BUSINESS CYCLE ASYMMETRIES: AN INVESTMENT COST APPROACH

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Abstract

In this paper, investment cost asymmetry is introduced in order to test wheter this kind of asymmetry can account for asymmetries in business cycles. By using a smooth transition function, asymmetric investment cost is modeled and introduced in a canonical RBC model. Simulations of the model with Perturbations Method (PM) are very close to simulations through Parameterized Expectations Algorithm (PEA), which allows the use of the former for the sake of time reduction and computational costs. Both symmetric and asymmetric models were simulated and compared. Deterministic and stochastic impulse-response excersices revealed that it is possible to adequately reproduce asymmetric business cycles by modeling asymmetric investment costs. Simulations also showed that higher order moments are insufficient to detect asymmetries. Instead, methods such as Generalized Impulse Response Analysis (GIRA) and Nonlinear Econometrics prove to be more efficient diagnostic tools.

1 Introduction

Traditional analyses on economic fluctuations have achieved certain consensus regarding business cycle causes with somewhat predictive and explicative power. Yet uncertain remain some relevant facts such as the one of the asymmetric behavior present in the GDP components along the business cycles. Asymmetries and nonlinearities can be seen through stylized facts, time varying amplitude in cyclical components of macroeconomic variables for instance. A simple way to easily identify such asymmetries is by calculating higher order moments for the distribution of cyclical components.

Overall, a very important challenge for economic models lies on data particularities, namely nonlinearities and asymmetries, particularly for the case of DSGE models. Despite being highly nonlinear, they seem to have symmetric behavior and symmetric transmission mechanisms as well as symmetric technology shocks. Models with these features are unable to adequately reproduce third and fourth moments of the empirical distributions of cyclical components of macroeconomic variables (Valderrama, 2007). On the basis of empirical analysis, business cycles asymmetries have been treated by Nonlinear Econometrics, and mostly through Switching Regime Econometrics. For example, Neftci (1984) uses Swithching Markov Estimation in order to study whether correlations of economic variables differ throughout the phases of business cycles. Supported on basic intuition, Neftci states that if a times series is symmetric along the business cycles and two regimes or states exist, the probability of remaining in state 1 is the same of remaining in state 2. Based on maximum likelihood and a Bayesian refinement of this, Neftci discovered that for unemployment series of the US economy the probability of remaining in a consecutive decrease state is higher than the probability of remaining in a consecutive increase state.

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Supported on concepts developed by Sichel (1993) and on the study by Clements and Krolzig (2003), Belaire-Franch and Contreras (2003) attempted to detect and estimate three kinds of asymmetries by means of a parametric test: deepness, stepness, and sharpness. Under the supposal that a time series is generated by a Markov Switching-Autorresive model with M regimes in the mean (MS-AR(p)), it was found that most of the countries sampled have certain asymmetry, except for the US and Germany. In turn, Gefang and Strachan (2010) employed a smooth transition VAR to measure the impact of international business cycles on the UK economy. The estimations were performed on the GDP growth rate. The countries involved in the analysis were the US, France, and Germany. It was found that the UK economy is influenced asymmetrically by other countries in the sample.

On DSGE modeling, there are Pytelarczyk (2005), Eo (2009), Belaygoned and Michel (2006), and Davig and Leeper (2005). These works have developed DSGE models with *ad hoc* switching regimes on the linearized dynamic equations of the model. Parameter estimations are used to perform impulse-response exercises. Other works such as Tristani (2008), Karagikli, Matheson, Smith and Valey (2007), and Bullard and Singh (2009) have developed DSGE models that introduce exogenous regime switching disturbances. Recent applications of Bayesian Econometrics have contributed to estimate parameters for DSGE models that include explicit regime switchings for the impulse-response matrix of coefficients as well as for time process of disturbances.

Modern Econometrics and, up to some extent, DSGE modeling have been concerned with nonlinearities and asymmetry of data generating processes. However, there is a further task for economists regarding the construction of models that take into account asymmetries as the result of endogenous optimal decision-making or, at least, include them in the basic behavioral equations of the models. Thus, in spite of the sophisticated tools used by the the authors aforementioned, a question remains unanswered: *Where do asymmetries come from?* The answer to this question might lie in modeling the behavior of firms and agents, considering that during booms they may behave differently than in recessions. That is to say, it is necessary to study the transmision mechanisms and behaviors that cause differences between phases of business cycles.

Asymmetries in production and productive factor utilization can be found in the literature. Nonetheless, some of them present controversial findings. Partial equilibrium models of representative firms and convex (symmetric) adjustment functions have been criticized as they ignore diverse features of firms, idiosincratic shocks, and microeconomic rigidities. These aspects have drawn more attention with their possible links to aggregate investment dynamics: fixed adjustment costs, irreversibilities, (S,s) dynamics, and lumpy investment. In this sense, Doms and Dunne (1998) found in a sample of firms that they adjust capital in lumpy ways, and fixed costs explain a significative part of firms' total investment expenditure, aggregate investment itself. A similar result has been obtained by Caballero and Engel (1994) through the estimation of a nonlinear model. Caballero and Engel (1991) with an extension of a (S,s) model also found that lumpy investment affects aggregate investment dynamics, thus showing analytically that the cross section distribution of firms' investment converges towards a long-run distribution. Caballero, Engel, and Haltiwanger (1995) also observe the asynchronicities of firms. By joining micro elements and aggregation, they deduce an inverse aggregate investment equation. Their estimations indicate that investment elasticities of shocks vary throughout time, which means that firms are willing to adjust capital when facing a high scarcity of it. With respect to the existence of micro rigidities, Cooper and Haltiwanger (2000) used an indirect inference method for a sample of firms.They found evidence supporting the joint existence of convex and non-convex costs, and irreversibilities.

In opposite direction, there are DSGE with micro rigidities, which have not encountered relative consensus of those works in partial equilibrium. Veracierto (2002) concludes that investment irreversibilities generate a small difference compared to a canonical RBC model. In a similar fashion, Thomas (2002) claims that lumpy investment does not have significative effects on aggregate investment. Khan and Thomas (2003) discovered that when fixing prices, it is possible to produce non-linear dynamics in aggregate investment, which then disappears by allowing price adjustment. Differently, Bachman, Caballero and Engel (2006a), and Bachman, Caballero and Engel (2006b) pointed out two existing smoothing mechanisms: pre-general equilibrium smoothing (which explains 60% of investment variance) and general equilibrium smoothing (which explains the remaining 40%). They have also demonstrated that the particular specification used by Khan and Thomas (2003) involves a small partial equilibrium effect which is reproduced in general equilibrium. A more realistic specification entails a big partial equilibrium effect that, as a consequence, implies an important aggregate effect on a general equilibrium model.

Since consensus between those studies has been unmet, this paper addresses a different and more "aggregate" modeling strategy. In this paper, asymmetric investment cost is introduced in order to test whether this asymmetry can account for asymmetries in business cycles. Among some works on asymmetries in factor demand and factor adjustment costs, an excellent contribution in this line, and roughly close to the present work, has been made by Palm and Pfann (1997). Their work addresses sources of asymmetry in production factors dynamics[1].They have indicated that linear-quadratic models and the implications of their simmetry is unable to pass statistical tests. Although they are not interested in the study of business cycles in a general equilibrium framework, their proposal poses two questions also addressed in the present paper: *What are the sources of the asymmetries?* and *Why do all tests for the underlying structures of adjustment costs are important for the aggregate production factors dynamics?*[2]. Their model for asymmetric production factor dynamics is built on the assumption that "*(...) firms, when making contingency plans on the use of factor imputs, account for differences in adjustment costs during different phases of business cycles*" .[3] A generalization of adjustment functions is proposed for both capital and labor. Given specific functional forms for production functions and adjustment costs (which nests the symmetric cost function), the model is estimated for first order conditions of profit maximization, and the null of symmetric cost function is rejected. Next, the estimated model is solved and simulated by means of Parameterized Expectations Algorithm (PEA) given the real prices of factors and productivity shocks. The aim of the simulation is to test whether the existence of external nonlinearity has some impact on dynamic factor input asymmetry of data. External nonlinearity is introduced by modeling real prices of factors as a nonlinear (quadratic) bivariate AR(1,1) process. A linear bivariate AR(1,1) is also modeled to serve the purpose of control framework. The main conclusion reveals that 50% of the dynamic factor demand asymmetry in the manufacturing sector of the Netherlands is explained by *behavioral or internal asymmetries* caused by asymmetric adjustment costs, while the remaining 50% is caused by *external nonlinearities* in real price factors.

Other studies have dealt with asymmetries in factor adjustment costs. Jaramillo et al. (1993) have worked on asymmetries for labor of the Italian industry, with firing costs being different to hiring costs. Their hypothesis was tested by a general model that nested symmetric costs, thus rejecting the null of symmetry. Pfann and Palm (1993) make a distinction between skilled and unskilled labor [4] for manufacturing sectors in the UK and the Netherlands. They found that data rejected the null of symmetric costs. Moreover, their results revealed a very interesting fact: hiring costs are higher than firing costs for unskilled labor, whereas the opposite is also true for skilled labor. About adjusting labor costs, Hamermesh and Pfann (1995) used a generalized cost function including gross and net changes in labor. Their estimations have revealed that this modeling is necessary to track down correctly labor demand dynamics of the US manufacturing sector.

2 A simple model with asymmetric investment costs

Let us suppose a hypothetical economy with neither technoloy nor population growth and cost of investment being asymmetric, which implies a greater disinvestment cost than investment. In the symmetric case, the investment cost assumes the very traditional form $\frac{\psi}{2}(\Delta k - \overline{in})^2$, being $\overline{in}$ net investment in steady state and $\Delta k = k_{+1} - k$ net investment. When $\Delta k = \overline{in}$, there is not any investment cost, i.e. the cost function reaches a threshold at that point. It is important to bear in mind that in an economy with neither population growth nor technological progress, net investment equals zero in the steady state.

Furthermore, it is known that the investment cost around that point, for instance $\Delta k = \epsilon$ or $\Delta k = -\epsilon$ ($\epsilon > 0$), is not the same in such a case where the economy is in recession, $\Delta k = -\epsilon$, and when it is in expansion, $\Delta k = \epsilon$. Thus, if we suppose that decreasing the investment is more costly than increasing it, the investment cost in each state is:

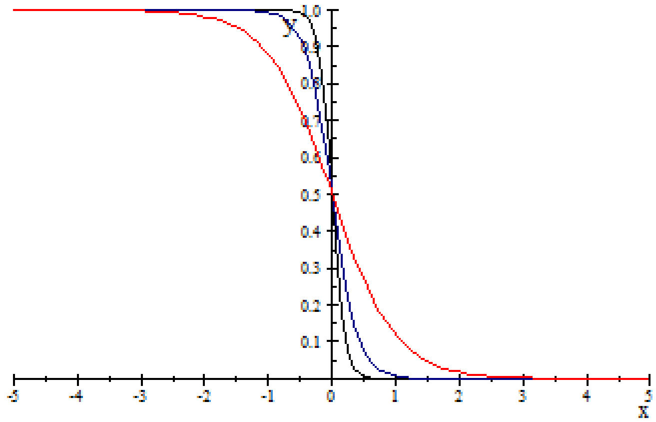
[1]This is exactly the title of their paper

[2]*I quote textually from Palm and Pfann (1997) pag. 362*

[3]*I quote textually from Palm and Pfann (1997) pag. 364*

[4]More precisely, they distinguish production and non-production workers.

Figure 1: Smooth transition function



$$C(\Delta k) = \left\{ \begin{array}{c} \frac{\psi_1}{2}(k_{+1} - k - \overline{in})^2, \text{if } \Delta k < 0 \\ \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2, \text{ if } \Delta k > 0 \end{array} \right\}, \psi_1 > \psi_2 \tag{1}$$

Let us suppose ϕ_t a smooth transition function between the states. If we define such transition function as an indicator function (or as a probability function), the regime switching cost function will be[5]:

$$C(\Delta k) = \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2 + \phi_t(\frac{\psi_1}{2}(k_{+1} - k - \overline{in})^2 - \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2) \tag{2}$$

where ϕ_t is a logistic one:

$$\phi_t = b/(1 + exp(\gamma(k_{+1} - k - \overline{in}))) \tag{3}$$

$$b = \left\{ \begin{array}{c} 1, \text{ if asymmetric beharior} \\ 0, \text{ if symmetric behavior} \end{array} \right\} \tag{4}$$

If $\gamma \to \infty, \phi$ has an almost instanteneuos change,if $\gamma \to 0$, $\phi \to 0.5$. if $k_{t+1} - k_t - \overline{in} < 0, \phi_t \to 1$, if $k_{t+1} - k_t - \overline{in} > 0, \phi_t \to 0$. figure 1 shows the transition function for $\gamma = 10, 5, 2.5$, and figure 2 shows symmetric and asymmetric (black line) cost functions for $\psi_2 = 1$ (red line), $\psi_1 = 4$ (green line) and $\gamma = 0.5$.

Thus, if we rewrite the capital cost adjustment we have:

$$C(\Delta k) = \varphi_t = \varphi_{2t} + \phi_t\left(\varphi_{1t} - \varphi_{2t}\right)$$

being

$$\varphi_{1t} = \frac{\psi_1}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{5}$$

$$\varphi_{2t} = \frac{\psi_2}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{6}$$

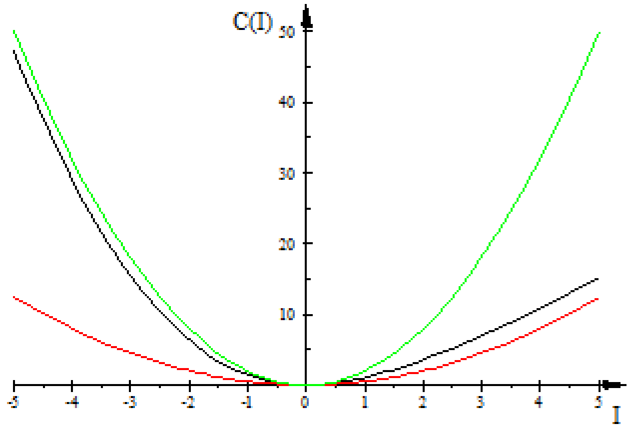
The marginal cost of adjusting capital in periods t and $t+1$ respectively will be

$$\frac{\partial \varphi_t}{\partial k_{t+1}} = \frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}}\left(\varphi_{1t} - \varphi_{2t}\right) + \phi_t\left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}}\right) \tag{7}$$

$$\frac{\partial \varphi_{t+1}}{\partial k_{t+1}} = \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}}\left(\varphi_{1t+1} - \varphi_{2t+1}\right) + \phi_{t+1}\left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}}\right) \tag{8}$$

[5]Pfann and Palm (1997) proppose a quadratic-exponential function to model asymmeties in cost functions $C(\Delta k) = \exp(\beta_k \Delta k) - 1 - \beta_k \Delta k + \frac{1}{2}\gamma_k(\beta \Delta k)^2$ and $C(\Delta n) = \exp(\beta_n \Delta n) - 1 - \beta_n \Delta n + \frac{1}{2}\gamma_n(\beta \Delta n)^2$ for capital and labor respectively.

Figure 2: Symmetric and asymmetric adjustment cost functions



Suppose that capital evolves as:[6]

$$k_{t+1} = (1-\delta)k + y - c - C(\Delta k) \tag{9}$$

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha} \tag{10}$$

$$1 = n_t + l_t \tag{11}$$

In other words, we assume neither population nor technological growth. The problem of the family, supposing a central planner perspective, is the standard one: choose consumption, leisure, and capital sequences to maximize the intertemporal utility function.

$$U(c) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\eta} l_t^{1-\eta}\right]^{1-\theta}}{1-\theta} \tag{12}$$

Subject to equations (2), (3) and (4). The lagrangean function for this problem is:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\eta} l_t^{1-\eta}\right]^{1-\theta}}{1-\theta} + \sum_{t=0}^{\infty} \lambda_t \beta^t \left[(1-\delta)k_t + y_t - c_t - C(i_t) - k_{t+1}\right] \right\} \tag{13}$$

First order conditions are:

$$\frac{\partial \mathcal{L}}{\delta c} = \left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} \eta c_t^{\eta-1} l_t^{1-\eta} - \lambda_t = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\delta k_{t+1}} = -\lambda_t - \lambda_t \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}} + \beta E_t \left\{ \lambda_{t+1} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}} \right] \right\} = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\delta n_t} = -\left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta} + \lambda_t (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\delta \lambda_t} = -k_{+1} + (1-\delta)k + y - i - C(\Delta k_{t+1}) = 0 \tag{17}$$

[6]This model is as simple as possible, the standard way to model costs of investment is to includ them into the entertemporal profit function of firms and then solve for the descentralised equilibrium. However altough is possible to do this so, is preferable to first solve and simulate this simple model and introduce more complex elements later.

By using the functional forms we have that:

$$\left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} c_t^{\eta-1} l_t^{1-\eta} \left\{1 + \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}}\right\} - \beta E_t \left\{ \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta}\right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}}\right]\right\} = 0 \quad (18)$$

$$-\left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta} + \left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} \eta c_t^{\eta-1} l_t^{1-\eta} (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} = 0 \quad (19)$$

$$k_{+1} - (1-\delta)k - y + c + \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2 + \phi_t\left(\frac{\psi_1}{2}(k_{+1} - k - \overline{in})^2 - \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2\right) = 0 \quad (20)$$

As we can see (18) is the Euler equation for consumption which seems to be quite similar to the traditional one. However, by taking into account that $\frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}}$ and $\frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}}$ are no longer linear expressions and, in fact, depend on the sign of Δk, if we replace the expressions corresponding to these derivatives within the Euler equation for consumption, we will have:

$$0 = \eta \left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} c_t^{\eta-1} l_t^{1-\eta} \left\{1 + \left(\frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}}(\varphi_{1t} - \varphi_{2t}) + \phi_t\left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}}\right)\right)\right\} - \beta E_t \left\{\eta \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta}\right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \left(\begin{array}{c} \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}}(\varphi_{1t+1} - \varphi_{2t+1}) \\ +\phi_{t+1}\left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}}\right) \end{array}\right)\right]\right\} \quad (21)$$

In this expression, it is possible to see that the transition probability between regimes ϕ_t does appear on both sides of the equation for t and $t+1$, and so does the change on this probability in interaction with the difference of adjustment costs $\frac{\partial \phi_t}{\partial k_{t+1}}(\varphi_{1t} - \varphi_{2t})$ [7] . In this line of reasoning, the equation for intratemporal optimality condition in the cannonical RBC will also be misspecified. By transforming the equivalent of $\left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta}(1-\eta)c_t^{\eta} l_t^{-\eta}$ from(18) into (19) we will have:

$$\left[c_t^{\eta} l_t^{1-\eta}\right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta} = \eta \left[\begin{array}{c} \beta\left\{1 + \left(\frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}}(\varphi_{1t} - \varphi_{2t}) + \phi_t\left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}}\right)\right)\right\}^{-1} \\ E_t\left\{\left[c_{t+1}^{\eta} l_{t+1}^{1-\eta}\right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta}\left[(1-\delta) + f'(k_{t+1}) - \left(\begin{array}{c} \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}}(\varphi_{1t+1} - \varphi_{2t+1}) \\ +\phi_{t+1}\left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}}\right)\end{array}\right)\right]\right\}\end{array}\right] \times (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} \quad (22)$$

Thus (22) shows that regime change probability and the interaction between probability derivative and adjustment costs difference also induce asymmetries.

3 Dynamics, calibration and simulation

Since the Euler equation of this model is nonlinear, as a regular DSGE's Euler equation, and asymmetric, it is necessary to use numerical methods to simulate it and solve it. Two alternative methods are addressed hereafter to show the inconvenience of using traditional log-linearization: Parameterized Expectations Approach

[7] If we admit that including,\textit{ad hoc},transition probabilities matrices into the dynamic systemofacanonical RBC model, there is still remaining a mispecification error, which is the expression related $\frac{\partial \phi_t}{\partial k_{t+1}}(\varphi_{1t} - \varphi_{2t})$

(PEA) and Perturbations Method (PM). PEA was formalized by Marcet and Marshall (1994) and is a global method consisting of approaching the expectations equations[8]. PM is a local procedure based on k-order Taylor approximations around a particular point (the steady state for the case of DSGE and RBC models). A very useful and powerful tool for this method is Dynare, which allows up to third-order approximations[9].

3.1 Loglinearisation

Lets suppose that $\eta = 0$ and that $n_t = 1$, we will rewrite the system as:

$$\varphi_{1t} = \frac{\psi_1}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{23}$$

$$\varphi_{2t} = \frac{\psi_2}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{24}$$

$$C(\Delta k) = \varphi_t = \varphi_{2t} + \phi_t\left(\varphi_{1t} - \varphi_{2t}\right) \tag{25}$$

$$k_{+1} = (1-\delta)k + y - c - C(\Delta k) \tag{26}$$

$$y_t = f(k_t) = A_t k_t^{\alpha} \tag{27}$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_{t,}\varepsilon \backsim (0, \sigma_{\varepsilon}^2) \tag{28}$$

first order conditions are:

$$c_t^{-\theta} = \lambda_t \tag{29}$$

$$\lambda_t + \lambda_t \frac{\partial \varphi_t}{\partial k_{t+1}} = \lambda_{t+1}\beta\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial \varphi_{t+1}}{\partial k_{t+1}}\right] \tag{30}$$

$$\frac{\partial \varphi_t}{\partial k_{t+1}} = \frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}}\left(\varphi_{1t} - \varphi_{2t}\right) + \phi_t\left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}}\right) \tag{31}$$

$$\frac{\partial \varphi_{t+1}}{\partial k_{t+1}} = \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}}\left(\varphi_{1t+1} - \varphi_{2t+1}\right) + \phi_{t+1}\left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}}\right) \tag{32}$$

Now, we consider the first order Taylor approximation around the log of the steady state for each regime, this is for $\Delta k > 0$ and for $\Delta k < 0$:

In the first regime or during a recession as$\Delta k < 0$, the log-linearized model is:

$$-\theta \hat{c}_t = \hat{\lambda}_t \tag{33}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t - \varphi\varphi_t \tag{34}$$

$$y\hat{y}_t = Ak^{\alpha}\hat{A}_t + \alpha Ak\hat{k}_t^{\alpha} \tag{35}$$

$$\varphi\hat{\varphi}_t = \varphi_1\hat{\varphi}_{1t} \tag{36}$$

$$\varphi_1\hat{\varphi}_{1t} = \psi_1\left(k_{t+1} - k_t\right)k\hat{k}_{t+1} - \psi_1\left(k_{t+1} - k_t\right)k\hat{k}_t \tag{37}$$

[8]An excelente and didactic reference about this method and its practical applications is Marcet and Lorenzoni (2001).
[9]The package also includes a Dynare++ module which allows up to seven-order approximation.

We will now take advantage of the fact that if we have a function $g(x)$, its log-linearisation becomes $g(X_t) \simeq g(X)(1+\eta x_t)$, being $x_t = \ln(X_t/X), \eta = \frac{\partial f(X)}{\partial X}\frac{X}{f(X)}$.

$$\begin{aligned}
&\lambda\hat{\lambda}_t + \lambda\frac{\partial\varphi(x)}{\partial k_{t+1}}\left(1+\eta_{11}\hat{k}_{t+1}\right) + \lambda\frac{\partial\varphi_t(x)}{\partial k_{t+1}}\left(1+\eta_{21}\hat{k}_{t+1}\right) \\
&= \beta\lambda\hat{\lambda}_{t+1}\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\right] \\
&+ \beta\lambda\left[\begin{array}{c} f'(k)\left(1+\eta_{31}\hat{k}_{t+1}\right) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_{41}\hat{k}_{t+1}\right) \\ -\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_{51}\hat{k}_{t+2}\right) \end{array}\right] + \beta\lambda f'(k)\left(1+\eta_{61}\hat{A}_{t+1}\right)
\end{aligned} \tag{38}$$

$$\begin{aligned}
\eta_{11} &= \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_t}\frac{k_t}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{21} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{31} = \frac{\partial^2 f(k_{t+1})}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{f'(k_{t+1})}, \\
\eta_{41} &= \frac{\partial^2\varphi_{t+1}}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}}, \eta_{51} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_{t+2}}\frac{k_{t+2}}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{61}\frac{\partial^2 f(k_{t+1})}{\partial k_{t+1}\partial A_{t+1}}\frac{A_{t+1}}{f'(k_{t+1})}
\end{aligned}$$

Thus, for the previous equations (evaluated in the steady state which implies $k_{t+1} = k_t = \bar{k}$,) we have:

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{39}$$

$$\bar{k}\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \bar{y}\hat{y}_t - \bar{c}\hat{c}_t \tag{40}$$

$$\hat{\lambda}_t = \beta\hat{\lambda}_{t+1}\left[(1-\delta) + f'(\bar{k})\right] + \beta f'(\bar{k})\left(1+\eta_{31}\hat{k}_{t+1}\right) + \beta\lambda f'(\bar{k})\left(1+\eta_{61}\hat{A}_{t+1}\right) \tag{41}$$

$$\bar{y}\hat{y}_t = \bar{A}\bar{k}\hat{A}_t + \alpha\bar{A}\bar{k}^{\alpha}\hat{k}_t \tag{42}$$

$$\ln A_t = \rho\ln A_{t-1} + \varepsilon_t, \varepsilon_t \backsim (0, \sigma_\varepsilon^2) \tag{43}$$

Notice that this linearised model for the recession regime is formed by linear equations.

In the second regime or during a boom as $\Delta k > 0$, the linearised model becomes:

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{44}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t - \varphi\varphi_t \tag{45}$$

$$y\hat{y}_t = Ak^{\alpha}\hat{A}_t + \alpha Ak\hat{k}_t^{\alpha} \tag{46}$$

$$\varphi\hat{\varphi}_t = \varphi_2\hat{\varphi}_{2t} \tag{47}$$

$$\varphi_2\hat{\varphi}_{2t} = \psi_2\left(k_{t+1} - k_t\right)k\hat{k}_{t+1} - \psi_2\left(k_{t+1} - k_t\right)k\hat{k}_t \tag{48}$$

Because this approximation is evaluated in the steady state, which impplies $k_{t+1} = k_t = \bar{k}$,

$$\begin{aligned}
&\lambda\hat{\lambda}_t + \lambda\frac{\partial\varphi(x)}{\partial k_{t+1}}\left(1+\eta_{12}\hat{k}_{t+1}\right) + \lambda\frac{\partial\varphi_t(x)}{\partial k_{t+1}}\left(1+\eta_{22}\hat{k}_{t+1}\right) \\
&= \beta\lambda\hat{\lambda}_{t+1}\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\right] \\
&+ \beta\lambda\left[\begin{array}{c} f'(k)\left(1+\eta_{32}\hat{k}_{t+1}\right) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_{42}\hat{k}_{t+1}\right) \\ -\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_{52}\hat{k}_{t+2}\right) \end{array}\right] + \beta\lambda f'(k)\left(1+\eta_{62}\hat{A}_{t+1}\right)
\end{aligned} \tag{49}$$

$$\eta_{12} = \frac{\partial^2 \varphi_t}{\partial k_{t+1} \partial k_t} \frac{k_t}{\frac{\partial \varphi_t(x)}{\partial k_{t+1}}}, \eta_{22} = \frac{\partial^2 \varphi_t}{\partial k_{t+1} \partial k_{t+1}} \frac{k_{t+1}}{\frac{\partial \varphi_t(x)}{\partial k_{t+1}}}, \eta_{32} = \frac{\partial^2 f(k_{t+1})}{\partial k_{t+1} \partial k_{t+1}} \frac{k_{t+1}}{f'(k_{t+1})},$$

$$\eta_{42} = \frac{\partial^2 \varphi_{t+1}}{\partial k_{t+1} \partial k_{t+1}} \frac{k_{t+1}}{\frac{\partial \varphi_{t+1}(x)}{\partial k_{t+1}}}, \eta_{52} = \frac{\partial^2 \varphi_t}{\partial k_{t+1} \partial k_{t+2}} \frac{k_{t+2}}{\frac{\partial \varphi_t(x)}{\partial k_{t+1}}}, \eta_{62} \frac{\partial^2 f(k_{t+1})}{\partial k_{t+1} \partial A_{t+1}} \frac{A_{t+1}}{f'(k_{t+1})}$$

Thus, for the previous equation (evaluated in the steady state implying $k_{t+1} = k_t = \bar{k}$), we have:

$$-\theta \hat{c}_t = \hat{\lambda}_t \tag{50}$$

$$\bar{k}\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \bar{y}\hat{y}_t - \bar{c}\hat{c}_t \tag{51}$$

$$\hat{\lambda}_t = \beta \hat{\lambda}_{t+1} \left[(1-\delta) + f'(\bar{k})\right] + \beta f'(\bar{k}) \left(1 + \eta_{32}\hat{k}_{t+1}\right) + \beta \lambda f'(\bar{k}) \left(1 + \eta_{62}\hat{A}_{t+1}\right) \tag{52}$$

$$\bar{y}\hat{y}_t = \bar{A}\bar{k}\hat{A}_t + \alpha \bar{A}\bar{k}^{\alpha}\hat{k}_t \tag{53}$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t, \varepsilon_t \backsim (0, \sigma_\varepsilon^2) \tag{54}$$

As seen, even for the recession regime, the linearization of the model in the boom regime leads us evidently to a set of linear equations with different coefficients. Then, in order to simulate the full model including the possibility of moving from one regime to the other, it would be necessary to model a transition probability matrix for all the equations in the system, which has the incovenience of being *ad hoc*. Therefore, it imposes transitions on the dynamics of all the equations, which is not modeled as the model's internal mechanisms of transmision. This makes it asymmetric by itself (Belaygoned & Michel, 2006; Davig & Leeper, 2005; Eo, 2009; and Pytelarczyk, 2005).

3.2 PEA algorithm

Now, PEA will be used in order to preserve the nonlinear features of the model.[10] With the goal of mapping the general form of PEA, the Euler equation and the capital transition equations are writen as in (18), (19) and (20), these conform the system as:

$$g(E_t \left[\Phi(z_{t+1}, z_t)\right], z_{t+1}, z_t, u_t) = 0$$

in this seting,

$$\Phi(z_{t+1}, z_t) = l_t^{-(1-\eta)(1-\theta)} E_t \left\{ \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta}\right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}}\right]\right\} \times \left\{1 + \frac{\partial C(\Delta k_t)}{\partial k_{t+1}}\right\}^{-1}$$

thus

$$c_t^{\eta(1-\theta)-1} = \beta \Phi(z_{t+1}, z_t)$$

$$z_t = (c_t, k_t, k_{t-1}, A_t)$$

$$z_{t+1} = (c_{t+1}, k_{t+1}, k_{t+2}, A_{t+1})$$

[10] convergence results, and algorithm basics are found in Marcet and Marshall (1994) and Marcet and Lorenzoni (1998).

$$x_t = (k_{t-1}, A_t)$$

The complete PEA algorith is as follows:

1. According to Marcet and Marshall (1994), it seems necessary to choose an adequate function $\Psi\left(\widetilde{\beta}, x_t\right)$ to approximate arbitrarily close to $\Phi(z_{t+1}, z_t)$. This will represent almost any function, except for a neural network. z_t is the vector of endogenous and exogenous variables as shown in the expectations function; x_t is a subset of variables used as regressor in the function Ψ; and $\widetilde{\beta}$ is a parameter vector in the approximation function Ψ.

2. Choose an initial $\widetilde{\beta}$,and for both initial values of state variables and a sequence of stochastic shocks compute

$$c_t = \left[\beta\Psi\left(\widetilde{\beta}, x_t\right)\right]^{1/(\eta(1-\theta)-1)} \tag{55}$$

β, η and θ are parameters of the utility function.

3. From step 2 we have series for c_t, and with k_t, and z_t; we are now to use Newton-Raphson (N-R) in order to appoximate l_t, from the equilibriun equation.

$$l_t = \frac{(1-\eta)c_t}{\eta(1-\alpha)z_t k_t^{\alpha}(1-l_t)^{-\alpha}} \tag{56}$$

4. From series obtained in steps 1 and 2 , obtain k_{t+1} from the motion equation of capital:

$$k_{t+1} - (1-\delta)k_t - y_t + c_t + \frac{\psi_2}{2}(k_{t+1} - k_t - \overline{in})^2 + \phi_t(\frac{\psi_1}{2}(k_{t+1} - k_t - \overline{in})^2 - \frac{\psi_2}{2}(k_{t+1} - k_t - \overline{in})^2) = 0 \tag{57}$$

then we have time series for c_{t+1}, $k_{t+1}, k_{t+2}, z_{t+1}$and l_t.

5. Define and compute:

$$c_t^{RE} = \left\{ \begin{array}{c} \beta l_t^{-(1-\eta)(1-\theta)} E_t \left\{ \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta}\right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}}\right] \right\} \\ \times \left\{1 + \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}}\right\}^{-1} \end{array} \right\}^{1/(\eta(1-\theta)-1)} \tag{58}$$

6. Regress $\frac{1}{\beta}(c_t^{RE})^{\eta(1-\theta)-1}$ on $\Psi\left(\widetilde{\beta}, x_t\right)$ and obtain new estimated values for $\widetilde{\beta}$.stop when you find a fixed point for $\widetilde{\beta}$ such that $\widetilde{\beta}_f = G(\widetilde{\beta}_f)$

being

$$G(\widetilde{\beta}) = \arg\min_{\zeta} \frac{1}{T}\sum_{t=0}^{T} \left\| \Phi(z_{t+1}\left(\widetilde{\beta}\right), z_t\left(\widetilde{\beta}\right)) - \Psi\left(\zeta, x_t\left(\widetilde{\beta}\right)\right) \right\|^2 \tag{59}$$

In order to capture nonlinearities and asymmetries from this model set up, it is needed a more flexible functional form:

$$\Psi\left(\widetilde{\beta}, x_t\right) = \exp\left(\Omega(\widetilde{\beta})\right) \tag{60}$$

$$\Omega(\widetilde{\beta}) = \widetilde{\beta}_1 + \widetilde{\beta}_2 \ln k_{t-1} + \widetilde{\beta}_3 \ln z_t \tag{61}$$

To stabilize the algorithm and to assist for convergence, steps 3 and 4 must be modified by imposing moving bands as suggested by Maliar and Maliar (2003).

Note that if we impose $b = 0$, or $\psi_1 = \psi_2$, we will obtain a standard DSGE model with symmetric adjustment costs. In this way, we can simulate both models for the the same time series of shocks and compare their time path as well as their higher order moments.Table 1 displays calibration parameters and steady state values in order to compare an asymmetric model with a symmetric one. Parameter cost for the symmetric model is calibrated as $\psi = 0.5\ (\psi_1 + \psi_2)$. Thus, the symmetric adjustment cost will be an intermediate case of low and high costs regime.

Figure 3: Asymmetric and symmetric models simulations

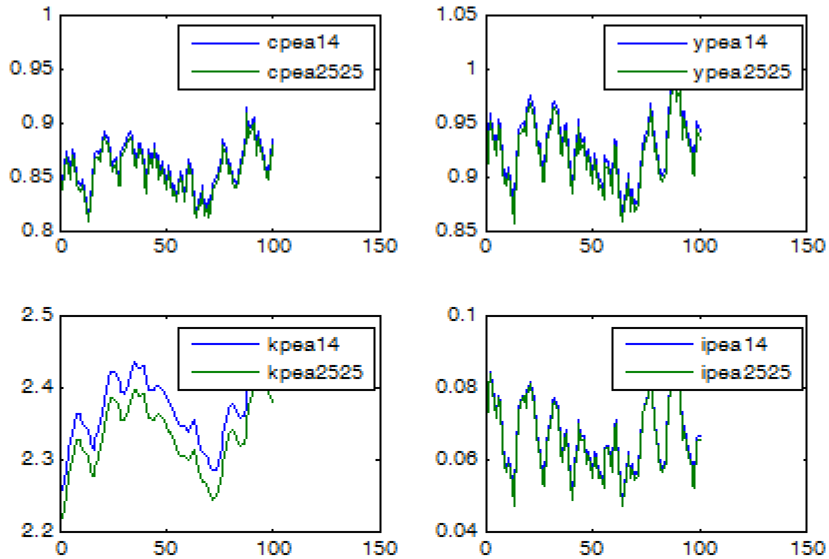


Figure 4: Asymmetric and symmetric models simulations

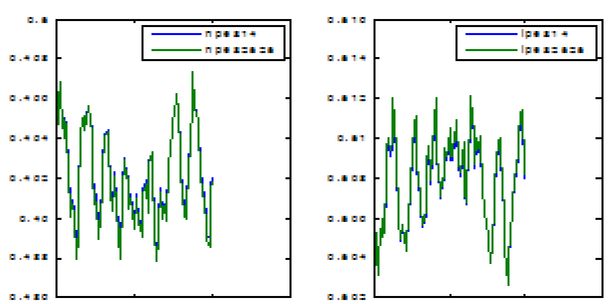


Table 1: Calibration

Parameter		Variable	
α	0.4	kss	2,329
ψ_1	4	yss	0,916
ψ_2	1	css	0,852
$\psi = 0.5(\psi_1 + \psi_2)$	2,5	$inss$	0
ρ	0.9	kss/yss	2.543
θ	2	rss	0.013
δ	0.0273	nss	0,492
σ_ε^2	0.018	$\beta = \frac{1}{1+rss}$	0.885
γ	500	$ibss$	0.0636

Table 2: Differences in moments of raw data from PEA for symmetric and asymmetric models simulations

	Difautocorr			Difsigrel		
variable	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	0.0005	0.0012	0.0024	-0.787	-0.1805	0.3864
'Yt'	-0.0004	0.0001	0.0009	0	0	0
'Kt'	-0.0005	-0.0001	0.0003	-0.8998	0.2043	2.3846
'Ibt'	-0.0031	-0.0018	-0.0009	-1.0718	0.3293	3.4855
'nt'	-0.0034	-0.0021	-0.0009	-1.6144	0.42	4.9376
'It'	-0.0034	-0.0021	-0.0009	-2.356	0.5032	5.22
Zr'	0	0	0	-2.3524	0.5942	5.9742
	DifsigrelHP			DifsigrelBK		
	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0125	-0.0119	-0.0112	-0.013	-0.0126	-0.0122
'Yt'	0	0	0	0	0	0
'Kt'	0.0067	0.0078	0.0087	0.006	0.0069	0.0076
'Ibt'	0.1368	0.158	0.1774	0.1423	0.164	0.1809
'nt'	0.0062	0.0065	0.0068	0.0066	0.0067	0.0069
'It'	0.0058	0.0062	0.0065	0.0062	0.0064	0.0066
Zr'	-0.0041	-0.0039	-0.0036	-0.0039	-0.0037	-0.0035

Note: for each variable, autocorrelation (for the raw data), relative standard deviations (for the raw data, HP filtered and BK filtered time series), are computed on the time series simulated by using PEA for the symmetric and the asymmetric versions of the model, then differendes were taken as follows: $\rho_x^{sim} - \rho_x^{asim}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{sim} - \left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{asim}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{sim} - \left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{asim}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{sim} - \left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{asim}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 3: Differences in Kurtosis and Skewness of HP filtered data from PEA for symmetric and asymmetric models simulations

	DifkurtHP			DifkurtHPneg			DifkurtHPpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0083	0.0011	0.0096	-0.0331	0.0029	0.0369	-0.0379	0.0019	0.0428
'Yt'	-0.0023	0.0005	0.0031	-0.0112	0.0011	0.0165	-0.0151	0.0009	0.0158
'Kt'	-0.0481	-0.0124	0.009	-0.1557	-0.0248	0.0508	-0.136	-0.0217	0.0407
'Ibt'	-0.2382	-0.0533	0.0127	-1.022	-0.1843	0.0485	-0.1751	-0.0267	0.0506
'nt'	-0.0223	-0.002	0.013	-0.069	-0.0019	0.0497	-0.0629	-0.0048	0.0409
'It'	-0.0214	-0.002	0.0136	-0.0627	-0.0035	0.0485	-0.0703	-0.003	0.0516
Zr'	0	0	0	0	0	0	0	0	0
	DifasimHP			DifasimHPneg			DifasimHPpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.2169	-0.0044	0.2121	-0.0111	-0.0009	0.0088	-0.0108	0.0007	0.0114
'Yt'	-0.2175	-0.0012	0.221	-0.0044	-0.0003	0.0039	-0.0045	0.0003	0.0046
'Kt'	-0.3106	-0.0086	0.2827	-0.0125	0.0068	0.0362	-0.0298	-0.0066	0.0123
'Ibt'	-0.7117	-0.3374	-0.0705	-0.0096	0.0297	0.1179	-0.036	-0.0091	0.0121
'nt'	-0.2282	0.0002	0.2665	-0.0145	0.0005	0.0183	-0.0181	-0.0011	0.0123
'It'	-0.2877	-0.0213	0.2044	-0.0128	0.0007	0.0162	-0.0187	-0.0009	0.0144
Zr'	-0.2085	0.0008	0.2183	0	0	0	0	0	0

Note: for each variable (simulated by using symmetric and asymmetric versions of the model), cyclical components were computed using HP filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{HP}^{sim} - (Kutosis)_{HP}^{asim}$, $(Skewness)_{HP}^{sim} - (Skewness)_{HP}^{asim}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

3.3 Perturbations algorithm

Although PEA algorithm and projections algorithm are generally time expensive, they are more precise as they are global approximation methods. However, it is possible to use a higher order [11] PM, which approximates the steady state and is less expensive than PEA algorithm. Through simulations carried out on Dynare, this latter method uses higher order derivatives of the dynamic system evaluated in the steady state.

3.4 COMPARING PEA AND PERTURBATIONS ALGORITHM

It is known that global approximation methods such as PEA are costly in terms of time and computation. However, local approximations such as log-linearisation and perturbations are less expensive. Notwithstanding, the issue of accuracy is yet a matter of concern. In this section, simulated time series with both algorithms are compared in order to asses accuracy and get an idea about how similar these algorithms are, making it possible to decide whether, without loss of accuracy, to use perturbations algorithm instead of a PEA algorithm. This experiment is performed by simulating pseudo-data for both methods in the following fashion: i) imposing symmetry in adjustment costs ($\psi_1 = \psi_2 = \overline{\psi} = 2.5$), and ii) imposing asymmetry ($\psi_1 = 4, \psi_2 = 1$). Other parameters remain the same as shown on table1.

3.4.1 Simulating the symmetric model

Table 3.4.1 hows the correlations of macro variables simulated by using PEA and PM (for the symmetric model). It can be seen that time series simulated by PEA move quite close to those simulated by PM.

[11] Log-linearisation is a first order Taylor approximation. Thus, higher-order approximation refers to second order, third order and so on.

Table 4: Differences in Kurtosis and Skewness of BK filtered data from PEA for symmetric and asymmetric models simulations

	DifcurktBK			DifkurtBKneg			DifkurtBKpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0064	0.0013	0.0078	-0.0275	0.0036	0.0353	-0.0346	0.0006	0.0331
'Yt'	-0.0017	0.0003	0.0024	-0.0104	0.0006	0.0135	-0.0126	0.0006	0.0184
'Kt'	-0.05	-0.0132	0.0093	-0.1721	-0.0247	0.0496	-0.1521	-0.0257	0.0445
'Ibt'	-0.2005	-0.0424	0.0199	-0.7958	-0.1522	0.0703	-0.1758	-0.0183	0.0607
'nt'	-0.0166	-0.0037	0.0042	-0.0591	-0.0063	0.0256	-0.0567	-0.0081	0.0212
'It'	-0.017	-0.0037	0.0046	-0.0584	-0.0073	0.0254	-0.0534	-0.0072	0.0241
Zr'	0	0	0	0	0	0	0	0	0
	DifasimBK			DifasimBKneg			DifasimBKpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0044	-0.0002	0.0042	-0.0091	-0.0009	0.0078	-0.0101	0.0002	0.0094
'Yt'	-0.0013	0	0.0013	-0.0039	-0.0002	0.0033	-0.0043	0.0002	0.005
'Kt'	-0.0114	-0.0008	0.0101	-0.0151	0.0068	0.0358	-0.0323	-0.0071	0.0117
'Ibt'	-0.0127	0.0016	0.0259	-0.0117	0.0259	0.1041	-0.0353	-0.0056	0.0146
'nt'	-0.0051	-0.0006	0.0037	-0.0076	0.0017	0.0117	-0.0136	-0.0022	0.0067
'It'	-0.0043	0	0.0046	-0.0066	0.0019	0.014	-0.0132	-0.002	0.0074
Zr'	0	0	0	0	0	0	0	0	0

Note: for each variable (simulated by using symmetric and asymmetric versions of the model), cyclical components were computed using BK filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{BK}^{sim} - (Kutosis)_{BK}^{asim}$, $(Skewness)_{BK}^{sim} - (Skewness)_{BK}^{asim}$. Sample periods were 500, replicated 500 times. Thus, estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 5:

	Correlations
	PEA/PM
'Ct'	0.9943
'Yt'	0.9953
'Kt'	0.9762
'Ibt'	0.9764
'nt'	0.9682
'It'	0.9682
Zr'	0.999

Note: each variable was simulated (in the symmetric model) by using both PEA and PM algorithms, then the correlations (of the raw data) are computed as follows: $corr(x_t^{PEA}, x_t^{PM})$. Sample periods were 500, replicated 500 times.

Table 6: Differences in moments of raw data from PEA and PM methods for the symmetric model simulations

	Difautocorr			Difsigrel		
variable	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0039	0	0.004	-1.8246	-0.225	0.1406
'Yt'	-0.0019	0.0025	0.0083	0	0	0
'Kt'	-0.0028	-0.0015	-0.0001	-0.9011	0.1371	1.5828
'Ibt'	0.0182	0.0339	0.0531	-1.0656	0.1881	3.7804
'nt'	0.0839	0.0527	0.0758	-1.169	0.2183	3.0755
'lt'	0.0839	0.0527	0.0758	-1.4631	0.2813	3.7006
Zr'	-0.0021	0	0.0027	-1.5636	0.3724	3.563
	Difsigrel HP			Difsigrel BK		
	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0875	-0.0298	-0.022	-0.0449	-0.039	-0.033
'Yt'	0	0	0	0	0	0
'Kt'	0.0408	0.0469	0.0535	0.0331	0.0387	0.0444
'Ibt'	0.2834	0.4052	0.5398	0.3993	0.5084	0.6308
'nt'	-0.0065	-0.0028	0.0009	-0.0028	0.0005	0.0035
'lt'	-0.0063	-0.0026	0.0009	-0.0025	0.0005	0.0034
Zr'	0.0069	0.0097	0.0122	0.0088	0.0108	0.0128

Note: for each variable in the symmetric model, autocorrelation (for the raw data), relative standard deviations (for the raw data, HP filtered and BK filtered time series), are computed on the time series simulated by using PEA and Perturbations Method, then differendes were taken as follows: $\rho_x^{PEA} - \rho_x^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: PEA simulations are very close to PM simulations.

displays differences in autocorrelations, relative variances (compared to σ_x/σ_{GDP}) computed on the raw data in the upper panels, and differences in relative variances computed on both HP and BK filtered data. Lower and upper bounds for a 95% confidence interval are also reported. In general, gross investment, labor, and leisure seem to have more persistence in the PEA than in the PM algorithm, whereas consumption and capital present lower persistence. Relative standard deviations seem to be quite similar for the two algorithms. For the case of relative standard deviations, they seem to be very similar because the mean value of their differences lies inside the confidence interval. The results for the differences of relative standard deviations are mixed: While both HP and BK filtered data of labor and leisure seem to have the same relative standard deviation, consumption seems to decrease and be higher for capital and investment. Tables7 and 3.4.1 show the differences in kurtosis and skewness of PEA data and PM data for HP and BK filtered data respectively. In these tables, an unabiguous result is evident: mean of differences in kurtosis and asymmetries are containend within a 95% confidence interval. Under the light of these results, it is possible to think of the Perturbations algorithm as one very close to PEA.

3.4.2 Simulating the asymmetric model:

Tables 9-3.4.2 show the same statistics as tables 3.4.1-3.4.1, but computed on the simulations of the asymmetric model ($\psi_1 = 4, \psi_2 = 1$). Several simulations were made for different values of γ (500, 100, 50, and 25),

Table 7: Differences in moments of HP filtered data from PEA and PM methods for the symmetric model simulations

	DifkurtHP			DifkurtHPneg			DifkurtHPpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0737	0.0021	0.0679	-0.2185	0.0247	0.2708	-0.2901	-0.0171	0.2035
'Yt'	-0.0236	-0.0011	0.0262	-0.1066	-0.0048	0.0855	-0.0892	-0.0012	0.1033
'Kt'	-0.1262	-0.0114	0.1178	-0.4507	-0.0439	0.3859	-0.4954	-0.0229	0.461
'Ibt'	-0.3172	0.0129	0.5085	-1.3138	0.0401	1.9176	-0.6787	-0.0415	0.6984
'nt'	-0.2352	-0.0206	0.2484	-0.9184	-0.1125	0.8092	-0.7168	0.0084	0.8834
'It'	-0.2277	-0.0162	0.2463	-0.7171	0.013	0.935	-0.895	-0.1051	0.7757
Zt'	-0.0028	0	0.0025	-0.0073	0	0.0084	-0.0077	0	0.0081
	DifasimHP			DifasimHPneg			DifasimHPpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.2094	-0.0041	0.2436	-0.0736	-0.0078	0.0552	-0.0684	-0.0054	0.0607
'Yt'	-0.2054	-0.001	0.2412	-0.0218	0.0011	0.0251	-0.024	0	0.0257
'Kt'	-0.2843	-0.0032	0.2766	-0.082	0.0148	0.11	-0.1203	-0.0025	0.1062
'Ibt'	-0.5988	-0.2793	0.0226	-0.2935	-0.0045	0.2472	-0.176	-0.0149	0.1611
'nt'	-0.214	0.0025	0.2627	-0.161	0.035	0.2265	-0.1854	0.0025	0.2122
'It'	-0.2788	-0.0211	0.1995	-0.2113	-0.0039	0.1846	-0.2256	-0.0334	0.1609
Zt'	-0.2001	0.0007	0.2352	-0.0027	0	0.0028	-0.0026	-0.0001	0.0025

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using HP filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{HP}^{PEA} - (Kutosis)_{HP}^{PM}$, $(Skewness)_{HP}^{PEA} - (Skewness)_{HP}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 8: Differences in moments of BK filtered data from PEA and PM methods for the symmetric model simulations

	DifcurktBK			DifkurtBKneg			DifkurtBKpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0506	-0.0004	0.0474	-0.1811	0.0242	0.2005	-0.2164	-0.0239	0.1631
'Yt'	-0.016	-0.0005	0.0176	-0.0803	-0.0049	0.0659	-0.0769	-0.0005	0.0821
'Kt'	-0.1404	-0.0078	0.1366	-0.5528	-0.0831	0.371	-0.3734	0.0365	0.5468
'Ibt'	-0.2135	0.0421	0.4355	-0.959	0.138	1.6097	-0.6609	-0.0706	0.4636
'nt'	-0.1429	-0.0116	0.1476	-0.5909	-0.0781	0.4252	-0.469	0.0069	0.5058
'It'	-0.1311	-0.0075	0.1474	-0.4571	0.017	0.5497	-0.5678	-0.0761	0.4148
Zr'	-0.004	-0.0001	0.002	-0.0128	-0.0001	0.0068	-0.0106	-0.0004	0.0085
	DifasimBK			DifasimBKneg			DifasimBKpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0329	-0.0099	0.0119	-0.0537	-0.0073	0.0421	-0.0584	-0.0072	0.0397
'Yt'	-0.0086	0.0017	0.012	-0.0194	0.0012	0.0207	-0.0208	0.0008	0.021
'Kt'	-0.0368	0.0332	0.1052	-0.0917	0.0249	0.1408	-0.107	0.0124	0.1441
'Ibt'	-0.1191	-0.0385	0.0175	-0.261	-0.029	0.175	-0.1387	-0.0198	0.1132
'nt'	-0.0206	0.0256	0.0708	-0.1224	0.023	0.1512	-0.1168	0.0064	0.1398
'It'	-0.0759	-0.0281	0.0182	-0.1404	-0.0093	0.1112	-0.1497	-0.0228	0.1185
Zr'	-0.0016	0	0.0016	-0.0061	0	0.0062	-0.0036	-0.0002	0.0029

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using BK filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{BK}^{PEA} - (Kutosis)_{BK}^{PM}, (Skewness)_{BK}^{PEA} - (Skewness)_{BK}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 9:

	Correlations PEA/PM
'Ct'	0.9941
'Yt'	0.9952
'Kt'	0.9754
'Ibt'	0.9726
'nt'	0.9648
'It'	0.9648
Zr'	0.999

Note: each variable was simulated (in the symmetric model) by using both PEA and PM algorithms, then the correlations (of the raw data) are computed as
follows: $corr(x_t^{PEA}, x_t^{PM})$. Sample periods were 500, replicated 500 times.

because for $\gamma \rightarrow \infty$ the smooth transition function becomes a step function; its derivative tends to infinite as well, and the model will lose its differentiability which is the corner stone of the PM algorithm. However, 100, 50 or 25 are still high values, thus the results reported in tables 9-3.4.2 are those for the simulations using $\gamma = 25$. In general, the means of the differences between moments of PEA and PM are contained within the 95% confidence interval as well as for the case of the results in the symmetric model, which means that PEA and PM are very close to each other.

3.5 Deterministic simulation

Moreover, in order to test the the model's construction consistency, deterministic simulations were performed imposing a deviation (negative and positive) of the technology process when simulating a one time shock. Instead of solving it by employing any approximation algorithm, Dynare's exact solver was used by imposing $a = 1.06$ and $a = 0.94$, which is equivalent to having $e = 0.058268908$ and $e = -0.061875404$ respectively.[12] Figures5 and 6 show a path time of key macro variables $c_t, y_t, k_t, in_t, n_t, ib_t, a_t, \phi_t$ (consumption, income, capital, net investment, labor, gross investment, technology, and transition function).

Figures 5 and 6 show re-scaled variables [13]. Given the calibration, several interesting behaviors were observed. For instance, the reaction of consumption towards a negative perturbation is stronger than when a positive shock occurs. This can be explained by the fact that disinvestment costs are higher than investment costs. It is important to note that the investment reaction during recession is lower that that during a boom.

[12] It would be also possible to impose a symmetric e (this is, the same size of the shock in absolute value) but there would not be a great difference in the results.

[13] Rescalation is necesary for comparison of the variables in a single plane. For simulated variables with a negative shock the computation is $abs(x_t) - max(x_t)$ ad for variables with a positive shock $x_t - \min(x_t)$.

Table 10: Differences in moments of the raw data from PEA and PM methods for the asymmetric model simulations

	Difautocorr			Difsigrel		
variable	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0043	-0.0003	0.0036	-2.0118	-0.2431	0.0924
'Yt'	-0.0023	0.0024	0.0079	0	0	0
'Kt'	-0.0028	-0.0014	0	-0.8857	0.1131	1.4397
'Ibt'	0.0196	0.0351	0.0537	-1.1679	0.1482	2.0336
'nt'	0.036	0.0543	0.076	-0.9365	0.2625	1.9434
'It'	0.036	0.0543	0.076	-1.1301	0.3046	2.8407
Zr'	-0.0021	0	0.0027	-1.207	0.3755	2.5604
	Difsigrel HP			Difsigrel BK		
	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0368	-0.0285	-0.0195	-0.0451	-0.0376	-0.0299
'Yt'	0	0	0	0	0	0
'Kt'	0.0379	0.0454	0.0539	0.0311	0.0372	0.0437
'Ibt'	2.1646	2.3645	2.5613	2.274	2.4673	2.6492
'nt'	-0.0089	-0.0045	-0.0001	-0.0056	-0.0013	0.0026
'It'	-0.009	-0.0047	-0.0005	-0.0058	-0.0016	0.0021
Zr'	0.0073	0.0104	0.0134	0.0089	0.0115	0.0139

Note: for each variable in the symmetric model, autocorrelation (for the raw data), relative standard deviations (for the raw data, HP filtered and BK filtered time series), are computed on the time series simulated by using PEA and Perturbations Method, then differendes were taken as follows: $\rho_x^{PEA} - \rho_x^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: PEA simulations are very close to PM simulations.

Table 11: Differences in moments of the HP filtered data from PEA and PM methods for the asymmetric model simulations

	DifkurtHP			DifkurtHPneg			DifkurtHPpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0783	0.0106	0.0986	-0.357	-0.0531	0.2102	-0.1892	0.1067	0.4452
'Yt'	-0.0423	-0.007	0.0236	-0.0897	0.0177	0.1484	-0.194	-0.0465	0.0648
'Kt'	-0.5809	-0.1519	0.1502	-1.8606	-0.2768	0.6609	-1.6199	-0.2642	0.5352
'Ibt'	-0.4188	0.2669	1.6097	-0.3533	1.4583	5.991	-2.6023	-0.6715	0.7754
'nt'	-0.8713	-0.2699	0.164	-0.8629	0.1674	1.2799	-3.1599	-0.9723	0.284
'It'	-0.9264	-0.3085	0.1206	-3.3604	-1.0421	0.2741	-0.9537	0.1363	1.221
'Zt'	-0.0027	0	0.0024	-0.0073	0	0.0077	-0.008	-0.0002	0.0081
	DifasimHP			DifasimHPneg			DifasimHPpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.2546	-0.0414	0.1885	-0.0544	0.0166	0.0885	-0.0429	0.031	0.1016
'Yt'	-0.1875	0.0149	0.2553	-0.0341	-0.0061	0.0205	-0.0424	-0.0134	0.015
'Kt'	-0.3123	-0.0098	0.2768	-0.1345	0.0743	0.3698	-0.2999	-0.0674	0.1161
'Ibt'	-0.0782	0.1349	0.3726	-1.0093	-0.3452	0.0448	-0.5171	-0.2029	0.122
'nt'	0.0168	0.2444	0.5063	-0.3063	-0.0579	0.2113	-0.5941	-0.248	0.0458
'It'	-0.5415	-0.2681	-0.035	-0.0263	0.2604	0.6197	-0.2156	0.0498	0.296
'Zt'	-0.2002	0.0011	0.2338	-0.0026	0	0.0027	-0.0026	-0.0001	0.0025

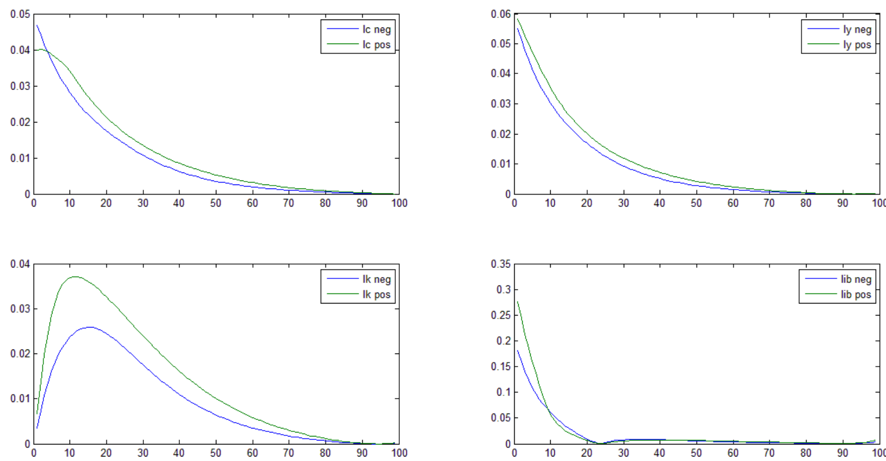
Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using HP filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{HP}^{PEA} - (Kutosis)_{HP}^{PM}$, $(Skewness)_{HP}^{PEA} - (Skewness)_{HP}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 12: Differences in moments of the BK filtered data from PEA and PM methods for the asymmetric model simulations

	DifcurktBK			DifkurtBKneg			DifkurtBKpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0756	0.0028	0.0751	-0.3304	-0.0829	0.1987	-0.168	0.0553	0.3464
'Yt'	-0.035	-0.0043	0.0246	-0.036	0.0151	0.1508	-0.1455	-0.0345	0.0471
'Kt'	-0.5868	-0.1501	0.1114	-2.1725	-0.3584	0.6827	-1.5992	-0.2695	0.351
'Ibt'	-0.4611	0.2389	1.4123	-0.7601	1.1683	5.0841	-2.4337	-0.47	1.065
'nt'	-0.7568	-0.2021	0.1379	-1.0124	0.0641	1.2095	-2.4905	-0.7405	0.189
'It'	-0.8197	-0.2317	0.109	-2.6465	-0.7921	0.1782	-1.1107	0.0253	1.0716
'Zt'	-0.003	-0.0001	0.002	-0.0124	-0.0002	0.0088	-0.0081	-0.0008	0.0085
	DifasimBK			DifasimBKneg			DifasimBKpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0012	0.0255	0.0543	-0.0465	0.011	0.0745	-0.0405	0.017	0.0788
'Yt'	-0.0237	-0.0114	-0.0001	-0.0323	-0.0051	0.0193	-0.0344	-0.0098	0.0121
'Kt'	-0.0725	0.0267	0.1188	-0.1342	0.0958	0.3973	-0.2817	-0.0675	0.0969
'Ibt'	-0.5756	-0.3421	-0.1955	-0.9432	-0.2919	0.097	-0.5102	-0.1423	0.1939
'nt'	-0.2996	-0.1749	-0.0867	-0.2617	-0.0279	0.2059	-0.4831	-0.1922	0.0365
'It'	0.0843	0.1748	0.3002	-0.0272	0.2023	0.5119	-0.2175	0.0173	0.2493
'Zt'	-0.0016	0	0.0016	-0.0029	0	0.0031	-0.0033	-0.0001	0.0029

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using BK filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)^{PEA}_{BK} - (Kutosis)^{PM}_{BK}$, $(Skewness)^{PEA}_{BK} - (Skewness)^{PM}_{BK}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Figure 5: Re-scaled variables



Consistently, income reaction during a recession is lower than during booms. Not only can this be explained by the investment decrease, but also by the labor decrease. The size of the adjustment in labor during recession is higher than during boom due to the fact that the decrease in wage during recession is not as big as the increase during boom. Thus, the models reproduces labor as well as wages increases during booms, and labor decreases and smaller wage reductions, which is all a signal of real rigidities in wages. Thus, most of the adjustment in this economy is led by consumption and labor. Obviously, in expansion periods, capital increases are bigger than capital decreases during recessions. It can also be seen that expansions are longer than recessions. This is also seemingly true for income and consumption. Figures 7 and 8 show time path deviations from the steady state.

3.6 Impulse response

Impulse Response (IR) is one of the most used analysis tools in macroeconometrics. However, it must be used carefully. Because the DSGE model studied in this paper is non-linear and asymmetric, IR analysis should not be performed as usual, assuming that the DGP is linear-multivariate. Moreover, it could be mistaken to simply shock technology once and then follow the whole system's adjustment. Therefore, in order to gauge asymmetric effects of shocks in this hypothetical economy, General Impulse Response Function (Koop et al., 1996) (GIRF hereafter) is to be adopted.[14].

Because of asymmetric DGP of this DSGE model, multivariate data simulated by using this very model lacks the following properties: symmetry property, linearity property, and history-independence property. Thus, linear impulse response functions (VAR-based) are not appropriate tools for analyzing the dynamics of such DSGE model. The GIRF, as defined by Koop et al. (1996), is conditioned by shocks and/or history:

$$GI_Y(n, v_t, \omega_{t-1}) = E[Y_{t+n}|v_t, \omega_{t-1}] - E[Y_{t+n}|\omega_{t-1}], \quad for \quad n = 0, 1...$$

Wherein Y_t is a vector of variables, v_t, a current shock ω_{t-1} is the history, and n is the forecast horizon. Koop et al. (1996) also describe a simple algorithm to compute these conditional expectations by means of Monte Carlo integration. According to this method, GIRF could be considered as a distribution of impulse responses for each period in the forecast horizon. Impulse responses computed in this way are calculated and reported by Dynare. By default, Dynare throws the first 100 observations and reports GIRF for a horizon

[14]Local Projections Impulse Response (Jordá, 2005) could also be used, but this technique is susceptible of symmetry, thus it would be not possible to detect asymmetry in data of this hipothetical model.

Figure 6: Re-scaled variables

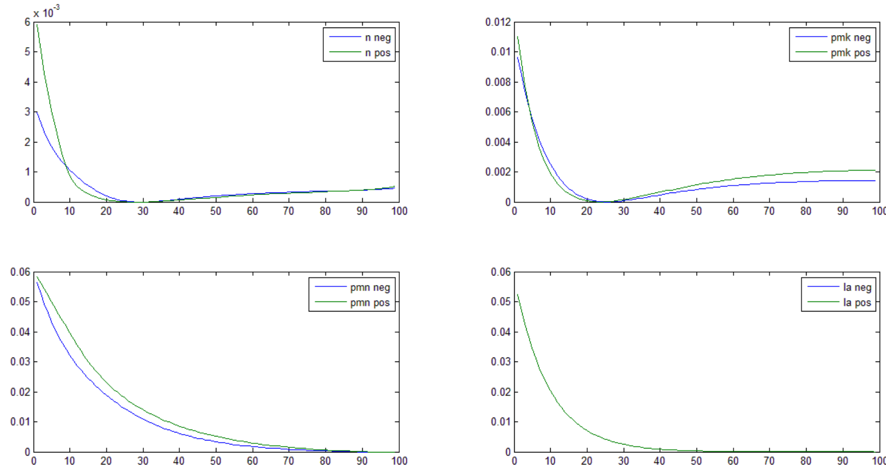


Figure 7: Deviations from the steady state

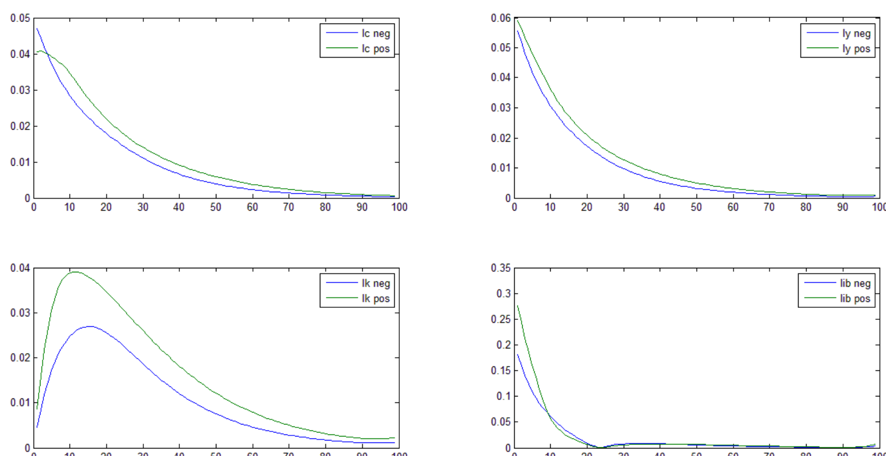
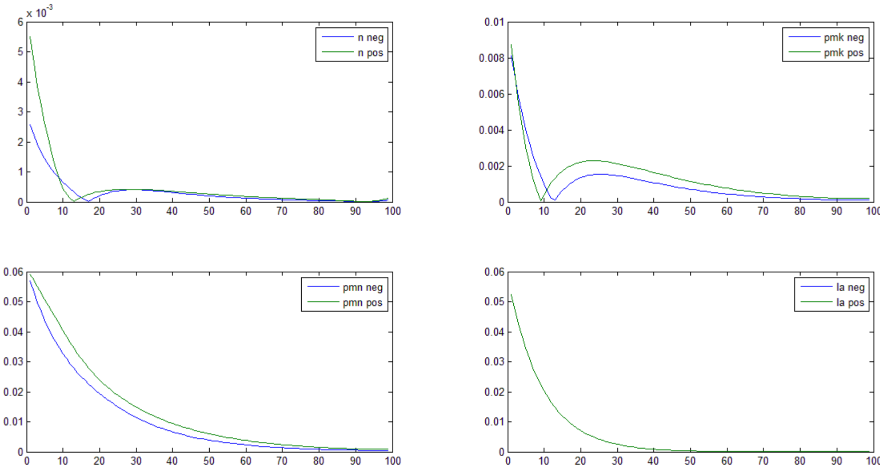


Figure 8: Deviations from the steady state



of 40 periods ahead.Figures 9 and 10 show impulse response (50 draws) for one standard deviation shock (positive and negative) on the perturbation term of the technology process; all variables except for labor and marginal products are considered as logarithms.

3.7 Conditioning on a particular shock

The first simulation exercise consisted of giving one standard deviation shock (positive and negative) to the technology process in the asymmetric investment cost model. The parameterization for this version of the model is the same as in table 1 fixing $\psi_1 = 4, \psi_2 = 1$ and $\gamma = 100$. The simulation was performed once (one replication); the response of macroeconomic variables in this hypothetical economy to negative shocks (in average) are asymmetric with respect to positive shocks (graphs 11 and 12 show the absolute values of responses of variables to negative shocks (blue) and to possible time shocks (green)). Replications of this excercise consisted in simulating 500 time series for the history of the model; this isω_{t-1} simulated 500 times. The economy was given the same standard deviation shock. Thus, the GIRF was computed as $GI_Y(n, v_t, \Omega_{t-1}) = E[Y_{t+n}|v_t, \Omega_{t-1}] - E[Y_{t+n}|\Omega_{t-1}]$ being Ω_{t-1} an information set of the previous history, and v_t a particular negative and positive standard deviation shock. Figures11 and 12 show these IR functions. The time paths for these impulse responses look softer, but this fact in no way affects the nature of the results. Figure 13 shows a Relative Intensity Indicator (RII), which means the ratio of impulse responses as shown in Figure 9; this is: impulse-response after positive shock divided by impulse-response after negative shock for each variable. If this indicator is greater than -1 and smaller than 0, negative shock is greater than the positive one; and the opposite occurs if the indicator is smaller than -1. On shock, negative impact on consumption, income, and labor are more intense than the positive impact, which is, however, more long-lasting than the negative, at least for consumption and income. For labor, negative effect is more intense and long-lasting. On the other hand, for capital positive shock, it is always more intense and long-lasting. This means that at short-term the adjustment is spread all over the variables, whereas at medium-term the adjustment is shared only between capital and labor.

3.8 Conditioning on a particular history

Due to the fact that asymmetric models are history-dependent, it is necessary to ask ourselves the question on what the time path of the economy would be when in a boom that is positively or negatively shocked, or when in a recession that is positively or negatively shocked. The results of simulating a positive shock as the

Figure 9: Impulse Response Function (50 replicas)

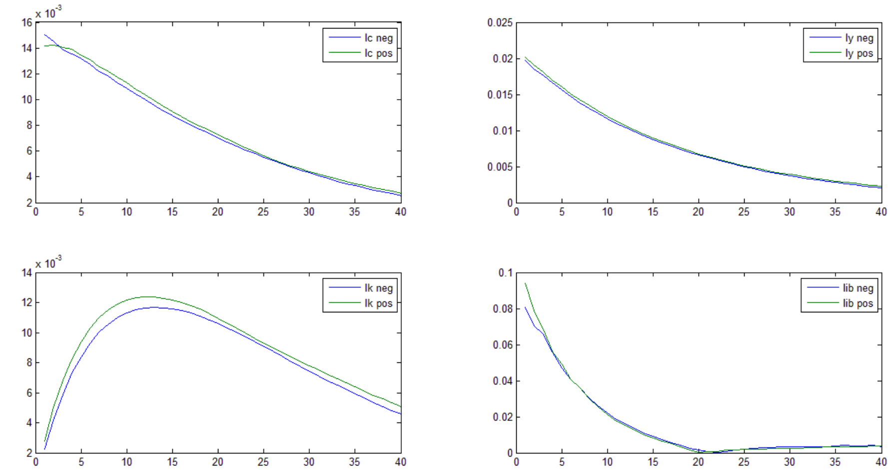


Figure 10: Impulse Response Function (50 replicas)

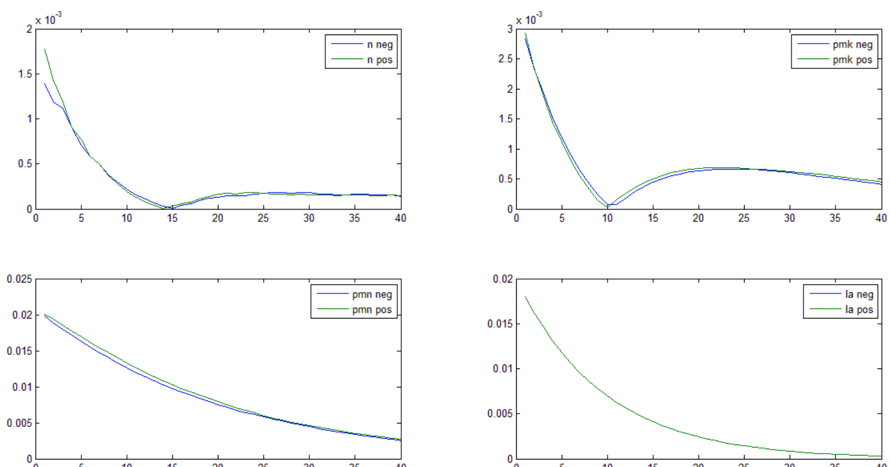


Figure 11: General Impulse Response Function (500 replicas)

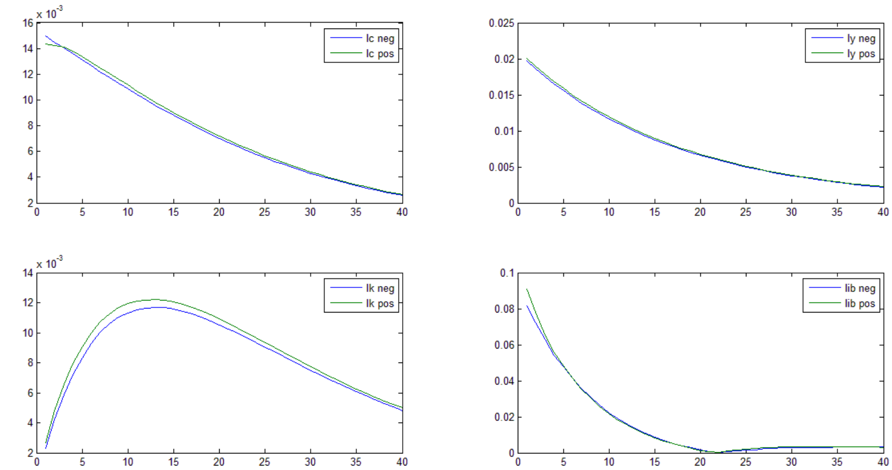


Figure 12: General Impulse Response Function (500 replicas)

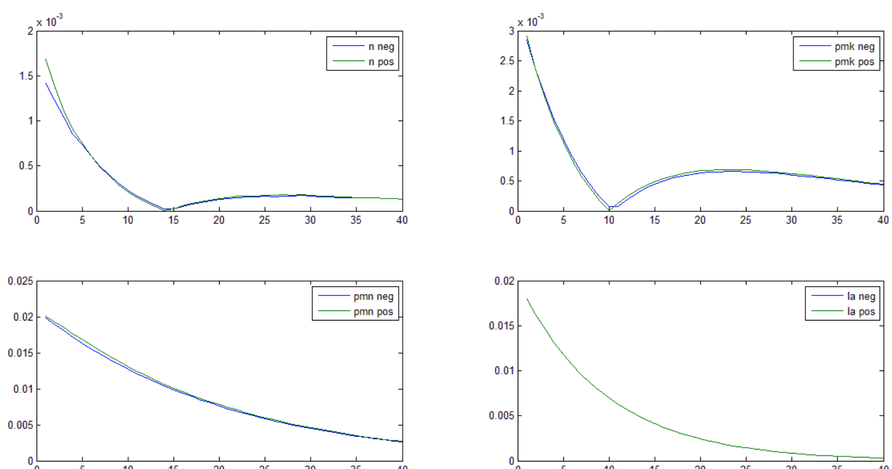
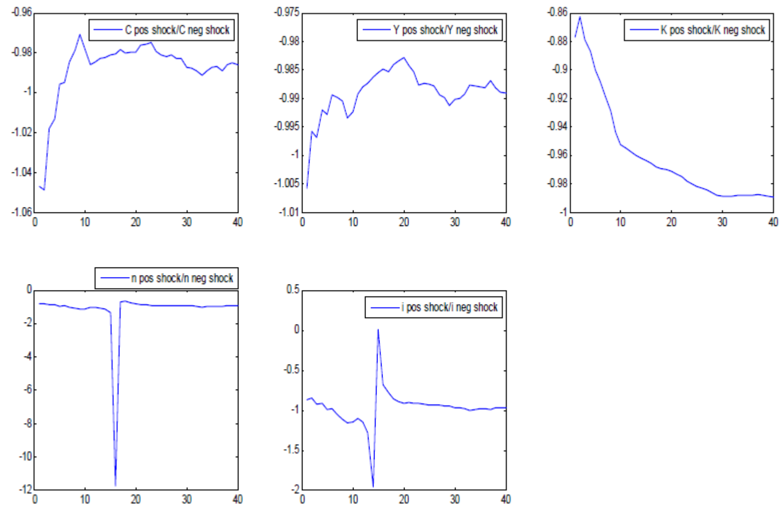


Figure 13: Relative Intensity Indicator



economy undergoes a boom or simulating a positive shock as the economy undergoes a recession are trivial: a recession deepening and boom sharpening. However, because of business cycles asymmetry, it would be necessary to perform the simulation in order to know the quantitative effects. In fact, it would be interesting to know the quantitative effects of a negative shock during a boom and a positive shock during a recession. To perform the exercise here proposed, it must be supposed that the economy is initially shocked (positively or negatively) in period one, and in period four it will receive a shock in the opposite direction to the one received in period one. Thus, the exercise deals with computing $GI_Y(n, v_t, \tilde{\Omega}_{t-1}) = E[Y_{t+n}|v_t, \tilde{\Omega}_{t-1}] - E[Y_{t+n}|\tilde{\Omega}_{t-1}]$ being $\tilde{\Omega}_{t-1}$ the state of the economy (either in boom or in recession) and v_t a positive or negative shock.

There is another important detail to consider: this exercise is time-dependent. This implies that the new position of the economy after the second shock would depend directly on how far it is from the steady state. That is to say, the longer the horizon of GIRF, the closer the economy will be to the steady state. Therefore, depending on the size of the shock (and on the economy's asymmetric structure) the economy could jump (suddenly perhaps) from a boom onto a recession, and vice versa. In order to standardize the timing problem, the exercise was performed as follows: the second (positive or negative) shock was introduced in a time $t0$ so that the technology gap were half of its initial value on shock. In this section, all variables have been measured in logarithms. In such a way, gaps between variables can be interpreted as log-deviations from the steady state.

3.8.1 A second shock in the opposite direction of the first shock

Figure 14 shows the GIRF of the economy after receiving a postive shock during a recession and a negative shock during a boom. In this exercise, it was very clear that a shock in the opposite direction pushes the economy to the next phase of the cycle, making it fall from a boom to a recession or making it jump from a recession to a boom. For the case of capital, it slowly reverses, nonetheless, the accumulation (deaccumulation) process induced by a positive (negative shock).

Because the model is asymmetric, the intensity of the fall will be different from the intensity of the jump. Then, it is necessary to compare the time paths after the second shock. Figure 15 shows the path of the economy after the second shock. Nevertheless, it is not conclusive about the asymmetries and the intensity of the shock. Figure 16 shows absolute deviation values from the steady state after the second shock. Also, figure 17 shows the intensity indicator (absolute values). This reveals that, on shock, the negative shock effect

Figure 14: GIRF for the first and the second shock

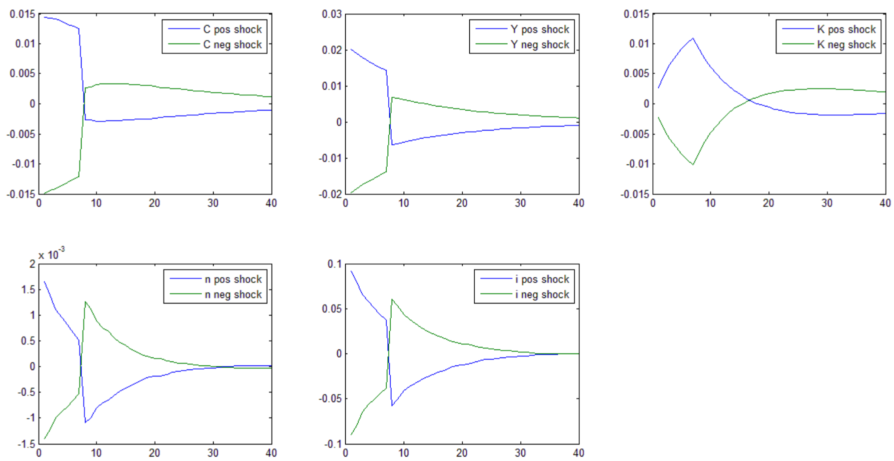


Table 13: Variation of the gap from the steady state after shocks

	Consumption		GDP		Capital		Investment		Labour		Leisure	
Period	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos
0	**1.443%**	**-1.498%**	**2.001%**	**-1.973%**	**0.262%**	**-0.224%**	**9.073%**	**-8.845%**	**0.338%**	**-0.285%**	**-0.330%**	**0.275%**
1	-0.012%	0.045%	-0.111%	0.116%	0.214%	-0.190%	-1.225%	1.134%	-0.060%	0.042%	0.059%	-0.041%
2	-0.034%	0.059%	-0.100%	0.102%	0.181%	-0.167%	-0.936%	0.786%	-0.040%	0.027%	0.039%	-0.025%
3	-0.024%	0.042%	-0.101%	0.101%	0.145%	-0.138%	-1.037%	0.996%	-0.046%	0.036%	0.046%	-0.035%
4	-0.032%	0.043%	-0.097%	0.095%	0.114%	-0.113%	-0.880%	0.806%	-0.039%	0.031%	0.038%	-0.030%
5	-0.043%	0.047%	-0.090%	0.088%	0.091%	-0.093%	-0.723%	0.747%	-0.029%	0.025%	0.028%	-0.024%
6	-0.042%	0.044%	-0.085%	0.084%	0.071%	-0.073%	-0.635%	0.632%	-0.026%	0.024%	0.026%	-0.023%
7	**-1.539%**	**1.505%**	**-2.055%**	**2.071%**	**-0.174%**	**0.194%**	**-9.299%**	**9.556%**	**-0.311%**	**0.341%**	**0.301%**	**-0.332%**
8	-0.003%	0.021%	0.041%	-0.038%	-0.152%	0.165%	0.641%	-0.724%	0.026%	-0.036%	-0.026%	0.035%
9	-0.006%	0.022%	0.041%	-0.041%	-0.129%	0.135%	0.724%	-0.792%	0.028%	-0.038%	-0.028%	0.038%
10	0.004%	0.004%	0.036%	-0.037%	-0.113%	0.115%	0.480%	-0.533%	0.020%	-0.025%	-0.019%	0.024%

during a boom is more intense than the one for consumption and income. Differently, the opposite takes place for labor investment and capital; besides, for medium-term effects of positive shock during recession, it seems to be more long-lasting.

Table 13 shows the variation of the gap after each shock. Gap variations when the economy is disturbed by a negative (positive) shock during a boom (recession), in absolute values, are greater only for consumption, whereas they are smaller for other variables; i.e., the pos/neg column is greater than the neg/pos column for consumption in period seven, while the opposite occurs for other variables. This takes place as investment decreases are more expensive than investment increases. As a consequence, consumption will suffer the major part of the adjustment on a negative shock. Hitherto, it could be concluded that during a recession the effect of a positive shock on the economy is more intense than the effect of a negative shock during a boom and this probably occurs because booms are more long-lasting than recessions.

Figure 15: GIRF for the second shock

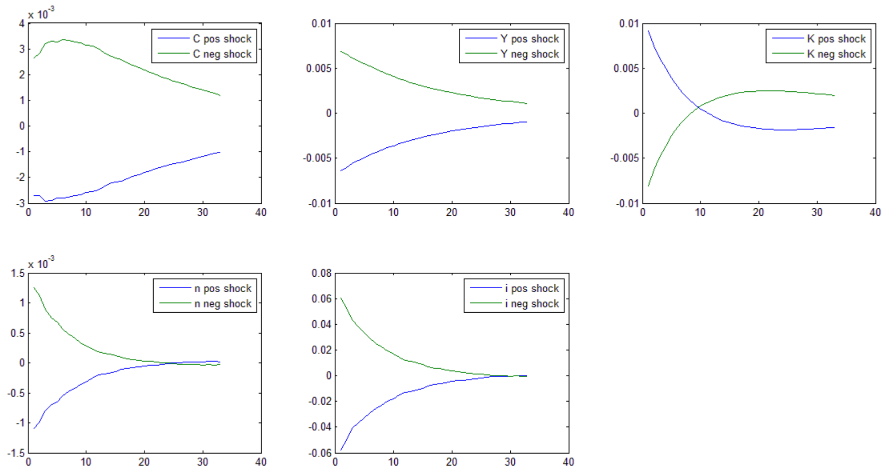


Figure 16: GIRF for the second shock (absolute values)

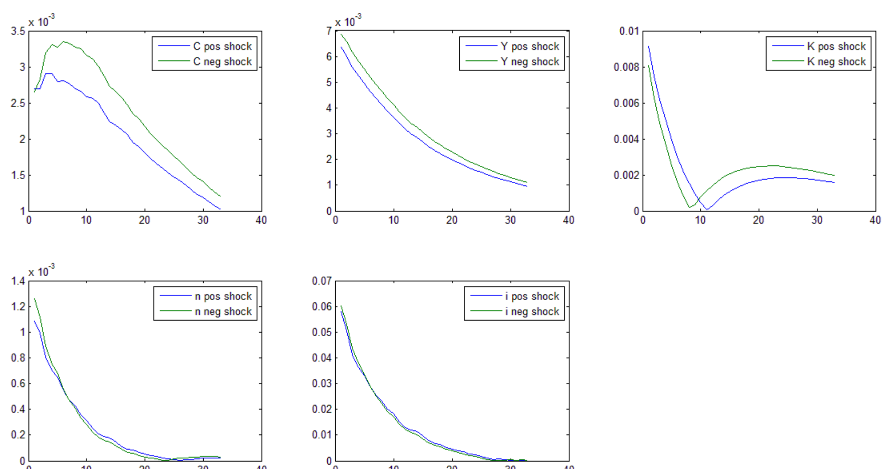
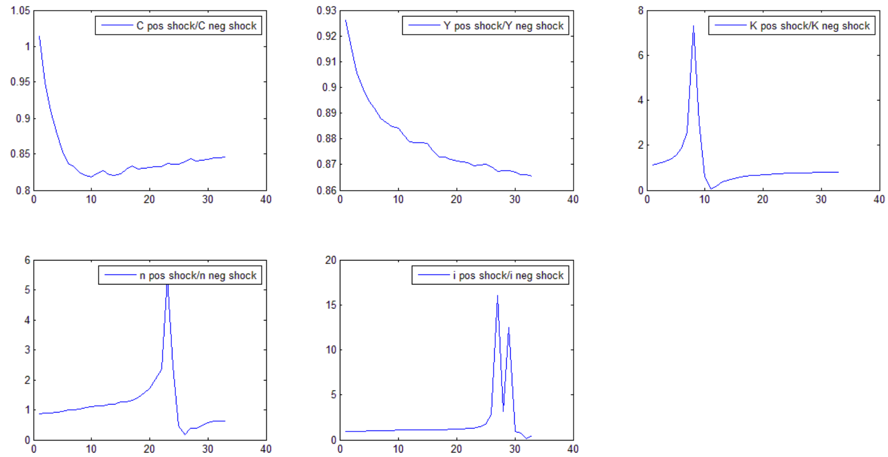


Figure 17: Relative Intensity Indicator for the second shock (absolute values)



3.8.2 A second shock in the same direction of the first shock

We might also wonder about the effect of a positive shock during a boom or about the effect of a negative shock during recession. To answer these questions, we have performed an excercise similar to the previous one. But, instead of giving a negative shock after a positive one, we give both a first and a second positive shocks. A first negative shock and a second negative shock are also simulated. Figures 18 to 20 show that when the economy is disturbed by a second positive shock, the boom regime protracts and the recession regime exacerbates.

This qualitative effect is expected, but what really concerns us here is its magnitude. Table 14 shows the size of the increase (decrease) of the gaps after the second positive (negative) shock. When the economy is in a boom and receives a positive perturbation (columns pos/pos), the variation value of the consumption gap in period 7 (in absolute values) is smaller than that when the economy is in a recession and receives a negative perturbation (columns neg/neg). For the other variables, the completely opposite case takes place. One more time, the explanation for this behavior is that the booms are more long-lasting than recessions. Also, because decreasing investment is more expensive than increasing it, the most part of the adjustment on shock relies on consumption.

4 Preliminary Conclusions

The DSGE model proposed here with asymmetric investment costs is able to generate asymmetric business cycles.

In general, recessions seem to be deeper (for consumption) than expansions and expansions seem to be more long-lasting than recessions. Thus, deepness and sharpness would be captured by this model's dynamics.

The adjustment intensity suffered by consumption and labor, with a smaller reaction in wages during recession and a greater increase in wage during booms, is an indicator that there is real rigidity on wages.

Asymmetries in RBC models could be more adequately captured by General Impulse Response Functions than by higher order moments. However, a more rigurous test for the properties of the asymmetric model proposed here (and in two papers proceeding from my doctoral thesis) could include the application of nonlinear econometric tools that could serve indeed a powerful tool for this purpose.

For the agenda: i) estimate parameters of the model for the real economy; ii) test for asymmetries in the time series and simulate with these nonlinear econometric models.

Figure 18: GIRF for the first and the second shocks (absoloute values)

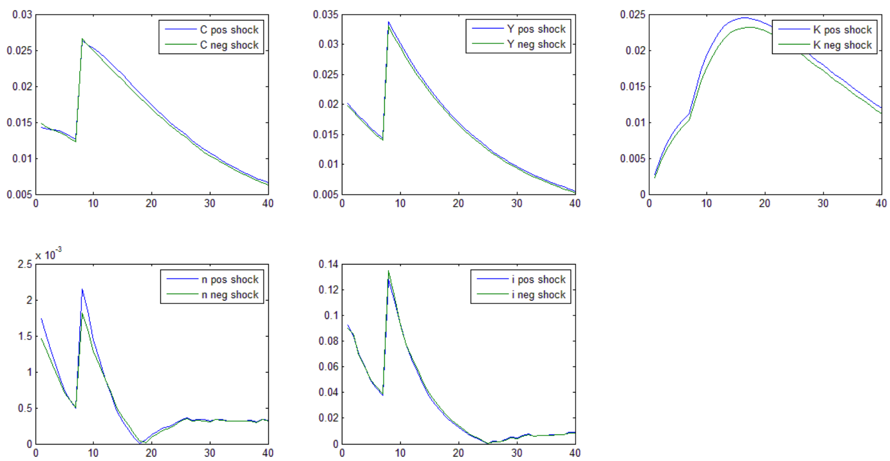


Figure 19: RII for the first and the second shocks

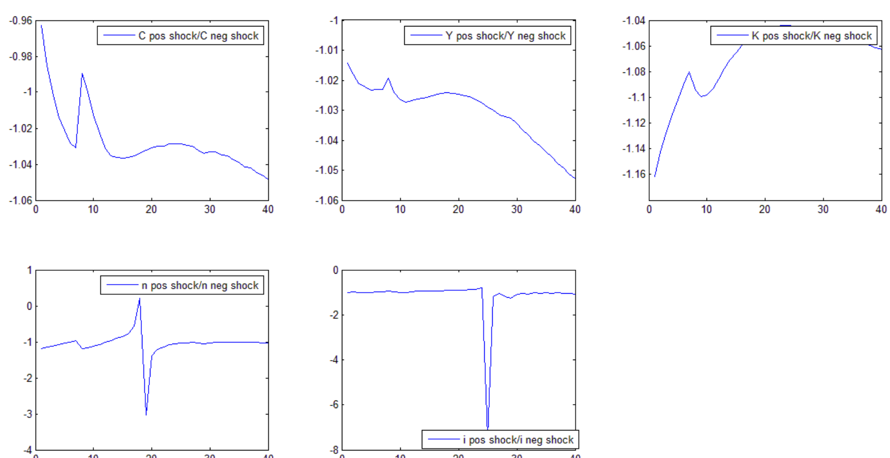


Figure 20: RII for second shock (absolute values)

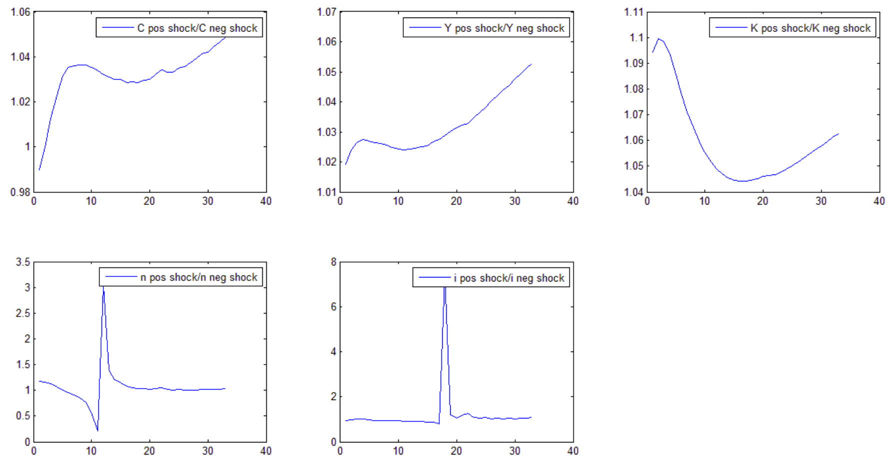


Table 14: Variation of the gap from the steady state after shocks

	Consumption		GDP		Capital		Investment		Labour		Leisure	
Period	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg
0	**1.428%**	**-1.477%**	**2.009%**	**-1.984%**	**0.270%**	**-0.236%**	**9.433%**	**-10.045%**	**0.351%**	**-0.302%**	**-0.343%**	**0.292%**
1	-0.004%	0.033%	-0.110%	0.116%	0.220%	-0.197%	-1.373%	1.935%	-0.064%	0.049%	0.063%	-0.047%
2	-0.015%	0.039%	-0.107%	0.109%	0.177%	-0.164%	-1.174%	1.170%	-0.056%	0.042%	0.055%	-0.041%
3	-0.027%	0.042%	-0.102%	0.102%	0.141%	-0.135%	-1.105%	1.120%	-0.045%	0.036%	0.044%	-0.034%
4	-0.038%	0.047%	-0.096%	0.094%	0.113%	-0.112%	-0.806%	0.750%	-0.035%	0.028%	0.035%	-0.028%
5	-0.042%	0.046%	-0.090%	0.089%	0.090%	-0.091%	-0.699%	0.694%	-0.030%	0.026%	0.029%	-0.026%
6	-0.047%	0.048%	-0.084%	0.083%	0.071%	-0.074%	-0.608%	0.611%	-0.023%	0.021%	0.022%	-0.020%
7	**1.380%**	**-1.449%**	**1.931%**	**-1.897%**	**0.327%**	**-0.283%**	**8.721%**	**-8.811%**	**0.333%**	**-0.265%**	**-0.327%**	**0.256%**
8	-0.053%	0.093%	-0.182%	0.188%	0.264%	-0.237%	-1.694%	1.781%	-0.079%	0.056%	0.077%	-0.054%
9	-0.049%	0.087%	-0.181%	0.178%	0.202%	-0.193%	-1.605%	1.255%	-0.080%	0.055%	0.079%	-0.054%
10	-0.076%	0.092%	-0.168%	0.165%	0.158%	-0.157%	-1.299%	1.388%	-0.055%	0.044%	0.054%	-0.042%

5 References

Abel, Andrew B; and Eberly, Janice C. (1994). A unified model of investment under uncertainty. The American Economic Review. Vol. 84, No. 5, December. Oo. 1369-1384.

Stéphane Adjemian, Houtan Bastani, Michel Juillard, Ferhat Mihoubi, George Perendia, Marco Ratto and Sébastien Villemot (2011), "Dynare: Reference Manual, Version 4," Dynare Working Papers, 1, CEPREMAP

Amisano, Gianni and Tristani, Oreste (2008). A DSGE model of the term structure with regime shifts.

An, Sungbae and Schorfheide,Frank (2007). BAYESIAN ANALYSIS OF DSGE MODELS. Econometric Reviews, 26(2–4):113–172,

Bachman, Ruediger, Caballero, Ricardo J. and Engle, Eduardo M.R.A. (2006a). Lumpy investment in dynamic general equilibrium. Cowles foundation discussion paper No.1566. June 2006.

Bachman, Ruediger, Caballero, Ricardo J. and Engle, Eduardo M.R.A. (2006a). Aggregate implications of lumpy investment: new evidence and a DSGE model. NBER working paper 12336, june 2006.

Barseghyan, Levon, Molinari, Francesca, O'Donoghue, Ted and Teitelbaum, Joshua (2010). The nature of risk preferences: evidence from insurance choices. July 21 2010.

Belaire-Franch and Contreras (2003). "An assessment of international business cycle asymmetries using Clements and Krolzig's Parametric approach". Studies in nonlinear dynamics and econometrics. Volume 6, issue 4.

Bullard, James and Singh,Aarti (2009). Learning and the Great Moderation. Federal Reserve Bank of St. Louis and University of Sydney respectively

Caballero, Ricardo J. and Engel, Eduardo (1991). Dynamic (S,s) economies. Econometrica, vol59, No.6, november. Pp. 1659-1686.

Caballero, Ricardo J. and Engel, Eduardo (1994). Explaining investment dynamics in U.S, manufacturin: a generalized (S.s) approach. NBER Working Paper 4887

Caballero, Ricardo J. , Engel, Eduardo and Haltiwanger, John C. (1995). Plant-level adjustment and aggregate investment dynamics. Brookings papers on economic activity, vol. 1995, No. 2. Pp 1-54.

Cooper, Russell W. and Haltiwanger, John C. (2000). On the nature of capital adjustment costs. NBER Working Paper 7925.

Davig, Troy and Leeper, Eric M. (2005). Fluctuating macro policies and the fiscal theory. NBER WP, 11212.

Doms, Mark and Dunne, Timothy (1998). Capital adjustment patterns in manufacturing plants. Review of economic dynamics 1, pp.409-429.

Eo, Yunjong (2009).Bayesian Analysis of DSGE Models with Regime Switching. Department of Economics Washington University in St. Louis, Job Market Paper. This Draft: February 11, 2009.

Gefang and Strachan (2010). Nonlinear impacts of international business cycles on the U.K.-A bayesian smooth transition VAR approach. Studies in nonlinear dynamics and econometrics. Volume 14, issue 1.

Hamermesh, D.S., Pfann, G.A. (1996). Turnover and the dynamics of labor demand. Economica 63, 359-368.

Jaramillo, F., Shciantarelli, F., Sembenelli, A. (1993). Are adjustment costs for labor asymmetric? An econometric test on panel data for Italy. Review of economics and statistics 75, 640-648.

Jordà, Òscar (2005). Estimation and inference of impulse responses by local projections. The american economic review, Vol.95, No. 1, pp.161-183.

Karagedikli, Özer; Matheson, Troy; Smith, Christie and Vahey, Shaun P. (2007). RBCs and DSGEs: The Computational Approach to Business Cycle Theory and Evidence. Reserve bank of Newzeland.

Khan, Aubhik and Thomas, Julia K. (2003). Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter? Journal of Monetary Economics 50, pp 331-360.

Khan, Aubhik and Thomas, Julia K. (2004). Idionsyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. Federal Reserve Bank of Minneapolis, Research Department Staff Report 352.

Koop, Gary; Pesaran, M. Hasem; and Potter, Simon M. (1996). Impulse response analysis in nonlinear multivariate models. Journal of Econometrics, 74, pp. 119-147.

Marcet, A. and Marshall, D. A. (1994). solving nonlinear rational expectations models by parameterized expectations: convergence and stationary solutions. Economics Working paper 76.

Marcet, A. and Lorenzoni, G. (2001). Parameterized Expectations Approach; some practical issues. Chapter book in: Computational methods for the study of dynamic economies. Edited by Ramon Marimon and Adrew Scott. Oxford University Press.

Miao, Jianjun and Wang, pengfei (2009). Does lumpy investment matter for business cycles? Working paper.

Neftci (1984): "are economic time series asymmetric over the business cycle?" in the journal of political economia, vol 92, •2, apr.1984.

Pfann, G.A., Palam, F.C., (1993). Asymmetric adjustment costs in nonlinear labour demand models for the Netherlands and U.K. manufacturing sectors. Review of Economic Studies 60, 297-312.

Palm, Franz C. and Pfann, Gerard A. (1997). Sources of asymmetry in production factor dynamics. Journal of Econometrics, 82, pp.361-392

Pytlarczyk, Ernest (2005). An Estimated DSGE Model for the German Economy within the Euro Area.

Thomas, Julia K. (2002). Is lumpy investment relevant for the business cycle? Federal Reserve Bank of Minneapolis, Research Department Staff Report 302. March 2002.

Valderrama, Diego (2002). Nonlinearities in international business cycles. Working paper -2002-23. Federal Reserve Bank of San Fransisco.

Valderrama, Diego (2007). Statistical nonlinearities in the business cycle: a challenge for the canonical RBC model. Journal of Economic dynamics and control (2007). 31, pp. 2957-2983.

Veracierto, Marcelo L. (2002). Plant-level irreversible investment and equilibrium business cycles. The American Economic Review. Vol 92,No 1.

6 Appendix

6.1 Loglinearising the model

When we linearise the full model including the asymmetric cost function of investment, we have as well as in the linearisation regime by regime, a set of linear equations and the nonlinear and asymmetric dynamics of the theoretical model originally constructed desappears.

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{62}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t - \varphi\varphi_t \tag{63}$$

$$y\hat{y}_t = Ak^{\alpha}\hat{A}_t + \alpha Ak\hat{k}_t^{\alpha} \tag{64}$$

$$\varphi\hat{\varphi}_t = \varphi_2\hat{\varphi}_{2t} + \phi\hat{\phi}_t\left(\varphi_1 - \varphi_2\right) + \phi\left(\varphi_1\hat{\varphi}_{1t} - \varphi_2\hat{\varphi}_{2t}\right) \tag{65}$$

$$\varphi_1\hat{\varphi}_{1t} = \psi_1\left(k_{t+1} - k_t\right)k\hat{k}_{t+1} - \psi_1\left(k_{t+1} - k_t\right)k\hat{k}_t \tag{66}$$

$$\varphi_2\hat{\varphi}_{2t} = \psi_2\left(k_{t+1} - k_t\right)k\hat{k}_{t+1} - \psi_2\left(k_{t+1} - k_t\right)k\hat{k}_t \tag{67}$$

$$\phi\hat{\phi}_t = \frac{\gamma exp(-\gamma(k-k))}{\left[1+exp(-\gamma(k-k))\right]^2}k\hat{k}_{t+1} - \frac{\gamma exp(-\gamma(k-k))}{\left[1+exp(-\gamma(k-k))\right]^2}k\hat{k}_t \tag{68}$$

$$\lambda\hat{\lambda}_t + \lambda\frac{\partial\varphi(x)}{\partial k_{t+1}}\left(1+\eta_1\hat{k}_{t+1}\right) + \lambda\frac{\partial\varphi_t(x)}{\partial k_{t+1}}\left(1+\eta_2\hat{k}_{t+1}\right) \tag{69}$$
$$= \beta\lambda\hat{\lambda}_{t+1}\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\right] + \beta\lambda\left[\begin{array}{c} f'(k)\left(1+\eta_3\hat{k}_{t+1}\right) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_4\hat{k}_{t+1}\right) \\ -\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_5\hat{k}_{t+2}\right) \end{array}\right]$$
$$+ \beta\lambda f'(k)\left(1+\eta_6\hat{A}_{t+1}\right)$$

This system is the same as the one we have when there is no asymmetries in the cost of investment. thus, nonlinear behavior of investment is not captured when using first order Taylor approximations. Thus we need a different numerical method to simulate and test this model.