Informality in the City: a Theoretical Analysis

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Informality in the City: a Theoretical Analysis
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Para Dios, y para mi Esposa, Hijos, Padres y Hermanos. Los amo.

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The informal sector occupies a significant portion of the labor market in cities in Latin America. According to recent estimates, the informal sector absorbs 57% of total workforce in the city of Bogotá, 50% in Medellin, 57% in Lima and 45% in Buenos Aires (Galvis, 2012; CNTPE, 2008; MTEySS, 2007). Part of the existing literature (Lewis, 1954; Harris and Todaro, 1970; Piore, 1980) interprets the informal sector as a residual sector or a buffer where rural-urban migrants queue for formal jobs. Under this view, the informal sector emerges because the inability of the modern (formal) sector to absorb the available labor supply. This inability in turn is explained by typical conditions present in markets of developing economies such as the lack of human and physical capital, the abundance of unskilled labor and the concentrated market structures, among others. Another important part of the literature (Maloney, 2004; Loayza, 1996; Rauch, 1991; Mejía and Posada, 2007) analyzes the causes of labor informality focusing on institutional reasons, and usually interprets the informal sector as a micro-entrepreneurial unregulated sector that offers intrinsic benefits, so that being informal is, to some extent, a matter of choice (Albrecht, et al., 2009).

The factors that drive economic development and institutional reforms are defined at the national or federal level and can only partly explain the incidence and persistence of urban labor informality[1]. Another set of determinants, which has not been yet extensively explored in the theoretical literature, comes from the characteristic of cities in which informal workers reside. The existence of urban and regional labor markets is a well-established fact (Zenou, 2009). In Colombia, for example, there is considerable evidence of regional segmentation (Galvis, 2010; Mesa, et al., 2008; Ortiz, et al., 2009) which means that workers and firms interact in much smaller markets than the national market. Because of this, local factors that define the particular structure of Latin America cities (such as transport systems, workers and firms locations, and rural urban migration) can have a deep impact on urban informal labor and vice versa.

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[1] For a review of the concept of informality see Guataquí, et al., (2011) and Garcia (2009).

Latin American cities display high levels of spatial segregation, usually in the form of large peripheral belts where there is concentration of low-income population, poor infrastructure provision, and difficult access to city centers (Bocarejo and Portilla, 2011; ONU-HABITAT, 2012; Secretaría Distrital de Planeación, 2013). In cities like Bogotá this segregation pattern is extended to the labor market. Informal workers live in the south and southeast of the city (Montoya, 2014), in zones that are far of central areas where formal jobs are generated (Bocarejo and Portilla, 2011). These zones are also characterized by the low quality both in the road network and the public transport system and as a result, periphery inhabitants make less than 1.5 trips per day, almost half than the high-income workers (Bocarejo and Oviedo, 2012). Additionally, informal jobs tend to be less centralized than the formal (Hernández and Gutiérrez, 2011, Gutierrez, 2011), reflecting that part of the informal economic activity is carried out inside the house or in the public space.

The first two chapters of this dissertation study how the particular structure of Latin America cities can affect the size of the informal sector. The first chapter builds a monocentric city model with a labor market characterized by search frictions and the presence of an informal sector. We assume that formal workers commute on a daily basis to the city center where formal economic activity is centralized, whereas informal workers commute less often and undertake some of their productive activities at home. Because of this, informal workers end up living on the periphery. We also assume that there is presence of informal housing. We show that, in order to induce workers to accept a job in the formal sector, formal firms must compensate them with the income that they could have obtained in the informal sector. Part of this income is the commuting cost savings obtained by the informal worker by commuting less often to the city center. The model shows that both a higher decentralization of informal jobs and a higher proportion of informal housing result in a higher informality rate in the labor market (informality rate from now on). This is because both situations increase the commuting cost savings and force formal firms to pay a larger compensation.

The model is extended by introducing rural-urban migration.[2] As before, a higher decentralization leads to a higher informality rate. This, in turn, pushes the expected income in the city downwards and reduces incentives for rural workers to migrate. However,

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[2] In Latin America rural-urban migration has explained 30% of urban growth between 1980 and 2010 (CEPAL, 2012).

surprisingly, rural urban migration increases. This is because a higher decentralization relaxes the competition for land near the city center which in turn reduces urban costs for all urban residents and effectively increases the expected income in the city. Finally, we use the extended model to compare the impact of search costs and transport policies on labor informality and welfare. The results show that an entry-cost subsidy has a higher impact on the informality rate than transport policies. This happens for two reasons. First, an entry-cost subsidy stimulates formal employment creation directly and indirectly (through a reduction in wages), while transport policies only affect formal employment creation indirectly. Second, the entry-cost subsidy affects rural-urban migration only through its effect on formal employment creation, while transport subsidies have an additional effect reducing urban costs, which ultimately lead to higher rural-urban migration and a higher saturation of the labor market. In other words, the Todaro paradox is more likely to happen when transport policies are in place.

In the second chapter, instead of imposing commuting differences, we allow informal workers to endogenously choose their commuting frequency knowing that commuting to the CBD implies a larger remuneration but also commuting costs, and staying at home implies a lower remuneration but no commuting costs. We assume that fixed commuting costs are high compared with the benefits of commuting to the CBD for informal workers. As a result a segmented city emerges in equilibrium, with formal workers residing in the centre, and informal workers residing in the periphery. As before, formal firms have to compensate unemployed workers with the income that they could have obtained outside the formal sector, which includes spatial costs savings. These savings do not depend on assumed commuting differences between different types of workers, but on the optimal commuting decision of informal workers. Our results show that location decisions have non-trivial effects on informality rates, through their effect of commuting and housing costs, so that they should be considered in policy designs aiming at reducing urban informality. We use the model to compare the impact and efficiency of four policy options: a subsidy on formal firms hiring-costs and a transport subsidy for either all workers, formal workers, or informal workers. We find that a transport subsidy targeted at informal workers is undesirable. We also find that a hiring-costs subsidy is superior to transport subsidies.

Informal housing and city structure

Cities in Latin America are often characterized by fast spatial expansion (Inostroza, et al., 2013), low buildings height and by peripheries where precarious short buildings concentrate (ONU-HABITAT, 2012; UN-HABITAT, 2010). On the explanation of these patterns, some authors have stressed the role of rural-urban migration under high unemployment, as the main force behind the emergence of the periphery and the rapid expansion of the city (Harris and Todaro, 1970; Zenou, 2011), and others have focused on the role of the migration under agglomeration economies, in turning cities into megacities (Krugman, 1991). However, less attention has been paid to the role of informal housing. If we consider slum settlements and durable self-constructions (that avoid taxes and urban standards) as informal constructions, the informal housing sector accounts, at least, for 25% of the housing market in Latin America.[3] Therefore, the informal housing sector occupies large extensions of urban land that could have been used by the formal sector, affecting the land market (Smolka and Biderman, 2011), the productivity in the housing industry, and finally the shape and the spatial structure of the city.

The previous theoretical literature on the economics of the informal housing sector has focused on the study of slums, emphasizing on issues such as illegal dwelling, or income differences combined with land regulations. For example, Jimenez (1985) and Brueckner and Selod (2009) study the economics of squatter settlements. Da Mata (2013) models slum growth on Brazil, focusing on the role of lack of property rights, while Heikkila and Lin (2013) study the implications of minimum lot size restrictions and the income differentials in the magnitude of slums. However, there is not a systematic approach that closely relates informal housing with the city structure. In particular, little is known about how land allocated to informal constructions changes with distance to the city center, how informal constructions can be compared with formal constructions, and how informality affects the size and tallness of the city.

The third chapter of this dissertation study how the structure of Latin American cities can be affected by (and can affect) the informal housing sector. More specifically, this chapter develops a monocentric city model with a formal and an informal sector in the housing

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[3] One in four inhabitants of Latin America lives in slum conditions (ONU-HABITAT, 2012).

industry. In this model, informal land developers evade taxes, use inefficient techniques, neglect the infrastructure for public services, and thereby expose to the risk of being punished. Given that intersectoral land mobility is assumed, a general equilibrium framework is used in the analysis of the land market. In contrast with the Muth-Mills model, here the relative price of land does not necessarily change with distance, implying that the traditional reason for a systematic variation of the buildings height over the urban space does not exist. However, a clear spatial pattern emerges. The model shows that as public infrastructure decreases with distance to the city center, the land allocated to produce short informal (tall formal) buildings increases (decreases) with distance. The model also shows that the spatial pattern of infrastructure is not necessarily the result of an (exogenous) lack of responsiveness of the State to fast urban growth. Instead, it is an (endogenous) optimal response of the local government to the way tax revenues and fines revenues change with distance to the city center. Another important result establishes that higher level of informal housing leads to a city with a large spatial size and shorter buildings.

Chapter one

Informal labor, city structure, and rural urban migration[4]

1.1.   Introduction

One of the most remarkable features of cities in Latin America is the existence and persistence of an informal sector in the labor market (Maloney, 2004; Mejía and Posada, 2007). In its most common form, informal employment refers to the case of workers who are not reported as such by their employers to the corresponding national authorities (e.g., they do not have a signed labor card). This implies that the informal sector is an unregulated sector (Albrecht, et al., 2009), where workers usually receive lower wages, do not pay taxes, do not contribute to social security system, have no record of job experience or opportunities for advancement, and have more difficulties accessing credit (Perry, et al., 2007). For the economy at large, the existence of informal employment implies losses, not only in tax revenues and a heavy social protection burden, but also in terms of productivity.

Most of the existing literature analyzing the causes of urban informality has focused on institutional reasons. There is some evidence indicating that over-regulation and red tape, a dual social security system, and lack of labor regulation enforcement all lead to higher informality rates (Perry, et al., 2007; Ferreira and Robalino, 2011). These aspects, which depend on institutional reforms at the national or federal level, can only partly explain the incidence and persistence of urban informality. Another important set of determinants, which has not been yet extensively explored in the literature, comes from the characteristic of cities in which informal workers reside.

Latin American cities show spatial segregation, usually in the form of large peripheral belts where there is concentration of low income population, poor infrastructure provision, and difficult access to city centers (Bocarejo and Portilla, 2011; ONU-HABITAT, 2012; Secretaría Distrital de Planeación, 2013). This segregation pattern also can be seen in the labor market. In Bogotá, for example, informal workers are concentrated in the south and southeast of the city (Montoya, 2014) where they face long distances to the city centers and the low quality both in the road network and the public transport system (Bocarejo and Portilla, 2011). All this means that periphery inhabitants experience very high costs in terms of money, time and energy when commuting to the central areas (ONU-HABITAT, 2012), where formal employment is mostly generated (Hernández and Gutiérrez, 2011; Gutiérrez, 2011). As a consequence, they commute less than the high income population.[5] Periphery inhabitants may be reluctant to take on a formal job because it implies a disproportionate increase in commuting costs. This is even more likely if informal employment is less centralized than formal employment. This is precisely the case in Bogotá (Hernández and Gutiérrez, 2011; Gutiérrez, 2011). Additionally, following the predictions of the Spatial Mismatch Literature, in a context of high segregation, the residents of the periphery may not get enough information about formal job opportunities, or could be discriminated by formal employment based on their place of residency (Ihlanfeldt and Sjoquist, 1998).

On the other hand, the pervasiveness of informal constructions in Latin American cities is a well known fact (ONU-HABITAT 2012). The proliferation of informal constructions is partly responsible for the fast sprawling experienced by Latin American cities, especially in the last 3 decades (Urban Age Program, 2009). New informal settlements have sprung out in the fringes of cities since the start of the industrialization process experienced by the region. The result of this continuous urban expansion is the consolidation of the peripheral belts. Although informal housing and informal employment are salient features of Latin American cities, there is not much available evidence on the relationship between the two.

The aim of this paper is to study the relationship between informal employment and city structure. We develop a spatial search model which includes relevant features of informal employment and informal housing in Latin America. Starting with a linear city with a unique

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[5] In Bogotá, for instance, poorer workers make on average 1.5 trips a day, less than the average of the rich (Bocarejo and Oviedo, 2012).

Central Business District (CBD) where all formal firms locate, we assume that informal workers do not pay taxes, are self-employed, and receive a transfer from the government in the form of subsidized healthcare. In this way, the informal sector can be considered as a micro-entrepreneurial unregulated sector that offers intrinsic benefits. This captures the view, relevant for the Latin America case, that being informal employed is, to some extent, a matter of choice (Albrecht, et al., 2009; Maloney, 2004, Perry, et al., 2007). Unlike formal workers, informal workers can undertake part of their productive activities at home, and consequently commute with less frequency to the CBD. As a result, informal workers live in the periphery. This structure reflects, first, that informal employment is more decentralized than formal employment and, second, the existence of segregation.

Next, following Posada (2015), we introduce informal housing in the urban land market. We assume two types of developers, formal and informal. Both types produce a homogeneous good (interior space) using urban land. Unlike formal developers, informal developers, while producing interior space, evade taxes, access public infrastructure in an irregular way, and use inefficient building techniques. They also assume a certain risk of being detected and punished. These assumptions aim to capture the most important features of informal land markets in Latin America.[6] Within this framework, we study the mechanisms relating informal housing and informal employment. The model allows us to draw predictions relating to the provision of basic public infrastructure, commuting costs and the probability of detection on both the incidence of informal housing and informal employment.

The results show that in order to induce workers to accept a job in the formal sector, formal firms must compensate them with the expected income obtained outside the formal sector. This expected income is a weighted sum of the income obtained as an unemployed (e.g. unemployment insurance) and the income obtained as an informal worker. Part of this income is the commuting cost saving obtained by the informal worker by commuting less often to the city center.[7] We demonstrate how higher informal job decentralization, a higher

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[6] According to Smolka and Biderman (2011), pag. 6: "Informality can open a gap for arbitrage, allowing informal developers to reap higher profits than formal developers because they avoid paying license fees and taxes, only partially provide infrastructure and services, devote smaller percentages of land to public uses, and offer below-minimum-size lots. These incentives stimulate supply of informal developments."

[7] This spatial compensation is different from the spatial compensation that arises when the commuting frequency of formal employees differs from that of the unemployed. The latter is the compensation that is present in the spatial search matching models of Zenou (2009), Zenou (2011), and Xiao (2014).

commuting cost, and a higher proportion of informal housing (as a result of lower infrastructure provision, or a lower probability of detection) all result in a higher informality rate. This is because all these situations increase the benefits of not commuting frequently to the CBD, and consequently increase the spatial compensation that formal firms have to offer as part of the formal wage.

Besides high levels of labor and housing informality, Latin American cities have showed large rural-urban migration flows. For instance, rural-urban migration has explained 30% of urban growth between 1980 and 2010 (CEPAL, 2012). Such migration has raised important questions because it has occurred precisely in cities where poverty, unemployment and informality are present. This is the basis of the seminal work of Harris and Todaro (1970), which explains how migration occurs under a persistent urban unemployment. Harris and Todaro recognized that when rural workers migrate to the city, they accept the possibility of being unemployed to get an urban job later. That is, Harris and Todaro recognized that decisions to migrate are based on differences between expected urban income and rural income. In their model, the equilibrating mechanism is the rate of urban unemployment. As the urban population increases the unemployment rate also increases. As a result, the expected income in the city decreases, reducing the incentives to migrate.

The Harris-Todaro model has been extended in many ways[8], and some of these extensions have been useful to understand the relationship between city structure and migration. For example, Brueckner and Zenou (1999) and Brueckner and Kim (2001) have incorporated an urban land market into the standard Harris-Todaro model and have showed that besides the unemployment rate, increasing urban costs also act as a factor deterring rural-urban migration. Zenou (2011) also formulates a rural urban migration model where the city is characterized by both a search matching labor market and an explicit land market. This allows him to analyze the impact of distinct urban policies on labor market outcomes. Even with all this progress, little is known about rural-urban migration under more complex city structures like one that incorporates a decentralized informal labor and informal housing. This is not a minor issue because typical rural-urban migrant will likely become an informal worker and live in an informal house.

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[8] For example, Zenou (2008), Satchi and Temple (2009), Sato (2004b) and Laing, et al. (2005) replaced the original assumption of minimum wage in urban areas by one of wage determination through bilateral negotiation on a framework of urban frictional unemployment.

In order to shed light on these issues, we extend our model for the case of an open city with rural-urban migration a-là Harris-Todaro. This extension allows us to capture simultaneously urban labor, land and housing market interactions, and at the same time, it allows us to determine their impact on rural-urban migration. In the model, both the unemployment rate and urban costs act as an equilibrating force. We find, as before, that higher informal job decentralization leads to a higher informality rate. This should lead to lower rural-urban migration. Surprisingly, the opposite holds: rural-urban migration actually increases. This happens because a more decentralized informal sector relaxes the competition for land near the CBD, reducing in this way the urban costs for all workers and effectively increasing the urban expected income.

Finally, we use the extended model to compare the impact of search costs and transport policies on labor informality and welfare.[9] The results show that an entry-cost subsidy has a higher impact on the informality rate than transport policies. This happens for two reasons. First, an entry-cost subsidy stimulates formal employment creation directly and indirectly (through a reduction in wages), while transport policies only affect formal employment creation indirectly. Second, the entry-cost subsidy affects rural-urban migration only through its effect on formal employment creation, while transport subsidies have an additional effect reducing urban costs, which ultimately lead to higher rural-urban migration and a higher saturation of the labor market. In other words, the Todaro paradox is more likely to happen with the transport policies.

The rest of the paper is organized as follows. In the second section we introduce the basic model for a closed city. In the third section, we develop the static comparative analysis for this model. In the fourth section, we introduce rural-urban migration, and show the comparative statics analysis for this open city model. In the fifth section we show the results of the policy analysis. The last section concludes.

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[9] According to Zenou (2011), only policies that take into account the complex interactions between markets can be successful.

1.2. A model of the informal city

1.2.1. The city

The city is linear, monocentric and closed, and its Central Business District (CBD) is at the origin (zero). There is differential job access and the commuting cost per unit of distance is τ. The total urban population is given by N ex-ante identical workers who live forever and have rational expectations. Of the total workforce, U are unemployed workers, L_F work in the formal sector and L_{IN} work in the informal sector. Then, the total urban population is equal to $N = L_F + L_{IN} + U$. Unemployed workers can obtain either a formal or an informal job, but only actively search for formal jobs. Each worker is a household that demands housing services, which are produced from land and public infrastructure by land developers (which can be formal or informal), in the housing industry. There is a local government that exogenously fixes taxes and fines, and provides infrastructure. Land developers income and government income are spent outside the city. Land is owned by absentee landlords. The urban land market is competitive, so at any distance x from the CBD, all agents take the housing rent $R_H(x)$, and the land rent $R(x)$, as given. In the city there is no vacant land, and land areas not used for residential purposes are assumed to be used for agricultural production. The agricultural rent is exogenous and equal to zero.

1.2.2. Households

A worker optimally chooses his place of residence between the CBD $(x = 0)$ and the city fringe $(x = x_f)$. A formal -informal- worker, residing at distance x from the CBD, receives a wage w_F - w_{IN} - pays a housing rent of $R_H(x)$, consumes z_F - z_{IN} - units of the non-spatial composite good (which is taken as the *numeraire*), and consumes one unit of housing. Also a formal -informal- worker have a commuting frequency to the CBD of s_F - s_{IN} - implying that pays a commuting cost per unit of distance of $s_F \tau$ - $s_{IN} \tau$ -. An unemployed worker, receives a fixed income w_U (derived from an unemployment subsidy, rent income or own savings), pays a housing rent $R_H(x)$, consumes z_U units of the composite good, consumes one unit of housing, and pays $s_U \tau$ in commuting costs per unit of distance, where s_U is the commuting frequency.

The budget constraint of a worker residing in the city at distance x from the CBD can be expressed as:

$$R_H(x) + s_i \tau x + z_i = w_i, \tag{1.1}$$

for $i = F, IN$ or U. Each worker is risk neutral, then the utility function is given by $\Omega(z_i) = z_i$, which combined with the budget constraint given by (1.1), implies that the instantaneous indirect utility function is:

$$W_i(x) = w_i - s_i \tau x - R_H(x), \tag{1.2}$$

for $i = F, IN$ or U. Using (1.2) it can be established the bid rent function of a worker is:

$$\Psi_i(x, W_i) = w_i - s_i \tau x - W_i, \tag{1.3}$$

which indicates the maximum land rent that a worker of category i is willing to pay in order to reach a utility level W_i.

1.2.3. Urban labor market

1.2.3.1. Formal sector

All formal firms locate in the CBD and consume no space. A formal worker commutes every day to the CBD to work, so his commuting frequency s_F takes the value of one. Also, the unemployed worker commutes every day to the CBD[10] but in order to search for a formal job, then $s_U = 1$. The commuting frequency of the unemployed worker is also his search efficiency. Because s_U is a constant, it follows that the average search efficiency $\bar{s}_U$ is also a constant and equal to 1. Each firm is a productive unit that hires a single worker. The hiring process is subject to search frictions, as defined in the standard search-matching framework (Pissarides,

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[10] This is only a simplifying assumption with no implications in the main results

2000). Firms fill a vacancy and unemployed workers find employment, according to a random Poisson process. In the aggregate, the number of contacts per unit of time between the worker and the firm sides of the market is determined by the following matching function:

$$d = d(\overline{s}_U U, V), \tag{1.4}$$

where V is the total number of vacancies. Following Zenou (2009), we have that $U=uN$ and $V=vM$, where M is the total mass of firms, v is the vacancy rate and u is the unemployment rate. We assume that (1.4) is increasing in its arguments, concave and homogeneous of degree 1. The rate at which vacancies are filled can be expressed as $d(\overline{s}_U U, V)/V \equiv q(\theta)$, where:

$$\theta = V / \overline{s}_U U, \tag{1.5}$$

is the labor market tightness in efficiency units. It is easy to show that $q'(\theta) < 0$. The rate at which an unemployed worker with search efficiency s_U leaves unemployment is $s_U d(\overline{s}_U U, V)/\overline{s}_U U \equiv s_U \theta q(\theta)$, and it can be shown that $[\theta q(\theta)]' \geq 0$. It is also possible to verify that:

$$\lim_{\theta \to 0} \theta q(\theta) = \lim_{\theta \to +\infty} q(\theta) = 0, \qquad \text{and}$$

$$\lim_{\theta \to +\infty} \theta q(\theta) = \lim_{\theta \to 0} q(\theta) = +\infty,$$

Which indicates, on the one hand, that whenever the number of unemployed workers is infinite, firms fill their vacancies instantaneously, and, on the other hand, that whenever the number of vacancies is infinite, unemployed workers find jobs instantaneously. When firms and unemployed workers meet, quantity y_F is produced and a wage rate w_F (to be determined in equilibrium) is negotiated. The formal firm pays a tax given by T, which means that when a firm hires a worker, its instantaneous profit is equal to: $y_F - w_F - T$. Finally, formal jobs are destroyed, following a random Poisson process, at an exogenous rate δ_F.

1.2.3.2. Informal sector

In Bogotá, informal workers live in the city periphery (Montoya, 2014) where they face long distances to the city centers and the low quality both in the road network and the public transport system (Bocarejo and Portilla, 2011). Additionally, a significant proportion of

informal employment is not generated at the city centers (Hernández and Gutiérrez, 2011; Gutiérrez, 2011). All this implies that periphery inhabitants commute less than the high income population (Bocarejo and Oviedo, 2012). To reflect these spatial aspects of the informality, we assume, first, that informal workers do not commute daily to the CBD $s_{IN} < 1$, and second, that the commuting frequency s_{IN} is also the fraction of the informal wage w_{IN} generated in the CBD (whereas $1 - s_{IN}$ is the fraction generated near or at home). Then, in the model, the informal sector is decentralized, and s_{IN} provides a measure of its degree of decentralization. Specifically, a decrease in the commuting frequency of informal workers is always followed by an increase of the fraction of the wage generated through home-based work. As we will show below these assumptions imply that in equilibrium informal workers live in the periphery.

Employment opportunities in the urban informal sector are created following a random Poisson process with exogenous creation rate denoted by α. Informal jobs are destroyed also following a random Poisson process with destruction rate given by δ_{IN}. The informal wage w_{IN} is assumed to be fixed and identical for all workers, and also lower than the productivity in the formal sector, $y_F > w_{IN}$. It is assumed that informal workers do not actively seek formal jobs. This can be thought as a consequence of labor market segregation.

Contrary to the situation in the formal sector, it is assumed that informal workers receive a transfer once at the beginning of their work period of a positive fixed amount b. This would reflect the coexistence of contributory and non-contributory health programs typical in countries such as Colombia or Brazil. Specifically, when workers are employed in the formal sector, they are part of the contributory program and have to give up to part of their salary to finance health services. On the other hand, when workers are employed in the informal sector, they are part of the non-contributory program, so they are not required to contribute and still have access to a set of free health services usually offered by the state (Perry, et al., 2007).[11]

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[11] In order to ensure that this difference is only captured in the fixed term b, we assume that all the contributions made by formal workers are recovered through access to services, whereas informal workers do not contribute to the system.

1.2.4. Housing industry

At any distance x to the CBD, interior living space (a homogeneous good) is produced both in the formal and the informal housing sectors. The technology in the formal housing sector is given by:

$$Q_{FH}(x) = A_{FH}\bar{Z}^{1-\sigma}l_{FH}(x)^{\sigma}, \tag{1.6}$$

where $Q_{FH}(x)$ is the formal production at x, $l_{FH}(x)$ is the land used in the formal housing sector at x (to be determined in equilibrium), and $\bar{Z} > 0$ is the level of the infrastructure for public services at x, which is exogenous and does not change with distance. Since $A_{FH} > 0$ and $0 < \sigma < 1$ are constants, it is clear that (1.6) shows a decreasing marginal productivity of land. The formal land developer pays a tax rate t over its sales. This rate is constant all over the city. In the informal housing sector, the technology is:

$$Q_{IH}(x) = A_{IH}l_{IH}(x), \tag{1.7}$$

where $Q_{IH}(x)$ is the informal production at x and $l_{IH}(x)$ is the land used in the informal housing sector at x (to be determined in equilibrium). Given that $A_{IH} > 0$ is constant, the land productivity is constant. The informal land developer may be detected evading taxes with a probability $1-\varphi$, where φ is between 0 and 1. This probability does not change with the distance to the CBD. The punishment applied to the informal land developer ads up to the product of the tax t and a fine $\rho > 1$. Therefore the punishment is equal to $t\rho$. For simplicity it is assumed that $\rho = 1/t$ which implies that the punishment consist in the confiscation of all the income. There is free intersectoral land mobility which links the two sectors in a general equilibrium framework. Land supply at each x is equal to one and must be fully employed, that is:

$$l_{FH}(x) + l_{IH}(x) = 1. \tag{1.8}$$

The land will always be more productive in the formal housing sector, then is always the case that:

$$\sigma A_{FH} \bar{Z}^{1-\sigma} > A_{IH} \,. \tag{1.9}$$

Finally, we assume that land developer's income and government's income (taxes and penalizations) are spent outside the city.

1.3. Land and housing market equilibrium

The urban equilibrium is characterized in two stages. First, given any arbitrary level of the housing rent, we find, at each distance x, the land rent $R^*(x)$, the land allocation between sectors, $l^*_{FH}(x)$ and $l^*_{IH}(x)$, and the total housing production $Q^*(x)$. Given the particular structure of the housing industry, these variables do not depend of the utility levels. This allows to uniquely determine the city fringe x^*_f and the frontier between the formal workers zone and the informal workers zone x^*_d. In the second stage, the instantaneous equilibrium utility levels W^*_F, W^*_U, W^*_{IN} and the equilibrium housing rent $R^*_H(x)$ are found. We assume that production will always be positive in both sectors at any distance x. Then, it is always the case that:

$$(1-t)\sigma A_{FH} \bar{Z}^{1-\sigma} < \phi A_{IH} \,. \tag{1.10}$$

Each profit maximizing land developer in the formal housing sector solves:

$$\max_{l_{FH}(x)} \left\{ \pi_{FH}(x) = R_H(x)(1-t)A_{FH}\bar{Z}^{1-\sigma}l_{FH}(x)^{\sigma} - R(x)l_{FH}(x) - v(x)\bar{Z} \right\} \quad \text{at each} \quad x \in (0, x_f] \,, \tag{1.11}$$

where $v(x)$ is the rental rate of the public infrastructure at x and x_f is the city fringe. Given that the housing industry operates under perfect competition, the first order condition of (1.11) yields:

$$R_H(x)(1-t)\sigma A_{FH}\bar{Z}^{1-\sigma}l_{FH}(x)^{\sigma-1} = R(x) \,. \tag{1.12}$$

This condition determines the land demand in the formal housing sector, and because the decreasing marginal productivity of land, shows an inverse relationship between $l_{FH}(x)$ and $R(x)$ for a fixed level of $R_H(x)$. This is illustrated by the $FH(x)$ curve in Figure 1. This curve is read from left-to-right, in a diagram where the vertical axis represents land quantities and

the horizontal axis represents the level of land rent. In the informal housing sector, each land developer maximizes the expected profits given by:

$$\max_{l_{IH}(x)}\{\pi_{IH}(x) = \varphi R_H(x)A_{IH}l_{IH}(x) - R(x)l_{IH}(x)\} \quad \text{at each} \quad x \in (0, x_f].$$

(1.13)

The first order condition of (1.13) yields:

$$\varphi R_H(x)A_{IH} = R(x), \quad (1.14)$$

which determines the demand for land in this sector. Since φ and A_{IH} are taken as given by land developers, when $\varphi R_H(x)A_{IH} < R(x)$ the informal land developer does not demand land, and when $\varphi R_H(x)A_{IH} > R(x)$ the demand does not exist. Finally, when $\varphi R_H(x)A_{IH} = R(x)$, any amount of land is demanded. This is illustrated by the $IH(x)$ curve in Figure 1. This curve is read from right-to-the left in the diagram. The free intersectoral land mobility establishes a common land rent in both sectors, which links them in a general equilibrium framework. In equilibrium, the land rent ensures that the value of marginal productivity of land is equal between sectors and that total land demand equals land supply. Thus, the equilibrium land allocation, at each x, is characterized by:

$$\varphi A_{IH}R_H(x) = R_H(x)(1-t)\sigma A_{FH}[\bar{Z}/l_{FH}^*(x)]^{1-\sigma}, \quad (1.15)$$

which can be represented graphically as the intersection of the $FH(x)$ and $IH(x)$ curves in Figure 1. In this diagram the fixed land supply (which is equal to one) is represented by the length of the horizontal axis.[12]

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[12] See Feenstra (2004) for a presentation of the Specific factors model.
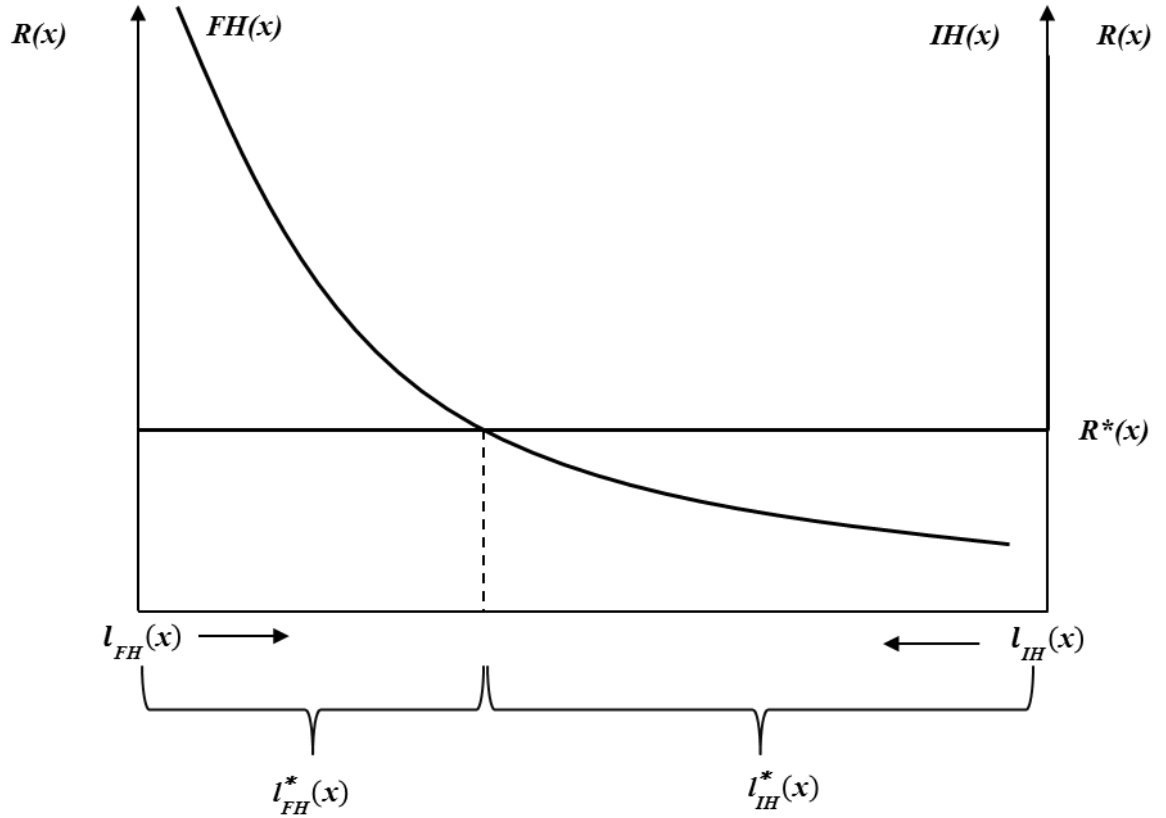
Figure 1. Land market equilibrium at distance x to the CBD

In equilibrium, the land rent is then given by:

$$R^*(x) = \varphi A_{IH} R_H(x),\qquad(1.16)$$

which is the value of the expected productivity in the informal housing sector.[13] Note from

(1.15) that it is always the case that: $\varphi/(1-t) = \sigma A_{FH}[\bar{Z}/l_{FH}^*(x)]^{1-\sigma}/A_{IH}$, which means that

in equilibrium the marginal rate of transformation (right side of the equation) is equal to the

relative price (for the producers) of the informal production in units of formal production (left

side of the equation). Because both expressions depend of parameters that do not change

with distance, it is clear that the equilibrium land allocation (as well the housing production)

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[13] The rental rate of the public infrastructure $v(x)$ is equal to the value of marginal productivity of public

infrastructure: $R_H(x)(1-\sigma)(1-t)A_{FH}\bar{Z}^{-\sigma}l_{FH}(x)^{\sigma}$.

does not change with distance.[14] Using (1.15), it is possible to obtain the equilibrium land allocation in the formal housing sector:

$$l_{FH}^* = \bar{Z}[(1-t)\sigma A_{FH} / \varphi A_{IH}]^{(1/1-\sigma)}, \qquad (1.17)$$

and the land allocation in the informal housing sector (recalling that the city is linear):

$$l_{IH}^* = 1 - \bar{Z}[(1-t)\sigma A_{FH} / \varphi A_{IH}]^{(1/1-\sigma)}. \qquad (1.18)$$

Replacing (1.17) in (1.6), we find the housing production in the formal housing sector:

$$Q_{FH}^* = \bar{Z}[(1-t)\sigma A_{FH}^{1/\sigma} / \varphi A_{IH}]^{(\sigma/1-\sigma)}, \qquad (1.19)$$

and replacing (1.18) in (1.7), we get the housing production in the informal housing sector:

$$Q_{IH}^* = A_{IH} - \bar{Z}[(1-t)\sigma A_{FH} / \varphi A_{IH}^\sigma]^{(1/1-\sigma)}. \qquad (1.20)$$

The total housing production at each distance x of the CBD is obtained adding the production of both sectors:

$$Q^* = A_{IH} + \bar{Z}[(1-t)\sigma A_{FH}^{1/\sigma} / \varphi A_{IH}]^{(\sigma/1-\sigma)}[1-(1-t)\sigma / \varphi]. \qquad (1.21)$$

Let $Q_{FH}^* / l_{FH}^* = \varphi A_{FH} / \sigma(1-t)$ be the building height in the formal housing sector and $Q_{IH}^* / l_{IH}^* = A_{IH}$ the building height in the informal housing sector. When (1.9) and (1.10) hold, it is possible to show that it is always the case that: $\varphi / (1-t)\sigma > 1$, implying that formal constructions are taller than informal constructions. Let $q^* = \left[\varphi A_{FH} / \sigma(1-t)\right] l_{FH}^* + A_{IH} l_{IH}^*$ be the average tallness of the buildings. It is easy to verify that this expression is identical to total housing production. Then, Q^* is also informative about the tallness of buildings. Assuming that formal and unemployed workers live close to the CBD, whereas informal workers live in

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[14] If we had assumed a spatial variation in the infrastructure or in the probability of detection (or in any other parameter), an intra-urban spatial pattern would have emerged for the informal housing. However, in this paper, it is not of our interest to study these intra-urban patterns. Instead we focus on the impacts of the informal housing over the city shape, and over the labor market.

the periphery (an equilibrium result that we will show below), the population constraints are given by:

$$\int_0^{x_d^*} Q^* dx = N - L_{IN},$$ (1.22)

and

$$\int_{x_d^*}^{x_f^*} Q^* dx = L_{IN}.$$ (1.23)

By rearranging (1.22) and using (1.21), it is possible to find the frontier between the formal workers zone and the informal workers zone:

$$x_d^* = \frac{N - L_{IN}}{Q^*},$$ (1.24)

whereas by rearranging (1.23) and using (1.22) and (1.21), it is possible to find the city fringe:

$$x_f^* = \frac{N}{Q^*}.$$ (1.25)

Note that the city fringe x_f^*, through the housing production, depends of taxes, infrastructure, the probability of detection, total factor productivity in the formal housing sector, and land productivity in the informal housing sector. Also note that the land allocation, the housing production, and the city fringe x_f^* do not depend on the utility levels. This is a consequence of the assumptions of fixed housing consumption and perfect substitutability between the formal and the informal production. Now we can find the equilibrium values of the instantaneous utilities W_F^*, W_U^*, W_{IN}^* and the equilibrium housing rent $R_H^*(x)$. The corresponding bid rents for each worker type are given by the following equations:

$$\Psi_F(x, W_F) = w_F - \tau x - W_F,$$ (1.26)

$$\Psi_{IN}(x, W_{IN}) = w_{IN} - s_{IN}\tau x - W_{IN},$$ (1.27)

$$\Psi_U(x, W_U) = w_U - \tau x - W_U.$$ (1.28)

These functions are linear and decreasing in x. The bid rent for a formal worker and the bid rent for an unemployed worker share the same level of inclination. Also both bid rents are steeper than the bid rent for an informal worker. Then, in equilibrium, formal workers and the unemployed live close to the CBD, whereas informal workers live in the periphery (Figure 2).

$$\Psi_F(x, \bar{I}_F, \bar{I}_U, \bar{I}_{IN}) = \Psi_U(x, \bar{I}_F, \bar{I}_U, \bar{I}_{IN})$$

$$\Psi_{IN}(x, \bar{I}_F, \bar{I}_U, \bar{I}_{IN})$$

CBD   Formal workers and $N\text{-}L_{IN}$ Informal workers N

$x = 0$ Unemployed
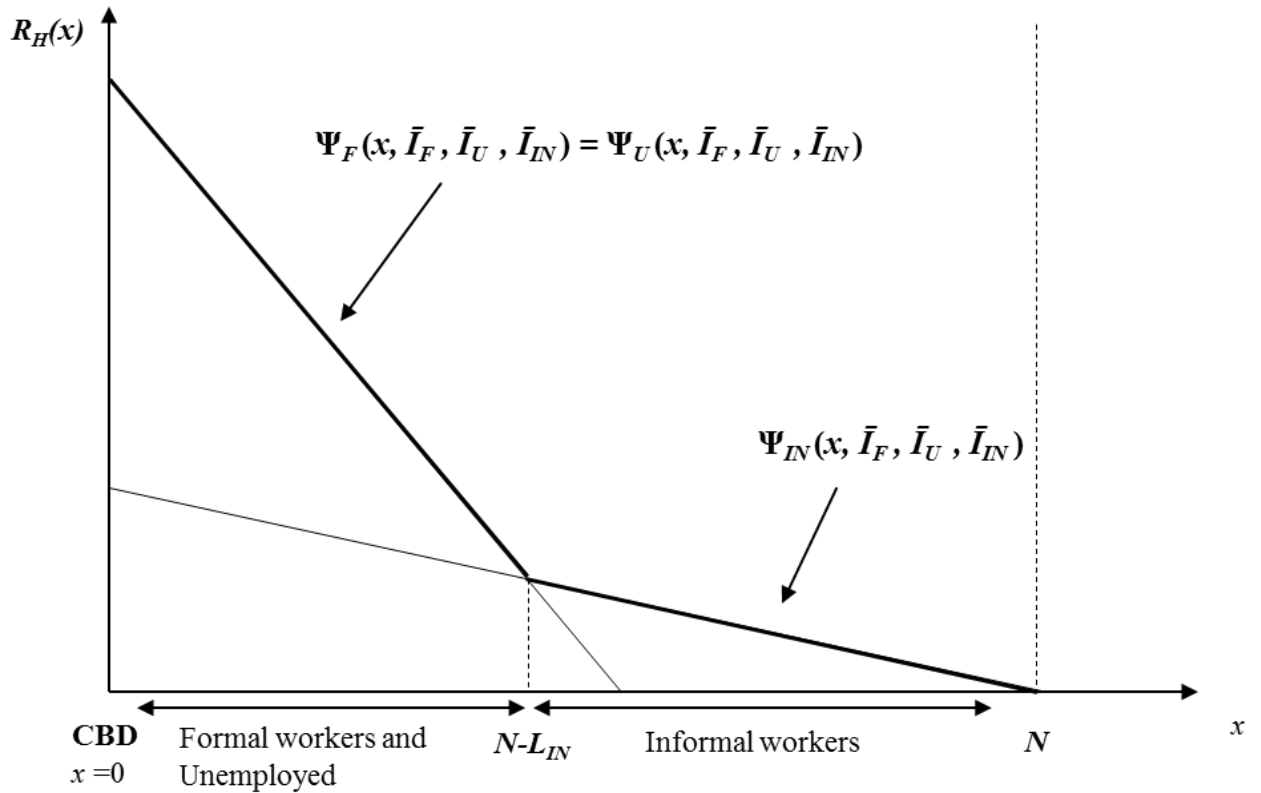
Figure 2. Urban land use equilibrium (The segregated city)

With this information it is possible to define the following equilibrium conditions (Zenou, 2009): an urban land use equilibrium, with fixed housing consumption, no relocation costs and formal, informal and unemployed workers is a n-tuple $\left(W_F^*, W_U^*, W_{IN}^*, R_H^*(x)\right)$ such that:

$$W_F^* - W_U^* = w_F - w_U, \tag{1.29}$$

$$w_U - \tau\left(\frac{N - L_{IN}}{Q^*}\right) - W_U^* = w_{IN} - s_{IN}\tau\left(\frac{N - L_{IN}}{Q^*}\right) - W_{IN}^*, \tag{1.30}$$

$$w_{IN} - s_{IN}\tau\frac{N}{Q^*} - W_{IN} = 0, \tag{1.31}$$

$$R_H^*(x) = \max\left\{\Psi_F(x, W_F^*), \Psi_U(x, W_U^*), \Psi_{IN}(x, W_{IN}^*), 0\right\} \text{ for each } x \in (0, x_f^*]. \tag{1.32}$$

Equation (1.29) implies that the bid rent for a formal worker is equal to the bid rent for an unemployed all over the city. Equation (1.30) implies that exactly at the frontier between the formal workers zone and the informal workers zone, x_d^*, the bid rent for a formal worker is equal to the bid rent for an informal worker. Equation(1.31), in turn, means that exactly at the city fringe the bid rent for an informal worker is equal to the agricultural rent (which is assumed to be zero). Equation(1.31), together with (1.16), guarantees that the equilibrium land rent is equal to agricultural rent at the city fringe. Finally, Equation (1.32) states that the equilibrium housing rent is equal to the upper envelope of the equilibrium bid rent curves of all workers and the agricultural rent line (Zenou, 2011). The equilibrium values of the instantaneous utilities for the formal, unemployed and informal workers can be obtained by using equations (1.29) to (1.31):

$$W_{IN}^* = w_{IN} - s_{IN}\frac{\tau}{Q^*}N, \tag{1.33}$$

$$W_U^* = w_U - \tau\frac{N}{Q^*} + (1 - s_{IN})\tau\frac{L_{IN}}{Q^*}, \tag{1.34}$$

$$W_F^* = w_F - \tau\frac{N}{Q^*} + (1 - s_{IN})\tau\frac{L_{IN}}{Q^*}. \tag{1.35}$$

By replacing (1.33), (1.34) and (1.35) in (1.26), (1.27) and (1.28) we can obtain the housing rent in equilibrium $R_H^*(x)$:

$$R_H^*(x) = \begin{cases} \tau\left(\dfrac{N}{Q^*} - x\right) - (1 - s_{IN})\tau \dfrac{L_{IN}}{Q^*} & \text{for} \quad 0 \le x \le \dfrac{N - L_{IN}}{Q^*}, \\[2ex] s_{IN}\tau\left(\dfrac{N}{Q^*} - x\right) & \text{for} \quad \dfrac{N - L_{IN}}{Q^*} \le x \le \dfrac{N}{Q^*}, \\[2ex] 0 & \text{for} \quad x \ge \dfrac{N}{Q^*}. \end{cases}$$

$$\tag{1.36}$$

Note that (1.36) together with (1.16) implies that the equilibrium land rent is described by:

$$R^*(x) = \begin{cases} \varphi A_{IH}\left[\tau\left(\dfrac{N}{Q^*} - x\right) - (1 - s_{IN})\tau \dfrac{L_{IN}}{Q^*}\right] & \text{for} \quad 0 \le x \le \dfrac{N - L_{IN}}{Q^*}, \\[2ex] \varphi A_{IH}\left[s_{IN}\tau\left(\dfrac{N}{Q^*} - x\right)\right] & \text{for} \quad \dfrac{N - L_{IN}}{Q^*} \le x \le \dfrac{N}{Q^*}, \\[2ex] 0 & \text{for} \quad x \ge \dfrac{N}{Q^*}. \end{cases}$$

$$\tag{1.37}$$

Finally, defining the urban cost for a worker as the sum of the housing rent and the commuting cost, we have that in equilibrium, the urban cost for a formal worker is $\tau(N/Q^*) - (1 - s_{IN})\tau(L_{IN}/Q^*)$, and for an informal worker it would be $s_{IN}\tau(N/Q^*)$.

1.4. Urban labor market equilibrium

1.4.1. Lifetime expected utility for workers and firms

Workers discount future values at a rate r. Let I_F, I_U, and I_{IN} denote the expected lifetime utility for a formal worker, for an unemployed worker and for an informal worker, respectively. In the steady-state, the Bellman equations for I_F and I_{IN} are given by:

$$rI_F = w_F - \tau\frac{N}{Q^*} + \left(1 - s_{IN}\right)\tau\frac{L_{IN}}{Q^*} - \delta_F\left(I_F - I_U\right), \tag{1.38}$$

$$rI_{IN} = w_{IN} - s_{IN}\tau\frac{N}{Q^*} + \delta_{IN}\left(I_U - I_{IN}\right), \tag{1.39}$$

which show that workers obtain their instantaneous utility in each period. Furthermore, formal workers can lose their jobs at a rate δ_F, which leads to a surplus loss of $I_F - I_U$. Informal workers, in turn, can lose their jobs at a rate δ_{IN}, so their change in surplus is given by $I_U - I_{IN}$. In the steady-state, the Bellman equation for I_U is given by:

$$rI_U = w_U - \tau \frac{N}{Q^*} + (1-s_{IN})\tau \frac{L_{IN}}{Q^*} + \theta q(\theta)(I_F - I_U) + \alpha(I_{IN} + b - I_U).$$

(1.40)

This equation shows that, besides obtaining her instantaneous utility, an unemployed worker can obtain either a job in the formal sector at a rate $\theta q(\theta)$ (since $s_U = 1$), with an accompanying increase in surplus of $I_F - I_U$, or a job in the informal sector at a rate α, with a surplus change of $I_{IN} + b - I_U$ (recall that b is the present value of the sum of all the transfers received while informally employed). Given that there are no relocation costs, in equilibrium all workers must reach the same utility level independently of their location in the city, therefore: $I_F = \bar{I}_F$, $I_U = \bar{I}_U$ and $I_{IN} = \bar{I}_{IN}$. Using (1.39) and (1.40) we obtain:

$$\bar{I}_U - \bar{I}_{IN} = \frac{1}{r + \alpha + \delta_{IN}}\left[w_U - w_{IN} - (1-s_{IN})\tau\left(\frac{N - L_{IN}}{Q^*}\right) + \alpha b + \theta q(\theta)(I_F - I_U)\right],$$

(1.41)

whereas by using (1.38) and (1.40) we get:

$$\bar{I}_F - \bar{I}_U = \frac{1}{r + \delta_F + \theta q(\theta)}\left[w_F - w_U - \alpha b + \alpha(I_U - I_{IN})\right]. \quad (1.42)$$

Then, replacing (1.41) in (1.42) we obtain:

$$\bar{I}_F - \bar{I}_U = \varphi(\theta)(r + \alpha + \delta_{IN})\left\{w_F - \gamma\left[w_{IN} + (1-s_{IN})\tau\left(\frac{N - L_{IN}}{Q^*}\right) + (r + \delta_{IN})b\right] - (1-\gamma)w_U\right\}$$

(1.43)

where $\varphi(\theta) = 1 / \left[(r + \delta_F)(r + \alpha + \delta_{IN}) + \theta q(\theta)(r + \delta_{IN}) \right]$ and $\gamma = \alpha / (r + \alpha + \delta_{IN})$.

Denoting the search cost per unit of time for a formal firms as c, we have that the value of a vacancy I_V and a filled vacancy I_O are given by:

$$rI_V = -c + q(\theta)(I_O - I_V),$$ (1.44)

$$rI_O = y_F - w_F - T - \delta_F(I_O - I_V).$$ (1.45)

Firms are free to post vacancies and they do until $I_V = 0$. This, together with (1.44) implies that:

$$I_O = \frac{c}{q(\theta)}.$$ (1.46)

If we use $I_V = 0$, and equations (1.45) and (1.46), we have the following equation determining job creation in the formal sector:

$$\frac{c}{q(\theta)} = \frac{y_F - w_F - T}{r + \delta_F}.$$ (1.47)

1.4.2. Wages in equilibrium

At each period, wages are set through a generalized Nash-bargaining process between firms and workers:

$$w_F = \underset{w_F}{\operatorname{argmax}} \left(I_F - I_U \right)^\beta \left(I_O - I_V \right)^{1-\beta},$$ (1.48)

where $0 \le \beta \le 1$ represents the bargaining power of workers. By solving (1.48) we obtain:

$$w_F = (1 - \beta) \left\{ \gamma \left[w_{IN} + (1 - s_{IN}) \tau \left(\frac{N - L_{IN}}{Q^*} \right) + (r + \delta_{IN}) b \right] + (1 - \gamma) w_U \right\} + \beta \left[y_F - T + (1 - \gamma) c\theta \right]$$

$.$ (1.49)

The first term in (1.49) is the compensation that firms must pay to induce workers to accept a job in the formal sector. The compensation is the fraction $(1-\beta)$ of the *expected* income obtained by the worker outside the formal sector, which is a weighted average of the unemployed *current* income w_U and the informal worker income, $w_{IN}+(1-s_{IN})\tau\left((N-L_{IN})/Q^*\right)+(r+\delta_{IN})b$, being $(1-\gamma)$ and γ the respective weights. This compensation is different from the one obtained in urban labor models without an informal sector or with a residual informal sector (Wasmer and Zenou, 2002; Zenou, 2008; and Zenou, 2011), where firms only compensate the unemployed worker with a fraction of his *current* income. This is because, in contrast with the mentioned models, here firms acknowledge that an unemployed worker can find a job in an informal sector that offers intrinsic benefits. One of these benefits is the *urban cost saving for the informal worker,* given by the difference between the urban cost for a formal worker and the urban cost for an informal worker: $(1-s_{IN})\tau\left((N-L_{IN})/Q^*\right)$. This difference is always positive, reflecting the fact that an informal worker commutes less often to the CBD.

The urban cost saving for an informal worker relates, for the first time in the literature, the wages in the formal sector (and the job creation rate), both with the degree of centralization of the informal sector s_{IN}, and with the sum of both the informal housing production and the formal housing production at each x, Q^*. The other benefits that make part of the informal sector income, and which play a secondary role in the analysis, are the wage w_{IN}, and the fraction $(r+\delta_{IN})$ of the transfers b. Finally, it is important to notice that if we had assumed that unemployed workers do not commute daily to the CBD, the urban cost for an unemployed worker would have been lower than the urban cost of a formal worker. This in turn would have created an additional urban cost saving. In previous models like Wasmer and Zenou (2002), and Zenou (2011) there is only one urban cost saving, because the informal sector does not exist or its spatial features are neglected. Therefore, the kind of informality studied here reveals a spatial compensation that has not been previously taken into account.

1.4.3. Informality and unemployment rates, and the steady-state equilibrium

The structure of the model allows for deriving explicit analytical expressions for the informality rate and the unemployment rate. In the steady-state, two conditions must hold: first, that the number of unemployed workers that find a formal job, $\theta q(\theta)U$, equals the number of workers that lose their formal jobs, $\delta_F L_F$, and second, that the number of unemployed workers that find an informal job, αU, equals the number of informal workers that lose their job, $\delta_{IN} L_{IN}$. Therefore, given that $U = N - L_F - L_{IN}$, the following must hold:

$$\delta_F L_F = \theta q(\theta)(N - L_F - L_{IN}),\qquad(1.50)$$

$$\delta_{IN} L_{IN} = \alpha(N - L_F - L_{IN}),\qquad(1.51)$$

which implies that the informality rate ϕ^* and unemployment rate u^* in the steady-state are given by:

$$\phi^* = \frac{\delta_F \alpha}{\delta_F(\delta_{IN} + \alpha) + \delta_{IN}\theta q(\theta)},\qquad(1.52)$$

$$u^* = \frac{\delta_F \delta_{IN}}{\delta_F(\delta_{IN} + \alpha) + \delta_{IN}\theta q(\theta)}.\qquad(1.53)$$

It is possible to verify that $(\partial\phi^*/\partial\theta) < 0$, $(\partial u^*/\partial\theta) < 0$, which shows that an increase in formal job creation leads to a decrease in both the informality and the unemployment rates.

Given a certain value of the labor market tightness θ, it is possible to determine all the endogenous variables in the model. Replacing (1.49) (the equilibrium wage in the formal sector) and (1.52) (the informality rate in the steady-state) into (1.47), we obtain the following condition determining the formal job creation rate in equilibrium θ^*:

$$(r + \delta_F)\frac{c}{q(\theta^*)} + \beta(1-\gamma)c\theta^* -$$

$$(1-\beta)\left\{ y_F - T - \gamma\left[w_{IN} + (1 - s_{IN})\tau(1 - \phi^*)\frac{N}{Q^*} + (r + \delta_{IN})b \right] - (1-\gamma)w_U \right\} = 0.$$

$$(1.54)$$

Let Λ be the expression on the left hand side of (1.54). Assuming that formal sector productivity is always higher than the expected income of the unemployed, i.e. assuming that:

$$y_F > \gamma \left[w_{IN} + \left(1 - s_{IN}\right) \tau \left(\frac{\delta_{IN}}{\delta_{IN} + \alpha} \right) \frac{N}{Q^*} + \left(r + \delta_{IN}\right) b \right] + \left(1 - \gamma\right) w_U + T ,$$

(1.55)

it is always the case that $\left(\partial \Lambda / \partial \theta\right) > 0,\ \lim_{\theta \to +\infty} \Lambda = \infty,\ \lim_{\theta \to 0} \Lambda < 0$. Thus, there must be a unique θ^* that satisfies equation (1.54), and consequently, there is a unique steady-state equilibrium.

1.5. Comparative statics

In this section, we investigate the effects of changes in the city's parameters over the job creation rate θ^*, the informality rate ϕ^*, the housing rent $R_H^*\left(x\right)$ and the land rent $R^*\left(x\right)$. We pay special attention on the parameters that give shape to the structure of the city: 1) the level of centralization of the informal sector s_{IN} in the labor market 2) the commuting cost per unit of distance τ, 3) the provision of the public infrastructure $\bar{Z}$, 4) the added value taxes t charged to formal land developers , and 5) the probability of detection of an informal land developer φ (see the Appendix A for formal proofs).

1.5.1. Decrease in the informal sector centralization

Recall that a decrease in the commuting frequency of an informal worker is always followed by an increase of the wage fraction generated through home-based work. Spatially, this can be seen as a decrease in the informal production at the center and a rise of the informal production in the periphery. As a consequence, the effects on the economy of a decrease in s_{IN} provide an idea of the effects on the economy of a lower degree of centralization of the informal sector.

When the commuting frequency s_{IN} decreases, the commuting cost for an informal worker decreases. This causes a reduction of the equilibrium housing rent all over the city and an

increase in the urban cost saving for the informal worker $(1-s_{IN})\tau((N-L_{IN})/Q^*)$. Consequently in the labor market, the wage paid by a formal firm increases, the job creation in the formal sector decreases, and the informality rate gets increased. In the land market, the increased labor informality makes the formal workers zone smaller. This relaxes the competition for housing and causes an additional reduction of the equilibrium housing rent at the formal workers zone. Finally, when the housing rent decreases, the value of marginal productivity of land decreases (both for the informal developers and the formal developers, at any distance x of the CBD). This is described in Figure 3 as a simultaneous downward shift of the $FH(x)$ curve$(FH(x)$ to $FH(x)')$ and the $IH(x)$ curve$(IH(x)$ to $IH(x)')$. Because these shifts are of the same proportion for both curves, there is a decrease in the equilibrium land rent, and no changes in the intersectoral housing production. The results are synthetized in proposition 1.
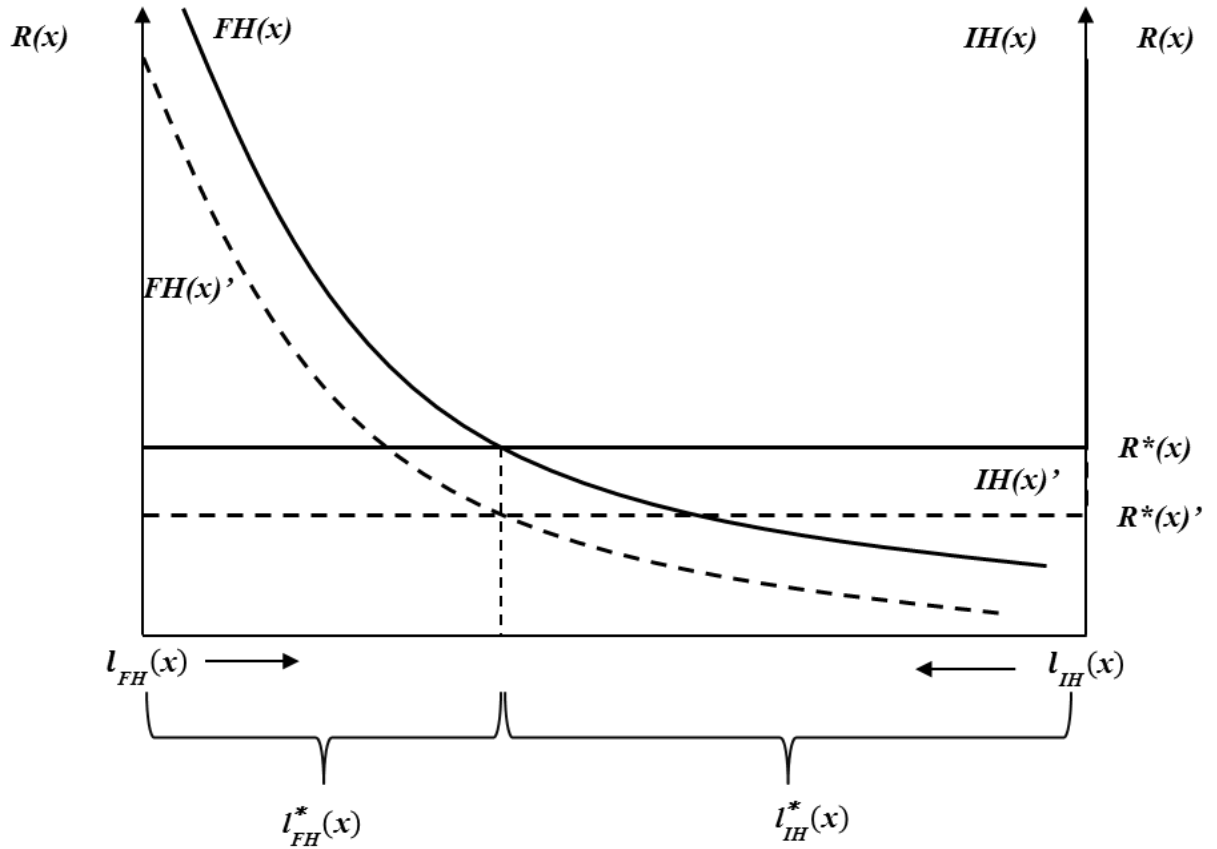
Figure 3. Effects of decentralization on the urban land market

Proposition 1

A decrease in the level of centralization of the informal sector s_{IN} leads to: a decrease in the job creation rate in the formal sector θ^*, an increase in the informality rate ϕ^*, and a decrease in the housing rent $R_H^*(x)$ and the land rent $R^*(x)$ at each $x \in \left[0, (N/Q^*)\right]$.

1.5.2. Increase in the commuting cost per unit of distance

An increase in the commuting cost per unit of distance τ leads to an increase in the total commuting cost for all workers, but in a higher proportion for the formal workers, as they commute more often to the CBD. As a consequence, there is a rise in the urban cost saving of informal workers $(1-s_{IN})\tau\left((N-L_{IN})/Q^*\right)$. As in the previous case, this implies that the wage paid by a formal firm rises, the job creation in the formal sector decreases, and the informality rate increases. These results lead to a key insight: *the informality rate can be affected by policies targeting intraurban mobility costs.* In the land market, the increase in commuting cost directly rises housing rents and land rents in the informal workers zone. In the formal workers zone, on the one hand, we have a direct effect that increases the housing rent and land rent. On the other hand, there is an indirect effect, leading to a decrease in these rents. The indirect effect exists because the formal workers zone gets reduced, relaxing the competition for housing. We have that the direct effect dominates near the CBD and the indirect effect dominates far from the CBD.

Proposition 2

An increase in the commuting cost per unit of distance τ leads to: a decrease in the job creation rate in the formal sector θ^*, and an increase in the informality rate ϕ^*. Also this variation implies that the housing rent $R_H^*(x)$ and land rent $R^*(x)$ both get increased at each

$$x \in \left[0, (N/Q)\left\{1-(1-s_{IN})\left(\tau(\partial\phi^*/\partial\tau)+\phi^*\right)\right\}\right] \quad and \quad x \in \left[((N-L_{IN})/Q^*), (N/Q^*)\right], \quad and$$

decrease at each $x \in \left[(N/Q)\left\{1-(1-s_{IN})\left(\tau(\partial\phi^*/\partial\tau)+\phi^*\right)\right\}, ((N-L_{IN})/Q^*)\right].$

1.5.3. Decrease in the provision of the public infrastructure (increase in the tax level charged to formal land developers)

At each distance $x \in \left[0, \left(N/Q^*\right)\right]$, a decrease in the provision of infrastructure $\bar{Z}$, reduces the marginal productivity of land for formal land developers, affecting their land demand given by (1.12). This is described in Figure 4 as a downward shift of the $FH(x)$ curve $\left(FH(x) \text{ to } FH(x)'\right)$. This leads to a decrease in the land allocation in the formal housing sector $l_{FH}^*(x)$, and to an increase in the land allocation in the informal housing sector $l_{IH}^*(x)$, whereas leaves the land rent $R^*(x)$ unaffected. Given the increased land allocation in the less productive sector, at each distance x the total housing production Q^* (and the average tallness of buildings q^*) decreases. Because of this, the city has to spread out to accommodate all the households. Therefore, the city fringe x_f^* and the frontier x_d^* are now farther from the CBD. In short, now the city is less tall and also more spread, i.e., it is less compact. As a consequence, the urban costs for all workers increases, but in a higher proportion for the formal workers because they commute more often to the CBD. As a result, the urban cost saving for the informal workers $\left(1 - s_{IN}\right) \tau \left(\left(N - L_{IN}\right)/Q^*\right)$ rises. Again this leads to a higher wage in the formal sector, to a lower job creation in the formal sector, and to a higher informality rate. Finally, in the informal workers zone, the housing rent and land rent rise because the city fringe x_f^* is now farther from the CBD. In the formal workers zone, on the one hand, the housing rent and land rent tend to rise because the higher informal housing, and on the other hand, these rents tend to fall because the formal employment falls. As a consequence of this, the change in the rents is ambiguous.

Proposition 3

A decrease in the provision of the public infrastructure $\bar{Z}$ (an increase in the taxes charged to the informal land developers) increases the informal housing all over the city, making the city less tall and more spread, i.e., less compact. This in turn leads to: a decrease in the job creation

rate in the formal sector θ^*, *an increase in the informality rate* ϕ^*, *an increase in the housing rent* $R_H^*(x)$ *and the land rent* $R^*(x)$ *at each* $x \in \left[\left(\left(N - L_{IN}\right)/Q^*\right), \left(N/Q^*\right)\right]$, *and an ambiguous change in these rents at each* $x \in \left[0, \left(\left(N - L_{IN}\right)/Q^*\right)\right]$.

Proposition 3 establishes a clear relationship between some of the most important features exhibited by cities in developing countries. *Remarkably, it shows that a higher level of informal housing leads to a larger spatial size of the city, and to a higher level of informality in the labor market.* It is important to notice that a rise in the level of taxes t, affects the land demand conditions in the same way that a decrease in the infrastructure. Thus, it generates exactly the same effects described in Proposition 3.
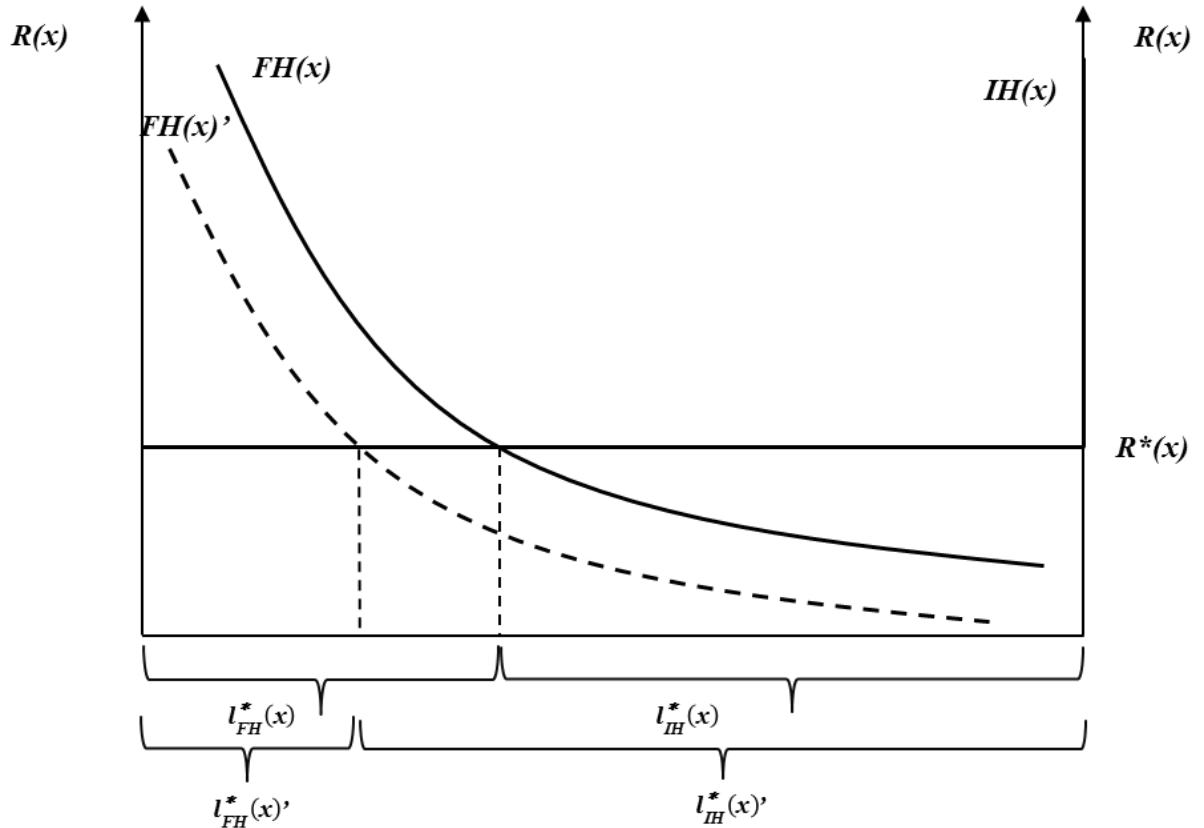
Figure 4. Initial effects of changes in the public infrastructure or the taxes on the land market

1.5.4. Effects of changes in the probability of detection of informal housing

At each distance $x \in \left[0, \left(N/Q^*\right)\right]$, a rise in the probability of detection $1-\varphi$, causes that the value of the expected marginal productivity of land in the informal housing sector to fall. Then, the $IH(x)$ curve shifts downwards in Figure 5 $\left(IH(x) \text{ to } IH(x)'\right)$. It is easy to see that this lowers the land rent $R^*(x)$ and the land allocated to the informal housing sector $l_{IH}^*(x)$. As a consequence, the total housing production Q^* and the average building height q^* increase. Because of this, the city is more compact now. This implies that the city fringe x_f^* and the frontier x_d^* are now closer to the CBD. As a result, the urban cost saving for an informal worker $\left(1-s_{IN}\right)\tau\left(\left(N-L_{IN}\right)/Q^*\right)$ falls. Therefore, the formal wage falls, formal job creation increases, and the informality rate decreases. Finally, in the informal workers zone, the housing rents and land rents fall whereas in the formal workers zone, the change in these rents is ambiguous.
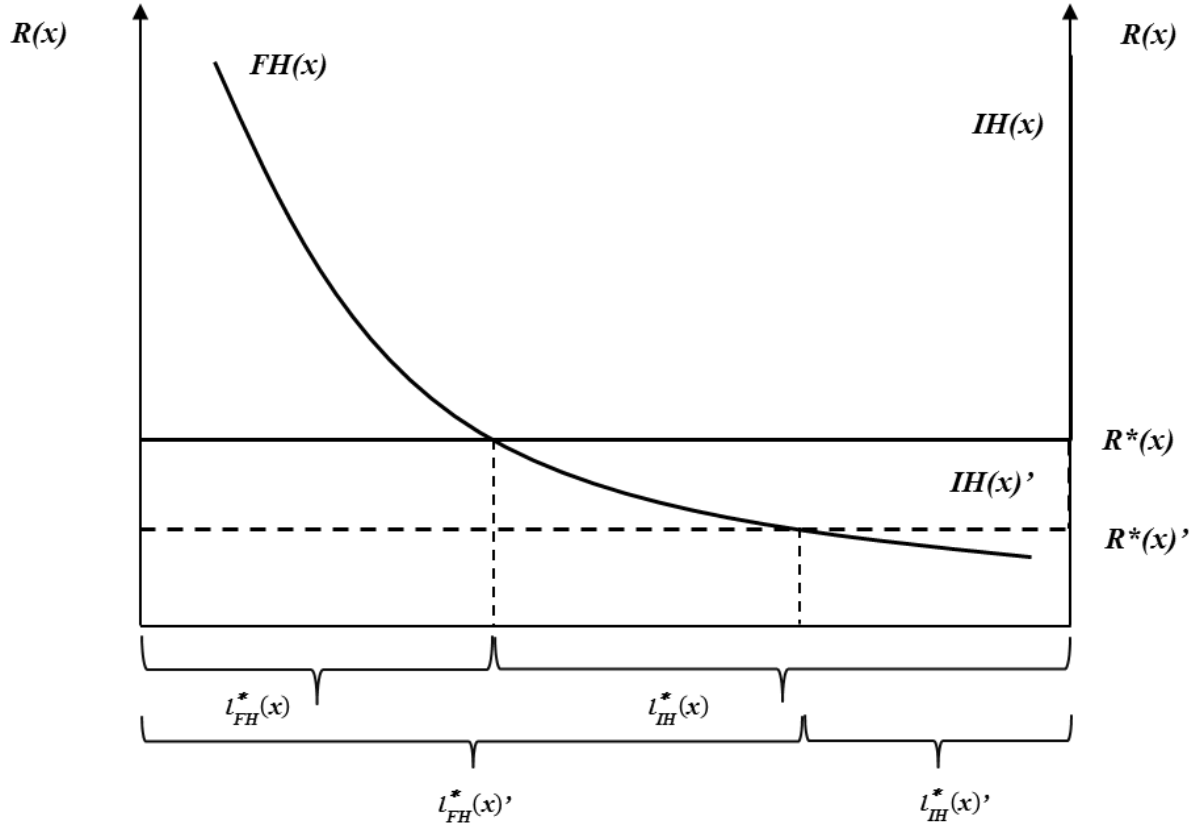
Figure 5. Initial effects of changes in the probability of detection on the land market

Proposition 4

A rise in the probability of detection $1-\varphi$, decreases the informal housing all over the city, making the city more compact. This in turn leads to: a rise in the job creation rate in the formal sector θ^*, a decrease in the informality rate ϕ^*, a decrease in the housing rent $R_H^*(x)$ and the land rent $R^*(x)$ at each $x \in \left[\left(\left(N-L_{IN}\right)/Q^*\right),\left(N/Q^*\right)\right]$, and an ambiguous change in these rents at each $x \in \left[0,\left(\left(N-L_{IN}\right)/Q^*\right)\right]$.

1.6.   Rural urban migration, informal labor, and informal housing.

Cities in developing countries display not only high levels of labor informality and unemployment, but also high rates of urban growth fueled by rural urban migration (Brueckner and Lall, 2014). For instance, in Latin America, rural-urban migration explained 30% of urban growth between 1980 and 2010 (CEPAL, 2012). The seminal work of Harris and Todaro (1970) shed light on why rural-urban migration occurs even when cities display large unemployment rates. Harris and Todaro's work highlights the fact that rural workers migrate to cities because urban *expected* incomes are higher. In their model, the equilibrating mechanism is the rate of urban unemployment. As the urban population increases the unemployment rate also increases. As a result, the expected income in the city decreases, reducing the incentives to migrate. Later extensions of Harris and Todaro's work (Brueckner and Zenou 1999; Brueckner and Kim 2001) consider that besides the unemployment rate, the increase in urban costs can also act as an equilibrating mechanism.

Despite these theoretical advances, it is still not clear how migration works when workers face not only the possibility of being unemployed, but also the possibility of working in a decentralized informal sector and living in informal housing. Next, we extend our model to the case of an open city where there is rural-urban migration a-là Harris-Todaro. With this extension, we aim at studying the relationship between the level of decentralization of the informal sector and the level of rural urban migration. We also use this extension to study in a detailed way how transport policies can affect labor informality. In particular, we seek to compare different policy options in terms of their efficiency.

1.6.1. Set up and calibration

Following Zenou (2011), we now consider a scenario where the non-housing good is produced in two regions: an urban and a rural. The total population is denoted by $\bar{N}$ and, as before, the urban population by N. The mass of rural workers (which as it will be shown below is also the total population in the rural region) is denoted by L_R. Therefore, the total population is $\bar{N} = L_F + L_{IN} + L_R + U$, the urban population is $N = \bar{N} - L_R$, and the unemployment level in the city is given by: $U = \bar{N} - L_F - L_{IN} - L_R$. The city structure is the

same as the one described before. In the rural region, the production function is $F_R(L_R)$ with $F_R'(L_R) > 0$ and $F_R''(L_R) < 0$. In equilibrium the rural wage is equal to the marginal product of rural labor:

$$w_R = F_R'(L_R). \tag{1.56}$$

As a consequence, there is no rural unemployment. Therefore, the total population in the rural region equals the mass of rural workers. Rural workers do not commute because they live close the fields that they cultivate. As a consequence, the urban cost (the commuting cost plus the housing rent) is higher than the rural cost (the rural worker pays the rural rent for the land he consumes). The instantaneous utility for a rural worker is given by the difference between the rural wage w_R and the rural rent (which is zero). Therefore, the expected lifetime utility of a rural worker is given by: $\int_0^\infty w_R e^{-rt} = (w_R/r)$.

As in the Harris-Todaro model, incentives for rural urban migration exist and workers base their migration decision on their expected incomes. Following Zenou (2011), we assume that a rural worker cannot search for an urban job from home, but she must first become unemployed in the city, and then search for an urban job. As a result, rural urban migration guarantees that, in equilibrium, the expected lifetime utility of a rural worker is equal to the expected lifetime utility of an unemployed worker. Formally, the migration condition is given by:

$$\bar{I}_U = \frac{w_R}{r}. \tag{1.57}$$

As before, the flows in and out unemployment must equalize:

$$\delta_F L_F = \theta q(\theta)(\bar{N} - L_F - L_{IN} - L_R), \tag{1.58}$$

$$\delta_{IN} L_{IN} = \alpha(\bar{N} - L_F - L_{IN} - L_R). \tag{1.59}$$

Therefore, the steady-state relationships between urban and rural employment are given by:

$$L_F = \frac{\delta_{IN}\theta q(\theta)}{\delta_F(\delta_{IN} + \alpha) + \delta_{IN}\theta q(\theta)}(\bar{N} - L_R), \tag{1.60}$$

$$L_{IN} = \frac{\delta_F \alpha}{\delta_F (\delta_{IN} + \alpha) + \delta_{IN} \theta q(\theta)} (\bar{N} - L_R). \qquad (1.61)$$

Because the urban population is precisely equal to $\bar{N} - L_R$, the informality rate ϕ^* and unemployment rate u^* in the city are still given by (1.52) and (1.53). Using the informality rate (1.52), we have that the equilibrium wage in the formal sector is now given by:

$$w_F = (1-\beta)\left\{ \gamma \left[w_{IN} + (1-s_{IN})\tau(1-\phi^*)\left(\frac{\bar{N} - L_R}{Q^*} \right) + (r + \delta_{IN})b \right] + (1-\gamma)w_U \right\} + \beta \left[y_F - T + (1-\gamma)c\theta \right]$$
.
$$(1.62)$$

whereas the difference between the expected lifetime utility of the formal worker and the expected lifetime utility of the unemployed worker is given by:

$$\bar{I}_F - \bar{I}_U = \varphi(\theta)(r + \alpha + \delta_{IN})\left\{ w_F - \gamma \left[w_{IN} + (1-s_{IN})\tau(1-\phi^*)\left(\frac{\bar{N} - L_R}{Q^*} \right) + (r + \delta_{IN})b \right] - (1-\gamma)w_U \right\}$$
$$(1.63)$$

Replacing the new wage (1.62) into (1.47), we obtain the new condition that determines the job creation rate as a function of the level of rural employment:

$$(r + \delta_F)\frac{c}{q(\theta^*)} + \beta(1-\gamma)c\theta^*$$
$$-(1-\beta)\left\{ y_F - T - \gamma \left[w_{IN} + (1-s_{IN})\tau(1-\phi^*)\frac{(\bar{N} - L_R)}{Q^*} + (r + \delta_{IN})b \right] - (1-\gamma)w_U \right\} = 0.$$
$$(1.64)$$

By using (1.62), (1.52), (1.56), and replacing (1.63) and (1.42) in (1.40), the migration condition (1.57) can be written as:

$$w_{IN} - s_{IN}\tau\left(\frac{\bar{N}-L_R}{Q^*}\right) + (1-\gamma)\left\{w_U + \alpha b - w_{IN} - (1-s_{IN})\tau(1-\phi^*)\left(\frac{\bar{N}-L_R}{Q^*}\right)\right\}$$

$$+\theta q(\theta)\varphi(\theta)(r+\delta_{IN})\beta\left\{y_F - T + (1-\gamma)c\theta - \gamma\left[w_{IN} + (1-s_{IN})\tau(1-\phi^*)\left(\frac{\bar{N}-L_R}{Q^*}\right) + (r+\delta_{IN})b\right] - (1-\gamma)w_U\right\} = F_R'(L_R)$$

<div align="right">(1.65)</div>

This condition determines the rural employment (the size of both regions) as a function of the job creation rate. By using equations (1.64) and (1.65), it is possible to find the equilibrium job creation rate θ^* and the equilibrium rural employment L_R^*. Thus, equations (1.64) and (1.65) characterize the steady-state equilibrium. The model is calibrated in order to reproduce relevant stylized facts of Latin American cities. We decided to solve the model in this way given that our interest is to study the implications of labor and housing informality on rural urban migration in a specific developing world context.

We begin by fixing parameters in the housing industry because the housing production is independent of any outcome in the labor market. We fix the total factor productivity in the formal housing sector A_{FH} at 1; whereas the land productivity in the informal housing sector A_{IH} is fixed at 0.5. The tax rate t is fixed at 0.3; the infrastructure $\bar{Z}$ at 1.3; and the probability of detection $1-\varphi$ at 0. The parameter of the formal production function σ is fixed in 0.5. Under the specified values, the total housing production at each x is 0.6479, whereas the formal housing production is 0.2275. As a result, the share of formal housing in the market is 35%. This is very close to estimated values for Colombia (Minvivienda, Gobierno de Colombia, 2010).

Now we focus on the parameters in the urban labor market. The parameter values (with a year as the implicit unit of time) are within the range of those used in previous studies (Albrecht, et al., 2009; Zenou, 2008; Zenou, 2011; Gómez, 2013; Gómez and Jaramillo, 2013). We start by fixing the discount rate r at 0.05; the unemployed worker current income w_U at 0; the informal job creation rate α at 1.5; the commuting costs per unit of distance τ at 0.01; and the total population $\bar{N}$ at 10. We then fix the productivity in the formal sector y_F at 1; the worker's bargaining power β at 0.5; the job destruction rate δ_F at 0.4. The tax level T is fixed at 0.3; and the cost of maintaining a vacancy c at 0.2. In the formal sector, we assume a

matching function of the following type: $d\left(\overline{S}_U U, V\right) = U^{0.5} V^{0.5}$. For the informal sector we fix both the wage w_{IN}, the amount of transfers (or value of social insurance subsidies) b, the job destruction rate δ_{IN}, and the decentralization rate s_{IN} at 0.5. Finally, in the rural region the production function is given by: $F_R\left(L_R\right) = L_R^{0.7}$.

Under the specified values, the model predicts an unemployment rate of 15.89% and an informality rate of 47.67%. This implies that approximately 56% of all employed workers are in the informal sector. This value is close to recent estimates for many Latin American cities, including Bogotá (57%), Medellín (50%), Lima (57%) and Buenos Aires (45%) (Galvis, 2012; CNTPE, 2008; MTEySS, 2007). The model also predicts that the rural population is about the 23.1% of the total population. Again, this value is close to recent estimates for many Latin American counties, including Colombia (24%), Peru (22%) and Brazil (15%) (World Bank, 2015).

1.6.2. Comparative statics

Now, we investigate the effects of changes in the city's parameters on rural urban migration. Again, we pay special attention on the parameters that shape the structure of the city: 1) the level of centralization of the informal sector s_{IN}; 2) the commuting cost per unit of distance τ; 3) and the parameters that determine informal housing production $\left(\overline{Z}, \text{and } t\right)$ (see the Appendix B for simulation results).

1.6.2.1. Decrease in the informal sector centralization

Figure 6 in the Appendix B shows the results of this analysis. As before, when the commuting frequency s_{IN} decreases, the job creation decreases. As a consequence, the informality and unemployment rate increase, and the probability to obtain a job in the formal sector $\theta q(\theta)$ decreases. This adverse situation in the urban labor market generates a downward pressure on the expected lifetime utility of the unemployed worker $r\overline{I}_U$. Therefore, recalling that the migration condition is given by $r\overline{I}_U = w_R$, it is expected a migration flow to the rural area (as

in a traditional Harris-Todaro setting). However, this is not the case. Surprisingly, more individuals are willing to work in the city. This is because when the informal sector becomes more decentralized, there is a strong direct effect on the urban *land* market that reduces the urban cost for *all workers*, leading to a significant increase of $r\overline{I}_U$. In more detail, reorganizing the left side of (1.65), it is possible to show that $r\overline{I}_U$ is a weighted average of the instantaneous utility levels in equilibrium: [15]

$$
r\overline{I}_U = \varphi(\theta)(r+\delta_{IN})\left\{
\begin{array}{l}
\theta q(\theta)\left\{w_F - [1-(1-s_{IN})]\phi^*\tau\left(\dfrac{\overline{N}-L_R}{Q^*}\right)\right\} + \alpha\dfrac{(r+\delta_F)}{(r+\delta_{IN})}\left\{w_{IN} + (r+\delta_{IN})b - s_{IN}\tau\left(\dfrac{\overline{N}-L_R}{Q^*}\right)\right\} + \\[3mm]
(r+\delta_F)\left\{w_U - [1-(1-s_{IN})]\phi^*\tau\left(\dfrac{\overline{N}-L_R}{Q^*}\right)\right\}
\end{array}
\right\}
$$

.
$$\text{(1.66)}$$

When the commuting frequency s_{IN} decreases, the competition for urban land is relaxed all over the city. As a consequence, the urban cost decreases for *all workers*, leading to an increase in their respective instantaneous utility levels. From (1.66) it can be seen that this causes an increase in the expected lifetime utility of the unemployed worker $r\overline{I}_U$. In conclusion, a more decentralized informal sector stimulates rural urban migration because, even if worsens the urban labor market, gives to the rural workers the advantage to live in a city with a lower cost of life (lower urban cost).

There are interesting implications of these results. First, even if the formality rate decreases, the level of formal work increases. Second, the unemployment rate and the informal rate can overreact to the increase in decentralization, because now they vary in response to both the lower job creation in the formal sector and the higher rural urban migration. Finally, it is worth noticing that both the level of formal employment and the level of unemployment increase. This suggests the existence of a "Todaro paradox".[16] However this is not the case, because both levels increase as a direct consequence of rural urban migration.

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[15] Recall that in equilibrium the instantaneous utility for a worker is given by de difference between its current income and the equilibrium urban cost (see equations (1.33), (1.34), and (1.35)).

[16] The Todaro paradox emerges when one extra minimum-wage job could induce more than one urban worker to migrate to the rural area, hence increasing the unemployment rate.

1.6.2.2.   Increase in the commuting cost per unit of distance

Figure 7 in the Appendix B shows the results of the analysis. As in the closed city case, an increase in the commuting cost per unit of distance τ leads to a lower job creation in the formal sector and to a higher informality rate. At the same time, the increase in the commuting cost increases the competition for urban land all over the city, making the urban cost higher for *all workers.* These adverse situations both in the land market and the labor market, lead to a lower level in the expected lifetime utility of the unemployed worker. As a consequence, the urban area is now less attractive. Therefore, the rural population increase and the urban population decrease.

1.6.2.3.   Decrease in the provision of the public infrastructure (increase in the tax level charged to formal land developers)

An increase in the taxes charged to formal land developers leads to lower formal housing and to lower housing production Q^*. This directly reduces the tallness of buildings and increases the spatial size of the city (i.e. pushes outwards the city fringe).[17] As a result, the urban cost saving for an informal worker rises, leading to a lower job creation, and to a higher informality rate (see Figure 8 in the Appendix B for the results). Because the city is now more spread, the competition for urban land increases, making the urban cost higher for *all workers.* Then, the urban area becomes less attractive and the rural population increases. As a consequence, there is an indirect pressure that reduces the spatial size of the city. At the end, the spatial size of the city effectively increases because this indirect pressure is not too strong. Then as before*, a higher level of informal housing leads to a larger spatial size of the city and to a higher level of informality in the labor market.*

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[17] The city fringe is given by: $x_f^* = \dfrac{N - L_R}{Q^*}$.

1.7. Policy analysis

In a previous analysis we have showed how the differences in commuting costs between formal and informal workers impact the urban labor market. In this section, we study in detail the ability of transport policies to reduce informality. On the one hand, we want to evaluate, in a comparative way, how they can impact welfare and informality levels. On the other hand, we want to identify how these policies should be implemented. To do so, we assume the government uses a limited amount of resources in one of three possible policy options: 1) an entry cost subsidy for formal firms; 2) a transport subsidy to all workers, and 3) a transport subsidy targeted to formal workers only. We assume that the government's budget is financed through a tax T on formal firms.[18]

Under the first policy option, the government assumes a fraction σ^c of the search costs of each formal firm, so that after the subsidy, the search cost is equal to $(1-\sigma^c)c$. Under the second policy option, the government assumes a fraction σ^τ of the total commuting costs of each worker, so the actual total commuting cost is $(1-\sigma^\tau)\tau x$ for formal workers and $(1-\sigma^\tau)s_{IN}\tau x$ for informal workers. This specification guarantees that, regardless of the traveled distance, formal workers face a commuting cost per unit distance equal to $(1-\sigma^\tau)\tau$, whereas informal workers one equal to $(1-\sigma^\tau)s_{IN}\tau$. Under the third policy option, the government gives to each formal worker a fraction $\sigma^{\tau F}$ of its total commuting costs, so that the actual total commuting cost becomes $(1-\sigma^{\tau F})\tau x$, while the commuting cost per unit of distance is $(1-\sigma^{\tau F})\tau$.

It is important to clarify that because we assume, first, that taxes are exogenous and fixed, and second, that no person receives a larger subsidy than other person, the subsidy has to adjust endogenously in order to meet the budget restriction. Lastly, we do not consider a subsidy to informal workers only, because generates exactly the same effects of an increase in the level

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[18] We simplify the analysis by assuming that housing production is exogenous and equal to 1 in all distances x with respect to the CBD.

of decentralization of the informal sector, and consequently results in an increase in the informality rate.

1.7.1. Policies under a budget restriction

Given that in equilibrium the number of vacancies is equal to $V^* = \theta^* u^* (N - L_R)$, we have that under the search cost subsidy, the government's budget restriction is $TL_F = \sigma^c c \theta^* u^* (N - L_R)$. This implies that the subsidy is endogenous and equal to:

$$\sigma^c = \frac{TL_F}{c\theta^* u^* (N - L_R)}. \tag{1.67}$$

Because we assume that $0 < \sigma^c < 1$, $TL_F < c\theta^* u^* N$ must hold. Equation(1.67) shows that the subsidy diminishes with an increase either in formal job creation, unemployment, the urban population or search-costs. The conditions that describe the equilibrium under this policy (and the conditions under the subsequent policy options) can be found in the Appendix C. Under a transport subsidy for all workers, the government's budget restriction is:

$$TL_F = \int_0^{(1-\phi^*)(N-L_R)} \sigma^\tau \tau x dx + \int_{(1-\phi^*)(N-L_R)}^{(N-L_R)} \sigma^\tau s_{IN} \tau x dx,$$

which is equivalent to: $TL_F = 0.5\sigma^\tau \tau (N - L_R)^2 \left[1 - (1 - S_{IN})(2\phi^* - \phi^{*2}) \right]$. This implies that the subsidy is given by:

$$\sigma^\tau = \frac{2TL_F}{\tau (N - L_R)^2 \left[1 - (1 - S_{IN})(2\phi^* - \phi^{*2}) \right]}. \tag{1.68}$$

Again, because we are assuming that $0 < \sigma^\tau < 1$, $2TL_F < \tau (N - L_R)^2 \left[1 - (1 - S_{IN})(2\phi^* - \phi^{*2}) \right]$ must hold. From (1.68), it can be seen that when the informality rate increases, the subsidy must increase in order to meet the budget

restriction. This happens because more informal workers are receiving the subsidy, and each of them faces lower commuting costs per unit of distance. Lastly, under a transport subsidy for formal workers only, the budget restriction is:

$$TL_F = \int_0^{\left(1-\phi^*\right)\left(N-L_R\right)} \sigma^{\tau F} \tau x \, dx \,,$$

or $TL_F = 0.5\sigma^{\tau F}\tau\left(N-L_R\right)^2\left(1-\phi^*\right)^2$. Then, the subsidy is given by:

$$\sigma^{\tau F} = \frac{2TL_F}{\tau\left(N-L_R\right)^2\left(1-\phi^*\right)^2} \,. \tag{1.69}$$

In this case, assuming that $0 < \sigma^{\tau F} < 1$, $2TL_F < \tau\left(N-L_R\right)^2\left(1-\phi^*\right)^2$ must hold. Because the subsidy is given to formal workers only, $1 - \sigma^{\tau F} > s_{IN}$ is the condition that guarantees the spatial equilibrium described in Figure 1. From (1.69) it can be seen that if the informality rate increases, the subsidy increases as well. This is because less formal workers receive the subsidy with same amount of public resources.

1.7.2. Comparative performance of transport policies

In order to evaluate and compare different policy options, we will use the change in the informality rate and the change in total welfare. Welfare is measured as the sum of the rural and urban output levels W (Sato, 2004; Zenou, 2009; Zenou, 2011):

$$W^* = \int_0^{+\infty} e^{-rt}\left\{y_F\left(1-u^*-\phi^*\right)N + w_U u^* N + w_{IN}\phi^* N - c\theta^* u^* N + \alpha u^* Nb - 0.5\tau\left(1-\phi^*\right)^2 N^2 - S_{IN}\tau\phi^* N^2\left(1-0.5\phi^*\right)\right\}dt +$$

$$\int_0^{+\infty} e^{-rt} F_R\left(L_R^*\right)dt.$$

$$\tag{1.70}$$

This expression is the sum of formal sector output, informal sector output, agricultural (rural) output, unemployment benefits, and government transfers to informal workers minus the costs of maintaining vacancies, commuting costs of formal workers and commuting costs of informal workers. Because wages, land and housing rents are pure transfers, they are not included in the calculation of welfare.

Table 1 shows the results. The first thing to note is that the search cost subsidy is the most efficient policy, because it achieves the largest level of welfare (163.38), the lowest unemployment rate (8.5), and the lowest informality rate (25.50). This happens for two reasons. On the one hand, the subsidy stimulates formal employment creation, both indirectly (reducing the formal wage) and directly (see equation (1.47)). All other policies only stimulate formal employment creation through a reduction in the formal wage.

Position		θ^*	ϕ^*	u^*	L_F^*	L_{IN}^*	U^*	L_R^*	W^*
	Base equilibrium	0.88	47.19	15.73	2.96	3.77	1.25	1.99	145.69
1	Search cost subsidy	9.64	25.50	8.50	5.59	2.16	0.72	1.52	163.38
3	Transport subsidy (all)	1.20	44.47	14.82	3.69	4.03	1.34	0.92	149.27
2	Transport subsidy (formal)	2.76	36.78	12.26	4.68	3.38	1.12	0.80	156.04

Table 1. Policy efficiency comparison

On the other hand, the search cost subsidy leads the economy to a level of rural-urban migration (and to a level of urban labor supply) that generates the lower saturation in the urban labor market. As a matter of fact, this policy generates the most significant reduction in both the informality and unemployment *levels*. This is because the entry cost subsidy induces rural-urban migration only through the improvement in the labor market, without directly affecting the land market and the urban costs. On the contrary, the transport subsidy to all workers significantly decreases the urban costs, inducing large rural-urban migration flows.

As a result the informality and unemployment *levels* increase. This result is similar to the "Todaro paradox".

The second thing to note is that the transport subsidy targeted to formal workers only outperforms the transport subsidy to all workers. Both subsidies directly affect the urban land market, leading to a large rural-urban migration. However, the targeted subsidy directly reduces the difference that originates the spatial compensation considered in the formal wage $(1 - s_{IN})$. Because of this, the effect of this subsidy on the labor market is strong enough to absorb the new migrants. In fact, under this subsidy, the levels of unemployment and informality decrease. In contrast, as note before, the transport subsidy to all workers does not reduce the informality and unemployment levels.

1.7.3. Sensitivity analysis

In order to check the robustness of the results, we compare the policies for different parameter values. The considered parameters are: the commuting costs per unit of distance, the search costs, the bargaining power and the level of decentralization. As can be seen in Figure 9, Figure 10, Figure 11 and Figure 12 in the Appendix D, the search cost subsidy generates the largest welfare value and the lowest informality rate regardless of the value these parameters take. Furthermore, the conclusion that the transport subsidy for formal workers only, is superior to the transport subsidy for all workers also holds for a wide range of parameter values.

1.8.   Conclusions

Most of the existing literature analyzing the causes of urban labor informality has focused on institutional reasons. Another important set of determinants, which has not been yet extensively explored in the literature, comes from the characteristic of cities in which informal workers reside. In order to investigate the impact of city structure in urban labor informality, this paper builds a monocentric city model with a labor market characterized by search frictions and the presence of an informal sector. To focus in the particular structure of the Latin American cities, we assume that: (1) formal workers commute on a daily basis to the city center where formal economic activity is centralized, whereas informal workers commute less often, live in the periphery and undertake some of their productive activities at home (informal jobs are in part decentralized), and (2) there is presence of informal housing.

The results show that in order to induce workers to accept a job in the formal sector, formal firms must compensate them with the expected income obtained outside the formal sector. This expected income is a weighted sum of the income obtained as an unemployed (e.g. unemployment insurance) and the income obtained as an informal worker. Part of this income is the commuting cost saving obtained by the informal worker by commuting less often to the city center. We demonstrate how higher informal job decentralization, a higher commuting cost, and a higher proportion of informal housing (as a result of lower infrastructure provision, or a lower probability of detection) all result in a higher informality rate. This is because all these situations increase the benefits of not commuting frequently to the CBD, and consequently increase the spatial compensation that formal firms have to offer as part of the formal wage.

We extend our model for the case of an open city with rural-urban migration a-là Harris-Todaro. This extension allows us to capture simultaneously urban labor, land and housing market interactions, and at the same time, it allows us to determine their impact on rural-urban migration. In the model, both the unemployment rate and urban costs act as an equilibrating force. We find, as before, that higher informal job decentralization leads to a higher informality rate. This should lead to lower rural-urban migration. Surprisingly, the opposite holds: rural-urban migration actually increases. This happens because a more decentralized informal sector relaxes the competition for land near the CBD, reducing in this way the urban costs for all workers and effectively increasing the urban expected income.

Finally, we use the extended model to compare the impact of search costs and transport policies on labor informality and welfare. The results show that an entry-cost subsidy has a higher impact on the informality rate than transport policies. This happens for two reasons. First, an entry-cost subsidy stimulates formal employment creation directly and indirectly (through a reduction in wages), while transport policies only affect formal employment creation indirectly. Second, the entry-cost subsidy affects rural-urban migration only through its effect on formal employment creation, while transport subsidies have an additional effect reducing urban costs, which ultimately lead to higher rural-urban migration and a higher saturation of the labor market.

1.9.   Appendix A

Proof of proposition 1

Consider a decrease in the level of centralization of the informal sector s_{IN}. First, totally differentiating equation (1.54) we have that: $\left(\partial\theta^*/\partial s_{IN}\right)=-\left(\partial\Lambda/\partial s_{IN}\right)/\left(\partial\Lambda/\partial\theta\right)$. Because $\left(\partial\Lambda/\partial\theta\right)>0$, the sign of $\left(\partial\theta^*/\partial s_{IN}\right)$ is the opposite of $\left(\partial\Lambda/\partial s_{IN}\right)$. Using (1.54) it is easy to show that $\left(\partial\Lambda/\partial s_{IN}\right)<0$, and therefore the job creation rate θ^* decreases with a decrease in the commuting frequency. Additionally we have that: $\left(\partial\phi^*/\partial s_{IN}\right)=\left(\partial\phi^*/\partial\theta^*\right)\left(\partial\theta^*/\partial s_{IN}\right)$, and because $\left(\partial\phi^*/\partial\theta^*\right)<0$, the sign of $\left(\partial\phi^*/\partial s_{IN}\right)$ is the same of $\left(\partial\Lambda/\partial s_{IN}\right)$. Thus, the informality rate ϕ^* increases with a decrease in the commuting frequency. We use (1.36) to see the effects over the housing rents. In the informal workers zone we have: $\left(\partial R_H^*\left(x\right)/\partial s_{IN}\right)=\tau\left(N/Q^*-x\right)>0$, whereas in the formal workers zone we have: $\left(\partial R_H^*\left(x\right)/\partial s_{IN}\right)=-\tau\left(N/Q^*\right)\left(\partial\phi^*/\partial s_{IN}\right)\left(1-s_{IN}\right)+\tau\left(N/Q^*\right)\phi^*>0$, which means that housing rents decrease all over the city with a decrease in the commuting frequency. Because the land rents are the housing rents times φA_{IH}, land rents also decrease with the commuting frequency.

Proof of proposition 2

Consider an increase in the commuting cost per unit of distance τ. Proceeding as before, we need to find the sign of $\left(\partial\Lambda/\partial\tau\right)$ to know how θ^* and ϕ^* change. Using (1.54), it can be shown that $\left(\partial\Lambda/\partial\tau\right)>0$, and therefore the job creation rate θ^* decreases and the informality rate ϕ^* increases with an increase in the commuting cost. We use (1.36) to see the effects over the housing rents. In the informal workers zone we have: $\left(\partial R_H^*\left(x\right)/\partial\tau\right)=s_{IN}\left(\left(N/Q^*\right)-x\right)>0$, whereas in the formal workers zone we have: $\left(\partial R_H^*\left(x\right)/\partial\tau\right)=\left(\left(N/Q^*\right)-x\right)-\left(1-s_{IN}\right)\left(N/Q^*\right)\left(\tau\left(\partial\phi^*/\partial\tau\right)+\phi^*\right)$. This implies that the

housing rent $R_H^*(x)$ and land rent $R^*(x)$ increase at each

$$x \in \left[0, (N/Q)\left\{1-(1-s_{IN})\left(\tau\left(\partial\phi^*/\partial\tau\right)+\phi^*\right)\right\}\right] \quad \text{and } x \in \left[\left((N-L_{IN})/Q^*\right),(N/Q^*)\right], \quad \text{and}$$

decrease at each $x \in \left[(N/Q)\left\{1-(1-s_{IN})\left(\tau\left(\partial\phi^*/\partial\tau\right)+\phi^*\right)\right\}, \left((N-L_{IN})/Q^*\right)\right]$.

Proof of proposition 3

Consider a decrease in the provision of the public infrastructure $\bar{Z}$. Using (1.54) it can be shown that $(\partial\Lambda/\partial\bar{Z})<0$, and therefore the job creation rate θ^* decreases and the informality rate ϕ^* increases with a decrease in the provision of the public infrastructure. In the informal workers zone we have that the land rents and housing rents increase. This is because it always holds that: $(\partial R_H^*(x)/\partial\bar{Z})=(\partial R_H^*(x)/\partial Q^*)(\partial Q^*/\partial\bar{Z})$, with $(\partial R_H^*(x)/\partial Q^*)<0$ and $(\partial Q^*/\partial\bar{Z})>0$. In the formal workers zone we have that:

$$\left(\partial R_H^*(x)/\partial\bar{Z}\right)=\left(-\tau N/\left(Q^*\right)^2\right)\left(\partial Q^*/\partial\bar{Z}\right)\left[1+(1-s_{IN})\phi^*\left\{\left(Q^*/\phi^*\right)\left(\partial\phi^*/\partial\bar{Z}\right)\left(1/\left(\partial Q^*/\partial\bar{Z}\right)\right)-1\right\}\right]$$

, which implies that the housing rent $R_H^*(x)$ and land rent $R^*(x)$ have an ambiguous change.

1.10. Appendix B
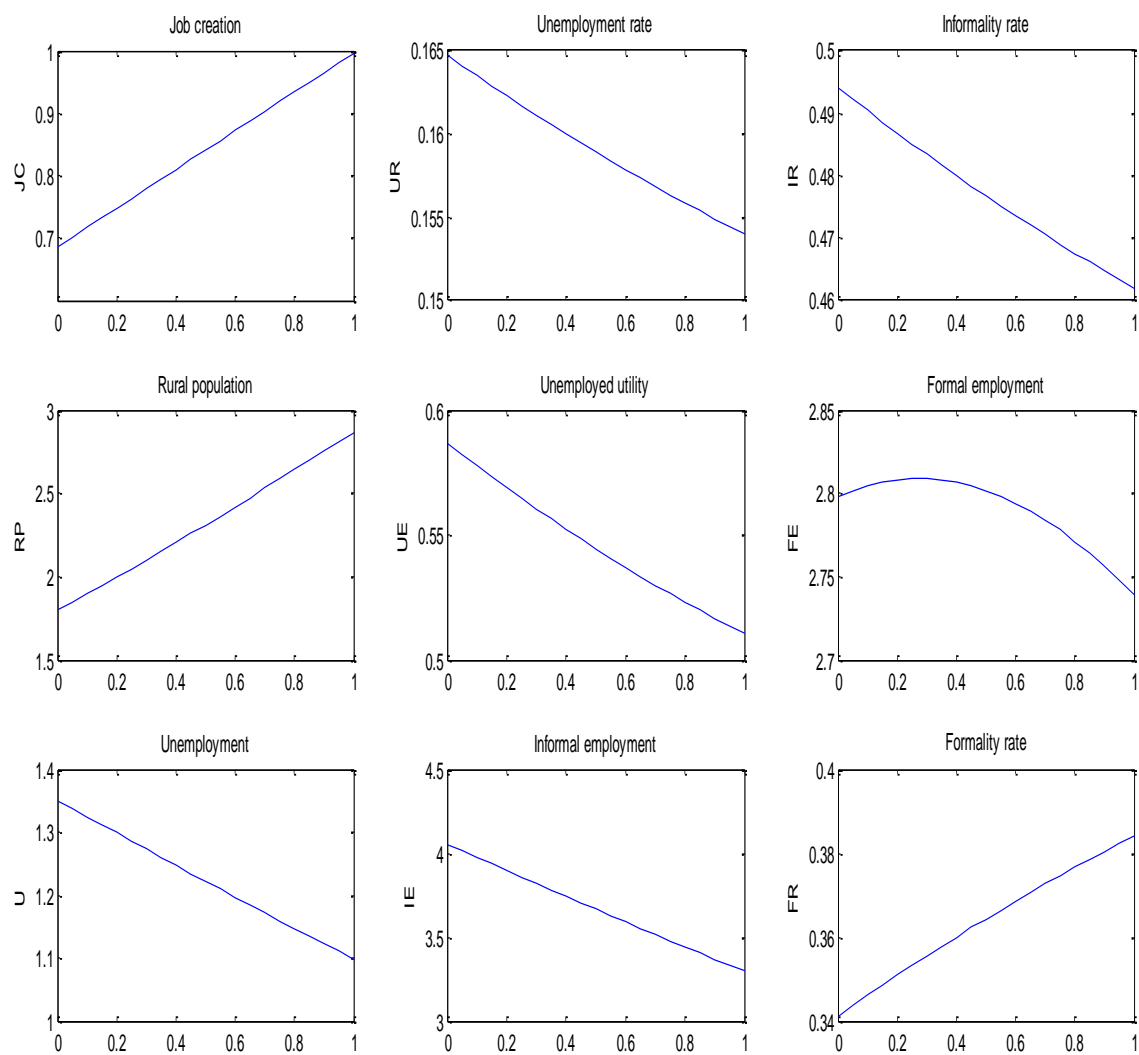
Figure 6. Effects of a higher decentralization of the informal sector
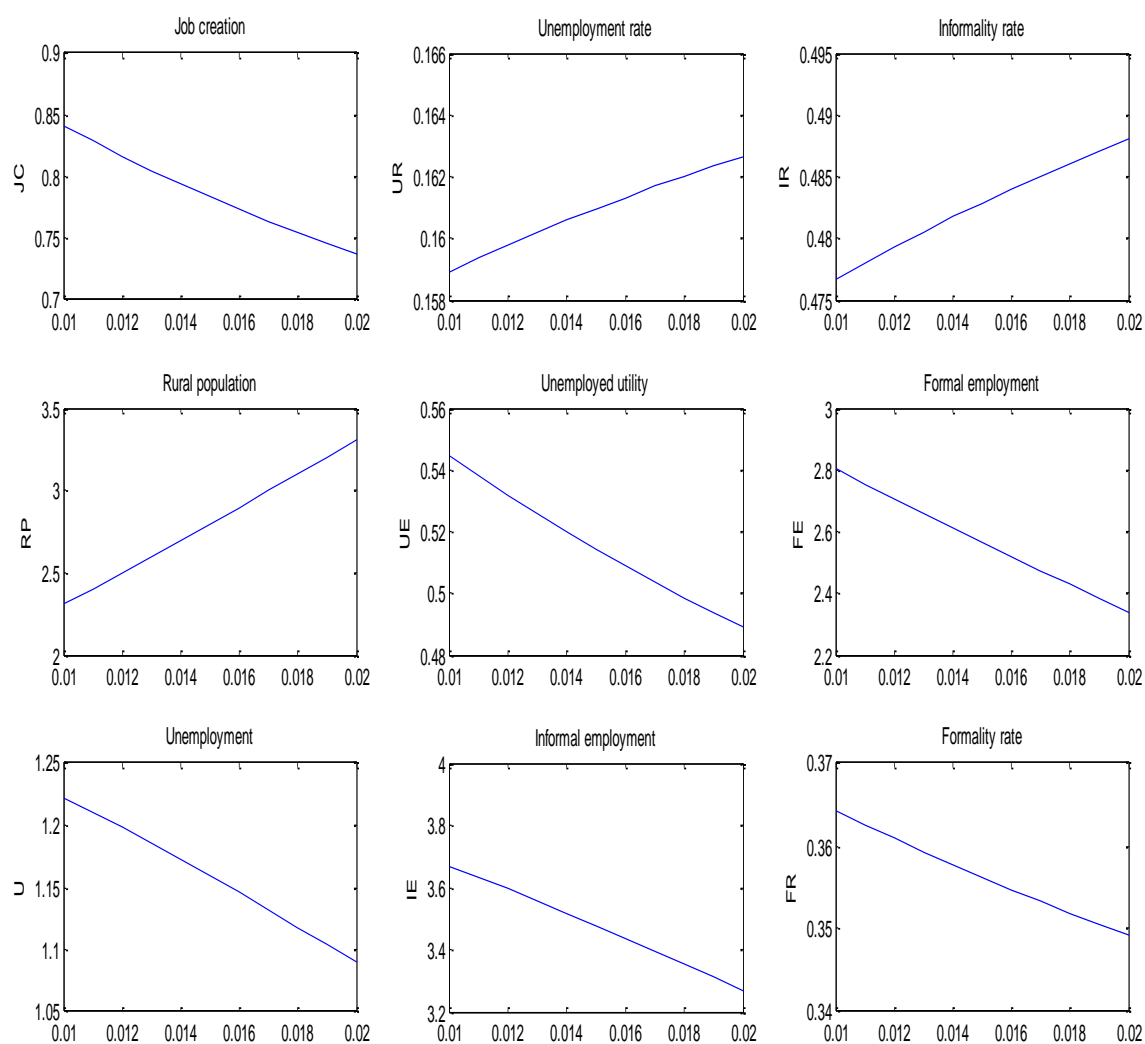
Figure 7. Effects of a rise in the commuting cost
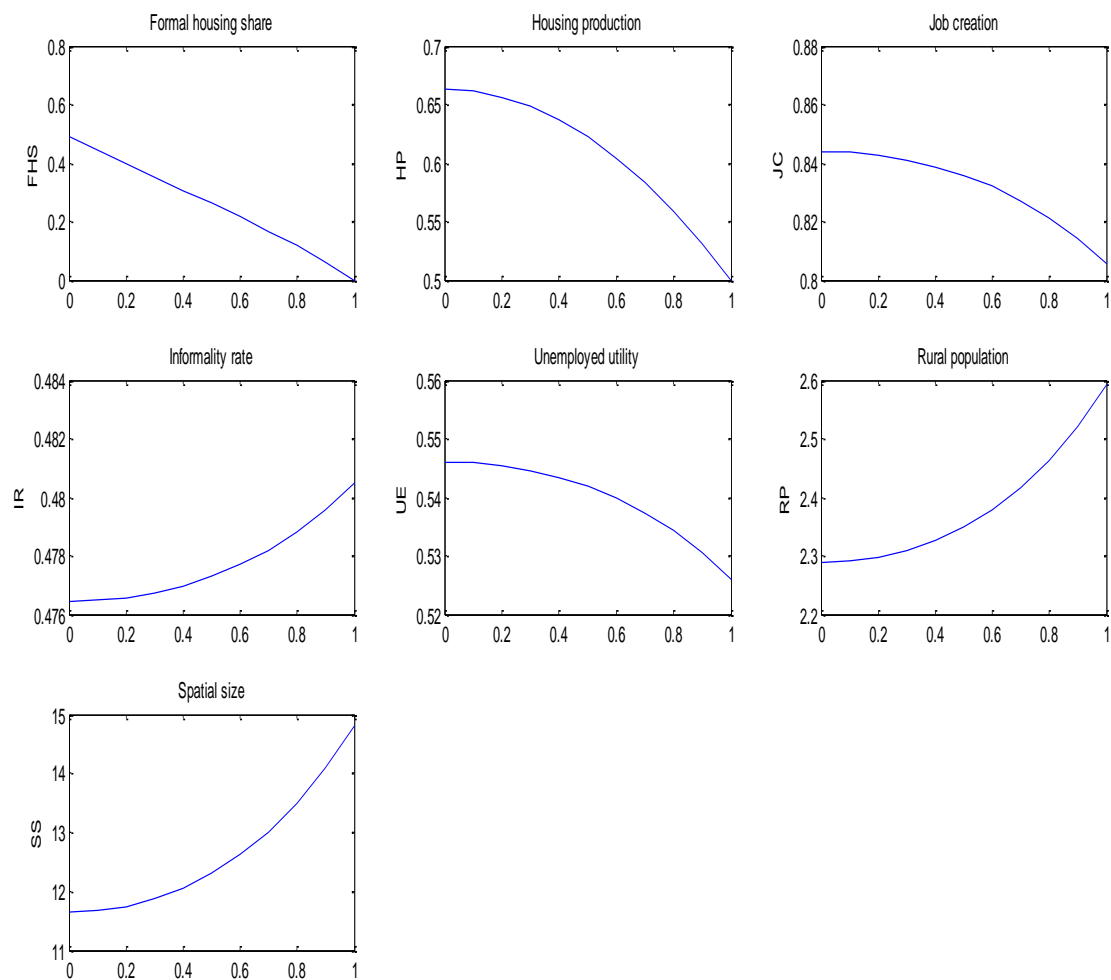
Figure 8. Effects of a decrease in the provision of public infrastructure (or an increase in the tax level charged to formal land developers)

1.11. Appendix C

Entry-costs subsidy:

$$(r+\delta_F)\frac{(1-\sigma^c)c}{q(\theta^*)}+\beta(1-\gamma)(1-\sigma^c)c\theta^*$$

$$-(1-\beta)\left\{y_F-T-\gamma\left[w_{IN}+(1-s_{IN})\tau(1-\phi^*)\frac{(\bar{N}-L_R)}{Q^*}+(r+\delta_{IN})b\right]-(1-\gamma)w_U\right\}=0$$

and

$$w_{IN}-s_{IN}\tau\left(\frac{\bar{N}-L_R}{Q^*}\right)+(1-\gamma)\left\{w_U+\alpha b-w_{IN}-(1-s_{IN})\tau(1-\phi^*)\left(\frac{\bar{N}-L_R}{Q^*}\right)\right\}$$

$$+\theta q(\theta)\varphi(\theta)(r+\delta_{IN})\beta\left\{y_F-T+(1-\gamma)(1-\sigma^c)c\theta-\gamma\left[w_{IN}+(1-s_{IN})\tau(1-\phi^*)\left(\frac{\bar{N}-L_R}{Q^*}\right)+(r+\delta_{IN})b\right]-(1-\gamma)w_U\right\}=F_R'(L_R)$$

where σ^c is given by (1.67).

Transport subsidy to all workers

$$(r+\delta_F)\frac{c}{q(\theta^*)}+\beta(1-\gamma)c\theta^*$$

$$-(1-\beta)\left\{y_F-T-\gamma\left[w_{IN}+(1-s_{IN})(1-\sigma^\tau)\tau(1-\phi^*)\frac{(\bar{N}-L_R)}{Q^*}+(r+\delta_{IN})b\right]-(1-\gamma)w_U\right\}=0$$

and

$$w_{IN} - s_{IN}\left(1-\sigma^{\tau}\right)\tau\left(\frac{\bar{N}-L_R}{Q^*}\right) + \left(1-\gamma\right)\left\{w_U + \alpha b - w_{IN} - \left(1-s_{IN}\right)\left(1-\sigma^{\tau}\right)\tau\left(1-\phi^*\right)\left(\frac{\bar{N}-L_R}{Q^*}\right)\right\}$$

$$+\theta q\left(\theta\right)\varphi\left(\theta\right)\left(r+\delta_{IN}\right)\beta\left\{y_F - T + \left(1-\gamma\right)c\theta - \gamma\left[w_{IN} + \left(1-s_{IN}\right)\left(1-\sigma^{\tau}\right)\tau\left(1-\phi^*\right)\left(\frac{\bar{N}-L_R}{Q^*}\right) + \left(r+\delta_{IN}\right)b\right] - \left(1-\gamma\right)w_U\right\} = F_R'\left(L_R\right)$$

where σ^{τ} is given by (1.68).

Transport subsidy to formal workers only

$$\left(r+\delta_F\right)\frac{c}{q\left(\theta^*\right)} + \beta\left(1-\gamma\right)c\theta^*$$

$$-\left(1-\beta\right)\left\{y_F - T - \gamma\left[w_{IN} + \left(1-\sigma^{\tau F} - s_{IN}\right)\tau\left(1-\phi^*\right)\frac{\left(\bar{N}-L_R\right)}{Q^*} + \left(r+\delta_{IN}\right)b\right] - \left(1-\gamma\right)w_U\right\} = 0$$

and

$$w_{IN} - s_{IN}\tau\left(\frac{\bar{N}-L_R}{Q^*}\right) + \left(1-\gamma\right)\left\{w_U + \alpha b - w_{IN} - \left(1-\sigma^{\tau F} - s_{IN}\right)\tau\left(1-\phi^*\right)\left(\frac{\bar{N}-L_R}{Q^*}\right)\right\}$$

$$+\theta q\left(\theta\right)\varphi\left(\theta\right)\left(r+\delta_{IN}\right)\beta\left\{y_F - T + \left(1-\gamma\right)c\theta - \gamma\left[w_{IN} + \left(1-\sigma^{\tau F} - s_{IN}\right)\tau\left(1-\phi^*\right)\left(\frac{\bar{N}-L_R}{Q^*}\right) + \left(r+\delta_{IN}\right)b\right] - \left(1-\gamma\right)w_U\right\} = F_R'\left(L_R\right)$$

where $\sigma^{\tau F}$ is given by (1.69).
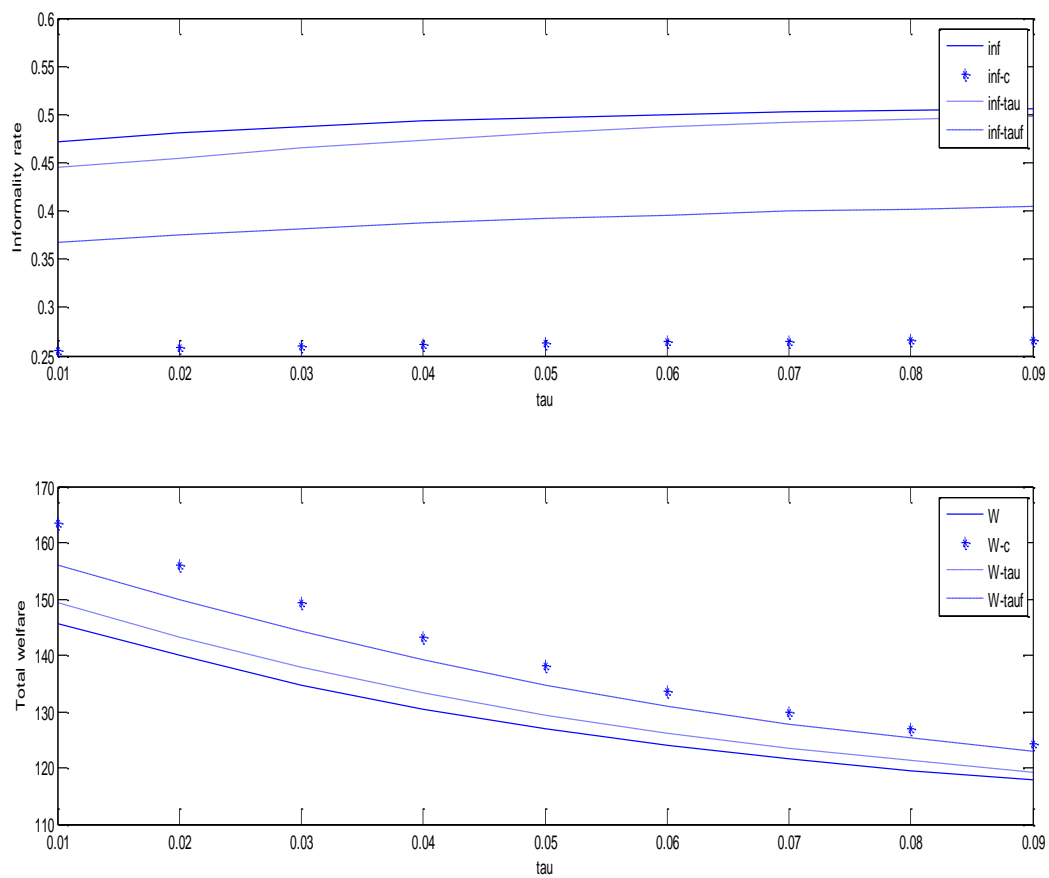
1.12. Appendix D

Figure 9. Informality rate vs. Commuting costs, and Total welfare vs. Commuting costs
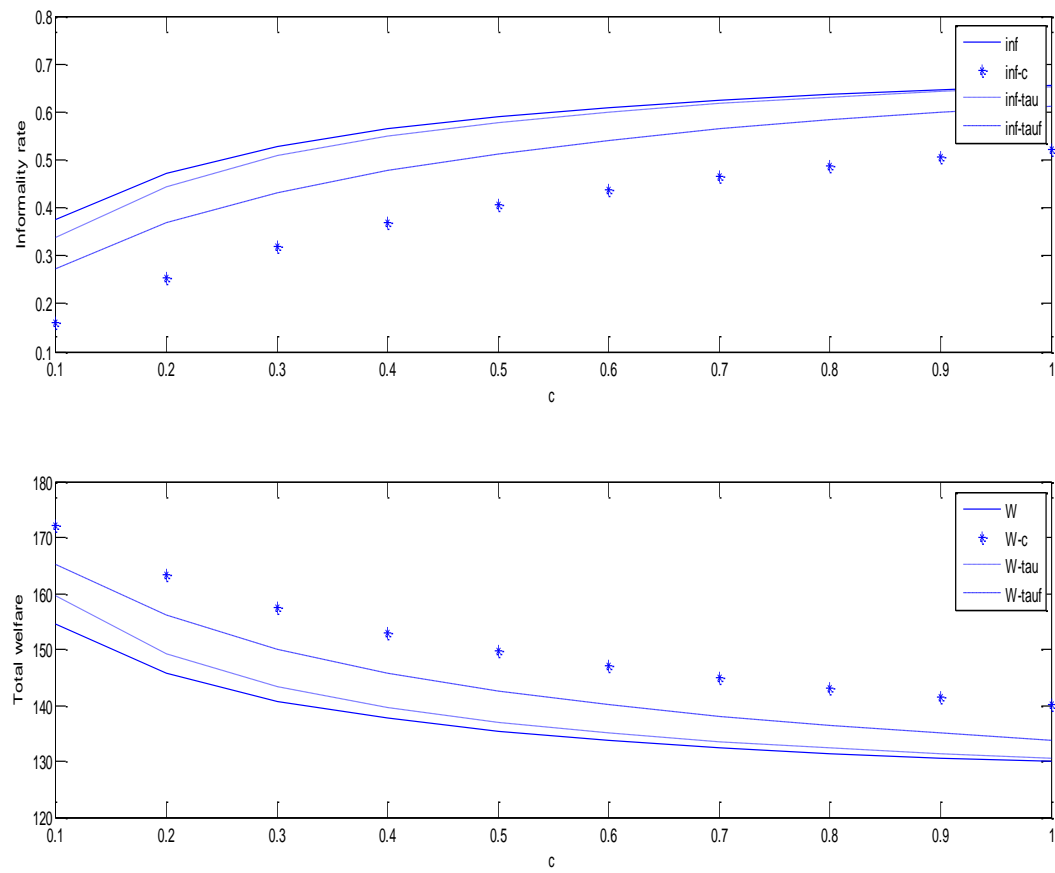
Figure 10. Informality rate vs. Entry costs, and Total welfare vs. Entry costs
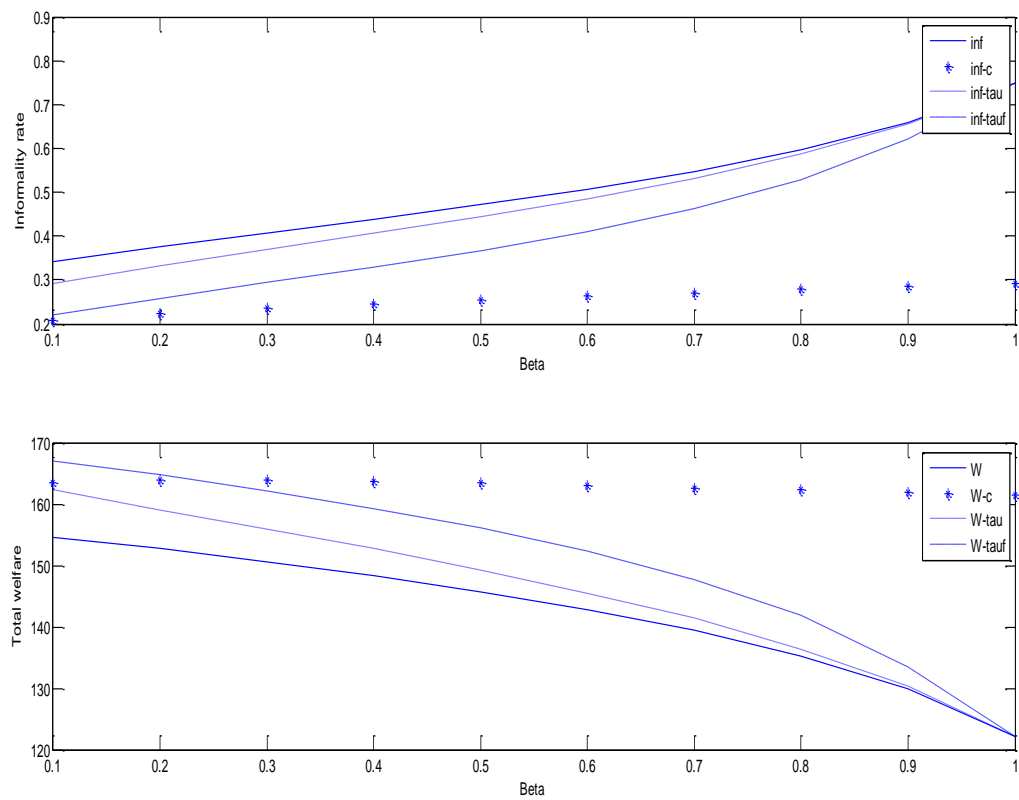
Figure 11. Informality rate vs. Bargaining power, and Total welfare vs. Bargaining power
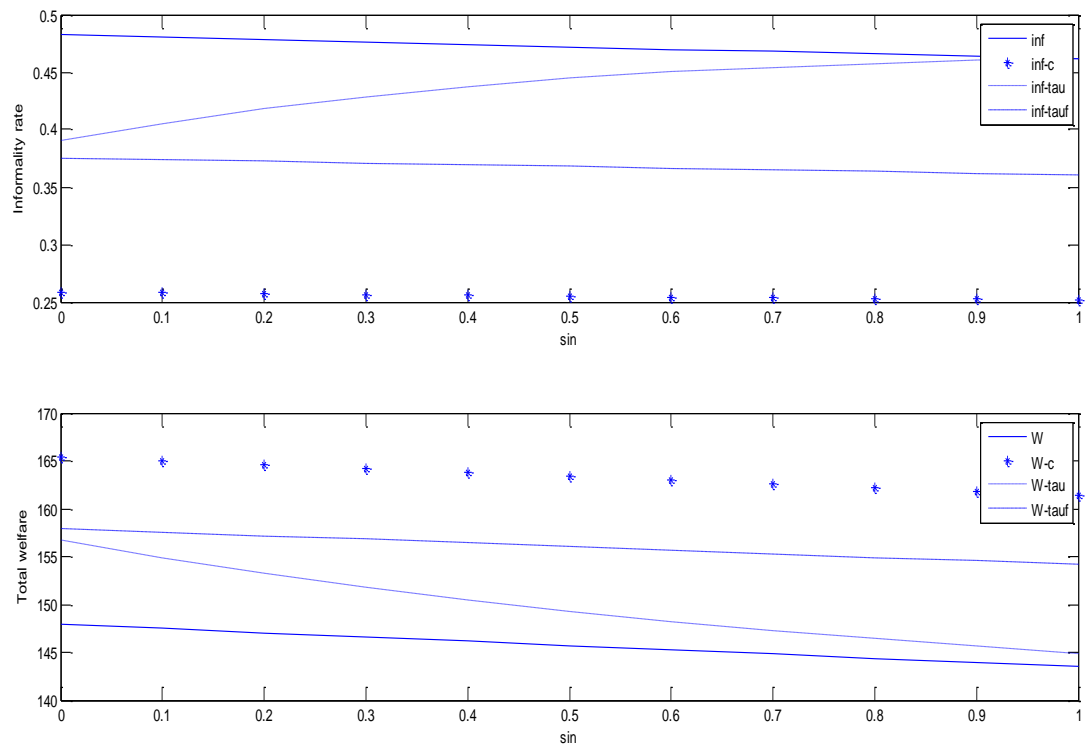
Figure 12. Informality rate vs. Decentralization, and Total welfare vs. Decentralization

Chapter two

On the Effect of Transport Subsidies on Informality Rates[19]

2.1.   Introduction

Cities in developing and emerging economies display a strong core-periphery split, with lower income groups located in peripheral areas of the city with restricted access to job opportunities, services, transport facilities and amenities (Rode, et al., 2009). In these cities, a large segment of the lower-income population has to bear not only longer commuting distances, but also longer commuting times for the same distance traveled. In Bogotá, for instance, the poor bear average commuting times up to half an hour longer per trip, and spend more than 20% of their income on transport, whereas the richest spend only 5%. Consequently, the poorest make less than 1.5 trips per day, almost half than the richest (Bocarejo and Oviedo, 2012). As a result of commuting differences, workers may opt for carrying out productive informal activities within or near home. According to recent estimates, informal employment accounts for more than half of non-agricultural employment in most developing regions of the world (Vanek, et al., 2014). Informality has a negative consequences not only in terms of productivity and incomes, but also in terms of the budget burden in countries where a large proportion of the population does not contribute to the health care and pension systems. The existence and persistence of an informal sector has been attributed mostly to institutional factors (Ferreira and Robalino, 2010; Perry, et al., 2007). Little is known, however, about the relationship between accessibility and informality, and whether transport policies can have an impact on informality rates.

In order to investigate how accessibility affects informality, we build a spatial search model. In the model there are three possible labor market statuses (formally employed, informally employed and unemployed); a formal sector where the hiring process is subject to search frictions which result in unemployment; and an informal sector where all workers are self-employed and have lower productivity than formal workers. Workers are distributed on a linear city with a unique Central Business District (CBD). Formal and unemployed workers commute every day to the CBD, where all formal activity is centralized. Instead of imposing commuting differences, we allow informal workers to endogenously choose their commuting frequency knowing that there are two informal sub-sectors: a CBD-based one, offering a certain remuneration, and a home-based one, offering a lower remuneration. A segmented city emerges in equilibrium, with formal workers residing in the CBD, and informal workers residing in the periphery. In order to attract workers, formal firms have to offer compensation for the lower commuting costs and social protection transfers workers would get if they were informally employed. We show that transport infrastructure investment have the capacity to reduce the informality rate, because lower commuting costs allow formal workers to offer a smaller spatial compensation to potential workers.

We use the model to compare the impact and efficiency of four policy options to reduce informality: a subsidy on formal firms hiring-costs, a transport subsidy for either all workers, formal workers only, or informal workers only. We focus our attention on the effects of the later, because to date, the effect of a subsidy targeted exclusively to informal workers has not been analyzed theoretically or empirically. Although these type of subsidies are not common, they have been implemented. An example is the recent transport subsidy program implemented Bogotá. The subsidy covering 40% of the cost of mass public transport is targeted exclusively to low-income beneficiaries of the subsidized social protection program (who are mostly informal workers). We find that a subsidy targeted exclusively to informal workers has no effect on the informality rate and slightly decreases welfare. The reason is that the subsidy affects the spatial compensation formal firms have to pay, and consequently results in lower job creation in the formal sector. Instead, a subsidy targeted to all workers, such as a general subsidy on the mass transport system flat-rate, does bring a reduction in the informality rate. However, we find that a hiring-costs subsidy implemented through, for instance, centralized employment agencies, is superior to transport subsidies in reducing informality. The results are robust to relaxing the assumption that unemployed workers

commute daily to the CBD.

Our paper is connected to a large body of literature analyzing the reasons behind the existence and persistence of an urban informal sector in developing and emerging economies (Camacho, et al., 2013; Ferreira and Robalino, 2010; Jütting, et al., 2008). Whereas recent contributions highlight the need to consider the heterogeneity of informal activities and the different motivations for choosing informality from the worker perspective (Günther and Launov, 2012; Maloney, 2004), the location decisions of informal workers are disregarded. In this paper we translate the heterogeneity of the urban informal sector into its spatial expression by considering the existence of two informal sub-sectors within an urban area. After considering a heterogeneous informal sector which offers intrinsic benefits, we show that location decisions have a non-trivial effects on informality rates, through their effect of commuting and housing costs.

Our paper is also connected to the Spatial Mismatch Hypothesis (SMH) literature. According to the SMH, the adverse labor outcomes of minorities are the result of the spatial disconnection between low-skilled jobs and the places where minorities reside (Kain, 1968). This hypothesis was inspired in the case of metropolitan areas of the US which, due to innovations in transportation, experienced increased residential suburbanization in the second half of the 20th century. At the same time, there was a process of dispersion of firms away from central areas within cities. Minorities allegedly relocated at a slower pace than jobs because they faced discrimination in the housing market or were subject to zoning regulations, leading to a concentration of minorities in inner-city areas where low-skilled job creation was slow (Ihlanfeldt and Sjoquist , 1998). In this literature, there is consensus on the view that distance to jobs is partly responsible for the unfavorable labor market outcomes of minorities (Selod, et al., 2007; Ihlanfeldt and Sjoquist , 1998). Our paper contributes to this literature by considering the case of urban areas in emerging and developing economies which display a different spatial pattern than US cities. Furthermore, it also considers the fact that, unlike the case of personal attributes such as race, informality is not a fixed attribute but a choice that offers intrinsic benefits (Albrecht, et al., 2009; Maloney, 2004).

Our model integrates two existing extensions of the standard search and matching framework: the inclusion of an informal sector and the integration of an urban land-use

market. Regarding the first extension, unlike existing works (Zenou, 2011; Zenou, 2008), we do not consider the informal sector to be a residual sector or a buffer where rural-urban migrants queue for formal jobs. In fact, we do not consider the effects of rural-urban migration at all, as we have in mind the case of consolidated urban areas. In our model, the informal sector is a micro-entrepreneurial unregulated sector that offers intrinsic benefits, such as health-care subsidies, so that being informal is, to some extent, a matter of choice (Albrecht, et al., 2009; Maloney, 2004). Regarding the second extension, previous works incorporate a spatial compensation paid by formal firms resulting from assumed commuting differences between unemployed and employed workers (Wasmer and Zenou, 2002; Smith and Zenou, 1995). In our framework, we consider instead commuting differences between formal and informal workers. Our contributions to existing theoretical models lie in making the commuting choices of informal workers endogenous. The article is structured as follows. Section 2 presents the model set up and shows the effect of changes in city structure and informal income sources on the informality rate. Section 3 compares the impact of the proposed policies. Section 4 discusses and concludes.

2.2. The Model

2.2.1. The city

The total urban population is equal to $N = L_F + L_{IN} + U$, where U is the unemployment level and L_F and L_{IN} are formal and informal employment levels. Unemployed workers can obtain either a formal or an informal job, but only actively search for formal jobs. In the reminder of the article we refer to the group of formal and unemployed workers $L_F + U$ as "formal workers". All workers live in a linear and closed city, with a unique Central Business District (CBD). We assume the density of residential land parcels to be unity, so that there are exactly x residential units within a distance x from the CBD. Workers optimally decide to reside at any point between the CBD $(x = 0)$ and the city fringe $(x = N)$. The commuting cost function is given by:

$$T(x) = T + \tau x, \tag{2.1}$$

which implies that commuters face a fixed cost T plus a variable cost τx, where τ is the commuting cost per unit of distance. T can be interpreted as a flat-rate charged for accessing the mass transport system, and τx as the leisure value lost in commuting. We choose this specification because our objective is to show the effect of a subsidy on a flat-rate charge rather than a subsidy on either all the units of distance traveled (e.g., a subsidy on gasoline consumption), or the first unit of distance traveled (Zenou, 2011).

2.2.2. Urban formal sector

Each formal firm is a productive unit that hires a single (previously unemployed) worker. We assume that all formal firms locate at the CBD. The hiring process is subject to search frictions, as defined in the standard search-matching framework (Pissarides, 2000). Firms fill a vacancy and unemployed workers find employment, following a random Poisson process. In the aggregate, the number of contacts per unit of time between the worker and the firm sides of the market is determined by the following matching function with exhibit Constant Returns to Scale:

$$d = d\left(\bar{S}_U U, V\right), \tag{2.2}$$

where V is the total number of vacancies and $\bar{S}_U$ is the average search efficiency of unemployed workers. M is the total mass of firms, v is the vacancy rate and u is the unemployment rate. The total number of unemployed is $U = uN$, and the total number of vacancies is $V = vM$. The rate at which vacancies are filled can be expressed as $d\left(\bar{S}_U U, V\right)/V \equiv q(\theta)$, where:

$$\theta = \frac{V}{\bar{S}_U U} \tag{2.3}$$

is the labor market tightness in efficiency units. The rate at which an unemployed worker with search efficiency S_U leaves unemployment is $S_U d\left(\bar{S}_U U, V\right)/\bar{S}_U U \equiv S_U \theta q(\theta)$. As can be verified, $q(\theta)$ and $\theta q(\theta)$ satisfy the standard properties: 1) $q'(\theta) < 0$; 2) $\left[\theta q(\theta)\right]' \geq 0$, and 3) $\lim_{\theta \to 0} \theta q(\theta) = \lim_{\theta \to +\infty} q(\theta) = 0$ and $\lim_{\theta \to +\infty} \theta q(\theta) = \lim_{\theta \to 0} q(\theta) = +\infty$, indicating, on the one hand, that whenever the number of unemployed workers is infinite, firms fill their vacancies

instantaneously, and, on the other hand, that whenever the number of vacancies is infinite, unemployed workers find jobs instantaneously. When firms and unemployed workers meet, a wage rate w_F is negotiated, and quantity y_F is produced.

2.2.3. Urban informal sector

The informal sector is composed of two informal sub-sectors: a fully agglomerated one at the CBD offering a remuneration w_c, and a home-based one offering a remuneration w_h. We assume that informal workers can generate more income at the CBD than at home, so that $w_c > w_h$. This could be the case if, for instance, there are positive net agglomeration economies accruing informal activities located at the CBD. Note that while formal activity is fully centralized at the CBD, informal activity is decentralized. The level of decentralization increases with the attractiveness of home-based productive activities with respect to CBD-based productive activities. Employment opportunities in the urban informal sector are created and destroyed following a random Poisson process with creation and destruction exogenous rates denoted by α and δ_{IN}, respectively. The informal remunerations w_c and w_h are assumed to be fixed and identical for all workers. Additionally the remuneration for the CBD-based subsector, w_c, is assumed to be lower than the productivity in the formal sector, $y_F > w_c$. We assume that informal workers spend all their available time in informal productive activities and consequently do not actively seek formal jobs (i.e., there is not on-the-job search).

Besides the wage, and contrary to the situation in the formal sector, we assume that informal workers receive a transfer once at the beginning of their work period of a positive fixed amount b. This fixed quantity can be interpreted as the present value of the sum of all the "gifts" received while informally employed that would have otherwise been paid had the worker been formally employed. We aim to capture the situation of countries such as Colombia and Brazil, where subsidized social protection (including pension and health insurance) is offered exclusively for the low income population (Perry, et al., 2007; Maloney,

2004).[20] In order to ensure that the difference in social protection costs is only captured in the fixed term b, we assume that all the contributions made by formal workers are recovered through access to services (e.g. health-care, insurance, etc.), while informal workers do not contribute to the system but can access these services for free, as needed.

2.2.4. Households

Formal workers are assumed to commute every day to work, so their trip frequency S_F takes the value of one (from a possible range between zero and one). Unemployed workers are assumed to have a trip frequency equal to their search efficiency S_U. Given that S_U is a constant, it follows that the average search efficiency $\bar{S}_U$ is also a constant. Because our interest lies on analyzing informal employment and not unemployment, for the sake of simplicity we assume that the unemployed commute daily to the CBD in search of work opportunities, so that $S_U = 1$. This assumption is not crucial to our analysis. In section 2.4.4 we show that our results hold also when we assume a lower commuting frequency for the unemployed.[21] Formal workers residing at a distance x from the CBD receive a wage w_F, pay a rent of $R(x)$, consume z_F units of a non-spatial composite good (produced both by formal and informal firms), consume one unit of land, and bear a commuting cost of $S_F(T + \tau x)$. Unemployed workers, in turn, receive a fixed income w_U (derived from an unemployment subsidy, rent income or own savings), pay a rent $R(x)$, consume z_U units of the composite good, consume one unit of land, and bear a commuting cost of $S_U(T + \tau x)$. After setting the composite non-spatial good as the numéraire, the budget constraint of each formal worker in the city can be expressed as:

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[20] In such dual systems, formal workers bear (a part of) their social protection costs, while informal workers do not bear such costs and are effectively subsidized by the government.

[21] To clarify the difference between unemployment and informal employment some additional remarks are in place. First, unemployment in the model is frictional, and can be considered a transitioning state between two different job statuses. It is, therefore, not related to cyclical unemployment. Second, in developing countries, people are often classified as "informal" even if they work on a family business for a small number of hours a week. The unemployed are usually classified as such if they are actively searching for jobs. Third, unemployment benefits, common in some European countries, are not common in emerging and developing economies, whereas social protection subsidies are.

$$R(x) + S_i(T + \tau x) + z_i = w_i, \tag{2.4}$$

for $i = F$ and U. Assuming individuals are risk neutral, the utility function of each type of worker, $\Omega(z_i) = z_i$, combined with the budget constraint given by (2.4), implies that the instantaneous indirect utility function is given by:

$$W_i(x) = w_i - S_i(T + \tau x) - R(x). \tag{2.5}$$

Once the worker becomes informal, she can optimally choose the commuting frequency S_{IN} knowing that the remunerations at the CBD and home-based informal sectors are w_c and w_h. It should be clear that commuting costs and the share of income generated at CBD increase with S_{IN}, whereas the share of income generated at home decreases with S_{IN}. Then, the instantaneous indirect utility level at x for an informal worker is given by:

$$W_{IN}(x) = S_{IN} w_c + (1 - S_{IN}) w_h - S_{IN}^2 (T + \tau x) - R(x). \tag{2.6}$$

Note that total commuting costs increase at a growing rate with S_{IN}. This reflects the increasing rate at which the commuter effort has to grow in order to ensure a rise in S_{IN}. Informal workers choose S_{IN} to maximize the instantaneous utility given by (2.6). The first order condition to this problem is:

$$w_c - w_h = 2S_{IN}(T + \tau x), \tag{2.7}$$

and the optimal commuting frequency is given by:

$$S_{IN}^* = \frac{w_c - w_h}{2(T + \tau x)}. \tag{2.8}$$

By commuting more frequently to the CBD, informal workers obtain a gain equal to the remuneration differential $(w_c - w_h)$, but also face commuting costs that increase at a rate

$2S_{IN}(T + \tau x)$. Informal workers stop commuting when these two quantities are equal. Note that the commuting frequency of informal workers decreases with distance: as informal workers move away from the CBD, the growth in their commuting costs exceeds their remuneration differential, so they optimally reduce their commuting frequency. Using the same reasoning, it can be shown that the commuting frequency, as well the level of centralization of the informal sector, decrease with: the informal home-based remuneration (w_h), the fixed part of the commuting cost (T), and the commuting cost per unit of distance (τ); and increase with the informal CBD remuneration (w_c). We replace (2.8) in (2.6) to obtain the instantaneous utility for informal workers:

$$W_{IN}(x) = w_h + \frac{(w_c - w_h)^2}{4(T + \tau x)} - R(x). \tag{2.9}$$

It is worth noticing that the expression $\left((w_c - w_h)^2 / 4(T + \tau x)\right)$ represents the net profits of commuting to the CBD. It also represents (in negative) the commuting costs for informal workers, which decrease with distance as a consequence of optimal reductions in the commuting frequency. For formal workers, on the other hand, commuting costs always increase with distance.

2.3.   Urban land use in equilibrium

We start by making a number of assumptions to simplify the analysis. In the city there is no vacant land, and land areas not used for residential purposes are used for agricultural production. Agricultural rents, denoted by R_A, are exogenous and equal to zero. The urban land market is competitive, so that all urban residents take $R(x)$ as given. Workers do not bear relocation costs, which are identical within each worker category. Land is owned by absentee landlords who earn land rents that are spent outside the city. Thus, in equilibrium, each category of worker -formal, informal and unemployed- reaches the same level of instantaneous utility, denoted by W_F, W_{IN} and W_U, respectively. The corresponding bid rents for each worker category are given by:

$$\Psi_F\left(x,W_F\right) = w_F - T - \tau x - W_F, \tag{2.10}$$

$$\Psi_{IN}\left(x,W_{IN}\right) = w_h + \frac{\left(w_c - w_h\right)^2}{4\left(T + \tau x\right)} - W_{IN}, \tag{2.11}$$

$$\Psi_U\left(x,W_U\right) = w_U - T - \tau x - W_U, \tag{2.12}$$

which indicate the maximum land rent that a worker of category i is willing to pay in order to reach a utility level W_i. It is possible to verify that the bid rents for formal and unemployed workers are linear, decreasing in x, and share the same level of inclination. The bid rents and urban costs for informal workers also decrease with distance. The explanation is as follows. When informal workers move away from the CBD, they enjoy lower commuting cost. However, at the same time, they lose productivity at a rate given by the wage difference between subsectors. Because the productivity losses are greater than the commuting costs savings, the bid rent has to decrease with distance. Given that commuting costs and the bid rent for informal workers decrease with distance, their urban costs also decrease. In order to guarantee that, all over the city, the bid rents for formal and unemployed workers are steeper than the bid rents for informal workers, we assume that fixed commuting costs are high compared with the benefits of commuting to the CBD for informal workers, so that $T > \left(\left(w_c - w_h\right)/2\right)$. We are now ready to define the equilibrium land use conditions (Zenou, 2009): an urban land use equilibrium, with fixed land consumption, no relocation costs, endogenous S_{IN} and formal, informal and unemployed workers is a n-tuple $\left(W_F^*, W_U^*, W_{IN}^*, R^*\left(x\right)\right)$ such that:

$$W_F^* - W_U^* = w_F - w_U, \tag{2.13}$$

$$w_U - T - \tau\left(N - L_{IN}\right) - W_U^* = w_h + \frac{\left(w_c - w_h\right)^2}{4\left[T + \tau\left(N - L_{IN}\right)\right]} - W_{IN}^*, \tag{2.14}$$

$$w_h + \frac{(w_c - w_h)^2}{4(T + \tau N)} - W_{IN}^* = 0, \qquad (2.15)$$

$$R^*(x) = max\left\{\Psi_F\left(x, W_F^*\right), \Psi_U\left(x, W_U^*\right), \Psi_{IN}\left(x, W_{IN}^*\right), 0\right\} \text{ for each } x \in (0, x]. \qquad (2.16)$$

Equation (2.13) states that the difference in utility levels of formal workers and the unemployed is equal to the difference between the level of income (and independent of the commuting distance to the CBD). The bid rents of these groups of workers are consequently identical across the city. Equation (2.14) implies that, exactly at the frontier that delimits the area where formal and unemployed workers reside $(N - L_{IN})$, the bid rents of formal and informal workers are the same. Equation (2.15), in turn, means that exactly at the fringe of the city, bid rents for informal workers are equal to agricultural rents (which are assumed to be zero). Finally, Equation (2.16) states that the equilibrium land rent is equal to the upper envelope of the equilibrium bid rent curves of all workers and the agricultural rent line (Zenou, 2011). We obtain the equilibrium values of the instantaneous utilities of formal, unemployed and informal workers using equations (2.13) to (2.15), so that:

$$W_{IN}^* = w_h + \frac{(w_c - w_h)^2}{4(T + \tau N)}, \qquad (2.17)$$

$$W_U^* = w_U - T - \tau(N - L_{IN}) - \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - L_{IN})} - \frac{1}{T + \tau N}\right], \qquad (2.18)$$

$$W_F^* = w_F - T - \tau(N - L_{IN}) - \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - L_{IN})} - \frac{1}{T + \tau N}\right]. \qquad (2.19)$$

By replacing (2.17), (2.18) and (2.19) in (2.11), (2.10) and (2.12), we can obtain the urban land rents in equilibrium $R^*(x)$:

$$R^*(x) = \begin{cases} \dfrac{(w_c - w_h)^2}{4}\left[\dfrac{1}{T + \tau(N - L_{IN})} - \dfrac{1}{T + \tau N}\right] + \tau(N - L_{IN} - x) & \text{for} \quad 0 \leq x \leq N - L_{IN}, \\[3mm] \dfrac{(w_c - w_h)^2}{4}\left[\dfrac{1}{T + \tau x} - \dfrac{1}{T + \tau N}\right] & \text{for} \quad N - L_{IN} \leq x \leq N, \\[3mm] 0 & \text{for} \quad x \geq N. \end{cases}$$

$$(2.20)$$

Figure 13 shows that a segmented city emerges in equilibrium, where formal workers and the unemployed reside in the area $(0, N - L_{IN}]$, and informal workers reside in the area $[N - L_{IN}, N]$.

$R(x)$

$\Psi_F(x, \bar{I}_F, \bar{I}_U, \bar{I}_{IN}) = \Psi_U(x, \bar{I}_F, \bar{I}_U, \bar{I}_{IN})$

$\Psi_{IN}(x, \bar{I}_F, \bar{I}_U, \bar{I}_{IN})$

CBD Formal Employed and $N\text{-}L_{IN}$ Informal Employed N Rural area x
$x = 0$ Unemployed
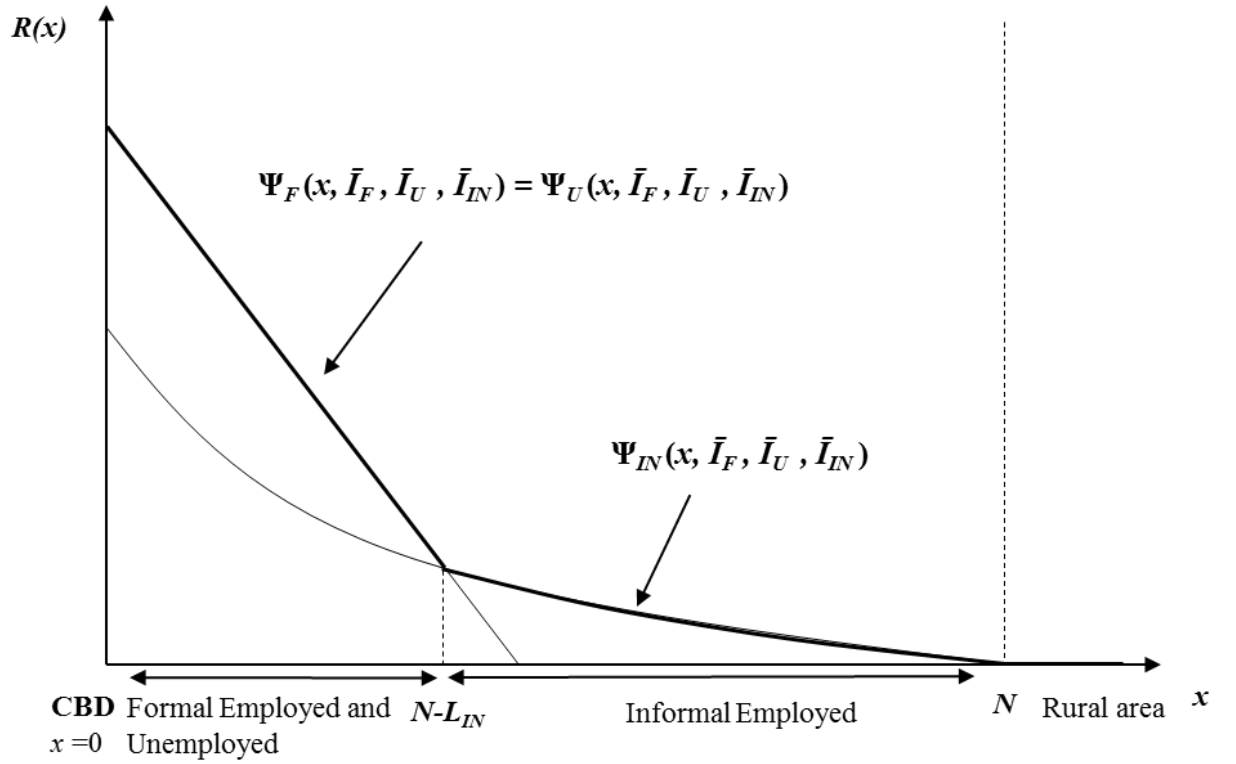
Figure 13. Urban land use equilibrium (the segregated city)

2.4. Urban labor market equilibrium

2.4.1. Lifetime expected utility for workers and firms

The model assumes that workers live indefinitely, have rational expectations and discount future values at a rate r. Let I_F, I_U and I_{IN} be, in turn, the expected utility of a formal worker, an unemployed and an informal worker. In the steady-state, the Bellman equations determining I_F and I_{IN} are given by:

$$rI_F = w_F - T - \tau(N - L_{IN}) - \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - L_{IN})} - \frac{1}{T + \tau N}\right] - \delta_F(I_F - I_U)$$

(2.21)

$$rI_{IN} = w_h + \frac{(w_c - w_h)^2}{4(T + \tau N)} + \delta_{IN}(I_U - I_{IN}),$$

(2.22)

which shows that workers obtain their instant utility in each period. Furthermore, formal workers can lose their jobs at a rate δ_F, which leads to a surplus loss of $I_F - I_U$. Informal workers, in turn, can lose their jobs at a rate δ_{IN}, so their change in surplus is given by $I_U - I_{IN}$. In the steady-state, the Bellman equation for I_U is given by:

$$rI_U = w_U - T - \tau(N - L_{IN}) - \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - L_{IN})} - \frac{1}{T + \tau N}\right] +$$
$$\theta q(\theta)(I_F - I_U) + \alpha(I_{IN} + b - I_U).$$

(2.23)

Equation (2.23) shows that, besides obtaining their instant utility, unemployed workers can obtain either a formal sector job at a rate $\theta q(\theta)$ (since $S_U = 1$), with an accompanying increase in surplus of $I_F - I_U$, or an informal job at a rate α, with a surplus change of

$I_{IN} + b - I_U$ (recall that b is the present value of the sum of all the transfers received while informally employed). Assuming no relocation costs, in equilibrium formal, unemployed and informal workers reach the same expected utility levels ($\bar{I}_F$, $\bar{I}_U$ and $\bar{I}_{IN}$) regardless of their intra-urban location. Using (2.22) and (2.23) we obtain:

$$\bar{I}_U - \bar{I}_{IN} = \frac{1}{r+\alpha+\delta_{IN}} \times$$

$$\left[w_U - w_h - T - \tau(N - L_{IN}) - \frac{(w_c - w_h)^2}{4} \left[\frac{1}{T + \tau(N - L_{IN})} \right] + \alpha b + \theta q(\theta)(I_F - I_U) \right],$$

$$(2.24)$$

whereas using (2.21) and (2.23) leads to:

$$\bar{I}_F - \bar{I}_U = \frac{1}{r + \delta_F + \theta q(\theta)} \left[w_F - w_U - \alpha b + \alpha(I_U - I_{IN}) \right]. \qquad (2.25)$$

Then, replacing (2.24) in (2.25) we obtain:

$$\bar{I}_F - \bar{I}_U = \varphi(\theta)(r + \alpha + \delta_{IN}) \times$$

$$\left\{ w_F - \gamma \left[w_h + T + \tau(N - L_{IN}) + \frac{(w_c - w_h)^2}{4} \left[\frac{1}{T + \tau(N - L_{IN})} \right] + (r + \delta_{IN})b \right] - (1 - \gamma)w_U \right\}, \qquad (2.26)$$

where $\varphi(\theta) = 1[(r + \delta_F)(r + \alpha + \delta_{IN}) + \theta q(\theta)(r + \delta_{IN})]$ and $\gamma = \alpha(r + \alpha + \delta_{IN})$. Hiring costs per unit of time for formal firms are given by c. It is important to clarify that these costs refer to costs incurred while a vacancy is open (those related to providing job information, publicity, etc.), and not those costs incurred once a match is made. Then, the value of a vacancy I_v and a filled vacancy I_o are given by:

$$rI_v = -c + q(\theta)(I_o - I_v) \qquad (2.27)$$

$$rI_o = y_F - w_F - \delta_F(I_o - I_v).$$

<div align="right">(2.28)</div>

Firms are free to post vacancies and they do so until $I_v = 0$, which, together with (2.27) implies:

$$I_o = \frac{c}{q(\theta)}.$$

<div align="right">(2.29)</div>

Using $I_v = 0$, (2.29) and (2.28) we have the following equation determining job creation in the formal sector:

$$\frac{c}{q(\theta)} = \frac{y_F - w_F}{r + \delta_F}.$$

<div align="right">(2.30)</div>

2.4.2. Wages and informality and unemployment rates in equilibrium

Each period wages are obtained through a generalized Nash-bargaining process between firms and workers, so that:

$$w_F = \underset{w_F}{argmax}(I_F - I_U)^{\beta}(I_o - I_v)^{1-\beta},$$

<div align="right">(2.31)</div>

where $0 \le \beta \le 1$ represents the bargaining power of workers. By solving (2.31) we obtain:

$$w_F = (1-\beta)\left\{\gamma\left[w_h + T + \tau(N - L_{IN}) + \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - L_{IN})}\right] + (r + \delta_{IN})b\right] + (1-\gamma)w_U\right\} +$$

$$\beta\{y + (1-\gamma)c\theta\}.$$

<div align="right">(2.32)</div>

This expression is key to understanding the model proposed here. The first part of (2.32) is the compensation that formal firms have to pay in order to induce workers to accept a job in the formal sector. This compensation is a fraction $(1-\beta)$ of the expected income of an

unemployed worker, which is a weighted average of current income w_U and informal sector income $w_h + T + \tau(N - L_{IN}) + \left((w_c - w_h)^2/4\right)\left[1/(T + \tau(N - L_{IN}))\right] + (r + \delta_{IN})b$, where $(1 - \gamma)$ and γ are the respective weights. Current income in the informal sector is given by the sum of: the home-based remuneration w_h, the fraction $(r + \delta_{IN})$ of the transfers (gifts) b received and a spatial compensation composed of net profits of commuting to the CBD $\left((w_c - w_h)^2/4\right)\left[1/(T + \tau N)\right]$ and the urban costs savings (commuting costs plus rents that would have been faced as a formal worker), $T + \tau(N - L_{IN}) + \left((w_c - w_h)^2/4\right)\left[\left(1/(T + \tau(N - L_{IN}))\right) - \left(1/(T + \tau N)\right)\right]$. These savings are a direct consequence of the possibility that informal workers have of working at home and optimally choosing their commuting frequency.

It is worth noting that, contrary to models that assume that the unemployed commute less than employed workers (Wasmer and Zenou, 2002; Smith and Zenou, 1995), our wage equation does not contain a direct spatial compensation paid to unemployed workers. Instead, formal firms have to compensate the unemployed worker with part of the expected informal sector income, which includes spatial costs savings. These savings do not depend on assumed commuting differences between different types of workers, but on the optimal commuting decision of informal workers that face the possibility of carrying their informal work either at the CBD or at home. We elaborate more on this result in section 2.4.4. Let us now consider formal and informal employment and unemployment in equilibrium. In the steady-state, two conditions must hold: first, that the number of unemployed workers that finds a formal job, $\theta q(\theta)U$, is equal to the number of workers that lose their formal jobs, $\delta_F L_F$, and second, that the number of unemployed workers that find an informal job, αU, is equal to the number of informal workers that lose their job, $\delta_{IN} L_{IN}$. Considering that $U = N - L_F - L_{IN}$, the following must hold:

$$\delta_F L_F = \theta q(\theta)(N - L_F - L_{IN}), \tag{2.33}$$

$$\delta_{IN} L_{IN} = \alpha(N - L_F - L_{IN}), \tag{2.34}$$

which implies that the informality rate ϕ^* and unemployment rate u^* in the steady-state are given by:

$$\phi^* = \frac{\delta_F \alpha}{\delta_F \left(\delta_{IN} + \alpha\right) + \delta_{IN} \theta q\left(\theta\right)}, \tag{2.35}$$

$$u^* = \frac{\delta_F \delta_{IN}}{\delta_F \left(\delta_{IN} + \alpha\right) + \delta_{IN} \theta q\left(\theta\right)}. \tag{2.36}$$

It is possible to verify that $\left(\partial \phi^* / \partial \theta\right) < 0$, $\left(\partial u^* / \partial \theta\right) < 0$, which shows that an increase in formal job creation leads to a decrease in both the informality and the unemployment rates. It can also be shown that $\lim_{\theta \to +\infty} \phi^* = \lim_{\theta \to +\infty} u^* = 0$, so that informality and unemployment disappear as formal sector employment creation reaches infinity. Thus, a necessary condition for the existence of informality is that search frictions in the formal sector exist, as explained in detail in Zenou (2008). Lastly we have $\lim_{\theta \to 0} \phi^* = \left(\alpha / \left(\delta_{IN} + \alpha\right)\right)$, and $\lim_{\theta \to 0} u^* = \left(\delta_{IN} / \left(\delta_{IN} + \alpha\right)\right)$, indicating the informality and unemployment rates prevalent when there is no formal sector.

2.4.3. Existence and uniqueness of the equilibrium

With the above results, and given a certain value of the labor market tightness parameter θ, it is possible to determine all the endogenous variables in the model. Replacing (2.32) (the formal sector equilibrium wage) and (2.35) (the informality rate in the steady-state) into (2.30), we obtain the following condition determining the formal job creation rate in equilibrium θ^*:

$$\left(r + \delta_F\right) \frac{c}{q\left(\theta^*\right)} + \beta\left(1 - \gamma\right) c \theta^* -$$

$$\left(1 - \beta\right) \times \left\{ y_F - \gamma \left[w_h + T + \tau\left(1 - \phi^*\right) N + \frac{\left(w_c - w_h\right)^2}{4} \left[\frac{1}{T + \tau\left(1 - \phi^*\right) N} \right] + \left(r + \delta_{IN}\right) b \right] - \left(1 - \gamma\right) w_U \right\} = 0. \tag{2.37}$$

Let Λ be the expression on the left hand side of (2.37). Assuming that formal sector productivity is always higher than the expected income of the unemployed, we have that:

$$y_F > \gamma \left[w_h + T + \tau \left(\frac{\delta_{IN}}{\delta_{IN} + \alpha} \right) N + \frac{\left(w_c - w_h \right)^2}{4} \left[\frac{1}{T + \tau \left(\frac{\delta_{IN}}{\delta_{IN} + \alpha} \right) N} \right] + \left(r + \delta_{IN} \right) b \right] + \left(1 - \gamma \right) w_U,$$

(2.38)

and consequently, it is always the case that $\left(\partial \Lambda / \partial \theta \right) > 0$, $\lim_{\theta \to +\infty} \Lambda = \infty$, $\lim_{\theta \to 0} \Lambda < 0$. Thus, there must be a unique θ^* that satisfies (2.37), and consequently, there is a unique steady-state equilibrium.

2.4.4. Wage equation under different assumptions for the commuting frequency of the unemployed

So far we have assumed that the commuting frequency of formal workers and the unemployed is the same $\left(S_F = S_U = 1 \right)$. We showed that in this case, the bid rents for formal and unemployed workers are the same, and that they are steeper than the bid rent of informal workers. This means that formal and informal workers share area $\left(0, N - L_{IN} \right]$, whereas informal workers live further away, in area $\left[N - L_{IN}, N \right]$ (see Figure 13). We now consider two alternative assumptions for the commuting frequency of the unemployed $\left(S_U \right)$, and show that our analysis holds for both cases. The first alternative is that the unemployed live in an intermediate area between formal and informal workers. Formally, this translates into assuming that $1 > S_U > \left(\left(w_c - w_h \right)^2 / 4T^2 \right)$, that is, that the bid rent of the unemployed is smaller than the bid rent of formal workers but larger than the bid rent of informal workers. In this case, it can be shown that the wage equals:

$$w_F = (1-\beta)\left\{ B + \gamma\left[w_h + S_U\left[T + \tau(N - L_{IN}) \right] + \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - L_{IN})} \right] + (r + \delta_{IN})b \right] + (1-\gamma)w_U \right\} +$$
$$\beta\left\{ y + (1-\gamma)c\theta \right\}.$$

(2.39)

where $B = (1 - S_U)(T + \tau L_F)$. Under the second alternative, the unemployed have a smaller bid rent than formal and informal workers, so that $T > \left((w_c - w_h)^2 / 4 \right) and \left((w_c - w_h)^2 / 4(T + \tau N)^2 \right) > S_U$. In this case, the wage is:

$$w_F = (1-\beta)\left\{ w_U + B + \left[\frac{(w_c - w_h)^2}{4} \frac{1}{(T + \tau L_F)[T + \tau(N - U)]} - S_U \right]\tau L_{IN} + \gamma\left[w_h + S_U[T + \tau(N - U)] + \frac{(w_c - w_h)^2}{4}\left[\frac{1}{T + \tau(N - U)} \right] + (r + \delta_{IN})b - w_U \right] \right\} +$$
$$\beta\left\{ y + (1-\gamma)c\theta \right\}.$$

(2.40)

Comparing equations (2.39) and (2.40) with equation (2.32), it becomes evident that the same spatial compensation term discussed previously appears in both cases. The difference is that now formal firms have to offer a spatial compensation also to the unemployed, given that they commute less frequently to the CBD $(S_U < 1)$. We can thus conclude that the spatial compensation linked to informal employment appears as long as formal workers live closer to the CBD than informal workers, independently of where the unemployed locate. Furthermore, the spatial compensation arising because of the existence of an informal sector is additional to any possible compensation offered to the unemployed when they commute less frequently to the CBD.

2.5.   Comparative statics

We now analyze the effects of changes in: 1) variables that represent the structure of the city (fixed commuting cost T, marginal commuting cost τ, and total population N), and; 2) variables that represent the direct income of informal workers (home-based remuneration w_h, CBD remuneration w_c, and social protection subsidies b), on the equilibrium formal job creation rate θ^* and the equilibrium informality rate ϕ^*. Totally differentiating equation

(2.37) leads to: $\left(\partial\theta^*/\partial p\right)=-\left(\left(\partial\Lambda/\partial p\right)/\left(\partial\Lambda/\partial\theta\right)\right)$, where p is any exogenous parameter. Given that $\left(\partial\Lambda/\partial\theta\right)>0$, the sign of $\left(\partial\theta^*/\partial p\right)$ is the opposite of $\left(\partial\Lambda/\partial p\right)$. Additionally we have that: $\left(\partial\phi^*/\partial p\right)=\left(\partial\phi^*/\partial\theta^*\right)\left(\partial\theta^*/\partial p\right)$, and because $\left(\partial\phi^*/\partial\theta^*\right)<0$, the sign of $\left(\partial\phi^*/\partial p\right)$ is the same of $\left(\partial\Lambda/\partial p\right)$. So, it is enough to find $\left(\partial\Lambda/\partial p\right)$ to establish in which direction θ^* and ϕ^* change.

Regarding the structure of the city, using (2.37) it is possible to show that $\left(\partial\Lambda/\partial T\right)>0$, which implies that a rise in T generates a decrease in θ^* and an increase in ϕ^*. The rise in fixed commuting costs increases formal urban costs, but at the same time decreases the net profit informal workers obtain by commuting to the CBD. Because the increase of the former is larger than the decrease of the latter, there is a larger spatial compensation, and a larger salary, paid by formal firms. Job creation in the formal sector is negatively affected by this larger wage, which ultimately results in a larger informal sector. This leads to a key insight of our model: reducing fixed commuting costs, by for instance improving the transportation infrastructure, can lead to a reduction in the informality rate. A similar analysis can be done for the case of changes in marginal commuting costs and changes in population size.

Regarding the direct income of informal workers, from (2.37) it is possible to show that $\left(\partial\Lambda/\partial w_h\right)>0$, which implies that a rise in w_h generates a decrease in θ^* and an increase in ϕ^*. A rise in the informal home-based remuneration increases the income for informal workers, and at the same time reduces formal urban costs (informal workers reduce their commuting frequency, which in turn relaxes the competition for land near the CBD). The latter effect is not large enough to offset the increase in informal income, so ultimately there is a larger spatial compensation, and a larger salary paid by formal firms. Again, job creation in the formal sector is negatively affected by this larger wage, which results in a larger informal sector.

The effect of a change in informal CBD remuneration has the same effect, but works through a different mechanism. From (2.37), we have that $\left(\partial\Lambda/\partial w_c\right)>0$. A rise in the CBD remuneration increases formal urban costs, because informal workers, in response, rise their

commuting frequency, intensifying the competition for land near the CBD. As a consequence, formal firms are forced to offer a higher wage. As in the previous case, job creation in the formal sector is negatively affected, resulting in a larger informal sector. Finally, an increase in social protection subsidies reduces job creation and increases the informality rate. Larger transfers to informal workers translate into higher income, consequently increasing the unemployed reserve wage, and in turn the compensation that formal firms must bear. We now turn to the analysis of policies.

2.6.   Informality and the impact of policies

An important policy objective in cities in developing countries is to diminish informality rates. The underlying idea is that a reduction in informality, or an increase in the size of the formal sector, unequivocally improves public welfare. Replacing informal jobs for formal ones has important consequences in terms of productivity and incomes, and also alleviates the subsidy budget burden in countries where a large proportion of the population does not contribute to social protection (Ferreira and Robalino, 2010). As we have shown in the previous section, reducing commuting costs is a way in which this policy objective can be achieved. However, are transport policies the most efficient way to reduce informality? And, which group, if any, should transport policies target? In this section we focus on these two questions by assuming that the local government has a fixed amount of money M to be used in one of four policies: 1) subsidizing a fraction of hiring costs of each formal firm; subsidizing a fraction of transport costs of 2) each worker, 3) each formal worker, or 4) each informal worker. It is important to note here that the amount of the subsidy given to each economic unit has to adjust endogenously to changes in the number of beneficiaries in order to always meet the budgetary restriction of the government. We assume that no person receives a higher or lower subsidy than the next.

2.6.1.   Four policy options compared

We now turn to the analysis of the four proposed policies in equilibrium. Under the first policy option, the government assumes fraction σ^c of the hiring costs of each firm, so that the effective costs diminish and become $(1-\sigma^c)c$. The subsidy has the effect of reducing

informality rates, as it can be shown that $\left(\partial\theta^*/\partial c\right)<0$. The budget constraint under the first policy option, a hiring-costs subsidy, can be expressed as $M=\sigma^c c\theta^* u^* N$, given that $V^*=\theta^* u^* N$. This implies that the hiring-costs subsidy is endogenous and equal to:

$$\sigma^c=\frac{M}{c\theta^* u^* N}. \tag{2.41}$$

Since $0<\sigma^c<1$, it must be the case that $M<c\theta^* u^* N$. Equation (2.41) shows that the amount of the subsidy granted diminishes with an increase in either formal job creation, unemployment, the total population or hiring-costs. The expression for the equilibrium condition for this and the subsequent policy options can be found in the appendix. Under the second policy option, the government assumes a fraction σ^τ of fixed commuting costs of each worker (independent of their status), so that effectively they become $\left(1-\sigma^\tau\right)T$. As shown in the comparative statics analysis, this subsidy has the effect of diminishing informality rates. Under this option, the government's budget constraint can be expressed as $M=\sigma^\tau TN$, implying that:

$$\sigma^\tau=\frac{M}{TN}. \tag{2.42}$$

Again, given that $0<\sigma^\tau<1$, it must be the case that $M<TN$. To ensure the segmented equilibrium described in Figure 13, it is necessary that $T\left(1-\sigma^\tau\right)>\left(\left(w_c-w_h\right)/2\right)$. Under the third policy option, the government gives each formal worker a fraction σ^F of her fixed commuting costs. The government's budget constraint under this option can be written as $M=\sigma^F T\left(1-\phi^*\right)N$, so that:

$$\sigma^F=\frac{M}{T\left(1-\phi^*\right)N}. \tag{2.43}$$

And as $0<\sigma^F<1$, $M<T\left(1-\phi^*\right)N$ must hold. Finally, under the fourth policy option the government gives each informal worker a fraction σ^{IN} of her fixed commuting costs. The

government's budget constraint in this case can be written as $M = \sigma^{IN} T \phi^* N$, so that:

$$\sigma^{IN} = \frac{M}{T \phi^* N}.$$ (2.44)

And as $0 < \sigma^{IN} < 1$, $M < T \phi^* N$ must hold. Again, to ensure the segmented equilibrium, it is necessary that $T\left(1 - \sigma^{IN}\right) > \left(\left(w_c - w_h\right)/2\right)$.

2.6.2. Base model calibration and comparative analysis of policies

The model is calibrated to produce levels of informality, formality and unemployment close to the levels of Latin American cities. The parameter values (with a year as the implicit unit of time) shown in Table 2 are within the range of those used in previous studies (Albrecht, et al., 2009; Zenou, 2008; Zenou 2011).

General	Value
Discount rate r	0.05
Unemployed current income w_U	0
Commuting costs p/u of distance τ	0.01
Fixed commuting costs T	0.11
Total population N	100
Formal	**Value**
Productivity y_F	1
Worker's bargaining power β	0.5
Formal job destruction rate δ_F	0.3
Cost of maintaining a vacancy c	0.2
Matching function $d\left(\bar{S}_U U, V\right)$	$U^{0.5}V^{0.5}$
Informal	**Value**
Informal job creation rate α	1.4
Informal job destruction rate δ_{IN}	0.5
Home-based remuneration w_h	0.3
CBD remuneration w_c	0.4
Transfers b	0.5

Table 2. Parameter values

The first row of Table 3 displays the steady-state equilibrium values under these conditions. Under the specified values, the model predicts an unemployment rate of 16.54% and an informality rate of 46.31%. For the total population values, the model predicts that approximately 54% of all employed workers are in the informal sector. This value is close to recent estimates for Latin American countries (Galvis, 2012; Jütting, et al., 2008; Vanek, et al., 2014). The informal/formal wage ratio according to the model is 0.41, slightly below estimates for Mexico, Argentina and Colombia (Albrecht, et al., 2009).

Pos.		θ^*	ϕ^*	u^*	V^*	W^*	Subsidy
	Base equilibrium	**0.45**	**46.31**	**16.54**	**7.51**	**868.90**	
1	Hiring-costs subsidy	1.53	35.34	12.62	19.31	886.07	0.51
2	Transport subsidy (all)	0.84	40.79	14.57	12.31	880.59	0.18
3	Transport subsidy (formal)	0.54	44.69	15.96	8.72	872.97	0.32
-	Transport subsidy (informal)	0.45	46.32	16.54	7.50	868.87	0.39

Table 3. Policy efficiency comparison

Next, we evaluate the performance of policies through two indicators: the change in the informality rate and the change in general welfare. Welfare is measured as the production level in the city W (Sato, 2004; Zenou, 2009), given by:

$$W^* = \int_0^{+\infty} e^{-rt} \left\{ \int_0^{\left(1-u^*-\phi^*\right)N} \left(y_F - T - \tau x\right)dx + \int_{\left(1-u^*-\phi^*\right)N}^{\left(1-\phi^*\right)N} \left(w_U - T - \tau x\right)dx + \int_{\left(1-\phi^*\right)N}^{N} \left[S_{IN}w_c + \left(1-S_{IN}\right)w_h - S_{IN}^2\left(T+\tau x\right)\right]dx + \left(r+\delta_{IN}\right)b\phi^*N - c\theta^*u^*N \right\}$$

$$(2.45)$$

Thus, the production level in equilibrium is the sum of formal and informal sector production, the income of the unemployed and the transfers made to informal workers minus the cost of maintaining vacancies, the commuting costs for formal workers and the commuting costs for informal workers. Wages in the formal sector and land rents are pure transfers, so they do not enter the production level calculations. We now compare the four proposed policies for a total budget of $M = 2$. Table 3 shows the results. The first thing to note is that subsidizing formal firms hiring-costs is the most efficient option because it generates the highest level of welfare, the largest number of vacancies, the lowest unemployment rate and the lowest informality rate. The subsidy causes lower wages in the formal sector, because now the current and future productivity matching (the second part of (2.32)) is smaller. Consequently, job creation is positively affected, which reduces informality. Additionally, as can be seen from equation (2.30), because the subsidy makes it cheaper to formal firms to keep vacancies, job creation is directly and positively affected. This is precisely why this policy is the most efficient, since any other option considered only works through wages.

Second, a transport subsidy targeted to informal workers is an undesirable policy because it does not affect the informality and unemployment rates, and because it leads to a small decrease in total welfare. With the subsidy, informal workers can access the CBD at a lower costs, which increases the net profit they derive from commuting to the CBD. Consequently, formal firms have to pay a larger spatial compensation in order to attract workers, which results in a decrease in formal job creation and an increase in the informality rate. The increase in informality, however, has an additional, offsetting effect. Because the number of formal workers has effectively diminished, there is less competition for land at the CBD, which lowers the urban costs and consequently the formal wage. With a lower formal wage, there is more formal employment creation and a lower informality rate. In the end, the informality rate is the same as it was initially, implying that the subsidy had no impact.

Third, in terms of efficiency, a transport subsidy for all workers outperforms a transport subsidy to formal workers only. Given the previous result, and the fact that the size of the transport subsidy per head for formal workers is larger than the transport subsidy per head for all workers, this result is unexpected. The explanation is as follows. First, the transport subsidy for formal workers decreases the spatial compensation. This encourages job creation and reduces informality. This rises the formal wage, which partially mitigates the informality reduction. With the transport subsidy to all workers, the spatial compensation also decreases, but the pressure generated on wages is lower because the subsidy to informal workers tends to lower the wage.

2.6.3. Sensitivity analysis

In order to check the robustness of the results, we compare the first three policy options for different parameter values. The solid curve represents the transport subsidy for formal workers, the dashed curve represents the transport subsidy for all workers and the dotted curve represents the hiring-costs subsidy. We first consider hiring costs. As can be seen in Figure 14, the hiring-costs subsidy generates the largest welfare value and the lowest informality rate regardless of the value that the hiring-cost parameter takes. Thus, the conclusion that the hiring-costs subsidy is the most efficient policy holds for any value of hiring costs. On the same vein, our main results regarding the transport subsidies hold: for

any given level of hiring costs, we still reach the conclusion that a transport subsidy for all workers outperforms a transport subsidy for formal workers. Finally, our results hold when we compare the three policies for a different amount of public resources (Figure 15) and for different bargaining power (Figure 16).
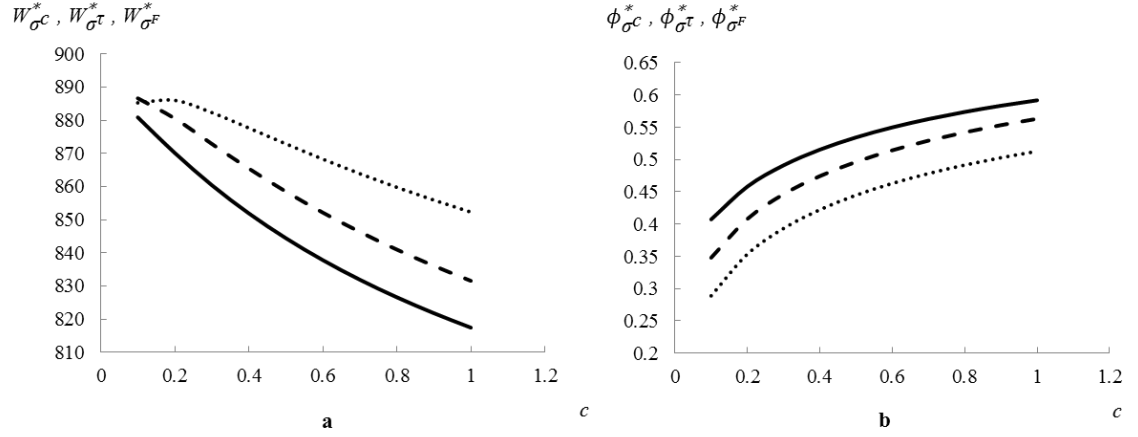

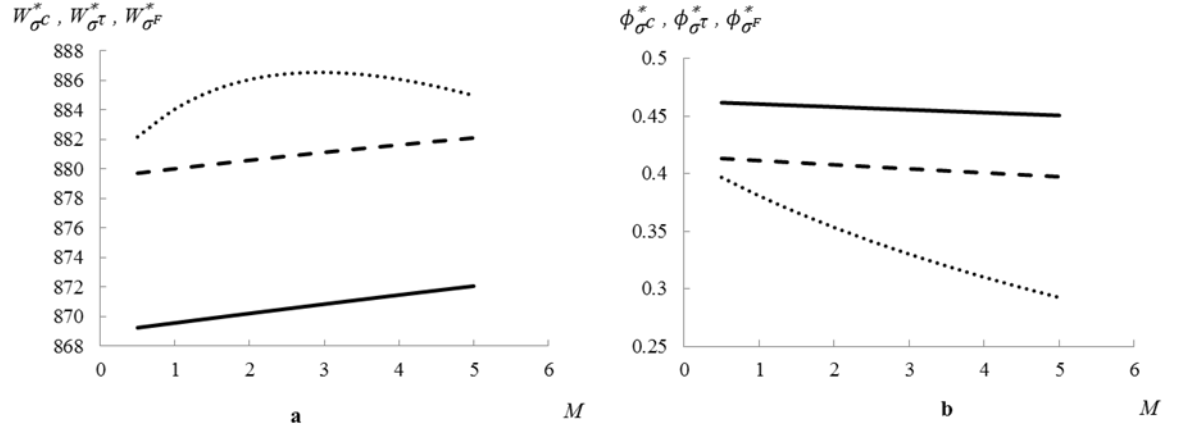**Figure 14. a: Total welfare vs Hiring costs. b: Informality rate vs Hiring costs**

Figure 15. a: Total welfare vs Public resources. b: Informality rate vs Public resources
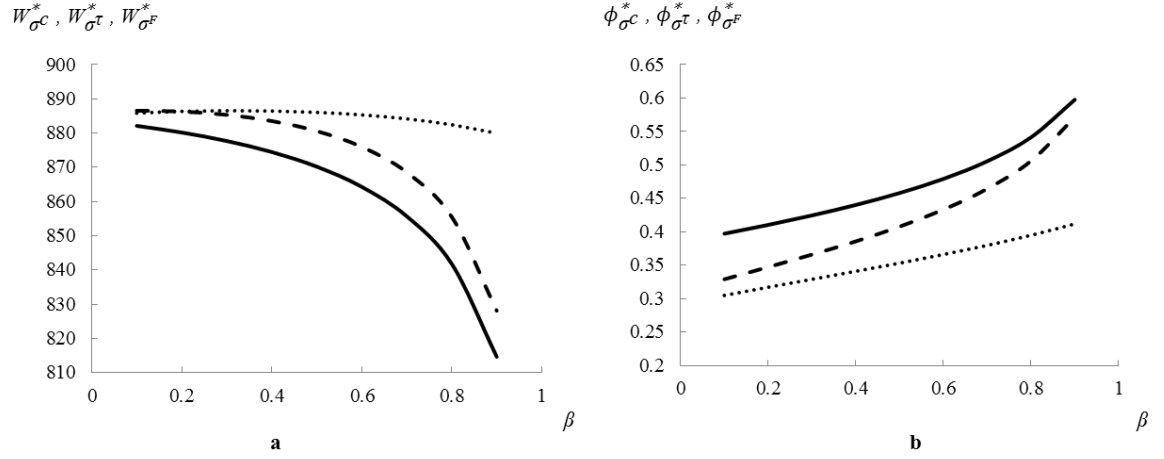
Figure 16. a: Total welfare vs Bargaining power. b: Informality rate vs Bargaining power

2.7.   Discussion and Conclusions

The decision to be informal may be not only related to income-related factors, such as the existence of social insurance subsidies, but also to accessibility (Moreno-Monroy, 2012). We have developed a search-matching model where formal workers commute every day to the CBD to work, and informal workers choose their optimal commuting frequency given the possibility to earn their income at home or at the CBD. A segmented city arises from the model, with formal workers and the unemployed located closer to the CBD, and informal workers located in the periphery. We characterized a steady-state equilibrium where the informality rate, land prices and wages are endogenously determined. According to our model, the informality rate is positively affected by an increase in fixed or variable commuting costs. This means that investments in improving accessibility, such as the expansion and development of an Integrated Transport System, can reduce the size of the urban informal sector.

We use this model to compare different policy options for reducing informality, and reach two main conclusions. First, we find a transport subsidy targeted exclusively to informal workers to be an undesirable policy because it has a neutral effect on the informality rate, and even a slightly negative effect on welfare. On the other hand, a subsidy to all workers does generate reductions in the informality rate and increases in welfare. This result is relevant for policy makers in cities where these type of targeted subsidies are being implemented or considered at the moment. Using transport accessibility analysis for Bogotá, (Bocarejo and Oviedo, 2012)

suggest that a cross-subsidy policy subsidizing fares for the lower-income population and taxing fares for the higher-income population would generate benefits in terms of equity given better access to job opportunities. However, their analysis does not consider the existence of informal (home-based) employment and related commuting decisions of informal workers. Our approach shows that mobility-related social inclusion policies of this type can potentially harm formal employment creation. Alternative policy designs could consider targeted subsidies linked to localized formal job creation programs, in order to ensure that better access also means better jobs. However, it is important to point out that subsidizing hiring costs is more efficient than subsidizing transport in reducing informality.

Finally, it is important to note that at present, our model assumes the transition between formal and informal jobs to be random, which is unrealistic. The extension of endogenous commuting choices for informal workers can also be refined further in order to also endogenize the return to different informal activities taking place within the city, and take into account likely effects such as congestion. Along these lines, more evidence is needed on the relationship between commuting frequencies and the choice of informal activities, in order to better understand the impact on transport policies on labor market outcomes.

2.8.   Appendix A

Equilibrium conditions of four policy options

Hiring-costs subsidy:

$$
(r+\delta_F)\frac{(1-\sigma^c)c}{q(\theta^*)}+\beta(1-\gamma)(1-\sigma^c)c\theta^*-
$$

$$
(1-\beta)\times\left\{y_F-\gamma\left[w_h+T+\tau(1-\phi^*)N+\frac{(w_c-w_h)^2}{4}\left[\frac{1}{T+\tau(1-\phi^*)N}\right]+(r+\delta_{IN})b\right]-(1-\gamma)w_U\right\}=0,
$$

$$
(2.46)
$$

where σ^c is determined by Equation (2.41).

Transport subsidy to all workers:

$$
(r+\delta_F)\frac{c}{q(\theta^*)}+\beta(1-\gamma)c\theta^*-
$$

$$
(1-\beta)\times\left\{y_F-\gamma\left[w_h+(1-\sigma^\tau)T+\tau(1-\phi^*)N+\frac{(w_c-w_h)^2}{4}\left[\frac{1}{(1-\sigma^\tau)T+\tau(1-\phi^*)N}\right]+(r+\delta_{IN})b\right]-(1-\gamma)w_U\right\}=0,
$$

$$
(2.47)
$$

where σ^τ is determined by Equation (2.42).

Transport subsidy to formal workers only:

$$(r+\delta_F)\frac{c}{q(\theta^*)} + \beta(1-\gamma)c\theta^* -$$

$$(1-\beta)\times\left\{y_F - \gamma\left[w_h + (1-\sigma^F)T + \tau(1-\phi^*)N + \frac{(w_c-w_h)^2}{4}\left[\frac{1}{T+\tau(1-\phi^*)N}\right] + (r+\delta_{IN})b\right] - (1-\gamma)w_U\right\} = 0,$$

$$(2.48)$$

where σ^F is determined by Equation (2.43).

Transport subsidy to informal workers only:

$$(r+\delta_F)\frac{c}{q(\theta^*)} + \beta(1-\gamma)c\theta^* -$$

$$(1-\beta)\times\left\{y_F - \gamma\left[w_h + T + \tau(1-\phi^*)N + \frac{(w_c-w_h)^2}{4}\left[\frac{1}{(1-\sigma^{IN})T+\tau(1-\phi^*)N}\right] + (r+\delta_{IN})b\right] - (1-\gamma)w_U\right\} = 0,$$

$$(2.49)$$

where σ^{IN} is determined by Equation (2.44).

Chapter three

Informal housing, spatial structure and city's compactness[22]

3.1.   Introduction

In its 2010/2011 State of the World´s Cities (SWC) report, UN-HABITAT defines a slum household as a group of individuals living in an urban area under the same roof, which lacks one or more of the following: durable housing, sufficient living space, easy access to safe water, access to adequate sanitation, and security of tenure. Given the high incidence of slums in the developing world (by 2010 the number of slum dwellers was estimated up to 828 million; UN-HABITAT, 2010) and its close relationship with urban poverty, slum reduction is an imperative for national and local authorities.

All through a combination of housing, land, infrastructure and poverty-reduction policies, a total of 227 million people moved out of slum conditions between 2000 and 2010, thus causing a fall in the proportion of the urban population living in slums from 39% to an estimated 32% (UN-HABITAT, 2010, pág. 32 and 42). Therefore, cities in developing countries are experiencing rapid urbanization in which slum dwellers are more integrated with the urban economy, are legalizing their tenure status, and are acquiring durable housing with basic services.

In Latin America, it is common that the production of new durable housing takes place outside government regulations. In Colombia, for example, through the Integral Neighborhood Improvement Program (*Mejoramiento Integral de Barrios)*, the authorities stimulated in-situ incremental house improvements and its connection to basic services. However, the connection to the infrastructure is usually remedial, and house improvements rarely fulfill

urban and taxation regulations. Specifically in Medellin, houses in neighborhoods called "*comunas*" are usually self-constructed, with inefficient technology, neglecting the payment of taxes, and connecting the house to the infrastructure of public services once it is built. This is not surprising given a general context where historically only 35% of houses have been formally generated and where there is excessive paperwork, and lack of transparency in the tax and planning system (Minvivienda, Gobierno de Colombia, 2010).

In this way, there is a variant of informal housing production which is not related with extreme poverty and illegal occupation of land, characterized by a remedial connection to infrastructure, non-compliance with urban standards and tax evasion. It is reasonable to think that this type of informality helps explaining important features shown by Latin America cities. Some of these are: the fast spatial expansion[23], the prevalence of short buildings, and the hosting of peripheries where low-income workers and precarious short buildings concentrate (UN-HABITAT, 2010; Zenou, 2011; Da Mata, 2013).

This is because informal constructions are close substitutes for formal constructions, implying that it occupy land that could have been used by the formal sector, consequently affecting the land market (Smolka and Biderman, 2011), the productivity in the housing industry, and finally the shape and the spatial structure of the city. In fact, informal constructions create a cost for the society. Its low levels of capital intensity, short height and use of inefficient technologies of production, lead to an inefficient use of urban land. This inefficiency can be seen in the form of rapid spreading short constructions which consume much more land than the required by formal developments.[24] This in turn makes the city less compact i.e. lower and wider. For example, the common story of slums formation and the proliferation of informal settlements is a story with occupation of large extensions of scarce land, by invasions, self-constructions, and pirate developments (Smolka and Biderman, 2011; UN-HABITAT, 2013). Informal housing is also a fundamental channel through which other urban variables affect the city. For instance, it has been argued that slow infrastructure growth, combined with fast population growth results in the proliferation of informal settlements (World Bank, 1993; UN-HABITAT, 2013).

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[23] Inostroza, et al., (2013) quantified the main parameters of expansion and sprawl for 10 Latin American cities between 1990 and 2010.   The authors show a pervasive spatial expansion, where most of the studied cities are expanding at fast rates with falling densities trend.

[24] In Bogotá and Medellin formal constructions usually take the form of apartment buildings.

To the best of our knowledge, there is no theoretical work that studies the causes of this kind of informality. Moreover, there is no theoretical work that studies its consequences (and the consequences of informal housing in general) over the city structure. In particular, little is known about how land allocated to informal constructions changes with distance to the city center, how informal constructions can be compared with formal constructions, and how informality affects the size and tallness of the city.[25] Additionally, little is known about how infrastructure and population growth affect the city through informal settlements.

Previous theoretical literature has focused on the study of slums, emphasizing on issues such as illegal dwelling, or income differences combined with land regulations. For example, Jimenez (1985) and Brueckner and Selod (2009) study the economics of squatter settlements. Da Mata (2013) models and quantifies slum growth for Brazil, focusing on the role of lack of property rights, while Heikkila and Lin (2013) study the implications of minimum lot size restrictions and the income differentials in the magnitude of slums. Additionally, in all these models, for the households' point of view, informal housing differs significantly form formal housing, and as a consequence there is an important emphasis on modeling the demand for land in the informal sector. This does not facilitate the construction of a framework that simultaneously considers intersectoral land mobility, bid rent functions, and differential job access, conditions required to include an interdependent informal sector into the urban space.

In this paper we develop a monocentric city model with and informal housing sector, characterized by remedial connection to the infrastructure, non-compliance with the urban standards and the evasion of taxes. To do so, we focus on the housing supply structure, because it allows explaining informality beyond poverty and illegal dwelling, and also allows to easily introducing differential job access. Specifically, the main driving force behind informality here, is the behavior of informal land developers combined with the behavior of the local government.

Informal production in developing countries can be the result of actions of economic agents that seek profits. Lack of regulation enforcement and the easy access to rudimentary

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[25] Duranton and Puga (2014) present a detailed review of the monocentric city model literature, together with its main extensions and empirical implications.

technologies create powerful incentives to produce outside regulations (Mejía and Posada, 2013). Of course, this is also the case in the housing industry, where the importance of informal land developers has been recognized. According to Smolka and Biderman (2011), pag. 6: "Informality can open a gap for arbitrage, allowing informal developers to reap higher profits than formal developers because they avoid paying license fees and taxes, only partially provide infrastructure and services, devote smaller percentages of land to public uses, and offer below-minimum-size lots. These incentives stimulate supply of informal developments".

Precisely, these incentives are determined by the local government, as it is responsible of taxation, provision of public services infrastructure, and law enforcement. Then, the government is a fundamental actor in the determination of the informality level in the housing market. So far, in the theoretical literature, the local government plays an absent or secondary role. In this sense, informality is, at least partially, the consequence of some sort of state incapacity. An interesting and alternative vision of the behavior of the authorities is represented in the macroeconomic analysis of Mejía and Posada (2013). The authors consider that spending in law enforcement and spending in infrastructure for public services are two different instruments used to maximize the level of aggregated formal production in the economy. Then, informality is not the result of the hypothetical weakness of the state, and it is instead the optimal level achieved and accepted by the government. In this paper, Mejía and Posada's (2013) view is used to represent the behavior of the local government. Thus for the first time in the informal housing literature, enforcement and infrastructure provision are endogenous and are fully characterized in the urban space.

More technically, this paper modifies the Muth-Mills model[26] adapting the housing industry to an environment of two sectors (formal and informal). On the supply side, both informal and formal land developers produce a homogenous good (interior living space) using land. However, unlike formals, informal producers evade taxes (value added taxes) and do not use the infrastructure for public services as an input (i.e. they do not pay infrastructure rent). This

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[26] Brueckner (1987) provides an excellent presentation of the Muth-Mills model. This is a model of urban spatial structure where land is an intermediate input in the production of housing. An important result obtained with the model is that structural density (capital-land ratio) is a decreasing function of the distance, so buildings are shorter farther from the CBD. According to Brueckner (1987), the explanation for this is that lower land rents are required at greater distances to compensate producers for the lower price per square foot of housing. The resulting decline with distance in the relative price of land causes producer substitution in its favor, leading to lower structural densities.

forces informal producers to use inefficient technology of production, and to bear the risk of being detected and fined for evading regulations. As intersectoral land mobility is assumed, a general equilibrium framework (analogous to the specific factor model of international trade) is used in the analysis of the land market. The land endowment is exogenous and fixed, must be fully employed, and is owned by absentee landlords. On the demand side, the households are renters, consume a fixed amount of housing, and perceive informal and formal production as perfect substitutes. To improve the model's predictions about socio-economic segregation and the density patterns within the city, two household's types are considered: the high-income and the low-income, where the latter compared with the former receives less income, consumes less housing and commutes with less frequency.

There is a local government that maximizes formal housing production through both spending in law enforcement and public services infrastructure. The increase in law enforcement raises the likelihood of detection (and thereby punishment) of informal land developers, whereas the rise in public services infrastructure increases the productivity of land in the formal sector. The government finances its spending through taxes raised on formal land developers, and fines charged to informal land developers. Both taxes and fines are considered exogenous variables. The analysis proceeds in two parts. First, it is assumed that infrastructure and law enforcement are exogenous. This allows focusing on the spatial patterns of the informality as well as its impact over the city structure. Then, the role of the local government is studied, which allows explaining the spatial variations of infrastructure and law enforcement.

In equilibrium, given that public infrastructure decreases with distance to the city center (this will be the optimal decision of the local government), the land allocated to produce short informal buildings increases with distance, while land allocated to produce tall formal buildings decreases. Thereby, the city's periphery shows high levels of housing informality, under the shape of short buildings.[27] At the same time, because low income workers commute with less frequency and consume less housing, the periphery gets dense and selectively inhabited by them. These characteristics reflect the situation of cities in Latin America and rationalize in a simple way how fast city growth combined with slow infrastructure growth originate the proliferation of informal housing.

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[27] This also implies low structural densities in the periphery

The comparative statics analysis shows that cities with either a low provision of infrastructure, high levels of taxes, or low levels of law enforcement all display a high level of housing informality. This implies an allocation of land in favor of the less productive sector, causing the city to be less compact (lower and wider), which in turn implies higher land and housing rents, as well as lower levels of utility at equilibrium, for city inhabitants. In this sense, informality is costly for society. With these results, the model establishes a clear connection between the proliferation of informal housing and the fast spatial expansion, and also rationalizes the common observation which holds that informal sector fixes a lower limit for land prices, raising the land price for the formal sector (Smolka, 2003). The analysis also shows that local governments can invest in infrastructure, can cut taxes or can strengthen the enforcement in order to encourage formalization and make the city more compact.

Regarding the local government behavior, given that the law enforcement is the opportunity cost of the infrastructure and vice versa, location at every point of the city comes with an optimal level of informal housing. As housing rents decrease with distance to the city center, there is a tax revenue decrease, which in turn leads to reduction in the optimal levels of both law enforcement and infrastructure, as distance increases. Thus, the slow growth of the public infrastructure is not necessarily the result of the lack of responsiveness of the State to the fast urban growth, instead, it is the result of an optimal decision of the government under resource restrictions.

The remainder of the paper is structured as follows: Section 2 presents the related literature. Section 3 presents the model set up, whereas Section 4 depicts the equilibrium. Comparative static experiments are displayed in Section 5. Section 6 introduces and analyzes the behavior of the local government and Section 7 studies the implications of informal housing on rural-urban migration. Section 8 summarizes and concludes.

3.2.   Related literature

This document is mainly related with the theories on the interlinked nature of the formal and informal sectors in the urban land market. Brueckner and Selod (2009) have been the first to deal with this issue. They have developed a general equilibrium framework where squatter settlements (which configure the informal housing sector) compete with formal households for the same land. The basic mechanism of the model works as follows: squatters settlements "squeeze" the formal land market, raising the formal price. This invites the landowners to consider the possibility of evicting the squatters. Then, in order to escape eviction, the squatter communities control the squeezing and the costs of eviction for the landowners. Through this model, the fraction of the urban land that goes to the formal sector and the fraction that goes to the informal sector can be established. Moreover, the model allows for studying the welfare effects of formalization policies. Specifically, the analysis shows that the squatting equilibrium is inefficient.

In the work of Heikkila and Lin (2013), rich households compete for land with poor ones. In this scenario, the authorities impose a minimum lot size restriction, which is always fulfilled by the rich while unaffordable for some (or all) of the poor. This forces the poor to leave the formal area, implying that they stop paying the formal land rent and only achieve the minimum consumption of land and other goods. In this way, an informal sector in the land market is created. The authors implement the "*everyone lives somewhere*" vision, using a Linear Expenditure System (LES) for the demand of housing and other goods that builds in minimum "basic needs" for livelihood. The analysis shows that while the informal sector is larger, the welfare gains for the wealthy households are larger. This is so because the model shows that the higher the number of poor households in the informal sector, the lower the aggregate demand for land in the formal sector is, leading to a rent subsidy in the formal sector. Finally, Brueckner (2013) extends the work of Brueckner and Selod (2009) by replacing the benevolent squatter organizer with competing, rent-seeking organizers. The analysis shows that the rent seeking behavior leads the market to assign more land to the squatter settlements.

On this document, in contrast with previous models, the interactions between the informal and the formal sector emerge in the supply side of the economy. More specifically, the

existence of informal land developers creates land competition between these and formal land developers. We study this competition with a basic neoclassical general equilibrium framework, analogous to the one used to study the labor market in the specific factors model of international trade. This allows, for a given level of the housing rent, to find a unique equilibrium land rent level as well a unique land allocation. Also, in contrast with previous models, here the role of housing demand is simplified. In particular, we assume that households consume a fixed quantity of housing and evaluate the formal and the informal production as perfect substitutes. This permits to easily incorporate differential job access. Using these elements together, we can calculate the bid rents (housing rents) and then determine at each distance of the city center, the land rent and the land allocation. Through this, for the first time in the literature, informal housing is placed within the city, making it possible to inquest into its spatial patterns as well as its implications over the city shape.

The simple structure of the model provides an additional advantage: the possibility to study the behavior of the local government. In the existing theoretical literature, the eviction of informal households is usually the way that land law is enforced. Because the law, regulations, and enforcement are defined at the government level, in this literature the local government should have played a crucial role in the determination of the level of the eviction. However, this was not the case; as a matter of fact, the local government seems to be clearly absent. In Jimenez (1984), Jimenez (1985) and Kapoor and Le Blanc (2008), eviction takes the form of an additional exogenous risk of the informal investments. In Turnbull (2008), eviction depends on the optimal decision of the landowners. In Brueckner and Selod (2009) and Brueckner (2013), it is the squatter organizer who drives the economy to achieve the no-eviction condition.

A similar situation is found when considering the provision of infrastructure and land regulations, which are other ways through which the government affects informality. Da Mata (2013), using a balanced budget constraint, considers the role of the public expenditures and taxes. Heikkila and Lin (2013) consider the role of minimum lot size restrictions. However, in none of these studies the local government has an objective function, which means that it plays a secondary role. Adapting the macroeconomic analysis of Mejía and Posada (2013), here the local government maximizes the formal production under income restrictions using the spending in the law enforcement and the spending in infrastructure. Thus, for the first

time in the informal housing literature, these variables are endogenous and fully characterized in the urban space.

3.3.   A model of informal housing

3.3.1. The city

The city is linear, monocentric and closed, and its Central Business District (CBD) is at the origin (zero). Urban residents demand housing services, which are produced from land and public infrastructure by the firms in the housing industry. There is a local government that charges taxes and fines, and provides law enforcement and infrastructure for public services. These variables are initially exogenous to facilitate the study of informality implications in the urban space. Land is owned by absentee landlords. The urban land market is assumed to be competitive. In this way, at any distance x from the CBD, all agents take the housing rent $R_H(x)$ and the land rent $R(x)$ as given. In the city there is no vacant land, and land areas not used for residential purposes are assumed to be used for agricultural production. The agricultural rent is exogenous and equal to zero.

3.3.2. Households

Each household contains one urban resident who commutes to a job in the CBD, consumes a fixed quantity of housing services, and consumes a composite non-spatial good (which is taken as the numeraire, so that its price is normalized to 1). The households are renters, and perceive informal and formal production as perfect substitutes. Following Xiao (2013), the model considers two households types: the high-income, labeled A, and the low income, labeled B. In comparison with the former, the later receives less income, consumes less housing, and commutes with less frequency. As in Heikkila and Lin (2013), this is only a simplification of the idea that there are "haves" and "have-nots" in cities. In the developing world, the "haves" can be thought as the workers in the formal sector and the "have-nots" can be thought as the workers in a decentralized informal sector. Whereas in Heikkila and Lin (2013) this heterogeneity plays a crucial role in explaining informality, in this work it does not, so the heterogeneity is only considered to improve the model predictions about socio-economic segregation and about density patterns.

The masses of the two household types are given by M_A and M_B. When an A resident lives in the city at distance x from the CBD, he receives a fixed income y_A, consumes z_A units of the non-spatial composite good, consumes one unit of housing, pays a housing rent $R_H(x)$ and pays τx in commuting costs, where τ is the cost per unit of distance. Then the budget constraint for this resident at distance x is:

$$R_H(x) + \tau x + z_A = y_A. \tag{2.50}$$

Assuming that the utility function is given by $U(z_A) = z_A$ and using the budget constraint, we have that the indirect utility function at distance x is:

$$u_A(x) = y_A - \tau x - R_H(x). \tag{2.51}$$

Similarly, when a B resident lives in the city at a distance x from the CBD, he receives a fixed income y_B, consumes z_B units of the non-spatial composite good, consumes d units of housing, pays a rent $dR_H(x)$ and pays $s\tau x$ in commuting costs, where $s\tau$ is the cost per unit of distance. It is assumed that $s < 1$, $d < 1$ and $y_B < y_A$, which reflects that this type of household, compared with the A type, receives less income, consumes less housing and commute with less frequency. It is also assumed that $(s/d) < 1$. This means that for a given level of housing consumption d, the low-income household commutes with little frequency. The budget constraint at distance x is:

$$dR_H(x) + s\tau x + z_B = y_B, \tag{2.52}$$

and again, given that the utility function is $U(z_B) = z_B$, the indirect utility at distance x is:

$$u_B(x) = y_B - s\tau x - dR_H(x). \tag{2.53}$$

The corresponding bid rents functions for each resident type are given by the following equations:

$$\Psi_A(x, u_A) = y_A - \tau x - u_A, \tag{2.54}$$

$$\Psi_B(x, u_B) = \frac{1}{d}(y_B - s\tau x - u_B), \tag{2.55}$$

which indicates the maximum land rent that a $A(B)$ resident type is willing to pay in order to reach a utility level $u_A(u_B)$. As it can be verified, these functions are linear and decreasing in x. It is also possible to verify that $\Psi_A(x, u_A)$ is steeper than $\Psi_B(x, u_B)$, given that $(s/d) < 1$. This means that, in equilibrium, A residents live close to the CBD, whereas B residents live in the periphery. This specification resembles a city with socio-economic segregation where lower-income workers are poorly connected with the CBD and are relegated to peripheral areas. This is often the case in the developing world (Da Mata, 2013).

The analysis will not consider the case of the "integrated" city equilibrium, which locates the high-income households at the periphery and the low- income households near the CBD (this equilibrium emerges when $(s/d) > 1$). This is because, as remarked above, households' land demand conditions have nothing to do with the levels of housing informality. For the analysis, the only implication of this scenario is that the initial equilibrium utility levels of the households change. Moreover, as it will be shown below, a rise in the housing informality always decreases the equilibrium utility levels of city inhabitants, regardless of the initial equilibrium location.

3.3.3. Housing industry

It is widely accepted that a significant part of informal production in developing countries is not necessarily related with poverty, and it is instead the result of economic agents' actions that seek profits. The lack of regulations enforcement and easy access to rudimentary

technologies create powerful incentives to produce outside regulations. In the housing industry, this is observed in informal developers who obtain benefits by providing housing through a process characterized by taxes evasion, incomplete provision of infrastructure and services, and the use of inefficient technologies (Smolka and Biderman, 2011). This production, however, creates a cost for the society because informal settlements use land that could have been used by the more efficient formal sector. Next, we use these elements in a synthetic way to represent a housing industry with an informal sector.

Consider, then, that at distance x to the CBD, interior living space (a homogeneous good) is produced in two sectors that coexist: the formal and the informal. In the formal sector, firms pay taxes (value added taxes), produce using land and infrastructure for public services, and have a high but decreasing marginal productivity of land. In contrast, in the informal sector, firms do not pay taxes, only use land to produce, have low and constant marginal productivity of land, and may be detected and fined for avoiding the regulations. The technology in the formal sector is given by:

$$Q_F(x) = A_F \bar{Z}(x)^{1-\alpha} L_F(x)^{\alpha}, \qquad (2.56)$$

where $Q_F(x)$ is the formal output at x, $L_F(x)$ is the land used in the formal sector at x (to be determined in equilibrium) and $\bar{Z}(x)$ is the level of the public infrastructure at x, which is exogenous (for now) and decreases with distance to the CBD according to:

$$\bar{Z}(x) = \bar{Z}e^{-x}. \qquad (2.57)$$

This reflects how infrastructure growth always goes behind city growth. As we will show below, this pattern comes as a result of the optimal decision of the local government that maximizes the formal production. Since $A_F > 0$ and $0 < \alpha < 1$, (2.56) shows constant returns to scale and decreasing marginal productivity of land. Denoting the tax rate with t, which is constant along the city, each profit maximizer firm in the formal sector solves:

$$\max_{L_F(x)} \left\{ \pi_F(x) = R_H(x)(1-t) A_F \bar{Z}(x)^{1-\alpha} L_F(x)^{\alpha} - R(x) L_F(x) - v(x) \bar{Z}(x) \right\}$$

at each $x \in \left(0, x_f\right]$,

(2.58)

where $R(x)$ is the land rent at x, $R_H(x)$ is the housing rent at x, $v(x)$ is the rental rate of the public infrastructure at x, and x_f is the city fringe, all of these to be determined in equilibrium. Given that the housing industry operates under perfect competition, the first order condition yields:

$$R_H(x)(1-t)\alpha A_F \bar{z}(x)^{1-\alpha} = R(x),$$

(2.59)

where $\bar{z}(x) = \bar{Z}(x)/L_F(x)$ is the infrastructure for land unit. This condition determines the land demand in the formal sector, and given a decreasing marginal productivity of land, it shows an inverse relationship between $L_F(x)$ and $R(x)$ for a fixed level of $R_H(x)$. This is illustrated by the $FR(x)$ curve in Figure 18, read from left-to-right, in a diagram where the vertical axis represents land quantities and the horizontal axis represents the level of land rent. In the informal sector, the technology is:

$$Q_{IN}(x) = A_{IN} L_{IN}(x),$$

(2.60)

where $Q_{IN}(x)$ is the informal output at x and $L_{IN}(x)$ is the land used in the informal sector at x (to be determined in equilibrium). Given that $A_{IN} > 0$, land productivity is constant. The informal firm can be detected when evading taxes with a probability $1-\phi$, where ϕ is between 0 and 1. This probability does not depend on the distance to the CBD and is exogenous. Later, when we study the behavior of the local government, this probability will be a function of the spending in law enforcement. For simplicity's sake, if the firm is detected, its income will be confiscated. Therefore, in this sector, each firm maximizes expected profits given by:

$$\max_{L_{IN}(x)}\left\{\pi_{IN}(x)=\phi R_{H}(x)A_{IN}L_{IN}(x)-R(x)L_{IN}(x)\right\} \quad \text{at each} \quad x\in(0,x_{f}], \qquad (2.61)$$

and again, given that the housing industry operates under perfect competition, the first order condition yields:

$$\phi R_{H}(x)A_{IN} = R(x), \qquad (2.62)$$

which determines the demand for land in this sector. Since ϕ and A_{IN} are fixed and exogenous, when $\phi R_{H}(x)A_{IN} < R(x)$ informal firms do not demand land, and when $\phi R_{H}(x)A_{IN} > R(x)$ the demand does not exist. Finally, when $\phi R_{H}(x)A_{IN} = R(x)$, any amount of land is demanded (this is illustrated by $IN(x)$ curve in Figure 18, which can be read from right-to-the left in the diagram). We assume that:

$$\alpha A_{F}\bar{Z}(x_{f})^{1-\alpha} > A_{IN}, \qquad (2.63)$$

which guarantees that land is always more productive in the formal sector. There is free intersectoral land mobility, which links the two sectors in a general equilibrium framework. Land supply at each x is equal to one and must be fully employed, that is:

$$L_{F}(x)+L_{IN}(x)=1. \qquad (2.64)$$

Finally, for simplicity we assume that land developers income and government income (taxes and penalizations) are spent outside the city.

3.4. Equilibrium and intra-city analysis

The analysis proceeds in two stages. First, given the city fringe x_{f}^{*}, and the frontier between A residents zone and B residents zone x_{b}^{*}, the equilibrium utility levels u_{A}^{*}, u_{B}^{*}, and the equilibrium housing rent $R_{H}^{*}(x)$ are found. Then, x_{f}^{*} and x_{b}^{*}, are found, after the land rent

$R^*(x)$, the land allocation between sectors, $L_F^*(x)$ and $L_{IN}^*(x)$, and the housing production $Q^*(x)$ are found at each distance x. As mentioned above, $\Psi_A(x, u_A)$ is steeper than $\Psi_B(x, u_B)$. Then, in equilibrium, A residents live close to the CBD, whereas B residents live in the periphery. Using this, in equilibrium we have:

$$\Psi_A\left(x_b^*, u_A^*\right) = \Psi_B\left(x_b^*, u_B^*\right), \tag{2.65}$$

$$\Psi_B\left(x_f^*, u_B^*\right) = 0, \tag{2.66}$$

$$R_H^*(x) = \max\left\{\Psi_A\left(x, u_A^*\right), \Psi_B\left(x, u_B^*\right), \, 0\right\} \quad \text{for each} \quad x \in \left(0, x_f^*\right]. \tag{2.67}$$

Therefore, given x_f^* and x_b^*, it is possible to find both the equilibrium utility levels u_A^* and u_B^* and the equilibrium housing rent $R_H^*(x)$. Equation (2.66) shows that the bid rent for B residents is zero at x_f^*. This, as it will be shown later, ensures that the equilibrium land rent is equal to agricultural rent (which is assumed to be zero) at x_f^*. Equation (2.65) implies that exactly at x_b^*, the bid rent of A residents is equal to the bid rent of B residents. Finally, equation (2.67) states that the equilibrium housing rent function is equal to the upper envelope of the equilibrium bid rent curves of all residents and the agricultural rent line. Using (2.65), (2.66), (2.54), and (2.55) we find that the equilibrium utility levels are:

$$u_B^* = y_B - s\tau x_f^*, \tag{2.68}$$

$$u_A^* = y_A - \left(1 - \frac{s}{d}\right)\tau x_b^* - \frac{s}{d}\tau x_f^*, \tag{2.69}$$

and replacing (2.68) and (2.69) in (2.54) and (2.55), the housing rent function in equilibrium $R_H^*(x)$ is:

$$
R_H^*\left(x\right)=
\begin{cases}
\left(1-\dfrac{s}{d}\right)\tau x_b^* +\dfrac{s}{d}\tau x_f^* -\tau x & \text{for} \quad 0\le x\le x_b^* \\[2ex]
\dfrac{s}{d}\tau\left(x_f^* -x\right) & \text{for} \quad x_b^*\le x\le x_f^* \qquad (2.70)\\[2ex]
0 & \text{for} \quad x\ge x_f^*.
\end{cases}
$$

This equilibrium is shown in Figure 17, which characterizes a *segregated city*, where A residents live in the area $\left(0, x_b^*\right]$, and B residents live in the area $\left[x_b^*, x_f^*\right]$. Then, as noted above, the city displays socio-economic segregation, where lower-income households are poorly connected with the CBD and are relegated to peripheral areas.
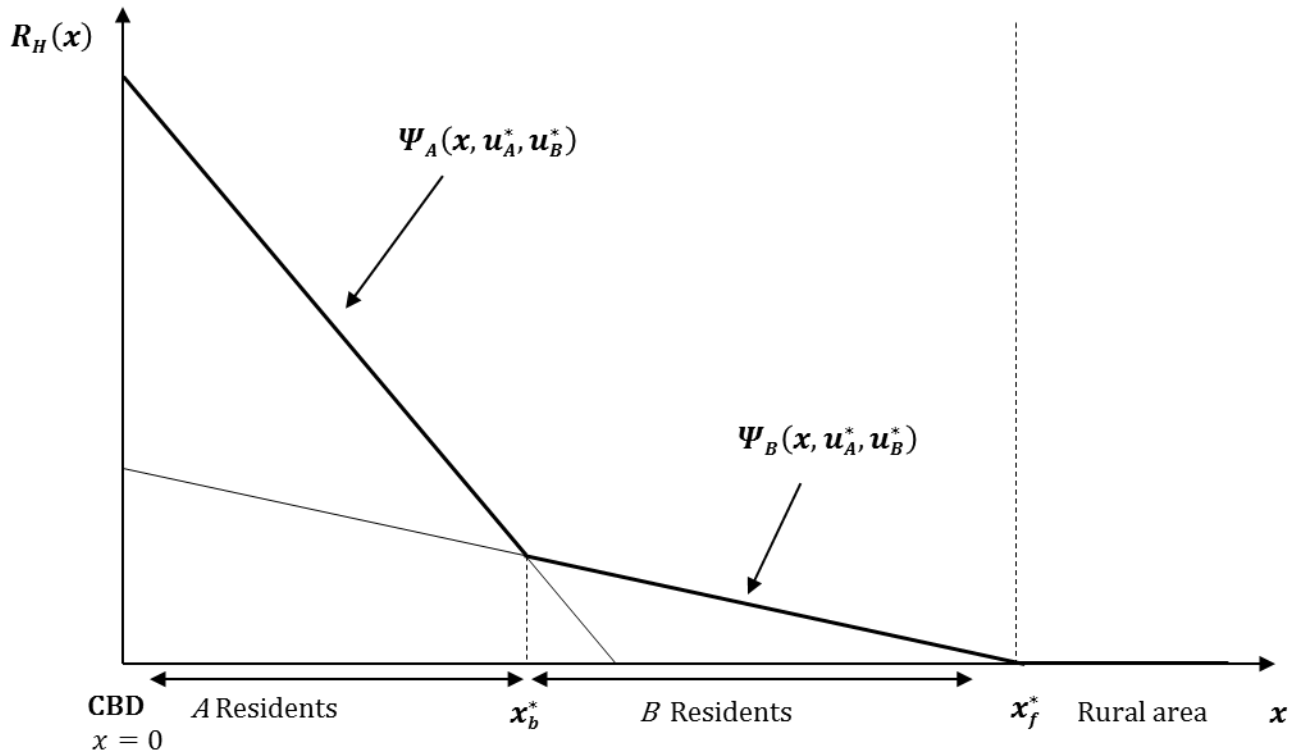

Figure 17. Housing rents in equilibrium (The segregated city)

In order to find x_b^* and x_f^*, it is necessary to find first the land rent, land allocation, and housing production at each distance x. The analysis proceeds in three stages. First, at any arbitrary distance x to the CBD, the land market equilibrium is characterized. This allows us to understand how the endogenous variables are determined. Second, a simple comparative static analysis shows how these variables evolve in the urban space. In this way we can establish the spatial patterns of the informal sector and the spatial patterns of the buildings height. Finally, we calculate the total housing production, which allows calculating x_b^* and x_f^*. It is assumed that:

$$(1-t)\alpha A_F \bar{Z}(0)^{1-\alpha} < \phi A_{IN}, \qquad (2.71)$$

which ensures that production will always be positive for both sectors at any distance x. The free intersectoral land mobility establishes a common land rent in both sectors, which links them in a general equilibrium framework. In equilibrium, the land rent ensures, first, that the value of marginal productivity of land is equal between sectors and that total land demand equals land supply. Thus, the equilibrium land allocation, at each x, is characterized by:

$$\phi A_{IN} R_H^*(x) = R_H^*(x)(1-t)\alpha A_F \left[\bar{Z}(x)/L_F^*(x)\right]^{1-\alpha}, \qquad (2.72)$$

which can be represented graphically in Figure 18 as the intersection of the $FR(x)$ and $IN(x)$ curves[28], at a diagram where the fixed supply of land (which is equal to one) is represented by the length of the horizontal axis. Note that the position of land demand curve in the formal sector depends on taxes, infrastructure and total factor productivity, whereas the position of land demand curve in the informal sector depends on land productivity, and the probability of detection. Therefore, the equilibrium land allocation depends on all previous variables mentioned. More formally, by restating (2.72), the land in the formal sector is:

$$L_F^*(x) = \theta^{(1/1-\alpha)}\bar{Z}(x), \qquad (2.73)$$

---

[28] See Feenstra (2003) for a presentation of the Specific factors model.

where $\theta = \left[(1-t)\alpha A_F / \phi A_{IN}\right]$. Replacing (2.73) in the formal sector production function (2.56) allows obtaining the equilibrium production level:

$$Q_F^*(x) = A_F \theta^{(\alpha/1-\alpha)} \overline{Z}(x). \qquad (2.74)$$

Because the city is linear, the land in the informal sector is given by:

$$L_{IN}^*(x) = 1 - \theta^{(1/1-\alpha)} \overline{Z}(x) \qquad (2.75)$$

and replacing (2.75) in the production function of the informal sector (2.60), the informal production level is:

$$Q_{IN}^*(x) = A_{IN} - A_{IN} \theta^{(1/1-\alpha)} \overline{Z}(x) . \qquad (2.76)$$

The equilibrium land rent is given by:

$$R^*(x) = \phi A_{IN} R_H^*(x), \qquad (2.77)$$

which simply is the value of the expected productivity of land in the informal sector. Finally, the rental rate of infrastructure $v^*(x)$ can be calculated as the value of its marginal productivity:

$$v^*(x) = R_H^*(x)(1-t)(1-\alpha)A_F \overline{z}^*(x)^{-\alpha}. \qquad (2.78)$$

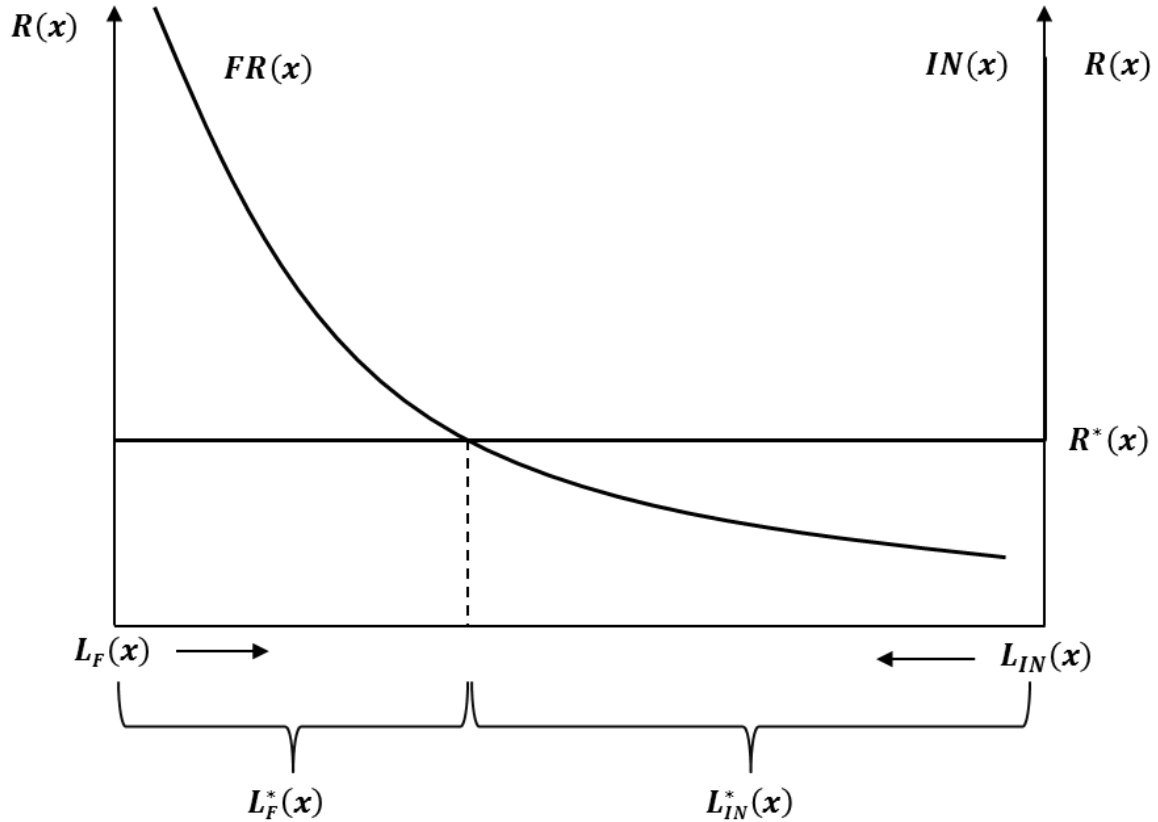Next, we establish the spatial pattern of the informal sector and the spatial pattern of the buildings height.

Figure 18. Land market equilibrium at distance x to the CBD

3.4.1. Intraurban spatial patterns of the informal housing

To proceed, let us consider the effects of a small movement away from the CBD, starting from any arbitrary location x in the city. First, as the housing rent decreases with distance, the value of marginal productivity of land in both sectors decreases. This is described in Figure 19 as a simultaneous downward shift of the $FR(x)$ curve $\left(FR(x) \text{ to } FR(x)'\right)$, and the $IN(x)$ curve $\left(IN(x) \text{ to } IN(x)'\right)$. Because these shifts are of the same proportion for both curves, there is a fall in the equilibrium land rent, and no changes in the land allocation. *Thus, equilibrium land rent decreases with distance from the CBD.* Now, because the level of infrastructure also decreases with distance, there is an additional downward shift of the

$FR(x)$ curve $\left(FR(x)' \text{ to } FR(x)'' \right)$. This occurs because a fall in the infrastructure reduces the value of marginal productivity of land in the formal sector. As a consequence, the land allocated to the informal sector rises and the land allocated to the formal sector falls, whereas the land rent remains as before.
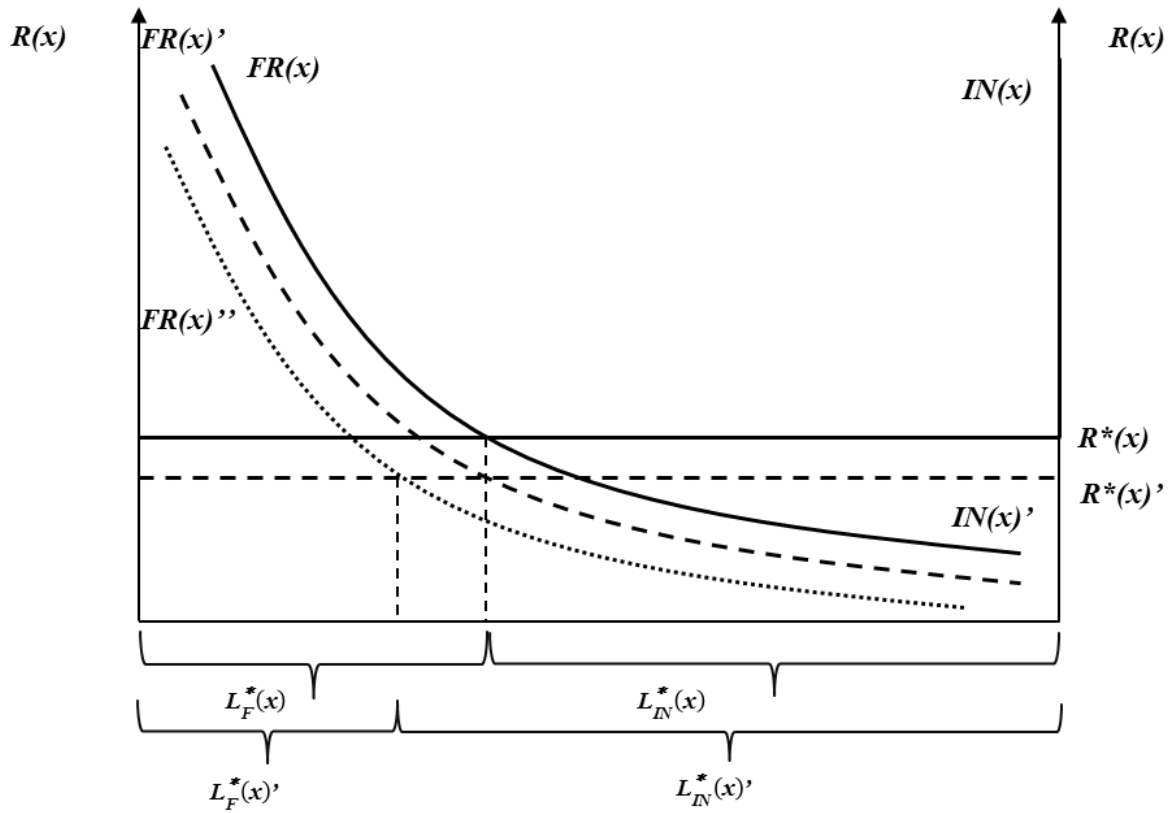

Figure 19. Effects of a rise in the distance to the CBD

Then, in short, informal housing increases with distance to the CBD. Now, what are the implications for the spatial structure of the buildings height in the city? Let $q_F^*(x) \equiv Q_F^*(x) / L_F^*(x)$ be the buildings height in the informal sector and $q_{IN}^*(x) \equiv Q_{IN}^*(x) / L_{IN}^*(x)$ the buildings height in the informal sector. Using (2.73), (2.74), (2.75) and (2.76) leads to:

$$q_F^*(x) = \phi A_{IN} / \alpha(1-t), \qquad (2.79)$$

and:

$$q_{IN}^{*}(x) = A_{IN}. \tag{2.80}$$

These quantities do not depend on distance from the CBD. Using (2.63) and (2.71), it can be shown that $\phi/(1-t)\alpha > 1$, which implies that formal constructions are taller than informal constructions. Thus, land allocated to produce short (tall) buildings increases (decreases) with distance to the CBD. Furthermore, by defining the average tallness of the buildings as:

$$q^{*}(x) = \left[\phi A_{IN} / \alpha(1-t)\right] L_{F}^{*}(x) + A_{IN} L_{IN}^{*}(x), \tag{2.81}$$

it is easy to see that it decreases with the distance to the CBD.

Proposition 1

Given that the infrastructure for public services decreases with distance to the CBD, the land allocated to produce short informal (tall formal) buildings increases (decreases) with distance to the CBD. This implies that the average tallness of buildings decrease with distance to the CBD.

Rearranging (2.77) it is possible to see that in equilibrium the relative price of land is equal to: $R^{*}(x)/R_{H}^{*}(x) = \phi A_{IN}$. As a consequence, it does not change with distance. From the point of view of the traditional Muth-Mills model, this implies that there is no reason for a systematic variation of the buildings height over the urban space.[29] However, in the model presented here, as we have just shown, a clear spatial pattern emerges, and is mainly explained by how land allocation (between sectors) changes with distance in response to the variations in infrastructure. This mechanism, which is new in the literature, can be suitable for explaining spatial regularities in cities where there is a large informal housing sector. Additional comments are in order. First, using (2.72) it is possible to show that:

$$\alpha A_{F} \bar{z}^{*}(x)^{1-\alpha} / A_{IN} = \phi/(1-t), \tag{2.82}$$

---

[29] In the Muth-Mills model lower land rents are required at greater distances to compensate producers for the lower price per square foot of housing. The resulting decline with distance in the relative price of land causes producer substitution in its favor, leading to lower structural densities.

which indicates that in equilibrium the marginal rate of transformation (left side of the equation) is equal to the relative (for the producers) price of the informal production, in units of formal production (right side of the equation). Because the relative price does not change with distance, the infrastructure per unit of land $\bar{z}(x)$ has to stay constant all over the city to guarantee the equilibrium. Thus, the land and the production in the formal sector decrease with distance at the same rate as the infrastructure. This explains why the buildings height in this sector does not change. Second, also using (2.82), it is possible to see that the relative price does not depend on utility levels. Therefore, land allocation and housing production do not depend on the equilibrium utility levels, which greatly simplify the analysis. Third, it is worth noting that the model contains other potential sources of the spatial variation of informality. For example, a similar analysis to the previous one can be done if spatial variations in the probability of detection are considered, instead of variations in infrastructure.

3.4.2. Closing the model

In equilibrium at the city fringe, the land rent must be equal to the agricultural rent (which is assumed equal to zero):

$$R^*\left(x_f^*\right)=0, \tag{2.83}$$

which holds only if the housing rent is equal to zero. This is precisely what the equilibrium condition (2.66) ensures. Note that (2.77) along with (2.70) imply that:

$$R^*(x)=\begin{cases} \phi A_{IN}\left[\left(1-\dfrac{s}{d}\right)\tau x_b^* + \dfrac{s}{d}\tau x_f^* - \tau x\right] & \text{for} \quad 0 \le x \le x_b^* \\[3mm] \phi A_{IN}\left[\dfrac{s}{d}\tau\left(x_f^* - x\right)\right] & \text{for} \quad x_b^* \le x \le x_f^* \\[3mm] 0 & \text{for} \quad x \ge x_f^*, \end{cases} \tag{2.84}$$

which describes the land rent along the city. The total housing production at x is obtained by adding the production in the formal sector and the production in the informal sector:

$$Q^*(x) = A_{IN} + A_F \theta^{(\alpha/1-\alpha)}\left[1-(1-t)\alpha/\phi\right]\bar{Z}(x). \qquad (2.85)$$

Total production of housing decreases with distance to the CBD. This happens because land allocated to the less productive sector increases with distance. Now it is possible to calculate the population density. In the A residents zone, $Q^*(x)$ is the density at each x, whereas in the B residents zone, the density is $Q^*(x)/d$. In canonical models, population density always decreases with distance. However, under this specification, just at the frontier between A residents zone and B residents zone, x_b^*, the density, jumps and may increase. For instance, when d is small near the frontier x_b^*, the B residents zone is more dense than the A residents zone.

To sum up, in equilibrium, we have these: the housing rent and land rent decrease with distance, as shown in canonical models; the city displays socio-economic segregation, where lower-income workers are poorly connected with the CBD and are relegated to dense peripheral areas; the land allocated to produce short informal (tall formal) buildings increases (decreases) with distance to the CBD; and public infrastructure, housing production and the average tallness of buildings decrease with distance. These characteristics fit well for cities in Latin America, which are characterized by dense peripheries where low income workers and precarious short buildings are concentrated, and where there is a poor provision of public infrastructure (UN-HABITAT, 2010; UN-HABITAT, 2013; Da Mata 2013). To close the model and find x_b^* and x_f^*, the following population constraints are used:

$$\int_0^{x_b^*} Q^*(x)\,dx = M_A, \qquad (2.86)$$

$$\int_{x_b^*}^{x_f^*} Q^*(x)\,dx = dM_B, \qquad (2.87)$$

where $Q^*(x)$ is defined by (2.85). It can be shown that (2.86) is equivalent to:

$$A_{IN}x_b^* + \Omega\left(1-e^{-x_b^*}\right) - M_A = 0, \qquad (2.88)$$

being $\Omega \equiv A_F\theta^{(\alpha/1-\alpha)}\left[1-(1-t)\alpha/\phi\right]\bar{Z}$. Let Λ_A be the expression on the left hand side of (2.88). It is always the case that:

$$\frac{\partial \Lambda_A}{\partial x_b} > 0, \quad \lim_{x_b \to +\infty} \Lambda_A = \infty, \quad \lim_{x_b \to 0} \Lambda_A < 0.$$

Thus, there must be a unique x_b^* that satisfies (2.88) and therefore (2.86). Using (2.86) and (2.87) it can be shown that (2.87) becomes:

$$\int_0^{x_f^*} Q^*(x)\,dx = M_A + dM_B. \qquad (2.89)$$

Proceeding in a similar fashion, it can be shown that there must be a unique x_f^* which satisfies (2.89). In this way it is shown that the equilibrium exists, and is unique.

3.5. Comparative statics and intercity analysis

In this section we investigate the effects of changes in the city's parameters over the informal housing and city structure. We focus on the provision of the public infrastructure, taxes, and the probability of detection, variables that directly depend on local governments. As in Brueckner (1987), we can use the conclusions to make intercity predictions, i.e., we can compare separate cities with parameter levels corresponding to the pre- and post-change values.

3.5.1. Effects of changes in the provision of the public infrastructure

At each distance $x \in \left(0, x_f^*\right]$, a fall in the provision of infrastructure $\bar{Z}$, reduces the value of marginal productivity of land in the formal sector, affecting the land demand given by (2.59), but leaving unaffected the land demand in the informal sector (2.62). This is described in Figure 20 as a downward shift of the $FR(x)$ curve $\left(FR(x) \text{ to } FR(x)''\right)$. Initially, this leaves the land rent $R^*(x)$ unaffected, decreases the land allocation in the formal sector $L_F^*(x)$ and rises its counterpart in the informal sector $L_{IN}^*(x)$. Given that land has been now allocated to the less productive sector, total housing production $Q^*(x)$ and the average height of buildings $q^*(x)$, decrease. By using (2.86) and (2.89), it can be seen that the frontier x_b^* and the city fringe x_f^* have to rise, which shows that as the housing production is lower at each x, the city has to spread to accommodate all the households. These changes lead utility levels u_A^* and u_B^* to fall, and both the housing rent $R_H^*(x)$ and the land rent $R^*(x)$, to increase. This is reflected in an upward shift of the $FR(x)$ curve $\left(FR(x)'' \text{ to } FR(x)'\right)$ and an upward shift of the $IN(x)$ curve $\left(IN(x) \text{ to } IN(x)'\right)$. There are no new changes in the rest of the equilibrium variables, as these shifts have the same proportion.

Proposition 2

A decrease in the level of public infrastructure increases informal housing all over the city, making the city less tall and more spread, i.e. less compact. This in turn generates higher housing rents, higher land rents and smaller utility levels.

An important implication of proposition 2 is that land consumption under a rapid proliferation of informal constructions is higher than land consumption under the proliferation of formal developments (recall that informal constructions are apartment's buildings). Then, proposition 2 established a clear connection between the proliferation of informal housing and the fast spatial expansion of the city. Precisely Inostroza, et al., (2013) have showed for 10 Latin American cities between 1990 and 2010, a pervasive spatial expansion, where most of the studied cities are expanding at fast rates. This occurs in a

context of continuous occupation of large extensions of scarce land, by invasions, self-constructions, and pirate developments (Smolka and Biderman, 2011; UN-HABITAT, 2013).

Proposition 2 also illustrates how the increase in the informality becomes costly for the society. Because the growth of informal constructions implies fast land consumption, the city fringe quickly gets farther from the CBD. This in turn implies a higher urban costs[30] and lower consumption for all city inhabitants. At the end a higher level of informality implies a lower level of the utility in the equilibrium. The model also explains the fact that the informal sector fixes a lower limit for land prices, raising the land price for the formal sector (Smolka, 2003). It is worth noticing that a rise in the level of taxes t affects the land demand conditions in the same way that a fall in the infrastructure. Thus, it generates exactly the same effects described in proposition 2.

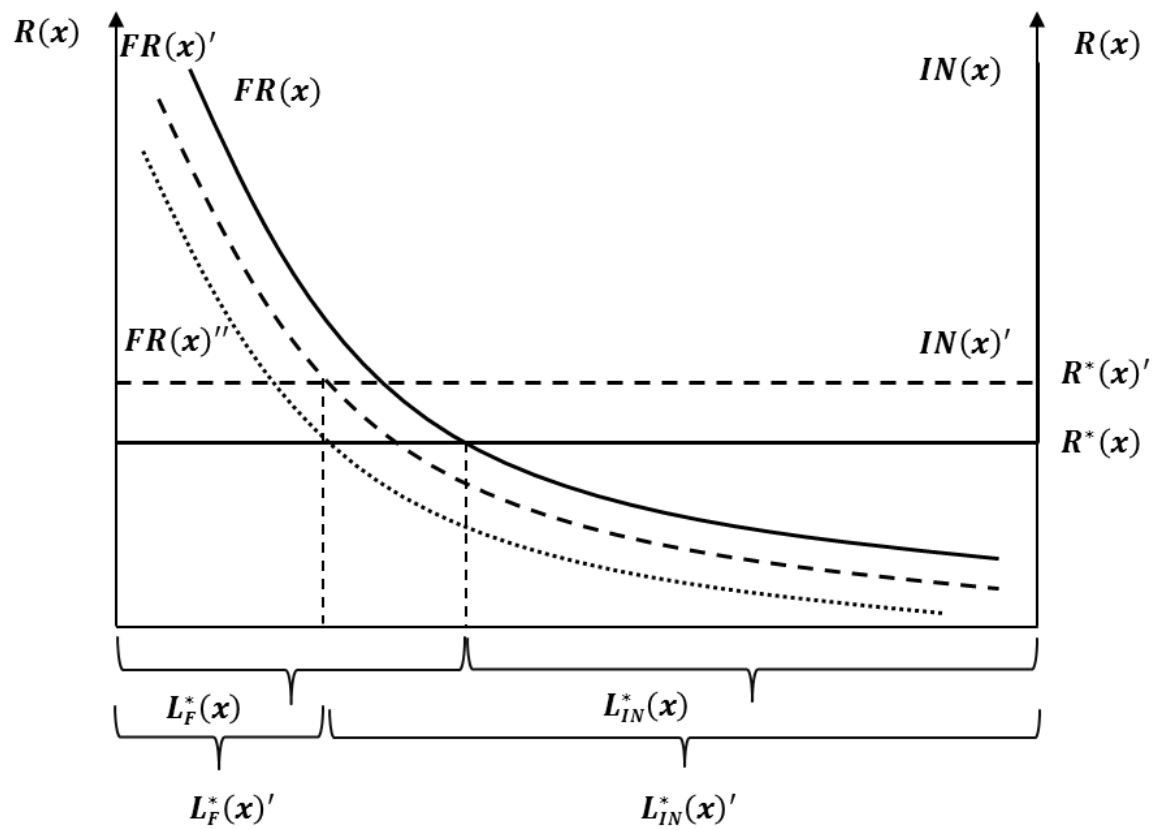[30] The urban costs are the commuting costs plus the housing rents

Figure 20. Effects of changes in the public infrastructure or the taxes
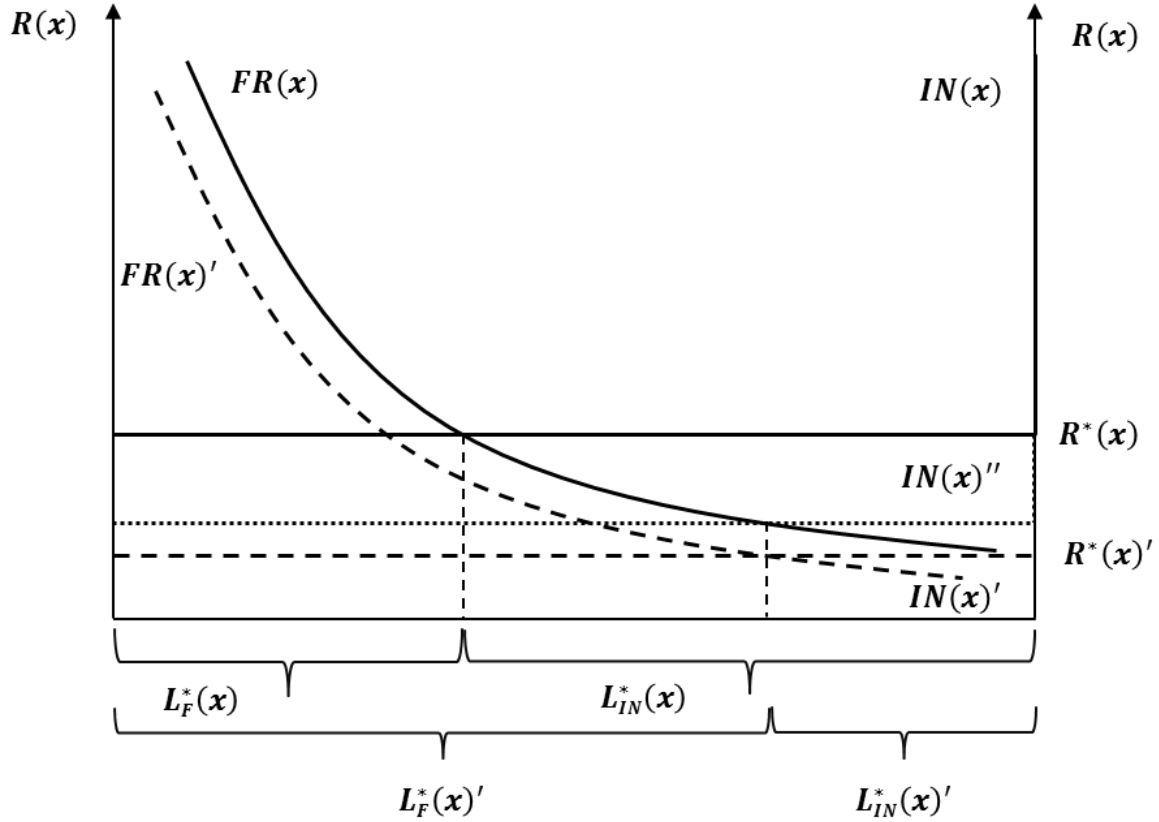
Figure 21. Effects of changes in the probability of detection

3.5.2. Effects of changes in the probability of detection of informal housing

At each distance $x \in (0, x_f^*]$, a rise in the probability of detection $1-\phi$, causes a fall in the value of the expected marginal productivity of land in the informal sector. Then, the $IN(x)$ curve shifts downwards in Figure 21 $\left(IN(x) \text{ to } IN(x)''\right)$. It is easy to see that this lowers the land rent $R^*(x)$ and the land allocated to the informal sector $L_{IN}^*(x)$. As a consequence, both the total housing production $Q^*(x)$ and the average height of buildings $q^*(x)$ increase. Then, the frontier x_b^* and the city fringe x_f^* fall, and the equilibrium utility levels u_A^* and u_B^* increase. All these changes cause the housing rent $R_H^*(x)$ to fall, which in turn leads the land

rent $R^*(x)$ to fall again. This is reflected in a downward shift of the $FR(x)$ curve $\left(FR(x) \text{ to } FR(x)' \right)$ and in a second downward shift of the $IN(x)$ curve $\left(IN(x)'' \text{ to } IN(x)' \right)$. Once again, because these shifts have the same proportion, there are no new changes in the rest of the equilibrium variables.

Proposition 3

An improvement in the probability of detection lowers informal housing all over the city, making the city more compact. This implies lower housing rents, lower land rents and higher utility levels.

3.6. The role of the local government

The local government, at each x, maximizes formal housing production, using spending in infrastructure for public services $\overline{Z}(x)$, and spending in law enforcement $E(x)$. The rise in the infrastructure for public services increases the productivity of land in the formal sector, whereas an increase in law enforcement increases the probability of detection (which is now endogenous) of the informal land developer, according to:

$$\phi(x) = E(x)^{-\beta}. \tag{2.90}$$

At each x, the government is subject to a budget constraint that establishes that the taxes paid by formal land developers plus the fines charged to informal land developers must be higher than or equal to the total spending. The tax rate and the level of fines are exogenous. Recall that we assumed that the punishment to the informal developers was the confiscation of all its income. This confiscation can be easily decomposed in tax payments and a fine: $t\rho$ where $\rho = (1/t) > 1$. We will now incorporate this formulation in order to facilitate the calibration necessary to solve the model later.

The economy works sequentially. First, the government chooses the level of spending in law enforcement and the level of spending in infrastructure for public services. Then, given these variables and the taxes, the land market equilibrium is established at each x. This resembles

the sequence of the Stackelberg model of oligopoly. Then, in order to fully characterize the equilibrium at each x; we proceed using backward induction. Let $\sigma_1 = \left[(1-t)\alpha A_F / A_{IN}\right]^{\alpha/1-\alpha}$, $\sigma_2 = \left[(1-t)\alpha A_F / A_{IN}\right]^{1/1-\sigma}$, and $\varepsilon(x) = \left[1 - t\rho + E(x)^{-\beta} t\rho\right]$. Thus, formally, the government's problem is:

$$\max_{\{E(x),Z(x)\}} Q_F^*(x) = A_F \sigma_1 \varepsilon(x)^{\alpha/1-\alpha} \bar{Z}(x), \text{ at each } x \in (0, x_f], \quad (2.91)$$

subject to:

$$tA_F \sigma_1 \varepsilon(x)^{\alpha/\alpha-1} \bar{Z}(x) R_H(x) + \left[1 - E(x)^{-\beta}\right] t\rho \left[A_{IN} - A_{IN}\sigma_2\varepsilon(x)^{1/\alpha-1}\bar{Z}(x)\right] R_H(x) -$$
$$\bar{Z}(x) P_z - E(x) P_E = 0$$
,

at each $x \in (0, x_f]$,

$$(2.92)$$

where P_z and P_E are the prices of the infrastructure and the enforcement, respectively. It is assumed that these prices are space invariant. The first order conditions of the problem are given by:

$$A_F \sigma_1 \varepsilon(x)^{\alpha/\alpha-1}\left[1 + t\lambda(x)R_H(x)\right] - \lambda(x)\left[1 - E(x)^{-\beta}\right]t\rho A_{IN}\sigma_2\varepsilon(x)^{1/\alpha-1} R_H(x) - \lambda(x)P_z = 0$$

$$(2.93)$$

$$\frac{\beta\alpha}{1-\alpha}A_F\sigma_1\varepsilon(x)^{1/\alpha-1}\bar{Z}(x)\left[1 + t\lambda(x)R_H(x)\right]E(x)^{-\beta-1}t\rho +$$
$$\lambda(x)t\rho A_{IN}R_H(x)\beta E(x)^{-\beta-1}\left\{1 - \sigma_2\bar{Z}(x)\frac{\left[1 - E(x)^{-\beta}\right]}{1-\alpha}\varepsilon(x)^{2-\alpha/\alpha-1}t\rho - \sigma_2\bar{Z}(x)\varepsilon(x)^{1/\alpha-1}\right\} - \lambda(x)P_E = 0$$

,

$$(2.94)$$

Using (2.92), (2.93) and (2.94), it is possible to find $\bar{Z}(x)$, $E(x)$ and $\lambda(x)$. Given the system's complexity, it is necessary to use numerical methods in order to obtain solutions and make comparative statics analysis. We can now establish the spatial patterns of infrastructure, enforcement, informal (formal) production and total production. To proceed, we calibrate the model and then we show how the endogenous variables change in response to variations in the housing rent, which is the exogenous variable that changes with distance (for the local government). The total factor productivity in the formal sector is fixed at 1, whereas the land productivity in the informal sector is fixed at 0.5. The tax rate is fixed at 0.3, and the prices for the infrastructure and the enforcement get 3 and 1 values respectively. Finally, the parameter for the detection technology is 0.7, whereas the parameter of the formal production function is fixed in 0.5. The chosen parameter values lie within the range of those used in Posada (2015), where the calibration is made to reproduce some aggregated stylized facts present in Latin American cities. Specifically the model reproduces an formality rate of 35% in the housing market. This is precisely the situation for Colombia where historically only 35% of houses have been formally generated (Minvivienda, Gobierno de Colombia, 2010). Figure 22 shows the results of the analysis.
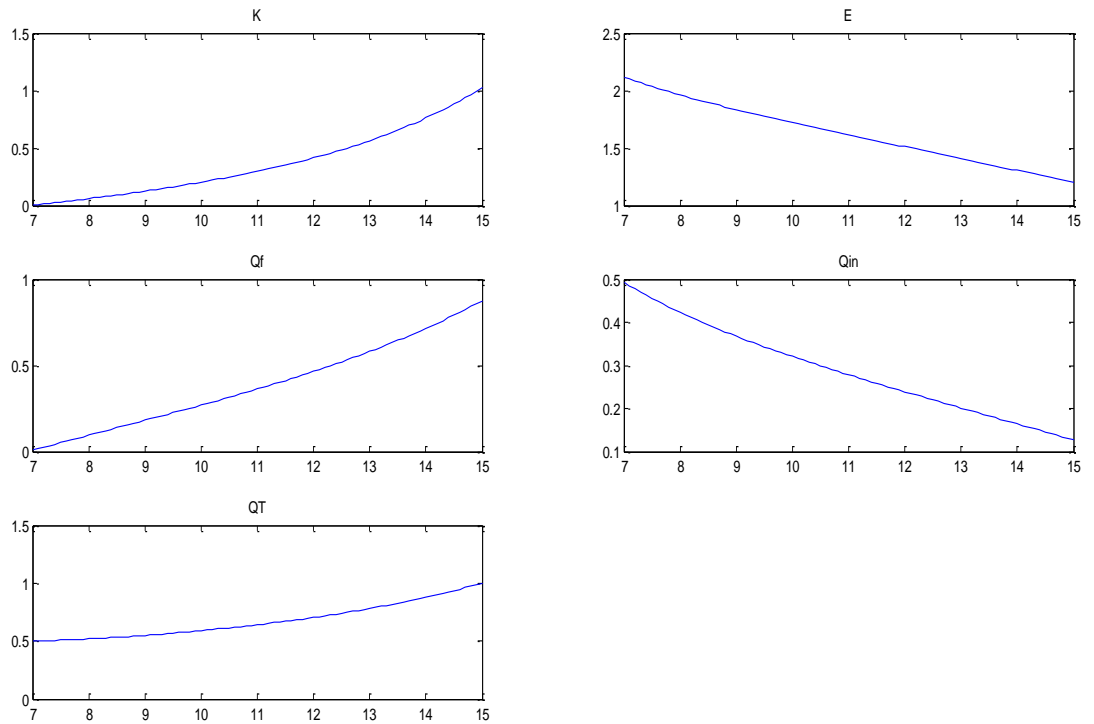
As can be seen, infrastructure, formal housing production, and total housing production all increase with the housing rent. Given that housing rents decrease with distance from the CBD, it follows immediately that these variables decrease with distance. This, in turn, implies that informal housing increases with distance. Regarding enforcement, it is easy to see that increases with distance. These results can be explained as follows: when the housing rent decreases, so it does the tax revenue, then, the government, trying to compensate this fall, increases enforcement. However, there is a net fall in government's income, which generates a decrease in the optimal level of infrastructure. As shown before, this raises the informality and reduces housing production. It is clear then, that the spatial pattern of the infrastructure is not necessarily the result of the lack of responsiveness of the government to the fast urban growth. Instead, it is the result of an optimal decision of the government, taken under resource restrictions. Then, the government is an active (endogenous) player in the determination of the level of informality.

3.7. Informal housing, city growth and rural urban migration

The seminal paper of Harris and Todaro (1970) was the first one to model rural urban migration in a context of urban unemployment. This analysis has been very useful to understand the case of developing countries where rural urban migration is a major source of city growth. The model starts with the assumption that *expected* urban earnings are higher than rural earnings. Because of this, migration emerges and only stops when the expected earnings are equal. In the model, the unemployment rate is the equilibrating mechanism because when the urban population increases, the urban unemployment rises, which in turn pushes the expected urban earnings downwards.

The work of Harris-Todaro improved significantly the understanding of the role of the unemployment in the path of migration and city growth, but says nothing about the role of the land market. Brueckner (1990) takes a first step in this direction and provides a rural urban migration model with an alternative equilibrating force: the escalation of urban costs. In short, when the city grows, the fringe gets farther from the CBD. Then, the worker who is farther away from the CBD has to incur in higher commuting costs. As a consequence, the

housing rent and the equilibrium urban cost rise for all workers. Although the model of Brueckner (1990) takes into account the role of the land market, it does not consider its peculiarities in developing countries. In this section, we take a first step in this direction and study the relationship between informal housing, city growth (close city) and rural urban migration (open city).

3.7.1. Effects of changes in population and housing consumption

A rise in the number of type A households M_A, a rise in the number of type B households M_B, or an increase in the housing consumption of type B households d causes a rise in total housing demand. Starting from the existing city fringe, the overall housing production has to increase in order to accommodate the new housing requirements in the city. Because the endowment of the public good decreases with distance, the rise in housing production is largely informal. Then, the city rapidly spreads to a new city fringe farther to the CBD. This can be seen formally by using (2.89). Finally, the utility levels u_A^* and u_B^* fall and the housing rent $R_H^*(x)$ and the land rent $R^*(x)$ increase.

Proposition 4
Population growth generates a fast proliferation of informal housing.

It has been accepted that one of the main reasons behind the configuration of slum settlements in cities in Latin America is the sustained city growth (in part because the voluminous rural-urban migration) combined with poverty and land use regulations (Da Mata, 2013). The result in proposition 4 describes a new mechanism through which city growth stimulates the proliferation of informality in the city. When the population grows, there is a new demand for housing services which is attended by the housing industry at the city fringe. Because at the city fringe there is a poor dotation of infrastructure, the new constructions are predominately informal. Therefore, in this mechanism, the slow growth of the infrastructure in the urban space plays a crucial role. The result resumed in proposition 4 is precisely a common observation in cities in the developing world. For instance, World Bank, (1993), and UN-HABITAT, (2013) have been argued that slow infrastructure growth, combined with fast population growth results in the proliferation of informal settlements.

Also Williamson, (1991), shows that: "The rapid growth in population is not matched by growth in delivery of land for housing, services, utilities and infrastructure important to sustain a reasonable quality of life. This is evident from the sprawl of informal settlements, increase in congestion, air and water pollution, poor and deteriorating infrastructure, and dilapidated housing".

3.7.2. Rural urban migration: informal housing in an open city

Now, we consider that the composite non-housing good may be produced in two regions: urban and rural. The total population is denoted with $\bar{N}$ and the urban population with N. Then, the rural population is given by $\bar{N} - N$. The urban region takes the form of a city like the one described in previous sections of the document. In the rural region, workers do not commute because they live close the fields that they cultivate. As a consequence, the urban cost (the commuting cost plus the housing rent) is higher than the rural cost (rural workers pay the rural rent for the land they consume). For simplicity, it is assumed that now in the city there is only one household type (the A type), and that the infrastructure endowment does not change with distance to the CBD. Because cities tend to host modern productive sectors, the urban productivity y is higher than the rural productivity y_r. As before, the indirect utility for a worker is given by the difference between the wage and the cost of living. Initially, the urban utility is higher than the rural utility; therefore incentives for rural urban migration exist. The rural urban migration guarantees that in equilibrium the utility for a rural worker is equal to the utility for an urban worker.

Because informal housing directly affects the urban cost, and our main purpose is to study the implication of informal housing on rural urban - migration and city growth, we focus on the urban cost as an equilibrating mechanism. The equilibrium housing production at each x and the city fringe are now given by:

$$Q^* = A_{IN} + \bar{Z}[(1-t)\alpha A_F^{1/\alpha} / \phi A_{IN}]^{(\alpha/1-\alpha)}[1-(1-t)\alpha / \phi]. \tag{2.95}$$

$$x_f^* = \frac{N}{Q^*}. \tag{2.96}$$

As a consequence, the migration equilibrium condition can be written as:

$$y - \tau \frac{N}{Q^*} = y_r \,, \qquad (2.97)$$

where $\tau\left(N/Q^*\right)$ is the equilibrium urban cost. This condition guarantees that the rural utility is equal that to the urban utility. Reorganizing (3) we can obtain the equilibrium urban population:

$$N = \frac{\left(y - y_r\right)Q^*}{\tau}. \qquad (2.98)$$

Now it is possible to study the effects of changes in the city's parameters on the level of urban population. When the infrastructure decreases (or the taxes increases, or the probability of detection decreases), informal housing production increases. Then, at each distance x, buildings grow shorter. As a consequence, the city fringe gets farther from the CBD, the urban cost rises, and the urban utility falls. This stimulates a decrease in urban population, which in turn reduces the urban cost to the point where the city and the rural area are equally attractive again. At the end, the city fringe and the urban utility do not change, but the urban population falls. *The previous analysis illustrates in a simple and intuitive way how the informal housing becomes an obstacle for more people to enjoy the advantages to live in a city.*

Finally, it is easy to see that the urban population and the city fringe (city size) increase with the productivity in the urban sector, and decrease with the productivity in the rural sector, and the commuting cost per unit of distance.

3.8.   Conclusions

Cities in Latin America are often characterized by fast spatial expansion, low buildings height and by peripheries where low- income workers and precarious short buildings are concentrated (UN-HABITAT, 2010; Zenou, 2011; Da Mata, 2013, Inostroza, et al., 2013). In explaining of these patterns, previous literature has stressed the role of rural-urban migration, under high unemployment, as the main force behind the emergence of the periphery and the large city size (Harris and Todaro, 1970; Zenou, 2011), or the role of the migration, under agglomeration economies, in turning cities into megacities (Krugman, 1991). However, less attention has been paid to the role of informal housing.

This paper develops a monocentric city model with a formal and an informal sector in the housing industry.   Here, informal land developers, evading regulations, using inefficient techniques, and using land that could have been used by the more efficient formal sector, find it profitable to produce a perfect substitute for formal production. In contrast with the Muth-Mills model, the model developed in the present work states that the relative price of land does not necessarily change with distance, implying that the traditional reason for a systematic variation of the buildings height over the urban space does not exist. However, a clear spatial pattern emerges. Given that the infrastructure for public services decreases with distance to the CBD, the land allocated to produce short informal (tall formal) buildings increases (decreases) with the distance to the CBD, implying that the average tallness of buildings decrease with the distance to the CBD. Then, the city periphery shows high levels of housing informality. This mechanism, which is new in the literature, can be suitable for explaining spatial regularities in developing world cities.

The model shows that a city with a low provision of infrastructure (or high taxes or low law enforcement) displays a high level of informal housing. This creates a city with a large size and a low buildings height. In turn, this implies high land rents, high housing rents and a low utility level in equilibrium for the city inhabitants. This result establishes a clear connection between the proliferation of informal housing and the fast spatial expansion of cities, two of the most important features exhibited by a representative city in Latin-America. It also illustrates how the increases in informality become costly for the society through its negative impact in the equilibrium utility levels for all the households. Finally, the model shows that

the spatial pattern of the infrastructure is not necessarily the result of an (exogenous) lack of responsiveness of the State to the fast urban growth. Instead, it is an (endogenous) optimal response of the local government to the way how tax revenues and fines revenues change with distance to the city center.

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