

Solving the problem of firm 2 to set prices, network decides on-net price p_{22} performing the maximization program:

$$\begin{aligned}\frac{\partial}{\partial p_{22}}\left\{\bar{\Pi}_2(\hat{x})\right\} &= \frac{\partial}{\partial p_{22}}\left\{\int \Pi_2(x,\hat{x})\,dx\right\}\\ &= \frac{\partial}{\partial p_{22}}\left\{\int_{\hat{x}}^{1} \pi_2(x,\hat{x}) + F_2 + R_{21}(x,\hat{x}) - f_2 dx + \right.\\ &\left.\int_{-1}^{-\hat{x}} \pi_2(x,\hat{x}) + F_2 + R_{21}(x,\hat{x}) - f_2 dx\right\}\\ &= \frac{\partial}{\partial p_{22}}\left\{(1-\hat{x})(F_2-f_2) + L_{22}(\hat{x})(p_{22}-c_{22})q(p_{22})\right.\\ &\left. + L_{21}(\hat{x})(p_{21}-c_{21})q(p_{21}) + L_{12}(\hat{x})(a_2-c_0)q(p_{12})\right\} = 0.\end{aligned} \tag{A-10}$$

Performing the derivative stated at equation (A-10), it is possible to establish the condition from which p_{22} can be found:

$$\frac{\partial\overline{\Pi}_2}{\partial p_{22}} = (1-\hat{x})\frac{\partial F_2}{\partial p_{22}} + L_{22}(\hat{x})\left[q(p_{22}) + (p_{22}-c_{22})q'(p_{22})\right] = 0$$

$$\begin{aligned}\frac{\partial F_2}{\partial p_{22}} &= \frac{\partial}{\partial p_{22}}\left\{F_1 + u_2(\hat{x},\hat{x}) - u_1(\hat{x},\hat{x}) - \tau(1-2\hat{x})\right\}\\ &= \left[1-G(\hat{x}|\hat{x})\right]\left(v'(p_{22}) + \gamma u'(q_{22})q'(p_{22})\right)\end{aligned}$$

defining

$$\hat{L}_{22} = (1-\hat{x})[1-G(\hat{x}|\hat{x})] \tag{A-11}$$

and substituting $\partial F_2/\partial p_{22}$ into derivative equation, first order condition is written as follows:

$$\begin{aligned}\frac{\partial\overline{\Pi}_2}{\partial p_{22}} = (1-\hat{x})\left[1-G(\hat{x}|\hat{x})\right]&\left[-q(p_{22})\right.\\ &\left.+\gamma p_{22}q'(p_{22})\right] + L_{22}(\hat{x})\left[q(p_{22}) + (p_{22}-c_{22})q'(p_{22})\right] = 0\end{aligned}$$

$$\begin{aligned}L_{22}(\hat{x})(p_{22}-c_{22})q'(p_{22}) &= \hat{L}_{22}(\hat{x})q(p_{22}) - \hat{L}_{22}(\hat{x})\gamma p_{22}q'(p_{22}) - L_{22}(\hat{x})q(p_{22})\\ L_{22}(\hat{x})(p_{22}-c_{22}) &= \hat{L}_{22}(\hat{x})\frac{q(p_{22})}{q'(p_{22})} - \hat{L}_{22}(\hat{x})\gamma p_{22} - L_{22}(\hat{x})\frac{q(p_{22})}{q'(p_{22})}\\ \frac{p_{22}-c_{22}}{p_{22}} &= \frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\frac{q(p_{22})}{q'(p_{22})p_{22}} - \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})} - \frac{q(p_{22})}{q'(p_{22})p_{22}}\\ \frac{p_{22}-c_{22}}{p_{22}} &= -\frac{1}{\eta}\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\frac{q(p_{22})}{q'(p_{22})p_{22}} + \frac{1}{\eta} - \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\\ \frac{p_{22}-c_{22}}{p_{22}} &= \frac{1}{\eta}\left[1 - \frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\right] - \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}.\end{aligned} \tag{A-12}$$

The latest expression is the equation (2-8) stated in Proposition 1 and can be easily written as follows:

$$p_{22} = \frac{\eta\, c_{22}\, L_{22}}{(\eta-1)L_{22} + (1+\gamma\eta)\hat{L}_{22}}. \tag{A-13}$$

or in a closed form:

$$p_{22} = \frac{\eta\, c_{22}\, \left((1-\lambda)(1-\hat{x})^2 + \lambda\left(1-\hat{x}-\frac{\epsilon}{4}\right)\right)}{(\eta-1)\left[(1-\lambda)(1-\hat{x})^2 + \lambda\left(1-\hat{x}-\frac{\epsilon}{4}\right)\right] + (1+\eta\gamma)\left[(1-\hat{x})\left(1-\hat{x}-\lambda\hat{x}-\frac{\lambda}{2}\right)\right]}. \tag{A-14}$$

A.2. Proof of Corollary 1.1

When calling circle weight λ increases, it is of interest to establish the magnitude of change in the factor accompanying call externality in the on-net Lerner indices, thus the next derivative is of interest:

$$\frac{\partial}{\partial\lambda}\left[\frac{\hat{L}_{ii}}{L_{ii}}\right] = \frac{1}{L_{ii}^2}\left(\frac{\partial \hat{L}_{ii}}{\partial\lambda}L_{ii} - \frac{\partial L_{ii}}{\partial\lambda}\hat{L}_{ii}\right) \tag{A-15}$$

where expression inside parenthesis defines the sign of the result. This expression can be proved to be negative using definitions for $\hat{L}_{ii}$ and L_{ii}, as follows:

$$\frac{\partial \hat{L}_{11}}{\partial\lambda}L_{11} - \frac{\partial L_{11}}{\partial\lambda}\hat{L}_{11} = \frac{\hat{x}^2}{4}(\varepsilon - 2\hat{x}) < 0$$
$$\frac{\partial \hat{L}_{22}}{\partial\lambda}L_{22} - \frac{\partial L_{22}}{\partial\lambda}\hat{L}_{22} = \frac{(1-\hat{x})^2}{4}(\varepsilon + 2\hat{x} - 2) < 0$$

where inequalities follow using Assumption 1.

A.3. Proof of Proposition 2

To find optimal p_{ij} both firms perform the following maximization program: $\max_{p_{ij}} \bar{\Pi}_i(\hat{x})$, which can be written as:

$$\max_{p_{ij}} \bar{\Pi}_i(\hat{x}) = \max_{p_{ij}} \int \Pi_i(x,\hat{x})\, dx = \frac{\partial}{\partial p_{ij}}\left\{\int \Pi_i(x,\hat{x})\, dx\right\}.$$

A.3.1. Network 1 problem to decide off-net price

Solving the problem of firm 1 to set prices, network decides off-net price p_{12} performing the maximization program:

$$\begin{aligned}\frac{\partial}{\partial p_{12}}\left\{\bar{\Pi}_1(\hat{x})\right\} &= \frac{\partial}{\partial p_{12}}\left\{\int_{-\hat{x}}^{\hat{x}} \Pi_1(x,\hat{x})\,dx\right\}\\ &= \frac{\partial}{\partial p_{12}}\left\{\int_{-\hat{x}}^{\hat{x}} \pi_1(x,\hat{x}) + F_1 + R_{12}(x,\hat{x}) - f\,dx\right\} = 0\\ &= \frac{\partial}{\partial p_{12}}\left\{\int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)(p_{11}-c_{11})q(p_{11}) + [1-G(\hat{x}|x)]\,(p_{12}-c_{12})q(p_{12})\right.\\ &\left. + F_1 + [1-G(\hat{x}|x)]\,(a-c_T)q_{p21} - f_1\,dx\right\} = 0\\ &= \frac{\partial}{\partial p_{12}}\left\{\hat{x}(F_1-f_1) + L_{11}(\hat{x})(p_{11}-c_{11})q(p_{11}) + \right.\\ &\left. L_{12}(\hat{x})(p_{12}-c_{12})q(p_{12}) + L_{21}(\hat{x})(a-c_0)q(p_{21})\right\} = 0\end{aligned} \tag{A-16}$$

where $L_{11}(\hat{x})$, $L_{12}(\hat{x})$ and $L_{21}(\hat{x})$ follow the definitions given in (A-2), (A-3) and (A-4). Performing the derivative stated at equation (A-16), it is possible to establish the condition from which p_{12} can be found:

$$\frac{\partial\overline{\Pi}_1}{\partial p_{12}} = \hat{x}\frac{\partial F_1}{\partial p_{12}} + L_{12}(\hat{x})\left[q(p_{12}) + (p_{12}-c_{12})q'(p_{12})\right] = 0 \tag{A-17}$$

$$\begin{aligned}\frac{\partial F_1}{\partial p_{12}} &= \frac{\partial}{\partial p_{12}}\left\{F_2 + u_1(\hat{x},\hat{x}) - u_2(\hat{x},\hat{x}) + \tau(1-2\hat{x})\right\}\\ &= \frac{\partial u_1(\hat{x},\hat{x})}{\partial p_{12}} - \frac{\partial u_2(\hat{x},\hat{x})}{\partial p_{12}}\\ &= [1-G(\hat{x},\hat{x})]v'(p_{12}) - G(\hat{x},\hat{x})\gamma u'(q_{12})q'(p_{12}).\end{aligned}$$

Substituting $\partial u_1(\hat{x},\hat{x})/\partial p_{12} - \partial u_2(\hat{x},\hat{x})/\partial p_{12}$ into (A-17):

$$\begin{aligned}\frac{\partial\overline{\Pi}_1}{\partial p_{12}} &= \hat{x}\left\{[1-G(\hat{x},\hat{x})]\,v'(p_{12}) - G(\hat{x},\hat{x})\gamma u'(q_{12})q'(p_{12})\right\}\\ &+ L_{12}(\hat{x})\left[q(p_{12}) + (p_{12}-c_{12})q'(p_{12})\right] = 0 \Leftrightarrow\\ &- q(p_{12})\hat{x}[1-G(\hat{x}|\hat{x})] - \hat{x}G(\hat{x}|\hat{x})\gamma u'(q_{12})q'(p_{12}) + L_{12}(\hat{x})q(p_{12})\\ &+ L_{12}(\hat{x})(p_{12}-c_{12})q'(p_{12}) = 0.\end{aligned}$$

Defining:

$$\hat{L}_{12} = \hat{x}[1-G(\hat{x}|\hat{x})], \tag{A-18}$$

former equation can be written as:

$$
\begin{aligned}
q(p_{12})\hat{L}_{12} + \hat{L}_{11}\gamma u'(q_{12})q'(p_{12}) &= L_{12}(\hat{x})q(p_{12}) + L_{12}(\hat{x})(p_{12}-c_{12})q'(p_{12})\\
q(p_{12})[\hat{L}_{12} - L_{12}] + \hat{L}_{11}\gamma p_{12} q'(p_{12}) &= L_{12}(\hat{x})(p_{12}-c_{12})q'(p_{12})\\
\frac{q(p_{12})}{p_{12}}[\hat{L}_{12} - L_{12}] + \hat{L}_{11}\gamma q'(p_{12}) &= L_{12}(\hat{x})q'(p_{12})\frac{p_{12}-c_{12}}{p_{12}}\\
\frac{q(p_{12})}{q'(p_{12})\,p_{12}}\left[\frac{\hat{L}_{12}}{L_{12}} - 1\right] + \gamma\frac{\hat{L}_{11}}{L_{12}} &= \frac{p_{12}-c_{12}}{p_{12}}\\
\frac{1}{\eta}\left[1 - \frac{\hat{L}_{12}}{L_{12}}\right] + \gamma\frac{\hat{L}_{11}}{L_{12}} &= \frac{p_{12}-c_{12}}{p_{12}}.
\end{aligned}
\tag{A-19}
$$

The latest expression is the equation (2-9) stated in Proposition 2 and can be easily written as follows:

$$
p_{12} = \frac{\eta\, c_{12}\, L_{12}}{(\eta-1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}} \tag{A-20}
$$

or in a closed form:

$$
p_{12} = \frac{\eta\, c_{11}\,\left(\hat{x} - (1-\lambda)\hat{x}^2 - \lambda\left(\frac{\varepsilon}{4} - \hat{x}\right)\right)}{(\eta-1)\left[\hat{x} - (1-\lambda)\hat{x}^2 - \lambda\left(\frac{\varepsilon}{4} - \hat{x}\right)\right] + \hat{x} - (1-\lambda)\hat{x}^2 - \frac{\lambda\hat{x}}{2} - \gamma\eta\left((1-\lambda)\hat{x}^2 + \frac{\lambda\hat{x}}{2}\right)}. \tag{A-21}
$$

A.3.2. Network 2 problem to decide off-net price

Solving the problem of firm 2 to set prices, network decides off-net price p_{21} performing the maximization program:

$$
\begin{aligned}
\frac{\partial}{\partial p_{21}}\left\{\int \Pi_2(x,\hat{x})\,dx\right\} &= \frac{\partial}{\partial p_{21}}\left\{\bar{\Pi}_2(\hat{x})\right\}\\
&= \frac{\partial}{\partial p_{21}}\left\{(1-\hat{x})(F_2 - f_2) + L_{22}(\hat{x})(p_{22}-c_{22})q(p_{22})\right.\\
&\left. + L_{21}(\hat{x})(p_{21}-c_{21})q(p_{21}) + L_{12}(\hat{x})(a_2 - c_0)q(p_{12})\right\} = 0.
\end{aligned}
\tag{A-22}
$$

Performing the derivative stated at equation (A-10), it is possible to establish the condition from which p_{21} can be found:

$$
\frac{\partial\overline{\Pi}_2}{\partial p_{21}} = (1-\hat{x})\frac{\partial F_2}{\partial p_{21}} + L_{22}(\hat{x})\left[q(p_{22}) + (p_{22}-c_{22})q'(p_{22})\right] = 0
$$

$$
\begin{aligned}
\frac{\partial F_2}{\partial p_{21}} &= \frac{\partial}{\partial p_{21}}\left\{F_1 + u_2(\hat{x},\hat{x}) - u_1(\hat{x},\hat{x}) - \tau(1-2\hat{x})\right\}\\
&= G(\hat{x}|\hat{x})v'(p_{21}) - [1 - G(\hat{x}|\hat{x})]\left[\gamma u'(q_{21})q'(p_{21})\right]
\end{aligned}
$$

defining

$$\hat{L}_{21} = (1-\hat{x})G(\hat{x}|\hat{x}) \tag{A-23}$$

substituting $\partial F_2/\partial p_{21}$ into derivative equation, and using definition given in equation (A-11), first order condition is written as follows:

$$-\hat{L}_{21}(\hat{x})q(p_{21}) - \hat{L}_{22}(\hat{x})\gamma p_{21}q'(p_{21}) + L_{21}(\hat{x})q(p_{21}) + L_{21}(\hat{x})\left[(p_{21}-c_{21})q'(p_{21})\right] = 0$$

$$L_{21}(\hat{x})(p_{21}-c_{21})q'(p_{21}) = \hat{L}_{21}(\hat{x})q(p_{21}) + \hat{L}_{22}(\hat{x})\gamma p_{21}q'(p_{21}) - L_{21}(\hat{x})q(p_{21})$$

$$L_{21}(\hat{x})\frac{p_{21}-c_{21}}{p_{21}} = \hat{L}_{21}(\hat{x})\frac{q(p_{21})}{q'(p_{21})p_{21}} + \gamma\hat{L}_{22}(\hat{x}) - L_{21}(\hat{x})\frac{q(p_{21})}{q'(p_{21})p_{21}}$$

$$\frac{p_{21}-c_{21}}{p_{21}} = -\frac{1}{\eta}\frac{\hat{L}_{21}(\hat{x})}{L_{21}(\hat{x})} + \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{21}(\hat{x})} + \frac{1}{\eta}$$

$$\frac{p_{21}-c_{21}}{p_{21}} = \frac{1}{\eta}\left[1-\frac{\hat{L}_{21}(\hat{x})}{L_{21}(\hat{x})}\right] + \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{21}(\hat{x})}. \tag{A-24}$$

The latest expression is the equation (2-10) stated in Proposition 2 and can be easily written as:

$$p_{21} = \frac{\eta\, c_{21}\, L_{21}}{(\eta-1)L_{21} + \hat{L}_{21} - \gamma\eta\hat{L}_{22}} \tag{A-25}$$

or in a closed form:

$$p_{21} = \frac{\eta\, c_{21}\left[1-\hat{x}+(1-\lambda)(1-\hat{x})^2 - \lambda\left(1-\hat{x}-\frac{\varepsilon}{4}\right)\right]}{\begin{array}{c}(\eta-1)\left[1-\hat{x}+(1-\lambda)(1-\hat{x})^2-\lambda\left(1-\hat{x}-\frac{\varepsilon}{4}\right)\right]+ \\ (1-\lambda)\hat{x}-(1-\lambda)\hat{x}^2+\frac{\lambda}{2}(1-\hat{x})-\gamma\eta\left[1-2\hat{x}+(1-\lambda)\hat{x}^2+\frac{\lambda\hat{x}}{2}+\lambda\left(\hat{x}-\frac{1}{2}\right)\right]\end{array}}. \tag{A-26}$$

A.4. Proof of Corollary 2.1

When calling circle weight (λ) increases, it is of interest to establish the magnitude of change in the factor accompanying call externality in the off-net Lerner index, thus the next derivative is of interest:

$$\frac{\partial}{\partial\lambda}\left[\frac{\hat{L}_{ii}}{L_{ij}}\right] = \frac{1}{L_{ij}^2}\left(\frac{\partial\hat{L}_{ii}}{\partial\lambda}L_{ij} - \frac{\partial L_{ij}}{\partial\lambda}\hat{L}_{ii}\right)$$

where expression inside parenthesis defines the sign of the result. This expression can be proved to be positive using definitions for $\hat{L}_{ii}$ and L_{ij}, as follows:

$$\frac{\partial\hat{L}_{11}}{\partial\lambda}L_{12} - \frac{\partial L_{12}}{\partial\lambda}\hat{L}_{11} = \frac{1}{4}\hat{x}^2(2(1-\hat{x})-\varepsilon) > 0$$

$$\frac{\partial\hat{L}_{22}}{\partial\lambda}L_{21} - \frac{\partial L_{21}}{\partial\lambda}\hat{L}_{22} = \frac{1}{4}(1-\hat{x})^2(2\hat{x}-\varepsilon) > 0$$

where inequalities follow using Assumption 1 .
To establish whether call externality effect is counterbalanced, inequality that follows is of interest:

$$\gamma\left(\frac{\partial\hat{L}_{ii}}{\partial\lambda}L_{ij}-\frac{\partial L_{ij}}{\partial\lambda}\hat{L}_{ii}\right)<\frac{1}{\eta}\left(\frac{\partial\hat{L}_{ij}}{\partial\lambda}L_{ij}-\frac{\partial L_{ij}}{\partial\lambda}\hat{L}_{ij}\right)$$

which turns into

$$\gamma\hat{x}(2-2\hat{x}-\varepsilon)\leq\frac{1}{\eta}(1-\hat{x})(2\hat{x}-\varepsilon) \tag{A-27}$$

for the case of network 1, and

$$\gamma(1-\hat{x})(2\hat{x}-\varepsilon)<\frac{1}{\eta}\hat{x}(2-2\hat{x}-\varepsilon) \tag{A-28}$$

for the case of network 2.
The previous expression will hold depending upon values of the parameters of the model. In particular, as long as call externality is low enough or price elasticity of demand is low enough, firms will set lower off-net prices in response to a stronger calling circle weight. However, the effect might be reversed for very strong call externality or very elastic markets, being the firm 2 the most likely to react with off-net price increase withstanding a strong calling circle weight.

A.5. Proof of Proposition 3

Recalling closed form for equations (2-7) and (2-8) -(A-8) and (A-13)-:

$$\begin{aligned}\frac{p_{11}}{p_{22}}&=\frac{\frac{\eta c_{11}L_{11}}{(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}}}{\frac{\eta c_{22}L_{22}}{(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}}}\\&=\frac{\eta c_{11}L_{11}\left[(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}\right]}{\eta c_{22}L_{22}\left[(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}\right]}\end{aligned}$$

assuming symmetric costs of the firms ($c_{ii}=c_O+c_T$), thus:

$$\frac{p_{11}}{p_{22}}=\frac{L_{11}\left[(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}\right]}{L_{22}\left[(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}\right]}.$$

$\hat{L}_{11}$, L_{22} and $\hat{L}_{22}$ can be written in terms of L_{11}:

$$\frac{p_{11}}{p_{22}}=\frac{L_{11}\left[(\eta-1)(L_{11}+1-2\hat{x})+(1+\gamma\eta)\left(1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right)\right]}{(L_{11}+1-2\hat{x})\left[(\eta-1)L_{11}+(1+\gamma\eta)\left(L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right)\right]}. \tag{A-29}$$

Ratio (A-29) must be compared against 1, which means that when prices are positive, the condition $p_{11} \geq p_{22}$ will hold when (A-29) is equivalent to

$$L_{11}\left[(\eta-1)(L_{11}+1-2\hat{x})+(1+\gamma\eta)\left(1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right)\right] \geq$$
$$(L_{11}+1-2\hat{x})\left[(\eta-1)L_{11}+(1+\gamma\eta)\left(L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right)\right]$$

$$L_{11}(1+\gamma\eta)\left[1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right] \geq$$
$$(L_{11}+1-2\hat{x})\left[(1+\gamma\eta)\left(L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right)\right]$$

$$\begin{aligned} L_{11}\left[1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right] &\geq (L_{11}+1-2\hat{x})\left[L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right] \\ L_{11}\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right) &\geq (L_{11}+1-2\hat{x})\left[-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right] \\ L_{11}(2\hat{x}-1) &\geq \hat{x}(2\hat{x}-1)+\varepsilon\left(\frac{1}{2}-\hat{x}\right) \\ L_{11} &\geq \hat{x}-\frac{\varepsilon}{2}. \end{aligned} \tag{A-30}$$

After procedural algebra on (A-30), next expression is the relationship of magnitude between on-net prices:

$$\lambda \geq \frac{\hat{x}(1-\hat{x})-\frac{\varepsilon}{2}}{\hat{x}(1-\hat{x})-\frac{\varepsilon}{4}} = \hat{\lambda}_{\text{on}}(\hat{x},\varepsilon) \tag{A-31}$$

which is the result of the Proposition.

A.6. Proof of Proposition 4

Recalling closed form for equations (2-8) and (2-10) -(A-20) and (A-25)-, p_{12}/p_{21} ratio is posed using expressions:

$$\begin{aligned} \frac{p_{12}}{p_{21}} &= \frac{\frac{\eta\, c_{12}\, L_{12}}{(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}}}{\frac{\eta\, c_{21}\, L_{21}}{(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}}} \\ &= \frac{\eta\, c_{12}\, L_{12}\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right]}{\eta\, c_{21}\, L_{21}\left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right]} \\ &= \frac{c_{12}\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right]}{c_{21}\left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right]} \end{aligned} \tag{A-32}$$

where the third equality follows since $L_{12} = L_{21}$. Ratio (A-32) must be compared against 1 which means that when prices are positive, the condition $p_{12} \geq p_{21}$ will hold when (A-32) is equivalent to:

$$(c_O + a)\left[(\eta - 1)L_{21} + \hat{L}_{21} - \gamma\eta\hat{L}_{22}\right] \geq (c_O + a)\left[(\eta - 1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right] \tag{A-33}$$

where $c_{12} = c_{21} = c_O + a$, given we asume firms are equally cost-efficient.

$$\begin{aligned}
\left[(\eta - 1)L_{21} + \hat{L}_{21} - \gamma\eta\hat{L}_{22}\right] &\geq \left[(\eta - 1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right] \\
\left[\hat{L}_{21} - \gamma\eta\hat{L}_{22}\right] &\geq \left[\hat{L}_{12} - \gamma\eta\hat{L}_{11}\right] \\
\hat{L}_{21} - \hat{L}_{12} &\geq \gamma\eta\left[\hat{L}_{22} - \hat{L}_{11}\right]
\end{aligned}$$

Using simplified expresions for $\hat{L}_{21} - \hat{L}_{12}$ as well as $\hat{L}_{22} - \hat{L}_{11}$, comparison under discussion turns out to be

$$\gamma\eta(2\hat{x} - 1) \;\geq\; \lambda\left(\hat{x} - \frac{1}{2}\right)(1 + \gamma\eta) \tag{A-34}$$

that finally leads to

$$\hat{\lambda}_{\text{off}}(\gamma, \eta) = \frac{2}{1 + \frac{1}{\gamma\eta}} \;\geq\; \lambda \tag{A-35}$$

which is the result stated in the Proposition.

A.7. Proof of Proposition 5

A.7.1. Network 1

Recalling closed form for equations (2-7) and (2-9) -(A-8) and (A-20)-, p_{11}/p_{12} ratio is posed:

$$\begin{aligned}
\frac{p_{11}}{p_{12}} &= \frac{\frac{\eta\, c_{11}\, L_{11}}{(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}}}{\frac{\eta\, c_{12}\, L_{12}}{(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}}} \\
&= \frac{\eta\, c_{11}\, L_{11}\left[(\eta - 1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right]}{\eta\, c_{12}\, L_{12}\left[(\eta - 1)L_{11} + (1 + \gamma\eta)\hat{L}_{11}\right]} \\
&= \frac{(c_O + c_T)\, L_{11}\left[(\eta - 1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right]}{(c_O + a)\, L_{12}\left[(\eta - 1)L_{11} + (1 + \gamma\eta)\hat{L}_{11}\right]}.
\end{aligned} \tag{A-36}$$

Ratio (A-36) must be compared against 1 which means that when prices are positive, the condition $p_{11} \geq p_{12}$ will hold when (A-36) is equivalent to:

$$(c_O+c_T)\,L_{11}\left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right] \;\geq\; (c_O+a)\,L_{12}\left[(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}\right]. \tag{A-37}$$

If symmetric costs are considered and $a = c_T$, (A-37) can be written as:

$$\begin{aligned}
L_{11}\left[\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right] \;&\geq\; L_{12}\left[(1+\gamma\eta)\hat{L}_{11}\right] \\
L_{11}\hat{L}_{12}-\gamma\eta L_{11}\hat{L}_{11} \;&\geq\; L_{12}\hat{L}_{11}+\gamma\eta L_{12}\hat{L}_{11} \\
L_{11}\hat{L}_{12}-(\hat{x}-L_{11})\hat{L}_{11} \;&\geq\; \gamma\eta\left[L_{11}\hat{L}_{11}+(\hat{x}-L_{11})\hat{L}_{11}\right] \\
L_{11} \;&\geq\; \hat{L}_{11}(1+\gamma\eta). \qquad \text{(A-38)}
\end{aligned}$$

Simplifying $L_{11}-\hat{L}_{11}$ and clearing for λ, (A-38) can be written as:

$$\lambda \;\geq\; \frac{\hat{x}^2}{\hat{x}\left(\hat{x}-\frac{1}{2}\right)+\frac{1}{2\gamma\eta}\left(\hat{x}-\frac{\varepsilon}{2}\right)} = \hat{\lambda}^1_{\text{dif}}(\hat{x},\gamma,\eta,\varepsilon) \tag{A-39}$$

which is the result stated in the Proposition.

A.7.2. Network 2

Recalling closed form for equations (2-8) and (2-10) -(A-13) and (A-25)-, p_{22}/p_{21} ratio is posed:

$$\begin{aligned}
\frac{p_{22}}{p_{21}} &= \frac{\frac{\eta\, c_{22}\, L_{22}}{(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}}}{\frac{\eta\, c_{21}\, L_{21}}{(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}}} \\
&= \frac{(c_O+c_T)\,L_{22}\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right]}{(c_O+a)\,L_{21}\left[(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}\right]}. \qquad \text{(A-40)}
\end{aligned}$$

Ratio (A-40) must be compared against 1 which means that when prices are positive, the condition $p_{22} \geq p_{21}$ will hold when (A-40) is equivalent to:

$$(c_O+c_T)\,L_{22}\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right] \;\geq\; (c_O+a)\,L_{21}\left[(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}\right]. \tag{A-41}$$

If symmetric costs are considered and $a = c_T$, (A-37) can be written as:

$$\begin{aligned}
L_{22}\left[\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right] \;&\geq\; L_{21}\left[(1+\gamma\eta)\hat{L}_{22}\right] \\
L_{22}\hat{L}_{21}-\gamma\eta L_{22}\hat{L}_{22} \;&\geq\; L_{21}\hat{L}_{22}+\gamma\eta L_{21}\hat{L}_{22} \\
L_{22}\hat{L}_{21}-L_{21}\hat{L}_{22} \;&\geq\; \gamma\eta\left[L_{22}\hat{L}_{22}+L_{21}\hat{L}_{22}\right] \\
L_{22}\hat{L}_{21}-(1-\hat{x}-L_{22})\hat{L}_{22} \;&\geq\; \gamma\eta\left[L_{22}+(1-\hat{x}-L_{22})\right]\hat{L}_{22} \\
L_{22} \;&\geq\; \hat{L}_{22}(1+\gamma\eta). \qquad \text{(A-42)}
\end{aligned}$$

Equation (A-42) can be written as:

$$\frac{\lambda}{2}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)\left(1+\frac{1}{\gamma\eta}\right) \geq L_{22}. \tag{A-43}$$

An intermediate important step to establish the relationship under consideration is the following: using the expresio for L_{22} in the right-hand side of (A-43)

$$\begin{aligned}
\frac{\lambda}{2}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)\left(1+\frac{1}{\gamma\eta}\right) &\geq (1-\lambda)(1-\hat{x})^2+\lambda\left(1-\hat{x}-\frac{\varepsilon}{4}\right)\\
\frac{\lambda}{2}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)+\frac{\lambda}{2\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &\geq (1-\lambda)(1-2\hat{x}+\hat{x}^2)+\lambda-\lambda\hat{x}-\frac{\lambda\varepsilon}{4}\\
\frac{\lambda}{2}\left[1-3\hat{x}+2\hat{x}^2+\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)\right] &\geq 1-2\hat{x}+\hat{x}^2.
\end{aligned}$$

Before solving for λ, term in square brackets $[\cdot]$ should be verified to be positive, so that both sides of the inequality can be multiplied by $1/[\cdot]$.

$$\begin{aligned}
1-3\hat{x}+2\hat{x}^2+\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &> 0\\
(1-\hat{x})(1-2\hat{x})+\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &> 0\\
\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &> (1-\hat{x})(2\hat{x}-1)\\
1-\hat{x}-\frac{\varepsilon}{2} &> \gamma\eta\,(1-\hat{x})(2\hat{x}-1)\\
1-\frac{\varepsilon}{2(1-\hat{x})} &> \gamma\eta\,(2\hat{x}-1)\\
\frac{1-\frac{\varepsilon}{2(1-\hat{x})}}{2\hat{x}-1} &> \gamma\eta.
\end{aligned} \tag{A-44}$$

When $\gamma = 0$, both sides of (A-43) can be multiplied by $1/[\cdot]$. The most stringent case would be $\gamma = 1$, that leads (A-44) into:

$$\frac{1-\frac{\varepsilon}{2(1-\hat{x})}}{2\hat{x}-1} > \eta. \tag{A-45}$$

Inequality (A-45) states a condition on η to be able to multiply both sides of (A-43) by $1/[\cdot]$ and solve for λ. However, it must also be taken into account that since the model is considering $\eta > 1$ given the functional forms for $v(\cdot)$ and $q(p)$, then left hand side of unequality (A-45) must be greater than one. This is satisfied if

$$\varepsilon < 4(1-\hat{x})^2. \tag{A-46}$$

Inequality (A-46) holds as long as $\hat{x} < 2/3$. This is obtained by using Assumption 1 and setting $\varepsilon = 2\hat{x}(1-\hat{x})$ as the most stringent case in the latest inequality. Using the expresion for L_{22} and assuming (A-45) and (A-46) hold, (A-43) is written as follows:

$$\lambda \geq \frac{(1-\hat{x})^2}{(1-\hat{x})\left(\frac{1}{2}-\hat{x}\right)+\frac{1}{2\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)} = \hat{\lambda}^2_{\text{dif}}(\hat{x},\gamma,\eta,\varepsilon) \tag{A-47}$$

which is the result stated in the Proposition.

A.8. Proof of Proposition 6

Recalling equation (A-1) and substituting off-net price for $p_{12} = p_{11} + \Delta$:

$$\begin{aligned}\frac{\partial \overline{\Pi}_1}{\partial p_{11}} =& \frac{\partial}{\partial p_{11}}\{\hat{x}(F_1 - f_1) + L_{11}(\hat{x})(p_{11}-c_{11})q(p_{11}) + \\ & L_{12}(\hat{x})(p_{11}+\Delta-c_{12})q(p_{11}+\Delta) + L_{21}(\hat{x})(a-c_0)q(p_{21})\} = 0\end{aligned}$$

$$\frac{\partial \overline{\Pi}_1}{\partial p_{11}} = \hat{x}\frac{\partial F_1}{\partial p_{11}} + L_{11}(\hat{x})\left[q(p_{11}) + (p_{11}-c_{11})q'(p_{11})\right] + L_{12}(\hat{x})\left[\frac{dq_{12}}{dp_{11}}(p_{11}+\Delta-c_{12}) + q_{12}\right] = 0$$

$$\begin{aligned}\frac{\partial F_1}{\partial p_{11}} &= \frac{\partial}{\partial p_{11}}\{F_2 + u_1(\hat{x},\hat{x}) - u_2(\hat{x},\hat{x}) - \tau\hat{x}\} \\ &= \frac{\partial u_1(\hat{x},\hat{x})}{\partial p_{11}} - \frac{\partial u_2(\hat{x},\hat{x})}{\partial p_{11}} \\ &= G(\hat{x}|\hat{x})\left[\upsilon'(p_{11}) + \gamma u'(q_{11})\right] + \left[1-G(\hat{x}|\hat{x})\right]\left[\upsilon'(p_{11}+\Delta)\right] - G(\hat{x}|\hat{x})[\gamma u'(q_{12})] \\ &= G(\hat{x}|\hat{x})\left[-q_{11} + \gamma(-\eta q_{11})\right] + [1-G(\hat{x}|\hat{x})][-q'_{12}] - G(\hat{x}|\hat{x})[\gamma(-\eta q'_{12})]\end{aligned}$$

where for the latest equality I have used:

$$\begin{aligned}\frac{du(q_{ii})}{dp_{ii}} &= \frac{du}{dq_{ii}}\frac{dq_{ii}}{dp_{ii}} \\ &= q_{ii}(-\eta)p_{ii}^{-\eta-1} \\ &= -\eta p_{ii}^{-\eta} \\ &= -\eta q_{ii}.\end{aligned}$$

Substituting $\partial F_1/\partial p_{11}$ into the maximization problem:

$$\begin{aligned}\frac{\partial \overline{\Pi}_1}{\partial p_{11}} =& \hat{x}\left\{G(\hat{x}|\hat{x})\left[-q_{11} + \gamma(-\eta q_{11})\right] + [1-G(\hat{x}|\hat{x})][-q'_{12}] - G(\hat{x}|\hat{x})[\gamma(-\eta q'_{12})]\right\} \\ & + L_{11}(\hat{x})\left[q_{11} + (p_{11}-c_{11})q'_{11}\right] + L_{12}(\hat{x})\left[q'_{12}(p_{11}+\Delta-c_{12}) + q_{12}\right] = 0\end{aligned}$$

$$L_{11}(\hat{x})(p_{11}-c_{11})q'_{11}+L_{12}(\hat{x})q'_{12}(p_{11}+\Delta-c_{12}) = \hat{x}G(\hat{x}|\hat{x})\left[q_{11}+\gamma\eta q_{11}\right]+\hat{x}[1-G(\hat{x}|\hat{x})]q'_{12} - \hat{x}G(\hat{x}|\hat{x})[\gamma\eta q_{12}]-L_{11}(\hat{x})q(p_{11})-L_{12}(\hat{x})q_{12}$$

Rearranging terms:

$$p_{11}\left[L_{11}(\hat{x})q'_{11}+L_{12}(\hat{x})q'_{12}\right] = L_{11}(\hat{x})c_{11}q'_{11}-L_{12}(\hat{x})(\Delta-c_{12})q'_{12}+p_{11}\left\{-\hat{x}G(\hat{x}|\hat{x})\frac{1+\gamma\eta}{\eta}q'_{11} +\gamma\hat{x}G(\hat{x}|\hat{x})q'_{12}+\frac{L_{11}(\hat{x})}{\eta}q'_{11}+\frac{L_{12}(\hat{x})}{\eta}q'_{12}\right\}+\hat{x}[1-G(\hat{x}|\hat{x})]q'_{12}+\gamma\hat{x}G(\hat{x}|\hat{x})q'_{12} + \frac{L_{12}(\hat{x})}{\eta}q'_{12}\Delta.$$

$$p_{11}\left\{q'_{11}\left[L_{11}(\hat{x})+\hat{x}G(\hat{x}|\hat{x})\frac{1+\gamma\eta}{\eta}-\frac{L_{11}(\hat{x})}{\eta}\right]+q'_{12}\left[L_{12}(\hat{x})-\gamma\hat{x}G(\hat{x}|\hat{x})-\frac{L_{12}(\hat{x})}{\eta}\right]\right\}=L_{11}(\hat{x})c_{11}q'_{11} + L_{12}(\hat{x})c_{12}q'_{12}+\hat{x}[1-G(\hat{x}|\hat{x})]q'_{12}+\left(\gamma\hat{x}G(\hat{x}|\hat{x})+\frac{L_{12}(\hat{x})}{\eta}-L_{12}(\hat{x})\right)q'_{12}\Delta.$$

Dividing both sides of the previous equality by $L_{11}(\hat{x})q'_{11}$ and using definitions (A-6) and (A-18):

$$p_{11}\left\{1-\frac{1}{\eta}+\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\frac{1+\gamma\eta}{\eta}+\frac{q'_{12}}{L_{11}(\hat{x})q'_{11}}\left[L_{12}(\hat{x})-\gamma\hat{L}_{11}(\hat{x})-\frac{L_{12}(\hat{x})}{\eta}\right]\right\}=c_{11} + c_{12}\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}}+\frac{\hat{L}_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}}+\frac{q'_{12}}{L_{11}(\hat{x})q'_{11}}\left(\gamma\hat{L}_{11}(\hat{x})+\frac{L_{12}(\hat{x})}{\eta}-L_{12}(\hat{x})\right)\Delta.$$

$$p_{11}\left\{1-\frac{1}{\eta}+\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\frac{1+\gamma\eta}{\eta}+\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}}\left[1-\frac{1}{\eta}-\gamma\frac{\hat{L}_{11}(\hat{x})}{L_{12}(\hat{x})}\right]\right\}=c_{11} + \frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}}\left\{c_{12}+\frac{\hat{L}_{12}(\hat{x})}{L_{12}(\hat{x})}-\Delta\left[1-\frac{1}{\eta}-\gamma\frac{\hat{L}_{11}(\hat{x})}{L_{12}(\hat{x})}\right]\right\}.$$

Clearing for p_{11} leads to result in Proposition 6.

A.9. Proof of Corollary 6.1

I am concerned about the case when the differential is set to zero, i.e. $\Delta = 0$. In this case, $q'_{11} = q'_{12}$, leading the the expression for uniform price stated in Corollary 6.1:

$$p_1^{\Delta} = \frac{\eta c_{11}L_{11}^2(\hat{x})+\eta c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+\eta L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}{L_{11}(\hat{x})\left\{(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x})\right\}}.$$

A.10. Proof of Corollary 6.2

Using the results of the previous section, I develop the algebra to establish how p_1^Δ behaves against p_{11}. I look for conditions when expression (2-16) is larger than (A-8):

$$\frac{\eta c_{11}L_{11}(\hat{x})+\eta c_{12}L_{12}(\hat{x})+\eta\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\hat{L}_{12}(\hat{x})}{(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x})}>\frac{\eta c_{11}L_{11}(\hat{x})}{(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})}$$

$$\begin{aligned}\eta c_{11}L_{11}(\hat{x})&\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]+\eta c_{12}L_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]\\&+\eta\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\hat{L}_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]\\&>\eta c_{11}L_{11}(\hat{x})\left[(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x})\right]\\\gamma\eta^2c_{11}L_{11}(\hat{x})\hat{L}_{11}(\hat{x})&+\eta c_{12}L_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]\\&+\eta\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\hat{L}_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]>\eta(\eta-1)c_{11}L_{11}(\hat{x})L_{12}(\hat{x}).\end{aligned}$$

Since the right hand side of the latest expression contains a single term $\eta(\eta-1)$, then I can just compare this with the coorresponding terms in the left hand side, given the rest of the left hand side expression is positive:

$$\begin{aligned}\eta(\eta-1)\left[c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]&>\eta(\eta-1)c_{11}L_{11}(\hat{x})L_{12}(\hat{x})\\c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+L_{12}(\hat{x})\hat{L}_{12}(\hat{x})&>c_{11}L_{11}(\hat{x})L_{12}(\hat{x})\\c_{12}L_{11}(\hat{x})+\hat{L}_{12}(\hat{x})&>c_{11}L_{11}(\hat{x})\\\hat{L}_{12}(\hat{x})&>0\end{aligned}$$

where I have assumed symmetric costs and cost-based access charges. Therefore, under these assumptions, $p_1^\Delta>p_{11}$.

A.11. Proof of Corollary 6.3

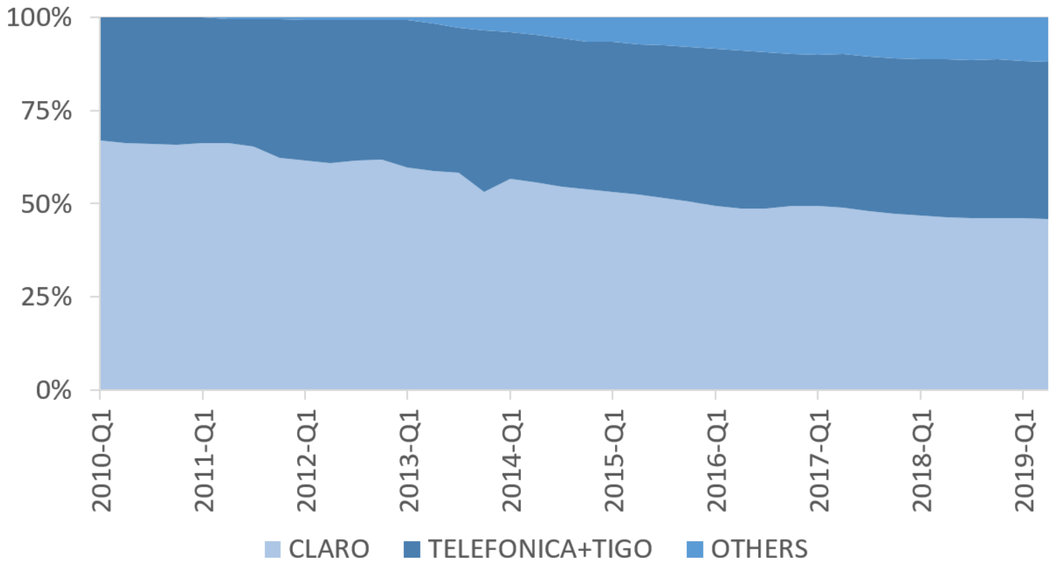
Using the results of Corollary 6.1, I now develop the algebra to establish how p_1^Δ behaves against p_{12}. I look for conditions when expression (2-16) is larger than (A-20):

$$\frac{\eta c_{11}L_{11}^2(\hat{x})+\eta c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+\eta L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}{L_{11}(\hat{x})\left\{(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x})\right\}}>\frac{\eta c_{12}L_{12}(\hat{x})}{(\eta-1)L_{12}(\hat{x})+\hat{L}_{12}(\hat{x})-\gamma\eta\hat{L}_{11}(\hat{x})}$$

$$\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]\left[(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \gamma\eta\hat{L}_{11}(\hat{x})\right] >$$
$$c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left\{(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right\}$$
$$\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]\left[(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x})\right]$$
$$-\gamma\eta\hat{L}_{11}(\hat{x})\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right] >$$
$$c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right]$$
$$\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]\left[(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x})\right]$$
$$-c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right] >$$
$$\gamma\eta\hat{L}_{11}(\hat{x})\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]$$
$$(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \frac{c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right]}{c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})} >$$
$$\gamma\eta\hat{L}_{11}(\hat{x})$$
$$\frac{1}{\eta\hat{L}_{11}(\hat{x})}\left\{(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \frac{c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[\eta\hat{x} - \hat{x} + \hat{L}_{11}(\hat{x})\right]}{c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}\right\} > \gamma$$
$$\frac{1}{\eta\hat{L}_{11}(\hat{x})}\left\{(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \frac{c_{11}L_{11}(\hat{x})L_{12}(\hat{x})\left[\eta\hat{x} - \hat{L}_{12}(\hat{x})\right]}{c_{11}\hat{x}L_{11}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}\right\} > \gamma.$$

In the latest inequality I have assumed termination rate is set equal to cost and firms are equally efficient. This is the result stated in Corollary 6.3.

A.12. Supporting figures



Market shares in the Colombian mobile market.
Source: MINTIC web portal and author's calculations.

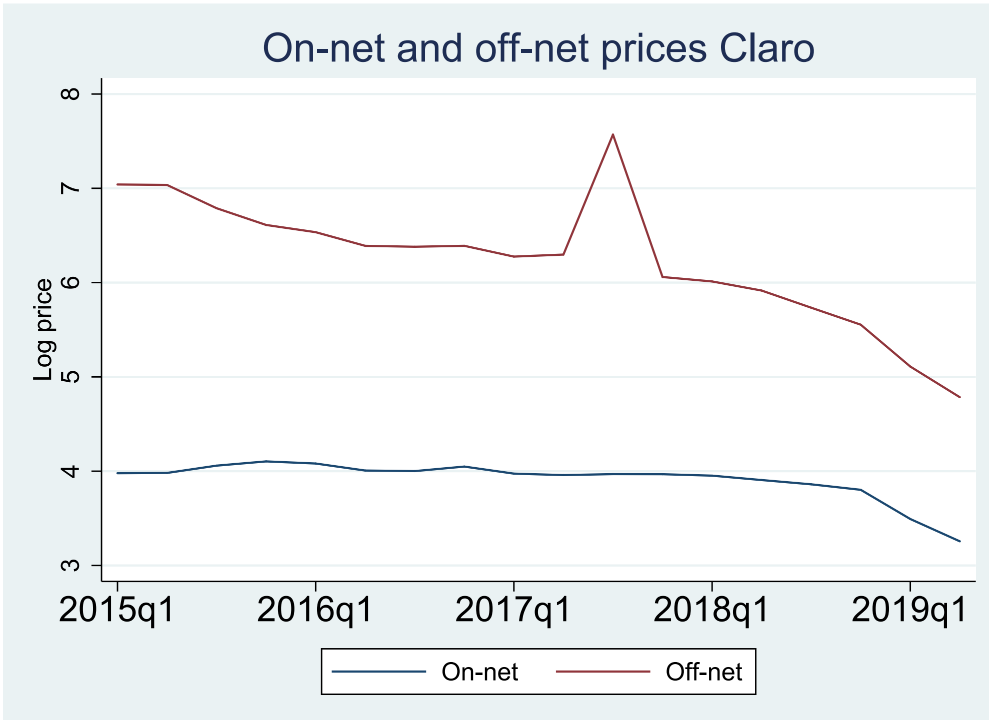


Figure **A-1**.: Mobile voice prices set by Comcel network in Colombia.
Source: MINTIC web portal and author's calculations.

A.13. Proofs of Lemmas and Propositions

Proof of Proposition 1: The incumbent's product introduction decision is shown in the text. Here, we elicit the incumbent's research decision.

Take $\gamma < \gamma^M$. The incumbents expected profits are given in (3-2). By taking the limit $dt \to 0$, we obtain $W^M = \frac{\lambda^I[p_0\bar{R}/r+(1-p_0)\underline{R}/r+(1-\gamma)R/r)-c]+R}{r+\lambda^I}$ or equivalently $W^M = \frac{\lambda^I[p_0\bar{R}/r+(1-p_0)\underline{R}/r-c]+R(1+(\lambda^I(1-}{r+\lambda^I}$ where we use $e^{-r\,dt} = 1 - r\,dt$. If the incumbent does not innovate, it obtains R/r. Then, the incumbent researches if $\frac{\lambda^I[p_0\bar{R}/r+(1-p_0)\underline{R}/r-c]+R(1+(\lambda^I(1-\gamma))/r)}{r+\lambda^I} > \frac{R}{r} \iff \frac{p_0\bar{R}/r+(1-p_0)\underline{R}/r-c}{R/r} > \gamma$. The right hand side increases with γ. Then, setting $\gamma \approx \underline{R}/R := \gamma^M$, we get $\frac{p_0\bar{R}/r+(1-p_0)\underline{R}/r-c}{R/r} > \frac{\underline{R}}{R} \iff p_0\bar{R} + (1-p_0)\underline{R} - c > \underline{R} \iff p_0 > \frac{c}{(\bar{R}-\underline{R})}$. This is true by Assumption 2 because $c/(\bar{R}-\underline{R}) > cr/(\bar{R}-R) \iff r < (\bar{R}-R)(\bar{R}-\underline{R})$.

Take $\gamma \geq \gamma^M$. The incumbents expected profits are given in (3-3). Solving, we get $W^M = \frac{\lambda^I[p_0(\bar{R}/r+(1-\gamma)R/r)+(1-p_0)R/r-c]+R}{r+\lambda^I}$, or equivalently $W^M = \frac{\lambda^I[p_0\bar{R}/r-c]+R(1+(\lambda^I(1-p_0\gamma))/r)}{r+\lambda^I}$. If the incumbent does not innovate, it obtains R/r. Then, the incumbent researches if $\frac{\lambda^I[p_0\bar{R}/r-c]+R(1+(\lambda^I(1-p_0\gamma))/r)}{r+\lambda^I} > \frac{R}{r} \iff \frac{p_0\bar{R}-rc}{p_0 R} > \gamma$. The right-hand increases with γ. Set $\gamma = 1$, we get $(p_0\bar{R}-rc)/p_0 R > 1 \iff p_0 > rc/(\bar{R}-R)$, which is true by Assumption 2.

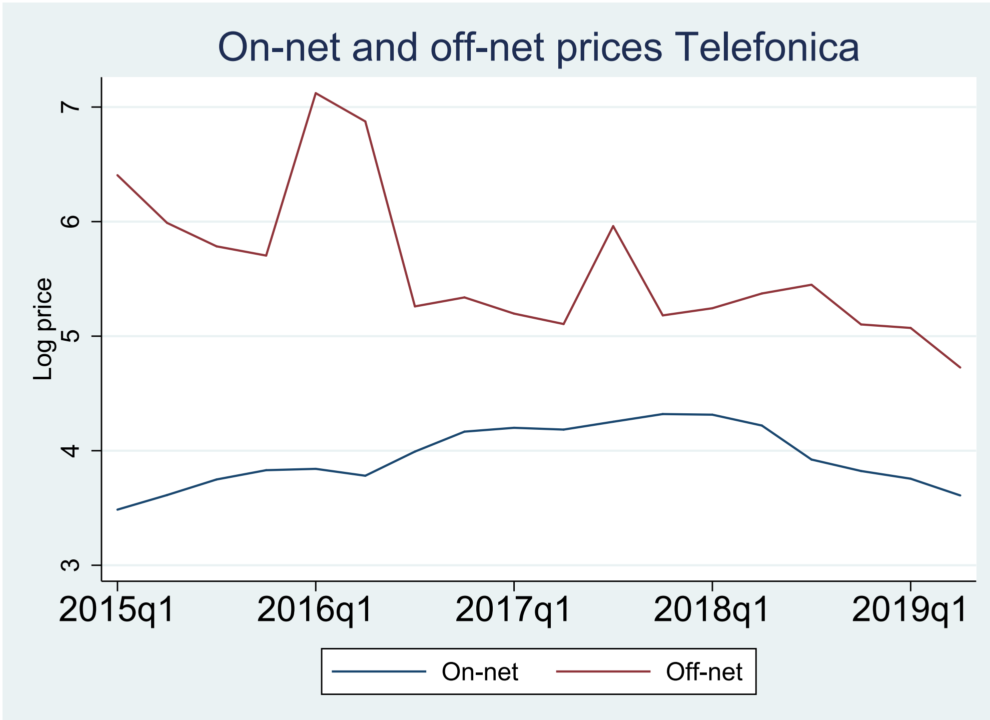


Figure **A-2**.: Mobile voice prices set by Telefonica network in Colombia.

Proof of Lemma 1: The law of motion $\dot{p}_t^i = -p_t^i(1-p_t^i)\alpha\lambda^{-i}$, where we have eliminated the superscripts for the ease of notation, can be transformed into a first-order linear differential equation $-\dot{z}_i^t + \alpha\lambda^{-i} z_i^t = \alpha\lambda^{-i}$ by setting $z_i^t = 1/p_t^i$. The differential equation $-\dot{z}_i^t + \alpha\lambda^{-i} z_i^t = \alpha\lambda^{-i}$ has a general solution $z_i^t = Ce^{\alpha\lambda^{-i}t} + 1$, where C is a constant to be determined. Then, because $p_t^i = 1/\left(Ce^{\alpha\lambda^{-i}t} + 1\right)$, and using the boundary condition that $p_{t=0}^i = p_0$, we obtain $C = (1-p_0)/p_0$. Finally, introducing this into the previous expression we obtain the particular solution in Expression (3-7) of the lemma. By setting $\alpha = 1$, we obtain Expression (3-6).

Take Expression (3-7), the change in p_t^i with respect to t and λ^{-i} are respectively $\frac{\partial p_t^i}{\partial t} = \frac{-\alpha\lambda^{-i}p_0(1-p_0)e^{-\alpha\lambda^{-i}t}}{\left(p_0+(1-p_0)e^{\alpha\lambda^{-i}t}\right)^2} < 0$; $\frac{\partial p_t^i}{\partial \lambda^{-i}} = \frac{-(1-p_0)p_0 e^{\alpha\lambda^{-i}t}\alpha t}{\left(p_0+(1-p_0)e^{\alpha\lambda^{-i}t}\right)^2} < 0$, and the same sign is obtained by setting $\alpha = 1$. Finally, differentiating Expression (3-7) with respect to α gives $\frac{\partial p_t^i}{\partial \alpha} = \frac{-p_0(1-p_0)\lambda^{-i}te^{\alpha\lambda^{-i}t}}{\left(p_0+(1-p_0)e^{\alpha\lambda^{-i}t}\right)^2} < 0$.

Proof of Lemma 3: To obtain the time $T^{i,INH}$, substitute $p^{i,INH}$ from Lemma 1 in the indifference condition in (3-12) and solving for t characterizes $T^{i,INH}$, the time firm i abandons research if no information arrives. We get $\frac{p_0}{p_0+(1-p_0)e^{\lambda^{-i}t}}\left[\lambda^i\left(V_{LH}^i - V_{LL}^i + \lambda^{-i}V_F^i\right)\right] = \lambda^i\left(c - V_{LL}^i\right)$ After re-arranging and taking the logs of both sides, we obtain the Expression in (3-13).

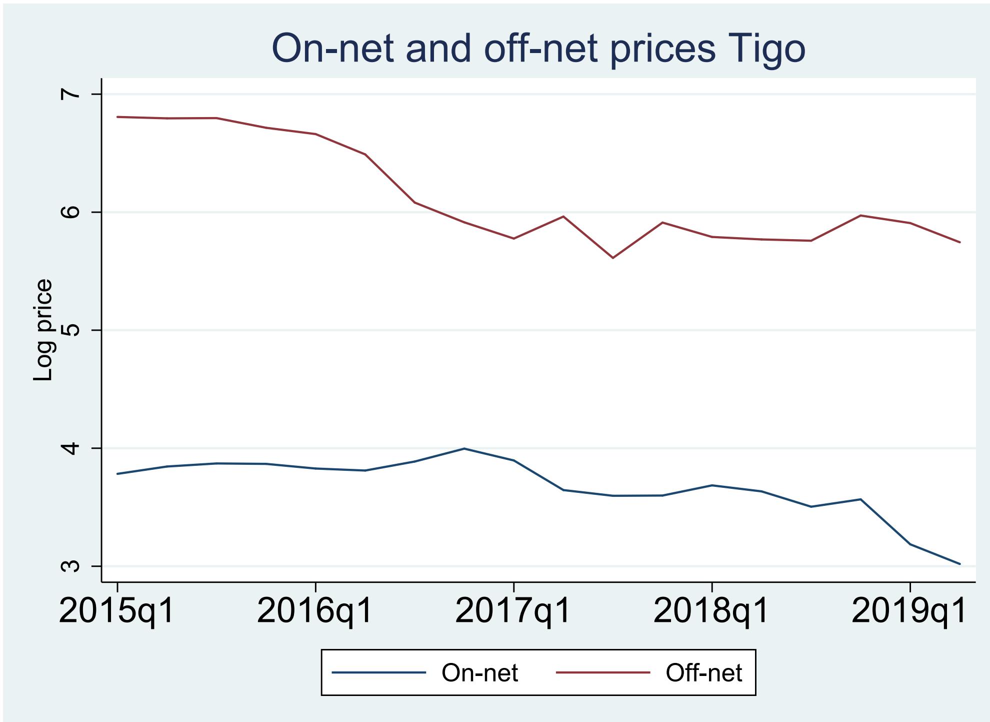


Figure **A-3**.: Mobile voice prices set by Tigo network in Colombia.
Source: MINTIC web portal and author's calculations.

Using the same procedure, we obtain the abandoning time $T^{i,INH\alpha}$ by introducing $p^{i,INH\alpha}$ from Lemma 1 in the indifference condition (3-12).

Proof of Proposition 2: Assume that the entrant is the first to complete research. Assume further than the market is highly profitable. If the entrant introduces the product and the incumbent continues with research the entrant obtains the profits as in (3-15). By not introducing the product, the entrant gets 0, which does not constitute a profitable deviation. Consider now delaying product introduction. In this case the entrant gets $V^E_{NI,dt} = e^{-rdt}\left[\lambda^I dt\bar{D}/r + (1-\lambda^I dt)V^E_I\right]$. With instantaneous probability $\lambda^I dt$, the incumbent finishes research and introduces the product, thus generating a discounted profit of $\bar{D}/r$. Comparing (3-15) and $V^E_{NI,dt}$, we get $e^{-rdt}\left[\lambda^I dt\bar{R}\right] > 0$, hence delaying product introduction is not a profitable deviation.
Consider now that in a highly profitable market, the incumbent abandons after the entrant has introduced the product. By introducing the product the entrant gets $V^E_I = e^{-rdt}\left[\bar{R}dt + V^E_I\right]$. Again delaying product introduction does not generate a profitable deviation because $V^E_{NI,dt} = e^{-rdt}\left[\lambda^I dt\bar{D}/r + (1-\lambda^I dt)V^E_I\right]$, and the difference between is $V^E_I - V^E_{NI,dt} = e^{-rdt}\left[\bar{R}dt + \lambda^I dt\left(\bar{R}/r - \bar{D}/r\right)\right]$, which is positive due to $\bar{R} > \bar{D}$. If the product is not introduced the entrant gets no profits.
Assume now that the market is of small profitability. If the entrant introduces the product, it gets $\underline{R}/r$. There is no profitable deviation consisting of either not introducing the product

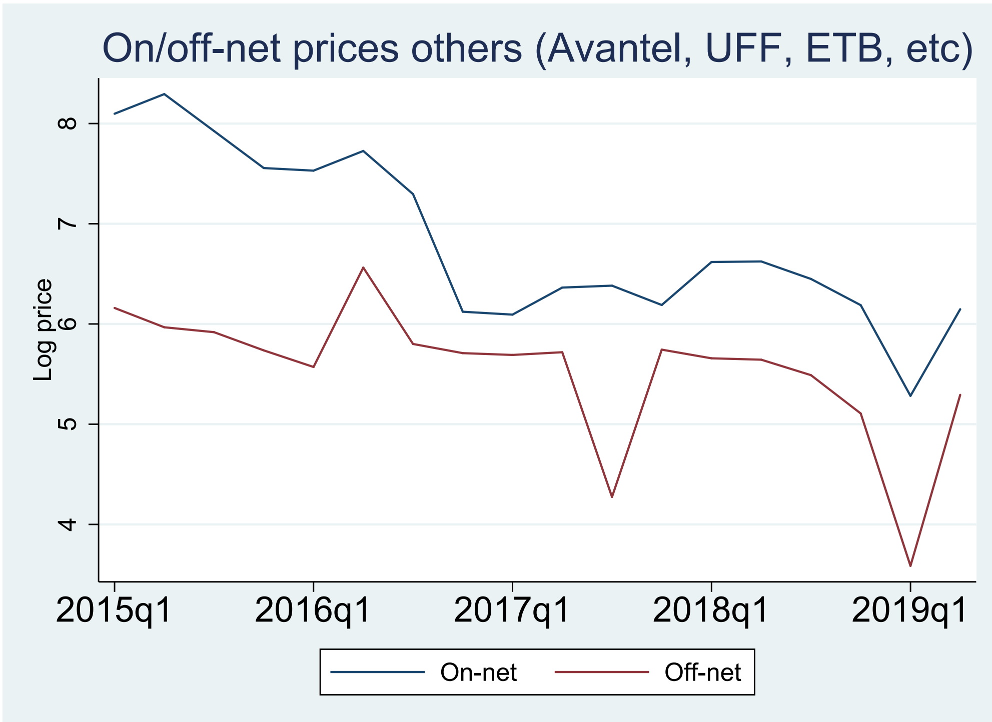


Figure **A-4**.: Mobile voice prices set by other networks in Colombia.
Source: MINTIC web portal and author's calculations.

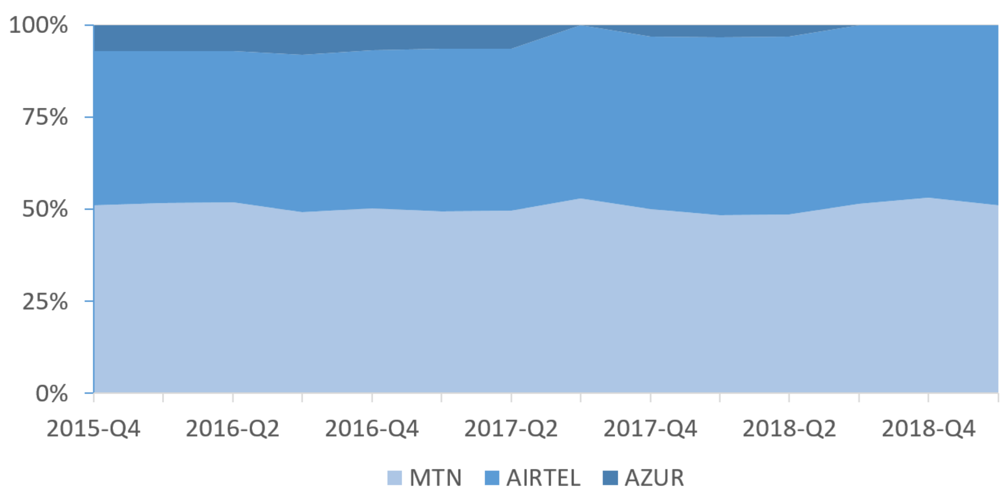
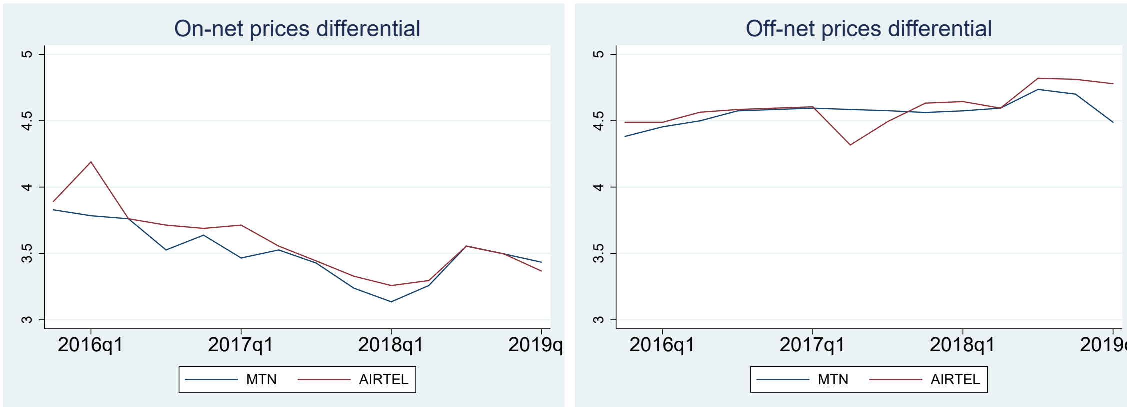


Figure **A-5**.: Mobile market shares in Republic of Congo.
Source: ARPCE and author's calculations.

or delaying the introduction. By not introducing the product the entrant gets 0, by delaying product introduction for a time $dt > 0$, the entrant gets $(1 - \lambda^I dt)(1 - rdt)\underline{R}/r$. Both cases generate strictly lower profits.

Proof of Proposition 3: We establish the incumbent's equilibrium product introduction



(a) On-net prices paths. (b) Off-net prices paths.

Figure **A-6.**: On-net and off-net price paths in Republic of Congo.
Source: ARPCE and author's calculations.

decisions for different values of γ. For $\gamma \leq \underline{\gamma}(\lambda^E)$, as $\underline{\gamma}(\lambda^E)$ as defined in (3-18), we have already shown that the incumbent always introduces the product regardless of market profitability. For $\gamma > \underline{\gamma}(\lambda^E)$ and $D \leq \tilde{D}$ where $\tilde{D}$ defined as in (3-10), we have shown that the incumbent does not introduce the product in a small profitability market, but it does when the market is highly profitable.

Then, it is only left to consider the case with $\gamma > \underline{\gamma}(\lambda^E)$ and $D > \tilde{D}$. Consider a candidate equilibrium in which the incumbent introduces the product in a highly profitable market and does not introduce the product in a small profitability market. In this candidate equilibrium, the entrant's beliefs evolve according to Expression 3-6 in Lemma 1. Then, if no product is introduced and no further discovery occurs, the entrant abandons research at time $T^{E,INH}$ as characterized in Lemma 3. In this equilibrium, when the incumbent finishes research at $\tau^I < T^{E,INH}$ and introduces the product the entrant learns high market profitability, $p^E_{\tau^I} = 1$, and for $\bar{D} > \tilde{D}$, it continues with research until it also makes a discovery. Hence, by introducing the product, the incumbent gets $\int_{\tau^I}^{\infty} e^{-(r+\lambda^E)t}\left[\bar{R} + (1-\gamma)R/r + \lambda^E \bar{D}/r\right] dt$. By not introducing the product, it obtains $\int_{\tau^I}^{T^{E,INH}} e^{-(r+\lambda^E)t}\left[R + \lambda^E \bar{D}/r\right] dt + e^{-(r+\lambda^E)(T^E-\tau^I)}\left[\bar{R}/r + (1-\gamma)R/r\right]$. Therefore, the incumbent is indifferent between both strategies when $\tau^I = \underline{T}$, when $\underline{T}$ is implicitly characterized by:

$$\overbrace{\int_{\underline{T}}^{\infty} e^{-(r+\lambda^E)t}\left[\bar{R} + (1-\gamma)R/r + \lambda^E \bar{D}/r\right] dt}^{\text{Inmediate product introduction}}$$
$$= \underbrace{\int_{\underline{T}}^{T^{E,INH}} e^{-(r+\lambda^E)t}\left[R + \lambda^E \bar{D}/r\right] dt + e^{-(r+\lambda^E)(T^{E,INH}-\underline{T})}\left[\bar{R}/r + (1-\gamma)R/r\right]}_{\text{No Product intoduction}}.$$

Denote $\eta \equiv \bar{R}/r + (1-\gamma)R/r$. Then, simple algebraic manipulations gives

$$\underline{T} = T^{E,INH} + \frac{1}{r+\lambda^E} \ln\Bigg[\frac{1}{[\eta]}\Bigg(\int_{\underline{T}}^{\infty} e^{-(r+\lambda^E)t}\big[\bar{R}+(1-\gamma)R/r+\lambda^E\bar{D}/r\big]dt - \int_{\underline{T}}^{T^{E,INH}} e^{-(r+\lambda^E)t}\big[R+\lambda^E\bar{D}/r\big]dt\Bigg)\Bigg]. \tag{A-48}$$

The time $\underline{T}$ exists and is unique. To see this, note that the left-hand side is increasing in $\underline{\mathrm{T}}$. The right-hand side is monotonically decreasing in $\underline{\mathrm{T}}$. This is so if $\ln[\cdot]$ decreases with $\underline{\mathrm{T}}$. This happens if the first integral decreases faster than the second. This occurs whenever:

$$e^{-(r+\lambda^E)t}\left[\bar{R} + (1-\gamma)R/r + \lambda^E\bar{D}/r\right] > e^{-(r+\lambda^E)t}\left[R + \lambda^E\bar{D}/r\right]$$
$$\iff \bar{R} + (1-\gamma)R/r + \lambda^E\bar{D}/r > R + \lambda^E\bar{D}/r \iff \bar{R} - R > -(1-\gamma)R/r,$$

which is always fulfilled.

From (A-48), we directly obtain that if the incumbent finishes reach at $\tau^I < \underline{T}(T^{E,INH})$, introducing the product immediately generates larger profits. Now, consider the case in which incumbent finishes research at $\tau^I \geq \underline{T}(T^{E,INH})$. In this case, not introducing the product in a highly profitable market cannot be an equilibrium. If the incumbent does not introduce the product, the entrant does not learn about the state of the market, and it never abandons research. Consequently, the incumbent obtains $\int_{\tau^I}^{T^{E,INH}} e^{-(r+\lambda^E)t}\left[R + \lambda^E\bar{D}/r\right] dt + e^{-(r+\lambda^E)T^{E,INH}} \int_0^{\infty} e^{-(r+\lambda^E)t}\left[\bar{R} + (1-\gamma)R/r + \lambda^E\bar{D}/r\right] dt$. Introducing the product constitutes a profitable deviation because

$$\overbrace{\int_{\tau^I}^{\infty} e^{-(r+\lambda^E)t}\left[\bar{R} + (1-\gamma)R/r + \lambda^E\bar{D}/r\right] dt}^{\text{Expected profits from immediate product introduction}}$$
$$-\left[\int_{\tau^I}^{T^{E,INH}} e^{-(r+\lambda^E)t}\left[R + \lambda^E\bar{D}/r\right] dt + e^{-(r+\lambda^E)T^{E,INH}} \int_0^{\infty} e^{-(r+\lambda^E)t}\left[\bar{R} + (1-\gamma)R/r + \lambda^E\bar{D}/r\right] dt\right]$$
$$= \int_{\tau^I}^{T^{E,INH}} e^{-(r+\lambda^E)t}\left[\bar{R} - R + (1-\gamma)R/r\right] dt > 0.$$

Hence, there must be an equilibrium in which the incumbent introduces the product with probability $\alpha_t \in (0,1)$. Since with α_t, the entrant becomes pessimistic about market profitability if no product is introduced, there must be a time $T^{E,INH\alpha}(\alpha_t)$ in which the entrant abandons research. For any time $t \in \big(\underline{T}(T^{E,INH}), T^{E,INH\alpha}(\alpha_t)\big)$, the probability α_t in which the incumbent introduces the product in a highly profitable market has to be such that at any time t the incumbent is indifferent between introducing the product immediately or waiting until the entrant abandons research. Therefore, α_t is implicitly characterized by the indifference condition:

$$\frac{\bar{R} + (1-\gamma)R/r + \lambda^E\bar{D}/r}{r+\lambda^E}$$
$$= \int_{\underline{T}(T^{E,INH})}^{T^{E,INH\alpha}(\alpha_t)} e^{-(r+\lambda^E)t}\left[\lambda^E(\bar{D}/r + (1-\gamma)R/r) + R\right] dt + e^{-(r+\lambda^E)T^{E,INH\alpha}(\alpha_t)}\left(\bar{R}/r + (1-\gamma)R/r\right).$$

(A-49)

The left-hand side states the expected profits of introducing the product immediately. The right-hand side gives the expected profits of not introducing the product until the entrant abandons research. Note that the incumbent strictly prefers delaying at any later time $t' > t$, since the time to wait until the entrant drops out is now less.

To show that α_t decreases over time, applying Leibniz's rule to Expression (A-49) we get:

$$\begin{aligned} 0 = & [R + \lambda^E(\bar{D}/r + (1-\gamma)R/r)]e^{-(r+\lambda^E)T^{E,INH\alpha}(\alpha_t)}\frac{dT^{E,INH\alpha}(\alpha_t)}{d\alpha_t}\frac{d\alpha_t}{dt} \\ & + [R + \lambda^E(\bar{D}/r + (1-\gamma)R/r))]\int_{\underline{\mathrm{T}}(T^{E,INH})}^{T^{E,INH\alpha}(\alpha_t)} \frac{\partial}{\partial t} e^{-(r+\lambda^E)\tau} d\tau \\ & - [\bar{R}/r + (1-\gamma)R/r](r+\lambda^E)e^{-(r+\lambda^E)T^{E,INH\alpha}(\alpha_t)}\frac{dT^{E,INH\alpha}(\alpha_t)}{d\alpha_t}\frac{d\alpha_t}{dt}. \end{aligned}$$

$$\implies \frac{d\alpha_t}{dt} = \frac{[R + \lambda^E(\bar{D}/r + (1-\gamma)R/r))]\int_{\underline{\mathrm{T}}(T^{E,INH})}^{T^{E,INH\alpha}(\alpha_t)} \frac{\partial}{\partial t} e^{-(r+\lambda^E)\tau} d\tau}{\left[(\bar{R}-R) + \lambda^E(\bar{R}/r - \bar{D}/r) + r(1-\gamma)R/r\right] e^{-(r+\lambda^E)T^{E,INH\alpha}(\alpha_t)}\frac{dT^E(\alpha_t)}{d\alpha_t}} < 0.$$

The last inequality comes from $dT^{E,INH\alpha}(\alpha_t)/d\alpha_t < 0$. If the incumbent introduces the product with a higher probability, the entrant becomes pessimistic faster and abandons earlier. After $T^{E,INH\alpha}(\alpha_t)$, the entrant abandons, and the incumbent introduces the product immediately after that.

Proof of Proposition 4: Consider first, the entrant's research decisions. Suppose first that $\gamma < \underline{\gamma}(\lambda^E)$, and $\bar{D} < \hat{D}$. Then, $V_F^E = 0$, $V_{LH}^E = \bar{R}/r$. Solving (3-22) gives $W^E = \left(\lambda^E\left(p_0\bar{R}/r + (1-p_0)\underline{R}/r - c\right)\right) / \left(r + \lambda^E + \lambda^I\right)$. Then, the entrant researches if $W^E > 0 \iff p_0 \times \bar{R}/r + (1-p_0) \times \underline{R}/r - c > 0$. The left-hand side increases in p_0. Hence, setting $p_0 = rc/\left(\bar{R} - \underline{R}\right)$, a necessary and sufficient condition is $\left(\frac{rc}{\bar{R}-R}\right)\frac{\bar{R}}{r} + \left(1 - \frac{rc}{\bar{R}-R}\right)\frac{\underline{R}}{r} - c > 0 \iff \frac{\underline{R}/r(\bar{R}-R)+c(R-\underline{R})}{\bar{R}-R} > 0$. The last inequality comes from $\bar{R} > R > \underline{R}$. Suppose $\bar{D} \geq \hat{D}$. Then, $V_{LH}^E = (\lambda^I\bar{D}/r + \bar{R})/(r+\lambda^I)$. Solving (3-22) gives $W^E = \big(\lambda^E\big(p_0(\lambda^I\bar{D}/r + \bar{R})/(r+\lambda^I) + (1-p_0)\underline{R}$
Because $V_F^E = \lambda^E(p_0\bar{D}/r - c)/(r+\lambda^E) > 0$, a necessary and sufficient condition for $W^E > 0$ is $p_0\left(\frac{\lambda^I\bar{D}/r+\bar{R}}{r+\lambda^I}\right) + (1-p_0)\underline{R}/r - c > 0$. Introducing $p_0 = rc/(\bar{R} - R)$ and $\bar{D} = \hat{D} = rc/p_0$, and simple algebraic manipulation gives $\frac{(r+\lambda^I)(\bar{R}-R-rc)\underline{R}/r+Rrc}{(r+\lambda^I)(\bar{R}-R)} > 0$. This is always fulfilled because $\bar{R} - R > c$ and $r < 1$.

Now suppose that With $\gamma \geq \underline{\gamma}(\lambda^E)$. In this case, the entrant begins with research because if the interior of the integral in Expressions (3-25) and (3-26) are positive. The same conditions in the previous point shows that this is indeed the case but with the threshold of $\tilde{D}$ instead of $\hat{D}$ to determine the expected profits of V_{LH}^E and V_F^E. Note that absent product introduction the entrant will eventually exit research.

Consider now the incumbent's research strategy. Suppose that $\gamma < \underline{\gamma}(\lambda^E)$, and $\bar{D} \leq \hat{D}$. Solving Expressions (3-23) and (3-28), we obtain that $W^I \geq W_{NR}^I$ if $\frac{\lambda^I\left[p_0\bar{R}/r+(1-p_0)\underline{R}/r-c\right]+(\lambda^I+\lambda^E)(1-\gamma)R+R}{r+\lambda^I+\lambda^E} \geq$

$\frac{\lambda^E(1-\gamma)R/r+R}{r+\lambda^E}$. Simple algebraic manipulation gives $p_0(\bar{R}-\underline{R})+\underline{R}-rc \geq \frac{r\gamma R}{r+\lambda^E} \iff p_0 \geq \frac{r\gamma R}{(r+\lambda^E)(\bar{R}-\underline{R})}+\frac{rc-\underline{R}}{\bar{R}-\underline{R}} := \tilde{p}(\gamma,\lambda^E)$. Because $\tilde{p}(\gamma,\lambda^E)$ increases with γ, by substituting $\gamma = \underline{\gamma}(\lambda^E) := \underline{R}(r+\lambda^E)/rR$, we obtain an upper-bound of $\bar{\tilde{p}} := rc/(\bar{R}-\underline{R})$. But $p_0 \geq \bar{\tilde{p}}$ is always fulfilled because $p_0 > \bar{\tilde{p}} \iff rc/\left(\bar{R}-R\right) > rc/\left(\bar{R}-\underline{R}\right) \iff R > \underline{R}$. Now suppose that $\bar{D} > \hat{D}$. Then, the solution of Expression (3-23) is $\frac{\lambda^I\left[p_0\left(\frac{\lambda^E\bar{D}/r+\bar{R}}{r+\lambda^E}\right)+(1-p_0)\underline{R}/r-c\right]+(\lambda^I+\lambda^E)(1-\gamma)R/r+\lambda^E\left(\frac{\lambda^I(p_0\bar{D}/r-c)}{r+\lambda^I}\right)+R}{r+\lambda^I+\lambda^E}$ Simple algebraic manipulation, and sing $\bar{D}=\hat{D}=rc/p_0$ establishes that $W^I-W^I_{NI}>0$ is equal to $p_0\bar{R}+(1-p_0)(r+\lambda^E)\underline{R}/r-rc \geq \gamma R$. Introducing the upper bound of $\underline{\gamma}(\lambda^E) := \underline{R}(r+\lambda^E)/rR$, a necessary and sufficient condition is $p_0 \geq rc/\left(\bar{R}-(r+\lambda^E)\underline{R}/r\right)$. The previous is always fulfilled when $rc/(\bar{R}-R) \geq (rc)/(\bar{R}-(r+\lambda^E)\underline{R}/r) \iff R \geq (r+\lambda^E)\underline{R}/r$, and with $\lambda^E = (r(R-\underline{R}))/\underline{R}$, we obtain $R \geq (1+(R-\underline{R})/\underline{R})\,\underline{R} \iff R \geq \underline{R}+(R-\underline{R})$.
Now suppose that $\gamma \geq \underline{\gamma}(\lambda^E)$ and $\bar{D} < \tilde{D}$. Because the incumbent becomes the monopolist after the entrant has abandoned research, $T^{E,INH}$, an lower bound of Expression (3-25) is $\frac{\lambda^I\left(p_0\left(\bar{R}/r+(1-\gamma)R/r\right)+(1-p_0)\left(\frac{\lambda^E(1-\gamma)R/r+R}{r+\lambda^E}\right)-c\right)+\lambda^E((1-\gamma)R/r+R)}{r+\lambda^I+\lambda^E}$. Simple algebraic manipulations gives that $W^I > W^I_{NR}$ is equivalent to $p_0 > \frac{(r+\lambda^E)c}{(r+\lambda^E)\bar{R}/r-R+(1-\gamma)R} := p'(\gamma,c,\lambda^E)$. Because $\partial p'(\gamma,c,\lambda^E)/\partial\gamma > 0$, the most stringent situation is $\gamma = 1$, and a necessary and sufficient condition for research is $\frac{rc}{\bar{R}-R} > \frac{(r+\lambda^E)c}{(r+\lambda^E)\bar{R}/r-R} \iff r(\bar{R}-R)+\lambda^E\bar{R} > (r+\lambda^E)(\bar{R}-R) \iff 0 > -\lambda^E$.
Now suppose that $\bar{D} > \tilde{D}$. In this case, the expected profits from undertaking research are given in (3-26). We now proceed to show if there are circumstances in which the incumbent decides not to research. To do that, consider a situation in which the entrant abandons research very late (this may happen with an inefficient incumbent's research λ^I or a cost of research $c \approx \underline{R}/r$). By researching, the incumbent gets profits

$$W^I \approx \int_0^\infty e^{-(r+\lambda^E+\lambda^I)t}\left\{\lambda^I\left[p_0\left(V^I_L+(1-\gamma)R/r\right)+(1-p_0)\left(\frac{\lambda^E(1-\gamma)R/r+R}{r+\lambda^E}\right)-c\right]\right.$$
$$\left.+\lambda^E\left[V^I_F+(1-\gamma)R/r\right]+R\right\}dt. \tag{A-50}$$

For $D \in \left(\tilde{D},\hat{D}\right]$, the solution of (A-50) is $\frac{\lambda^I}{r+\lambda^E+\lambda^I}\left[p_0\left(\frac{\bar{R}+\lambda^E c}{r+\lambda^E}+(1-\gamma)\frac{R}{r}\right)+(1-p_0)\left(\frac{\lambda^E(1-\gamma)R/r+R}{r+\lambda^E}\right)-c\right]+\frac{\lambda^E}{r+\lambda^E+\lambda^I}\left[\frac{\lambda^I(p_0-1)c}{r+\lambda^I}+(1-\gamma)\frac{R}{r}\right]+R$. Simple algebraic manipulation gives s that $W^I \geq W^I_{NR}$ if $p_0 > \frac{(r+\lambda^E)c+(r+\lambda^E)/(r+\lambda^I)\lambda^E c}{\bar{R}+\lambda^E c-\gamma R+(r+\lambda^E)/(r+\lambda^I)\lambda^E c} := \tilde{p}(\cdot)$. Because $\partial\tilde{p}(\gamma,c,\lambda^E)/\partial\gamma > 0$, the most stringent situation happens for $\gamma = 1$, and there exists a prior $p_0 < \tilde{p}(\gamma=1,c,\lambda^E)$ in which the incumbent does not research if $\tilde{p}(\gamma=1,c,\lambda_E) = \frac{(r+\lambda^E)c+(r+\lambda^E)/(r+\lambda^I)\lambda^E c}{\bar{R}+\lambda^E c-\gamma R+(r+\lambda^E)/(r+\lambda^I)\lambda^E c} > \frac{rc}{\bar{R}-R} = p_0$. Algebraic manipulation gives that this occurs when $\bar{R}-R > rc$, which is true by Assumption 2.
For $\bar{D} > \hat{D}$, $\frac{\lambda^I\left[p_0\left(\frac{\bar{R}+\lambda^E\bar{D}/r}{r+\lambda^E}+(1-\gamma)R/r\right)+(1-p_0)\left(\frac{\lambda^E(1-\gamma)R/r+R}{r+\lambda^E}\right)-c\right]+\lambda^E\left[\frac{\lambda^I(p_0(\bar{D}/r)-c)}{r+\lambda^I}+(1-\gamma)R/r\right]+R}{r+\lambda^I+\lambda^E}$ is the solution to (A-50). Since W^I decrease with $\bar{D}$, a lower bound of W^I is obtained with $\bar{D}=\hat{D}=rc/p_0$. Then, simple algebraic manipulation gives that $W^I > W^I_{NR}$ if $p_0 > rc/(\bar{R}-\gamma R) := \hat{p}(\gamma,c)$. Because $\partial\hat{p}(\gamma,c)/\partial\gamma > 0$, the most stringent situation arises for

$\gamma = 1$, which gives $\bar{\hat{p}} := rc/(\bar{R} - R)$, and because $p_0 \geq \bar{\hat{p}}$, the incumbent always researches.

Proof of Proposition 5: For $\gamma < \underline{\gamma}(\lambda^E)$, Proposition 4 states that the incumbent always researches in equilibrium. With a RJV in which research outcomes are public, the incumbent researches

$$\int_0^\infty e^{-(r+\lambda^E+\lambda^I)t} \left\{ \lambda^I \left[p_0 \left(\frac{\bar{R} + \lambda^E \bar{D}/r}{r + \lambda^E} + (1-\gamma)R/r \right) + (1-p_0)\left(\underline{R}/r + (1-\gamma)R/r\right) - c \right] + \lambda^E \left[\frac{\lambda^I(\bar{D}/r - c)}{r + \lambda^I} + (1-\gamma)R/r \right] + R \right\} dt > \frac{R + \lambda^E(1-\gamma)R/r}{r + \lambda^E}.$$

Since the LHS icreases with $\bar{D}$, substituting $\bar{D} \approx \tilde{D} = rc$, researching gives $\frac{\lambda^I\left[p_0(\bar{R}+\lambda^E c)/(r+\lambda^E)+(1-p_0)\underline{R}/r\right]+(}{r+\lambda^E+\lambda^I}$
Then, simple algebraic manipulation establishes research when $p_0 > \frac{(r+\lambda^E)c+\gamma R-(r+\lambda^E)\underline{R}/r}{\bar{R}+\lambda^E c-(r+\lambda^E)\underline{R}/r} :=$ $\tilde{p}^{PI}$. The right-hand side increases with γ. Substituting $\gamma \approx \underline{R}/R$ we obtain $p_0 > \left((r+\lambda^E)c - \lambda^E \underline{R}/r\right)$ ($\lambda^E c)$. Then, for a small enough discount rate, $r < \frac{\lambda^E(\bar{R}-R)(c-\underline{R}/c)}{c(R+\lambda^E c)} := \underline{r}(\lambda^E, c)$, the incumbent does not research.
For $\gamma \geq \underline{\gamma}(\lambda^E)$, with public information, the incumbent obtains positive expected profits after the entrant introduces the product $V^I_{F,PI} = \lambda_I(\bar{D}/r - c)/(r + \lambda^I)$ (those profits are zero under private information $V^I_F = 0$). Under public information, the incumbent does not obtain the expected monopolistic profits after time $T^{E,INH\alpha}(\alpha_t)$; the entrant never abandons research. For $\bar{D} \approx rc$, we obtain $V^I_{F,PI} \approx 0 = V^I_F$. Becoming the monopoly after the entrant drops out generates too much research under private research outcomes. Because $V^I_{F,PI}$ increases with $\bar{D}$, there exist $\hat{D} \in (rc, rc/p)$ such that $\tilde{p}^{PI}(\cdot, \hat{D}) = \tilde{p}(\cdot, \hat{D})$. Therefore, for $\bar{D} < \hat{D}$, we have $\tilde{p}^{PI}(\cdot, \hat{D}) < \tilde{p}(\cdot, \hat{D})$ and the opposite happens for $\bar{D} > \hat{D}$. This explains the over-investment and the under-investment with private outcomes.

A.14. Tables

A.14.1. DiD regressions

Voice

Internet

A.15. Klemperer (1987a) and Network Externalities for Boundedly Rational Consumers and $\rho = 1$

A.15.1. Model Description

I build on Klemperer (1987a) to propose a model with network externalities. The setting considers consumers derive utility in direct proportion to the market size of the firm. This

Table **A-1**.: Differences in differences specifications for placebo tests.

	Dependent variable: arpu					
	Voice service			Internet access		
	(1)	(2)	(3)	(4)	(5)	(6)
Treated*Ban	15758.0 (10294.4)	12571.6 (9940.2)	11419.1 (9698.2)	336481.2 (204346.6)	348600.8* (200251.5)	297482.9 (209801.3)
Ban	-31656.3 (22205.3)	-37643.2 (23710.8)	-45787.3* (24334.8)	-1612706.3** (704105.3)	-748076.3* (441730.4)	-468632.5 (438360.0)
Treated	-43712.2*** (13314.5)	-42847.8*** (13350.0)	-42421.5*** (13214.9)	465955.3** (224311.3)	451682.9** (224236.5)	441706.1* (225188.2)
Access Charge	461.3* (275.9)	552.3* (292.4)	608.6** (306.6)			
Exchange rate	3.98 (16.8)	6.96 (17.2)	5.67 (18.0)	-805.3* (426.3)	-211.3 (194.9)	-29.2 (192.4)
RAN Data				-29169.7*** (8352.6)	-27112.1*** (8009.7)	-25245.6*** (8015.3)
Constant	45748.3 (44592.7)	34653.0 (46417.9)	37705.3 (49764.2)	2845736.7** (1315787.5)	912065.1* (505187.7)	313514.9 (495665.3)
Operator FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	757	757	757	891	891	891
R^2	0.329	0.329	0.328	0.815	0.815	0.815

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
Columns (1)-(3) display results for regressions using voice data. Columns (4)-(6) display results for regressions using Internet access data.

framework supports the fact that mobile markets display network externalities. I look to understand consumers and firms reactions to switching costs change in the presence of described externalities.

The game has two stages $t \in \{1, 2\}$. In stage 1, firms decide first period prices and consumers make purchase decisions. In second period, firms set prices and consumers purchase. The market is fully covered every period and both firms and consumers use discount factor λ.

There are two firms $i \in \{A, B\}$ that compete *à la Bertrand*. They sell an undifferentiated product. Each firm maximizes expected profits. The market size is $\rho = 1$. Firm $i = A$ locates at position 0 of the Hotelling line, and firm $i = B$ sits at position 1 of the market.

Consumers are located on a Hotelling line. Each consumer buys one unit of the product each period. A consumer located at x purchasing from firm $i = A$ at period t derives utility,

$$U_t^A = r - p_t^A - x + \alpha q_t^A,$$

where α accounts for network externality strength and q_t^i is the number of units sold by the firm from which consumer x purchases. A similar utility definition applies for consumers purchasing the product of firm $i = B$. Consumers take into account prices in second period, but can not fully internalize the reaction of firms in the second period to foresee future market shares. This makes them boundedly rational.

The consumers' preferences can change. A proportion ν of consumers leave the market after purchasing in period 1. A proportion μ of consumers have changing preferences at second

Table **A-2**.: Mobile voice difference in differences with all operators and different time span.

	Dependent variable: Voice ARPU with all operators 2012-2018				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	3244.2	4412.8	6855.0	8259.4*	8817.0*
	(4698.8)	(4637.9)	(4651.1)	(4569.8)	(4512.8)
Ban	13031.3	12735.2	12117.1	11712.8	11498.4
	(15912.9)	(15890.6)	(15889.6)	(15890.7)	(15897.4)
Treated	-76786.2***	-77084.2***	-77709.6***	-12087.1***	-78924.1***
	(3025.3)	(3096.0)	(3214.0)	(2806.2)	(3513.7)
Constant	82367.3***	82540.9***	82899.1***	16956.3	83495.3***
	(15075.9)	(15076.5)	(15079.9)	(14973.6)	(15103.2)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	686	686	686	686	686
R^2	0.795	0.795	0.795	0.795	0.796
	Dependent variable: Voice ARPU with all operators 2012-2019				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	3609.4	4764.6	7200.1	8590.3*	9169.0*
	(4928.0)	(4890.2)	(4891.7)	(4786.5)	(4689.2)
Ban	25094.2***	24795.6***	24171.6***	23765.5**	23541.4**
	(9377.6)	(9354.0)	(9335.5)	(9320.0)	(9307.5)
Treated	-75161.1***	-75454.5***	-76052.2***	-76662.2***	-77201.4***
	(2997.1)	(3055.1)	(3152.5)	(3275.6)	(3396.8)
Constant	70145.8***	70332.0***	70708.2***	71051.2***	71337.4***
	(7889.4)	(7886.2)	(7882.0)	(7885.0)	(7885.5)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	761	761	761	761	761
R^2	0.799	0.799	0.799	0.800	0.800
	Dependent variable: Voice ARPU with all operators 2012-2020				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	3759.8	4903.8	7353.8	8755.0*	9353.5**
	(5013.5)	(4976.0)	(4974.8)	(4864.2)	(4755.8)
Ban	26001.1**	25818.0**	26416.2***	25234.5**	25181.5**
	(11551.9)	(11535.9)	(8423.3)	(11512.3)	(11501.3)
Treated	-74875.9***	-75170.7***	-75768.2***	-76376.9***	-76918.6***
	(2985.8)	(3042.3)	(3136.8)	(3256.5)	(3372.7)
Constant	69173.0***	69248.0***	68403.4***	69524.2***	69640.9***
	(10337.3)	(10338.4)	(6674.7)	(10346.0)	(10348.4)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	788	788	788	788	788
R^2	0.801	0.801	0.802	0.802	0.802

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
.

period. The remaining proportion $(1 - \mu - \nu)$ of consumers have the same preferences in both periods.

Assumption 3.

$$\alpha < \frac{2(2\mu s + 1)}{3(\mu + \nu)} - \frac{4}{3}.$$

Table **A-3**.: Mobile voice difference in differences without any operator participating in both markets and different time span.

	Dependent variable: Voice ARPU without any operator participating in both markets 2012-2018				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	608.2	3639.4	7720.3	9871.4	10999.2
	(8012.1)	(7761.0)	(7563.2)	(7239.3)	(7003.8)
Ban	2645.2	2294.4	1753.6	1389.3	1125.7
	(20961.5)	(20952.3)	(20952.0)	(20949.1)	(20947.5)
Treated	-79136.3***	-79424.0***	-80159.3***	-80947.9***	-81713.9***
	(3281.7)	(3367.5)	(3526.4)	(3721.3)	(3932.7)
Constant	90668.7***	90810.8***	91126.4***	91437.1***	91737.1***
	(20124.6)	(20121.5)	(20115.8)	(20113.3)	(20112.4)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	546	546	546	546	546
R^2	0.785	0.785	0.786	0.786	0.787
	Dependent variable: Voice ARPU without any operator participating in both markets 2012-2019				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	966.5	4102.6	8280.3	10483.3	11683.8
	(8622.4)	(8396.9)	(8171.9)	(7806.9)	(7518.1)
Ban	23985.8***	23629.5***	23093.2***	22736.7**	22475.0**
	(8922.8)	(8909.7)	(8899.5)	(8887.4)	(8873.9)
Treated	-9524.9***	-12342.5***	-16089.9***	-13877.6***	-12144.2***
	(2731.6)	(2588.9)	(2873.9)	(2899.6)	(3308.0)
Constant	758.8	3438.9	6810.5	4171.8	2017.4
	(6632.4)	(6529.5)	(6571.1)	(6488.7)	(6578.6)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	601	601	601	601	601
R^2	0.790	0.790	0.790	0.791	0.791
	Dependent variable: Voice ARPU without any operator participating in both markets 2012-2020				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	1099.8	4257.7	8481.4	10715.6	11952.9
	(8782.4)	(8561.5)	(8333.3)	(7960.6)	(7658.8)
Ban	14243.8	19250.3*	18693.8*	13468.2	15174.1
	(12965.8)	(10658.8)	(10642.2)	(12955.3)	(11720.4)
Treated	-9346.6***	-9680.4***	-10437.1***	-11212.4***	-11968.0***
	(2719.6)	(2794.6)	(2947.8)	(3118.8)	(3274.2)
Constant	10417.7	5250.1	5654.7	10877.9	9246.2
	(11396.9)	(8662.6)	(8646.2)	(11411.6)	(9908.7)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	621	621	621	621	621
R^2	0.793	0.793	0.794	0.794	0.795

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
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A.15.2. Second Period

The decisions of consumers in second period must consider the purchase decision in first period and eventual change in preferences.

Proportion ν of consumers: They leave the market and enter again. Therefore, they are not

Table **A-4**.: Mobile voice difference in differences without any operator participating in both markets and different time span.

	Dependent variable: Voice ARPU without any operator participating in both markets 2012-2018				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	12208.1*	13528.6**	15138.1**	16306.3***	16306.3***
	(6663.9)	(6381.2)	(6232.6)	(6161.7)	(6161.7)
Ban	864.7	530.5	120.5	-241.7	-241.7
	(20940.3)	(20938.6)	(20936.9)	(20942.8)	(20942.8)
Treated	-27606.3***	-83662.6***	-84966.9***	-86240.7***	-86240.7***
	(3565.8)	(4393.3)	(4694.6)	(5008.2)	(5008.2)
Constant	37039.2*	92431.0***	92917.1***	93387.6***	93387.6***
	(19995.2)	(20113.3)	(20123.7)	(20146.6)	(20146.6)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	546	546	546	546	546
R^2	0.787	0.787	0.788	0.788	0.788
[1em]	Dependent variable: Voice ARPU without any operator participating in both markets 2012-2019				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	13013.3*	14287.6**	15802.9**	16859.8***	16859.8***
	(7114.5)	(6745.1)	(6498.8)	(6288.6)	(6288.6)
Ban	14999.7	21900.6**	14274.9	21208.6**	21208.6**
	(11935.3)	(8852.6)	(11910.9)	(8848.6)	(8848.6)
Treated	-13020.1***	-16501.3***	-17699.0***	-16360.7***	-16360.7***
	(3490.9)	(3440.5)	(3678.8)	(4125.7)	(4125.7)
Constant	9565.3	5256.7	13011.4	3743.8	3743.8
	(10274.5)	(6416.3)	(10166.3)	(6552.4)	(6552.4)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	601	601	601	601	601
R^2	0.791	0.792	0.792	0.793	0.793
[1em]	Dependent variable: Voice ARPU without any operator participating in both markets 2012-2020				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	13322.0*	14603.8**	16118.2**	17172.9***	17172.9***
	(7242.9)	(6858.1)	(6595.3)	(6360.9)	(6360.9)
Ban	13305.7	13226.7	17033.9	16694.5	16694.5
	(12946.3)	(12941.8)	(10549.6)	(10540.1)	(10540.1)
Treated	-12845.6***	-13834.3***	-15019.2***	-16154.7***	-16154.7***
	(3449.4)	(3632.0)	(3845.2)	(4053.0)	(4053.0)
Constant	11191.4	11382.4	7711.1	8209.7	8209.7
	(11404.9)	(11403.5)	(8535.5)	(8528.9)	(8528.9)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	621	621	621	621	621
R^2	0.795	0.795	0.796	0.796	0.796

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
.

subject to switching costs in second period. The indifference condition follows:

$$p_2^A + x - \alpha\sigma^A \leq p_2^B + (1-x) - \alpha\sigma^B + s$$
$$x \leq \frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha\sigma^A\rho\right]$$

Proportion μ of consumers: They have changing preferences. Therefore, they are subject to switching costs in second period. Also, the indifference condition depends on purchase

Table **A-5**.: Mobile Internet difference in differences with all operators and different time span.

	Dependent variable: Internet ARPU with all operators 2012-2018				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	377869.7*	329873.5*	231543.4	231663.2	70334.6
	(203999.3)	(195523.2)	(194555.6)	(196902.1)	(219020.0)
Ban	-334500.8	-942024.3*	-262377.5	-1392709.4**	-1362408.4**
	(417074.9)	(569624.3)	(408160.8)	(652405.2)	(653641.4)
Treated	-6569296.3***	-6580503.4***	-6584970.3***	-6593291.5***	-6578520.6***
	(695864.3)	(696246.5)	(696719.6)	(697251.4)	(697435.2)
Constant	6526878.3***	7141835.5***	6479248.9***	7608866.3***	7608743.2***
	(706290.8)	(814883.9)	(705762.8)	(871238.4)	(871487.9)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	891	891	891	891	891
R^2	0.815	0.815	0.815	0.815	0.814
	Dependent variable: Internet ARPU with all operators 2012-2019				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	359128.2*	310166.0	213128.8	185173.6	5089.6
	(197534.3)	(189472.5)	(188482.3)	(190441.6)	(218079.3)
Ban	-328684.0	-1044549.4*	-357107.4	-1023196.3*	-1447498.7**
	(410560.0)	(612236.9)	(414386.5)	(611309.6)	(709873.4)
Treated	-9889423.0***	-9898307.0***	-9901608.1***	-9905529.4***	-9889484.0***
	(1266918.6)	(1267092.2)	(1267276.2)	(1267365.6)	(1267064.2)
Constant	9898219.4***	10621818.4***	9951341.8***	10622062.1***	11080223.3***
	(1274898.1)	(1366350.6)	(1284252.0)	(1366339.7)	(1410403.9)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	1039	1039	1039	1039	1039
R^2	0.774	0.774	0.774	0.774	0.774
	Dependent variable: Internet ARPU with all operators 2012-2020				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	355174.0*	305962.0	208807.9	180570.1	713.9
	(197301.4)	(189232.2)	(188208.7)	(190111.9)	(217753.8)
Ban	-327583.9	-559601.8	-356011.7	-1022022.8*	-1446461.3**
	(410624.7)	(412370.3)	(414442.4)	(611392.0)	(709965.2)
Treated	728154.5	719656.0	716953.4	713375.6	730150.8
	(534450.2)	(535001.4)	(535706.2)	(536521.1)	(537153.2)
Constant	-713764.1	-473944.1	-660700.3	9955.1	467788.1
	(537228.4)	(541267.2)	(544528.0)	(712470.4)	(797336.8)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	1048	1048	1048	1048	1048
R^2	0.774	0.774	0.774	0.774	0.774

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
.

decisions in the first period. The indifference condition for buyers of A at period one:

$$p_2^A + x - \alpha\sigma^A \leq p_2^B + (1-x) - \alpha\sigma^B + s$$
$$x \leq \frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha\sigma^A + s\right]$$

Table **A-6**.: Mobile Internet difference in differences without any operator participating in both markets and different time span.

	Dependent variable: Internet ARPU without any operator participating in both markets 2012-2018				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	417303.1*	338780.3	203855.3	216166.1	13860.1
	(235353.2)	(224978.6)	(227071.2)	(228557.4)	(263244.2)
Ban	-669795.5	-1113323.4	-307937.9	-1107618.6	-1644168.7**
	(518803.6)	(695879.4)	(508562.5)	(696022.4)	(788875.7)
Treated	666220.3	653504.6	651005.7	641644.8	664623.5
	(569392.1)	(570315.8)	(571584.8)	(573160.3)	(574394.1)
Constant	-423567.1	22961.0	-774676.0	23039.2	574827.4
	(577020.9)	(748134.3)	(573032.8)	(748116.9)	(832012.2)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	751	751	751	751	751
R^2	0.814	0.814	0.814	0.814	0.814
	Dependent variable: Internet ARPU without any operator participating in both markets 2012-2019				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	481973.6**	387690.6*	245978.0	216838.3	-15662.2
	(237139.3)	(227174.0)	(229054.2)	(229451.6)	(271753.9)
Ban	-664295.0	-660308.0	-1230339.5*	-1769000.1**	-124351.9
	(512819.1)	(512093.0)	(743816.0)	(851931.6)	(515177.4)
Treated	850906.8	838551.5	835175.8	829942.5	852990.4
	(616756.3)	(617267.0)	(618021.4)	(619022.4)	(619572.2)
Constant	-542246.5	-542402.0	35554.0	575229.6	-1052338.8*
	(625041.3)	(625000.8)	(833931.7)	(931067.8)	(615592.5)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	879	879	879	879	879
R^2	0.773	0.773	0.773	0.773	0.773
	Dependent variable: Internet ARPU without any operator participating in both markets 2012-2020				
	2013-q3	2013-q4	2014-q1	2014-q2	2014-q3
Treated*Ban	480020.4**	384178.7*	241477.1	211508.7	-21339.9
	(237251.9)	(227248.6)	(229024.2)	(229238.5)	(271541.4)
Ban	-81391.9	-81243.4	-73777.5	-74248.6	-48423.4
	(512042.0)	(512888.1)	(513810.9)	(515321.6)	(516462.7)
Treated	861111.9	849175.6	846075.5	841194.6	863942.4
	(616716.4)	(617212.8)	(617943.4)	(618915.7)	(619470.0)
Constant	-1124351.3*	-1120556.3*	-1120014.0*	-1118445.2*	-1127155.4*
	(630982.2)	(631337.2)	(631528.6)	(631888.0)	(631600.0)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	885	885	885	885	885
R^2	0.773	0.773	0.773	0.773	0.773

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
.

The indifference condition for buyers of B at period one:

$$p_2^B + (1 - x) - \alpha\sigma^B \leq p_2^A + x - \alpha\sigma^A + s$$

$$x \geq \frac{1}{2}\left[p_2^B - p_2^A + (1 - \alpha) + 2\alpha\sigma^A - s\right]$$

Proportion $(1 - \mu - \nu)$ of consumers: These consumers have the same preferences in second period. Therefore, the number of consumers is $(1 - \mu - \nu)\sigma^A$.

Table **A-7**.: Mobile Internet difference in differences without any operator participating in both markets and different time span.

	Dependent variable: Internet ARPU without any operator participating in both markets 2012-2018				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	-295113.4	-449588.5	-629296.0**	-890554.4***	-890554.4***
	(298233.9)	(298665.7)	(319599.5)	(303616.3)	(303616.3)
Ban	-50794.1	-31573.5	-6797.9	29112.2	29112.2
	(547771.2)	(548549.9)	(549136.1)	(545175.7)	(545175.7)
Treated	723550.4	769853.3	949839.9	1099091.5*	1099091.5*
	(574024.0)	(575385.2)	(645898.6)	(627876.5)	(627876.5)
Constant	-992211.3*	-994545.5*	-1113085.6*	-1164720.2*	-1164720.2*
	(597343.2)	(597405.1)	(623933.4)	(620736.1)	(620736.1)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	751	751	751	751	751
R^2	0.814	0.814	0.814	0.815	0.815
[1em]	Dependent variable: Internet ARPU without any operator participating in both markets 2012-2019				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	-350093.6	-528178.9*	-724901.2**	-990304.1***	-990304.1***
	(314610.6)	(314182.7)	(328423.1)	(310008.5)	(310008.5)
Ban	-92927.1	-71182.5	-45206.8	-8907.5	-8907.5
	(515938.8)	(516801.0)	(517238.7)	(514068.5)	(514068.5)
Treated	907361.2	952575.7	1146013.0*	1283721.0*	1283721.0*
	(618852.7)	(619447.5)	(682262.3)	(666276.3)	(666276.3)
Constant	-1056239.2*	-1059581.3*	-1195945.2*	-1249943.5*	-1249943.5*
	(615661.5)	(615862.0)	(642839.7)	(640289.2)	(640289.2)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	879	879	879	879	879
R^2	0.773	0.774	0.774	0.774	0.774
[1em]	Dependent variable: Internet ARPU without any operator participating in both markets 2012-2020				
	2014-q4	2015-q1	2015-q2	2015-q3	2015-q4
Treated*Ban	-355428.0	-533358.7*	-729774.7**	-994488.4***	-994488.4***
	(314419.0)	(313913.3)	(328175.1)	(309767.3)	(309767.3)
Ban	959.0	43481.9	100388.5	182914.4	182914.4
	(517299.2)	(519675.1)	(522733.6)	(505866.9)	(505866.9)
Treated	917276.6	961899.2	1155437.7*	1291736.2*	1291736.2*
	(618852.4)	(619513.0)	(681706.8)	(666005.2)	(666005.2)
Constant	-1149043.5*	-1173182.4*	-1341429.8**	-1441672.0**	-1441672.0**
	(630886.4)	(630930.9)	(676102.2)	(663888.6)	(663888.6)
Operator FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	885	885	885	885	885
R^2	0.773	0.774	0.774	0.774	0.774

Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.
.

I put the expressions together to find the total sales of firm A in second period, q_2^A:

$$q_2^A = \nu \frac{1}{2}\left[p_2^B - p_2^A + 2\alpha\sigma^A + (1-\alpha)\right] + \mu \left\{ \sigma^A \frac{1}{2}\left[p_2^B - p_2^A + 2\alpha\sigma^A + (1-\alpha) + s\right] + \sigma^B \frac{1}{2}\left[p_2^B - p_2^A + 2\alpha\sigma^A + (1-\alpha) - s\right] \right\} + (1-\mu-\nu)\sigma^A.$$

Latest expression can be rewritten to account for common term $(p_2^B - p_2^A)$:

$$q_2^A = \frac{1}{2}\left\{(2\sigma^A - 1)\left[(1-\mu-\nu) + \mu s + \alpha(\mu+\nu)\right] + 1 + (\mu+\nu)(p_2^B - p_2^A)\right\}. \qquad \text{(A-51)}$$

Equation (A-51) replicates equation (1) of the paper Klemperer (1987a) when $\alpha = 0$.
Next, solve the problem of the firms in second period. Since each firm maximizes profits with respect to price, firm i solves:

$$\max_{p_2^i} \ \pi_2^i = \max_{p_2^i} \ (p_2^i - c)q_2^i.$$

The solution to this problem follows. The solution for price of firm $i = A$:

$$p_2^A = \frac{1}{3\,(\mu+\nu)}\{2 + 3\,c\,\mu + 3\,c\,\nu - \mu\,s + \mu + \nu + 2\,\sigma_A - \alpha\,\mu - \alpha\,\nu$$
$$+2\,\mu\,s\,\sigma_A - 2\,\mu\,\sigma_A - 2\,\nu\,\sigma_A + 2\,\alpha\,\mu\,\sigma_A + 2\,\alpha\,\nu\,\sigma_A\}$$

$$\begin{aligned} p_2^A &= c + \frac{1}{\mu+\nu}\left\{1 + \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu) + \mu s + \alpha(\mu+\nu)\right]\right\} \\ &= c + \frac{1}{\mu+\nu}\left\{1 + \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu) + \mu s\right]\right\} + \frac{2\sigma^A - 1}{3}\alpha \end{aligned}$$

This expression replicates the result for p_2^A of Klemperer (1987a) when $\alpha = 0$.
The solution for price of firm $i = B$:

$$p_2^B = \frac{1}{3\,(\mu+\nu)}\{4 + 3\,c\,\mu + 3\,c\,\nu + \mu\,s - \mu - \nu - 2\,\sigma_A + \alpha\,\mu + \alpha\,\nu - 2\,\mu\,s\,\sigma_A + 2\,\mu\,\sigma_A$$
$$+2\,\nu\,\sigma_A - 2\,\alpha\,\mu\,\sigma_A - 2\,\alpha\,\nu\,\sigma_A\}$$

$$\begin{aligned} p_2^B &= c + \frac{1}{\mu+\nu}\left\{1 - \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu) + \mu s + \alpha(\mu+\nu)\right]\right\} \\ &= c + \frac{1}{\mu+\nu}\left\{1 - \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu) + \mu s\right]\right\} - \frac{2\sigma^A - 1}{3}\alpha \end{aligned}$$

A useful expression is the difference $p_2^B - p_2^A$:

$$p_2^B - p_2^A = -\frac{2\,(2\,\sigma_A - 1)\,(1 + \mu\,s - \mu - \nu + \alpha\,\mu + \alpha\,\nu)}{3\,(\mu+\nu)}$$

$$p_2^B - p_2^A = -\frac{2(2\sigma^A - 1)}{3(\mu+\nu)}\left[(1-\mu-\nu) + \mu s + \alpha(\mu+\nu)\right]$$

Using the difference $p_2^B - p_2^A$, I rewrite equation (A-51) as follows:

$$q_2^A = \frac{1}{2}\left\{1 + \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu) + \mu s + \alpha(\mu+\nu)\right]\right\}.$$

A.15.3. First Period

Consumer profits in first period must account for probability of changing preferences due to rational behavior. In addition to first period profits, the consumer forms expectations of payoffs for second period. If preferences change with probability μ, the consumer can switch firm. The consumer can also keep preferences with probability $(1-\mu-\nu)$ and keeps on the same location. If the consumer leaves the market with probability ν, the expected payoffs from second period are null.

Consumers discount future payoffs by considering the likelihood of purchasing from any of the firms, given their first period decisions. There are two cases to be analysed: if a consumer buys from A in first period, and if a consumer buys from B in first period. If a consumer located at z buys from firm A in period one, the total payoffs in first period are

$$U_1^A = r - p_1^A - z + \alpha\sigma^A + \mu\lambda \left\{ \int_0^{x_1^A} r - p_2^A - x + \alpha\sigma^A \, dx + \int_{x_1^A}^1 r - p_2^B - (1-x) + \alpha\sigma^B - s \, dx \right\} + \lambda(1-\mu-\nu)(r - p_2^A - z + \alpha\sigma^A).$$

The integral limits denoted as x_1^A, define the indifferent location of the consumer that purchases from A in first period and switches to B in second period:

$$\begin{aligned} r - p_2^A - x_1^A + \alpha\sigma^A &= r - p_2^B - (1 - x_1^A) + \alpha\sigma^B - s \\ 2x_1^A &= p_2^B - p_2^A + \alpha\sigma^A - \alpha\sigma^B + 1 + s \\ x_1^A &= \frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha\sigma^A + s\right]. \end{aligned}$$

Using the latest expression for x_1^A, I solve the integrals to find U_1^A

$$U_1^A = r - p_1^A - z + \alpha\sigma^A + \mu\lambda \left\{ \frac{1}{4}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha\sigma^A + s\right]^2 + \left[r - p_2^B + \left(\alpha - \frac{1}{2}\right) - \alpha\sigma^A - s \right] \right\} + \lambda(1-\mu-\nu)(r - p_2^A - z + \alpha\sigma^A).$$

I simplify the previous expression and set $z = \sigma^A$ since that is the indifference condition that will be used to clear for σ^A later on:

$$U_1^A = r - p_1^A - (1-\alpha)\,\sigma^A + \lambda\,\mu \left\{ \frac{1}{4}\left[p_2^B - p_2^A + (1-\alpha) + 2\,\alpha\,\sigma^A + s\right]^2 + \left[r - p_2^B + \left(\alpha - \frac{1}{2}\right) - \alpha\,\sigma^A - s \right] \right\} + \lambda\,(1-\mu-\nu)\,\left(r - p_2^A - (1-\alpha)\,\sigma^A\right)$$

I work out now the second case. A consumer that purchases from $i = B$ in the first period and eventually switches to $i = A$ in second period gets the next expected utility:

$$U_1^B = r - p_1^B - (1-z) + \alpha\sigma^B + \mu\lambda \left\{ \int_0^{x_1^B} r - p_2^A - x + \alpha\sigma^A - s\,dx + \int_{x_1^B}^1 r - p_2^B - (1-x) + \alpha\sigma^B\,dx \right\} + \lambda(1-\mu-\nu)(r - p_2^B - (1-z) + \alpha\sigma^B).$$

The integral limits denoted as x_1^B, define the indifferent location of the consumer that purchases from B in first period and switches to A in second period:

$$\begin{aligned} r - p_2^B - (1 - x_1^B) + \alpha\sigma^B &= r - p_2^A - x_1^B + \alpha\sigma^A - s \\ 2x_1^B &= p_2^B - p_2^A + \alpha\sigma^A - \alpha\sigma^B + 1 - s \\ x_1^B &= \frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha\sigma^A - s\right]. \end{aligned}$$

Using the latest expression for x_1^B, I solve the integrals to find U_1^B

$$U_1^B = r - p_1^B - (1-z) + \alpha\sigma^B + \mu\lambda \left\{ \frac{1}{4}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha\sigma^A - s\right]^2 + \left[r - p_2^B + \left(\alpha - \frac{1}{2}\right) - \alpha\sigma^A\right] \right\} + \lambda(1-\mu-\nu)(r - p_2^B - (1-z) + \alpha\sigma^B).$$

I look for the indifferent consumer in the first stage of the game. The indifferent consumer is the one located at $z = \sigma^A$ such that $U_1^A = U_1^B$. Solving this problem, it is possible to find σ^A:

$$\sigma^A = \frac{1}{2} + \frac{p_1^B - p_1^A}{2} \left\{ \frac{2\lambda}{3(\mu+\nu)} \left[(1-\mu-\nu) + \mu s\right]^2 + (1-\alpha)\lambda\left[(1-\mu-\nu) + \mu s\right] + (1-\alpha) - \lambda\mu s \right\}^{-1}.$$

I proceed to solve first period problem of the firms. Each firm maximizes discounted profits with respect to price. Firm i solves:

$$\max_{p_1^i} \Pi^i = \max_{p_1^i}\ \pi_1^i + \lambda\pi_2^i = \max_{p_1^i}\ [p_1^i - c]\sigma^i(p_1^i) + \lambda[p_2^i(\sigma^i(p_1^i)) - c]q_2^i(\sigma^i(p_1^i)).$$

The solution to this problem follows. The solution for price of firm $i = A$:

$$p_1^A = c + 1 + \frac{2\lambda\mu s(\mu s + 1)}{3(\mu+\nu)} + \frac{1}{3}\left\{\lambda(1-\mu-\nu) - 4\lambda\mu s - 3\alpha\left[1 - \lambda(\mu+\nu)\right] - 5\alpha\lambda - 3\alpha\lambda\mu s\right\}$$

The solution for price of firm $i = B$:

$$p_1^B = c + 1 + \frac{2\lambda\mu s(\mu s + 1)}{3(\mu+\nu)} + \frac{1}{3}\left\{\lambda(1-\mu-\nu) - 4\lambda\mu s - 3\alpha\left[1 - \lambda(\mu+\nu)\right] - 5\alpha\lambda - 3\alpha\lambda\mu s\right\}$$

$$p_1^A = p_1^B = c + 1 + \frac{2\lambda\mu s(\mu s+1)}{3(\mu+\nu)} + \frac{1}{3}\left\{(1-\mu-\nu)\lambda - 4\lambda\mu s - \alpha\left[3(1-\mu-\nu)\lambda + 3\lambda\mu s + 3 + 2\lambda\right]\right\}$$

$$\sigma^A = \frac{1}{2} + \frac{p_1^B - p_1^A}{2\phi},$$

where ϕ is defined as follows

$$\phi = \frac{2\lambda}{3(\mu+\nu)}\left[1-\mu-\nu+\mu s\right]^2 + (1-\alpha)\left[1+(1-\mu-\nu)\lambda\right] - \alpha\lambda\mu s.$$

A.15.4. Comparative Statics

I provide a comparative statics analysis to reflect the behavior of the market. First, I provide the analysis for first period prices, then for second period prices in the symmetric equilibrium, i.e. $\sigma^A = 1/2$. If switching costs change, the price of firm A changes according to:

$$\frac{\partial p_1^A}{\partial s} = \frac{2(2\mu s+1)}{3(\mu+\nu)} - \alpha - \frac{4}{3}.$$

Therefore, p_1^A increases as long as Assumption 3 holds. Interestingly, network externality exacerbates the effect of changes in switching costs. Equally important, the effect is larger the larger the switching cost magnitude.
In the second period, prices in the symmetric equilibrium are the same for both firms. These prices do not depend on switching costs, therefore

$$\frac{\partial p_2^A}{\partial s} = 0.$$

A.16. Klemperer (1987a) and Network Externalities for Fully Rational Consumers and $\rho = 1$

A.16.1. Model Description

I build on from Klemperer (1987a) to propose a model with network externalities. The setting considers consumers derive utility in direct proportion to the market size of the firm. This framework supports the fact that mobile markets display network externalities. I look to understand consumers and firms reactions to switching costs change in the presence of described externalities.
The game has two stages $t \in \{1, 2\}$. In stage 1, firms decide first period prices and consumers make purchase decisions. In second period, firms set prices and consumers purchase. The market is fully covered every period and both firms and consumers use discount factor λ.
There are two firms $i \in \{A, B\}$ that compete *à la Bertrand*. They sell an undifferentiated product. Each firm maximizes expected profits. The market size is $\rho = 1$. Firm $i = A$

locates at position 0 of the Hotelling line, and firm $i = B$ sits at position $\rho = 1$ of the market[1].
Consumers are located on a Hotelling line. Each consumer buys one unit of the product each period. A consumer located at x purchasing from firm $i = A$ at period t derives utility,

$$U_t^A = r - p_t^A - x + \alpha q_t^A,$$

where α accounts for network externality strength and q_t^i is the number of units sold by the firm from which consumer x purchases. A similar utility definition applies for consumers purchasing the product of firm $i = B$. Consumers take into account prices in second period, and fully internalize the reaction of firms in the second period to foresee future market shares. This makes them fully rational.
The consumers' preferences can change. A proportion ν of consumers leave the market after purchasing in period 1. A proportion μ of consumers have changing preferences at second period. The remaining proportion $(1 - \mu - \nu)$ of consumers have the same preferences in both periods.

Assumption 4.

$$\frac{2}{3(\mu + \nu)} < \alpha < \frac{1}{\mu + \nu}.$$

A.16.2. Second Period

The decisions of consumers in second period must consider the purchase decision in first period and eventual change in preferences.
Proportion ν of consumers: They leave the market and enter again. Therefore, they are not subject to switching costs in second period. The indifference condition follows:

$$\begin{aligned} p_2^A + x - \alpha q_2^A &\leq p_2^B + (1 - x) - \alpha q_2^B \\ x &\leq \frac{1}{2}\left[p_2^B - p_2^A + 2\alpha q_2^A + (1 - \alpha)\right] \end{aligned}$$

Proportion μ of consumers: They have changing preferences. Therefore, they are subject to switching costs in second period. Also, the indifference condition depends on purchase decisions in the first period. The indifference condition for buyers of A at period one:

$$\begin{aligned} p_2^A + x - \alpha q_2^A &\leq p_2^B + (1 - x) - \alpha q_2^B + s \\ x &\leq \frac{1}{2}\left[p_2^B - p_2^A + 2\alpha q_2^A + (1 - \alpha) + s\right] \end{aligned}$$

[1] The market size is normalized to $\rho = 1$, just like Doganoglu and Grzybowski (2004) to make the model tractable.

The indifference condition for buyers of B at period one:

$$p_2^B + (1-x) - \alpha q_2^B \leq p_2^A + x - \alpha q_2^A + s$$
$$x \geq \frac{1}{2}\left[p_2^B - p_2^A + 2\alpha q_2^A + (1-\alpha) - s\right]$$

Proportion $(1-\mu-\nu)$ of consumers: These consumers have the same preferences in second period. Therefore, the number of consumers is $(1-\mu-\nu)\sigma^A$.
I put the expressions together to find the total sales of firm A in second period, q_2^A:

$$q_2^A = \nu\frac{1}{2}\left[p_2^B - p_2^A + 2\alpha q_2^A + (1-\alpha)\right] + \mu\left\{\sigma^A\frac{1}{2}\left[p_2^B - p_2^A + 2\alpha q_2^A + (1-\alpha) + s\right] + \sigma^B\frac{1}{2}\left[p_2^B - p_2^A + 2\alpha q_2^A + (1-\alpha) - s\right]\right\} + (1-\mu-\nu)\sigma^A.$$

Clearing for q_2^A leads to:

$$q_2^A = \frac{1}{1-\alpha(\mu+\nu)}\left\{\nu\frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha)\right] + \mu\left\{\sigma^A\frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + s\right] + \sigma^B\frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) - s\right]\right\} + (1-\mu-\nu)\sigma^A\right\}.$$

Latest expression can be rewritten to account for common term $(p_2^B - p_2^A)$:

$$q_2^A = \frac{1}{1-\alpha(\mu+\nu)}\left\{\frac{1}{2}(\mu+\nu)\left[p_2^B - p_2^A + (1-\alpha)\right] + \mu\frac{s}{2}(2\sigma_A - 1) + (1-\mu-\nu)\sigma_A\right\}. \tag{A-52}$$

Equation (A-52) replicates equation (1) of the paper Klemperer (1987a) when $\alpha = 0$.
Next, solve the problem of the firms in second period. Since each firm maximizes profits with respect to price, firm i solves:

$$\max_{p_2^i} \pi_2^i = \max_{p_2^i} (p_2^i - c)q_2^i.$$

The solution to this problem follows. The solution for price of firm $i = A$:

$$p_2^A = \frac{1}{3\,(\nu + \mu\,\sigma_A + \mu\,\sigma_B)}\,(2 + 3\,c\,\nu + \nu + 2\,\sigma_A - 2\,\alpha\,\mu + 3\,c\,\mu\,\sigma_A + 3\,c\,\mu\,\sigma_B - 3\,\alpha\,\nu + \mu\,s\,\sigma_A - \mu\,s\,\sigma_B - \mu\,\sigma_A + \mu\,\sigma_B - 2\,\nu\,\sigma_A - \alpha\,\mu\,\sigma_A - \alpha\,\mu\,\sigma_B)$$

$$p_2^A = \frac{1}{3\,(\nu+\mu)}\,(2 + 3\,c\,\nu + \nu + 2\,\sigma_A - 2\,\alpha\,\mu + 3\,c\,\mu(\sigma_A + \sigma_B) - 3\,\alpha\,\nu + \mu\,s(\sigma_A - \sigma_B) - \mu(\sigma_A - \sigma_B) - 2\,\nu\,\sigma_A - \alpha\,\mu(\sigma_A + \sigma_B))$$

$$p_2^A = \frac{1}{3\,(\nu+\mu)}\left[2 + 3\,(\mu+\nu)(c-\alpha) + \nu + 2\,\sigma_A\,(1+\mu(s-1)-\nu) - \mu(s-1)\right]$$
$$= c + \frac{1}{\mu+\nu}\left\{1 + \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu)+\mu s\right] - \alpha(\mu+\nu)\right\}$$

I can also write the solution in the second period to p_2^A as follows:

$$\begin{aligned}
p_2^A &= \frac{2\,(1+\mu(s-1)-\nu)}{3\,(\nu+\mu)}\sigma_A + \frac{2+3\,(\mu+\nu)(c-\alpha)+\nu-\mu(s-1)}{3\,(\nu+\mu)}\\
&= \frac{2\,(1+\mu(s-1)-\nu)}{3\,(\nu+\mu)}\sigma_A + \frac{2+3\,(\mu+\nu)c+\nu-\mu(s-1)}{3\,(\nu+\mu)} - \alpha\\
&= \frac{2\left[(1-\mu-\nu)+\mu s\right]}{3\,(\mu+\nu)}\sigma_A + \frac{2+3\,(\mu+\nu)c+\nu-\mu(s-1)}{3\,(\mu+\nu)} - \alpha\\
&= \theta_2^A\sigma_A + \xi_2^A.
\end{aligned}$$

This expression replicates the result for p_2^A of Klemperer (1987a) when $\alpha = 0$.
The solution for price of firm $i = B$:

$$p_2^B = \frac{1}{3\,(\nu+\mu\,\sigma_A+\mu\,\sigma_B)}\left[4 + 3\,c\,\nu - \nu - 2\,\sigma_A - 4\,\alpha\,\mu + 3\,c\,\mu\,\sigma_A + 3\,c\,\mu\,\sigma_B - 3\,\alpha\,\nu - \mu\,s\,\sigma_A + \mu\,s\,\sigma_B \right.$$
$$\left. +\mu\,\sigma_A - \mu\,\sigma_B + 2\,\nu\,\sigma_A + \alpha\,\mu\,\sigma_A + \alpha\,\mu\,\sigma_B\right]$$

$$p_2^B = \frac{1}{3\,(\nu+\mu)}\left[4 + 3\,c(\mu+\nu) - \nu - 2\,\sigma_A - 3\,\alpha(\mu+\nu) + \mu(\sigma_A-\sigma_B)(1-s) + 2\,\nu\,\sigma_A\right]$$
$$= c + \frac{1}{\mu+\nu}\left\{1 - \frac{2\sigma^A - 1}{3}\left[(1-\mu-\nu)+\mu s\right] - \alpha(\mu+\nu)\right\}$$

I can also write the solution in the second period to p_2^B as follows:

$$\begin{aligned}
p_2^B &= \frac{-2\,(1+\mu(s-1)-\nu)}{3\,(\nu+\mu)}\sigma_A + \frac{4+3(\mu+\nu)(c-\alpha)-\nu+\mu(s-1)}{3\,(\nu+\mu)}\\
&= \frac{-2\,(1+\mu(s-1)-\nu)}{3\,(\nu+\mu)}\sigma_A + \frac{4+3(\mu+\nu)c-\nu+\mu(s-1)}{3\,(\nu+\mu)} - \alpha\\
&= \frac{-2\left[(1-\mu-\nu)+\mu s\right]}{3\,(\mu+\nu)}\sigma_A + \frac{4+3(\mu+\nu)c-\nu+\mu(s-1)}{3\,(\mu+\nu)} - \alpha\\
&= -\theta_2^A\sigma^A + \xi_2^B. \qquad \text{(A-53)}
\end{aligned}$$

A useful expression is the difference $p_2^B - p_2^A$:

$$p_2^B - p_2^A = \frac{1}{3\,(\nu+\mu\,\sigma_A+\mu\,\sigma_B)}2\,\left(1 - \nu - 2\,\sigma_A - \alpha\,\mu - \mu\,s\,\sigma_A + \mu\,s\,\sigma_B + \mu\,\sigma_A - \mu\,\sigma_B + \right.$$
$$\left. 2\,\nu\,\sigma_A + \alpha\,\mu\,\sigma_A + \alpha\,\mu\,\sigma_B\right)$$

$$p_2^B - p_2^A = \frac{2(1-2\,\sigma_A)}{3\;(\mu+\nu)}\,[(1-\mu-\nu)+\mu s]$$

I have prices in second period. Now, I can attempt the solution to first period problem. It will be useful to write equation (A-52) in a more simplified way, using previous result for $p_2^B - p_2^A$:

$$q_2^A = \frac{1}{6[1-\alpha(\mu+\nu)]}\left[2-\mu(s-1)+\nu+2\sigma^A(1+\mu(s-1)-\nu)-3\alpha(\mu+\nu)\right]$$

$$q_2^A = \frac{1}{2[1-\alpha(\mu+\nu)]}\left\{1+\frac{2\sigma^A-1}{3}\,[(1-\mu-\nu)+\mu s]-\alpha(\mu+\nu)\right\}$$

This solution for q_2^A matches equation (3) of the paper Klemperer (1987a) when $\alpha = 0$. I can also write the solution as follows

$$\begin{aligned}
q_2^A &= \frac{(1-\nu)+\mu(s-1)}{3[1-\alpha(\mu+\nu)]}\sigma^A + \frac{2-\mu(s-1)+\nu-3\alpha(\mu+\nu)}{6[1-\alpha(\mu+\nu)]}\\
&= \frac{[(1-\nu-\nu)+\mu s]}{3[1-\alpha(\mu+\nu)]}\sigma^A + \frac{2\rho-\mu(s-1)+\nu-3\alpha(\mu+\nu)}{6[1-\alpha(\mu+\nu)]}\\
&= \frac{[(1-\nu-\nu)+\mu s]}{3[1-\alpha(\mu+\nu)]}\sigma^A - \frac{[(1-\mu-\nu)+\mu s]}{6[1-\alpha(\mu+\nu)]} + \frac{1}{2}
\end{aligned}$$

$$q_2^A = \phi\sigma^A + \omega. \tag{A-54}$$

A.16.3. First Period

Consumer profits in first period must account for probability of changing preferences due to rational behavior. In addition to first period profits, the consumer forms expectations of payoffs for second period. If preferences change with probability μ, the consumer can switch firm. The consumer can also keep preferences with probability $(1-\mu-\nu)$ and keeps on the same location. If the consumer leaves the market with probability ν, the expected payoffs from second period are null.

Consumers discount future payoffs by considering the likelihood of purchasing from any of the firms, given their first period decisions. There are two cases to be analysed: if a consumer buys from A in first period, and if a consumer buys from B in first period. If a consumer located at z buys from firm A in period one, the total payoffs in first period are

$$\begin{aligned}
U_1^A = r-p_1^A-z+\alpha q_1^A+\mu\lambda\left\{\int_0^{x_2} r-p_2^A-x+\alpha q_2^A\,dx+\int_{x_2}^1 r-p_2^B-(1-x)+\alpha q_2^B-s\,dx\right\}\\
+\lambda(1-\mu-\nu)(r-p_2^A-z+\alpha q_2^A).
\end{aligned}$$

The integral limits denoted as x_2, define the indifferent location of the consumer that purchases from A in first period and switches to B in second period:

$$\begin{aligned} r - p_2^A - x_2 + \alpha q_2^A &= r - p_2^B - (1 - x_2) + \alpha q_2^B - s \\ 2x_2 &= p_2^B - p_2^A + \alpha q_2^A - \alpha q_2^B + 1 + s \\ x_2 &= \frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha q_2^A + s\right]. \end{aligned}$$

Using the latest expression for x_2, I solve the integrals to find U_1^A

$$\begin{aligned} U_1^A = r - p_1^A - z + \alpha q_1^A + \mu\lambda &\left\{ \frac{1}{4}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha q_2^A + s\right]^2 \right. \\ &\left. + \left[r - p_2^B + \left(\alpha - \frac{1}{2}\right) - \alpha q_2^A - s \right] \right\} + \lambda(1-\mu-\nu)(r - p_2^A - z + \alpha q_2^A). \end{aligned}$$

I work out now the second case. A consumer that purchases from $i = B$ in the first period and eventually switches to $i = A$ in second period gets the next expected utility:

$$\begin{aligned} U_1^B = r - p_1^B - (1-z) + \alpha q_1^B + \mu\lambda &\left\{ \int_0^{x_2} r - p_2^A - x + \alpha q_2^A - s\, dx + \int_{x_2}^1 r - p_2^B - (1-x) + \alpha q_2^B\, dx \right\} \\ &+ \lambda(1-\mu-\nu)\left(r - p_2^B - (1-z) + \alpha q_2^B\right). \end{aligned}$$

The integral limits denoted as x_2, define the indifferent location of the consumer that purchases from B in first period and switches to A in second period:

$$\begin{aligned} r - p_2^B - (1 - x_2) + \alpha q_2^B &= r - p_2^A - x_2 + \alpha q_2^A - s \\ 2x_2 &= p_2^B - p_2^A + \alpha q_2^A - \alpha q_2^B + 1 - s \\ x_2 &= \frac{1}{2}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha q_2^A - s\right]. \end{aligned}$$

Using the latest expression for x_2, I solve the integrals to find U_1^B

$$\begin{aligned} U_1^B = r - p_1^B - (1-z) + \alpha q_1^B + \mu\lambda &\left\{ \frac{1}{4}\left[p_2^B - p_2^A + (1-\alpha) + 2\alpha q_2^A - s\right]^2 \right. \\ &\left. + \left[r - p_2^B + \left(\alpha - \frac{1}{2}\right) - \alpha q_2^A \right] \right\} + \lambda(1-\mu-\nu)\left(r - p_2^B - (1-z) + \alpha q_2^B\right). \end{aligned}$$

I look for the indifferent consumer in the first stage of the game. The indifferent consumer is the one located at $z = \sigma^A$ such that $U_1^A = U_1^B$. Solving this problem, it is possible to find σ^A:

$$\begin{aligned} \sigma^A = &\frac{1}{(\alpha + \lambda\,(\mu+\nu-1) - 1) - \frac{\lambda\,(3\,\alpha\,(\mu+\nu)-2)\,((\mu+\nu-1)-\mu\,s)^2}{3\,(\mu+\nu)\,(\alpha\,(\mu+\nu)-1)}} \left\{ \frac{p_{1\mathrm{A}}}{2} - \frac{p_{1\mathrm{B}}}{2} - \frac{(\lambda\,(\mu+\nu-1)-1)\,(\alpha-1)}{2} \right. \\ &\left. + \frac{\alpha\,\lambda\,\mu\,s}{2} + \frac{\lambda\,((\mu+\nu-1)-\mu\,s)^2}{3\,(\mu+\nu)\,(\alpha\,(\mu+\nu)-1)} + \frac{\alpha\,\lambda\,(\mu\,s + (\mu+\nu)\,(\alpha-1))\,((\mu+\nu-1)-\mu\,s)}{2\,(\alpha\,(\mu+\nu)-1)} \right\} \end{aligned}$$

$$\sigma^A = \frac{1}{2}\frac{1}{(1-\alpha+\lambda\,(1-\mu-\nu)) - \frac{\lambda\,(3\,\alpha\,(\mu+\nu)-2)\,((\mu+\nu-1)-\mu\,s)^2}{3\,(\mu+\nu)\,(1-\alpha\,(\mu+\nu))}}\{p_{1\mathrm{B}} - p_{1\mathrm{A}} + (1+\lambda\,(1-\mu-\nu))\,(1-\alpha)$$
$$-\alpha\,\lambda\,\mu\,s + \frac{2\lambda\,((\mu+\nu-1)-\mu\,s)^2}{3\,(\mu+\nu)\,(1-\alpha\,(\mu+\nu))} + \frac{\alpha\,\lambda\,((\mu+\nu)\,(1-\alpha)-\mu\,s)\,((1-\mu-\nu)+\mu\,s)}{1-\alpha\,(\mu+\nu)}\Bigg\}$$

$$\sigma^A = \frac{1}{2\delta}(p_1^B - p_1^A + \gamma). \tag{A-55}$$

I can also write σ^A as follows:

$$\begin{aligned}\sigma^A &= \frac{1}{2\delta}\left\{p_1^B - p_1^A + \delta\right\}\\ &= \frac{1}{2} + \frac{p_1^B - p_1^A}{2\delta}\end{aligned}$$

where δ is defined as follows:

$$\delta = 1-\alpha+\lambda\,(1-\mu-\nu) - \frac{\lambda\,(3\,\alpha\,(\mu+\nu)-2)\,((\mu+\nu-1)-\mu\,s)^2}{3\,(\mu+\nu)\,(1-\alpha\,(\mu+\nu))} \tag{A-56}$$

I proceed to solve first period problem of the firms. Each firm maximizes discounted profits with respect to price. Firm i solves:

$$\max_{p_1^i}\Pi^i = \max_{p_1^i}\ \pi_1^i + \lambda\pi_2^i = \max_{p_1^i}\ [p_1^i - c]\sigma^i(p_1^i) + \lambda[p_2^i(\sigma^i(p_1^i)) - c]q_2^i(\sigma^i(p_1^i)).$$

The solution for first period prices follow:

$$\begin{aligned}p_1^A = p_1^B &= \frac{3\,c\,\mu - 2\,\lambda + 3\,c\,\nu + 3\,\delta\,\mu + 3\,\delta\,\nu + 2\,\lambda\,\mu + 2\,\lambda\,\nu - 2\,\lambda\,\mu\,s}{3\,(\mu+\nu)}\\ &= \frac{3\,c(\mu+\nu) + 3\,\delta(\mu+\nu) - 2\,\lambda(1-\mu-\nu+\mu s)}{3\,(\mu+\nu)}\\ &= c + \delta - \frac{2\,\lambda(1-\mu-\nu+\mu s)}{3\,(\mu+\nu)},\end{aligned}$$

where δ follows the definition of equation (A-56).

A.16.4. Comparative Statics

I provide a comparative statics analysis to reflect the behavior of the market. First, I provide the analysis for first period prices, then for second period prices in the symmetric equilibrium, i.e. $\sigma^A = 1/2$. If switching costs change, the price of firm A changes according to:

$$\frac{\partial p_1^A}{\partial s} = \frac{\lambda\,[3\alpha(\mu+\nu)-2]}{3(\mu+\nu)\,[1-\alpha(\mu+\nu)]}2(\mu+\nu-1-\mu s)(\mu) - \frac{2\lambda\mu}{3(\mu+\nu)}.$$

Therefore, p_1^A increases as long as Assumption 4 holds. Interestingly, network externality exacerbates the effect of changes in switching costs. Less notably, but equally important, the effect is larger the larger the switching cost magnitude.
The price in the second period reacts to market shares of the first period. The competition intensity of the first period translates into second period strategies of the firms. Given market shares of first period, the price of the leader in the second period changes as follows

$$\frac{\partial p_2^A}{\partial s} = 0.$$

Since market shares are even in the first period, the second period prices are not sensitive to switching costs.

A.17. Comparing models of Appendix A.16 and Appendix A.15

	Fully rational	Boundedly rational
p_1^A p_1^B	$p_1^A = p_1^B = c + \delta - \frac{2\lambda(1-\mu-\nu+\mu s)}{3(\mu+\nu)}$	$c + 1 + \frac{2\lambda\mu s(\mu s+1)}{3(\mu+\nu)} + \frac{1}{3}\{(1-\mu-\nu)\lambda - 4\lambda\mu s - \alpha[3(1-\mu-\nu)\lambda + 3\lambda\mu s + 3 + 2\lambda]\}$
σ_A	$\frac{1}{2} + \frac{p_1^B - p_1^A}{2\delta}$	$\frac{1}{2} + \frac{p_1^B - p_1^A}{2\phi}$
p_2^A	$c + \frac{1}{\mu+\nu}\left\{1 + \frac{2\sigma^A-1}{3}[1-\mu-\nu+\mu s] - \alpha(\mu+\nu)\right\}$	$c + \frac{1}{\mu+\nu}\left\{1 + \frac{2\sigma^A-1}{3}[1-\mu-\nu+\mu s + \alpha(\mu+\nu)]\right\}$
p_2^B	$c + \frac{1}{\mu+\nu}\left\{1 - \frac{2\sigma^A-1}{3}[1-\mu-\nu+\mu s] - \alpha(\mu+\nu)\right\}$	$c + \frac{1}{\mu+\nu}\left\{1 - \frac{2\sigma^A-1}{3}[1-\mu-\nu+\mu s + \alpha(\mu+\nu)]\right\}$
q_2^A	$\frac{1}{2[1-\alpha(\mu+\nu)]}\left\{1 + \frac{2\sigma^A-1}{3}[1-\mu-\nu+\mu s] - \alpha(\mu+\nu)\right\}$	$\frac{1}{2}\left\{1 + \frac{2\sigma^A-1}{3}[1-\mu-\nu+\mu s + \alpha(\mu+\nu)]\right\}$

Table **A-8**.: Main results for models of Appendix A.16 and Appendix A.15.

The definition of δ follows the equation (A-56).

A.17.1. Comparative Statics

I assume an asymmetric market and perform some comparative statics for leader firm. This is the case that captures my interest, since the market I analyze is consistently asymmetric along time. Let $\sigma_1^A > 1/2$, so that firm A holds the larger market share. The comparative statics for prices in the first and the second period is shown in Table **A-9**.
Besides analyzing the asymmetric case predictions of my models, I make a narrower focus on the second period prices set by the large firm. I use this approach to resemble the strategic reaction of the firms when switching costs are removed.
Second period prices of the leader firm in both models increase with switching costs. Depending on their preferences, consumers switch and internalize the costs of switching. If consumers are eager to switch, they are eager to pay even a higher price to move. In particular, the consumers with changing preferences are the ones subject to switching costs, i.e.

	Fully Rational	Boundedly Rational
$\partial p_1^A/\partial s$	$\frac{\lambda[3\alpha(\mu+\nu)-2]}{3(\mu+\nu)[1-\alpha(\mu+\nu)]}2(\mu+\nu-1-\mu s)(\mu)-\frac{2\lambda\mu}{3(\mu+\nu)}$	$\frac{2(2\mu s+1)}{3(\mu+\nu)}-\alpha-\frac{4}{3}$
$\partial p_1^A/\partial s > 0$	$\alpha > \frac{1}{\mu+\nu}\left[1+\frac{1-\mu-\nu+\mu s}{1-3(1-\mu-\nu+\mu s)}\right]$	$\alpha < \bar{\alpha} = \frac{2(2\mu s+1)}{3(\mu+\nu)}-\frac{4}{3}$
$\partial p_2^A/\partial s$	$\frac{2\sigma^A-1}{3(\mu+\nu)}\mu$	$\frac{2\sigma^A-1}{3(\mu+\nu)}\mu$

Table **A-9**.: Comparative statics for prices of leader firm (firm A) when $\sigma_1^A > 1/2$ and switching cost changes.

the proportion μ of consumers in the market. Since the burden of switch does not depend on the size of any firm, the dis-utility for consumers is the same regardless of the size of the previous or the new firm. This is why in the case of boundedly rational consumers the change in second period price when switching cost changes is the same as for fully rational consumers.

Network externalities do not shape the change in prices of the second period when switching costs are removed. The introduction of network externalities into Klemperer (1987a) model, affects price levels without regard to switching costs.

When network externalities increase, the second period prices decrease for fully rational consumers and increase for boundedly rational consumers. For consumers that fully foresee market shares in second period, the expected utility due to firm size is correct. This implies consumers are not subject to any strategy of the firm due to lack of information. For boundedly rational consumers, they expect a different payoff due to network externalities, which translates in a bad purchase decision since actual market shares in second period do not match the ones of the first period.

	Fully Rational	Boundedly Rational
$\partial p_1^A/\partial\alpha$	$-1-\frac{\lambda(1-\mu-\nu+\mu s)^2}{3[1-\alpha(\mu+\nu)]^2}$	$-\left[(1-\mu-\nu)\lambda+\lambda\mu s+1+\frac{2}{3}\lambda\right]$
$\partial p_2^A/\partial\alpha$	-1	$\frac{2\sigma^A-1}{3}$

Table **A-10**.: Comparative statics for prices of leader firm (firm A) when $\sigma_1^A > 1/2$ and network externality changes.

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