PUBLIC TRANSPORT CONCESSION IN BOGOTÁ: A MORAL HAZARD AND ADVERSE SELECTION PROBLEM

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Public transport concession in Bogotá: a moral hazard and adverse selection problem.*

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Abstract

This paper analyzes the contractual model of public transportation in Bogota based on the effects of moral hazard and adverse selection. The emphasis is given to the rules of operation and how these can affect the system's quality. It is analyzed based on the effort made by the concessionaire and whether they are efficient or not. First, it is shown that efficient concessionaires have incentives to pretend to have higher costs in a perfect information contract, which implies higher costs for the district. Then, I analyze the moral hazard problem and show that the district should warrant higher payments to guarantee that the firms undertake the effort. Finally, I study the imperfect information contract where the most economical contract is the one that provides effort only for the efficient agent. Hidden action and hidden characteristics generate higher costs for the principal that depend on the observed quality and type of the agent. In any case, under specific parameters over operator's costs, the district offers a contract where the concessionaire does not exert effort.

Keywords: Public Transportation, Moral Hazard, Adverse Selection

*To my parents.

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1 Introduction

Public transport systems are tools to reduce travel times and forge a more productive city. In addition, 2.5 billion people will move to cities in developing countries by 2050 (Tsivanidis, 2018), so urban transportation needs to optimize travel time to guarantee to reduce inefficient costs associated with population displacement within an expanding city. While technical aspects, such as the speed and capacity of different means of transport, are essential, the rules with which transportation concessionaires work are also vital. Still, if it works through a contractual model with perverse incentives, there will be lower quality, its use will be discouraged, and higher costs will be generated for the district budget. This document focuses on how the public transport of Bogotá, Colombia, can be affected by a contractual model.

Economic and social changes in Bogotá generate the need to develop passenger transport. A Bus Rapid Transit (BRT) system called Transmilenio is the central axis in the mobility of Bogotá. It works in a Public-Private Partnership (PPP) contract that consists of private operators that administer the system and the district government that guarantees the infrastructure necessary for the operation (Pachon, 2016). This paper seeks to analyse theoretically of the current contractual model in Transmilenio. Although the system is considered financially sustainable in the contract, earnings from the collection have not been sufficient to generate remuneration for all agents in the system, and it has created a financial deficit in the system (Pachon, 2016). In consequence, the District Development Plan 2020-2023 of Bogotá opened on the possibility of the district operating the system directly by creating a public transportation company. Therefore, it is essential to understand the conditions under which the city's public transport works today and its consequences. These aspects lead us to the research question: Can the contractual model of a public transportation system affect the quality of the system in the case of the Bus Rapid Transit Transmilenio in Bogotá?

This work seeks to apply contract theory to study the Transmilenio's case. The focus is on the contract, although studies of Transmilenio have focused on the technical aspects of the operation of the system and its development over time (Tsivanidis, 2018; Echeverry et al., 2005; Akbar & Duranton, 2017; Bocarejo et al., 2013; Lleras, 2003). It means that theory has extensively studied bus speed, congestion, user satisfaction, the comparison between mobility before and after the existence of Transmilenio. Hence, this document proposes to open a theoretical discussion on the contract. Specially because the system's unpopularity has been evidenced and the need to take measures regarding the mobility of the capital of Colombia has become evident ("The troubles of Bogotá's TransMilenio", 2020; "Bogotá's rise and fall", 2011).

The main contribution of this work is to study how the contractual model of public transportation can affect the agent's interactions and thus the quality of the system, together with the costs and benefits of the operation. Concessionaires' effort, operational costs, and system quality reflect the social welfare that determines the results of the contractual model. Likewise, it shows that the difference in information between operators and the district and the definitions of profit margins stipulated in the contractual model affect mobility and increase operating costs.

This model has had a public tender and, in fact, distributions of different companies according to the lines. It let us simplify the problem by taking just one road and one concessionaire into account. Therefore, we consider a scenario where a company can operate the transport system and the district offers a contract to delegate this action. It is interesting to study from economic theory the moral hazard incentive and the asymmetric information. The public authority does not have the same capacity as the concessionaires to know or evaluate the technological efficiency or the intention of the operator to reduce its costs (Gagnepain et al., 2009). Hence, I analyze an adverse selection and moral hazard problem.

First, I analyze when the district knows the type of concessionaire (efficient or inefficient) and the effort of the agent. The parameters for which effort or no effort is generated are determined. It is also evident how efficient concessionaires, or those with low operational costs, have incentives to receive payments as if they had high costs, which would make the system more costly for the district government. Then, I analyze the case if the district cannot observe the concessionaire's effort. This results in higher payments since the district must assume these costs in the contract.

The most realistic case is the one in which it is not observable neither how much effort is exerted, nor whether the concessionaire is efficient or not. For this purpose, several scenarios were developed in this paper, in which the principal offers different self-selection contracts. The contracts that generate a separating equilibrium are two: first, the contract when the effort is guaranteed for the efficient agent and the inefficient one. Second, the contract with effort is just for the efficient type. Therefore, it can be analyzed that moral hazard and adverse selection generate that the district should pay more for the case where both agents make an effort. Nevertheless, the payments are reduced if a contract is taken where only the efficient one exerts effort. Therefore, the principal selects this economic contract.

The asymmetric information is of interest to economists because they are essential in economic analysis due to the malfunctioning of markets. According to Monsalve (2018), this concern was accentuated with neo-Walrasian model from Arrow & Debreu (1954) developed under conditions of uncertainty. In Mussa & Rosen (1978) and Eric et al. (1984), adverse selection emerges from a context where a seller does not know the buyer's willingness to pay, resulting from the deal's preconditions. The analysis of moral hazard has been developed with contract theory through works that consider cases in economics where there is hidden action (Holmstrom & Bengt, 1982; Mirrlees & A., 1976; Grossman et al., 1983). Furthermore, incentive theory and, of particular interest to this paper, models of adverse selection contracts and moral hazard are extensively developed in Laffont & Martimort (2002) and Bolton & Dewatripont (2004). This situation happens in the Transmilenio concession, where the district will establish the contract, but the private operator participates based on preconditions unknown to the district. A similar approach applied to urban transport in France is found in Gagnepain et al. (2009). In this case, based on a model of similar context to that proposed for our case study, it seeks to create a dynamic model that allows the local government to generate a menu of contracts for companies to "self-select". Initially, this approach is distanced from our case since I seek to apply the contract theory to focus on the moral hazard and adverse selection problem. In that sense, it is planned to review a static model with a cost-based regulation such as the one proposed in William (2003).

The literature on urban transport in Bogotá has focused on empirical analysis. (Tsivanidis, 2018; Echeverry et al., 2005; Akbar & Duranton, 2017; Bocarejo et al., 2013; Lleras, 2003). In general, these sources give an important emphasis about a comparison in two cases. First, the mobility in Bogotá before and after Transmilenio's system. Second, the results of the implementation if Trnasmilenio's system with respect to bus speed, congestion, and user satisfaction. Particularly, in Tsivanidis (2018) social welfare is studied from a population, employment, and house prices due to Transmilenio system's evolution. One of the main findings is that high-skilled workers benefited more than low-skilled workers for reallocation residence. For instance, an important implication is that the greater the distance, the travel time is longer. Echeverry et al. (2005) proposes a Cost-Benefit Analysis (CBA), where they find that the CBA is negative because of spillovers of corridors are not covered by Transmilenio. Akbar & Duranton (2017) provides a relationship for the time cost of travel on the demand for travel. This paper shows the implications of cost on user behavioral decisions. These references have studied the transport model as it has been granted. However, our investigation seeks to open a discussion regarding how the quality (which is a reflection of factors like costs, speed, and user satisfaction) can be impacted by the concession model.

This paper consists of five sections, including this introduction. Section 2 describes the context of the contract. In section 3 is found the basic environment of the model, utility functions, and the timing of the game. Section 4 is about the analysis with perfect information, moral hazard and incomplete information contract. Finally, section 5 provides conclusions for the paper.

2 Background

The model developed in this paper simplifies the complexity of Bogotá's public transportation system. Firstly, since 2000, private concessions have been granted for the operation of bus lines transportation systems in different phases. So far, three phases have been developed. However, the developed model simplifies the problem to a single-phase where the concessionaire operates the system, generates quality, and receives a transfer from this. However, let us take a closer look at how it works. In phases I and II, the concessionaires are in charge of purchasing the buses and guaranteeing the operation of public transportation. The main difference with the third phase is that there are two agents, one that operates the system and another that is in charge of ensuring that the buses are ready to operate. But in general terms, in the three phases, in order to guarantee the operation of the system, the agents must comply with the services, frequencies, and schedules assigned by Transmilenio S.A. -which represents the district-. The supply of the rolling equipment necessary for operation, control, and maintenance of its automotive fleet and the surveillance of the parking areas of the operation yards. The endowment, administration, maintenance, and operation of the technical support areas that Transmilenio S.A. grants in concession.

Secondly, the model does not establish conditions on the infrastructure necessary for providing the service but assumes its existence. The agent would not be able to operate without it. This assumption is valid because it is effectively guaranteed in the contractual model, only that the district administration guarantees it. Let us see how it works in detail. The District responds to the infrastructure and maintenance of the road network, cleanliness, and security. The district government must ensure specialized trunk corridors, where it will primarily provide the system with lanes for exclusive use, stations, bridges, and pedestrian access areas. It also grants in concession the infrastructure constituted by the technical support areas that are part of the operating yards.

Thirdly, social welfare, in this case, is related to an increase in the mass of users. First, the more people use public transport, the more revenue will come into the district via commercial rates (rate defined by political decisions). Second, in terms of public policies on travel times, public space, environmental aspects, and others, it is more beneficial for the city if more people use public transport. Nevertheless, there are two types of fees: the technical and the commercial ones. The technical rate depends on the kilometers traveled and the costs for the sustainability of the system. However, the commercial rate is lower than the technical ones, and to maintain the profits to the agent agreed in the contract, the amount missed must be assumed by the district budget. The accumulated budget must include costs, operating expenses, and utilities that the system requires to provide the service under the contract parameters. The rate includes the operating costs of the trunk system and feeder buses, the cost of collecting fees, the cost of the administering agency, and the cost of the fiduciary administration of resources. As previously mentioned, the transfer to the agent in the model is based on quality, correlated to the kilometers traveled.

3 Model

Bogota's mass public transport concession model is an excellent example of a contract with moral hazard and adverse selection. It starts with the adverse selection problem. The company operates the system according to its previous conditions that the district does not know. As a result, the concessionaire can be efficient or inefficient, and oversight of the actual operating costs has not been guaranteed. In practice, the district trusts the fees stated by the concessionaires. It implies that the district cannot differentiate an efficient company from an inefficient one. As a consequence, the capital may incur higher costs and affect the district budget.

In the second place, there is a moral hazard problem. Considering the payment is made from the request for kilometers traveled, the operator benefits from a higher requirement. However, since the competent entity does not implement a compliance control, the operator may not perform the requested kilometers traveled and therefore not comply with the required quality. Thus, even the operator could request a payoff based on a remarkable effort than the one made. This case was evident from the pandemic of Covid-19.[1] The urgency of confinement and social distancing needed a maximum capacity public transport operation. However, as observed in mid-2020, private operators decided to reduce the buses in service given lower users' entry to the system due to the confinement. It was evident by the agglomerations that were generated.[2] This situation revealed covert action in the operation of the

[1] As a measure to prevent overcrowding and thus prevent the spread of Coronavirus, the Mayor's Office of Bogotá defined in 2020 that public transportation should be carried out with 100% of the fleet working and at 35% of its capacity (https://bit.ly/3zN644x).

[2] On March 25, 2020, Bogotá Mayor Claudia López denounced on her Twitter account that some Trans-

system, which additionally endangered users.

These considerations lead us to model this contract as an adverse selection and moral hazard problem.

3.1 Basic environment

Consider a principal-agent problem. where the principal is the district that wants to delegate the operation of public transport and hire it. The agent operates the service and decides whether or not to accept the contract. In this contract, the principal offers a contract to the agent, but if the agent does not accept it, the game is over. In case of acceptance, the agent operates making an effort that implies system quality. Given the quality of the service, the principal generates a payment.

There are two types of the agent: efficient and inefficient. Each type of the agent has a different operating costs. The efficient type has lower operating costs θ_L. Efficient types exist in the economy with a proportion v. The inefficient has larger operation costs θ_H. Where $\theta_L < \theta_H$. Inefficient agents exist in the economy with a proportion $(1-v)$. Each type of agent decides to exert a costly effort e, which can take two values, $e = 0$ and $e = 1$.

Undertaking effort e generates a disutility for the agent $\psi(e)$. If the agent makes an effort, he obtains a disutility $\psi(1) = \psi$. However, if the agent does not exert effort, there is

milenio operators prioritize their business over citizens' lives by generating crowds. By this, she refers to the fact that despite having indications to operate with a maximum capacity of 35 percent, they reduced the frequency of buses (https://bit.ly/2YvS9Cn).

no disutility $\psi(0) = 0$.

In probability, making an effort generates a higher quality of the system and not making an effort generates a lower quality. The lower the quality of the system, the more users use another mode of transport. The stochastic quality $\widetilde{\gamma}$ can be a low-quality $\underline{\gamma}$ or a high one $\overline{\gamma}$ with $\overline{\gamma} > \underline{\gamma}$. The probability of having a high quality given that the agent does not exert effort is $Pr(\widetilde{\gamma} = \overline{\gamma}|e = 0) = \pi_0$ and when the agent does exert effort is $Pr(\widetilde{\gamma} = \overline{\gamma}|e = 1) = \pi_1$. Where the principal is most likely to have high quality when the agent strives, $\pi_1 > \pi_0$. The difference between those probabilities is $\Delta\pi = \pi_1 - \pi_0$. The probability of having low quality given that the agent does not exert effort is $Pr(\widetilde{\gamma} = \underline{\gamma}|e = 0) = 1 - \pi_0$, and when the agent exerts effort, the probability of having low quality is $Pr(\widetilde{\gamma} = \underline{\gamma}|e = 1) = 1 - \pi_1$. In this case, $1 - \pi_1 < 1 - \pi_0$. Hence, it is more likely to have low quality without effort.

3.2 Utility functions

The utility function for the principal is a function of the social welfare, $S(m)$, and it increases due to the mass of users, $m > 0$. This function is strictly increasing and concave. It means that $S'(m) > 0$ and $S''(m) < 0$.

The principal offers a contract based on the quality of the system. So, the principal will give a transfer to the agent contingent on the quality of the final product. The compensation to the agent is high $\bar{t}$ when the quality is high $\overline{\gamma}$ and low $\underline{t}$ with low-quality $\underline{\gamma}$. The principal will offer a menu of contracts. In a separating equilibrium, the agent will select the contract designed for him. The efficient agent gets $\{\bar{t}_L, \underline{t}_L\}$ and the inefficient agent gets $\{\bar{t}_H, \underline{t}_H\}$.

Therefore, the principal's utility function depends on the social welfare and the transfer to the agent according to the effort and the agent's type. The utility function for the principal is,

$$V = \begin{array}{c} v(\pi(e)(S(\overline{m}) - \bar{t}_L) + (1-\pi(e))(S(\underline{m}) - \underline{t}_L)) \\ +(1-v)(\pi(e)(S(\overline{m}) - \bar{t}_H) + (1-\pi(e))(S(\underline{m}) - \underline{t}_H)). \end{array} \tag{1}$$

An agent has a linear utility that implies risk neutrality, which depends on a transfer according to quality. The agent's utility also depends on operating costs and the disutility generated by the effort made. The agent's utility depends on the transfer provided by the principal t, subtracting the marginal cost of producing quality (θ_i) and the disutility of making an effort ψ. Where $\theta_H > \theta_L$. Accordingly, the efficient agent obtains expected profits if it accepts the contract equal to

$$U_L(e) = \pi(e)(\bar{t}_L - \theta_L\overline{\gamma}) + (1-\pi(e))(\underline{t}_L - \theta_L\underline{\gamma}) - \psi \times e. \tag{2}$$

The utility function for the inefficient agent is

$$U_H(e) = \pi(e)(\bar{t}_H - \theta_H\overline{\gamma}) + (1-\pi(e))(\underline{t}_H - \theta_H\underline{\gamma}) - \psi \times e. \tag{3}$$

3.3 Timing of the game

At t_0, the agent discovers his type θ_L or θ_H. At t_1, the principal offers a contract with a transfer to the agent according to the quality $t(\gamma)$. The principal can observe the speed of the buses, location, and the general state of the operation, which means that the quality is observable and can be part of the contract. At t_2, the agent accepts or rejects the contract. At t_3, the agent exerts an effort e and gets his payoff $t(\gamma)$, so Transmilenio operates.

The timing of the contractual game is as in Figure 1:

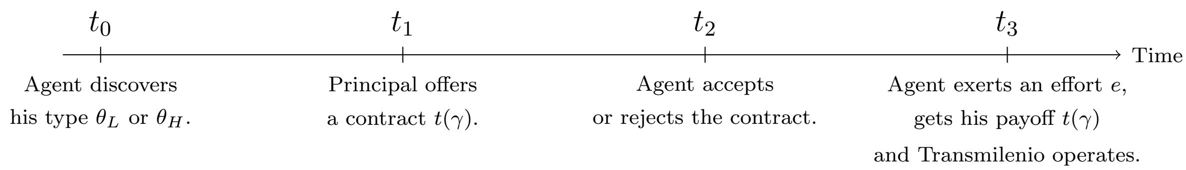

Figure 1: Timing of the game.

4 Analysis

In this section, three contracts are analyzed to understand the modeling of the problem under study. The first sub-section examines the case of perfect information, where the district can distinguish the type of agent and observe the effort made. In the second sub-section, the principal can indicate the type of agent with whom he contracts but cannot observe the effort he makes. Finally, the third sub-section deals with imperfect information. The principal does not know if the agent is efficient or inefficient and the effort made.

4.1 Perfect information

It is considered the complete information problem as a benchmark. Both the agent type and effort can be observable by the principal. Therefore, it is possible to establish a specific contract for the scenario where the agent exerts effort and another contract where the agent does not exert effort. Then, the principal will offer a contract $C_L^{FB} = (\bar{t}_L, \underline{t}_L, e)$ to the efficient agent and a contract $C_H^{FB} = (\bar{t}_H, \underline{t}_H, e)$ to the inefficient one. In both cases, the principal will give a high payment when there is high quality and a low payment when the quality is low. However, the principal will distinguish the payments if the agent makes an effort or does not. Finally, we will review the contract with perfect information for the efficient and the inefficient agent.

Let us begin with the principal's problem for the efficient agent. The principal's problem in this case is:

$$\max_{(\bar{t}_L, \underline{t}_L, e)} \pi(e)(S(\overline{m}) - \bar{t}_L) + (1 - \pi(e))(S(\underline{m}) - \underline{t}_L) \tag{4}$$

Subject to the participation constraint for the efficient agent where the efficient agent will accept the contract specifying he undertakes effort.

$$PC_{e=1} : \pi_1(\bar{t}_L - \theta_L \overline{\gamma}) + (1 - \pi_1)(\underline{t}_L - \theta_L \underline{\gamma}) \geq \psi \tag{5}$$

The effort is part of the contract, but the agent needs to receive a payment to accept the contract that will specify the effort. It can be found that when $\underline{t}_L^{FB} = \theta_L \underline{\gamma}$ in equation 5, then $\pi_1(\bar{t}_L - \theta_L \overline{\gamma}) = \psi$. Thus, the payment when there is high quality needs to be $\bar{t}_L^{FB} = \frac{\psi}{\pi_1} + \theta_L \overline{\gamma}$ to exert effort.

Now assume the principal is offering a contract with zero effort, so there is no disutility ($\psi = 0$). In this case, the participation constraint will be given by,

$$PC_{e=0} : \pi_0(\bar{t}_L - \theta_L \overline{\gamma}) + (1 - \pi_0)(\underline{t}_L - \theta_L \underline{\gamma}) \geq 0. \tag{6}$$

In an effortless contract, the principal gives the agent $\bar{t}_L^{FB} = \theta_L \overline{\gamma}$ when there is large quality and $\underline{t}_L^{FB} = \theta_L \underline{\gamma}$ with low quality. With these results, we obtain from equation 4 the following inequality:

$$\begin{aligned} &\pi_1(S(\overline{m}) - (\frac{\psi}{\pi_1} + \theta_L\overline{\gamma})) + (1-\pi_1)(S(\underline{m}) - \theta_L\underline{\gamma}) \\ &\geq \pi_0(S(\overline{m}) - \theta_L\overline{\gamma}) + (1-\pi_0)(S(\underline{m}) - \theta_L\underline{\gamma}) \end{aligned} \tag{7}$$

Arranging terms, according to equation 8 defined below, the expected profit of the principal corresponds to the difference between $S(\overline{m})$ and $S(\underline{m})$. That is, the return of the high mass with respect to the low mass. As follows,

$$\underbrace{\Delta\pi(S(\overline{m}) - S(\underline{m}))}_{\text{Expected benefits.}} \geq \underbrace{\psi + \Delta\pi\theta_L(\overline{\gamma} - \underline{\gamma})}_{\text{Expected costs.}} \tag{8}$$

This inequality is the condition for generating a contract with effort for the efficient type. Otherwise, an effortless contract is generated. Accordingly, the payment of the principal will be greater than or equal to the costs incurred. Else, the agent does not want to provide an effort because it is too costly compared to the benefit it entails.

The inefficient agent is solved similarly. The only change in the result is that now the operating cost is θ_H. Therefore, we obtain the following condition to guarantee the effort of the inefficient agent:

The shape for the perfect information contract is established, as indicated in the following proposition.

Proposition 1: The perfect information contract.

i). *When* $\frac{\Delta\pi(S(\overline{m})-S(\underline{m}))-\psi}{\Delta\pi(\overline{\gamma}-\underline{\gamma})} \geq \theta_H$ *the first-best contract that is offered to the efficient agent is* $C_L^{FB} = (\overline{t}_L^{FB}, \underline{t}_L^{FB}, e) = (\theta_L\overline{\gamma} + \frac{\psi}{\pi_1}, \theta_L\underline{\gamma}, 1)$ *and the first-best contract offered to the inefficient agent is* $C_H^{FB} = (\overline{t}_H^{FB}, \underline{t}_H^{FB}, e) = (\theta_H\overline{\gamma} + \frac{\psi}{\pi_1}, \theta_H\underline{\gamma}, 1)$.

ii). *When* $\theta_H > \frac{\Delta\pi(S(\overline{m})-S(\underline{m}))-\psi}{\Delta\pi(\overline{\gamma}-\underline{\gamma})} \geq \theta_L$ *the first-best contract that is offered to the efficient agent is* $C_L^{FB} = (\bar{t}_L^{FB}, \underline{t}_L^{FB}, e) = (\theta_L\overline{\gamma} + \frac{\psi}{\pi_1}, \theta_L\underline{\gamma}, 1)$ *and the first-best contract offered to the inefficient agent is* $C_H^{FB} = (\bar{t}_H^{FB}, \underline{t}_H^{FB}, e) = (\theta_H\overline{\gamma}, \theta_H\underline{\gamma}, 0)$.

iii). *When* $\frac{\Delta\pi(S(\overline{m})-S(\underline{m}))-\psi}{\Delta\pi(\overline{\gamma}-\underline{\gamma})} < \theta_L$ *the first-best contract offered to the efficient agent is* $C_L^{FB} = (\bar{t}_L^{FB}, \underline{t}_L^{FB}, e) = (\theta_L\overline{\gamma}, \theta_L\underline{\gamma}, 0)$ *and the first-best contract for the inefficient agent is* $C_H^{FB} = (\bar{t}_H^{FB}, \underline{t}_H^{FB}, e) = (\theta_H\overline{\gamma}, \theta_H\underline{\gamma}, 0)$.

According to proposition 1, when the principal's profit (guaranteeing payment for the disutility of the effort ψ and given a high quality) is greater than θ_H, as indicated in item (**i**), the principal will offer a high-effort contract to both agents. However, it is cheaper for the principal to pay the efficient agent than the inefficient agent due to marginal production costs. Therefore, when the principal's profit lies between θ_H and θ_L, as in item (**ii**), the payment will include effort for the efficient agent but not for the inefficient one. Finally, suppose the principal makes a profit less than θ_L, as in item (**iii**). In that case, it decides to offer an effortless contract to both types of agents.

This result is interesting because there is a discussion about whether the operators of the Transmilenio system would have incentives to lie in their costs and increase them, despite complying with the requested operation. Consequently, a perfect information contract shows that the district must pay an inefficient concessionaire larger costs when it meets the expected quality.

4.2 Semi-perfect information: Moral Hazard

In this case, the principal can differentiate the efficient agent from the inefficient but does not know the effort made by each one of them. Therefore, the principal wants to induce effort. However, this measure will not be part of the contract because it is not observable. As in the previous section, the principal solves this problem for each agent type. The principal's problem when the agent is efficient is

$$\max_{(\bar{t}_L, \underline{t}_L)} \pi(e)(S(\overline{m}) - \bar{t}_L) + (1 - \pi(e))(S(\underline{m}) - \underline{t}_L). \tag{9}$$

Subject to the participation constraint

$$\pi(e)(\bar{t}_L - \theta_L \overline{\gamma}) + (1 - \pi(e))(\underline{t}_L - \theta_L \underline{\gamma}) - \psi \geq 0. \tag{PC}$$

The moral hazard constraint ensures that the agent gets incentives to make an effort $e = 1$ greater than or equal to the disutility ψ. This restriction occurs when the utility of the agent is greater when making an effort $(U_L(e|e = 1) \geq U_L(e|e = 0))$. This constraint is subsequently:

$$\pi_1(\bar{t}_L - \theta_L \overline{\gamma}) + (1 - \pi_1)(\underline{t}_L - \theta_L \underline{\gamma}) - \psi \geq \pi_0(\bar{t}_L - \theta_L \overline{\gamma}) + (1 - \pi_0)(\underline{t}_L - \theta_L \underline{\gamma}) \tag{10}$$

This expression specifies the relationship in the transfers to induce the agent to make an effort. The agent's profit - which arises from the transfer generated by the principal - must guarantee to be at least equal to the costs incurred by the agent given the quality it produces.

The participation constraint if the agent does effort is the following:

$$\pi_1(\bar{t}_L - \theta_L \overline{\gamma}) + (1 - \pi_1)(\underline{t}_L - \theta_L \underline{\gamma}) - \psi \geq 0 \tag{11}$$

The principal can take the agent's moral hazard constraint to zero utility from the transfers and solve the constraints with equality. Therefore, from 10, the moral hazard constraint leads us to,

$$\bar{t}_L = \frac{\psi}{\Delta\pi} + \theta_L(\overline{\gamma} - \underline{\gamma}) + \underline{t}_L \tag{12}$$

Without loss of generality $\underline{t}_L^{MH} = \theta_L\underline{\gamma}$. Using the low optimal transfer in equation 12, we obtain the high optimal transfer:

$$\bar{t}_L^{MH} = \theta_L\overline{\gamma} + \frac{\psi}{\Delta\pi} \tag{13}$$

If the principal does not want to offer a contract that incentives the agent to exert effort, only the participation constraint needs to be fulfilled. Then:

$$\pi_0(\bar{t}_L - \theta_L\overline{\gamma}) + (1 - \pi_0)(\underline{t}_L - \theta_L\underline{\gamma}) \geq 0 \tag{14}$$

If the principal does not want to provide effort, he offers a contract that the agent accepts. However, if there is no effort, the principal will give the first-best contract in the case of no effort. Those payoffs are $\theta_L\overline{\gamma}$ and $\theta_L\underline{\gamma}$ for the efficient agent. Therefore, from equation 10, the circumstance in which the principal will provide a contract with an effort to the efficient agent will be as follows:

$$\begin{aligned} &\pi_1(S(\overline{m}) - (\theta_L\overline{\gamma} + \frac{\psi}{\Delta\pi})) + (1 - \pi_1)(S(\underline{m}) - \theta_L\underline{\gamma}) \\ &\geq \pi_0(S(\overline{m}) - \theta_L\overline{\gamma}) + (1 - \pi_0)(S(\underline{m}) - \theta_L\underline{\gamma}) \end{aligned} \tag{15}$$

Arranging terms gives,

$$\underbrace{\Delta\pi(S(\overline{m}) - S(\underline{m}))}_{\text{Expected benefits.}} \geq \underbrace{\frac{\pi_1\psi}{\Delta\pi} + \Delta\pi\theta_L(\overline{\gamma} - \underline{\gamma})}_{\text{Expected costs.}} \tag{16}$$

Under this condition, the principal will offer a contract to the efficient agent with effort. If not fulfilled, the principal will pay the agent without effort. It is worth noting that perfect information analysis gives a lower weight to the expected costs. It is because analyzing on the RHS of equations 17 and 8, $\frac{\pi_1\psi}{\Delta\pi} \geq \psi$. In order to offer a contract that provides effort. This is because moral hazard gives positive rents to the agent and the principal obtains a lower expected profit.

The inefficient agent is solved in the same way. Whereby the principal gives a transfer $\bar{t}_H^{MH}$ or $\underline{t}_H^{MH}$ depending on the quality and his inequality to provide effort is:

$$\underbrace{\Delta\pi(S(\overline{m}) - S(\underline{m}))}_{\text{Expected benefits.}} \geq \underbrace{\frac{\pi_1\psi}{\Delta\pi} + \Delta\pi\theta_H(\overline{\gamma} - \underline{\gamma})}_{\text{Expected costs.}} \tag{17}$$

The moral hazard contract is established in the following proposition.

Proposition 2: Semi-perfect information contract - Moral Hazard.

i). *When* $\frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2(\overline{\gamma}-\underline{\gamma})} \geq \theta_H$ *the contract offered to the efficient agent is* $C_L^{MH} = (\bar{t}_L^{MH}, \underline{t}_L^{MH}) = (\theta_L\overline{\gamma} + \frac{\psi}{\Delta\pi}, \theta_L\underline{\gamma})$ *and the contract offered to the inefficient agent is* $C_H^{MH} = (\bar{t}_H^{MH}, \underline{t}_H^{MH}) = (\theta_H\overline{\gamma} + \frac{\psi}{\Delta\pi}, \theta_H\underline{\gamma})$.

ii). *When* $\theta_H > \frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2(\overline{\gamma}-\underline{\gamma})} \geq \theta_L$ *the contract offered to the efficient agent is* $C_L^{MH} = (\bar{t}_L^{MH}, \underline{t}_L^{MH}) = (\theta_L\overline{\gamma} + \frac{\psi}{\Delta\pi}, \theta_L\underline{\gamma})$ *and the contract offered to the inefficient agent is* $C_H^{MH} = (\bar{t}_H^{MH}, \underline{t}_H^{MH}) = (\theta_H\overline{\gamma}, \theta_H\underline{\gamma})$.

iii). *When* $\frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2(\overline{\gamma}-\underline{\gamma})} < \theta_L$ *the contract offered to the efficient agent is* $C_L^{MH} = (\bar{t}_L^{MH}, \underline{t}_L^{MH}) = (\theta_L\overline{\gamma}, \theta_L\underline{\gamma})$ *and the contract for the inefficient agent is* $C_H^{MH} = (\bar{t}_H^{MH}, \underline{t}_H^{MH}) = (\theta_H\overline{\gamma}, \theta_H\underline{\gamma})$.

According to proposition 2, the principal will pay the agent the marginal production and the cost of effort only if the payments to the principal are higher than its costs. Conversely, when there is low quality, the principal will only pay the agent the production costs.

Comparing the payments received by the agents in proposition 2 to proposition 1, it is found that the payoffs from the principal to the agent in a moral hazard contract are higher than with a perfect information contract ($\frac{\psi}{\Delta\pi} \geq \frac{\psi}{\pi_1}$). The reason is that the principal has to offer a contract to provide effort. Therefore, this proposition implies that the principal obtains a lower utility with a moral hazard contract than if he generated a perfect information contract. Another implication is that effort occurs in equilibrium for a lower range of parameters.

The possibility of hidden actions in Transmilenio makes it more expensive for the District's expected benefit to generate a contract that guarantees that the system operator will make an effort. Also, the contract is more costly for the district since, as stated, the contract must provide the effort.

4.3 Incomplete information: Adverse Selection and Moral Hazard

With imperfect information, a contract can be designed for each agent type, but agents must self-select. Another critical aspect is that the effort will not be part of the contract considering it is not observable either. Then, the principal's problem is:

$$\max_{\{(\bar{t}_L,\underline{t}_L);(\bar{t}_H,\underline{t}_H)\}} \begin{array}{c} v(\pi(e)(S(\overline{m})-\bar{t}_L)+(1-\pi(e))(S(\underline{m})-\underline{t}_L)) \\ +(1-v)(\pi(e)(S(\overline{m})-\bar{t}_H)+(1-\pi(e))(S(\underline{m})-\underline{t}_H)) \end{array} \tag{18}$$

Subject to Individual Rationality:

$$\pi(e)(\bar{t}_L - \theta_L\overline{\gamma}) + (1-\pi(e))(\underline{t}_L - \theta_L\underline{\gamma}) - \psi \times e \geq 0 \tag{IR_L}$$

$$\pi(e)(\bar{t}_H - \theta_H\overline{\gamma}) + (1-\pi(e))(\underline{t}_H - \theta_H\underline{\gamma}) - \psi \times e \geq 0 \tag{IR_H}$$

It also has an Incentive Compatibility constraint which, when met, ensures that a separating equilibrium is obtained where each type chooses the contract that was designed for the agent. This constraint is:

$$\pi(e)(\bar{t}_L - \theta_L\overline{\gamma}) + (1-\pi(e))(\underline{t}_L - \theta_L\underline{\gamma}) \geq \pi(e)(\bar{t}_H - \theta_L\overline{\gamma}) + (1-\pi(e))(\underline{t}_H - \theta_L\underline{\gamma}) \tag{IC_L}$$

$$\pi(e)(\bar{t}_H - \theta_H\overline{\gamma}) + (1-\pi(e))(\underline{t}_H - \theta_H\underline{\gamma}) \geq \pi(e)(\bar{t}_L - \theta_H\overline{\gamma}) + (1-\pi(e))(\underline{t}_L - \theta_H\underline{\gamma}) \tag{IC_H}$$

And subject to the Moral Hazard constraints, ensure that agent types make an effort:

$$\pi_1(\bar{t}_L - \theta_L\overline{\gamma}) + (1-\pi_1)(\underline{t}_L - \theta_L\underline{\gamma}) - \psi \geq \pi_0(\bar{t}_L - \theta_L\overline{\gamma}) + (1-\pi_0)(\underline{t}_L - \theta_L\underline{\gamma}) \tag{MH_L}$$

$$\pi_1(\bar{t}_H - \theta_H\overline{\gamma}) + (1-\pi_1)(\underline{t}_H - \theta_H\underline{\gamma}) - \psi \geq \pi_0(\bar{t}_H - \theta_H\overline{\gamma}) + (1-\pi_0)(\underline{t}_H - \theta_L\underline{\gamma}) \tag{MH_H}$$

With these constraints in place, I analyze the different separating contracts that the principal can offer. First, I analyze the case where both agents undertake effort, then the case where neither agent undertakes effort, and those where only one of them undertakes effort. Finally, I analyze under what conditions the principal will offer a contract with effort and define which contract the principal will offer.

4.3.1 Separating contracts where both agents undertake effort

I will analyze the case where both types of agents make an effort. Then, the most economical way to satisfy the effort is to meet the two moral hazard constraints with equality. This constraint for the efficient agent would then be:

$$\bar{t}_L = \frac{\psi}{\Delta\pi} + \theta_L(\overline{\gamma} - \underline{\gamma}) + \underline{t}_L \tag{19}$$

Without loss of generality, we can guarantee that when there is low quality, the payment when both agents type exert effort (EE) is $\underline{t}_L^{EE} = \theta_L\underline{\gamma}$. Then, the payoff with high quality is:

$$\bar{t}_L^{EE} = \frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma} \tag{20}$$

Given that the contract designed for the efficient type makes him undertake effort in equilibrium, the individual rationality of the efficient type is fulfilled, and, therefore, it can be eliminated. Since the efficient agent is guaranteed rents that include the disutility of effort, this fixed agent will have utilities greater than zero. Replacing $\bar{t}_L^{EE}$ and $\underline{t}_L^{EE}$ on the Individual Rationality constraint for the efficient agent, we find that there is a moral hazard rent $\pi_0\psi \geq 0$.

The payoffs of the inefficient agent will now be established from the constraint MH_H as follows:

$$\bar{t}_H = \frac{\psi}{\Delta\pi} + \theta_H(\overline{\gamma} - \underline{\gamma}) + \underline{t}_H \tag{21}$$

As can be seen, the payoffs under the moral hazard constraint of the inefficient agent are solved in the same way as with the efficient agent. The only difference is that the inefficient agent's costs and transfers are used. Therefore, the payments of the inefficient agent are $(\bar{t}_H^{EE}, \underline{t}_H^{EE}) = (\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma}, \theta_H\underline{\gamma})$.

Furthermore, as demonstrated, by meeting the moral hazard constraints, the Individual Rationality constraints are also met with a moral hazard rent of $\pi_0\psi \geq 0$. For this reason, the two constraints IR_L and IR_H can be eliminated.

Let us now analyze the Incentive Compatibility constraints for this case where both types of agents strive. Replacing the obtained payoffs $\{(\bar{t}_L^{EE}, \underline{t}_L^{EE}); (\bar{t}_H^{EE}, \underline{t}_H^{EE})\}$, we find that the IC_L inequality is

$$\pi_1\bar{t}_L^{EE} + (1-\pi_1)\underline{t}_L^{EE} \geq \pi_1\bar{t}_H^{EE} + (1-\pi_1)\underline{t}_H^{EE}. \tag{22}$$

Then, as we get $-\pi_1\overline{\gamma} \geq (1-\pi_1)\underline{\gamma}$, the IC_L inequality is not satisfied. It means that in this context, the efficient agent has incentives to deviate and take the contract of the inefficient type. This situation is explained by the fact that the payment contemplates the costs of performing a certain quality. Thus, the efficient agent has incentives to take the payoff as inefficient since he would be paid for higher costs. On the contrary, when analyzing constraint IC_H, I obtain

$$\pi_1\bar{t}_H^{EE} + (1-\pi_1)\underline{t}_H^{EE} \geq \pi_1\bar{t}_L^{EE} + (1-\pi_1)\underline{t}_L^{EE}. \tag{23}$$

So, the inequality is satisfied with $\overline{\gamma} \geq \underline{\gamma} - \underline{\gamma}/\pi_1$. Therefore, the inefficient agent has no incentive to deviate.

We have shown that there is no separating equilibrium in the most economical way to provide effort. However, there is a separating equilibrium in which raising the transfers of the efficient type without the inefficient type deviating. For this reason, the payment of the efficient type will now be sought by fulfilling the incentive compatibility (IC_L) with equality. That is

$$\pi_1 \bar{t}_L + (1-\pi_1)\underline{t}_L = \pi_1 \bar{t}_H + (1-\pi_1)\underline{t}_H. \tag{24}$$

Using the payments of the inefficient agent previously obtained ($\bar{t}_H^{EE}$, $\underline{t}_H^{EE}$), the expression is

$$\bar{t}_L = \frac{\psi}{\Delta\pi} + \theta_H \overline{\gamma} + \frac{(1-\pi_1)}{\pi_1}\theta_H \underline{\gamma} - \frac{(1-\pi_1)}{\pi_1}\underline{t}_L. \tag{25}$$

Without loss of generality $\underline{t}_L^{EE} = \theta_L \underline{\gamma}$. So,

$$\bar{t}_L^{EE} = \frac{\psi}{\Delta\pi} + \theta_H \overline{\gamma} + \frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H - \theta_L). \tag{26}$$

This payment is more costly for the principal when compared to the one found in equation 21. Therefore, as the IC_L constraint is met with equality, the moral hazard constraint is also met.

Now, let us verify if the inefficient agent wants to accept the contract designed for him. Replacing $\bar{t}_L^{EE}$, $\underline{t}_L^{EE}$, $\bar{t}_H^{EE}$ and $\underline{t}_H^{EE}$ in IC_H, I get

$$\pi_1(\frac{\psi}{\Delta\pi} + \theta_H \overline{\gamma}) + (1-\pi_1)\theta_H \underline{\gamma} \geq \pi_1(\frac{\psi}{\Delta\pi} + \theta_H \overline{\gamma} + \frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H - \theta_L)) + (1-\pi_1)\theta_L \underline{\gamma}. \tag{27}$$

Arranging terms, the constraint is fulfilled.

It implies that in a contract with incomplete information, when both agents make an effort, a costly contract for the efficient agent must be guaranteed. Otherwise, if the principal offers the cheapest contract for the efficient type, the efficient type has incentives to deviate and take the contract designed for the inefficient one.

Proposition 3: Incomplete information contract - Separating contracts where both agents undertake effort.

The principal offers the contract

$$\{(\bar{t}_L^{EE}, \underline{t}_L^{EE}); (\bar{t}_H^{EE}, \underline{t}_H^{EE})\} = \{(\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma} + \frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H - \theta_L), \theta_L\underline{\gamma}); (\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma}, \theta_H\underline{\gamma})\}. \tag{28}$$

There is a separating equilibrium in which both agents exert effort. The principal will then offer an expensive separating contract. It is easily observable that when comparing these payments with those obtained in proposition 2, the contract, when there is low quality, offers the same payment. Similar occurs in equilibrium for the inefficient agent with high quality. However, the contract for the efficient agent implies a higher payoff when there is high quality. This growth is due to the additional cost that the principal must incur to ensure that the efficient type makes an effort in equilibrium and does not deviate from the contract designed for itself. The additional transfer for the efficient type with high quality is $\frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H - \theta_L)$. Given that by definition $\frac{(1-\pi_1)}{\pi_1} \leq 1$, it is a smaller proportion of the low quality.

If the district generates a contract for both types of agents to strive for, it cannot offer the cheapest contract and will have to resort to a more expensive one.

4.3.2 Separating contract where none of the agents undertake effort

Next, we will analyze the contract when neither of the two agents makes an effort. When this happens, we remove the moral hazard constraints. Then, the cheapest way is found when the individual rationality constraints are satisfied with equality. Analyzing IR_L, we find:

$$\bar{t}_L \geq \theta_L \overline{\gamma} - \frac{(1-\pi_0)}{\pi_0}(\underline{t}_L - \theta_L \underline{\gamma}) \tag{29}$$

Without loss of generality, the transfer of the efficient agent with low quality, when none of the agents undertake effort (NN), is $\underline{t}_L^{NN} = \theta_L \underline{\gamma}$. Then, according to the equation 29, the payment of the efficient agent with high quality is $\bar{t}_L^{NN} = \theta_L \overline{\gamma}$. On the same way, analysing IR_H the corresponding payments are $\bar{t}_H^{NN} = \theta_H \overline{\gamma}$ and $\underline{t}_H^{NN} = \theta_H \underline{\gamma}$. With these payments, the incentive compatibility constraint for the efficient agent (IC_L^{NN}) is

$$\pi_0 \theta_L \overline{\gamma} + (1-\pi_0)\theta_L \underline{\gamma} \geq \pi_0 \theta_H \overline{\gamma} + (1-\pi_0)\theta_H \underline{\gamma}. \tag{30}$$

Then, with $-\pi_0 \overline{\gamma} \geq (1-\pi_0)\underline{\gamma}$, IC_L^{NN} is not satisfied. For the inefficient agent, IC_H^{NN} is

$$\pi_0 \theta_H \overline{\gamma} + (1-\pi_0)\theta_H \underline{\gamma} \geq \pi_0 \theta_L \overline{\gamma} + (1-\pi_0)\theta_L \underline{\gamma}. \tag{31}$$

So, with $\pi_0 \overline{\gamma} \geq -(1-\pi_0)\underline{\gamma}$, the constraint is fulfilled. The efficient agent has an incentive to deviate and take the inefficient agent's contract. However, this does not occur for the efficient agent.

Then, keeping the payments of the inefficient agent, a costly contract for the principal will be analyzed when the constraint IC_L is met with equality. As follows:

$$\pi_0 \bar{t}_L + (1-\pi_0)\underline{t}_L = \pi_0 \bar{t}_H + (1-\pi_0)\underline{t}_H \tag{32}$$

With the payments to the inefficient agent $(\bar{t}_H^{NN} = \theta_H \overline{\gamma}$ and $\underline{t}_H^{NN} = \theta_H \underline{\gamma})$, and with $\underline{t}_L^{NN} = \theta_L \underline{\gamma}$. The IC_L constraint is

$$\bar{t}_L = \theta_H \overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L). \tag{33}$$

By construction this payment satisfies the constraint IR_L. Now we will analyze with the payments found, if the inefficient agent has incentives to deviate with constraint IC_H.

$$\pi_0(\theta_H\overline{\gamma} - \theta_H\overline{\gamma}) + (1-\pi_0)(\theta_H\underline{\gamma} - \theta_H\underline{\gamma}) \geq \pi_0(\theta_H\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L) - \theta_H\overline{\gamma}) + (1-\pi_0)(\theta_L\underline{\gamma} - \theta_H\underline{\gamma}) \tag{34}$$

Then, the constraint is fulfilled. Therefore, with a more costly contract with the efficient agent equilibrium is found and the principal offers this separating contract.

Proposition 4: Incomplete information contract - Separating contract where none of the agents undertake effort.

The principal offers the contract

$$\{(\bar{t}_L^{NN}, \underline{t}_L^{NN}); (\bar{t}_H^{NN}, \underline{t}_H^{NN})\} = \{(\theta_H\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L), \theta_L\underline{\gamma}); (\theta_H\overline{\gamma}, \theta_H\underline{\gamma})\}. \tag{35}$$

In order to generate an equilibrium contract where neither agent undertakes effort, the principal must generate a costly contract where the efficient type has no incentive to deviate. The principal seeks that the agents strive for the system to operate. This contract establishes the payments to set the conditions for the principal to guarantee a contract with effort.

4.3.3 Separating contracts where the efficient type undertakes effort and the inefficient types does not

It is cheaper for the principal to generate a separating equilibrium where the efficient agent will exert effort, and the inefficient agent will not. Since the inefficient agent will not exert effort, his moral hazard constraint can be eliminated. The moral hazard constraint is satisfied with equality for the efficient type since it is the cheapest way to generate effort in equilibrium. In the same way of section 4.3.1, the moral hazard constraint lead us to the contract for the efficient agent. As the previously shown, the contract for the efficient agent when undertakes effort and the inefficient does not (EN) is $(\bar{t}_L^{EN}, \underline{t}_L^{EN}) = (\frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma}, \theta_L\underline{\gamma})$. Then, the constraint IR_L is satisfied due to the moral hazard rent $\pi_0\psi \geq 0$.

Having characterized these transfers, let us consider the following relaxed problem. In this relaxed problem, we will omit the Incentive-compatibility of the efficient type and work only with the Incentive-compatibility of the inefficient type. We will see what is the contract for the inefficient agent to choose the contract designed for himself, and not the other. The most economical way to do it would be that the Incentive Compatibility of the inefficient agent (IC_H) is fulfilled with equality. As follows:

$$\pi_0\bar{t}_H + (1-\pi_0)\underline{t}_H = \pi_0\bar{t}_L^{EN} + (1-\pi_0)\underline{t}_L^{EN} \tag{36}$$

Arranging terms gives,

$$\bar{t}_H = \frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\theta_L\underline{\gamma} - \frac{(1-\pi_0)}{\pi_0}\underline{t}_H \tag{37}$$

As with the efficient agent, without loss of generality, it is established that $\underline{t}_H^{EN} = \theta_H\underline{\gamma}$. Therefore, the transfer of the inefficient agent with high quality is:

$$\bar{t}_H^{EN} = \frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma} - \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L) \tag{38}$$

Now, let us verify that agents do not deviate. The contract will be verified with the incentive compatibility of the efficient agent and the individual rationality of the inefficient agent. Starting with IR_H, we get:

$$\frac{\psi}{\Delta\pi} - (\theta_H - \theta_L)(\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}) \geq 0 \tag{39}$$

Establishing that $\psi \geq \overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}$, we can conclude that the transfers are found to belong to a separating equilibrium. Now, verifying IC_L gives

$$\frac{\pi_1}{(1-\pi_1)} \geq \frac{\pi_0}{(1-\pi_0)}. \tag{40}$$

Since π_1 is greater than π_0, it is found that the incentive compatibility for the efficient agent is satisfied and, therefore, the efficient agent does not deviate from the contract designed for him.

Proposition 5: Incomplete information contract - Separating contracts where the efficient type undertakes effort and the inefficient types does not.

The principal offers the contract

$$\{(\bar{t}_L^{EN}, \underline{t}_L^{EN}); (\bar{t}_H^{EN}, \underline{t}_H^{EN})\} = \{(\frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma}, \theta_L\underline{\gamma}); (\frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma} - \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L), \theta_H\underline{\gamma})\}. \tag{41}$$

Comparing proposition 5 to Proposition 3, the contract for the efficient type with high quality is more expensive in Proposition 3. This means that for the principal, it is more economical a contract where the efficient type makes an effort, but the inefficient one does not. In the contract of Proposition 3 (both types make an effort), a proportion is added to the payment of the efficient type to guarantee equilibrium. On the contrary, in proposition 5 (only the Efficient type strives), a proportion is subtracted from the payment of the efficient type when there is high quality. It is interesting to note that the proportion that is subtracted is the same as that added in proposition 4 to guarantee equilibrium. In other words, while in Proposition 3, the expected payment becomes more expensive for the principal, in Proposition 5, it becomes cheaper. Thus, this contract for the efficient type ends up being even cheaper than the Moral Hazard contract as stated in proposition 2.

For the principal, the contract for the inefficient type when there is high quality is more economical, compared to the efficient agent contract.

4.3.4 Separating contracts where the inefficient type undertakes effort and the efficient types does not

Finally, I will review the equilibrium where the efficient agent does not make an effort, but the efficient agent does. Since the efficient agent does not exert effort, his moral hazard constraint is eliminated. The most economical way to satisfy the moral hazard constraint of the inefficient agent is when it is satisfied with equality. Therefore, solving similarly as in the section 4.3.3, from MH_H we find the contract when the inefficient agent undertakes effort: $\bar{t}_H^{NE} = \frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma}$ and $\underline{t}_H^{NE} = \theta_H\underline{\gamma}$. Those payoffs are binding with IR_H due to the moral hazard rent. From the constraint IC_L with equality, we obtain the contract for the efficient agent $\underline{t}_L^{NE} = \theta_L\underline{\gamma}$ and $\bar{t}_L^{NE} = \frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L)$.

Arranging terms with IR_L constraint, I get

$$\frac{\psi}{\Delta\pi} + (\theta_H - \theta_L)\overline{\gamma} + (\theta_H - \theta_L)\frac{(1-\pi_0)}{\pi_0}\underline{\gamma} \geq 0. \tag{42}$$

Therefore, the efficient agent accepts the contract. However, when parsing IC_H, I get

$$\frac{\pi_0}{(1-\pi_0)} \geq \frac{\pi_1}{(1-\pi_1)}. \tag{43}$$

According to this result, the inequality is not satisfied, and therefore, the inefficient agent has an incentive to deviate and take the efficient agent's contract.

Proposition 6: Incomplete information contract - Separating contracts where the inefficient type undertakes effort and the efficient types does not.

There cannot be a separating contract in which the inefficient type undertakes effort and the efficient does not.

This separating contract is costly in itself, as it only provides effort for the inefficient agent, who has the highest marginal costs. Now, the inefficient type has incentives to deviate, so there is no equilibrium and there is no separating contract.

4.3.5 The Principal offers a contract with effort to both agents type

We will now analyze under what conditions the principal offers a contract with effort for both agents type. For this we use the contracts determined in proposition 3 and proposition 4, where the first corresponds when both agents exert effort and the latter when neither agent does.

From equation 19, the principal will provide a contract with effort to both agent types when his utility is bigger than without exerting effort. As follows:

$$\begin{aligned}
v(\pi_1(S(\overline{m}) - \bar{t}_L^{EE}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_L^{EE})) \\
+(1-v)(\pi_1(S(\overline{m}) - \bar{t}_H^{EE}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_H^{EE})) \\
\geq \\
v(\pi_0(S(\overline{m}) - \bar{t}_L^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_L^{NN})) \\
+(1-v)(\pi_0(S(\overline{m}) - \bar{t}_H^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_H^{NN}))
\end{aligned} \tag{44}$$

Arranging terms[3], I get

$$\underbrace{\Delta\pi(S(\overline{m}) - S(\underline{m}))}_{\text{Expected benefits.}} \geq \underbrace{\pi_1 \frac{\psi}{\Delta\pi} + \Delta\pi\theta_H(\overline{\gamma} - \underline{\gamma})}_{\text{Expected costs.}}. \tag{45}$$

This is the necessary condition for the principal to offer a contract with effort. It is interesting to note that the principal offers a contract with effort for both types of agent regardless the agent type. Interestingly, this is the same constraint as in the moral hazard contract for the inefficient type.

Proposition 7: The Principal offers a contract with effort to both agents type.

***i)**. When $\frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2(\overline{\gamma}-\underline{\gamma})} \geq \theta_H$ the contract offered is*

$$\{(\overline{t}_L^{EE}, \underline{t}_L^{EE}); (\overline{t}_H^{EE}, \underline{t}_H^{EE})\} = \{(\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma} + \frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H - \theta_L), \theta_L\underline{\gamma}); (\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma}, \theta_H\underline{\gamma})\}.$$

***ii)**. When $\frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2(\overline{\gamma}-\underline{\gamma})} < \theta_H$ the contract offered is*

$$\{(\overline{t}_L^{NN}, \underline{t}_L^{NN}); (\overline{t}_H^{NN}, \underline{t}_H^{NN})\} = \{(\theta_H\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L), \theta_L\underline{\gamma}); (\theta_H\overline{\gamma}, \theta_H\underline{\gamma})\}.$$

Although the condition under which the principal will offer an effort contract is the same in this proposition as in the moral hazard one, the efficient payments are higher when there is high quality. It is mainly because the principal must generate an expensive contract to ensure equilibrium. Thus, given $\frac{(1-\pi_0)}{\pi_0} > \frac{(1-\pi_1)}{\pi_1}$, the addition in the contract of the efficient type is less than that of the inefficient.

[3] Appendix A.

4.3.6 The Principal offers a contract with effort to the efficient agent and an effortless contract to the inefficient one

Now, the principal offers a contract to provide effort when the following inequality is satisfied:

$$
\begin{aligned}
&v(\pi_1(S(\overline{m}) - \overline{t}_L^{EN}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_L^{EN})) \\
&+(1-v)(\pi_1(S(\overline{m}) - \overline{t}_H^{EN}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_H^{EN})) \\
&\geq \\
&v(\pi_0(S(\overline{m}) - \overline{t}_L^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_L^{NN})) \\
&+(1-v)(\pi_0(S(\overline{m}) - \overline{t}_H^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_H^{NN}))
\end{aligned}
\tag{46}
$$

Arranging terms[4], the principal will offer an effort contract if the next inequality is satisfied:

$$
\underbrace{\Delta\pi(S(\overline{m}) - S(\underline{m}))}_{\text{Expected benefits.}} \geq \underbrace{\pi_1 \frac{\psi}{\Delta\pi} + (\pi_1\theta_L - \pi_0\theta_H)\overline{\gamma} - \Delta\pi\theta_H\underline{\gamma} - v(1-\pi_1)\underline{\gamma}(\theta_H - \theta_L) - (1-v)\pi_1\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L)}_{\text{Expected costs.}} \tag{47}
$$

Conditioned to this constraint, the principal offers a contract with effort just for the efficient type. These expected costs are lower than those used in Proposition 7. The condition for the principal to offer an effort contract is greater in Proposition 7 than in Proposition 8.

In this case, the expected costs consider the disutility of the effort and a proportion of high quality. However, there is a reduction in these expected costs for generating a low quality that depends on π_i or the proportion of the type of agent v and $(1-v)$.

[4] Appendix B.

Proposition 8: ***The Principal offers a contract with effort to the efficient agent and an effortless contract to the inefficient one.***

i)*. When*

$$\frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2} \geq \frac{(\pi_1\theta_L-\pi_0\theta_H)}{\Delta\pi}\overline{\gamma}-\theta_H\underline{\gamma}-v\frac{(1-\pi_1)}{\Delta\pi}\underline{\gamma}(\theta_H-\theta_L)-(1-v)\pi_1\frac{(1-\pi_0)}{\pi_0\Delta\pi}\underline{\gamma}(\theta_H-\theta_L),$$

the contract offered is

$$\{(\overline{t}_L^{EN},\underline{t}_L^{EN});(\overline{t}_H^{EN},\underline{t}_H^{EN})\}=\{(\frac{\psi}{\Delta\pi}+\theta_L\overline{\gamma},\theta_L\underline{\gamma});(\frac{\psi}{\Delta\pi}+\theta_L\overline{\gamma}-\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H-\theta_L),\theta_H\underline{\gamma})\}.$$

ii)*. When*

$$\frac{\Delta\pi^2(S(\overline{m})-S(\underline{m}))-\pi_1\psi}{\Delta\pi^2} < \frac{(\pi_1\theta_L-\pi_0\theta_H)}{\Delta\pi}\overline{\gamma}-\theta_H\underline{\gamma}-v\frac{(1-\pi_1)}{\Delta\pi}\underline{\gamma}(\theta_H-\theta_L)-(1-v)\pi_1\frac{(1-\pi_0)}{\pi_0\Delta\pi}\underline{\gamma}(\theta_H-\theta_L),$$

the contract offered is

$$\{(\overline{t}_L^{NN},\underline{t}_L^{NN});(\overline{t}_H^{NN},\underline{t}_H^{NN})\}=\{(\theta_H\overline{\gamma}+\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H-\theta_L),\theta_L\underline{\gamma});(\theta_H\overline{\gamma},\theta_H\underline{\gamma})\}.$$

In a contract with effort only for the efficient type, the conditions by which the principal guarantees an effort contract are less than in a contract where both agents make an effort. On the other hand, as observed, the expected payments are also lower. For this reason, it is cheaper for the principal to have this type of contract.

When expected costs are less than expected benefits, the principal will offer an effort contract. Otherwise, he will offer an effortless contract.

Therefore, in a scenario of imperfect information, the principal chooses the contract where the efficient strives and the inefficient does not.

5 Conclusion

As the research has shown, the contractual model of a public transportation system can affect the quality of the system. It was analyzed based on the moral hazard and adverse selection constraints. The model analysis shows that under specific parameters, the principal (the district) has incentives to provide an effortless contract for the agent (the concessionaire). This consideration arises mainly from the costs incurred by the agent to operate the system and which the district must provide, as well as the self-selection of efficient and inefficient agents. Given the uncertainty, it is more costly for the district to provide this contract because it does not have effective control over the costs incurred by the concessionaire and the effort it makes. Because they are not observable, the principal must provide for these costs in the contract to incentivize good performance. In fact, in most of the cases analyzed, the efficient operator has incentives to deviate and take the inefficient type of contract. Even if the agent is efficient, he receives a higher payment if he had the payment of the inefficient one. This behavior produced the Principal increasing the payments for the efficient agent's contract, which increased his expected costs. The only case where this did not happen was the last case analyzed. The principal offered a contract with effort only for the efficient agent, which implied reducing the payment of the inefficient type.

This work leaves the door open to further evaluate the variables that impact the contractual model and its implication on the system's users. For example, advancing in microfunding the mass of users of the system and how elements such as the price of transport or the ratio of travel distances, using Transmilenio versus using another mode of transport can increase or decrease the number of users of the system. This paper primarily analyzes the game's rules (the contract) beyond the technical aspects, such as the system's operation, bus speed, overcrowding, and track prioritization. Although this paper dedicates to analyzing the perverse incentives that the current system model has in general terms. This paper also seeks to feed the political discussion on renegotiating contracts whether private or public agents should develop passenger transport operation through theoretical elements. The argument considers that the Mayor's Office of Bogotá recently proposed and initiated the development of a public concessionaire that could operate the system.

Appendices

Proof of proposition 7

We analyze the case when the principal offers a contract exerting effort to both agents types. The principal provides a contract with effort if the inequality of equation 45 is satisfied. As follows:

$$\begin{aligned} v(\pi_1(S(\overline{m}) - \bar{t}_L^{EE}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_L^{EE})) \\ +(1-v)(\pi_1(S(\overline{m}) - \bar{t}_H^{EE}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_H^{EE})) \\ \geq \\ v(\pi_0(S(\overline{m}) - \bar{t}_L^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_L^{NN})) \\ +(1-v)(\pi_0(S(\overline{m}) - \bar{t}_H^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_H^{NN})) \end{aligned} \tag{48}$$

Arranging terms, I get:

$$\begin{aligned} v(\pi_1(S(\overline{m}) - (\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma} + \frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H - \theta_L))) + (1-\pi_1)(S(\underline{m}) - \theta_L\underline{\gamma})) \\ +(1-v)(\pi_1(S(\overline{m}) - (\frac{\psi}{\Delta\pi} + \theta_H\overline{\gamma})) + (1-\pi_1)(S(\underline{m}) - \theta_H\underline{\gamma})) \\ \geq \\ v(\pi_0(S(\overline{m}) - (\theta_H\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L))) + (1-\pi_0)(S(\underline{m}) - \theta_L\underline{\gamma})) \\ +(1-v)(\pi_0(S(\overline{m}) - \theta_H\overline{\gamma}) + (1-\pi_0)(S(\underline{m}) - \theta_H\underline{\gamma})) \end{aligned} \tag{49}$$

Then, clearing the internal parentheses, arranging the proportion $\pi(e)$ and arranging the proportion v, the inequality is

$$\begin{aligned}
v\pi_1 S(\overline{m}) - v\pi_1\frac{\psi}{\Delta\pi} - v\pi_1\theta_H\overline{\gamma} - v\pi_1\frac{(1-\pi_1)}{\pi_1}\underline{\gamma}(\theta_H-\theta_L) + v(1-\pi_1)S(\underline{m}) - v(1-\pi_1)\theta_L\underline{\gamma}\\
+\pi_1 S(\overline{m}) - \pi_1\frac{\psi}{\Delta\pi} - \pi_1\theta_H\overline{\gamma} + (1-\pi_1)S(\underline{m}) - (1-\pi_1)\theta_H\underline{\gamma}\\
-v\pi_1 S(\overline{m}) + v\pi_1\frac{\psi}{\Delta\pi} + v\pi_1\theta_H\overline{\gamma} - v(1-\pi_1)S(\underline{m}) + v(1-\pi_1)\theta_H\underline{\gamma})\\
\geq \\
v\pi_0 S(\overline{m}) - v\pi_0\theta_H\overline{\gamma} - v\pi_0\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H-\theta_L) + v(1-\pi_0)S(\underline{m}) - v(1-\pi_0)\theta_L\underline{\gamma}\\
+\pi_0 S(\overline{m}) - \pi_0\theta_H\overline{\gamma} + (1-\pi_0)S(\underline{m}) - (1-\pi_0)\theta_H\underline{\gamma}\\
-v\pi_0 S(\overline{m}) + v\pi_0\theta_H\overline{\gamma} - v(1-\pi_0)S(\underline{m}) + v(1-\pi_0)\theta_H\underline{\gamma})
\end{aligned} \tag{50}$$

Finally, eliminating equal expressions, the condition which the principal offers a contract with effort to both agents type is:

$$\Delta\pi(S(\overline{m}) - S(\underline{m})) \geq \pi_1\frac{\psi}{\Delta\pi} + \Delta\pi\theta_H(\overline{\gamma} - \underline{\gamma}) \tag{51}$$

Arranging terms, the condition is also

$$\frac{\Delta\pi^2(S(\overline{m}) - S(\underline{m})) - \pi_1\psi}{\Delta\pi^2(\overline{\gamma} - \underline{\gamma})} \geq \theta_H. \tag{52}$$

Therefore, if this condition is greater than θ_H, the principal will offer an effortless contract.

Proof of Proposition 8

Now, we analyze the case when the principal offers a contract with effort to the efficient agent and an effortless contract to the inefficient one. The principal offers a contract to provide effort when the following inequality is satisfied:

$$\begin{aligned} & v(\pi_1(S(\overline{m}) - \bar{t}_L^{EN}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_L^{EN})) \\ & +(1-v)(\pi_1(S(\overline{m}) - \bar{t}_H^{EN}) + (1-\pi_1)(S(\underline{m}) - \underline{t}_H^{EN})) \\ & \geq \\ & v(\pi_0(S(\overline{m}) - \bar{t}_L^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_L^{NN})) \\ & +(1-v)(\pi_0(S(\overline{m}) - \bar{t}_H^{NN}) + (1-\pi_0)(S(\underline{m}) - \underline{t}_H^{NN})) \end{aligned} \tag{53}$$

Replacing terms, the inequality to satisfy is

$$\begin{aligned} & v(\pi_1(S(\overline{m}) - (\frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma})) + (1-\pi_1)(S(\underline{m}) - \theta_L\underline{\gamma})) \\ & +(1-v)(\pi_1(S(\overline{m}) - (\frac{\psi}{\Delta\pi} + \theta_L\overline{\gamma} - \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L))) + (1-\pi_1)(S(\underline{m}) - \theta_H\underline{\gamma})) \\ & \geq \\ & v(\pi_0(S(\overline{m}) - (\theta_H\overline{\gamma} + \frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L))) + (1-\pi_0)(S(\underline{m}) - \theta_L\underline{\gamma})) \\ & +(1-v)(\pi_0(S(\overline{m}) - \theta_H\overline{\gamma}) + (1-\pi_0)(S(\underline{m}) - \theta_H\underline{\gamma})). \end{aligned} \tag{54}$$

So, arranging the internal parentheses and arranging v, I get

$$\begin{aligned} & v\pi_1 S(\overline{m}) - v\pi_1\frac{\psi}{\Delta\pi} - v\pi_1\theta_L\overline{\gamma} + v(1-\pi_1)S(\underline{m}) - v(1-\pi_1)\theta_L\underline{\gamma} \\ & +\pi_1 S(\overline{m}) - \pi_1\frac{\psi}{\Delta\pi} - \pi_1\theta_L\overline{\gamma} + \pi_1\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L) + (1-\pi_1)S(\underline{m}) - (1-\pi_1)\theta_H\underline{\gamma} \\ & -v\pi_1 S(\overline{m}) + v\pi_1\frac{\psi}{\Delta\pi} + v\pi_1\theta_L\overline{\gamma} - v\pi_1\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L) - v(1-\pi_1)S(\underline{m}) + v(1-\pi_1)\theta_H\underline{\gamma} \\ & \geq \\ & v\pi_0 S(\overline{m}) - v\pi_0\theta_H\overline{\gamma} - v\pi_0\frac{(1-\pi_0)}{\pi_0}\underline{\gamma}(\theta_H - \theta_L) + v(1-\pi_0)S(\underline{m}) - v(1-\pi_0)\theta_L\underline{\gamma} \\ & +\pi_0 S(\overline{m}) - \pi_0\theta_H\overline{\gamma} + (1-\pi_0)S(\underline{m}) - (1-\pi_0)\theta_H\underline{\gamma} \\ & -v\pi_0 S(\overline{m}) + v\pi_0\theta_H\overline{\gamma} - v(1-\pi_0)S(\underline{m}) + v(1-\pi_0)\theta_H\underline{\gamma}. \end{aligned} \tag{55}$$

Finally, eliminating equal expressions and organizing the inequality according to expected benefits and costs, the condition is the following:

$$\frac{\Delta\pi^2(S(\overline{m}) - S(\underline{m})) - \pi_1\psi}{\Delta\pi^2} \geq \frac{(\pi_1\theta_L - \pi_0\theta_H)}{\Delta\pi}\overline{\gamma} - \theta_H\underline{\gamma} - v\frac{(1-\pi_1)}{\Delta\pi}\underline{\gamma}(\theta_H - \theta_L) - (1-v)\pi_1\frac{(1-\pi_0)}{\pi_0\Delta\pi}\underline{\gamma}(\theta_H - \theta_L) \tag{56}$$

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