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Strategic Licensing with Retail Competition: an Innovation Theory of Harm*

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Abstract

We study incentives to license an innovation in a vertically related market, where upstream innovations can sell new technology to downstream retailers that sell differentiated products to final consumers. In a context with private actions, private outcomes, and uncertainty regarding the innovation's success, the decision to license depends on the nature of the downstream products. When goods are strong complements, an innovator licenses its technology to signal the feasibility of the innovation, allowing the competing innovator to complete the research process. When goods are sufficiently weak complements or substitutes (so that the innovator's bargaining position worsens when both retailers adopt), an innovator may either license immediately, thereby foreclosing competing innovations, or delay licensing to induce pessimism in the rival innovator, potentially causing them to abandon the race prematurely. Therefore, our model proposes a new theory of harm in which innovators can strategically delay licensing to manipulate rival's learning process.

JEL Codes: D83; O31; O38

Keyword: Licensing, research uncertainty, private actions, theory of harm, complementary goods, substitute goods.

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1 Introduction

The innovation process in the tech industry is often characterized by strategies designed to disadvantage potential competitors with the intention of monopolizing the market. With killer acquisitions, firms acquire promising innovations from smaller competitors to shelve them, thus preventing rivals from benefiting from such advancements (Cunningham et al. 2021). Non-Practicing Entities (NPEs) involve firms that acquire patents without licensing the underlying technologies, limiting the ability of competitors to innovate or enter the market (Choi and Gerlach 2018). In pay-for-delay agreements, incumbent firms pay potential competitors to delay the introduction of innovations, reducing competitive pressure (Bokhari 2013). In the practice of technology "vaporware," companies announce non-existent products to deter competitors, demonstrating how strategic withholding and lack of transparency can influence rivals' R&D decisions (United States v. Microsoft Corp., 1998).[1] To address these practices, competition authorities worldwide are increasingly implementing tools and frameworks to assess potential theories of harm.

This article explores an additional strategy that an innovator can employ to deter potential rival innovators from entering the market. An innovator who has completed the development of a new technology can choose to keep the innovation secret and refrain from licensing it to other firms. By withholding innovation, the innovator creates uncertainty about the project's feasibility, leading rival innovators to abandon the innovation race prematurely. This strategy is in line with the "Who killed the electric car?" controversy, in which General Motors was accused of shelving the EV1 electric vehicle project, possibly discouraging competitors from pursuing similar technologies at the time (Paine 2006).

There has been a renewed discussion on the impact of vertical structures on innovation. The Illumina/GRAIL prohibition, in September 2022, marks the first instance in which the Commission has blocked a vertical merger based on an innovation theory of harm (European Commission 2022).[2] This theory builds on the presumption that when an upstream innovator owns key assets to develop an innovation that downstream firms and their rivals depend on, the vertical relationship may create incentives to foreclose access to the technology and refuse to license it to competitors. This could diminish product competition in the downstream market and reduce the ability of the rival to invest in innovation (Eben and Reader 2023).

[1] See the case and its resolution at: https://www.justice.gov/atr/complaint-us-v-microsoft-corp

[2] The merger involved two US-based companies: Illumina operates in global genomics and is the supplier of next-generation sequencing (NGS) systems; GRAIL is a healthcare organization that develops blood-based cancer detection tests that utilize genomic sequencing.

To provide an explanation of the previous phenomenon, we present a simple model of innovation in a market involving two retailers selling differentiated products and two innovators competing to develop a technology for the retailers. The innovation process consists of a single stage and successful completion results in a technology that lowers production costs. However, there is uncertainty surrounding the feasibility of the innovation. If innovation is unfeasible, innovators eventually become too pessimistic and decide to discontinue their research efforts. If innovation is feasible, the innovator has the option to license the technology to one of the retailers.

A key assumption in the model is that an innovator can exclusively license its technology to a single retailer through an exclusive technology use contract, preventing licensing to multiple retailers at the same time, which is not uncommon.[3] For example, in 2019, Boeing partnered with Aerion to develop the AS2, a supersonic business jet (Lynch 2021). This partnership was structured to leverage Boeing's expertise in aviation with Aerion's supersonic technology innovations. An exclusive technology use clause ensured that Boeing was the only firm with access to Aerion's innovations for commercial aviation purposes. Another example involves Sony and Taiwan Semiconductor Manufacturing Company (TSMC), which established a semiconductor fabrication plant in Japan (Kageyama 2024). This strategic collaboration aimed to secure a stable supply of semiconductors, which is crucial for Sony's diverse range of electronic products. An exclusive technology use agreement here would ensure Sony's priority access to TSMC's cutting-edge manufacturing capabilities, securing Sony's supply chain against global semiconductor shortages.

In our model, licensing the innovation serves not only as a means for an innovator to generate profits but also as a strategic tool to influence the rival's learning process regarding project feasibility. To illustrate this, consider two scenarios: (i) the innovation is licensed immediately after the research stage is completed, or (ii) after discovery, the innovator withholds the new technology without licensing.

In the case of licensing (i), when an innovator licenses the technology, it signals to the rival that the project is feasible. If no licensing has been observed, the innovator infers that the competing innovator is still active in research and has not yet completed the innovation. Over time, as no signals of success (e.g., licensing) emerge, both innovators become increasingly pessimistic, becoming more convinced that the project may not be feasible. This learning arises from the firm's own experimentation and the observations of the rival's lack of licensing.

With no licensing (ii), the absence of licensing from the rival innovator offers no indica-

[3] This assumption avoids potential renegotiation issues with competing retailers.

tion of whether the project is feasible. As a result, an innovator can only learn about the project's feasibility through its own experimentation. The lack of licensing by the rival fails to deliver the positive signal about project feasibility that typically follows licensing. Without this reinforcement effect, the likelihood of the innovator abandoning the race increases significantly.

This dynamic creates an interesting intertemporal trade-off for an innovator that has just completed the innovation. Licensing the technology signals to the rival that the project is viable, encouraging the rival to continue to research until it also completes and licenses the innovation to the competing retailer. Conversely, withholding licensing keeps the rival uncertain about the project's feasibility, potentially leading the rival to abandon research prematurely.

We find that equilibrium licensing decisions depend on the characteristics of the products sold in the market. When retailers compete with sufficiently strong complementary products, a rival retailer that owns the new technology generates a positive externality, which translates into higher overall profits. In this case, the innovator can charge a higher licensing fee when both retailers adopt the technology compared to when only one does. The improved bargaining position of the innovator incentivizes the licensing of the innovation after completion, signaling to the rival innovator that the project is feasible. This positive reinforcement effect encourages the rival innovator to continue its research until the project is completed and licensed to the competing retailer.

If competition in the retail market involves substitute or weak complements products, the reverse occurs. A single retailer with the technology gains a competitive advantage over the retailer with the older technology. In this scenario, the innovator prefers to be the sole firm that licenses to a retailer rather than a situation in which both retailers hold the technology. This is because when both retailers hold, the technology competition would intensify and the bargaining position of an innovator would deteriorate.

A strategy that allows the innovator to monopolize the innovation is to shelve the technology and refrain from licensing it to a retailer. By withholding licensing, the rival innovator does not receive the positive signal about the feasibility of the project that would typically follow licensing. Over time, if no discovery is made, the rival becomes increasingly pessimistic about the feasibility of the innovation and eventually abandons the race.

However, for an innovator that has completed research, delaying licensing is costly, as licensing remains its only source of profit. This makes the decision to withhold licensing to be time-sensitive. If the innovation is discovered early, the innovator licenses immediately, as delaying until the rival abandons research would result in substantial losses from foregone profit. If the innovation is discovered later, the innovator may opt to delay the licensing. With the rival innovator nearing abandonment, waiting becomes advantageous, allowing the innovator to secure greater market power in the future. Interestingly, the timing threshold at which an innovator shifts from licensing to waiting depends on the level of product substitutability. This threshold decreases with the level of product substituability, and when products are strong complements the initial innovator has no incentive to delay and will always license immediately.

The proposed innovation model is based on well-established innovation frameworks as presented by Choi (1991) and Malueg and Tsutsui (1997). These studies consider an R&D competition characterized by both the unpredictable timing of discoveries and the uncertainty surrounding the effectiveness of R&D efforts. A common element in these models is the assumption that successful outcomes become publicly known, omitting the strategic aspect of revealing research findings. Instead, our paper examines the implications of strategically revealing research outcomes through licensing on the decisions and learning processes of competing innovators.

To this end, our paper is closely related to Bag and Dasgupta (1995) who consider a model of information sharing in an innovation process. In a model of uncertainty regarding the viability of the project, the authors argue that if research outcomes emerge sufficiently early, firms are inclined to disclose them. This strategic disclosure serves as a signal to the rival that the firm may be a high type. In line with their findings, our model suggests that licensing is also pursued when discoveries are made early. However, our rationale differs; in our scenario, early disclosure happens because the cost of concealing licensing until competitors cease their research is very high.

To date, the literature has examined how strategic choices of firms can engender pessimism among competitors, leading rivals to withdraw from competition. However, our approach diverges from existing frameworks, which have primarily concentrated on firms' positions in the race (Gill 2008 and Jansen 2010), or on the strategic revelation of partial research findings (Akcigit and Liu 2015). Instead, our model emphasizes the role of information dissemination through the timing of licensing.

Our model adds a new dimension to the literature on strategic delay in research decisions across various scenarios. Katz and Shapiro (1987) investigate competition in innovation, where firms gain by emulating each other's products, resulting in a deliberate postponement of research efforts, transforming the innovation race into a waiting game. Similarly, Boch and

Markowitz (1996) explore how a policy of delayed disclosure affects the pace of discovery and investment motivation in a multiphase innovation process. Weeds (2002) investigates firms that delay R&D investments to avoid potential innovation contests. Banerjee and Sarvary (2009) study the dynamics between an incumbent and a newcomer in an innovation race, where strategic decisions on investment levels and timing of product launches are made. In their analysis, firms postpone product launches to maximize the growth potential of existing offerings. Our model differs from these studies by not focusing on technological spillovers or imitation. Instead, it highlights a scenario where firms deliberately delay product licensing. This strategic move aims to make rival innovators pessimistic and eventually cause them to abandon the race.

A key departure of our model from the existing literature on innovation is its microfoundation of the retail market, where innovations are targeted. In this regard, our model relates to the literature studying the adoption of technology by downstream firms in vertically related markets. In these models, the adoption of technology provides downstream firms with a competitive advantage over rivals, by providing access to improved production technology, but the adoption process incurs costs that gradually decrease over time.[4] Numerous factors determine the equilibrium time for adoption. Alipranti et al. (2015) demonstrate that the mere existence of a vertical agreement may foster the adoption of technology. An upstream innovator has incentives to increase its downstream competitiveness by charging a lower wholesale price and extracting the resulting higher profits through a fixed fee. Better contract terms for technology adoption -aimed at reinforcing efficiency and increasing profits-lead downstream firms to outsource input to adopt technology sooner than those producing in-house.

Alipranti and Petrakis (2022) further consider the effect of the upstream market structure. With an upstream monopolist, downstream firms adopt the technology earlier than when faced with a competitive upstream. The lack of commitment by a monopolist to refrain from opportunistic behavior induces downstream firms to obtain the input under more favorable trading terms than those offered by separate suppliers, giving incentives for earlier adoption. Similarly to those models, our model features an optimal time for technology diffusion, although the upstream innovator makes the decisions. However, our model differs by focusing on the innovation process itself, which these other models do not explicitly explore.

[4]Decreasing adoption costs is a common assumption in the literature, as in Reinganum (1981), Fudenberg and Tirole (1985) and Quirmbach (1986).

2 A Simple Innovation Model with Retail Competition

We analyze a continuous-time innovation model in which two potential innovators, I_1 and I_2, compete to develop a technology aimed at reducing production costs for retailers who compete in a duopolistic market selling differentiated products. For example, two software companies might intend to create an advanced inventory management system that helps retailers minimize storage and handling expenses. Similarly, two renewable energy firms could compete to develop more efficient solar panel technology, allowing retailers to reduce their energy costs.

We assume that the retail market is composed by two retailers, R_1 and R_2, each offering differentiated products and facing a constant marginal production cost $c > 0$. If a retailer adopts the new technology from the innovator, it achieves a cost reduction denoted by the exogenous parameter ε, resulting in a new marginal cost of $c' = c - \varepsilon$. By producing at a lower cost, the retailer gains a competitive advantage over its competitor.

We examine a scenario in which an innovator can license the innovation to a single retailer. Without loss of generality, we assume that the innovator i can only license to retailer i and not to retailer $j \neq i$. This exclusivity restriction may result from either the innovator and the retailer signing an exclusive contract or the innovation being specifically tailored to the unique needs of the selected retailer.[5] In our model, we abstract from strategic considerations and incentives to sign such exclusive contracts but take them as given. This assumption grants the innovator full bargaining power in setting the licensing fees for its innovation with the retailer.[6]

Instances of exclusive contracts are not uncommon. For example, Apple invested $45 million in Corning to develop an exclusive Ceramic Shield for the iPhone 12 lineup (Apple Newsroom, May 10, 2021).[7] Similarly, Tesla entered into a three-year pricing agreement with Panasonic for the supply of lithium ion battery cells (Reuters, 17 June 2020).[8] Another example is the partnership between Coca-Cola and McDonald's, where Coca-Cola developed a drink technology designed specifically for McDonald's restaurants.

[5]This exclusivity assumption has been considered in other models; see, for example: Horn and Wolinsky (1988), Gal-Or (1991), and Milliou and Petrakis (2007).

[6]There is a substantial body of literature that studies the implications of exclusive contracts on investment incentives. See Segal and Whinston (2000), Simpson and Wickelgren (2007), and Chen and Sappington (2011).

[7]See: https://www.apple.com/newsroom/2021/05/apple-awards-corning-45-million-from-its-advanced-manufacturing-fund/

[8]See: https://www.reuters.com/article/business/tesla-signs-three-year-pricing-deal-with-battery-cell-maker-panasonic-idUSKBN23O0AV/

Figure 1 illustrates the market structure, featuring a vertical relationship with two competing innovating firms, each capable of exclusively licensing their technology to a specific retailer operating in a duopolistic market.

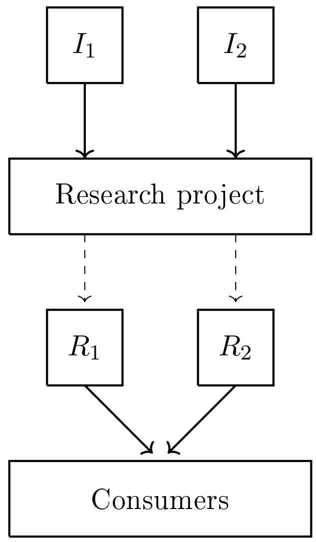

Figure 1: An innovation model with retail competition and exclusive technology use.

2.1 Retail Market

We model our duopolistic retail market with differentiated goods built on the frameworks of Singh and Vives (1984) and Erkal (2005). The market consists of a continuum of consumers whose utility function is given by

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{q_1^2 + 2\theta q_1 q_2 + q_2^2}{2}, \tag{1}$$

where q_1 and q_2 represent the output levels of retailers R_1 and R_2, respectively.

The parameter a represents the level of utility derived from consumption. The parameter $\theta \in (-1, 1)$ captures the degree of product differentiation: goods are substitutes when $\theta > 0$, independent when $\theta = 0$, and complements when $\theta < 0$. The optimal consumption decision for a typical consumer is derived by maximizing the utility function net of the expenditure on goods, $U(q_1, q_2) - p_1 q_1 - p_2 q_2$, where p_1 and p_2 represent the prices set by the retailers.

2.2 Research Environment and Information

Time is continuous, and the innovation process involves the completion of a single stage governed by a Poisson discovery process with hazard rate λ. We assume that there is uncertainty about the hazard rate, which can take one of two values: λ_H and λ_L, where $\lambda_H > \lambda_L = 0$. A hazard rate of $\lambda_L = 0$ implies that innovation is infeasible.[9] Innovators are uncertain about the true value of λ and share a common prior, p_0, representing the probability that innovation is feasible ($\lambda = \lambda_H$). This framework introduces two sources of uncertainty in the innovation process: the stochastic timing of discoveries and the feasibility of innovation.

We assume perfect symmetry between the two innovators, who incur the same continuous flow cost γ, while actively involved in the process and share a common discount rate r. The decision to stop paying the flow cost is irreversible and implies a withdrawal from the innovation race.

We make two key assumptions about the information available to innovators regarding their rivals: (i) *private learning*, where research outcomes are not publicly observable, and (ii) *private actions*, where investment and exit decisions remain hidden, keeping rivals unaware of each other's progress. The only observable action is when a rival licenses the innovation to a retailer. Although each innovation is licensed to a specific retailer, we assume that both innovators are working on the same type of innovation. Consequently, if an innovator licenses the innovation, the rival learns that the project is feasible. Thus, the decision to license directly impacts the rival's learning about the feasibility of the innovation.

2.3 Strategies and Equilibrium

An equilibrium consists of a strategy profile for each player (innovators and retailers) and a belief system for each innovator, such that every player maximizes their expected payoff given the strategy profile and belief system of their rival.

At any point in time, a strategy may in principle depend on the entire history of the game. To simplify the analysis, we restrict our attention to Markov strategies, in which past play influences current decisions only through a payoff-relevant state variable. In our setting, the state of the game at time t is given by the triplet $s_t = (\tau_t, p_t, \alpha_t)$. Then, τ_t is the state of technology, indicating how many retailers have adopted the new technology, p_t is the probability that the innovation is feasible and α_t is the probability that the rival innovator

[9] The uncertainty in the Poisson rate is borrowed from Choi (1991).

remains active in research.

With Markov strategies, an innovator's strategy is a mapping from the current state s_t to a set of possible actions. These actions include whether to continue incurring research costs or abandon the innovation process. Once the innovation is complete, the strategy prescribes whether and when to license it to the designated retailer and, if so, specifies the license fee. Each innovator's decision must be sequentially rational: given the state s_t, the innovator chooses an action that maximizes its expected payoff, taking as given (i) the current state, (ii) the rival's strategy, and (iii) its beliefs regarding feasibility and the rival's possible actions. In addition, these beliefs must be consistent with the observed state transitions and the rival's observable actions.

Finally, at any time t, and given the current available technology, both retailers set prices simultaneously and non-cooperatively.

3 Competition in the Retail Market

We analyze three scenarios regarding competition in the retail market: 1) no innovation is introduced, and both retailers continue to produce with the old technology at a marginal cost of c; 2) a single innovator succeeds in the innovation and licenses it to its respective retailer, enabling that retailer to produce at a reduced marginal cost of $c' = c - \varepsilon$, while the other retailer remains dependent on the old technology with a marginal cost of c; 3) both innovators succeed and license their innovations to their respective retailers, resulting in both retailers benefiting from the new technology and achieving a reduced marginal cost of c'.

Regarding the efficiency gains provided by the new technology, we make the following assumption.

Assumption 1. $a > c > \varepsilon > 0$.

The flow profit of a retailer is denoted by $\pi^{j,\#}$, where $j = Y, N$ indicates whether the retailer has access to the new technology (Y for yes, N for no), and $\# = 0, 1, 2$ represents the number of retailers utilizing the new technology. If neither retailer adopts the new technology, both compete using the old technology and earn profits of $\pi^{N,0}$. If one retailer adopts the new technology while the other relies on the old technology, their respective profits are $\pi^{Y,1}$ and $\pi^{N,1}$. If both retailers adopt the new technology, their profits are $\pi^{Y,2}$. The interested reader will find the demand functions for the retailers and the equilibrium prices, quantities, and profits in the Appendix.

A key question that arises when one retailer has a more efficient technology is if possessing such technology may foreclose the competing retailer, which operates with a less efficient technology. This scenario suggests that innovation is drastic, making the retailer with less efficient technology unable to compete against its more efficient rival. To analyze this, we define the relative efficiency of the new technology with respect to the size of the market as $\delta = \frac{\varepsilon}{a-c}$, where a higher value of δ indicates a greater effectiveness of the new technology. Proposition 1 establishes the conditions under which the innovation is non-drastic, ensuring that all retailers remain active in the market.

Proposition 1. *If the goods are complements or independent, then the innovation is always non-drastic. If the goods are substitutes and the relative efficiency is such that* $\delta < \frac{(2+\theta)(1-\theta)}{\theta}$*, then the innovation is non-drastic. In other words, for each* δ*, there is a unique* $\overline{\theta}$ *such that the innovation is non-drastic when the degree of substitutability is low enough, i.e.,* $\theta < \overline{\theta}$*.*

We later discuss the implications of a drastic innovation; however, most of the subsequent analysis focuses on scenarios where the innovation is non-drastic.

Assumption 2. *The innovation is non-drastic, i.e.,* $\delta = \frac{\varepsilon}{a-c} < \frac{(2+\theta)(1-\theta)}{\theta}$*.*

With the assumption that innovation is non-drastic, the following proposition outlines the profit ranking of retailers based on the number of retailers adopting the new technology and the nature of the goods, that is, whether they are substitutes or complements.

Proposition 2. *The ranking of retailer's profits is as follows:*

i) If the goods are substitutes or independent, then $\pi^{Y,1} \geq \pi^{Y,2} > \pi^{N,0} \geq \pi^{N,1}$*.*

ii) If the goods are complements, then $\pi^{Y,2} > \pi^{Y,1} > \pi^{N,1} > \pi^{N,0}$*.*

From the proposition, one can see that being the sole retailer with the new technology yields the highest profits when competition is with substitute goods. Having a better technology than the rival provides a competitive advantage. This is not the case when competition involves complementary goods. In this situation, an increase in the efficiency of a rival firm generates a positive externality for the other retailer, resulting in both being better off as they become more efficient. Similarly, for substitute goods, firms are better off when neither adopts the new technology, as the profits of a retailer without the technology are lower when its competitor possesses it. The opposite occurs with complementary goods.

The profits of retailers are crucial in determining the licensing fees that innovators can charge for their innovation. As discussed previously, we focus on the case where each retailer can only purchase the innovation from a specific innovator who, in each period, makes a take-it-or-leave-it offer. This assumption gives the innovator full bargaining power. Consequently, when setting the licensing fee, the innovator only needs to compensate the retailer according to its outside option. The outside option of the retailer is constituted by its profit absent the new technology ($\pi^{N,\#}$), depending on whether its competitor has gained access to the new technology through licensing. Hence, the fixed fee is equal to i) $\pi^{Y,1}-\pi^{N,0}$, if the rival retailer does not have the technology, and ii) $\pi^{Y,2}-\pi^{N,1}$ if it does. Interestingly, whether licensing fees are higher or lower when the rival retailer possesses the new technology compared to when it does not, that is, $\pi^{Y,1}-\pi^{N,0} > \pi^{Y,2}-\pi^{N,1}$, depends on the underlying fundamentals of the economy: the effectiveness of innovation, δ, and the degree of substitutability of the goods, θ. This result is formalized below in Proposition 3. To this end, we first introduce a variable that captures the relative licensing fees difference when the rival retailer does or does not hold the new technology. We refer to this as the innovators' relative change in its bargaining position.

Definition. *The relative change in the bargaining position is* $\eta = \frac{\pi^{Y,1}-\pi^{N,0}}{\pi^{Y,2}-\pi^{N,1}}$.

i) If $\eta < 1$, then the relative change in the bargaining position of the innovator increases when the rival innovator also licenses.

ii) If $\eta > 1$, then the relative change in the bargaining position of the innovator decreases when the rival innovator also licenses.

iii) If $\eta = 1$, then there is no relative change in the bargaining position of the innovator.

We can relate the previous definition to the potential externality generated by the rival retailer when it adopts the technology, as this affects the change in the licensing fees that the first innovator can charge its respective retailer. The externality is considered positive if the licensing fees increase (the bargaining position increases), and negative if the licensing fees decrease (the bargaining position decreases). If the licensing fees remain unchanged, the adoption of the new technology by the rival retailer does not generate any externality. This static measure of externalities will be the crucial determinant of the innovator's dynamic incentive to signal project feasibility or induce rival pessimism.

Proposition 3. *The change in the relative bargaining position deteriorates with the degree of substitution between the goods, i.e., $\partial\eta/\partial\theta > 0$. Moreover, when goods are complementary, there is a positive externality, $\eta < 1$, and if goods are substitutes, then there is a negative externality, $\eta > 1$. If the goods are independent, then there is no externality.*

The proposition states that changes in an innovator's relative bargaining position depend on whether the competition in the retail market is between substitutes or complements goods. In the case of substitute goods, the innovator benefits most by being the sole licensor, as its licensed retailer gains a competitive advantage with respect to the rival retailer that allows for higher licensing fees. This advantage becomes more pronounced as the goods become more substitutable, explaining the improvement in relative bargaining position ($\partial\eta/\partial\theta$). However, once a rival innovator also licenses its technology, the licensing fees must be drastically reduced due to intensified competition in the retail market. In contrast, when retail competition is between complementary goods, the opposite effect holds. An innovator can obtain larger licensing fees when both retailers have the innovation and want the competitive innovator to complete the research.

An example of the substitute goods scenario is Qualcomm's licensing practices in the mobile chip market, where it has historically provided essential technologies such as CDMA, 3G, 4G, and 5G patents to handset manufacturers. When Qualcomm was effectively the sole provider of these high-performance chip sets, it leveraged its dominant position to negotiate elevated royalty rates. However, as alternative chip suppliers such as Intel and MediaTek entered the market with similar technologies, Qualcomm's bargaining power diminished, forcing it to lower its fees in response to increased competition *Federal Trade Commission v. Qualcomm Inc.* (n.d.). In contrast, the video game console industry illustrates the complementary goods scenario. In this market, console hardware and software are interdependent, meaning the value of a console increases as more third-party developers license software development kits (SDKs) or publishing rights from the console maker. Recognizing this, console makers often reduce licensing fees for major game publishers to incentivize them to develop exclusive or high-quality titles for their platform, thus increasing the attractiveness of consoles to consumers and driving hardware sales (Corts and Lederman 2009).

4 Competition in the Innovation Race

Given the equilibrium in the downstream market and the licensing fees established by the innovators, we now proceed to analyze the research and licensing decisions undertaken by the innovators. Interestingly, we show that the decision to license has a direct impact on the innovation learning process regarding the feasibility of the innovation. This observation is crucial to understanding the strategic foreclosure that innovators can implement.

To construct an equilibrium, we have the following steps:

1. Conjecture a licensing decision.

2. Establish the state transitions consistent with the licensing conjecture.

3. Characterize the optimal players' strategies for the state of the game.

4. Verify that the licensing conjecture is optimal given the players' strategies.

These steps will establish the existence of an equilibrium. To demonstrate uniqueness, it is necessary to show that no alternative licensing decision can be optimal.

4.1 An Inclusive Equilibrium

In this section, we construct an equilibrium in which the first innovator licenses the technology to its respective retailer after completing the research. Following the licensing, the competing innovator learns that the innovation is feasible and chooses to continue to research until it also completes the innovation and licenses it to the other retailer. In this equilibrium, the primary objective of licensing is to signal the feasibility of the project, encouraging the rival innovator to continue to research.

We begin the analysis by considering a subgame in which one of the innovators has successfully completed the innovation and must decide whether or not to license it to its respective retailer. If the innovator licenses, the state of the game transitions to $s_t = (\tau_t, p_t, \alpha_t) = (1, 1, \alpha_t)$, where $\tau_t = 1$ indicates that one retailer has adopted the technology, $p_t = 1$ reflects the certainty that the project is feasible – as it has already been completed by one of the innovators – and $\alpha_t \in \{0, 1\}$ means that the rival innovator is no longer active or still active in the research stage, respectively.

Assume first that the rival innovator continues with research until completion,that is $s_t = (\tau_t, p_t, \alpha_t) = (1, 1, 1)$. A situation in which the rival abandons the race, $s_t = (\tau_t, p_t, \alpha_t) = (1, 1, 0)$, will later be defined as a "de facto" foreclosure and will be considered in Section 4.2. Then, when the follower innovator continues, by licensing, the first innovator obtains the present value of expected profit:

$$V^L = \int_0^\infty e^{-(r+\lambda)t} \left[(\pi^{Y,1} - \pi^{N,0}) + \lambda(\pi^{Y,2} - \pi^{N,1}) \right] dt = \frac{(\pi^{Y,1} - \pi^{N,0}) + \lambda(\pi^{Y,2} - \pi^{N,1})}{r + \lambda}. \quad (2)$$

Observe from the expression that, as long as the first innovator is the only one licensing the technology to its retailer, the licensing contract generates a flow profit of $\pi^{Y,1} - \pi^{N,0}$. However, once the rival innovator successfully develops the innovation and licenses the new technology to the competing retailer, where, in a Poisson arrival process, this occurs with probability λ. Then, the licensing contract instead generates a flow profit of $\pi^{Y,2} - \pi^{N,1}$.

Now, consider that the first innovator decides not to license. In this case, the state of the game, as perceived by the following innovator, is $s_t = (\tau_t, p_t, \alpha_t) = (0, p_t, \alpha_t)$, where the only certainty is that at time t, none of the retailers has the innovation. The beliefs about the feasibility of the project, p_t, and the probability that the rival innovator remains active in the research α_t at any time t, must be consistent with the inclusive equilibrium in which the innovators license the innovation.

We now proceed to characterize the equilibrium path beliefs. The characterization of α_t is straightforward. In the equilibrium path, an innovator expects the rival innovator to license the technology upon completing the innovation. Therefore, if no license has been issued by time t, it implies that the rival innovator is still in the innovation stage. Hence, the consistent beliefs are $\alpha_t = 1$.

Regarding beliefs about innovation feasibility p_t, the equilibrium path assumes that the rival innovator has not yet succeeded in research; otherwise, the innovation would already have been licensed. If innovation is feasible, the probability that it will not be discovered within an interval dt is $(1 - \lambda dt)$. The belief that neither innovator has made a discovery leads to a combined probability of no discovery given by $(1 - \lambda dt)(1 - \lambda dt)$, which simplifies to $(1 - 2\lambda dt)$. If the project is not feasible, the probability that neither innovator develops the technology is $(1 - p_t)$. Using Bayes' rule, the probability that the innovation is feasible given no license in dt is:

$$p_{t+dt} = \frac{p_t(1 - 2\lambda dt)}{p_t(1 - 2\lambda dt) + (1 - p_t)}.$$

This generates a law of motion given by $\dot{p}_t = -p_t(1 - p_t)2\lambda$, which is derived by subtracting p_t from both sides, dividing by dt, and taking the limit as $dt \to 0$. In the Appendix, we show that the closed form solution is:

$$p_t^L = \frac{p_0 e^{-2\lambda t}}{p_0 e^{-2\lambda t} + (1 - p_0)} = \frac{p_0}{p_0 + (1 - p_0)e^{2\lambda t}}. \tag{3}$$

This equation shows that, as long as no firm licenses the innovation, the innovator still engaged in the research becomes increasingly pessimistic about the feasibility of the innovation. Over time, if neither the innovator nor its rival manages to license the innovation, the belief in the feasibility of the innovation diminishes, leading the innovator to grow more convinced that the innovation may not be feasible. Under the licensing conjecture, "no news" means that neither of the two independent research efforts has succeeded. This combination of information (or lack thereof) causes pessimism to grow at a rate of 2λ. This contrasts sharply with the no-licensing scenario where an innovator learns only from their own failure, causing pessimism to grow more slowly (at a rate of λ).

An implication of this result is that the innovator will eventually become sufficiently pessimistic to abandon research altogether. To determine the point in time when this occurs, we need to compare the expected profits from continuing research with those from abandoning.

Along the equilibrium path, the follower innovator assumes that its rival has not yet completed research. It also believes that if it is the first to license innovation, it will earn the profit V^L, as defined in (2). What remains to be determined is the profit that the innovator will earn if, at some future point, the rival innovator successfully licenses the innovation. In this scenario, the game transitions to the state $s_t = (\tau_t, p_t, \alpha_t) = (1, 1, 0)$, and the present value of expected profit for the follower innovator is given by the following:

$$V^F = \int_0^\infty e^{-(r+\lambda)t} \left[\lambda(\pi^{Y,2} - \pi^{N,1}) - \gamma\right] dt = \frac{\lambda(\pi^{Y,2} - \pi^{N,1}) - \gamma}{r + \lambda}. \tag{4}$$

For an innovator to abandon research precisely at time t, it must be indifferent between abandoning at time t and continuing research for an additional small interval of time dt before abandoning. The expected payoff for such a strategy is given by:

$$(1 - rdt) \left[\lambda dt \left(p_t V^L\right) + \lambda dt \left(p_t V^F\right)\right] - \gamma dt.$$

With an instantaneous probability of λdt, the innovator becomes the leader after completing the innovation, earning V^L. If the rival licenses (this is perceived as the belief that the rival innovator will complete the innovation, which occurs with a probability of λdt), then it becomes the follower, earning V^F. If neither firm completes the innovation, which occurs with a probability of $(1 - \lambda dt)(1 - \lambda dt)$, the innovator abandons the project in the next period and makes no profits.

Dividing the expression by dt and taking the limit as $dt \to 0$, and because leaving research generates no profits, the indifference condition between continuing or abandoning research becomes:

$$\lambda p_t(V^L + V^F) = \gamma, \tag{5}$$

where the solution to this equation determines the time at which the innovator abandons research.

The next lemma states that the abandoning time depends on the parameters of the model.

Lemma 1. *If there is no licensing in an inclusive equilibrium, then there are specific points in time when innovators abandon research if no information arrives.*

i) For $\eta > 2r$, then the first innovator abandons at the unique time T^{L1} and the second at the unique time T^{L2} with $T^{L2} > T^{L1}$.

ii) Otherwise, both innovators abandon at T^{L1}.

The characterization of these abandonment times is provided in the proof of the lemma in the Appendix, where we show that those times are earlier when the arrival rate λ is large or when the research cost γ is high.

Having characterized the abandonment times in the absence of licensing, we can now derive the present value of expected profit: that a first innovator will obtain if it decides not to license. To this end, assume that the discovery occurs at time $\tau < T^{L1}$.[10] Then, the present value of expected profit of not licensing is:

$$\begin{aligned} V^{NL}(\tau) &= \int_0^{T^{L1}-\tau} e^{-(r+\lambda)t}\left[\lambda(\pi^{Y,2}-\pi^{N,1})\right]dt + e^{-(r+\lambda)(T^{L1}-\tau)}\left(\frac{\pi^{Y,1}-\pi^{N,0}}{r}\right) \\ &= \left(\frac{1-e^{-(r+\lambda)(T^{L1}-\tau)}}{r+\lambda}\right)\lambda(\pi^{Y,2}-\pi^{N,1}) + \left(\frac{e^{-(r+\lambda)(T^{L1}-\tau)}}{r}\right)\left(\pi^{Y,1}-\pi^{N,0}\right). \end{aligned} \tag{6}$$

Without license, the innovator does not earn any profits; it only begins to earn profits when the rival innovator completes the innovation and licenses it to the competing retailer. This occurs with an instantaneous probability of λdt, and the licensing contract provides flow profits of $\pi^{Y,2} - \pi^{N,1}$. If the competing innovator does not complete the innovation by time T^{L1}, which occurs with probability $e^{-(r+\lambda)(T^{L1}-\tau)}$, it abandons research. In this case, the innovator licenses the innovation to its respective retailer, obtaining the present value of expected profit of $(\pi^{Y,1} - \pi^{N,0})/r$.

A direct comparison between the present value of expected profits from not licensing and the expected profits from licensing in (2), reveals that when goods are complements or weak substitutes, licensing is more profitable than not licensing.

This result, presented in Proposition 4, is straightforward. When goods are strong complements, the bargaining position of an innovator improves if the rival retailer adopts the

[10] If $\eta > 2r$ and the discovery occurs between T^{L1} and T^{L2}, the innovator will license the innovation, as the rival innovator would have abandoned the race at this time.

new technology. A strategic way to encourage this adoption is through licensing. Licensing innovation signals to the rival innovator that innovation is feasible. This information encourages the rival innovator to continue its research efforts until completion and eventual licensing to its retailer. Conversely, if innovation is not licensed, the rival innovator becomes increasingly pessimistic and will eventually abandon research if no discovery is made.

When goods are weak complements or substitutes, the reverse occurs: being the sole innovator licensing the innovation to its respective retailer provides a stronger bargaining position and, consequently, larger profits compared to the scenario where the rival innovator also licenses. However, not licensing innovation is costly because of the foregone profits from the fees that could be obtained through licensing. As a result, if the goods are only weak substitutes, the foregone profits effect dominates, and the innovator decides to license. However, as we show later in Section 4.2, this result does not hold when the level of substitutability of the goods becomes significantly higher.

Proposition 4. *When goods are strong complements, i.e., for $\eta < \lambda r/(r+\lambda) < 1$, there is an inclusive equilibrium in which:*

- *The innovator licenses the innovation to its respective retailer after discovery.*
- *If a discovery occurs before T^{L1}, both retailers adopt the innovation.*
- *If no discovery occurs before T^{L2}, no retailer adopts the technology and only one adopts if discovery occurs between T^{L1} and T^{L2}.*

It is straightforward to argue that the results stated in the proposition are unique. First, note that the abandonment times, as presented in Lemma 1, are unique. Additionally, suppose that there exists an equilibrium in which the innovator chooses not to license. The only rationale for such a decision would be to induce abandonment by the rival innovator, thereby becoming the sole firm licensing to the retailer (a similar argument, as previously provided, suggests that the rival innovator would eventually abandon research from becoming pessimistic about innovation feasibility). However, this cannot constitute an equilibrium when goods are complements. The reason is that an innovator is better off when the rival retailer also has the technology. Consequently, there is always a profitable deviation that involves licensing the innovation.

4.2 An Equilibrium with Foreclosure

In this section, we characterize situations where the first innovator aims to become the sole developer of the innovation and explores strategies to foreclose the competing innovator. We consider two possibilities: *de facto foreclosure*, where the rival innovator abandons the race after the first innovator licenses the technology, and *strategic foreclosure*, where the first innovator delays licensing to strategically influence the rival innovator's learning process, inducing premature exit from innovation. Next, we analyze both equilibria.

4.2.1 De Facto Foreclosure

The occurrence of de facto foreclosure is the easiest to characterize. This happens when the rival innovator abandons the race after the first innovator licenses the technology. De facto foreclosure arises in two cases: (i) when the innovation is drastic, making the rival retailer unable to compete in the market, leading to its exit, or (ii) when the flow profits from licensing to the second retailer are lower than the flow cost of research. The latter case occurs when $\gamma > \lambda(\pi^{Y,2} - \pi^{N,1})$, making V^F, as defined in (4), becomes negative.[11]

Our model predicts that *de facto foreclosure* is more likely to emerge in markets with strong substitutes, as this intensifies competition in the retail market, making $(\pi^{Y,2} - \pi^{N,1})$ very small. There is evidence supporting this prediction in the smartphone operating system market. Google's Android became a dominant substitute for other operating systems like BlackBerry OS and Nokia Symbian. The licensing of Android to device manufacturers drastically reduced the profitability of competing systems. As a result, BlackBerry and Nokia were unable to sustain their R&D efforts and ultimately exited the market.

4.2.2 Strategic Foreclosure

The previous section established that under strong complementary goods, there exists a unique equilibrium in which the first innovator licenses innovation to signal project feasibility to the rival innovator, leading to adoption of innovation for the rival retailer (inclusive equilibrium). This section restricts the analysis to the case where goods are weak complements or substitutes, that is, $\eta \geq \frac{r\lambda}{r+\lambda}$. In this context, we demonstrate that other types of equilibrium may arise. Specifically, we focus on an equilibrium in which the first innovator keeps the

[11] While we do not elaborate extensively on this equilibrium, note that the learning process will remain the same as in the inclusive equilibrium, as licensing always occurs after discovery, and the abandonment time, in the absence of information arrival, will be unique and earlier than in the previous section, since there are no profits associated with being the follower innovator.

innovation secret (i.e., no licensing) to influence the rival innovator's learning process. The objective is to make the rival increasingly pessimistic about the feasibility of the innovation, ultimately inducing premature exit from the race.

We conjecture an equilibrium with no licensing. To establish the optimal research strategy, we first need to characterize how the state of the game evolves over time, consistent with an equilibrium without licensing, i.e., $s_t = (\tau_t, p_t, \alpha_t)$. In the absence of licensing, $\tau_t = 0$ as no retailer holds the technology. The characterization of p_t (the probability of the innovation being feasible) and α_t (the belief about the rival innovator's stage) is more complex, and we proceed to address this next.

For the characterization of p_t, note that if an innovator goes through a time interval dt without receiving any information from its competitor, and conditional on the project being feasible, two distinct events could have occurred: (i) the rival innovator has not developed the innovation, or (ii) the rival innovator successfully developed the innovation but did not license it. However, the innovator cannot distinguish between these two scenarios. Thus, conditional on the project being feasible, the innovator remains uninformed unless it successfully develops the innovation itself, which happens with probability $1 - \lambda dt$. In contrast, if the project is not feasible, no information is obtained. Applying Bayes' rule, the probability that the innovation is feasible in the absence of information arrival is given by:

$$p_{t+dt} = \frac{p_t(1 - \lambda dt)}{p_t(1 - \lambda dt) + (1 - p_t)}.$$

Then, one can easily obtain the law of motion $\dot{p}_t = -p_t(1 - p_t)\lambda$, which has a solution equal to:

$$p_t^{NL} = \frac{p_0 e^{-\lambda t}}{p_0 e^{-\lambda t} + (1 - p_0)} = \frac{p_0}{p_0 + (1 - p_0)e^{\lambda t}}. \tag{7}$$

Observe that, unlike the equilibrium with licensing, beliefs evolve at a slower pace. In the absence of licensing, an innovator does not learn from the rival's experimentation and relies only on its own progress to update beliefs. Despite the slower pace of pessimism, we later show that, in the absence of information, innovators will eventually abandon research.

For the characterization of α_t, an innovator never learns with certainty whether the rival innovator is still actively researching. The rival could have already completed the innovation (if an arrival has occurred) or could still be engaged in research (if no arrival has occurred).

The probability of the first event (the completion of innovation) at any time t is given by:

$$\int_0^t e^{-\lambda s} \lambda ds. \tag{8}$$

In the expression, $e^{-\lambda s}$ represents the probability that no discovery is made before time s and λds is the probability that a discovery is made in the interval $[s, s+ds]$. Therefore, $e^{-\lambda s}\lambda ds$ is the probability that a discovery is made by time t, as seen from time zero. Solving for the integral yields $1 - e^{-\lambda t}$. Then, the probability that the innovator is still active in the innovation process conditional on not obtaining information up to time t is $\alpha_t = \frac{1-e^{-\lambda t}}{1-e^{-\lambda t}+e^{-\lambda t}} = 1 - e^{-\lambda t}$, and it has completed the innovation with probability $e^{-\lambda t}$.

Using the probabilities regarding the feasibility of the innovation and the probability that the rival innovator is still in the innovation stage, we can determine the time at which innovators will decide to abandon the race, conditional on no information arriving. The expected payoff from continuing research for a duration dt before abandoning is given by:

$$(1 - rdt)\left[\lambda dt p_t^{NL}\left(\alpha_t V^1 + (1-\alpha_t)V^2\right)\right] - \gamma dt,$$

with probability λdt, the innovator completes the innovation and licenses it before abandoning. This can yield profits $V^1 = \frac{\pi^{Y,1} - \pi^{N,0}}{r}$ if the competitor has not completed the innovation, resulting in a single retailer holding the innovation. Alternatively, it can yield profits $V^2 = \frac{\pi^{Y,2} - \pi^{N,1}}{r}$ if the competitor has also completed the innovation, which allows both retailers to hold the innovation.

The indifference condition between continuing and abandoning the research becomes:

$$\lambda p_t^{NL}\left(e^{-\lambda t}V^1 + (1 - e^{-\lambda t})V^2\right) = \gamma. \tag{9}$$

The next lemma shows that innovators do not abandon the race at the same time.

Lemma 2. *If no licensing takes place in a strategic foreclosure equilibrium, innovators abandon research at different times if no information arrives. The first innovator abandons at the unique time T^{NL1} while the second innovator abandons at the unique time T^{NL2} where $T^{NL2} > T^{NL1}$.*

When goods are weak complements or substitutes, i.e., $\eta \geq \frac{r\lambda}{r+\lambda}$, the bargaining position of an innovator deteriorates if the rival innovator completes the innovation and licenses it to its retailer. This implies that an innovator will pursue strategies to become the sole technology provider (monopolist) in the retail market.

From the previous lemma, an innovator that has completed research will monopolize the retail market if no licensing carried out before T^{NL1}. Instead, if the innovator decides to license, the rival innovator will continue researching until completion. This will eventually increase competition in the retail market as both retailers will hold the technology.

In its decision to license or wait until the rival abandons, an innovator must trade off the increased profits from becoming the monopolist against the losses incurred by delaying licensing until the rival innovator abandons the race. The option value of becoming a monopolist depends on the level of substitutability, with a higher substitutability increasing its value. Conversely, the profits that must be foregone depend on the timing of the innovation's discovery, with earlier discoveries incurring larger costs. Thus, the decision to delay the licensing depends on both factors. The next proposition formalizes this result.

Proposition 5. *Let* $\bar{\eta} = \left(r\lambda e^{-(r+\lambda)T^{NL1}}\right) / \left((r+\lambda)e^{-(r+\lambda)T^{NL1}} - r\right)$*, and* $\tilde{T}(\eta)$ *be the time such that* $V^L = V^{NL}(\tilde{T}(\eta))$*. Then, with weak compliments and substitute goods, i.e.,* $\eta \geq \frac{r\lambda}{r+\lambda}$*, there exists a foreclosure equilibrium in which:*

i) For $\eta < \bar{\eta}$*, the licensing decision depends on the time of discovery:*

- *a) If the discovery occurs at time* $\tau \in \left[0, \tilde{T}(\eta)\right)$*, then the innovator licenses.*
- *b) If the discovery occurs at time* $\tau \in \left[\tilde{T}(\eta), T^{NL1}\right)$*, then the innovator does not license before* T^{NL1}*.*
- *c) If the discovery occurs at time* $\tau \geq T^{NL1}$*, then the innovator licenses.*

ii) For $\eta \geq \bar{\eta}$*, the innovator does not license before* T^{NL1}*.*

The proposition indicates that for an intermediate value of η, the decision of the initial innovator to license the innovation is time dependent. In this case, the equilibrium has a form of license-wait-license structure defined by two specific time thresholds.[12] If the initial innovator completes its research before reaching the first threshold, it immediately licenses the innovation. Waiting for the rival innovator to abandon its research results in a loss of potential profits that outweighs the benefits of becoming a monopolist after the rival's withdrawal. If the innovator completes the project between the first and second

[12] Das et al. (2020) investigate the process of learning via experimentation within the framework of the bandit model and explore how players utilize various technologies to examine the uncertain option and find that only the firm with a less advanced technology adopts threshold-based strategies

thresholds, it withholds licensing the product. Waiting to license happens because the rival is closer to abandoning its research, which shifts the cost-benefit analysis in favor of waiting. Once the rival withdraws at the second threshold, the initial innovator promptly licenses the innovation, assured of gaining monopolistic control, and eliminating any incentive to postpone further. This rationale remains valid if the project is completed after the second threshold where immediate licensing occurs.

Interestingly, the proposition indicates that the initial decision-making threshold for licensing by the innovator is influenced by the parameter η. In the appendix, we demonstrate that $\partial\tilde{T}(\eta)/\partial\eta < 0$. This implies that the greater the potential loss that the initial innovator faces due to a competing innovation in the downstream market, the sooner the innovation must come for it to decide to license. Furthermore, we establish that for sufficiently large values of η, it results in $\tilde{\eta} < 0$, indicating that the initial innovator refrains from licensing until the competing innovators withdraw from the research project.

5 Concluding Remarks

This paper has examined the dynamics of product innovation among two upstream innovators whose new technology, if proven viable, can be licensed downstream to lower the marginal cost of production for competing retailers. We have demonstrated that the decisions to license or withhold the innovation influence the rival innovator's learning process regarding the innovation's feasibility. An initial innovator can strategically utilize its private knowledge about the feasibility of the project to foster pessimism in the rival innovator, thereby prompting an early withdrawal from the research project.

Our results demonstrate that withholding licensing is less frequent when innovations are highly effective, i.e., drastic, or when downstream competition involves firms producing strong complementary products. This suggests that competition authorities should focus on cases in which downstream products are substitutes or weak complements and where innovations are incremental, since such scenarios create stronger incentives for firms to withhold innovations to strategically manipulate the learning process of rival innovator, resulting in the foreclosure of competing innovators.

Competition authorities should establish specialized divisions to oversee licensing practices in industries with rapid innovation. In particular, these divisions should monitor licensing decisions in sectors where incremental innovation is prevalent and downstream products are substitutes, as these areas present a higher risk of anti-competitive behavior. Additionally, authorities must develop expertise in evaluating whether non-licensing decisions are strategically designed to suppress competition rather than serving legitimate business interests.

The proposed theory builds upon existing theories of harm in innovation, such as reduced innovation incentives and market monopolization, which are actively considered by competition authorities. For example, through the Digital Markets Act (DMA), the European Commission is implementing regulatory measures to mitigate competition concerns in digital markets. In addition, competition authorities, including the FTC, CMA, and the European Commission, are currently adapting their strategies to address challenges in the digital economy, particularly in AI and data-driven markets. In July 2024, these authorities issued a joint statement emphasizing the need for antitrust oversight in the development of generative AI, underscoring the importance of preventing monopolistic control while incentivizing innovation.[13]

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[13]See: https://www.gov.uk/government/publications/joint-statement-on-competition-in-generative-ai-foundation-models-and-ai-products/joint-statement-on-competition-in-generative-ai-foundation-models-and-ai-products.

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Appendix

Retailer market

(1) *Before Innovation.*

Consider the following differentiated duopoly, where two retailers, i and $j \in \{1,2\}$ and $i \neq j$, have a constant marginal cost c. Let the representative consumer have the utility function $U(q_i,q_j) = a(q_i+q_j) - (q_i^2 + 2\theta q_i q_j + q_j^2)/2$, where q_i and q_j are the output levels of firm i and j, respectively, and $a > 0$ and $-1 < \theta < 1$. The first–order conditions of the utility maximization problem are

$$\frac{\partial U}{\partial q_i} = 0 \Longrightarrow a - \frac{2q_i + 2\theta q_j}{2} - p_i = 0 \text{ and}$$
$$\frac{\partial U}{\partial q_j} = 0 \Longrightarrow a - \frac{2q_j + 2\theta q_j}{2} - p_j = 0.$$

Hence, the demand functions are

$$q_i = \frac{(1-\theta)a - p_i + \theta p_j}{1-\theta^2} \text{ and}$$
$$q_j = \frac{(1-\theta)a - p_j + \theta p_i}{1-\theta^2}.$$

The retailer i maximizes $\pi_i(p_i,p_j) = (p_i - c)q_i = (p_i - c)\frac{(1-\theta)a - p_i + \theta p_j}{1-\theta^2}$ by choosing p_i. Then, the first–order condition to this problem is:

$$\frac{\partial \pi_i(p_i,p_j)}{\partial p_i} = 0 \Longrightarrow \frac{(1-\theta)a - 2p_i + \theta p_j + c}{1-\theta^2} = 0.$$

Hence, we have $p_i = \frac{(1-\theta)a + \theta p_j + c}{2}$.

Similarly, the retailer firm j maximizes $\pi_j(p_i,p_j) = (p_j - c)q_j = (p_j - c)\frac{(1-\theta)a - p_j + \theta p_i}{1-\theta^2}$ by choosing p_j. Then, the first–order condition for this problem is:

$$\frac{\partial \pi_j(p_i,p_j)}{\partial p_j} = 0 \Longrightarrow \frac{(1-\theta)a - 2p_j + \theta p_i + c}{1-\theta^2} = 0.$$

Hence, we have $p_j = \frac{(1-\theta)a+\theta p_i+c}{2}$.
Therefore, we have

$$\begin{aligned} p^{\mathrm{N},0} &= p_i = p_j = \frac{(1-\theta)a+c}{2-\theta}, \\ q^{\mathrm{N},0} &= q_i = q_j = \frac{a-c}{(1+\theta)(2-\theta)}, \text{ and} \\ \pi^{\mathrm{N},0} &= \pi_i = \pi_j = \frac{[(2+\theta)(1-\theta)(a-c)]^2}{(1-\theta^2)(4-\theta^2)^2}. \end{aligned}$$

Note that since $a > c$, we have $p^{N,0} > c$. It is easy to see that, since $a > c$ and $-1 < \theta < 1$, we have $\pi^{N,0} > 0$.

(2) *Innovation licensed to only one retailer.*

A single innovator creates a new technology and grants exclusive license to a retailer that experiences a marginal cost $c' = c - \varepsilon$, while the other retailer maintains the marginal cost of c. Without loss of generality, assume that it licenses the new technology to the firm i. The retailer i maximizes $\pi_i^{\mathrm{Y},1}(p_i^{\mathrm{Y},1}, p_j^{\mathrm{N},1}) = (p_i^{\mathrm{Y},1} - c')q_i = [p_i^{\mathrm{Y},1} - (c-\varepsilon)]\frac{(1-\theta)a - p_i^{\mathrm{Y},1} + \theta p_j^{\mathrm{N},1}}{1-\theta^2}$ by choosing $p_i^{\mathrm{Y},1}$. Then, the first–order condition to this problem is:

$$\frac{\partial \pi_i^{\mathrm{Y},1}(p_i^{\mathrm{Y},1}, p_j^{\mathrm{N},1})}{\partial p_i^{\mathrm{Y},1}} = 0 \Longrightarrow \frac{(1-\theta)a - 2p_i^{\mathrm{Y},1} + \theta p_j^{\mathrm{N},1} + c - \varepsilon}{1-\theta^2} = 0.$$

Hence, we have $p_i^{\mathrm{Y},1} = \frac{(1-\theta)a+\theta p_j^{\mathrm{N},1}+c-\varepsilon}{2}$.
Similarly, the retailer j maximizes $\pi_j^{\mathrm{N},1}(p_i^{\mathrm{Y},1}, p_j^{\mathrm{N},1}) = (p_j^{\mathrm{N},1} - c)q_j^{\mathrm{I},1} = (p_j^{\mathrm{N},1} - c)\frac{(1-\theta)a - p_j^{\mathrm{N},1} + \theta p_i^{\mathrm{Y},1}}{1-\theta^2}$ by choosing $p_j^{\mathrm{N},1}$. Then, the first–order condition to this problem is:

$$\frac{\partial \pi_j^{\mathrm{N},1}(p_i^{\mathrm{Y},1}, p_j^{\mathrm{N},1})}{\partial p_j^{\mathrm{N},1}} = 0 \Longrightarrow \frac{(1-\theta)a - 2p_j^{\mathrm{N},1} + \theta p_i^{\mathrm{Y},1} + c}{1-\theta^2} = 0.$$

Hence, we have $p_j^{\mathrm{N},1} = \frac{(1-\theta)a+\theta p_i^{\mathrm{Y},1}+c}{2}$.
Therefore, we have

$$\begin{aligned} p^{\mathrm{Y},1} &= p_i^{\mathrm{Y},1} = \frac{(2+\theta)((1-\theta)a+c) - 2\varepsilon}{4-\theta^2}, \text{ and } p^{\mathrm{N},1} = p_j^{\mathrm{N},1} = \frac{(2+\theta)((1-\theta)a+c) - \theta\varepsilon}{4-\theta^2}, \\ q^{\mathrm{Y},1} &= q_i^{\mathrm{Y},1} = \frac{(2+\theta)(1-\theta)(a-c) + (2-\theta^2)\varepsilon}{(1-\theta^2)(4-\theta^2)}, \text{ and } q^{\mathrm{N},1} = q_j^{\mathrm{N},1} = \frac{(2+\theta)(1-\theta)(a-c) - \theta\varepsilon}{(1-\theta^2)(4-\theta^2)}, \\ \pi^{\mathrm{Y},1} &= \pi_i^{\mathrm{Y},1} = \frac{[(2+\theta)(1-\theta)(a-c) + (2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2}, \text{ and } \pi^{\mathrm{N},1} = \pi_j^{\mathrm{N},1} = \frac{[(2+\theta)(1-\theta)(a-c) - \theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2}. \end{aligned}$$

(3) *Innovation licensed to both retailers*

Each innovator develops their own new technology and licenses it to their respective retailers, which both benefit from a marginal cost reduction of ε, leading to a new marginal cost of $c' = c - \varepsilon$. Then, we have

$$p^{\mathrm{Y},2} = p_i^{\mathrm{Y},2} = p_j^{\mathrm{Y},2} = \frac{(1-\theta)a + c - \varepsilon}{2-\theta},$$
$$q^{\mathrm{Y},2} = q_i^{\mathrm{Y},2} = q_j^{\mathrm{Y},2} = \frac{a-c+\varepsilon}{(1+\theta)(2-\theta)}, \text{ and}$$
$$\pi^{\mathrm{Y},2} = \pi_i^{\mathrm{Y},2} = \pi_j^{\mathrm{Y},2} = \frac{[(2+\theta)(1-\theta)(a-c+\varepsilon)]^2}{(1-\theta^2)(4-\theta^2)^2}.$$

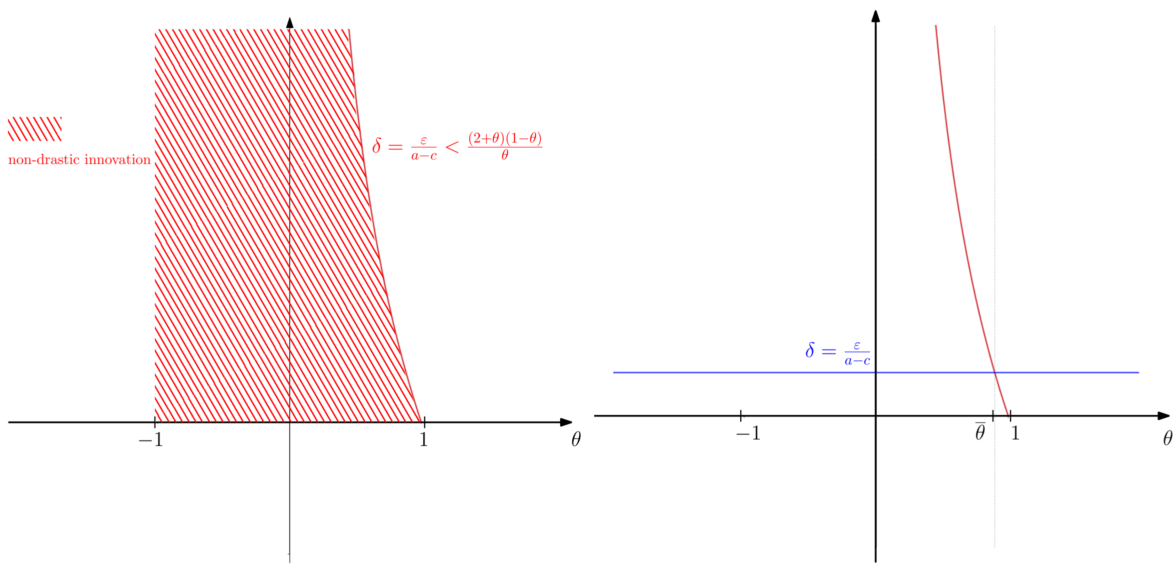

Figure 2: The region for non-drastic innovation and the greatest degree of subtitution $\bar{\theta}$ with non-drastic innovation, where $a = 3$, $c = 1.5$, and $\varepsilon = 1$, resulting $\delta = \frac{\varepsilon}{a-c} = 0.66$ and $\bar{\theta} = 0.80$.

Proof. **(Proposition 1)** If all retailers are active, then the innovation is non-drastic. The quantities that retailers are producing are $q^{\mathrm{N},0} = \frac{a-c}{(1+\theta)(2-\theta)}$, $q^{\mathrm{Y},2} = \frac{a-c+\varepsilon}{(1+\theta)(2-\theta)}$, $q^{\mathrm{Y},1} = \frac{(2+\theta)(1-\theta)(a-c)+(2-\theta^2)\varepsilon}{(1-\theta^2)(4-\theta^2)}$, and $q^{\mathrm{N},1} = \frac{(2+\theta)(1-\theta)(a-c)-\theta\varepsilon}{(1-\theta^2)(4-\theta^2)}$. Since $a > c$, $\varepsilon > 0$, and $\theta \in (-1, 1)$, we have $q^{\mathrm{N},0} > 0$ and $q^{\mathrm{Y},2} > 0$. Furthermore, we have $q^{\mathrm{Y},1} > q^{\mathrm{N},1}$. Therefore, all retailers are active if and only if $q^{\mathrm{N},1} > 0$. Then, we have $\frac{\varepsilon}{a-c} = \delta < \frac{(2+\theta)(1-\theta)}{\theta}$. Let $\bar{\theta}$ be such that $\delta = \frac{(2+\bar{\theta})(1-\bar{\theta})}{\bar{\theta}}$. Then, $\bar{\theta} = \frac{\sqrt{(\delta+1)^2+8}-(\delta+1)}{2}$. Since $\delta > 0$, we have $\bar{\theta} > 0$ and $\lim_{\varepsilon \to 0} \bar{\theta} = 1$. Note that if $\theta < \bar{\theta}$, then the innovation is non-drastic. Hence, since $\bar{\theta} > 0$, if the goods are complements or independent, then the innovation is non-drastic. If the goods are substi-

Before Innovation	Innovation licensed to only one retailer	Innovation licensed to both retailers
$p^{\mathrm{N},0} = \frac{(1-\theta)a+c}{2-\theta}$	$p^{\mathrm{Y},1} = \frac{(2+\theta)((1-\theta)a+c)-2\varepsilon}{4-\theta^2}$	$p^{\mathrm{Y},2} = \frac{(1-\theta)a+c-\varepsilon}{2-\theta}$
	$p^{\mathrm{N},1} = \frac{(2+\theta)((1-\theta)a+c)-\theta\varepsilon}{4-\theta^2}$	
$q^{\mathrm{N},0} = \frac{a-c}{(1+\theta)(2-\theta)}$	$q^{\mathrm{Y},1} = \frac{(2+\theta)(1-\theta)(a-c)+(2-\theta^2)\varepsilon}{(1-\theta^2)(4-\theta^2)}$	$q^{\mathrm{Y},2} = \frac{a-c+\varepsilon}{(1+\theta)(2-\theta)}$
	$q^{\mathrm{N},1} = \frac{(2+\theta)(1-\theta)(a-c)-\theta\varepsilon}{(1-\theta^2)(4-\theta^2)}$	
$\pi^{\mathrm{N},0} = \frac{[(2+\theta)(1-\theta)(a-c)]^2}{(1-\theta^2)(4-\theta^2)^2}$	$\pi^{\mathrm{Y},1} = \frac{[(2+\theta)(1-\theta)(a-c)+(2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2}$	$\pi^{\mathrm{Y},2} = \frac{[(2+\theta)(1-\theta)(a-c+\varepsilon)]^2}{(1-\theta^2)(4-\theta^2)^2}$
	$\pi^{\mathrm{N},1} = \frac{[(2+\theta)(1-\theta)(a-c)-\theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2}$	

Table 1: Equilibrium prices, quantities, and profits before and after the innovation, if the innovation is licensed to one or to both retailers.

tutes and the new technology is not highly effective, i.e., $\delta < \frac{(2+\theta)(1-\theta)}{\theta}$ or $\theta \in (0, \overline{\theta})$, then the innovation is non-drastic. Otherwise, the innovation is drastic. □

Table 1 summarizes of the equilibrium prices, quantities, and profits. Next, we compare the prices and profits in different scenarios.

Lemma 3. *The price after innovation is lower than the price before innovation, i.e.,* $p^{Y,1} < p^{N,0}$ *and* $p^{Y,2} < p^{N,0}$*. When innovation is licensed to one single firm, the price of that firm is lower than the price of the firm that does not have innovation:* $p^{Y,1} < p^{N,1}$*. When innovation is licensed to both firms, the price of is lower than the price of the firm that does not have innovation:* $p^{Y,2} < p^{N,1}$*. However, if the goods are substitutes or independent, then* $p^{Y,2} \leq p^{Y,1}$ *and* $p^{N,1} \leq p^{N,0}$*. If the goods are complements, then* $p^{Y,1} < p^{Y,2}$ *and* $p^{N,0} < p^{N,1}$*.*

Similarly, we have $\pi^{Y,1} > \pi^{N,0}$ *and* $\pi^{Y,2} > \pi^{N,0}$*. Furthermore, we have* $\pi^{Y,1} > \pi^{N,1}$ *and* $\pi^{Y,2} > \pi^{N,1}$*. If the goods are substitutes or independent, we have* $\pi^{Y,1} \geq \pi^{Y,2}$ *and* $\pi^{N,0} \geq \pi^{N,1}$*. If the goods are complements, then we have* $\pi^{Y,2} > \pi^{Y,1}$ *and* $\pi^{N,1} > \pi^{N,0}$*.*

Proof. We compare the prices in different scenarios. First, we show that the price after the innovation is lower than the price before the innovation. Since $\varepsilon > 0$, we have $p^{\mathrm{Y},1} =$

$\frac{(2+\theta)((1-\theta)a+c)-2\varepsilon}{4-\theta^2} < \frac{(2+\theta)((1-\theta)a+c)}{4-\theta^2} = \frac{(1-\theta)a+c}{2-\theta} = p^{\mathrm{N},0}$ and $p^{\mathrm{Y},2} = \frac{(1-\theta)a+c-\varepsilon}{2-\theta} < \frac{(1-\theta)a+c}{2-\theta} = p^{\mathrm{N},0}$. Second, we show that when the innovation is licensed to only one firm, the price of that firm is lower than the price of the firm that does not have the innovation. Since $\theta \in (-1,1)$, we have $p^{\mathrm{Y},1} = \frac{(2+\theta)((1-\theta)a+c)-2\varepsilon}{4-\theta^2} < \frac{(2+\theta)((1-\theta)a+c)-\theta\varepsilon}{4-\theta^2} = p^{\mathrm{N},1}$. Third, we show that when the innovation is licensed to both firms, the price of is lower than the price of the firm that does not have the innovation. It is easy to see that $p^{\mathrm{Y},2} = \frac{(1-\theta)a+c-\varepsilon}{2-\theta} = \frac{(2+\theta)((1-\theta)a+c)-(2+\theta)\varepsilon}{4-\theta^2} < \frac{(2+\theta)((1-\theta)a+c)-\theta\varepsilon}{4-\theta^2} = p^{\mathrm{N},1}$. Finally, if the goods are substitutes or independent, i.e., $\theta \in [0,1)$, then we have $p^{\mathrm{Y},2} = \frac{(1-\theta)a+c-\varepsilon}{2-\theta} = \frac{(2+\theta)((1-\theta)a+c)-(2+\theta)\varepsilon}{4-\theta^2} \leq \frac{(2+\theta)((1-\theta)a+c)-2\varepsilon}{4-\theta^2} = p^{\mathrm{Y},1}$ and $p^{\mathrm{N},1} = \frac{(2+\theta)((1-\theta)a+c)-\theta\varepsilon}{4-\theta^2} \leq \frac{(2+\theta)((1-\theta)a+c)}{4-\theta^2} = \frac{(1-\theta)a+c}{2-\theta} = p^{\mathrm{N},0}$. If the goods are complements, i.e., $\theta \in (-1,0)$, then we have $p^{\mathrm{Y},1} = \frac{(2+\theta)((1-\theta)a+c)-2\varepsilon}{4-\theta^2} < \frac{(2+\theta)((1-\theta)a+c)-(2+\theta)\varepsilon}{4-\theta^2} = \frac{(1-\theta)a+c-\varepsilon}{2-\theta} = p^{\mathrm{Y},2}$ and $p^{\mathrm{N},0} = \frac{(1-\theta)a+c}{2-\theta} = \frac{(2+\theta)((1-\theta)a+c)}{4-\theta^2} < \frac{(2+\theta)((1-\theta)a+c)-\theta\varepsilon}{4-\theta^2} = p^{\mathrm{N},1}$.

We compare the profits in different scenarios. First, since $\theta \in (-1,1)$, we have $\pi^{\mathrm{Y},1} = \frac{[(a-c)(2+\theta)(1-\theta)+(2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{N},0}$, and since $\varepsilon > 0$, $\pi^{\mathrm{Y},2} = \frac{[(a-c+\varepsilon)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{N},0}$. Second, since $\theta \in (-1,1)$ and $\varepsilon > 0$, $\pi^{\mathrm{Y},1} = \frac{[(a-c)(2+\theta)(1-\theta)+(2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)-\theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{N},1}$, and $\pi^{\mathrm{Y},2} = \frac{[(a-c+\varepsilon)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)-\theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{N},1}$. Finally, if the goods are substitutes or independent, then since $\theta \in [0,1)$ and $\varepsilon > 0$, we have $\pi^{\mathrm{Y},1} = \frac{[(a-c)(2+\theta)(1-\theta)+(2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} \geq \frac{[(a-c+\varepsilon)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{Y},2}$ and $\pi^{\mathrm{N},0} = \frac{[(a-c)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)-\theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{N},1}$. If the goods are complements, then since $\theta \in (-1,0)$ and $\varepsilon > 0$, we have $\pi^{\mathrm{Y},2} = \frac{[(a-c+\varepsilon)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)+(2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{Y},1}$ and $\pi^{\mathrm{N},1} = \frac{[(a-c)(2+\theta)(1-\theta)-\theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} > \frac{[(a-c)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} = \pi^{\mathrm{N},0}$. □

Proof. **(Proposition 2)** By Lemma 3, as a corollary, we have if the goods are substitutes or independent, then we have $\pi^{\mathrm{Y},1} \geq \pi^{\mathrm{Y},2} > \pi^{\mathrm{N},0} \geq \pi^{\mathrm{N},1}$ and if the goods are complements, then we have $\pi^{\mathrm{Y},2} > \pi^{\mathrm{Y},1} > \pi^{\mathrm{N},1} > \pi^{\mathrm{N},0}$. □

Proof. **(Proposition 3)** The change in the relative bargaining position is $\eta = \frac{\pi^{Y,1}-\pi^{N,0}}{\pi^{Y,2}-\pi^{N,1}}$. Then,

$$\begin{aligned}
\pi^{\mathrm{Y},1} - \pi^{\mathrm{N},0} &= \frac{[(a-c)(2+\theta)(1-\theta)+(2-\theta^2)\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} - \frac{[(a-c)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} \\
&= \frac{-2(a-c)(2+\theta)(1-\theta)(2-\theta^2)\varepsilon + (2-\theta^2)^2\varepsilon^2}{(1-\theta^2)(4-\theta^2)^2} \text{ and} \\
\pi^{\mathrm{Y},2} - \pi^{\mathrm{N},1} &= \frac{[(a-c+\varepsilon)(2+\theta)(1-\theta)]^2}{(1-\theta^2)(4-\theta^2)^2} - \frac{[(a-c)(2+\theta)(1-\theta)-\theta\varepsilon]^2}{(1-\theta^2)(4-\theta^2)^2} \\
&= \frac{2(a-c)(2+\theta)^2(1-\theta)^2\varepsilon + (2+\theta)^2(1-\theta)^2\varepsilon^2 + 2(a-c)(2+\theta)(1-\theta)\theta\varepsilon - \theta^2\varepsilon^2}{(1-\theta^2)(4-\theta^2)^2}.
\end{aligned}$$

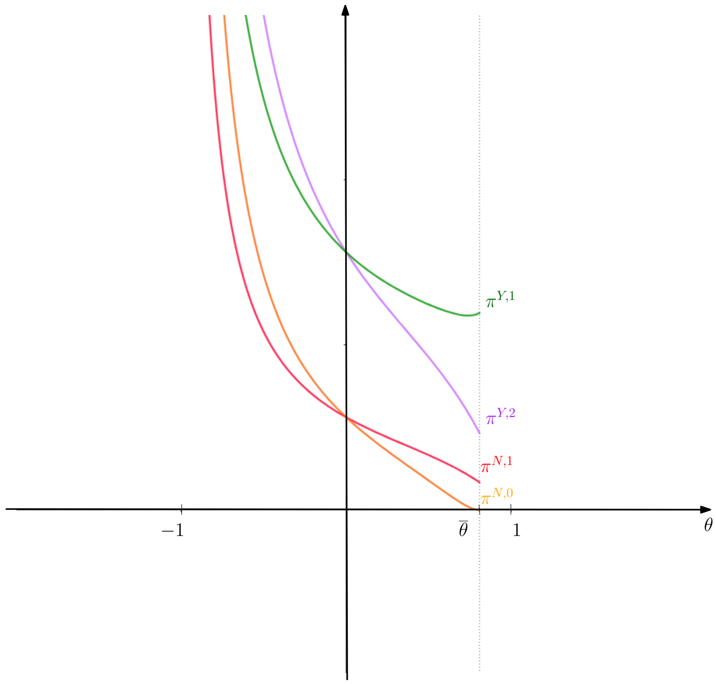

Figure 3: The profits when $\pi^{Y,1}$, $\pi^{Y,2}$, $\pi^{Y,1}$, and $\pi^{\mathrm{N},0}$ where $a = 3$, $c = 1.5$, $\varepsilon = 1$, and $\bar{\theta} = 0.80$.

Then, we have

$$\begin{aligned}\eta &= \frac{-2(a-c)(2+\theta)(1-\theta)(2-\theta^2)\varepsilon + (2-\theta^2)^2\varepsilon^2}{2(a-c)(2+\theta)^2(1-\theta)^2\varepsilon + (2+\theta)^2(1-\theta)^2\varepsilon^2 + 2(a-c)(2+\theta)(1-\theta)\theta\varepsilon - \theta^2\varepsilon^2} \\ &= \frac{2(a-c)(2-\theta-\theta^2) + (2-\theta^2)\varepsilon}{2(a-c)(2-\theta-\theta^2) + (2-2\theta-\theta^2)\varepsilon}\end{aligned}$$

By differentiating η with respect to θ, since $a > c$ and $\varepsilon > 0$, we get

$$\frac{\partial \eta}{\partial \theta} = \frac{(2(a-c)+\varepsilon)(2\varepsilon(\theta^2+2))}{[2(a-c)(2-\theta-\theta^2) + (2-2\theta-\theta^2)\varepsilon]^2} > 0$$

Hence, we show that the relative bargaining position is an increasing function of the degree of substitution, θ.

Note that η has an asymptote at $\tilde{\theta}$ where $2(a-c)(2-\tilde{\theta}-\tilde{\theta}^2) + (2-2\tilde{\theta}-\tilde{\theta}^2)\varepsilon = 0$. For each $\delta = \frac{\varepsilon}{a-c}$, there is $\tilde{\theta} = \frac{-(\delta+1)+\sqrt{3\delta^2+10\delta+9}}{\delta+2}$, where $0 < \tilde{\theta} < 1$, $\lim_{\theta \to \tilde{\theta}^-} \eta = +\infty$, and $\lim_{\theta \to \tilde{\theta}^+} \eta = -\infty$. Also, we have $\overline{\theta} = \frac{\sqrt{(\delta+1)^2+8}-(\delta+1)}{2} < \frac{-(\delta+1)+\sqrt{3\delta^2+10\delta+9}}{\delta+2} = \tilde{\theta}$.

Therefore, if $\theta \in (-1, 0)$, then the relative bargaining position, $\eta < 1$, i.e., there is a positive externality. If $\theta = 0$, then the relative bargaining position, $\eta = 1$, i.e., there is no externality.

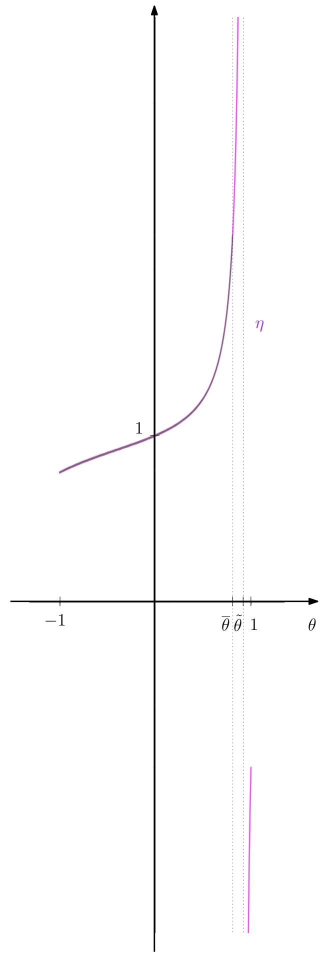

Figure 4: The relative bargaining η under non-drastic innovation where $\theta \in (-1, \overline{\theta})$.

If $\theta \in (0, \tilde{\theta})$, then the relative bargaining position, $\eta > 1$, i.e., there is a negative externality. □

Innovators' competition

Proof. **Proof of** (3) **and** (7) We can re-write the law of motion $\dot{p}_t = -p_t(1-p_t)2\lambda$ as $\dot{p}_t/(p_t)^2 + 2\lambda/p_t = 2\lambda$ with the initial value p_0. Using substitution $z_t = 1/p_t$, we have a non-homogeneous first-order linear differential equation $-\dot{z}_t + 2\lambda z_t = 2\lambda$ with the initial value $z_0 = 1/p_0$. Then, the general solution of the differential equation is $z_t = ce^{2\lambda t} + 1$ where $c = (1-p_0)/p_0$. Hence, we obtain $p_t^L = \frac{p_0 e^{-2\lambda t}}{p_0 e^{-2\lambda t} + (1-p_0)} = \frac{p_0}{p_0 + (1-p_0)e^{2\lambda t}}$. The law of motion at (7) is obtained similarly using substitution, $-\dot{z}_t + \lambda z_t = \lambda$ with the initial value $z_0 = 1/p_0$, and we get $p_t^{NL} = \frac{p_0 e^{-\lambda t}}{p_0 e^{-\lambda t} + (1-p_0)} = \frac{p_0}{p_0 + (1-p_0)e^{\lambda t}}$. □

Proof. **(Lemma 1)** Let T^{L1} be the abandoning time that solves the indifference condition (5) such that $\lambda p_{T^{L1}}(V^L + V^F) = \gamma$. Since $\partial p_t/\partial t < 0$, the left hand side of the equation (5) is monotonically decreasing. Hence, T^{L1} exists and is unique. To explicitly characterize T^{L1}, introduce p_t in (3), V^L in (2), and V^F in (4) into (5) to obtain

$$T^{L1} = \frac{1}{2\lambda} \ln \left[\left(\frac{p_0}{1-p_0} \right) \left(\frac{\lambda(V^L + V^F)}{\gamma} - 1 \right) \right]$$
$$= \frac{1}{2\lambda} ln \left[\left(\frac{p_0}{1-p_0} \right) \frac{\lambda \left[\left(\pi^{Y,1} - \pi^{N,0} \right) + 2\lambda \left(\pi^{Y,2} - \pi^{N,1} \right) \right] - (r+2\lambda)\gamma}{(r+\lambda)\gamma} \right].$$

At $t = T^{L1}$, the rival innovator continues the research if $p_{T^{L1}} \lambda \bar{V} > \gamma$ where $\bar{V} = (\pi^{Y,1} - \pi^{N,0})/r$. This occurs if $\bar{V} > V^L + V^F$:

$$\frac{\pi^{Y,1} - \pi^{N,0}}{r} > \frac{\pi^{Y,1} - \pi^{N,0} + \lambda(\pi^{Y,2} - \pi^{N,1})}{r+\lambda} + \frac{\lambda(\pi^{Y,2} - \pi^{N,1})}{r+\lambda}.$$

Since $\eta = (\pi^{Y,1} - \pi^{N,0})/(\pi^{Y,2} - \pi^{N,1})$, the above inequality is equivalent to $\eta > 2r$.

In this case, the first innovator abandons at the time T^{L1}, while the second does so at the time

$$T^{L2} = T^{L1} + \frac{1}{\lambda} \ln \left[\left(\frac{p_{T^{L1}}}{1 - p_{T^{L1}}} \right) \frac{\lambda \left(\pi^{Y,1} - \pi^{N,0} \right) - (r+\lambda)\gamma}{(r+\lambda)\gamma} \right], \tag{10}$$

where the second part of the right hand side is obtained by solving $p_{T^{L1}+t} \lambda \bar{V} = \gamma$.

For $\eta < 2r$, note that at $t = T^{L1}$, the term $p_{T^{L1}} \lambda$ is smaller than γ. Hence, if no discovery occurs, both innovators would abandon their projects at the same time $t = T^{L1}$. □

Proof. **(Proposition 4)** The difference in the present value of expected profit between licensing and not licensing is:

$$V^L - V^{NL} = \frac{e^{-(r+\lambda)\left(T^{L1}-\tau\right)}}{r+\lambda} \lambda \left(\pi^{Y,2} - \pi^{N,1} \right) + \frac{\left(\pi^{Y,1} - \pi^{N,0} \right)}{(r+\lambda)} - e^{-(r+\lambda)\left(T^{L1}-\tau\right)} \frac{\left(\pi^{Y,1} - \pi^{N,0} \right)}{r}$$
$$= \left(\frac{1}{r+\lambda} + \left(\frac{1}{\eta} - \frac{r+\lambda}{\lambda r} \right) \mu(\tau) \right) \left(\pi^{Y,1} - \pi^{N,0} \right)$$

where $\mu(\tau) = (\lambda/(r+\lambda))e^{-(r+\lambda)\left(T^{L1}-\tau\right)}$ and $\eta = (\pi^{Y,1} - \pi^{N,0})/(\pi^{Y,2} - \pi^{N,1})$. Then, it is easy to see that $V^L - V^{NL} \geq 0$ when $\eta \leq \frac{\lambda r}{r+\lambda}$ since $r > 0$, $\lambda \geq 0$, for each $\tau > 0$, we have $\mu(\tau) = (\lambda/(r+\lambda))e^{-(r+\lambda)\left(T^{L1}-\tau\right)} > 0$, and $\pi^{Y,1} > \pi^{N,0}$. □

Proof. **(Lemma 2)** Let T^{NL1} be the abandoning time that solves the indifference condition in (9) such that $\lambda p^{NL}_{T^{NL1}}\left(e^{-\lambda T^{NL1}}V^1+(1-e^{-\lambda T^{NL1}})V^2\right)=\gamma$. Since $\partial p^{NL}_t/\partial t<0$, $\partial\alpha_t/\partial t=\lambda e^{-\lambda t}>0$, and $V^1>V^2$, if $\eta=(\pi^{Y,1}-\pi^{N,0})/(\pi^{Y,2}-\pi^{N,1})>1$, then the left hand side of the equation is monotonically decreasing. Hence, T^{NL1} exists and is unique. The characterization of T^{NL1} is obtained by introducing p^{NL}_t in (7), V^1, V^2, and α_t into (9) to obtain:

$$\left(\frac{p_0}{p_0+(1-p_0)e^{\lambda T^{NL1}}}\right)\lambda\left[e^{-\lambda T^{NL1}}\left(\frac{\pi^{Y,1}-\pi^{N,0}}{r}\right)+(1-e^{-\lambda T^{NL1}})\left(\frac{\pi^{Y,2}-\pi^{N,1}}{r}\right)\right]=\gamma. \tag{11}$$

At $t=T^{NL1}$, the rival innovator continues the research if $p^{NL}_{T^{NL1}}\lambda V^1$ is larger than the left hand side of the indifference condition in (9) evaluated at T^{NL1}. However, this occurs since $V^1>V^2$. Then, the first innovator abandons at the time T^{NL1}, while the second does so at the time

$$T^{NL2}=T^{NL1}+\frac{1}{\lambda}ln\left[\left(\frac{p^{NL}_{T^{NL1}}}{1-p^{NL}_{T^{NL1}}}\right)\frac{\lambda\left(\pi^{Y,1}-\pi^{N,0}\right)-(r+\lambda)\gamma}{(r+\lambda)\gamma}\right], \tag{12}$$

where the second part of the right hand side is obtained by solving $p^{NL}_{T^{NL1}+t}\lambda V^1=\gamma$. □

Proof. **(Proposition 5)** Note that the present value of expected profit of not licensing is given by (6) but replacing T^{L1} by T^{NL1}. We start by eliciting the change in the expected profits of not licensing with respect to the time of discovery. Then:

$$\frac{\partial V^{NL}(\tau)}{\partial\tau}=(r+\lambda)e^{-(r+\lambda)(T^{NL1}-\tau)}\left[\frac{\pi^{Y,1}-\pi^{N,0}}{r}-\frac{\lambda(\pi^{Y,2}-\pi^{N,1})}{r+\lambda}\right].$$

Then, we have $\partial V^{NL}(\tau)/\partial\tau>0$ if $\eta>r\lambda/(r+\lambda)$, and $\partial V^{NL}(\tau)/\partial\tau<0$, otherwise.

First, consider the case where $\eta<r\lambda/(r+\lambda)$. Since $\partial V^{NL}(\tau)/\partial\tau<0$, there is always licensing in equilibrium if $V^L>V^{NL}(\tau=0)$, that is:

$$\begin{aligned}V^L&=\frac{(\pi^{Y,1}-\pi^{N,0})+\lambda(\pi^{Y,2}-\pi^{N,1})}{r+\lambda}\\&>\left(\frac{1-e^{-(r+\lambda)T^{NL1}}}{r+\lambda}\right)\lambda(\pi^{Y,2}-\pi^{N,1})+\frac{e^{-(r+\lambda)T^{NL1}}}{r}\left(\pi^{Y,1}-\pi^{N,0}\right)=V^{NP}(\tau=0).\end{aligned}$$

With simple algebraic manipulations, the previous is equivalent to

$$\frac{\pi^{Y,1}-\pi^{N,0}}{r+\lambda}>e^{-(r+\lambda)T^{NL1}}\left(\frac{\pi^{Y,1}-\pi^{N,0}}{r}-\frac{\lambda(\pi^{Y,2}-\pi^{N,1})}{r+\lambda}\right).$$

Since $e^{-(r+\lambda)T^{NL1}} < 1$, a sufficient condition for the previous expression to be fulfilled is that

$$\frac{\pi^{Y,1} - \pi^{N,0}}{r+\lambda} > \frac{\pi^{Y,1} - \pi^{N,0}}{r} - \frac{\lambda(\pi^{Y,2} - \pi^{N,1})}{r+\lambda},$$

which is equivalent to $\eta < r$, but since $r > r\lambda/(r+\lambda)$, there is always licensing in this region.

Next, consider the case where $\eta \geq r\lambda/(r+\lambda)$. Since $\partial V^{NL}(\tau)/\partial\tau \geq 0$, there is always licensing in equilibrium if $V^L > V^{NL}(\tau = T^{NL1})$, that is,

$$V^L = \frac{(\pi^{Y,1} - \pi^{N,0}) + \lambda(\pi^{Y,2} - \pi^{N,1})}{r+\lambda} > \frac{\pi^{Y,1} - \pi^{N,0}}{r} = V^{NL}(\tau = T^{NL1}),$$

which is true if $\eta < r$. Therefore, in the case of complementary goods, an equilibrium without licensing cannot arise.. Now, consider the case where $\eta > r$. To this end, we first define the time $\tilde{T}$ such that $V^L = V^{NL}(\tau = \tilde{T})$. To characterize $\tilde{T}$, observe that

$$\begin{aligned}
V^L &= \frac{(\pi^{Y,1} - \pi^{N,0}) + \lambda(\pi^{Y,2} - \pi^{N,1})}{r+\lambda} \\
&= \left(\frac{1 - e^{-(r+\lambda)(T^{NL1} - \tilde{T})}}{r+\lambda}\right)\lambda(\pi^{Y,2} - \pi^{N,1}) + e^{-(r+\lambda)(T^{NL1} - \tilde{T})}\left(\frac{\pi^{Y,1} - \pi^{N,0}}{r}\right) = V^{NL}(\tau = \tilde{T}). \\
\iff &\quad \frac{\eta}{r+\lambda} + \frac{\lambda}{r+\lambda} = \frac{\lambda}{r+\lambda}(1 - e^{-(r+\lambda)(T^{NL1} - \tilde{T})}) + \left(\frac{\eta}{r}\right)e^{-(r+\lambda)(T^{NL1} - \tilde{T})} \\
\iff &\quad \tilde{T}(\eta) = \left(\frac{1}{r+\lambda}\right) ln\left[\frac{1}{r+\lambda - r\lambda/\eta}\right] + (r+\lambda)T^{NL1},
\end{aligned}$$

where the second line comes from dividing both parts by $(\pi^{Y,2} - \pi^{N,1})$, and the last line comes from solving for $\tilde{T}$.

Differentiating with respect to η gives:

$$\frac{\partial\tilde{T}(\eta)}{\partial\eta} = -\frac{1}{r+\lambda}\left(r + \lambda - \frac{r\lambda}{\eta}\right)\left(\frac{r\lambda}{(r+\lambda\eta) - r\lambda)^2}\right) < 0,$$

where the negative sign is because $(r + \lambda - r\lambda/\eta) > 0$ since $\eta > r\lambda/(r+\lambda)$.

Since $\partial\tilde{T}(\eta)/\partial\eta < 0$, there exists a value of η such that $\tilde{T}(\bar{\eta}) = 0$. In this case, the innovator never licenses until the rival firm abandons the research. To obtain $\bar{\eta}$, we need to solve $V^{NL}(\tilde{T}=0) = V^L$. Therefore,

$$\begin{aligned} V^{NL}(\tilde{T}=0) &= \left(\frac{1-e^{-(r+\lambda)T^{NL1}}}{r+\lambda}\right)\lambda(\pi^{Y,2}-\pi^{N,1}) + e^{-(r+\lambda)T^{NL1}}\left(\frac{\pi^{Y,1}-\pi^{N,0}}{r}\right) \\ &= \frac{(\pi^{Y,1}-\pi^{N,0}) + \lambda(\pi^{Y,2}-\pi^{N,1})}{r+\lambda} = V^L \\ \iff \quad & e^{-(r+\lambda)T^{NL1}}\left(\frac{\bar{\eta}}{r} - \frac{\lambda}{r+\lambda}\right) = \frac{\bar{\eta}}{r+\lambda} \iff \bar{\eta} := \frac{r\lambda e^{-(r+\lambda)T^{NL1}}}{(r+\lambda)e^{-(r+\lambda)T^{NL1}} - r}. \end{aligned}$$

The second line comes from dividing both parts by $(\pi^{Y,2}-\pi^{N,1})$, and the last line comes from solving for $\bar{\eta}$.

Since $\partial V^{NL}(\tau)/\partial\tau > 0$, the previous analysis shows that for $\eta \geq \bar{\eta}$, the leading innovator never licenses until the rival innovator abandons the research.

From the previous analysis, it is direct to obtain that for $\eta \in \left[\frac{r\lambda}{r+\lambda}, \bar{\eta}\right)$, the discovery occurs at time $\tau \in \left[0, \tilde{T}(\eta)\right)$, the leading innovator licenses since $V^L > V^{NL}(\tau)$. If the discovery occurs at time $\tau \in \left[\tilde{T}(\eta), T^{NL1}\right)$, the leading innovator does not license before T^{NL1} since $V^{NL}(\tau) > V^L$.

□