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SEASONAL ADJUSTMENT AND COINTEGRATION*

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ABSTRACT

We examine the effects of seasonal adjustment filters on the size and power of ADF and PP residual-based cointegration tests via a Monte Carlo and an empirical application. Our results indicate that the use of filters distorts the size and reduces the power of these tests.

Key words: Seasonal adjustment, linear filters, cointegration.

JEL Classification: C12, C15, C22

RESUMEN

El documento examina el efecto de filtros de ajuste estacional en el tamaño y poder de pruebas de cointegración, que usan los residuales como las pruebas ADF y PP, mediante procedimientos MonteCarlo y una aplicación empírica. Nuestros resultados indican que el uso de filtros distorsiona el tamaño y reduce el poder de estas pruebas.

Palabras clave: ajuste estacional, filtros lineales, cointegración.

Clasificación JEL: C12, C15, C22

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1. INTRODUCTION

One of the issues that arise in econometric modelling when high frequency data are used, is whether to conduct the econometric analysis on data that have been subjected to seasonal adjustment, or in terms of unadjusted data. The effects of seasonal adjustment filters on linear regression models have been analysed by Wallis (1974). In the context of nonstationary series, Ghysels (1990) and Ghysels and Perron (1993) explore, from both analytical and simulation perspectives, the effects of the Henderson moving average filter and the linear approximation of the X-11 filter, in their quarterly and monthly versions, on the power of the augmented Dickey and Fuller (ADF) and Phillips and Perron (PP) unit root tests. They find that these filters substantially reduce the power of the tests compared to the case where the data are not seasonally adjusted.

This paper examines the effects of seasonal adjustment filters on the size and power of the residual-based cointegration tests of Dickey and Fuller (1979, 1981), and Phillips and Perron (1988). The results of the Monte Carlo simulations provide a justification for using seasonally unadjusted data, since the power of the cointegration tests is adversely affected by the use of seasonal adjustment filters. The outline of the paper is as follows. Section 2 presents a money demand modelling exercise for the United States using seasonally adjusted and unadjusted data. Based on the results of section 2, section 3 presents a Monte Carlo study of the size and power properties of the Dickey and Fuller and Phillips and Perron cointegration tests with seasonally adjusted and unadjusted data. Section 4 offers some concluding remarks.

2. THE DEMAND FOR MONEY IN THE UNITED STATES

This section estimates long-run money demand equations for the United States. We are particularly interested in whether the finding of cointegration changes depending on the use of seasonally adjusted or unadjusted data. The long-run money demand equation is given by:

$$m^d = a_0 + a_1 y + a_2 p - a_3 r \tag{1}$$

where m^d is money in nominal terms, y is a measure of the volume of real transactions, p is an appropriate price level, and r is an interest rate on the alternatives of holding money.

We use the M2 definition of money for the monetary aggregate over the period 1959.1 to 1987.4. The scale variable (denoted Y) corresponds to Gross National Product in 1987 dollars, and the price level (denoted P) corresponds to the consumer price index. There are two interest rates, the 6-month treasury bill rate (denoted R_6) and the long-term U.S. government bond yield (denoted R_L). All series are considered in logarithms and denoted m_2, y and p.[1] The interest rate series are not considered in logarithms in order to allow the interest rate elasticity to vary with the level of the interest rate.

[1] The M_2, price and interest rate series were downloaded from the Federal Reserve Economic Database (FRED) of the Federal Reserve Bank of St. Louis (Internet site www.stls.frb.org). The series for seasonally adjusted GNP in 1987 dollars was taken from U.S. Department of Commerce (1992) and for seasonally unadjusted GNP, following Barsky and Miron (1989) and Ghysels (1990), we use the series of nominal GNP series, taken from U.S. Department of Commerce (1992), divided by the consumer price index (unadjusted).

ADF tests are used to determine the order of integration of the series. The number of lags selected follows the approach of Campbell and Perron (1991), starting with an upper bound of 6 lags and testing down. Centred seasonal dummies are used for unadjusted data. The results of the unit root tests, not reported here, indicate that all series contain a unit root.[2]

The top panel of Table 1 presents the results of estimating different cointegration regressions among the adjusted series using OLS. We examine the possibility of cointegration among M2, output and prices; M2, output, prices and R_6; and M2, output, prices and R_L. Our results suggest the presence of cointegration among M2, output, prices and R_6.[3] Imposing homogeneity in prices, and homogeneity in prices and income, does not improve the results, since in no case can the hypothesis of non-cointegration be rejected.

Next, the bottom panel of Table 1 presents the results of estimating the cointegration regressions using unadjusted data. We find evidence of cointegration in all the seven cases considered. The finding of cointegration for the M2 definition of money is robust to the variables included in the cointegration regression. Lastly, it is worth noticing that in the first five specifications both price and income homogeneity are accepted.

3. SEASONAL ADJUSTMENT AND COINTEGRATION: A MONTE CARLO STUDY

To investigate the power properties of the ADF and PP cointegration tests with seasonally adjusted and unadjusted data, we constructed the following experiment based on the long-run unadjusted relationship between m_2, y, p and R_6. Imposing price and income homogeneity, the simulation results are based on:

$$V_{2,t} = \alpha_0 + \alpha_1 R_{6,t} + \upsilon_t, \qquad (2a)$$

$$(1-L)R_{6,t} = \eta_t + \theta_1\eta_{t-1} + \theta_2\eta_{t-2}, \qquad (2b)$$

where $V_{2,t} = m_{2,t} - p_t - y_1$; $(1-\rho_1 L)(1-\rho_4 L^4)\upsilon_t = \varepsilon_t$; $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$, and $\eta_t \sim N(0,\sigma_\eta^2)$. The parameters are $\alpha_0 = 0.937$, $\alpha_1 = -0.944$, $\rho_1 = 0.65$, $\rho_4 = 0.45$, $\theta_1 = 0.27$, $\theta_2 = -0.27$, $\sigma_\eta^2 = 0.008319^2$ and $\sigma_\varepsilon^2 = 0.00017$. Thus R_6 is an ARIMA(0,1,2) process, and V_2 and R_6 are cointegrated with u_t as a stationary seasonal ARMA process. For the Monte Carlo simulations, we generated 1,000 replications of V_2 and R_6 of length n = 120, defined by (2a) and (2b). However, to obtain 120 observations for the filtered series, it is necessary to generate additional data points; therefore, both the unadjusted and adjusted series are based on a total sample size of 320 observations.

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[2] Following Ericsson, Hendry and Tran (1994) and Hendry (1995, chapter 15), all seasonal adjusted and unadjusted series are cointegrated with vector [1, -1]; that is, the seasonal component of the series appears to be stationary (these results are not reported but are available upon request).

[3] Miller (1991) examined the possibility of cointegration among alternative monetary aggregates, output, prices and interest rates in the United States, using adjusted data for the same sample period. Miller only found cointegration among M2, real GNP, GNP deflator and the four to six-month commercial-paper rate.

In a further set of experiments ρ_1 is increased to 0.75 and 0.85 (with ρ_4 fixed), and ρ_4 is increased to 0.65 (with ρ_1 fixed); when the ρ's are changed, σ_υ^2 remains fixed and σ_ε^2 is adjusted to the new parameter values.

We analyse two linear symmetric time-invariant filters:

- The linear approximation of the quarterly version of the X-11 filter, as given by Laroque (1977, Table 1); see also Ghysels and Perron (1993, Table A.2).
- The quarterly version of the Henderson moving average filter, as given by Shiskin et. al. (1967, Appendix B, Table 3), and also used by Ghysels and Perron (1993, Equation 2.7), which is one of the key elements in the X-11 filter.

The effects of the two adjustment filters on the size and power of the ADF and PP tests, applied to the residuals of the cointegrating relationship between V_t and R_t, are investigated in Tables 2 and 3, respectively. These tables report the empirical sizes and powers of the two residual-based unit root tests when the data is unadjusted as opposed to when: (i) both the dependent variable (V_t) and the explanatory variables (R_t) are filtered, denoted 2-sided; and (ii) the dependent variable is filtered, denoted 1-sided.

The choice of the lag length (p) in all experiments is crucial and so results are reported for a range of alternative lag lengths for both the ADF and PP tests. However, in most cases the choice p was invariant to a range of selection criteria which are used to determine the lag length, and p is selected such that (i) Akaike Information Criterion (AIC) is minimised; (ii) Schwarz criterion (SC) is minimised; (iii) sequentially dropping insignificant lags until one rejects H_0; and (iv) the errors are serially uncorrelated.

In general, for the unadjusted data a lag length of four (p=4) was selected by all 4 criteria. For the X-11 filtered data a lag length of zero (p=0) was selected by all criteria, irrespective of whether 2-sided or 1-sided filtering was used. For the Henderson filter the optimal lag length varied across alternative experiments, although four (p=4) and eight (p=8) seemed to be the two best alternatives.

Using seasonally unadjusted data when ρ_1=1 and ρ_4=0.45, the empirical size probability of the ADF test approaches the theoretical significance level of 5% for p=3, 4, 5 and 6, while it is too small for $p<3$ and too large for $p>6$. For the PP test the empirical size is too small (around 2.7%) for all lag lengths. When the X-11 filter is used, the empirical size probabilities of the ADF and PP tests are always too small (1.8% for the ADF(0) test, and below 2.1% for the PP test) regardless of the lag length.

Increasing ρ_4 to 0.65, so that ρ_1=1 and ρ_4=0.65, the empirical size probability of the ADF test has generally fallen and is now too small up to p=5 and too large for p³8 when seasonally unadjusted data are used. Applying the X-11 filter to the data lowers the empirical size of the ADF(0) test. In the case of the PP test, the empirical size probabilities are too small both for the seasonally unadjusted data and the X-11 filtered data. The use of the Henderson filter has qualitatively similar results on the size of the ADF and PP tests.

Using the seasonally unadjusted data when ρ_1 = 0.65 and ρ_4 = 0.45, the ADF(4) (ADF(8)) test correctly rejects the null hypothesis of no cointegration around 66% (70%) of the time. However, this compares unfavourably with the more powerful PP test, which has power of at least 98%.[4] The use of the X-11 filter has a limited effect on the power of the ADF test, as long as the optimal value ADF(0) is used, although power now falls more steeply with increases in the lag length. Applying the PP test to the X-11 filtered data reduces the power of the test compared to that observed for the unadjusted data.

Increasing ρ_1, so that $\rho_1 = 0.75$ and $\rho_4 = 0.45$, the ADF(4) (ADF(8)) test applied to the unadjusted data now has power of 50% (56%), whereas the PP maintains its higher power of at least 75%. Applying the X-11 filter, the power of the ADF(0) test has fallen sharply to 27%, more seriously the PP test has a minimum power of only 31%. Increasing ρ_1 further, so that $\rho_1 = 0.55$ and $\rho_4 = 0.45$, leaves the filtered X-11 data with no power to correctly reject the null of no cointegration, compared with power of at least 27% for the unadjusted data.

Increasing ρ_4, so that $\rho_1 = \rho_4 = 0.65$, the PP test still has high power for the unadjusted data of at least 95%, while the filtered X-11 series has maximum power for the PP (and ADF(0)) test less than 47%.[5]

The use of the 1-sided filter, compared with the 2-sided filter, slightly increases the power of both the ADF and PP tests in almost all cases, although the qualitative pattern of results discussed above remains unaltered.

The results from the Henderson filter are qualitatively similar to those obtained from the application of the X-11 filter. However, the Henderson filter lowers the power of the PP test for the adjusted series even more relative to that of the unadjusted series (except when $\rho_1 = \rho_4 = 0.65$); in the case of the ADF(4) test this occurs when $\rho_1 = 0.65$ and $\rho_4 = 0.65$, and when $\rho_1 = \rho_4 = 0.65$. For example, when $\rho_1 = 0.65$ and $\rho_4 = 0.45$, such that the errors from the regression of V_t on R_t ought to appear very stationary, the use of the Henderson filter reduces the power of the ADF(4) and PP tests to less than 45% and 60%, respectively, compared to 65% and 98% for the unadjusted series.

4. CONCLUDING REMARKS

In this paper we have estimated long-run money demand equations for the US over the period 1959.1-1987.4 using the M2 definition of money. We found that using seasonally adjusted data, compared with seasonally unadjusted data, reduces the probability of discovering a cointegrating (equilibrium) relationship. Based on the empirical estimates for the money demand relationship involving seasonally unadjusted M2, we conducted a Monte Carlo simulation, which showed that the application of the (linearised) X-11 filter to the dependent variable markedly reduces the power of the ADF and PP residual-based cointegration tests. These results suggest that researchers ought to be careful even when estimating long-run relationships using seasonally adjusted data and ought, where possible, to confirm their finding using seasonally unadjusted data.

[4] The power of the PP test would be slightly higher if it were correctly sized.

[5] Using the trace and l-max tests of Johansen to determine the number of cointegrating vectors does not overturn these results as these tests were invariably less powerful than either the ADF or PP tests.

TABLE 1. COINTEGRATION REGRESSIONS

SEASONALLY ADJUSTED DATA

Model	Constant	y	p	R_6	R_L	ADF		PP(8)
						Lags	Test	
m_2	-1.127	1.282	0.815			1	-2.962	-2.751
m_2	-1.342	1.332	0.852	-1.438		2	-3.182	-4.809***
m_2	-0.745	1.261	0.936		-2.113	0	-3.208	-3.477
m_2-p	0.413	1.089		-1.713		2	-1.650	-2.645
m_2-p	-0.186	1.190			-2.583	0	-3.100	-3.298
m_2-p-y	0.969			-1.155		2	-2.205	-2.654
m_2-p-y	0.975				-1.140	0	-2.418	-2.662

SEASONALLY UNADJUSTED DATA

Model	Constant	y	p	R_6	R_L	ADF		PP(8)
						Lags	Test	
m_2	0.753	1.014	0.961			4	-3.579*	-3.063
m_2	0.692	1.039	1.000	-1.183		6	-3.877	-5.405***
m_2	1.100	0.993	1.064		-1.770	4	-3.377	-4.101*
m_2-p	0.689	1.039		-1.183		6	-3.877**	-5.410***
m_2-p	0.617	1.051			-1.180	4	-3.324	-4.067**
m_2-p-y	0.937			-0.944		4	-3.338*	-4.316***
m_2-p-y	0.933				-0.808	4	-3.456**	-3.445**

Notes:
The ADF test includes seasonal dummies for unadjusted data. Prior to the application of the PP test, the residuals of the cointegration equation for unadjusted data were regressed on a constant a seasonal dummies. *, ** and *** denote significance at the 10, 5 and 1 per cent levels, respectively, based on the critical values tabulated by MacKinnon (1991).

TABLE 2. SIZE OF UNIT ROOT TESTS

Coeffs.	Test	Unadj.	X-11 filter		Henderson filter	
			2-sided	1-sided	2-sided	1-sided
$\rho_1 = 1.00$ $\rho_4 = 0.45$	ADF(0)	2.3	1.8	1.8	2.0	1.9
	ADF(2)	2.9	1.7	1.5	1.9	1.5
	ADF(4)	5.1	1.3	1.6	2.6	2.5
	ADF(8)	9.6	0.8	1.4	4.1	2.1
	PP(4)	2.7	1.4	1.9	1.4	2.0
	PP(6)	2.6	1.5	2.0	1.4	2.2
	PP(8)	2.7	1.6	2.1	1.3	2.3
	PP(10)	2.6	1.5	1.9	1.3	2.1
$\rho_1 = 1.00$ $\rho_4 = 0.65$	ADF(0)	2.2	2.8	2.7	2.8	2.9
	ADF(2)	3.4	2.0	2.1	2.6	2.3
	ADF(4)	3.4	1.3	1.4	3.4	2.3
	ADF(8)	6.1	1.0	1.0	3.4	2.2
	PP(4)	2.7	2.0	2.4	2.0	2.4
	PP(6)	2.6	1.6	2.2	1.6	2.1
	PP(8)	2.8	1.6	1.9	1.6	2.0
	PP(10)	2.8	1.8	2.0	1.5	1.7

TABLE 3. POWER OF UNIT ROOT TESTS

Coeffs.	Test	Unadj.	X-11 filter		Henderson filter	
			2-sided	1-sided	2-sided	1-sided
ρ_1= 0.65 ρ_4= 0.45	ADF(0)	99.1	74.4	78.4	35.4	50.8
	ADF(2)	73.3	33.5	31.7	1.5	8.1
	ADF(4)	65.6	17.5	21.2	45.5	42.2
	ADF(8)	69.7	6.9	9.1	28.8	19.4
	PP(4)	98.2	72.6	77.3	43.5	54.7
	PP(6)	98.7	77.3	80.3	46.6	59.6
	PP(8)	99.2	77.9	82.4	41.1	57.0
	PP(10)	99.5	77.9	82.3	37.7	54.5
ρ_1= 0.75 ρ_4= 0.45	ADF(0)	78.7	27.3	34.2	8.8	18.6
	ADF(2)	38.4	14.6	11.9	0.8	4.0
	ADF(4)	49.4	13.1	15.0	34.3	31.5
	ADF(8)	55.8	5.0	7.6	25.7	16.4
	PP(4)	74.5	31.5	36.0	15.1	23.7
	PP(6)	78.7	35.3	41.6	16.6	26.9
	PP(8)	81.3	34.9	41.4	13.9	24.1
	PP(10)	83.5	32.7	40.4	11.5	22.1
ρ_1= 0.85 ρ_4= 0.45	ADF(0)	28.4	5.7	8.3	2.2	5.9
	ADF(2)	10.2	6.0	3.3	0.3	1.4
	ADF(4)	27.7	6.7	10.4	21.1	15.8
	ADF(8)	40.1	2.9	5.8	18.7	10.9
	PP(4)	26.9	8.0	10.2	4.0	7.5
	PP(6)	31.4	8.6	12.0	4.3	8.5
	PP(8)	33.9	7.9	11.9	3.7	8.3
	PP(10)	34.7	6.8	11.0	3.3	7.8
ρ_1= 0.65 ρ_4= 0.65	ADF(0)	97.5	41.6	48.5	39.6	50.6
	ADF(2)	51.8	11.1	9.8	0.5	2.5
	ADF(4)	29.4	7.8	9.9	36.7	23.5
	ADF(8)	39.5	3.1	4.3	18.5	12.5
	PP(4)	94.3	38.6	43.6	37.2	47.2
	PP(6)	96.3	44.5	49.5	42.7	52.4
	PP(8)	97.5	44.2	51.5	41.8	53.3
	PP(10)	98.2	46.6	53.0	42.4	55.7

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