A Mathematical Model Under Uncertainty for Optimizing Medicine Logistics in Hospitals

Carlos Franco[1(✉)], Eduyn Ramiro López-Santana[2], and Juan Carlos Figueroa-García[2]

[1] Universidad del Rosario, Bogotá, Colombia
carlosa.franco@urosario.edu.co
[2] Universidad Distrital Francisco José de Caldas, Bogotá, Colombia
{erlopezs,jcfigueroa}@udistrital.edu.co

Abstract. Managing resources in hospitals is one of the most challenging duties in healthcare. The complexity of supply chain management in hospitals is high due to different factors such as life cycle of medicines, demand uncertainty, variation of prices, monetary resources, space constraints, among others. The main important factor of the supply chain in hospitals is the welfare of patients which depends of the correct management and administration of medicines, in this way backorders or stockouts are not allowed. In this paper we propose a mathematical model to make real planning over a health care supply chain considering real factors face by decision makers. For testing results we have used real data considering different sources of uncertainty. We have choose 5 different types of medicines and run the optimization model to determine the optimal solution over a set of scenarios generated for modeling uncertainty. For testing the results, we have compare over a year planning the results obtained by our policy and the results obtained by the hospital, improving the results in terms of costs.

Keywords: Optimization · Robust optimization · Optimization under scenarios Hospital planning · Pharmaceutical logistics

1 Introduction

Pharmaceutical costs are representative expenses in hospitals. It can vary between 25 to 30% of the total costs [1], also logistic costs associate to moving, conditioning, handling and dispatching medicines are a very big portion of the total logistic costs, they can vary between 35 to 40% of the total logistic costs [2]. Pharmaceutical expenses are one of the most important global issues for planning because in the world can be estimated that medicines consume 20%–30% of global health spending [3]. However, pharmaceutical supply chain management is more difficult than typical applications within industrial companies since medicines and surgical supplies must be available for use always and methods are not developed for this type of industries [4].

The managing of medicines in hospitals represents considerable challenges like the needs to store medicines and surgical supplies that required to be of sufficient quantity and availability for the staff to use when necessary, inventory policies and service levels among others.

J. C. Figueroa-García et al. (Eds.): WEA 2018, CCIS 916, pp. 53–60, 2018.
https://doi.org/10.1007/978-3-030-00353-1_5

To model uncertainty in any situation, it can be adjusted by a distribution function but in some cases these values of uncertainty cannot be adjusted by a specific distribution function. In this study we have model the uncertainty generating scenarios that allow to model the real behavior of the random parameters. In the managing of medicines two types of different risks can occur: stockouts or surplus inventories [5].

In this paper we have developed and optimization model that consider some random aspects to model the real aspects of decision makers in hospitals that allows to obtain robust solutions that improve the performance of management the medicines. The paper is organized as follows: Sect. 2 introduce some literature review, in Sect. 3 the mathematical model is presented and in Sect. 4 some results are analyzed. Finally, in Sect. 5 the conclusions are presented.

2 Overview of Related Literature

A review of different models about pharmaceutical supply chain is introduced in [6] where the proposed taxonomy is made by differentiated different echelons of the chain and different levels. The proposed literature is divided into network optimization, inventory models and optimization of distribution of medicines. A first conclusion is that in inventory models the most source of uncertainty is the demand and it has been deeply studied but some other sources has not been included in these studies. A first approximation of inventory models with medicines was proposed in [7] where an extension of the periodic review model is proposed, they implement space constraints in the model, but it is included in the objective function. Also, in [4], a space constraint is used. The proposal considers the volume of the medicines to include as a constraint of space limitation. Some other models have developed approximations to the same problem, but these models are not developed for medicines or the management of hospitals [8–12].

A stochastic and periodic review model is presented in [13]. Objective function is formulated in terms of stock-out and budget. Also, in [14] a Markov chain model is proposed using the order up to level policies and considering stochastic demand, batching, emergency deliveries, and service levels, also a heuristic is proposed to reduce the computational complexity. Little and Coughlan [4] develop a constraint-based model for determining stock levels for all products at a storage location with space constraints, which considers the criticality of medicines. This model is an extension of a previous article presented by [7].

In [15] an extension of the (R, s, S) model is proposed. It is denoted as the (R, s, c, S) model based on the classic EOQ model. Another inventory model has been developed in [16]. Two models are proposed, one based on (s, S) model and the second one is formulated in terms of optimal allocation. Also, an approximation via simulation is presented in [17]. Two stages are considered, a Markov decision process to represent medicine demand and the use of simulation to evaluate the inventory policies characterized in the first phase. Another approximation using system dynamics was developed by Wang et al. [18].

Also, some approaches use RFID systems as those presented in [19, 20]. A different objective function is used in [21] the maximization of the total net profit is considered. A mixed integer linear programming is used. Also, a proposal for testing inventory policies by considering characteristics of medicines is developed in [1] where for testing the policies, a simulation model is developed.

3 Problem Definition

The objective of the mathematical model is to ensure that decisions of purchasing medicines consider the costs associated with prices of medicines, purchases not planed, also the problem considers constraints associated to human resources capacities, satisfaction of demand and availability of medicines in the market.

3.1 Mathematical Model

The proposed mathematical model is as follows:

Sets

- T = Time periods in the planning horizon
- P = Set of type of medicines
- S = Set of suppliers
- L = Set of medicine's life cycle
- K = Set of scenarios

Parameters

- d_{ptk} = Demand of each type of medicine in each scenario
- lt_{spk} = Lead time of each supplier for each medicine in each scenario
- c_{psk} = Cost of each medicine in each scenario
- ls_p = Lot size of each medicine
- av_{sp} = Availability of each medicine by each supplier
- ut_p = Unit doses time for each type of medicine
- lc_p = Life cycle for each medicine
- ec_p = Cost of each medicine for avoid unsatisfied demand
- cap = Availability of human resources in hours
- mna = Number allowed for making supplies, this parameter implies that there is a limitation in the number of orders because of administrative capacity

Variables

- Q^s_{pt} = Number of lots of medicines for each period required to each supplier

$$y^s_{pt} = \left\{ \begin{array}{c} 1\, \textit{if the requirement of medicine p to supplier is made} \\ 0\, \textit{otherwise} \end{array} \right\}$$

- IP^{lk}_{pt} = Inventory level for each medicine with specific life cycle in each time in each scenario

- I_{pt}^{k} = Net Inventory level of medicines for each time period and each scenario, this variable totalized the previous variable per period time
- RP_{pt}^{lk} = Amount of medicine distributed for each period time for each scenario
- R_{pt}^{k} = Net amount of medicine distributed
- EQ_{pt} = Number of lots purchased in emergency cases for each period time

$$\begin{aligned}\boldsymbol{MinZ} = \Big(&\sum\nolimits_{k\in K}\sum\nolimits_{t\in T}\sum\nolimits_{s\in S}\sum\nolimits_{p\in P} c_{spt} * Q_{pt}^{sk} + \sum\nolimits_{t\in T}\sum\nolimits_{p\in P} ec_p * EQ_{pt} \\ &+ \sum\nolimits_{s\in S}\sum\nolimits_{k\in K}\sum\nolimits_{t\in T}\sum\nolimits_{p\in P}\sum\nolimits_{l\in L|(t-l)\rangle lt_{spk}-1} c_{psk} * IP_{pt}^{lk}\Big)/|\boldsymbol{K}|\end{aligned} \tag{1}$$

$$IP_{pt}^{lk} = IP_{p\,t-1}^{l-1k} - RP_{pt}^{lk} \quad \forall t, \forall p, \forall l | l \leq t\, and\, t - l \leq lc_p - 1, \forall k \tag{2}$$

$$IP_{pt}^{lk} = \sum_{s\in S} ls_p * Q_{pt-lt_{spk}}^{s} - RP_{pt}^{lk} \quad \forall t, \forall p, \forall l = 1, \forall k \tag{3}$$

$$I_{pt}^{k} = \sum_{l\in 1..t} IP_{pt}^{lk} \quad \forall t, \forall p, \forall k \tag{4}$$

$$R_{pt}^{k} = \sum_{l\in 1..t} RP_{pt}^{lk} \quad \forall t, \forall p, \forall k \tag{5}$$

$$Q_{pt}^{s} \leq M * y_{pt}^{s} \quad \forall t, \forall p, \forall s \tag{6}$$

$$y_{pt}^{s} \leq av_{sp} \quad \forall t, \forall p, \forall s \tag{7}$$

$$\sum_{p\in P}\sum_{s\in S} ut_p * ls_p * Q_{pt}^{s} \leq cap \quad \forall t \tag{8}$$

$$R_{pt}^{k} + ls_p * EQ_{pt} = d_{Ptk} \quad \forall t, \forall p, \forall k \tag{9}$$

$$\sum_{p\in P}\sum_{s\in S} y_{pt}^{s} \leq mna \quad \forall t, \forall k \tag{10}$$

$$RP_{pt}^{lk} = 0 \quad \forall t, \forall p, \forall l | t - l > lc_p - 1, \forall k \tag{11}$$

$$\begin{gathered} Q_{pt}^{s} \geq 0, y_{pt}^{s} \geq 0 \quad \forall t, \forall p, \forall s \\ IP_{pt}^{lk} \geq 0, RP_{pt}^{lk} \geq 0 \quad \forall t, \forall p, \forall l, \forall k \\ R_{,pt}^{k} I_{pt}^{k} \geq 0 \quad \forall t, \forall p, \forall k \\ EQ_{pt} \geq 0 \quad \forall t, \forall p \end{gathered} \tag{12}$$

The objective function consists in minimizing the average expected total costs overall scenarios. It contains the costs of regular purchases, the costs of emergency purchases and the costs of loss of medicines, Eq. (1).

For modeling the inventory levels of medicines considering life cycle we proposed two different types of constraints, in Eq. (2) we define the inventory level for each type of medicine for each scenario in the corresponding age of life, this means that in a specific period of time for a specific medicine there are amounts of the medicine that bellows to a different age, for example in period 2 for medicine 1 can be medicines with age 1 and/or age 2. In this way, the constraints for an specific medicine in an specific period time in a specific age is equal to the amount of medicine in the previous period that has the one year less of life minus the amount of medicine given to satisfy the demand in an specific cycle life, this means that it can be selected which age of medicine it is going to satisfy the demand.

Equation (3) is complementary to Eq. (2) because it models the age of medicines, when a purchase is made the age of medicines in inventory are 1, in this way we can model the age of medicines when they increase the period of life. Finally, the amount of medicines with age one given to satisfy the demand is subtracted. Constraints (4) contains the net inventory for a specific type of medicine in every period time in each scenario as the total amount of medicines in a specific period time in a specific scenario for each type of medicine.

Similarly to Eq. (4), in Eq. (5) the amount of medicines distributed to satisfy the demand is totalized. Equation (6) guarantees that only it is possible to purchase medicines if the binary variable is activated. The availability of medicines is modeled as the relationship between the binary variable that defines if a specific amount of medicine is supply by a specific company and the parameter that indicates if the company has in its portfolio a specific medicine, this constraint is modeled in Eq. (7).

By regulation every medicine must be put in unit-doses packages, so people for pharmacy are involved in this task. Human resource capacity is modeled in Eq. (8).

The amount of medicines given to satisfy the demand must be the exactly demand because patients cannot wait until suppliers provide medicines and this is because of the health of patients. The maximum number of orders allowed to made in a month is modeled in Eq. (10) and it is not allowed to distribute medicines out of the life cycle as presented in Eq. (11). Finally, the types of variables are modeled with Eq. (12).

4 Results

4.1 Instances Description

For testing the proposed model, we have used real data for a hospital that allowed us to analyze and generate different types of scenarios considering real life conditions that can not be modeled with the traditional models. Some random parameters were generated for generating scenarios and provide robustness to the solution. We have selected 5 types of different medicines to generate scenarios and analyze the results of the application of our model and the solution generated by the hospital. For each type of medicine, we have generated 30 different scenarios varying the selling price of suppliers, lead times and demand where we consider a full year divided into months. For analyzing the results also, we have run our model in a year planning to compare the results of the optimal solution provided by our approach and the solution made by the hospital.

4.2 Results and Analysis

For analyzing the results, we first summarize the gap between the solutions. These results are summarized in Table 1 as follows: column one contains the type of medicine, column two shows the worst scenario in costs over the total scenarios (in Colombian pesos), column three presents the best scenario in costs over the 30 scenarios (in Colombian pesos), and finally the gap as a percentage and the average results are presented.

Table 1. Scenario results

Medicine	Worst scenario	Best scenario	Gap	Average scenario
1	$ 6,202,767	$ 6,100,822	1.67%	$ 6,151,239
2	$ 5,603,429	$ 4,967,355	12.81%	$ 5,570,349
3	$ 3,522,535	$ 3,512,620	0.28%	$ 3,518,658
4	$ 12,173,578	$ 12,156,288	0.14%	$ 12,164,385
5	$ 26,502	$ 23,814	11.29%	$ 25,097
Average	$ 1,304,395,581	$ 1,295,714,056	0.67%	$ 5,485,946

In Table 1, it can be concluded that the average distance between solutions doesn't exceed 1%, in that case we can say that our method hasn't a big variation between the results, therefore the method obtain a good quality solution for the real case application. Now for comparing the results to see the differences between the real situation and the results obtained by our model, we have run our model with different data and we have compared our solution generated without consider the new data and the decisions taken by the hospital's manager. These results are summarized in Fig. 1. Where the real policy is presented in costs (by color blue) and the optimal policy obtained by our model is presented in color orange. Results of real values are not presented because of internal policies of the hospital.

The improvement of the planning for each medicine is 3.71%, 99.63%, 10.20%, 0.26% and 12.97% respectively. This means that in average in 25% is improved the policy of managing medicines in the hospital. Also it can be concluded that for 4 of the 5 medicines the improvement is at least 10% except for medicine 1 and 4, also for medicine 2 the improvement is over 90% presented a big reduction of the total costs if the model were implemented.

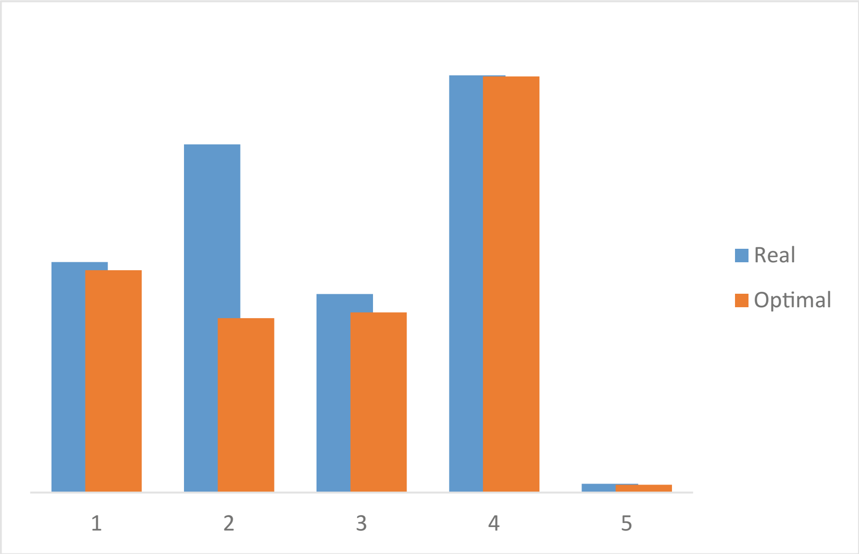

Fig. 1. Comparison between optimal policy and real policy (Color figure online)

5 Conclusions

In this paper we have studied the problem of planning medicines in the case of a hospital. Some sources of uncertainty were considered considering real situation presented in the planning of resources such as demand, lead times and life cycle. 30 different scenarios were considered for modeling uncertainty and for each parameter these scenarios were adapted for considering different variations of values. A total of five medicines were considered for testing the results obtaining improvements in the total costs.

Future works will consider development of different approaches as stochastic optimization, simulation optimization and also other sources of uncertainty. Also, testing the model over a big number of medicines for considering the real case of application in the decision making.

Acknowledgments. We thank Fair Isaac Corporation (FICO) for providing us with Xpress-MP licenses under the Academic Partner Program subscribed with Universidad Distrital Francisco Jose de Caldas (Colombia).

References

1. Gebicki, M., Mooney, E., Chen, S.-J., Mazur, L.M.: Evaluation of hospital medication inventory policies. Health Care Manag. Sci. **17**(3), 215–229 (2014)
2. McKone-Sweet, K.E., Hamilton, P., Willis, S.B.: The ailing healthcare supply chain: a prescription for change. J. Supply Chain Manag. **41**(1), 4–17 (2005)

3. World Health Organization. The World Health Report Health System Financing: The Path to Universal Coverage (2010)
4. Little, J., Coughlan, B.: Optimal inventory policy within hospital space constraints. Health Care Manag. Sci. **11**, 117–183 (2008)
5. Zepeda, E.D., Nyaga, G.N., Young, G.J.: Supply chain risk management and hospital inventory: effects of system affiliation. J. Oper. Manag. **44**, 30–47 (2016)
6. Franco, C., Alfonso-Lizarazo, E.: A structured review of quantitative models of the pharmaceutical supply chain. Complexity **2017**, 1–13 (2017)
7. Vincent, V., Ranton, M.: Hospital pharmacy inventory management: economic order quantity model with space limitation. Hosp. Mater. Manag. Q. **5**(3), 82–86 (1984)
8. Ouyang, L.-Y., Ho, C.-H., Su, C.-H., Yang, C.-T.: An integrated inventory model with capacity constraint and order-size dependent trade credit. Comput. Ind. Eng. **84**, 133–143 (2015)
9. Chou, S.-Y., Julian, P.C., Hung, K.-C.: A note on fuzzy inventory model with storage space and budget constraints. Appl. Math. Model. **33**, 4069–4077 (2009)
10. Chung, K.-J.: The correct proofs for the optimal ordering policy with trade credit under two different payment methods in a supply chain system. TOP **20**(3), 768–776 (2012)
11. Tsai, S.C., Zheng, Y.-X.: A simulation optimization approach for a two-echelon inventory system with service level constraints. Eur. J. Oper. Res. **229**, 364–374 (2013)
12. Priyan, S., Uthayakumar, R.: Two-echelon multi-product multi-constraint product returns inventory model with permissible delay in payments and variable lead time. J. Manuf. Syst. **36**, 244–262 (2015)
13. Şatir, A., Cengiz, D.: Medicinal inventory control in a university health centre. J. Oper. Res. Soc. **38**(5), 387–395 (1987)
14. Guerrero, W.J., Yeung, T., Guéret, C.: Joint-optimization of inventory policies on a multi-product multi-echelon pharmaceutical system with batching and ordering constraints. Eur. J. Oper. Res. **231**, 98–108 (2013)
15. Dellaert, N., Van De Poel, E.: Global inventory control in an academic hospital. Int. J. Prod. Econ. **46–47**, 277–284 (1996)
16. Kelle, P., Woosley, J., Schneider, H.: Pharmaceutical supply chain specifics and inventory solutions for a hospital case. Oper. Res. Health Care **1**(2–3), 54–63 (2012)
17. Vila-Parrish, A.R., Ivy, J.S., King, R.E., Fitts, E.P.: A simulation-based approach for inventory modeling of perishable pharmaceuticals. In: Winter Simulation Conference, pp. 7–15 (2008)
18. Wang, L.-C., Cheng, C.-Y., Tseng, Y.-T., Liu, Y.-F.: Demand-pull replenishment model for hospital inventory management: a dynamic buffer-adjustment approach. Int. J. Prod. Res. **53** (24), 7533–7546 (2015)
19. Çakıcı, Ö.E., Groenevelt, H., Seidmann, A., Simon, W.E.: Using RFID for the management of pharmaceutical inventory—system optimization and shrinkage control. Decis. Support Syst. **51**(4), 842–852 (2011)
20. Schapranow, M.-P., Mu¨ller, J., Zeier, A., Hasso, P.: Costs of authentic pharmaceuticals: research on qualitative and quantitative aspects of enabling anti-counterfeiting in RFID-aided supply chains. Pers. Ubiquit. Comput. **16**, 271–289 (2012)
21. Gökçe, C., Harun, Y.R.: A novel approach for inventory problem in the pharmaceutical supply chain. J. Pharm. Sci. **24**(4), 1–16 (2016)