# Job Polarization and the Informal Labor Larket

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**Abstract**

This paper analyses the incidence of job polarization in developing and emerging countries, where a substantial fraction of the urban labor force works in the informal sector. I build a general equilibrium model with informality and endogenous occupational choice. Workers in the informal sector do not pay taxes, are less productive, and have the same ability to perform manual tasks. The analytical solution of the model shows that job polarization, driven by a Routine-Biased Technological Change (RBTC), could lead to a decrease in the share of employment in the informal sector and a reduction in the wage inequality at the bottom of the skill distribution.

## 1 Introduction

The polarization of employment in industrialized countries has been a widely studied phenomenon over the last couple of decades. It documents a simultaneous growth in employment and wages of high-skill (problem-solving, creativity, situational adaptability, and in-person interactions) occupations and low-skill (personal services) occupations, compared to middle-skill (production, clerical, and sales) occupations. The main explanation about the drivers of job polarization is the Routine- Biased Technological Change hypothesis, first introduced by Autor et al. (2003), which suggests that technological progress tends to substitute for workers who operate routine tasks. Simultaneously, it increases the relative demand for workers who perform complementary non-routine tasks (abstract and manual tasks).

Most of this literature (see Autor et al. 2003, Autor et al. 2006, Autor and Dorn 2013, Goos et al. 2014, Michaels et al. 2014, Feng and Graetz 2015) focus on analyzing the job polarization in the United States and several European countries. There are a few studies that focus on analyzing job polarization in developing countries. According to the World Development Report (2016), there are signs that employment is also polarizing in several low and middle-income countries. This study finds that the average decline in the share of routine employment has been 7.8

percentage points for the period 1995-2012. Reijnders and de Vries (2018) also find evidence of an increase in the share of non-routine jobs in total employment for a group of advanced and major emerging countries during the period 1999-2007. They find that for all these countries, technological change was the main force behind employment changes.

Labor markets in developing and emerging economies are characterized by the existence of a large informal sector[1]. The informal economy refers to activities that are partially or fully outside the regulatory frameworks. Most workers in this sector are self-employed, and their income comes from operating small unincorporated enterprises. These include activities such as trading on the streets or in markets; sales of cooked food from kiosks; the transport of people or goods by pedal-power or motorbikes; repairing clothes, shoes, or motor scooters; dwelling construction or adding extensions to them; scavenge for reusable waste; or providing a range of personal services like hairdressing, shoe cleaning, street theater, house cleaning, and the like (Blades et al. 2011). In sum, the informal sector can be described as a labor-intensive sector with poor working conditions and relatively lower productivity compared with the formal economy. This sector contributes significantly to employment creation, production, and income generation in developing countries. Nevertheless, a large informal sector has negative consequences for competitiveness and growth and may also be the source of further economic retardation.

This paper aims to analyze the incidence of job polarization in developing countries where, different from developed countries, a substantial fraction of the urban labor force is self-employed in the informal sector performing labor-intensive activities. In order to analyze how technological change, which is one of the leading explanations for job polarization, could affect the structure of employment and wages in emerging countries, I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorm (2013). I consider a labor market in which some workers are low-skill while others are high-skill. I assume that there are three sectors in this economy: the goods sector uses capital and employs workers to perform abstract and routine tasks; the formal service sector employs workers to perform manual tasks, and; the informal service sector employs workers also to perform manual tasks. Workers in the informal service sector can avoid taxation, but are less productive.

A key feature of the model is that households can produce services in the informal sector[2], which are substitutes for services produced in the formal sector, and complement for goods. Additionally, each worker is characterized by a set of skills in performing abstract, routine, and manual tasks. High-skill workers only perform abstract tasks, and low-skill workers can perform both routine and manual tasks. I assume that low-skill workers have the same ability to

[1]Informal employment accounts for more than half of non-agricultural employment in most developing countries: around 68 percent in Asia and the Pacific, 68 percent in the Arab States, 66 percent in sub-Saharan Africa, 65 percent in East and Southeast Asia (excluding China) and around 51 percent in Latin America (International Labor Office, 2018).

[2]Since informal sector is labor-intensive, I assume that informal workers are employed in manual task occupations

accomplish manual tasks in the informal sector, while they are heterogeneous in their ability to perform a routine task or a manual task in the formal sector. This feature implies that workers moving from the goods sector to the formal service sector can keep some of their abilities, while the ones moving to the informal sector will have the same ability as all informal workers.

Additionally, the lack of taxation in the informal sector leads to an inefficient reallocation of employment between the formal and informal service sectors. At the same time, it allows the fiscal policy to have an asymmetric effect on the reallocation of labor between these two sectors.

The analytical solution (asymptotic solution) of the model shows that when the elasticity of substitution between capital and routine labor is higher than the elasticity of substitution between goods and services, the constant decrease in prices of automating routine tasks eventually causes low-skill labor flows from routine tasks to manual tasks. In this case, Routine-biased technological change (RBTC), affecting mainly the production of goods, can increase aggregate demand for services and eventually increase employment and wages in service occupations. Additionally, when goods and services are complements, wages also polarize. These conditions for job and wage polarization are the same found by Author and Dorn 2013, in the case without informality.

I find that employment and wages in both the formal and the informal service sectors increase due to the increased demand for services. The allocation of labor in the service sector depends on the level of labor income taxes, the degree of substitution between the two types of services, and the level of efficiency in the formal service sector. I find that the relative units of efficient labor in the informal sector (as well as the share of informal employment in the service sector) decrease with technological progress. This result is driven by the assumption that workers, previously working in routine tasks, still can use some of their skills when they move to the formal service sector, while all workers in the informal sector have the same ability to accomplish manual tasks. Therefore, the increasing demand for labor in the informal service sector requires a higher increase in wages to compensate for the loss of skills from working in this sector. As a consequence, relative wages in the informal sector increase, as well as their relative prices. It follows that the increase in relative prices of informal services decreases their relative demand and therefore the relative demand for workers in the service sector.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the analytical solution of the model for the labor allocation. Section 4 derives the analytical solution of the model for the relative wages. Section 5 presents the results of the model simulations, and Section 6 concludes.

# 2 Model

In this section, I develop a general equilibrium model with three sectors and endogenous occupational choice based on Author and Dorn (2013). The goods sector employs high-skilled workers in abstract jobs $L_a$, low-skill workers in routine tasks $L_r$, and Capital $K$. The service sector is composed of two sectors: formal and informal, which use only unskilled labor in manual tasks, $L_{sf}$ and $L_{si}$ respectively. I assume that high-skilled workers have homogeneous skills at performing abstract tasks, while their skills are heterogeneous in performing routine tasks and manual tasks in the formal sector. As in Author and Dorn (2013), task-biased technological change (TBTC) is modeled as an exogenous fall in the price of capital $P_{k.}$

## 2.1 The goods sector

The goods sector is perfectly competitive and uses abstract tasks $L_a$, routine tasks $L_r$, and capital $K$ to produce $Y_g$ units of goods. As in Autor and Dorn (2013), I assume the good sector uses the following technology:

$$Y_g = A_g \left(L_a\right)^{(1-\beta)} X^{\beta} = A_g \left(L_a\right)^{(1-\beta)} \left[\left(\mu_r L_r\right)^{\nu} + \left(\mu_k K\right)^{\nu}\right]^{\frac{\beta}{\nu}}, \tag{1}$$

where

$$X_t = \left[\left(\mu_r L_r\right)^{\nu} + \left(\mu_k K\right)^{\nu}\right]^{\frac{1}{\nu}},$$

with $\beta \in (0,1)$ and $\nu \in (0,1)$. From equation (1), the elasticity of substitution between abstract labor and the total routine task is equal to 1, while the elasticity of substitution between routine labor and computer capital is $\frac{1}{1-\nu} > 1$. As a result, $K$ is a relative complement to abstract labor and a relative substitute for routine labor.

Firms in the goods sector solve the following maximization problem:

$$Max\ \Pi_t^G = P_g A_g L_a^{1-\beta} X^{\beta} - p_k K - w_r L_r - w_a L_a \tag{2}$$

The first-order conditions for problem (2) with respect to abstrac labor $L_a$, routine labor $L_r$, and capital $K$ respectively are given by

$$(1-\beta)\frac{Y_g}{L_a} = w_a, \tag{3}$$

$$\kappa_R \frac{Y_g}{L^r}\left(\frac{L_r}{X}\right)^{\nu} = w_r, \tag{4}$$

$$\kappa_k \frac{Y_g}{K} \left( \frac{K}{X} \right)^{\nu} = p_t^k, \tag{5}$$

where $\kappa_R = \beta\mu_r^\nu$ and $\kappa_k = \beta\mu_k^\nu$. I have normalized $P_g = 1$.

As in Autor and Dorn (2013), TBTC is modeled as an exogenous fall in $P_{k.}$

## 2.2 The service sector

The service sector uses manual tasks to produce services in a competitive environment. There are two types of firms in this sector: formal and informal. Workers and firms in the formal service sector have to pay labor income taxes, but due to the better employment conditions workers are more productive. Workers in the informal sector can avoid taxes but are less productive.

### 2.2.1 Formal service sector

The formal service sector uses only manual labor as input. The production function writes:

$$Y_{sf} = A_{sf} L_{sf}$$

where $L_{sf}$ is the total efficient units of manual labor employed in the formal service sector, and $A_{sf}$ is the common labor productivity in this sector.

Firms solve the following maximization problem:

$$Max\ \Pi_{sf} = P_{sf} Y_{sf} - w_{sf} L_{sf},$$

In equilibrium, wage per efficiency unit of labor in the formal service sector is equal to their marginal productivity:

$$w_{sf} = P_{sf} A_{sf} \tag{6}$$

Since workers in the formal service sector differs in their skill to perform manual task activities, the wage of a worker with $\eta_i^\theta$ efficiency units of manual labor is $\eta_i^\theta w_{sf}$.

### 2.2.2 Informal service sector

The informal sector is labor-intensive, and uses only manual labor as input. The total amount of informal services produced in the economy is:

$$Y_{si} = A_{si}L_{si}$$

where $L_{si}$ is the total units of low-skill labor employed in the informal service sector, and $A_{si}$ is the labor productivity in this sector. With $A_{si} < A_{sf}$. I assume that each worker is equally talented in providing low-skilled informal services.

The profit maximization problem writes:

$$Max\ \Pi_{si} = P_{si}Y_{si} - w_{si}L_{si}$$

The first-order condition with respect to $L_{si}$ implies:

$$w_{si} = P_{si}A_{si} \tag{7}$$

Note that since everyone has the same skill in this sector, everyone working in the low-skilled informal service sector earns the same wage.

## 2.3 Capital

Capital is produced and supplied in a competitive framework. As in Author and Dorn (2013), The production technology of capital is described by

$$K_t = Y_k \frac{e^{\delta_k t}}{\Theta} \tag{8}$$

where $Y_k$ is the amount of final goods used to produce capital, $\delta_k > 0$, and $\theta = e^{\delta_k}$ is an efficiency term. Capital fully depreciates at each period. $\delta_k$ represents the growth rate of capital productivity, and $t$ represents the period of time.

The price of capital is equal to its marginal cost.

$$\begin{aligned} p_k &= \tfrac{Y_k}{K} \\ &= \Theta e^{-\delta_k t} \end{aligned} \tag{9}$$

Note that as time passes, the price of capital falls to zero asymptotically.

## 2.4 Occupational choice

I assume that every member of the household works full-time in one of the three market sectors. Low-skill workers are heterogeneous in their endowment of efficiency units of labor $\eta$, which is drawn from a time-invariant distribution $f(\eta)$. The endowment $\eta$ determines the productivity of each individual in each sector. I assume that $\eta$ denotes the worker's efficiency units of labor in routine tasks, while $\eta^\theta$ denotes the worker's efficiency units of labor in manual tasks in the formal service sector. Workers in the informal service sector have homogeneous skills in performing manual tasks, each worker in this sector supply a unit mass of manual labor. This assumption implies that a worker with endowment $\eta$ has individual productivity, measured in efficiency units, of $\eta$ performing routine tasks, $\eta^\theta$ performing manual tasks in the formal service sector, with $\theta \in (0, 1)$, and 1 performing manual tasks in the informal service sector. Note that workers with an endowment $\eta > 1$ are more efficient in the formal service sector than in the informal service sector. This assumption is motivated by the fact that most of the workers in the formal sector work in larger companies with better working conditions, and in most cases, they have access to training programs. While workers in the informal sector are self-employed. Therefore, it is realistic to assume that some workers are more skilled performing manual task activities in the formal sector than in the informal sector.

I assume that workers in the formal service sector (goods and formal services) have to pay labor income taxes, while workers in the informal sector can avoid taxation.

Since any low-skill worker can work in any of the three sectors, it is optimal for each worker to choose the type of work and the sector that provides her with the highest wage. Therefore, it is optimal for an individual $i$ endowed with $\eta_i$ efficiency units of labor to work in the goods sector only if:

$$\eta_i(1-\tau)W_r \geq max\left[\eta_i^\theta(1-\tau)W_{sf}, W_{si}\right]$$

$$\eta^* = \left(\frac{W_{sf}}{W_r}\right)^{\frac{1}{1-\theta}} \tag{10}$$

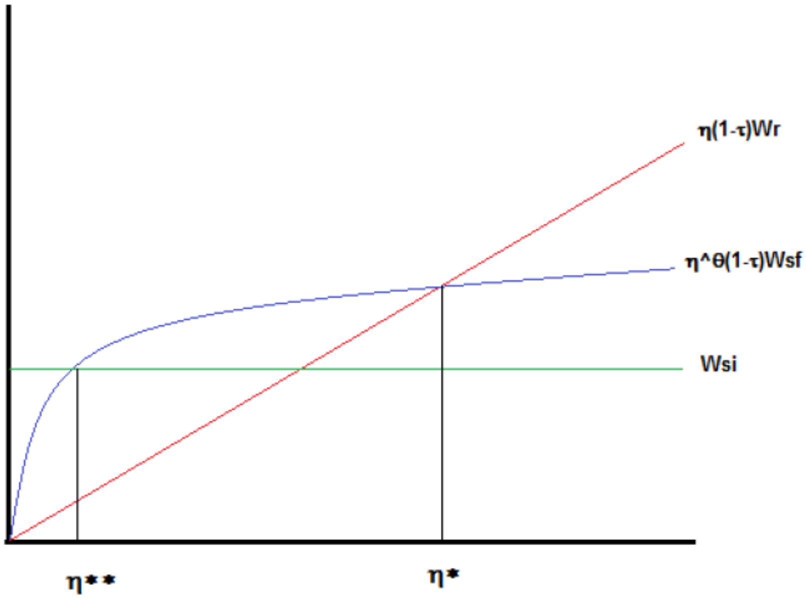
Workers whose efficiency level is lower than $\eta^*$ have to decide whether to work in the formal service sector or the informal service sector. Then, it is optimal for an individual to work in the formal service sector only if:

$$\eta_i^\theta(1-\tau)W_{sf} \geq W_{si}$$

$$\eta^{**} = \left(\frac{W_{si}}{(1-\tau)W_{sf}}\right)^{\frac{1}{\theta}} \tag{11}$$

Figure 1 shows this endogenous occupational choice. Low-skill workers whose efficiency units of labor are higher than $\eta^*$ sort themselves into the goods to perform routine tasks, and those with efficiency units of labor lower than $\eta^{**}$ sort themselves into the informal service sector.

**Figure 1. Optimal Labor Choice**




I assume that $\eta$ is distributed Uniform on the interval $[0,\ \eta^{max}]$, with density and distribution functions $F(\eta)$ and $f(\eta)$ defined as follows:

$$F(\eta) = \begin{cases} \frac{\eta}{\eta^{max}} & 0 \leq \eta \leq \eta^{max} \\ 1 & \eta > \eta^{max} \end{cases} \tag{12}$$

$$f(\eta) = \begin{cases} \frac{1}{\eta^{max}} & 0 \leq \eta \leq \eta^{max} \\ 0 & \eta < 0 \; or \; \eta > \eta^{max} \end{cases}$$

The endogenous occupational choice of low-skill workers determines the effective labor supply in each sector. The aggregate efficiency units supplied to the routine tasks, manual tasks in the formal and informal service sector can be written, respectively, as follows:

$$L_r = \int_{\eta^*}^{\eta^{max}} \frac{\eta}{\eta^{max}} d\eta = \left[ \frac{(\eta^{max})}{2} - \frac{(\eta^*)^2}{2\eta^{max}} \right]. \tag{13}$$

$$L_{sf} = \int_{\eta^{**}}^{\eta^*} \frac{\eta^\theta}{\eta^{max}} d\eta = \left[ \frac{(\eta^*)^{\theta+1}}{(\theta+1)\,\eta^{max}} - \frac{(\eta^{**})^{\theta+1}}{(\theta+1)\,\eta^{max}} \right], \tag{14}$$

$$L_{si} = \int_0^{\eta^{**}} \frac{1}{\eta^{max}} d\eta = \frac{\eta^{**}}{\eta^{max}} \tag{15}$$

## 2.5 Households

I assume a representative household, whose members derive utility from the consumption of goods and formal and informal services. The household collects the wages of all its members and allocate total income to maximize the following utility function:

$$U = Ln\left(\left[\gamma_g C_g^{\rho} + \gamma_s C_s^{\rho}\right]^{\frac{1}{\rho}}\right), \tag{16}$$

with

$$C_s = \left(a_f C_{sf}^{\psi} + a_i C_{si}^{\psi}\right)^{\frac{1}{\psi}}$$

subject to the budget constraint.

$$P_g C_g + P_{sf} C_{sf} + P_{si} C_{si} = (1-\tau)\left(w_a L^a + w_r L^r + w_{sf} L^{sf}\right) + w_{si} L^{si} + T \tag{17}$$

The elasticity of substitution between $C_g$ and $C_s$ is equal to $\sigma_c = \frac{1}{1-\rho}$. The elasticity of substitution between $C_{sf}$ and $C_{si}$ is equal to $\sigma_s = \frac{1}{1-\psi}$. I assume that goods and services are complements, $\sigma_c < 1$, while formal and informal services are substitutes, $\sigma_s > 1$.

The First order conditions of this maximization problem are as follows:

$$\frac{C_g}{C_s} = \left(C_{sf}^{1-\psi} \frac{\gamma_g}{\psi a_f \gamma_s} \frac{P_{sf}}{P_g}\right)^{\sigma_c} \tag{18}$$

$$\frac{C_{si}}{C_{sf}} = \left(\frac{a_i}{a_f} \frac{P_{sf}}{P_{si}}\right)^{\sigma_s} \tag{19}$$

The left-hand side of the equation (18) represents the relative supply, while the right-hand side is the relative demand for goods compared to services. In the same way, equation (19) equals the relative supply of informal services, compared to formal services, with its relative demand.

## 2.6 Government

Government always runs a balanced budget. Therefore, in each period, government budget constraint is as follows[3]:

$$T = \tau\left(w_a L_a + w_r L_r + w_{sf} L_{sf}\right) \tag{20}$$

[3]For comparative reasons, in the simulation of the model, I also analyze the case when the tax rate varies in order to balance the constant subsidies and the fluctuating tax base.

## 2.7 Clearing conditions

$$C_g = Y^g - p_k K \tag{21}$$

$$C_{sf} = Y_{sf} \tag{22}$$

$$C_{si} = Y_{si} \tag{23}$$

# 3 Asymptotic Labor Allocation

In this section, I determine the log run allocation (asymptotic equilibrium) of low-skill workers in the goods, formal and informal service sectors. Given that the price of computer capital $p_k$ converges to zero asymptotically, computer capital converges to infinity:

$$\lim_{t\to\infty} K_t = \infty \tag{24}$$

Since the maximum value of $L^r$ is $\frac{(\eta^{max})}{2}$, the production of $X$ will be asymptotically determined by the capital level ($X \sim \alpha_k K$). It implies that:

$$\lim_{t\to\infty} \frac{X_t}{\mu_k K_t} = 1 \tag{25}$$

and

$$Y_g \sim (\mu_k K)^{\beta}. \tag{26}$$

Replacing equation (26) into equation (21), and using equations (24) and (25), I show in Appendix A1, that the solution for the asymptotic supply of low-skilled labor in the goods and formal and informal service sector is as follows[4]:

$$L_r = \begin{cases} 0 & if \ \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \overline{L}_r \in \left(0, \frac{\eta^{max}}{2}\right) & if \ \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ \frac{\eta^{max}}{2} & if \ \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \tag{27}$$

$$\tau = \frac{T}{(w_a L_a + w_r L_r + w_{sf} L_{sf})}$$

[4]Here $\hat{L}_{si}$ is the solution to the equation $(\eta^{max})^{\theta+1} = (\theta+1)\,\eta^{max}\left(\frac{\hat{L}_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}\hat{L}_{si}\right)^{\theta+1}$. with $C_{11} = \left(\left(\frac{A_{si}}{A_{sf}}\right)^{\psi} \frac{a_i}{a_f(\eta^{max})^{\theta}(1-\tau)}\right)^{\frac{1}{\theta+1-\psi}}$. And $\bar{L}_{si}$ is the solution to the equation $\frac{gg(L_{si})^{1-\nu}(C_s(L_{si}))^{(\rho-1)}(L_{sf})^{\psi-1}}{\left((\theta+1)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max}L_{si})^{\theta+1}\right)^{-1}} = \Psi_1$, where $\Psi_1 = \frac{\gamma_g \kappa_R A_g \mu_k^{\beta(\rho-1)+(\beta-\nu)}}{(1-\beta)^{1-\rho}\psi a_f \gamma_s A_{sf}^{\psi}}$.

where $\bar{L}_r = gg(\bar{L}_{si})$.

$$L_{si} = \begin{cases} \hat{L}_{si} & if \ \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}, \\ \bar{L}_{si} \in \left(0, \ \hat{L}_{si}\right) & if \ \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \ \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \tag{28}$$

$$L_{sf} = \begin{cases} \hat{L}_{sf} & if \ \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \bar{L}_{sf} \in \left(0, \ \hat{L}_{sf}\right) = \Psi & if \ \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \ \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \tag{29}$$

where $\hat{L}_{sf} = \left(\frac{\hat{L}_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}$ and $\bar{L}_{sf} = \left(\frac{\bar{L}_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}$. $C_{11} = \left(\left(\frac{A_{si}}{A_{sf}}\right)^{\psi} \frac{a_i}{a_f(\eta^{max})^{\theta}(1-\tau)}\right)^{\frac{1}{\theta+1-\psi}}$

Equations (28), (29), and (27) show that, as in Author and Dorn (2013), the allocation of low-skill labor between manual tasks and routine tasks depends on the relative magnitudes of the consumption and production elasticities, scaled by the share of the routine aggregate in goods production ($\beta$). When $\frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}$ (the production elasticity, scaled by $\beta$, exceeds the consumption elasticity) the demand for routine labor decreases with technological progress, and the relative demand for low-skill labor in both the formal and the informal service sector increases. Hence, the constant decrease in prices of automating routine tasks eventually causes all low-skill labor to flow from routine tasks to manual tasks. As a result, employment in the formal and informal service sectors increases.

The allocation of labor between the formal and the informal service sector is determined by the equation $L_{sf} = \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}$. It depends on the differences in productivity ($A_{si}$ and $A_{sf}$), the level of labor income taxes paid by formal workers $\tau$, consumer preferences ($\psi$, $a_f$, and $a_i$), and efficiency of labor in the formal sector $(\eta^{max})^{\theta}$. The higher the aggregate labor productivity and efficiency in the formal service sector, and the lower the labor income taxes, the higher the relative allocation of labor in the formal service sector.

Replacing equations (24) and (25) into equation (19), I have that the evolution of the ratio between the efficient units of labor in the informal and the formal sector depends exclusively on the evolution of the wage ratio between the two sectors.

$$\frac{L_{si}}{L_{sf}} = \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{a_i}{a_f}\frac{w_{sf}}{w_{si}}\right)^{\sigma_s}. \tag{30}$$

Additionally, from equation (11 ) the wage ratio $\frac{w_{si}}{w_{sf}}$ can be expressed as follows:

$$\frac{w_{si}}{w_{sf}} = (\eta^{**})^{\theta} (1 - \tau) \tag{31}$$

The wage ratio between the formal and the informal service sector depends on the level of labor income tax paid by formal workers and the efficiency level of the marginal worker, $(\eta^{**})^{\theta}$. The assumption that workers in the formal service sector are heterogeneous in their skill to perform manual tasks implies that this ratio is not constant and varies with the efficiency units of labor in the formal service sector. The higher the value of $\theta$, the higher the worker's skills in the formal service sector (for those workers whose skill level is $\eta > 1$) compared with their skills in the informal service sector, and therefore the higher the relative informal wage. Note that for the case when all workers have the same ability to perform manual tasks activities in both service sectors (when $\theta = 0$ ), the wage ratio in the service sector $\frac{w_{si}}{w_{sf}}$ and hence the employment ratio $\frac{L_{si}}{L_{sf}}$ are constant and independent from technological progress.

Low-skill workers leaving the goods sector and entering the formal service sector are the ones that have relatively high efficiency, and those leaving the formal service sector are the ones that have relatively low efficiency. As a consequence, the average efficiency in the formal service sector increases.

# 4 Asymptotic wage inequality.

In this section, I study the evolution of wage inequality, measured by the evolution of manual to abstract, and manual to routine wage ratios, as well as the evolution of formal to informal wage ratios in the service sector. Using equations (4) and (18), and replacing the optimal conditions for wages in the service sector, (6) and (7), I show in Appendix A2 that the wage ratio $\frac{w_{sf}}{w_r}$ can be written as follows:

$$\frac{w_{sf}}{w_r} = \left( (\theta + 1) \eta^{max} CC_1 L_{sf} + (\eta^{max} L_{si})^{\theta+1} \right)^{-1}$$

Given the asymptotic labor allocation when $K_t \to \infty$

$$\frac{w_{sf}}{w_r} = \begin{cases} (\eta^{max})^{1-\theta} & if \;\; \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \left( (\theta + 1) \eta^{max} CC_1 \bar{L}_{sf} + \left(\eta^{max} \bar{L}_{si}\right)^{\theta+1} \right)^{-1} \in \left(0, (\eta^{max})^{1-\theta}\right) & if \;\; \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \;\; \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \tag{32}$$

On the other side, the wage ratio $\frac{w_{si}}{w_r}$ can be written as follows (see appendix A2 for the complete derivation):

$$\frac{w_{si}}{w_r} = \left(\frac{L_{si}}{L_{sf}}\right)^{\psi-1} \Psi_1 \left(\left(\theta+1\right)\eta^{max} L_{sf} + \left(\eta^{max} L_{si}\right)^{\theta+1}\right)^{-1}$$

Similarly, given the asymptotic labor allocation when $K_t \to \infty$ I obtain:

$$\frac{w_{si}}{w_r} = \begin{cases} \left(\left(\theta+1\right)\eta^{max}\hat{L}_{sf} + \left(\eta^{max}\hat{L}_{si}\right)^{\theta+1}\right)^{-1} \left(\frac{\hat{L}_{si}}{\hat{L}_{sf}}\right)^{\psi-1} \Psi_1 & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \left(\left(\theta+1\right)\eta^{max}\bar{L}_{sf} + \left(\eta^{max}\bar{L}_{si}\right)^{\theta+1}\right)^{-1} \left(\frac{\bar{L}_{si}}{\bar{L}_{sf}}\right)^{\psi-1} \Psi_1 & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \tag{33}$$

Equations (32) and (33) imply that the relative wage paid to manual tasks versus routine tasks increases with technological progress, for the case when the production elasticity (scaled by $\beta$) excess the consumption elasticity. Since Task-biased technological progress affects mainly the production of goods, and goods and services are complements, the aggregate demand for services also increases, consequently increasing the unit wage in service occupations relative to the unit wage of routine labor in the goods sector.

Additionally, from equations (31) and (15), the unit wage of manual labor in the informal service sector relative to the unit wage of manual labor in the formal service sector can be expressed as follows:

$$\frac{w_{si}}{w_{sf}} = \left(\eta^{max} L_{si}\right)^{\theta} \left(1-\tau\right) \tag{34}$$

The evolution of the wage ratio $\frac{w_{si}}{w_{sf}}$ is thus determined by the evolution of the employment share in the informal service sector $L_{si}$. Scaled by the level of taxes ($\tau$) and the efficiency levels in the formal service sector ($\theta$). From equations (34) and (28), the asymptotic wage ratio $\frac{w_{si}}{w_{sf}}$ is determined as follows

$$\frac{w_{si}}{w_{sf}} = \begin{cases} \left(\eta^{max}\hat{L}_{si}\right)^{\theta} \left(1-\tau\right) & if \quad \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ \left(\eta^{max}\bar{L}_{si}\right)^{\theta} \left(1-\tau\right) & if \quad \frac{1}{\sigma_c} = \frac{\beta-\nu}{\beta} \\ 0 & if \quad \frac{1}{\sigma_c} < \frac{\beta-\nu}{\beta} \end{cases} \tag{35}$$

Given that $\hat{L}_{si} > \bar{L}_{si}$, equation (35) implies that the wage ratio between informal and formal occupations in the service sector increases with technological progress, for the case when $\frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}$. When the production elasticity (scaled by $\beta$) is bigger than the consumption elasticity, technological progress raises the relative demand for low-skill labor in the formal and the informal service sectors, which requires a rise in wages in both sectors. Note that a worker endowed with $\eta_i$ efficiency units of labor has individual labor productivity of $\eta_i^{\theta}$ when he works in the formal service sector, while the individual labor productivity of the same worker in the informal service sector is equal to 1. Since a worker with $\eta > 1$, is more efficient when he works in the formal service sector, the increase in wages in the informal service sector has to be higher than the one in the formal service sector in order to compensate for this loss of skills. The higher the worker's skill in the formal service sector ($\theta$), the higher the increase in the wage ratio $\frac{w_{si}}{w_{sf}}$.

From equation (30), it is now possible to analyze the evolution of the employment composition in the service sector. This equation shows that when formal and informal services are substitutes (when $\frac{1}{1-\psi} > 1$), the efficiency units of labor employed in the informal sector relative to those employed in the formal service sector $\frac{L_{si}}{L_{sf}}$ decrease with the relative wage in the informal sector. The increase in $\frac{w_{si}}{w_{sf}}$ increases the relative price of informal services, thus lowering the relative demand for these services and therefore the relative demand for informal labor.

Finally, I determine the evolution of the wage ratio between wages in the service sector and wages in the abstract task. As Author and Dorn (2013), wage polarization will occurs when $\frac{w_{sf}}{w_r}$ and $\frac{w_{sf}}{w_r}$ rises, and $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ are either stable or declining.

In Appendix A2, I show that the wage ratios $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{sI}}$ can be written as follows:

$$\frac{w_a}{w_{sf}} = \kappa_{af} K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)} C_s^{1-\rho} L_{sf}^{1-\psi}$$

$$\frac{w_a}{w_{si}} = \kappa_{ai} K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)} C_s^{1-\rho 1} L_{si}^{1-\psi}$$

with $\kappa_{af} = \frac{(1-\beta)(\mu_k)^\beta \gamma_g \left((1-\beta)(\mu_k)^\beta\right)^{\rho-1}}{(\gamma_s) a_f \psi A_{sf}^\psi}$, and $\kappa_{ai} = \kappa_{af}\left(\frac{A_{sf}}{A_{si}}\right)^\psi$.

As capital converge to infinity, the evolution of $\frac{w_a}{w_{sf}}$ is determined by the elasticity of substitution between goods and services, $\sigma_c$. Specifically,

$$\frac{w_a}{w_{sf}} = \begin{cases} \infty & if \quad \sigma_c > 1, \\ \kappa_{af} \hat{C}_s^{1-\rho} \hat{L}_{sf}^{1-\psi} & if \quad \sigma_c = 1 \quad with\ \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ 0 & if \quad \sigma_c < 1 \end{cases} \tag{36}$$

$$\frac{w_a}{w_{sf}} = \begin{cases} \infty & if \quad \sigma_c > 1, \\ \kappa_{ai} \hat{C}_s^{1-\rho} \hat{L}_{si}^{1-\psi} & if \quad \sigma_c = 1 \quad with\ \frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta} \\ 0 & if \quad \sigma_c < 1 \end{cases} \tag{37}$$

Equations (36) and (37) show that when goods and services are gross complements $\sigma_c < 1$, the ratio between abstract tasks wages and manual tasks wages converge to zero. As a result, the model implies overall wage and job polarization due to a task-biased technological change.

# 5 Model simulations

In this section, I analyze the evolution of employment shares and sectoral wages as the price of capital converges to zero. I simulate the evolution of employment and wages under different values of the elasticity of substitution between formal and informal services $\sigma_s$. As equation (30) shows, the elasticity of the relative employment in the informal sector to the relative informal wages depends on $\sigma_s$. I also consider a different fiscal policy where labor income taxes are variable. The purpose of this exercise is to analyze how the job polarization process in developing and emerging countries, would affect the informal sector depending on preferences, and tax policies within each country.

All parameters are time-invariant, and the only exogenous change over time is the price of capital. I simulate the model for the case when the the production elasticity (scaled by $\beta$) excess the consumption elasticity (it means when $\frac{1}{\sigma_c} > \frac{\beta-\nu}{\beta}$). Under this scenario, independent from the values $\sigma_c$, $\beta$, and $\nu$, technological progress always leads to job polarization. For the parameters describing preferences, I set the elasticity of substitution between goods and services at $\sigma_c = 0.4$, and the elasticity of substitution between formal and informal services at $\sigma_s = \frac{1}{0.7}$. The share of routine aggregate in goods production is set at $\beta = 0.6$. Labor income taxes are fix at $\tau = 0.18$. The rest of the parameters are set at values commonly found in the literature.

**Table 1. Parameter values: benchmark calibration**

| Name | Symbol | Value |
|---|---|---|
| Share of routine aggregate in goods production | $\beta$ | 0.6 |
| Elasticity of substitution between consumption and services | $\sigma_c$ | 0.4 |
| Elasticity of substitution between formal and informal services | $\sigma_s$ | 1/0.7 |
| Inverse of the elasticity of substitution between routine labor and capital | $\nu$ | 0.5 |
| Parameter associated to the skill level of formal workers in the service sector | $\theta$ | 0.7 |
| Maximum skill level | $\eta^{max}$ | 2 |
| Labor income tax | $\tau$ | 0.18 |
| Parameter reflecting technological progress | $\delta_k$ | 0.01 |
| Relative weigh of goods ans services in the utility function | $\gamma_g = \gamma_s$ | 0.5 |
| Relative weigh of formal and informal services in the utility function | $a_f = a_i$ | 0.5 |
| Relative weigh of routine labor and capital in the production function of goods | $\mu_r = \mu_k$ | 0.5 |
| Transfers | $T$ | 0.145 |
| Aggregate labor productivity in the formal service sector | $A_{sf}$ | 1 |
| Aggregate labor productivity in the informal service sector | $A_{si}$ | 0.7 |

Note that $L_j$ represents the total amount of efficiency unit of labor employed in sector $j$, and $w_j$ represents the wage per efficiency unit of labor in that sector, $j \in (r,\ si,\ sf)$. For the simulation, I also analyze the evolution of the employment shares and the average wages in each sector.

The low-skill employment share $N_j$ is the mass of individuals who supply their labor in sector $j$, which are defined as follows:

$$N_r = \int_{\eta^*}^{\eta^{max}} f(\eta)d\eta = \left[1 - \frac{\eta^*}{\eta^{max}}\right] \tag{38}$$

$$N_{sf} = \int_{\eta^{**}}^{\eta^*} f(\eta)d\eta = \frac{1}{\eta^{max}}\left[\eta^* - \eta^{**}\right] \tag{39}$$

$$N_{si} = \int_{0}^{\eta^{**}} f(\eta)d\eta = \frac{\eta^{**}}{\eta^{max}} \tag{40}$$

Additionally, the average wage in sector $j$ is defined as the total labor income in sector $j$, divided by the mass of people working in this sector:

$$\bar{w}_j = \frac{w_j L_j}{N_j}, \qquad for \quad j \in (r, sf, si).$$

and in terms of the wage ratio between different sectors:

$$\frac{\bar{w}_j}{\bar{w}_i} = \frac{w_j}{w_i}\frac{L_j}{N_j}/\frac{L_i}{N_i}, \quad for \quad j \neq i.$$

Figure 2 shows the evolution of low-skill employment in the goods and the service sectors, under the benchmark calibration, when the price of capital converges to zero. In the goods sector, routine labor is substituted with capital, which decreases employment and the efficiency units of routine labor ($N_r$ and $L_r$) in this sector. At the same time, the increase in the aggregate demand for services increases both raw employment ($N_{si}$ and $N_{sf}$) and the efficiency units of labor in the formal and informal sectors ($L_{si}$ and $L_{sf}$) . Consequently, the constant decrease in prices of automating routine tasks eventually causes all low-skill labor to flow from routine tasks to manual tasks. Moreover, it is worth noticing from Figure 2 that under this scenario, job polarization leads to a decrease in the ratio between the efficient units of labor in the informal and the formal sector $L_{si}/L_{sf}$. This decrease is due to the increase of relative informal unit wages in the service sector (see equation (30) and Figure 3). Additionally, by comparing the evolution of $L_{si}/L_{sf}$ versus the relative employment share of the informal sector $N_{si}/N_{sf}$, it is possible to notice that the decrease on $L_{si}/L_{sf}$ is higher than the decrease on $N_{si}/N_{sf}$, which implies that the formal service sector becomes more efficient.

**Figure 2. Evolution of employment**

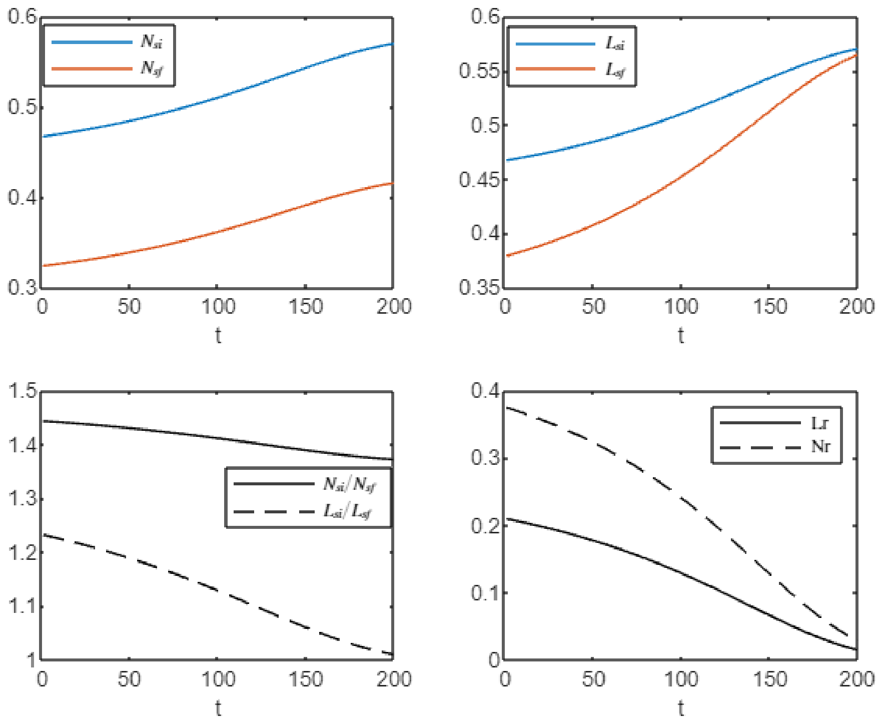



Figure 3 shows the evolution of relative prices and wages as the price of capital decreases. It shows that the ratio between informal and formal wages in the service sector ($w_{si}/w_{sf}$) increases. As was stated previously in section 4, technological progress raises the relative demand for low-skill labor in both service sectors, which requires a rise in wages in each sector. Workers with ability $\eta > 1$ are more skilled when they work in the formal sector. Therefore, the increase in wages in the informal service sector has to be higher than the increase in the formal service sector to compensate for this loss of skills. As a result, the increase in the relative informal wage increases the relative price of informal services and, as a consequence, it reduces their relative demand.

In terms of average wages, it is worth noticing that the relative average wage in the informal service sector versus the average wage in the formal sector, $\bar{w}_{si}/\bar{w}_{sf}$, is almost constant when the price of capital decreases. It implies that, on average, the wage differences between formal and informal workers remain constant with technological progress. Notice also that the wage ratio between manual tasks in the service sector and routine tasks increases, while the wage ratio between abstract tasks and manual tasks increases initially and then converges to zero. This result shows that when $\sigma_c < 1$, wages polarize in the long run(see equation (36)).

**Figure 3. Evolution of relative wages and prices in the service sector**

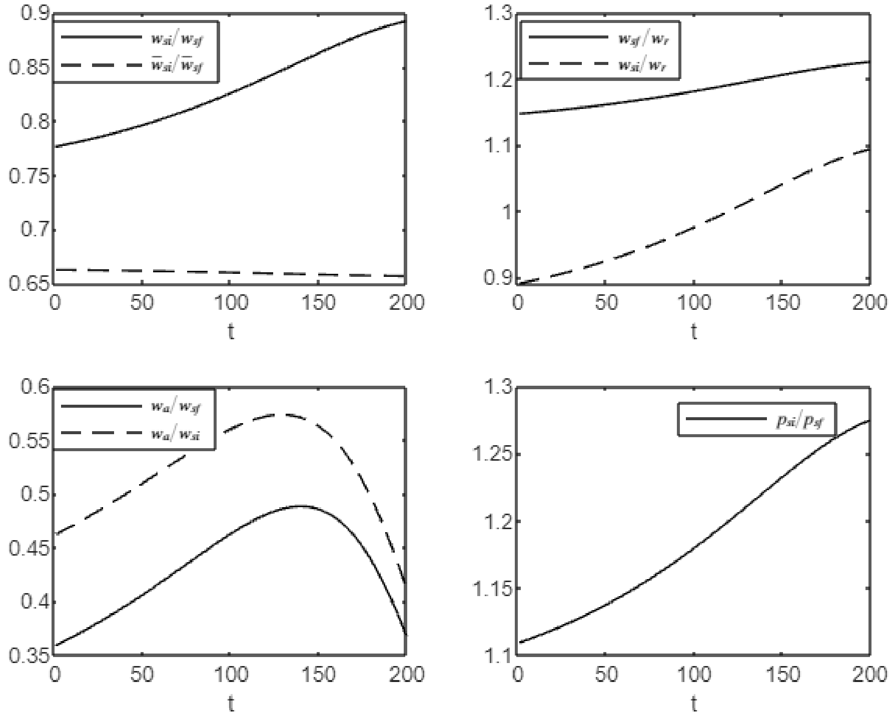
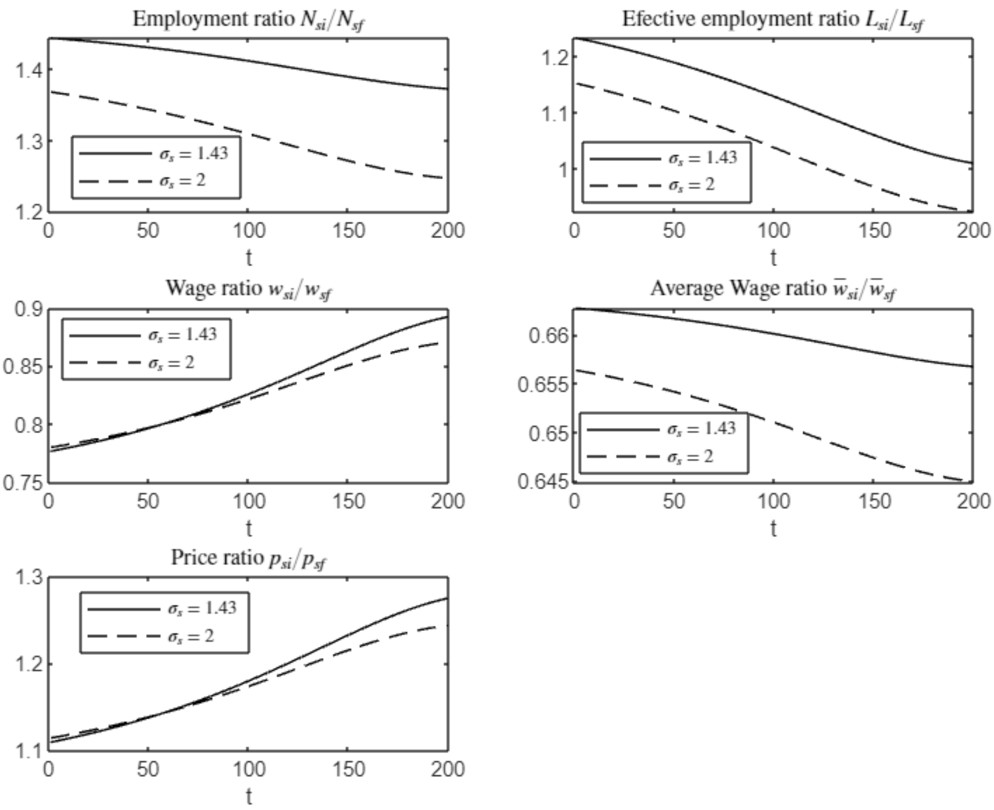


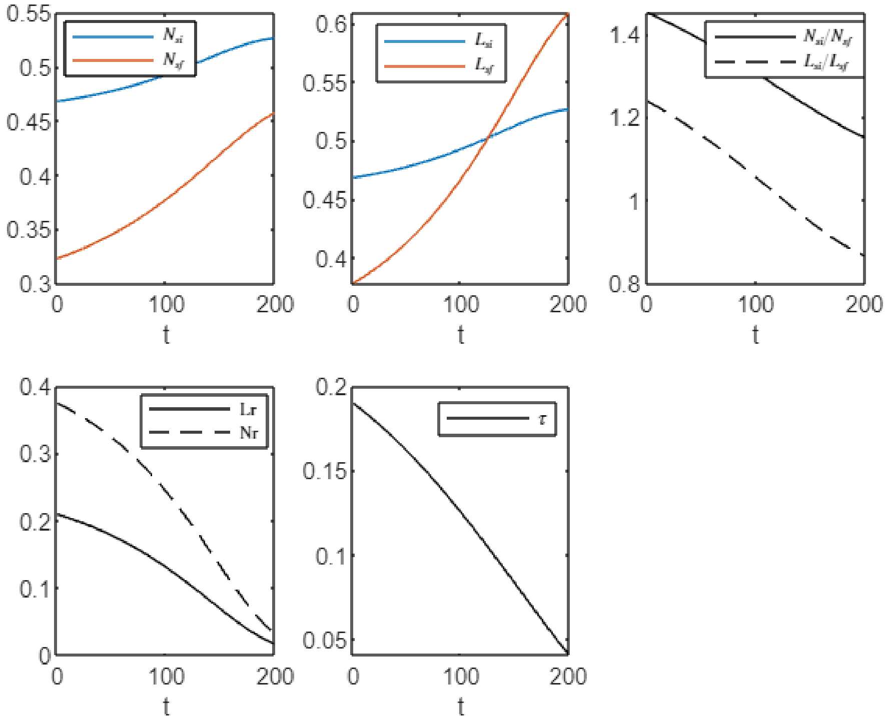

Figure 4 shows the evolution of relative employment, prices, and wages in the service sector when the price of capital decreases and the degree of substitution between formal and informal services is higher. When $\sigma_s$ is higher (increases from 1.43 to 2), the elasticity of the relative informal employment with respect to relative informal wages increases. Therefore, the size of the informal sector is lower, since consumers' demand for formal services increases.

**Figure 4. Evolution of employment, prices and wages in the service sector for different values of $\sigma_s$**



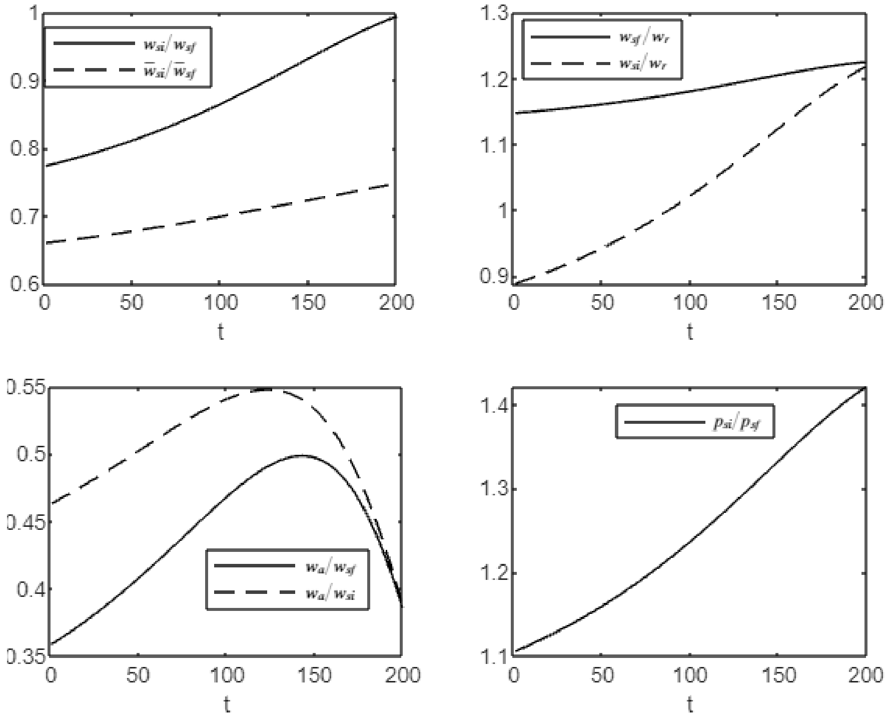

Finally, Figures 5 and 6 show the effect of technological progress on the evolution of employment, wages, and prices when taxes are variable. I assume that the tax rate varies in order to balance the constant subsidies and the fluctuating tax base, $\tau = \frac{T}{(w_a L_a + w_r L_r + w_{sf} L_{sf})}$. The initial value for the tax rate is the same as the benchmark calibration. As time passes, the price of capital decreases, and wages in the goods sector for workers performing abstract tasks and wages in the formal service sector increase. The increase on $w_a L_a + w_{sf} L_{sf}$ is higher than the decrease on $w_r L_r$. This implies a decrease of the labor income tax paid by formal workers. Notice, by comparing Figures 2 and 5, that the increase in the efficient units of labor and the employment share in the formal service sector is higher since workers have more incentives to go to the formal service sector, due to lower taxes. Under this policy, the model predicts a higher decrease in the informal sector than in the case with constant taxes.

**Figure 5 Evolution of employment when $\tau$ is variable**




Similarly, by comparing figures 3 and 6, it is possible to notice that the increase in relative wages and prices in the informal service sector is lower for the case when the labor income taxes decrease with technological progress.

**Figure 6. Evolution of wages and prices in the service sector when $\tau$ is variable**




# 6 Conclusions

This paper analyzes how the incidence of job polarization affects the distribution of employment and wages in the presence of a large informal sector. I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorm (2013). I assume that there are three sectors in this economy: the goods sector, the formal service sector, and the informal service sector. Workers in the informal service sector are at the bottom of the skill distribution, are less productive, and can avoid taxation.

The analytical solution of the model implies that when the elasticity of substitution between capital and routine labor is higher than the elasticity of substitution between goods and services, the constant decrease in the price of capital eventually causes low-skill labor flows from routine tasks to manual tasks. This condition for job polarization is the same found by Author and Dorn (2013). In this case, Task-biased technological progress can increase aggregate demand for services and eventually increase employment and wages in service occupations. Additionally, when goods and services are complements, wages also polarize.

Additionally, I find that the efficient units of labor, as well as the number of workers, hired in the informal sector increase as a result of the increasing demand for informal services. However, the optimal composition of employment in the service sector depends on the level of taxes in the formal service sector $\tau$, the degree of substitution between the two types of services $\sigma_s$, and the level of efficiency in the formal service sector $\eta^{\theta}$. I find that share of informal employment in the service sector decreases with technological progress. This result is explained by the fact that some workers, whose skill level is $\eta > 1$, are more skilled when they work in the formal sector. Therefore, the increase in the demand for labor in the informal service sector requires a higher increase in wages to compensate for this loss of skills. As a consequence, the increase in relative wages in the informal sector increases their relative prices, which in turn decreases their relative demand and hence the relative demand for workers in the informal service sector.

I simulate the model for different values of the elasticity of substitution between formal and informal services and also for the case when labor income taxes are variable. I find that, when the elasticity of substitution between formal and informal goods increases, the effect of technological progress on the reduction of the share of informal employment is higher. This is because consumers are more likely to substitute informal services when their relative price increases. For the case where the labor income tax varies to balance the constant subsidies and the fluctuating tax base, I find that technological progress leads to a decrease in taxes due to the increase in wages in the formal sector. Therefore, the increase in the efficiency units of labor and the employment share in the formal service sector is higher compared to the case where taxes are constant. Under this scenario, workers will have more incentives to stay in the formal service sector due to lower taxes. Under variable taxes, the model predicts a higher decrease in informality due to technological progress.

The previous results are mainly driven by the assumption of flexible labor markets in all sectors and by the fact that some workers are more skilled when they work in the formal sector. One interesting extension of the model will be the introduction of wage and employment rigidities in the formal sector, while the informal sector is frictionless. Under this scenario, the positive effect of job polarization outlined in this paper can be diminished or even reversed. When task-biased technological progress leads to an increase in labor demand in the service sector, the presence of real wage rigidities and search and matching frictions in the formal sector, could lead to a higher increase in employment in the informal service sector.

# Appendix A1: Asymptotic labor allocation

From the optimization conditions for the households I have

$$\max_{\{C_g, C_{sf}, C_{si}\}} U = Ln\left(C\right)$$

$$U = Ln\left(C\right), \tag{41}$$

with

$$C = \left[\gamma_g C_g^{\rho} + \gamma_s C_s^{\rho}\right]^{\frac{1}{\rho}}, \tag{42}$$

$$C_s = \left(a_f C_{sf}^{\psi} + a_i C_{si}^{\psi}\right)^{\frac{1}{\psi}}$$

$$P_g C_g + P_{sf} C_{sf} + P_{si} C_{si} = (1-\tau)\left(w_a L^a + w_r L^r + w_{sf} L_{sf}\right) + w_{si} L_{si} + T$$

The Lagrangian of this problem writes:

$$\mathcal{L} = ln\left[\gamma_g C_g^{\rho} + \gamma_s \left(a C_{sf}^{\psi} + (a_i) C_{si}^{\psi}\right)^{\frac{\rho}{\psi}}\right]^{\frac{1}{\rho}}$$
$$+\lambda\left((1-\tau)\left(w_a L_a + w_r L_r + w_{sf} L_{sf}\right) + w_{si} L^{si} + T - P_g C_g - P_{sf} C_{sf} - P_{si} C_{si}\right)$$

F.O.C

$$\{C_g\} \qquad [...]^{\frac{1}{\rho}-1} \gamma_g C_g^{\rho-1} = P_g \lambda \tag{43}$$

$$\{C_{sf}\} \qquad [...]^{\frac{1}{\rho}-1} \gamma_s C_s^{\rho-1} a_f \psi C_{sf}^{\psi-1} = \lambda P_{sf} \tag{44}$$

$$\{C_{si}\} \qquad [...]^{\frac{1}{\rho}-1} \gamma_s C_s^{\rho-1} (a_i) \psi C_{si}^{\psi-1} = \lambda P_{si} \tag{45}$$

By dividing equations ( 43) and (44), and equations (45) and (44)I obtain:

$$\frac{C_g}{C_s} = \left(C_{sf}^{1-\psi} \frac{\gamma_g}{\psi a_f \gamma_s} \frac{P_{sf}}{P_g}\right)^{\sigma_c} \tag{46}$$

$$\frac{C_{si}}{C_{sf}} = \left(\frac{a_i}{a_f} \frac{P_{sf}}{P_{si}}\right)^{\sigma_s} \tag{47}$$

Replacing $C_{si} = A_{si} L_{si}$ and $C_{sf} = A_{sf} L_{sf}$ into equation (47) I obtain

$$L_{si} = L_{sf} \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{a_i}{a_f} \frac{w_{sf}}{w_{si}}\right)^{\frac{1}{1-\psi}}$$

from equation (11) I have $\frac{w_{sf}}{w_{si}} = \frac{1}{(\eta^{**})^{\theta}(1-\tau)}$. Replacing this expression into the previous equation:

$$L_{si} = L_{sf} \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}} \left(\frac{(a_i)}{a_f} \frac{1}{(\eta^{**})^{\theta}(1-\tau)}\right)^{\frac{1}{1-\psi}}$$

Using equation (15) I can express $\eta^{**} = \eta^{max} L_{si}$,then

$$L_{si} = L_{sf}\left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}}\left(\frac{(a_i)}{a_f}\frac{1}{\left(\eta^{max}L_{si}\right)^{\theta}(1-\tau)}\right)^{\frac{1}{1-\psi}}$$

$$L_{si}^{\frac{\theta+1-\psi}{1-\psi}} = L_{sf}\left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{1-\psi}}\left(\frac{a_i}{a_f\left(\eta^{max}\right)^{\theta}(1-\tau)}\right)^{\frac{1}{1-\psi}}$$

$$L_{si} = L_{sf}^{\frac{1-\psi}{\theta+1-\psi}}C_{11} \tag{48}$$

where $C_{11} = \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi}{\theta+1-\psi}}\left(\frac{a_i}{a_f(\eta^{max})^{\theta}(1-\tau)}\right)^{\frac{1}{\theta+1-\psi}}$

On the other side, replacing equations (4), (21), and (26) into equation (46) I have:

$$\frac{C_g}{C_s} = \left(C_{sf}^{1-\psi}\frac{\gamma_g}{\psi a_f\gamma_s}\frac{P_{sf}}{P_g}\right)^{\sigma_c}$$

$$\gamma_g\left(Y^g - p_kK\right)^{\frac{-1}{\sigma_c}}\frac{P_{sf}}{P_g} = \gamma_s C_s^{\frac{-1}{\sigma_c}}\psi a_f C_{sf}^{\psi-1}$$

$$\gamma_g\left(Y^g - p_kK\right)^{\rho-1}\frac{\kappa_R A_g L_a^{(1-\beta)}X^{\beta-\nu}L_r^{\nu-1}}{A_{sf}}\frac{w_{sf}}{w_r} = \gamma_s\left(C_s(L_{si})\right)^{(\rho-1)}\psi a_f\left(A_{sf}L_{sf}\right)^{\psi-1}$$

From (10) I have that $\frac{w_{sf}}{w_r} = \left(\eta^*\right)^{1-\theta}$. replacing this equation into the previous equation I obtain:

$$\gamma_g\left(Y^g - p_kK\right)^{\rho-1}\frac{\kappa_R A_g L_a^{(1-\beta)}X^{\beta-\nu}L_r^{\nu-1}}{A_{sf}}\left(\eta^*\right)^{1-\theta} = \gamma_s\left(C_s(L_{si})\right)^{(\rho-1)}\psi a_f\left(A_{sf}L_{sf}\right)^{\psi-1}$$

with $Y_g - p_kK = (1-\beta)\left(\mu_kK\right)^{\beta}$ and $\frac{X_t}{\mu_kK_t} = 1$, $L_{si} = L_{sf}^{\frac{1-\psi}{\theta+1-\psi}}C_{11}$

$$\gamma_g\left((1-\beta)\left(\mu_kK\right)^{\beta}\right)^{\rho-1}\frac{\kappa_R A_g L_a^{(1-\beta)}\left(\mu_kK_t\right)^{\beta-\nu}L_r^{\nu-1}}{A_{sf}}\left(\eta^*\right)^{1-\theta} = \gamma_s\left(C_s(L_{si})\right)^{(\rho-1)}\psi a_f\left(A_{sf}L_{sf}\right)^{\psi-1}$$

$$\frac{L_r^{1-\nu}}{\left(\eta^*\right)^{1-\theta}}\left(C_s(L_{si})\right)^{(\rho-1)}L_{sf}^{\psi-1} = K_t^{-(\beta)(1-\rho)+(\beta-\nu)}\Psi_1 \tag{49}$$

with $\Psi_1 = \frac{\gamma_g\kappa_R A_g\mu_k^{\beta(\rho-1)+(\beta-\nu)}}{(1-\beta)^{1-\rho}\psi a_f\gamma_s A_{sf}^{\psi}}$.

Now I want to express equation (49) in terms of $L_{si}$ only.

Combining equations (39) , (40) and (48) I obtain:

$$\eta^* = \left((\theta+1)\,\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}L_{si}\right)^{\theta+1}\right)^{\frac{1}{\theta+1}} \tag{50}$$

Replacing (50) into (13) I am able to express $L_r$as a function of $L_{si}$:

$$L_r = \left[ \frac{(\eta^{max})}{2} - \frac{\left( \left( (\theta+1)\,\eta^{max} \left( \frac{L_{si}}{C_{11}} \right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max} L_{si})^{\theta+1} \right)^{\frac{1}{\theta+1}} \right)^2}{2\eta^{max}} \right] = gg(L_{si}) \tag{51}$$

Therefore, equation (49) can be written as follows:

$$\frac{gg(L_{si})^{1-\nu} \left( C_s(L_{si}) \right)^{(\rho-1)} \left( \left( \frac{L_{si}}{C_{11}} \right)^{\frac{1+\theta-\psi}{1-\psi}} \right)^{\psi-1}}{\left( (\theta+1)\,\eta^{max} \left( \frac{L_{si}}{C_{11}} \right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max} L_{si})^{\theta+1} \right)^{\frac{1-\theta}{\theta+1}}} = K_t^{-(\beta)(1-\rho)+(\beta-\nu)} \Psi_1$$

with $\sigma_c = \frac{1}{1-\rho}$

$$\frac{gg(L_{si})^{1-\nu} \left( C_s(L_{si}) \right)^{(\rho-1)} \left( \left( \frac{L_{si}}{C_{11}} \right)^{\frac{1+\theta-\psi}{1-\psi}} \right)^{\psi-1}}{\left( (\theta+1)\,\eta^{max} \left( \frac{L_{si}}{C_{11}} \right)^{\frac{1+\theta-\psi}{1-\psi}} + (\eta^{max} L_{si})^{\theta+1} \right)^{\frac{1-\theta}{\theta+1}}} = K_t^{-\frac{(\beta)}{\sigma_c}+(\beta-\nu)} \Psi_1 \tag{52}$$

# Appendix A2: Asymptotic wage inequality

From equations (43) and (44) I find

$$[...]^{\frac{1}{\rho}-1} (\gamma_s)\, C_s^{\rho-1} a\psi C_{sf}^{\psi-1} = [...]^{\frac{1}{\rho}-1} \gamma_g C_g^{\rho-1} P_{sf}$$

$$(\gamma_s)\, C_s^{\rho-1} a\psi C_{sf}^{\psi-1} = \gamma_g \left( (1-\beta)\,(\mu_k K)^{\beta} \right)^{\rho-1} \frac{w_{sf}}{A_{sf}}$$

$$w_{sf} = \frac{\gamma_s C_s^{\rho-1} a\psi C_{sf}^{\psi-1} A_{sf}}{\gamma_g \left( (1-\beta)\,(\mu_k K)^{\beta} \right)^{\rho-1}} \tag{53}$$

From the optimization problem for the firm in the goods and service sector, I have

$$w_r = \kappa_R\, (\mu_k K)^{\beta-\nu}\, g(L_{sf})^{\nu-1} \tag{54}$$

$$w_a = P_g\,(1-\beta)\,\frac{Y_g}{L_a} = (1-\beta)\,(\mu_k K)^{\beta} \tag{55}$$

$$w_{sf} = P_{sf} A_{sf}$$

$$w_{si} = P_{si} A_{si}$$

from equations (53) and (54) I obtain:

$$\frac{w_{sf}}{w_r} = \frac{C_s^{\rho-1} \left(L_{sf}\right)^{\psi-1} \left(g(L_{sf})\right)^{1-\nu}}{\kappa_{sf} \left(K_t\right)^{-\beta(1-\rho)+\beta-\nu}}$$

$$\frac{w_{sf}}{w_r} = \frac{C_s^{\rho-1} \left(L_{sf}\right)^{\psi-1} \left(g(L_{sf})\right)^{1-\nu}}{\kappa_{sf} \left(K_t\right)^{-\beta(1-\rho)+\beta-\nu}} \tag{56}$$

with $\kappa_{sf} = \frac{\alpha^{\rho-1}\kappa_R(\mu_k)^{(\beta)(\rho-1)+\beta-\nu}}{(A_{sf})^{\psi-1}\left(\psi\alpha\frac{\gamma_s}{\gamma_g}\right)^2}$

Replacing the expression $L_{si} = L_{sf}^{\frac{1-\psi}{\theta+1-\psi}} C_{11}$

$$\frac{w_{sf}}{w_r} = \frac{C_s^{\rho-1} \left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{\psi-1} \left(gg(L_{si})\right)^{1-\nu}}{\kappa_{sf} \left(K_t\right)^{-\beta(1-\rho)+\beta-\nu}}$$

Replacing equation (52) I have

$$\frac{w_r}{w_{sf}} = \frac{\left((\theta+1)\,\eta^{max} C C_1 \left(\bar{L}_{si}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}\bar{L}_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}} \left(K_t\right)^{(\beta)(\rho-1)+\beta-\nu}\, \Psi_1}{\kappa_{sf} K_t^{-(\beta)(1-\rho)+(\beta-\nu)}}$$

$$\frac{w_r}{w_{sf}} = \left((\theta+1)\,\eta^{max} C C_1 \left(\bar{L}_{si}\right)^{\frac{1+\theta-\psi}{1-\psi}} + \left(\eta^{max}\bar{L}_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}} \frac{\Psi_1}{\kappa_{sf}}$$

Similarly, the wage ratio between informal manual tasks and routine tasks $\frac{w_{si}}{w_r}$ can be determined as follows:

By dividing equation (43) into (45) I obtain:

$$\frac{\gamma C_g^{\rho-1}}{\gamma_s C_s^{\rho-1} \psi\left(\beta\right) C_{si}^{\psi-1}} = \frac{P_g}{P_{si}}$$

Replacing $w_{si} = P_{si} A_{si}$, and $C_g = Y^g - p_k K = \alpha\left(\mu_k K\right)^{\beta}$ I have

$$\frac{\left((1-\beta)\left(\mu_k K\right)^{\beta}\right)^{\rho-1}}{\left[C_s\right]^{\rho-1} \psi\beta\left(A_{si} L_{si}\right)^{\psi-1}} = \frac{\gamma_s A_{si}}{\gamma_g w_{si}}$$

$$w_{si} = C_s^{\rho-1} \psi \frac{\gamma_s \beta}{\gamma_g} A_{si}^{\psi} L_{si}^{\psi-1} \left((1-\beta)\left(\mu_k K\right)^{\beta}\right)^{1-\rho} \tag{57}$$

Dividing equation (57) into (54), I obtain

$$\frac{w_{si}}{w_r} = \frac{C_s^{\rho-1}\psi\frac{\gamma_s\beta}{\gamma_g}A_{si}^{\psi}L_{si}^{\psi-1}}{\left((1-\beta)\left(\mu_k K\right)^{\beta}\right)^{\rho-1}\kappa_R\left(\mu_k K\right)^{\beta-\nu}g(L_{sf})^{\nu-1}}$$

$$\frac{w_{si}}{w_r} = \frac{\kappa_{si}C_s^{\rho-1}L_{si}^{\psi-1}gg(L_{si})^{\nu-1}}{K^{-\beta(1-\rho)+(\beta-\nu)}}$$

with $\kappa_{si} = \frac{\psi\beta\gamma_s A_{si}^{\psi}\left((1-\beta)(\mu_k)^{\beta}\right)^{1-\rho}}{\gamma_g\kappa_R(\mu_k)^{\beta-\nu}}$

Replacing equation (52) I have

$$\frac{gg(L_{si})^{1-\nu}\left(C_s\right)^{(\rho-1)}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{\psi-1}}{\left(\left(\theta+1\right)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+\left(\eta^{max}L_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}} = K_t^{-\frac{(\beta)}{\sigma_c}+(\beta-\nu)}\Psi_1$$

$$\frac{w_{si}}{w_r} = \frac{\kappa_{si}C_s^{\rho-1}L_{si}^{\psi-1}gg(L_{si})^{1-\nu}}{K^{-\beta(1-\rho)+(\beta-\nu)}}$$

$$\frac{w_{si}}{w_r} = \frac{\left(\left(\theta+1\right)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+\left(\eta^{max}L_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}K_t^{-\frac{(\beta)}{\sigma_c}+(\beta-\nu)}\Psi_1 L_{si}^{\psi-1}}{K^{-\beta(1-\rho)+(\beta-\nu)}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{\psi-1}}$$

$$\frac{w_{si}}{w_r} = \left(\left(\theta+1\right)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+\left(\eta^{max}L_{si}\right)^{\theta+1}\right)^{\frac{1-\theta}{\theta+1}}\left(\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}\right)^{1-\psi}\kappa_{si}\Psi_1 L_{si}^{\psi-1}$$

$$\frac{w_{si}}{w_r} = \left(\left(\theta+1\right)\eta^{max}\left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\psi}}+\left(\eta^{max}L_{si}\right)^{\theta+1}\right)^{-1}\left(\frac{L_{sf}}{L_{si}}\right)^{1-\psi}\kappa_{si}\Psi_1$$

or

$$\frac{w_{si}}{w_r} = \left(\left(\theta+1\right)\eta^{max}L_{sf}+\left(\eta^{max}L_{si}\right)^{\theta+1}\right)^{-1}\left(\frac{L_{sf}}{L_{si}}\right)^{1-\psi}\kappa_{si}\Psi_1$$

Finally, by dividing equation (55) into (53), and equation (55) into (57) I obtain the wage ratios $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ respectively:

$$\frac{w_a}{w_{sf}} = \frac{(1-\beta)\left(\mu_k\right)^{\beta}\gamma_g\left((1-\beta)\left(\mu_k\right)^{\beta}\right)^{\rho-1}K^{\beta\left(1-\frac{1}{\sigma_c}\right)}}{\left(\gamma_s\right)C_s^{\rho-1}a\psi\left(A_{sf}L_{sf}\right)^{\psi-1}A_{sf}}$$

$$\frac{w_a}{w_{sf}} = \kappa_{af}K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)}C_s^{1-\rho}L_{sf}^{1-\psi}$$

where $\kappa_{af} = \frac{(1-\beta)(\mu_k)^{\beta}\gamma_g\left((1-\beta)(\mu_k)^{\beta}\right)^{\rho-1}}{(\gamma_s)a_f\psi A_{sf}^{\psi}}$

$$\frac{w_a}{w_{si}} = \frac{(1-\beta)\left(\mu_k K\right)^{\beta}}{C_s^{\rho-1}\psi\frac{\gamma_s\beta}{\gamma_g}A_{si}^{\psi}L_{si}^{\psi-1}\left(\alpha\left(\mu_k K\right)^{\beta}\right)^{1-\rho}}$$

$$\frac{w_a}{w_{si}} = \frac{(1-\beta)\,\mu_k^{\beta}K^{\beta\left(1-\frac{1}{\sigma_c}\right)}}{C_s^{\rho-1}\psi\frac{\gamma_s\beta}{\gamma_g}A_{si}^{\psi}L_{si}^{\psi-1}\left((1-\beta)\,\mu_k^{\beta}\right)^{1-\rho}}$$

$$\frac{w_a}{w_{si}} = \frac{\kappa_{ai}K^{\beta\left(\frac{\sigma_c-1}{\sigma_c}\right)}}{C_s^{\rho-1}L_{si}^{\psi-1}}$$

where $\kappa_{ai} = \frac{(1-\beta)(\mu_k)^{\beta}\gamma_g\left((1-\beta)(\mu_k)^{\beta}\right)^{\rho-1}}{(\gamma_s)a_f\psi A_{si}^{\psi}}$

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