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Price discrimination under non-uniform calling circles and call externalities*

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Abstract

This work develops a competition model between two asymmetrical networks with calling circles, allowing subscribers to derive utility by receiving calls. Unlike the traditional literature predictions, in equilibrium firms have strategies to set off-net price below on-net price. In markets where consumers display strongly concentrated calling patterns, firms can only extract limited surplus from off-net calls. This is reinforced if consumers display weak call externalities, languishing the price strategies to discourage off-net calls. Furthermore, regulating price differential of the large firm can lead consumers to face higher fees compared to discriminatory setting. Therefore, regulators should broaden efforts to measure call externalities and calling circles strength before making decisions on retail tariff regulation.

Keywords: Calling circles; Call externalities; Network competition; Price differentials.
JEL Classification: D43, D62, L14.

1 Introduction

The operators in mobile telecommunications markets have pursued discriminatory practices in which they set prices for calls inside their own network (on-net) below the prices for calls outside their network (off-net) as a strategy to stimulate on-net calls consumption. High off-net tariffs discourage calls outside their network creating a network externality. Yepes et al. (2012) show that in Colombia average off-net prices were 50% higher than on-net. In Peru, price differential could be as high as 842% [Loaiza and Barriga (2014)], while for Chile prices for calls outside the network could be as high as 232% the price for calls inside the network [Loaiza and Barriga (2014) and TDLC (2012)].

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In some countries price discrimination is not present anymore, due to regulation or market evolution, but in others remains. In Congo mobile market, for example, off-net prices can be as high as 100% on-net tariffs.[1] Hence, price discrimination in mobile services is a strategy still in place.[2]

Some regulators devote efforts to study this behavior and make decisions regarding the on/off-net gap. The argument to support this type of settlement lies on the deepening of the network externality effect due to the existence of the differential [Harbord and Pagnozzi (2010)]. The regulatory body for the telecommunications mobile market in Colombia, Comisión de Regulación de Comunicaciones -CRC-, mandated the largest operator (Claro) to set off-net prices equal to or lower than on-net prices, since January 2013.[3] The Chilean antitrust authority -TDLC- made a similar decision on 2012 mandating the mobile operators to offer plans without price differential, since January 2014.[4] Conversely, operators in countries like Peru [Loaiza and Barriga (2014)] and Congo are not constrained to set retail prices.

The decisions that set limits on the price gap brought mixed consequences to the markets. In Colombia quarterly average prepaid on-net traffic of Claro decreased from 10 billion minutes in 2012 to 7.6 billion minutes in 2013, while off-net traffic increased from 298 million minutes to 403 million minutes in the same time frame. Looking at prices for prepaid service, by the second quarter of 2013 the average price for prepaid calls of Claro -both on-net and off-net- rised 28% compared to the last quarter of 2012.[5] In Chile, a market with an akin oligopoly structure by the time when the ban was issued, the effects of a similar regulation on retail pricing show that on-net prices displayed increases as high as 198% and a reduction of 50% for off-net prices [Loaiza and Barriga (2014)].

The on-net price increase together with off/net price decrease effects after the retail regulation, pose the question about the underlying mechanism leading to these outcomes. In particular, how the strategic decisions of a network regulated at the retail level depend on the interaction of calling circles and call externalites. To answer these inquiries, I develop a theoretical model of network competition and discuss price discrimination for the case of unregulated firms. Then, I extend my model to introduce the retail price regulation. The outcomes of my model convey a message against retail price regulation in mobile telecommunications markets.[6]

This paper also talks about features of markets where the traditional price differential rule, i.e. off-net price higher than on-net, is not met. Hence, my approach unveils the relevant features of markets where different than usual on/off-net pricing discrimination behavior is observed. My setting is a model of network competition

[1] See for example, `https://www.mtn.cg/particuliers/plans-tarifaires/free/`, `https://www.mtn.cg/particuliers/plans-tarifaires/mtn-boss/` and `https://www.airtel.cg/appel_locaux`. Last accessed on may 2021.

[2] Short message services are also subject to price discrimination. These services are widely used in Africa for surveys Lau et al. (2019), election monitoring, health information campaigns, disaster relief operations, and mobile banking Jack and Suri (2014), Aker et al. (2017).

[3] Resolution 4050, issued by the CRC, became effective on Jan. 01, 2013.

[4] See https://www.subtel.gob.cl/gobierno-establece-una-reduccion-de-73-para-las-tarifas-de-cargos-de-acceso-moviles/.

[5] There is no disaggregated data for on-net and off-net revenues, only prepaid and postpaid revenues. Traffic and revenue information is available at https://postdata.gov.co/.

[6] This policy implication applies in markets such as Congo, where price differentials are in place.

with consumers located on a Salop circle and two networks competing for consumers, as well as strategically setting retail prices according to the calling party pays -CPP- principle.[7] Market shares are different, which is the only source of asymmetry between networks. For the case of access charges, I assume these are set by a regulator at the symmetric cost level.[8]

Consumers decide consumption of outgoing calls, and derive utility from calls made to destinations either inside their network or in the rival network. Calls are made to destinations in a non-uniform pattern such that more calls are made to certain destinations than to others -calling circles-. In addition to calling circles, I allow consumers to derive utility by receiving calls so that consumer overall utility will be shaped by outgoing as well as incoming calls. The equilibrium prices I obtain are such that the corresponding on-net Lerner indices are dampened the higher the call externality, while off-net Lerner indices increase when call externality is considered.

My model extends previous results in the literature. I show that, if consumers call heavily to destinations in their circle, the large firm sets a higher on-net price compared to the on-net price set by the small network. However, the larger network can still set a higher on-net price than its rival if the market is sufficiently concentrated. This result constitutes theoretical support to regulators' concerns about the inconvenience of a highly concentrated market. For off-net prices, the large firm sets a higher price than the small network if calling patterns are not strongly concentrated. This condition depends only on call externality and price elasticity of demand.

Regarding on/off-net price differential, my model explains both traditional and non-traditional outcomes. I find a series of conditions on structural parameters such that traditional lower on-net than off-net prices and non-traditional on-net higher than off-net prices can be explained within the same theoretical framework. Hoernig et al. (2014) find a condition for which on-net prices will be lower than off-net prices without distinction of price differentials for each network. Their result holds for a profit maximizing access charge. In my setting I find conditions when access charge is set at the cost level. Therefore, my contribution fills this gap considering the effect that the interaction of calling circles and call externalities have on price differentials on a per firm basis when access charges are regulated at cost.

The equilibrium off-net price in a market where calling circles are present is lower compared to a market where consumers call evenly to any destination. If consumers make a significant amount of calls to destinations in their calling circle, the demand for calls outside the circle is weak. Since off-net destinations are located outside the circle, the previous rationale implies that demand for off-net calls is also weak.

The low off-net pricing behavior just described is reinforced if consumers display a low call externality. When consumers value incoming calls much less than outgoing calls, the amount of surplus derived by consumers for incoming calls is very small. Hence, outgoing off-net calls yield a small amount of utility for consumers in the rival network making the firm less willing to discourage off-net calls through a higher price. In summary, either

[7] This approach means that only the caller pays for the call.

[8] By the same argument as in López and Rey (2016), asymmetrical access charges are fading away as a regulatory tool to enhance competition in the market.

strong calling circles or low call externalities draw off-net equilibrium prices closer to zero.

Traditional models predict that, after differential regulation, on-net prices increase and off-net prices decrease. The main contribution of this paper states that the retail regulated price can be higher than both on-net and off-net unregulated price outcomes. In markets with consumers that heavily place calls to destinations in their circles and do not highly benefit from incoming calls, the regulated firm sets a higher uniform price compared to both on-net and off-net prices. This means that retail price regulation should be avoided in markets that do not display these features, since consumers will face higher prices for calls to any destination after the price differential ban.

My model is extended to account for retail price differential ban of the large network. The regulated price is always higher than on-net discriminating price. In markets with consumers who make evenly distributed calls among destinations and derive high utility from incoming calls, the regulated price is lower than off-net discriminating price. If calls are strongly concentrated in the circles and consumers value incoming calls much less than outgoing calls, regulated price shifts to lower values, even below off-net discriminating price. Lastly, the larger the market share of the large network the larger the space for which the regulated price is lower than off-net discriminating price.

The theoretical literature about network competition has shifted from models that consider consumers calling uniformly to every destination, towards settings in which consumers make calls to certain destinations more heavily than to others, also known as calling circles. Armstrong (1998), Laffont et al. (1998a and 1998b) constitute the groundwork in network competition. In these models, two firms compete in prices for consumers located along a Hotelling line. The networks set either a linear or a two-part tariff while consumers make calls uniformly to any other destination in the market. Further contributions, Gabrielsen and Vagstad (2008) and Calzada and Valletti (2008), account for calling circles by considering each consumer makes a proportion of calls to certain destinations while the remaining calls are uniformly distributed among any destination in the market. My work is closer to the framework of Hoernig et al. (2014), who model calling circles as a probability distribution centered around each consumer. However, they do not incorporate in their setting the call externality to establish how pricing strategies of the networks are shaped either in the discrimination case nor in the retail regulated scenario.

Implications on competition when consumers derive utility for receiving calls were first discussed in the literature related to competition under receiver party pays -RPP- principle since the relevance of call externality was clearer in this context.[9,10] For the case of CPP, Berger (2005) takes call externality into consideration to analyse the problem of access pricing, while Hoernig (2007) analyses retail price behavior of firms when consumers derive utility from incoming calls. In my model, I take call externality into account and depart from the access charge price problem by considering the compensation for the usage of the rival network is set at

[9] Under RPP, the callee party pays a fee when answering the call.

[10] For further references about RPP see DeGraba (2003), Hermalin and Katz (2001, 2004), Jeon et al. (2004) and Kim and Lim (2001).

cost.

Retail price regulation has been subject of analysis aimed to establish implications on welfare. From a theoretical perspective, Hoernig (2008) proposes a model of network competition considering calling circles, that predicts a strategic reaction of the regulated firm increasing on-net prices and reducing off-net prices. Hoernig's model takes into consideration call externalities but not calling circles effects. Unlike Hoernig, my model is able to predict simultanous price increase (on-net and off-net) when the regulator bans the retail price differential. Empirical analysis by Rojas (2015) finds how consumer surplus and profits change according to different retail price regulation rules in Chile. He finds that consumers might be harmed while firms are better-off with uniform prices. Similarly, Agostini et al. (2017) carry out the analysis for the Chilean market aiming to identify if retail differentials are a vehicle of predatory behavior. These empirical contributions set their framework on theoretical models considering call externalities but disregarding calling circles.

The rest of this paper is organized as follows. Section 2 contains the detailed description of the model. Discriminatory price outcomes are discussed in section 3. Section 4 contains price differential results. I discuss retail price regulation in section 5. Finally, section 6 concludes.

2 Model set-up

This work develops a competiton model between two networks whose sizes are exogenously given by the amount of consumers subscribed to each. Figure 1 depicts concepts related to market shares, on-net call, off-net call and corresponding prices.

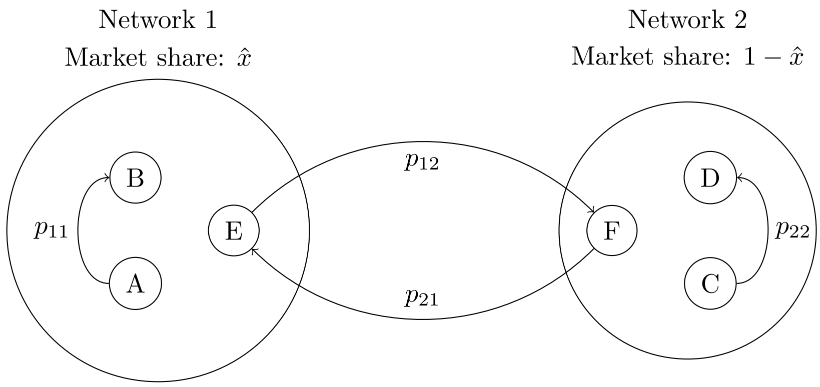


Figure 1: Basic elements for the model.

The networks ($i = 1, 2$) behave like profit maximizing firms by competing in the market and facing marginal costs for originating (c_O) and terminating calls (c_T) which are the same for both firms, i.e., firms are considered equally efficient when providing on-net and off-net calls. Furthermore, each firm holds a portion of consumers that constitutes its market share. These market shares are given as follows: $\hat{x} > 1/2$ corresponds to the market share of network 1 and network 2 holds a market share $1-\hat{x}$. $\hat{x}$ represents the location of the consumer that is indifferent between offers of network 1 and network 2. This location is considered exogenous in the model (see Figure 2).

Consumers can either make on-net or off-net calls. On-net calls are those calls terminated in the same network they were initiated (i.e., calls originated in network i whose destination is network i) which are charged by the originating firm at a price p_{ii}. Off-net calls are those calls initiated on any network and terminated on the rival network (i.e., calls originated in network i whose destination is network j), which are charged at a price p_{ij} by the originating network.

Specifically, I am interested in exploring on-net/off-net pricing strategy firms follow when call externality (consumer derived utility by receiving calls) is considered together with non-uniform call patterns.

2.1 Consumers

Consumers in my model locate uniformly on a salop cirle in the interval $[-1,1]$ while firms are located in $x=0$ and $x=1$, as displayed in Figure 2:

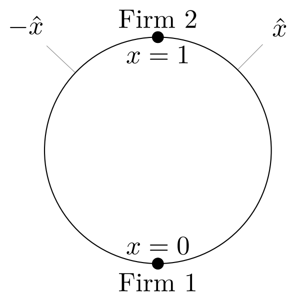


Figure 2: Consumers' location.

Regarding consumers location and subscription to networks, consumers can belong to one of two firms present in the market: consumers belonging to network 1 are located in the interval $[-\hat{x},\hat{x}]$ of the circle, while consumers belonging to network 2 are located in the interval $[-1,-\hat{x}]\cup[\hat{x},1]$. One of the key features of the model is the consumer behavior when making calls: each consumer calls with a higher probability to consumers located close to her, representing a higher probability of calls to consumers with similar preferences. This behavior is known as calling circles, where the circle means a set of destinations to which the consumer calls more frequently. [11]

Calling circles are modeled using a function $G(y|x)$, where $G(\cdot)$ is a cumulative distribution function -CDF-,

[11] According to Hoernig et al. (2014), this behavior can be explained by the affinity consumers feel with a specific brand thanks to market effort developed by the firms, or just the coverage level offered in a given area. Authors like Birke and Swann (2010), even point the possibility that consumers take into consideration subscribing the network where their relatives or acquaintances belong to.

indicating the likelihood a consumer located at x calls every other subscriber located below point y (consumers located at $y' < y$). Consumers y' can belong either to network 1 or network 2, which means that calling pattern for each subscriber is a CDF that averages the probability of making calls uniformly to every subscriber in the market (uniform CDF described by $U(\cdot)$) and the probability of making calls to subscribers in the circle (described by $H(\cdot)$), as follows:

$$G(y|x) = (1 - \lambda)U(y) + \lambda H(y - x), \tag{1}$$

where $0 \leq \lambda \leq 1$ is the weight of the calling circles in the calling pattern of the consumer, and $H(z)$ is a uniform CDF with density $h(\cdot)$ with the functional form:

$$h(z) = \frac{1}{\varepsilon}, 0 \leq z \leq \varepsilon. \tag{2}$$

This function describes the calling circle centered around each consumer where ε is the calling circle size for each consumer. The calling circle size in my model is considered to be small enough compared to the market size of each network, as stated in Assumption 1.

Assumption 1. *Calling circle size is small enough compared to the market shares of the firms:* $\varepsilon < 2\hat{x}(1 - \hat{x})$.

Given on-net and off-net prices, consumers demand calls with duration $q_{ii} = q(p_{ii})$ and $q_{ij} = q(p_{ij})$ deriving an indirect utility given by:

$$\upsilon(p) = \max_q \{u(q) - pq\},$$

where,

$$u(q) = \frac{q^{1-\frac{1}{\eta}}}{1 - \frac{1}{\eta}},$$

and parameter η stands for the price elasticity of demand for both on-net and off-net calls.

Given this framework, I allow for the possibility for the consumers to derive utility not only by making calls but also by receiving calls: utility function for every consumer must include the corresponding terms allowing this novel conjoint approach; therefore, the utility a consumer located at x derives follows the next functional form:

$$U_1 = G(\hat{x}|x)\left[\upsilon(p_{11}) + \gamma u(q_{11})\right] + \left[1 - G(\hat{x}|x)\right]\left[\upsilon(p_{12}) + \gamma u(q_{21})\right] + u_0 - F_1 - \tau x, \tag{3}$$

for the case of a consumer belonging to network 1, and

$$U_2 = \left[1 - G(\hat{x}|x)\right]\left[\upsilon(p_{22}) + \gamma u(q_{22})\right] + G(\hat{x}|x)\left[\upsilon(p_{21}) + \gamma u(q_{12})\right] + u_0 - F_2 - \tau(1 - x), \tag{4}$$

for the case of a consumer in network 2. These expressions account for the intrinsic utility of subscribing to the network (u_0) which I consider high enough to have a fully covered market, utility for making calls ($\upsilon(p)$) and utility for receiving calls ($\gamma u(q)$)[12]. Note that the indirect utility is used for the case of calls made by the agent

[12] The difference $u_0 - \tau x$ can be thought of as the utility a consumer derives to be able to use basic services such as emergency calls, before making calls to other consumers in the market. See Jeon et al. (2004)

since it reflects the reaction to any decision of the firms on prices, while direct utility is used when accounting for utility of received calls since the agent does not make any decision on the duration of received calls.[13] F_1 and F_2 correspond to the fixed fee of the two-part tariff set by the firms, and τ represents the disutility the consumer bears due to the distance between his location and the one of the network he is subscribed to.

2.2 Firms

There are two firms in the market which are located at points $x = 0$ and $x = 1$ on the Salop circle and behave as profit maximizing agents competing *à la Bertrand* by making decisions on fixed fees and prices offered to consumers,

$$T_i = F_i + q_{ii}p_{ii} + q_{ij}p_{ij},$$

meaning that for on-net calls inside network i firm charges a price p_{ii}, while for off-net calls originated at network i and terminated at network j firm i charges a price p_{ij} to consumers, besides the fixed fee F_i.

Since my model considers consumers located along the Salop circle, competitive dynamics in the interval $[0, 1]$ are mirrored in the interval $[-1, 0]$, in consequence the analysis in the interval $[0, 1]$ explains the behavior of the whole market. In this work the large network will also be regarded as network 1, big network or large firm. Likewise, the small network will be regarded as network 2 or small firm.

Total benefits of firm 1 from any consumer x are

$$\Pi_1(x, \hat{x}) = \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f,$$

where π_1 are the payoffs generated by consumer x to the firm 1:

$$\pi_1(x, \hat{x}) = G(\hat{x}|x)(p_{11} - c_{11})q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c_{12})q(p_{12}),$$

F_1 is the fixed fee of the two-part tariff offered to consumers subscribed to network 1, R_{12} are the perceived benefits of the firm due to access charges received for terminating calls originated in network 2:[14]

$$R_{12}(x, \hat{x}) = [1 - G(\hat{x}|x)]\,(a - c_T)q_{p21},$$

and f is the fixed cost of serving a subscriber. Recalling that analysis for $x \in [0, 1]$ explains the competitive dynamics in the whole market and integrating for all consumers, total benefits for network 1 are:

$$\bar{\Pi}_1(\hat{x}) = 2\int_0^{\hat{x}} \Pi_1(x, \hat{x})\,dx. \tag{5}$$

A similar definition applies for network 2 benefits, considering its market share are the consumers located in the interval $\hat{x} \leq x \leq 1$ and $-1 \leq x \leq -\hat{x}$ on the Salop circle:

$$\bar{\Pi}_2(\hat{x}) = 2\int_{\hat{x}}^1 [\pi_2(x, \hat{x}) + F_2 + R_{21}(x, \hat{x}) - f_2]\,dx \tag{6}$$

[13] Call externalities are considered in the literature as the ratio of incoming call utility to outgoing call, see Jeon et al. (2004) and Sobolewski and Czajkowski (2018).

[14] Access charges correspond to money paid by network 2 to network 1 by using the latter's infrastructure to terminate calls originated in the former

Networks maximize profits given by equations (5) and (6), a process that requires the definition of some terms that will be useful later on. Along section 2.1, calling circles were defined by means of a CDF -denoted $G(\hat{x}|x)$- that considers the likelihood for each consumer to call destinations in her circle as well as the likelihood to make calls towards destinations outside her circle.

Following Hoernig et al. (2014), the amount of on-net calls generated in network 1 can be computed by summing up for every consumer x belonging to this network:

$$L_{11}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)\frac{1}{2}dx.$$

Given that I am interested in the case when there is a big network, the aggregate number of on-net calls in network 1 when $\hat{x} > 1/2$ is given by:

$$L_{11}(\hat{x}) = \int_{0}^{2-2\hat{x}} H(z)dz + 2\hat{x} - 1.$$

In a similar fashion, the aggregate number of off-net calls originated in network 1 when $\hat{x} > 1/2$ is given by the expression that follows:

$$L_{12}(\hat{x}) = \hat{x} - L_{11}(\hat{x}).$$

For the case of network 2, a similar approach can be followed to define both aggregate number of on-net calls (L_{22}) and aggregate number of off-net calls originated by consumers of this network (L_{21}).

Lastly, the case of the calling pattern for the marginal subscriber $G(\hat{x}|\hat{x})$ is considered. If every inframarginal consumer had the same calling pattern of the marginal consumer, the on-net amount of calls of network 1 are given by the next expression:

$$\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x}).$$

Consequently, off-net calls originating in network 1 if every inframarginal consumer holds the same calling pattern as marginal consumer would be given by

$$\hat{L}_{12}(\hat{x}) = \hat{x}\left[1 - G(\hat{x}|\hat{x})\right].$$

Similar definitions apply for $\hat{L}_{22}(\hat{x})$ and $\hat{L}_{21}(\hat{x})$.

2.3 Timing and Equilibrium Concept

Regarding the timing of the game, at $t = 1$ firms set fixed fees given marginal consumer $\hat{x}$; in $t = 2$ firms decide prices, consumers make calls and payoffs are realized.

The game I develop consists of complete information, therefore the strategic decisions of the firms will lead to a Nash equilibrium in pure strategies. The equilibrium consists of a strategy profile that formulates the on-net price and the off-net price set by the network.

3 Market outcomes

3.1 Equilibrium on-net prices

Given the framework along section 2 for the game between firms and consumers, equilibrium on-net prices set by the firms can be obtained after maximizing the profits for each network (equations (5) and (6)), as stated in Proposition 1.

Proposition 1. *Equilibrium on-net Lerner indices are given by the next expressions:*

$$\frac{p_{11} - c_{11}}{p_{11}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{11}}{L_{11}}\right] \underbrace{-\gamma\frac{\hat{L}_{11}}{L_{11}}}_{C.E.\ effect}, \tag{7}$$

and

$$\frac{p_{22} - c_{22}}{p_{22}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{22}}{L_{22}}\right] \underbrace{-\gamma\frac{\hat{L}_{22}}{L_{22}}}_{C.E.\ effect}. \tag{8}$$

Proof. See section A in the appendix. □

From the Lerner indices given in (7) and (8), some interesting features about the optimal on-net prices can be discussed. Regarding price elasticity of demand (η) it can be seen in the last term of each expression that this is not shaping the behavior of the firms due to the presence of call externalities, resembling the results obtained by Hoernig (2007). This reflects how call externalities affect consumer surplus through utility derived due to received calls length but not as a result of consumer decisions on indirect utility given prices in the market. In other words, the term $1/\eta$ explains the reaction of the firm in response to consumers' behavior when deciding the length of calls to consume by virtue of price set by the network. Meanwhile, the call externality in the second term of the expressions explains how the firm takes into account utility derived by consumer x as a result of other consumers calling him/her, a utility that is not a result of consumers' own decision.

Absent call externalities, my results resemble the Lerner indices obtained by Hoernig et al. (2014); furthermore, for the case of uniform calling pattern, lerner indices of Hoernig (2007) are recoverd when accounting for two-part tariff equilibrium.[15] My result states that as long as call externality is greater than zero, consumers will derive benefits not only for making calls but also from receiving calls, and this will trigger the incentives of firms to encourage a higher volume of calls inside their networks since more calls means more consumer surplus, leading to more benefits that firms can extract. On-net calls consumption can be fostered by reducing the on-net call price, which is the mechanism at work explained by the negative term preceding the call externality in on-net Lerner indices.

The ratio $\hat{L}_{ii}/L_{ii}$ mandates how strong is the effect of call externality for consumers in network i. $\hat{L}_{ii}$ represents the utility of the marginal consumer for incoming calls when on-net price decreases, being this

[15]Uniform calling pattern means $\lambda = 0$, which leads to $\hat{L}_{ii} = L_{ii}$.

consumer the one that benefits the least among the consumers in the network. Thereby, a price decrease translates into an additional surplus for marginal consumer that can be extracted through fixed fee. For inframarginal consumers, a price decrease will set them better-off than marginal consumer, however, lower on-net price could lead to a reduction of profits despite the growth in on-net call volume, then firms will face a limitation to exploit call externality effects on consumer surplus. That is why L_{ii} appears in the denominator of the ratio being discussed. Taken together, absent calling circles, quotient equals to one and decreases in magnitude as calling circles become stronger. This means that the effect of the calling circle becomes weaker and firms set higher on/net prices. These insights are stated at Corollary 1.1 as follows.

Corollary 1.1. *Call externality effect on on-net Lerner index is weaker the larger the calling circle weight (λ).*

Proof. See section B in the appendix. □

3.2 Equilibrium off-net prices

Equilibrium off-net prices set by the firms can be obtained after maximizing the profits for each network (equations (5) and (6)), as stated in Proposition 2.

Proposition 2. *Equilibrium off-net lerner indices are given by the next expressions:*

$$\frac{p_{12} - c_{12}}{p_{12}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{12}}{L_{12}}\right] \underbrace{+\gamma\frac{\hat{L}_{11}}{L_{12}}}_{C.E.\ effect}, \tag{9}$$

and

$$\frac{p_{21} - c_{21}}{p_{21}} = \frac{1}{\eta}\left[1 - \frac{\hat{L}_{21}}{L_{21}}\right] \underbrace{+\gamma\frac{\hat{L}_{22}}{L_{21}}}_{C.E.\ effect}, \tag{10}$$

Proof. See section C in the appendix. □

In a similar fashion as for on-net Lerner indices, price elasticity of demand is not shaping the effects of call externality on the behavior of the firm when setting off-net prices. This conduct is implied by a mechanism akin to the one described for on-net prices. Consumers derive utility for making off-net calls such that duration of these calls is decided by consumers according to prices set by the firms, which is reflected by the expression including $1/\eta$; meanwhile, utility due to outgoing calls that consumers of network i generate to reach consumers in network j is explained by the term containing the call externality.

From the Lerner indices given in (9) and (10), absent call externalities, I obtain the same results as Proposition 1 in Hoernig et al. (2014); furthermore, for the case of uniform calling pattern[16], results of Hoernig (2007) are recovered when accounting for two-part tariff. Off-net Lerner indices are shaped by call externality in the opposite direction to on-net ones, stemming from firms' reaction to the utility that outgoing calls generate for

[16] Uniform calling pattern means $\lambda = 0$, which leads to $\hat{L}_{ij} = L_{ji}$, $\hat{L}_{11}/L_{12} = \hat{x}/(1-\hat{x})$ and $\hat{L}_{22}/L_{21} = (1-\hat{x})/\hat{x}$.

consumers in the rival network. If call externality is greater than zero, consumers in network i derive utility by making off-net calls, but will also allow subscribers receiving this type of calls in the rival network to derive utility; this effect is internalized by firm i which reacts by refraining consumers to make off-net calls through and increase in p_{ij}. This mechanism becomes evident by the positive sign of the factor containing the call externality in the off-net Lerner indices.

The magnitude of $\hat{L}_{ii}/L_{ij}$ ratio can be understood as follows: $\hat{L}_{ii}$ represents the benefit derived by marginal subscriber of network j due to received calls generated by subscribers in network i. This subscriber is the one that derives the highest benefits from receiving calls generated by consumers in network i such that p_{ij} is the instrument at hand for the firms to extract this surplus. Hence, raising off-net price will shift marginal consumer utility downwards and this will be reflected as a smaller disutility for network i that becomes a higher fixed fee. This is the reason why the term multiplying call externality in off-net lerner index is positive. However, the firm also internalizes the reduction of outgoing calls generated by their own subscribers if price is raised, which is accounted by the denominator L_{ij}.

For a calling circle weight increase, marginal consumer reduces off-net consumption less strongly than inframarginal subscribers which translates into a higher off-net calls aggregate reduction if inframarginal consumers have a different calling pattern than marginal consumer. Thus, as calling circle weight rises, the proportion of off-net calls generated by marginal consumer is higher than for inframarginal, such that inframarginal consumers drive aggregate reduction demand of off-net calls leading to a price decrease. The described reduction in off-net calls has two effects due to utility derived by consumers of the network and utility generated for consumers in the rival network: when calling circle weight is higher, more on-net calls are placed and demand for off-net calls decreases, then firms react by setting lower off-net prices to foster consumption (elasticity effect). Fostered consumption, however, will also raise benefits derived by rival network's consumers, an effect that is internalized by originating network by means of price increase because of the increased amount of outgoing calls (call externality effect). Then, there is a tradeoff between reducing off-net price due to the elasticity effect and increasing the off-net price because of call externality effect. These two forces act against each other and one will overcome the opposite depending on call externality magnitude and elasticity strength. For inelastic markets, consumers react mildly to price changes and elasticity effect dominates giving the firms more power to extract higher benefits from consumers by an overall price reduction. On the other hand, if call externality is very strong, utility provided to rival network is large enough so that firms react by increasing prices to deplet off-net consumption. This analysis is stated at Corollary 2.1, as follows.

Corollary 2.1. *The call externality effect on off-net Lerner index is stronger the larger the calling circle weight (λ).*

Proof. See section D in the appendix. □

4 Price differentials

In this section I study the relationship between on-net prices, as well as on-net/off-net price differential. Thus, sections 4.1, 4.2 and 4.3 discuss findings from the model.

4.1 On-net prices ratio (p_{11}/p_{22})

Equilibrium on-net prices reflect the decision adopted by each network to extract benefits from their subscriber base given the utility each consumer derives from calls made, as well as calls received. Depending upon consumer attributes, on-net prices will vary and the mechanism behind those decisions is of interest to further establish the strategies under retail regulation. The condition under which $p_{11} \geq p_{22}$ is stated in Proposition 3.

Proposition 3. *On-net price set by the large network is greater than or equal to the on-net price of the small network if the next inequality on calling circle weight holds:*

$$\lambda \geq \frac{\hat{x}(1-\hat{x}) - \frac{\varepsilon}{2}}{\hat{x}(1-\hat{x}) - \frac{\varepsilon}{4}} = \hat{\lambda}_{on}(\hat{x}, \varepsilon). \tag{11}$$

Proof. See section E in the appendix. □

Corollary 3.1. $\hat{\lambda}_{on}(\hat{x}, \varepsilon)$ *is always positive and less than one.*

Proof. Numerator and denominator of $\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon)$ are positive from Assumption 1, while numerator is less than the denominator, then this ratio is positive and less than 1. □

For the particular case of $\lambda = 1$ and using the result given in Corollary 3.1, the on-net prices relationship turns out to be: $\lambda = 1 \Rightarrow p_{11} > p_{22}$. This condition means that if consumers call to destinations in the calling circle only, then the largest firm finds incentives to charge a higher on-net price compared to on-net price set by network 2. In this scenario, consumers place no calls outside their calling circle leading to a greater amount of on-net calls and less off-net calls, then competition is weakened and more benefits can be extracted from consumers by means of on-net pricing strategy.

In addition, it can be seen that the interval $[\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon), 1]$ expands when either $\hat{x}$ or ε increases, therefore the larger the market share of network 1 allows for a wider range of vales of λ for which the on-net price of network 1 will be higher than on-net price of network 2. A similar behavior for $\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon)$ holds when calling circle size increases. This result supports the claim of regulators about the concern that a highly concentrated market can have on the price outcomes of the large firm. Larger firm 1 market share or larger calling circle size translates into a higher probability of making on-net calls for subscribers in network 1, which is a market force -stronger demand- that raises incentives for this firm to set higher on-net prices. Observe the relationship between on-net prices does not depend upon price elasticity of demand nor the magnitude of call externality, only market shares and calling circle size are involved. This is explained because both firms are internalizing the utility consumers derive for making and receiving calls inside their corresponding networks.

The parameter λ represents consumer preferences, so the analysis outlined means that if the consumers in the market display strong preferences towards making calls to destinations located in their calling circle, the larger network on-net pricing strategy will exploit this behavior by increasing on-net price at a higher rate than small network will be allowed to. The other way around, if consumers in the market display more even patterns of calls (among on-net and off-net calls), then λ will be located far from 1 and on-net price of large network will be lower than on-net price of the small network. In this case, larger network size and calling circle should be big enough so as to allow on-net price of network 1 to be set above the corresponding price of network 2.

If the calling circle size approaches to zero in the limit and consumers only make calls to their circle ($\lambda = 1$), there will only be on-net calls and both firms will set the same on-net price. This resembles monopoly level prices, as only on-net calls will flow in the networks. In this case, there will be two separate markets, one for each network, and the firms behave as monopolies on each market. Departing from $\lambda = 1$ on-net price of bigger network will be less than on-net price of small network, a behavior explained by the fact that the larger network loses a greater amount of on-net calls than its rival, which translates into a stronger on-net price reduction than on-net price reduction of network 2. These insights are summarized in Corollary 3.2.

Corollary 3.2. *On-net prices approach the monopoly level if calling circle size approaches zero and consumers make calls to destinations in their circle only.*

4.2 Off-net prices ratio (p_{12}/p_{21})

Now, I analyse the off-net prices differential of the firms present in the market, i.e., the differential between p_{12} and p_{21} by comparing the relative magnitudes of these prices (p_{12}/p_{21}). The condition under which $p_{12} \geq p_{21}$ holds is stated in Proposition 4 as follows:

Proposition 4. *Off-net price set by the large network is greater than or equal to the off-net price set by the small network if the next inequality on calling circle weight holds:*

$$\lambda \leq \frac{2}{1 + \frac{1}{\gamma\eta}} = \hat{\lambda}_{off}(\gamma, \eta). \tag{12}$$

Proof. See section F in the appendix. □

Inequality (12) states that p_{12} will be greater than or equal to p_{21} as long as $\lambda \leq \hat{\lambda}_{\mathrm{off}}(\gamma, \eta)$; in the limit, when call externality approaches zero, p_{12} is less than p_{21}. There are two forces at work here: from one side, the consumers derive surplus by making calls and this surplus is higher when calling circle weight is weak, which means originated off-net calls are an important source of benefits to be extracted by the network. From the other, if the call externality is high, rival consumers derive higher benefits due to received off-net calls, therefore the large network internalizes this and increases off-net price to shrink the surplus rival consumers derive and, in consequence, the amount of surplus the rival network can extract. The off-net price increase mechanism for the large network will weaken if either the calling circle weight is high enough, such that originated off-net calls are not a major source of benefits or if call externality is low enough such that benefits derived by rival consumers do not constitute an incentive to harm the rival.

From network 2 perspective, the off-net price increase incentive is similar to the one faced by network 1 although it differs in magnitude. The utility for consumers in the rival network is internalized by each firm through the fixed fee of the tariff, so the amount of off-net calls the marginal consumer of network i recevies can also be seen as the amount of on-net calls that would take place in network j if every consumer had the same calling pattern as the marginal consumer. That is why the quotients of the call externality effect in equations (9) and (10) account for the terms $\hat{L}_{ii}$. Hence an increase in γ leads to a faster increase in p_{12} compared to the increase in p_{21} because $\gamma\hat{L}_{11}/L_{12}$ is greater than $\gamma\hat{L}_{22}/L_{21}$.

Hoernig (2007) describes a similar mechanism in terms of the Lerner indices for on-net and off-net prices: when call externality is taken into consideration, the incentives to raise off-net prices come into play due to additional utility derived by the consumers of the rival network therefore enhancing its competitive position. To the best of my knowledge, the existing literature has only highlighted the role of call externality (see Hoernig (2007) and Hoernig (2008)), but not the calling circles counterforce lessening the strength of call externality effects. This dampening response is evident from the behavior of $\hat{\lambda}_{\text{off}}(\gamma, \eta)$, which states that calling circle weight can surpass the relevance of call externality explaining the differential between off-net prices. Thus, inequality (12) is stating that call externality encourages a competitive behavior in the market that becomes apparent by off-net price raising, however this incentive is dampened by the existence of calling circles, which means that consumer calling preferences is the mechanism through which firms moderate the described conduct.

Regarding the consumers' elasticity, the effect is the same as for the case of call externality: higher price elasticity of demand leads to a higher threshold $\hat{\lambda}_{\text{off}}(\gamma, \eta)$ which means that a wider range of calling circle weight values (i.e. interval $[0, \hat{\lambda}_{\text{off}}(\gamma, \eta)]$) is allowed for the network 1 to set a higher off-net price than network 2. In this case, a high price elasticity of demand means that consumers are highly sensitive to price changes, hence a small increase in off-net price of either firm will turn into a reduced consumption for this type of calls. However, if the calling circle weight value is low (consumer preferences are stronger to make off-net calls), networks can take advantage of this and extract higher benefits from its subscribers by means of a higher off-net call price, refraining also its consumers from improving the welfare position of consumers in the rival network.

Higher price elasticity of demand implies that consumers will, more than proportionally, increase off-net consumption when faced against off-net price decrease but this consumption can also rise when calling circle weight is high, meaning that if consumer preferences for off-net calls are strong then networks react by increasing these prices in order to improve their revenue position. Since price reduction could lead to more than proportional consumption increase, which in turn will lead to increased benefits of received calls for consumers in the rival network, when call externality is high the described mechanism effect is magnified and the incentives to reduce off-net prices are weakened. This magnification is accounted by the term $\gamma\eta$ present in inequality (12). For the case of big network, the amount of additional calls that could be delivered to the small network would give rise to more benefits for rival consumers due to call externality.

4.3 On/off-net prices differential (p_{ii}/p_{ij})

Unlike thresholds $\hat{\lambda}_{\text{on}}(\hat{x}, \varepsilon)$ and $\hat{\lambda}_{\text{off}}(\gamma, \eta)$, the differentials for prices set by each network depend on every structural parameter of the model, namely, call externality, price elasticity of demand, market shares and calling circle size. For the case of call externality and price elasticity of demand, dependence is algebraically the same as for $\hat{\lambda}_{\text{off}}(\gamma, \eta)$ but this effect is now shaped by market shares and calling circle size. The condition for the relationship $p_{ii} \geq p_{ij}$ to hold is given in Proposition 5.

Proposition 5. *On-net price set by network i will be greater than or equal to its own off-net price if the next inequalities on calling circle weight hold.*

For network 1:

$$\lambda \geq \frac{\hat{x}^2}{\hat{x}\left(\hat{x} - \frac{1}{2}\right) + \frac{1}{2\gamma\eta}\left(\hat{x} - \frac{\varepsilon}{2}\right)} = \hat{\lambda}^1_{dif}(\hat{x}, \gamma, \eta, \varepsilon). \tag{13}$$

For network 2:

$$\lambda \geq \frac{(1-\hat{x})^2}{(1-\hat{x})\left(\frac{1}{2} - \hat{x}\right) + \frac{1}{2\gamma\eta}\left(1 - \hat{x} - \frac{\varepsilon}{2}\right)} = \hat{\lambda}^2_{dif}(\hat{x}, \gamma, \eta, \varepsilon). \tag{14}$$

Proof. See section G in the appendix. □

Proposition 5 states that when call externality approaches zero $p_{ii} \geq p_{ij}$, resembling the case in which networks behave according to Hoernig et al. (2014). Subject to these conditions, networks do not face incentives to extract benefits from consumers due to incoming on-net calls and do not face incentives to discourage calls whose destination are consumers in the rival network. Each firm is taking advantage of rent extraction due to generated calls: calls inside its network because of calling circles strength -p_{ii} grows- and promoted calls to destinations in the rival network by decreasing p_{ij}.

As call externality strength increases, consumers derive utility from incoming calls but also trigger additional utility for callees in both the own and the rival network. This induces the firms to set strategies to exploit benefits of own consumers (p_{ii} drops) and lessen derived utility by consumers of the rival (p_{ij} rises). However, this force might be outweighed by calling circles if these are strong enough so that call volume displays the behavior that follows. Originated calls in network 1 consolidate inside the network (which correspondingly holds in network 2), leading outgoing off-net calls volume to decline and off-net prices to drop. Since calling pattern strenghtening yields an increase of on-net call volume, which can be understood as an on-net demand increase, on-net prices rise. Thus, if λ is high enough, the price outcomes for high call externality are reversed.

When large firm market share increases, interval $\left[\hat{\lambda}^1_{\text{dif}}, 1\right]$ shrinks while interval $\left[\hat{\lambda}^2_{\text{dif}}, 1\right]$ widens. As market share of the big network becomes even larger, the proportion of on-net calls is higher and the incentives to exploit call externalities (by means of a lower on-net price) from its consumers are more relevant than the incentives to discourage off-net calls (by means of higher off-net price) due to the same externality. In consequence, on-net price can be lower than off-net. These outcomes can only be offset if calling circle weight is high enough, i.e. $\lambda > \hat{\lambda}^1_{\text{dif}}$.

For the case of network 2 the effect is the opposite. Shrinking this network market share leads to a lower proportion of on-net calls and the incentives to exploit own consumers' call externality is weaker. Therefore, the network exploits calling circles more heavily (by means of a higher on-net price), and becomes more aware about call externality effects on consumers of the rival (increase off-net price). In this case it is not required to have very high values of calling circle weight to observe $p_{22} > p_{21}$, i.e. $\hat{\lambda}^2_{\text{dif}}$ decreases. A graphical interpretation is provided in Figure 3.

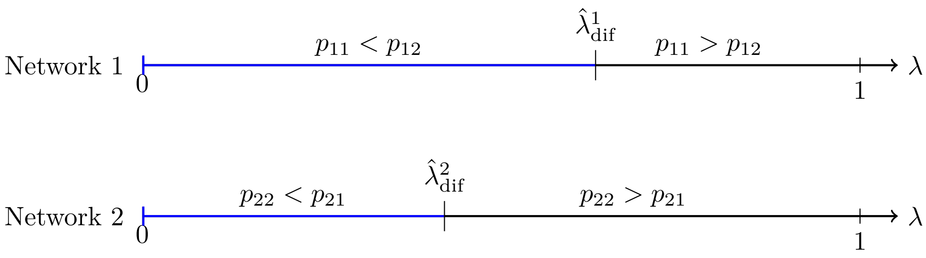


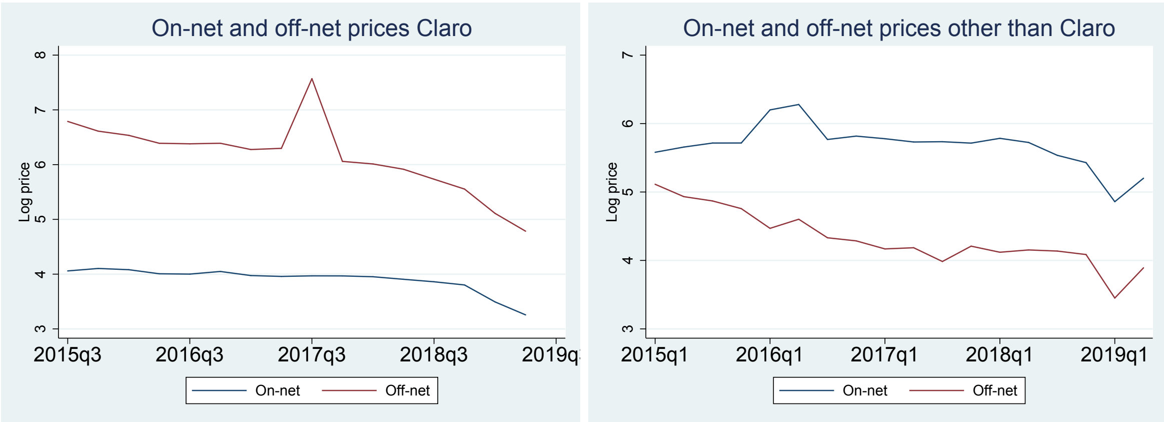
Figure 3: Threshold illustration for $\hat{\lambda}^i_{\text{dif}}$.

4.4 Price differentials discussion

To provide some insights on the purpose of my model, I recall mobile prepaid price figures for Colombian and Congolian markets.[17,18] The price evolution paths show that for the largest operator in Colombia on-net price is always set below off-net (Figure 4a) which is a classic prediction in the literature of network competition, while for the rest of the competitors (Figure 4b) on-net tariffs are set above off-net ones, being this behavior persistent over time. An additional finding from these data is the difference in on-net prices among operators (as well as off-net differential): on-net price set by the large network is always less than the on-net price of its competitors and off-net price set by Claro is always greater than off-net tariffs of its rivals. On/off-net classic differential predictions are mantained for Tigo and Telefonica, while for smaller firms the differential is reversed so that off-net prices are set above on-net prices (Figures 7 through 9 and 10 in the appendix). In average, off-net price differential is reversed for Comcel competitors (as shown in Figure 4) mainly due to small networks in the market.

Looking at the mobile prepaid market from The Republic of Congo, three salient features of the behavior of on-net and off-net prices in this country are highlighted. First, on-net prices are persistently lower than off-net, which is a classic prediction in the literature of network competition. Second, the on-net price set by MTN is

[17] Data for Colombia can be found at https://colombiatic.mintic.gov.co/.
[18] Data for The Republic of Congo can be found at http://www.arpce.cg/telecharger-observatoires.



(a) Claro.

(b) Other operators than Claro.

Figure 4: Prepaid prices in the Colombian mobile market.
Source: MINTIC web portal and author's calculations.

always lower than the corresponding price set by the small network (check left hand side panel of Figure 12 in the Appendix). Finally, off-net price of the big network is lower than that of the small network.

With these figures at hand, some conclusions can be drawn in light of my model. On/off-net differential in The Republic of Congo allows to say that calling circle weight for consumers in this market is low enough so as to make off-net prices of both large and small firm to be higher than on-net prices, a prediction from Proposition 5. For the case of Colombia, given the differential of average prices is reversed for smallest firms, casts evidence to conclude that consumers have a calling circle weight not as low compared to the consumers in The Republic of Congo. This means that a market where off-net prices are lower than on-net, becomes evidence of a less concentrated calling patterns than markets where the opposite holds.

Nonetheless, the call externality is playing a key role, since the presence of on-net prices below off-net for the large network but a reverse outcome for the small one implies the existence of the blue line regions in Figure 3. This means that externality effects are not negligible in the market under consideration and also, that market is highly concentrated, which is an equivalence to the Colombian case.

5 Regulating price differential

I model the regulation of the on/off-net price differential as in Hoernig (2008), by assuming that off-net price equals on-net price plus a difference Δ: $p_{ij} = p_{ii} + \Delta$. I am interested in the setting for differential regulation on the prices for network 1, while network 2 can still price discriminate on-net and off-net calls.

Proposition 6. *When price differential is regulated, retail uniform price set by network 1 will be as follows:*

$$p_{11} = \frac{c_{11} + \frac{L_{12}}{L_{11}}\frac{q'_{12}}{q'_{11}}\left\{c_{12} + \frac{\hat{L}_{12}}{L_{11}} - \Delta\left[1 - \frac{1}{\eta} - \frac{\hat{L}_{11}}{L_{12}}\gamma\right]\right\}}{1 - \frac{1}{\eta} + \frac{\hat{L}_{11}}{L_{11}}\frac{1+\gamma\eta}{\eta} + \frac{L_{12}}{L_{11}}\frac{q'_{12}}{q'_{11}}\left[1 - \frac{1}{\eta} - \frac{\hat{L}_{11}}{L_{12}}\gamma\right]}. \tag{15}$$

Proof. See section H in the appendix. □

Results from Proposition 6 represent an implicit solution for the uniform price that network 1 would set if price differential is regulated. I am particularly interested in the case when the regulator bans the price discrimination for the larger network, a decision that was made by the regulator in Colombia. In such a scenario, the uniform price set by the regulated network is stated in Corollary 6.1.

Corollary 6.1. *When price differential is regulated to zero* $(\Delta = 0)$*, uniform price set by the large network will be as follows:*

$$p_1^{\Delta} = \frac{\eta c_{11} L_{11}^2 + \eta c_{12} L_{11} L_{12} + \eta L_{12}\hat{L}_{12}}{L_{11}\left\{(\eta - 1)\left[L_{11} + L_{12}\right] + \hat{L}_{11}\right\}}. \tag{16}$$

Proof. See section I in the appendix. □

Price elasticity of demand shapes the behavior of uniform price in the expected way: highly elastic markets foster competition strength leading to overall lower prices, which can be easily checked in Equation 16. Regarding call externality my results show that, under a price discrimination ban, this parameter becomes irrelevant in the price setting strategy for the regulated firm resembling a previous result obtained by Hoernig (2008). This is so because the marginal utility of the off-net calls received by consumers in the small network -regarded as a cost- when the differential approaches zero, is the same as the marginal utility derived by consumers in the large network due to received calls originated by consumers belonging to this same firm.

Meanwhile, calling circles involve an interesting implication for the firm subject to price discrimination forbiddance. Previous literature such as Hoernig et al. (2014) has shown how the regulation under discussion could be beneficial for the market, but under some circumstances this could not be the case. In what follows I state a couple of results from my model that allow me to state how uniform price is set and how it compares to on/off-net discrimination prices.

Corollary 6.2. *When price differential is regulated to zero* $(\Delta = 0)$*, uniform price set by the large network is greater than on-net price set under price discrimination*

Proof. See section J in the appendix. □

Corollary 6.3. *In markets with equally efficient firms and access charges set at the cost level, equilibrium uniform price of large network is set above the off-net discrimination price when call externality is weak enough:*

$$\gamma < \gamma_{\Delta}(\hat{x}, \eta, \lambda, \varepsilon) = \frac{1}{\eta\hat{L}_{11}}\left[(\eta - 1)L_{12} + \hat{L}_{12} - \frac{c_{11}L_{11}L_{12}\left(\eta\hat{x} - \hat{L}_{12}\right)}{c_{11}\hat{x}L_{11} + L_{12}\hat{L}_{12}}\right] \tag{17}$$

Proof. See section K in the appendix. □

The equilibrium uniform price set by the bigger network is higher than the on-net discriminatory price. This result means that under regulation consumers in this network would pay a higher fee for calls whose destination belongs to the same network.

Conversely, the uniform price is set above the off-net discriminatory price if conditions on call externality and calling circle weight are met. Leader network will set a uniform price above off-net price in markets where consumers display a low valuation for incoming calls. For this type of markets, the firm is not highly concerned about utility derived by consumers of the rival network, which is a more evident position when the market also displays a consumer behavior towards making calls to calling circles in a high proportion.

If call externality is high, in the discriminating scenario utility derived by consumers in the rival network turns more relevant and the off-net pricing will reflect this by a higher price level, but in the regulated scenario the firm is not concerned about this, which means a uniform price can be set below discriminating off-net one. Under these conditions, calling circles turn out to be more relevant to consider: if consumers heavily place calls to destinations in their circle, uniform price will reflect the firm's strategy to exploit calling behavior of its own consumers.[19] This strategy is not longer in place if calling to destinations in the circle becomes a weaker behavior, so that the firm adopts a different strategy by encouraging its consumers to make more calls without concerning about derived utility of callees in the rival network. This is the case when uniform price is set below off-net discriminating one.

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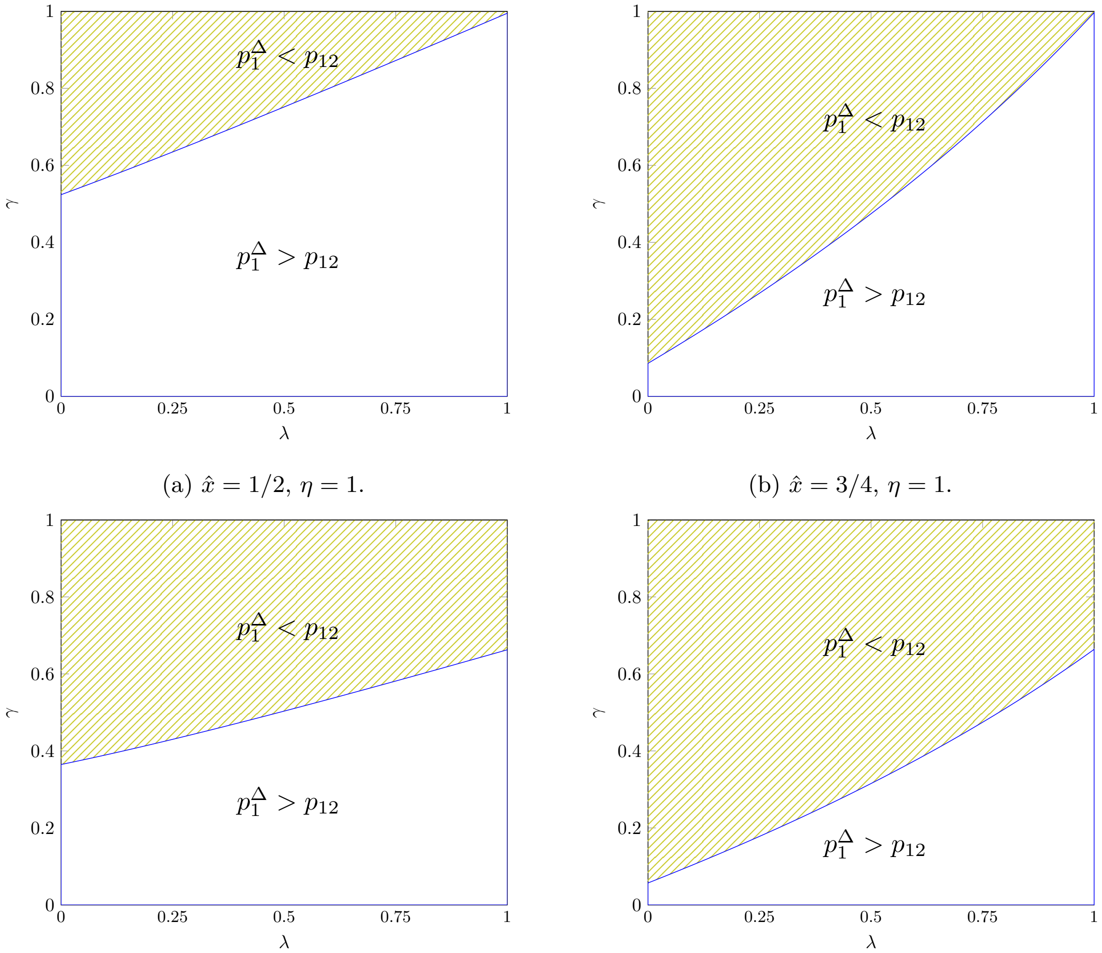
There is still a third force to consider. A concentrated market implies that consumers in the large network will more likely place calls to destinations in the same network, allowing the firm to extract rents by fostering these calls with lower prices. The more concentrated the market is, the lower will be the uniform price as long as calling circles are weak. In such a case, uniform price decrease in concentrated markets leads to a wider range of call externality such that this price is lower.

Therefore, in non-concentrated markets where consumers display a low valuation for incoming calls and calls are made towards destinations with a heavy weight to circles, regulation would lead consumers in the large network to face a higher price for making calls towards destinations located in the small network. This is indeed an undesirable outcome, given that consumers are also facing a higher on-net price in any case. Regulating price differentials without considering important features in the market such as calling circles and call externalities can lead to outcomes such as the ones described. This means that in order to lay such a regulatory rule, accurate measurements about call externalities and calling circles strength should be available at hand, prior to making decisions related to retail pricing regulation. This is at odds to policy recommendations raised to the CRC related to keep the price differential regulation active.[20] Such statements highlighted the benefits of a more even on-net off-net traffic distribution, and a presumed decrease of off-net prices, but do not take into account

[19]It can be proved that (16) approaches discriminating on-net pricing under these conditions and highly concentrated markets.
[20]See TMG (2016).

that the decrease of 2.4 billion on-net minutes per quarter was poorly compensated with barely 100 million off-net minutes quarterly.

Figure 5 summarizes different regulatory scenarios depending on the structure of the market and the fundamental behavior of consumers. As stated formerly, more concentrated markets and consumers with lower valuation for calls as well as lighter calling circles will lead to regions where regulated uniform price will be lower than off-net discriminatory price.



(a) $\hat{x} = 1/2$, $\eta = 1$.

(b) $\hat{x} = 3/4$, $\eta = 1$.

(c) $\hat{x} = 1/2$, $\eta = 1.5$.

(d) $\hat{x} = 3/4$, $\eta = 1.5$.

Figure 5: Regions for uniform price higher than discriminating off-net price level.

6 Conclusion

This paper analyzed how pricing behavior is shaped under a setting of network competition between two firms when both calling circles and call externality are considered. Traditional outcomes state that equilibrium off-net prices are set above on-net prices, however I propose a different approach to establish the reasons why that might be otherwise.

Call externality plays a role that counteracts calling circle effects for the on/off-net price decisions the firms must deal with. Existence of calling circles drives call demand towards on-net destinations in a more relevant way than just uniformly distributed demand, providing firms incentives to exert their power by increasing on-net prices as calling circle weight rises. On-net prices differential is driven by market shares and calling circle size. Call externality and price elasticity of demand do not play a role in this differential since on-net consumers derive their utility of calls originated and terminated in the same network they belong to.

The call externality effect is relevant for off-net price differentials. Comparing off-net prices set by the firms, I found that call externality is shaping the incentives of the firms to extract surplus derived by consumers from received calls and these incentives are strengthened as the call externality increases. Looking at on/off-net differentials, absent call externality, off-net prices would be set above on-net prices for any value of calling circle weight. However, high values of call externality could allow to obtain a reversed price differential.

These results are meaningful since they allow to explain how consumers' attributes -calling circles and call externality- can lead to non-traditional outcomes where the small firm in the market sets on-net price below off-net, while its rival acts the opposite. This type of outcome is expected to be observed in concentrated markets where consumers have a non zero valuation for receiving calls and the proportion of calls to circles is moderated. Besides, this model also allows outcomes in which both firms set on-net price above off-net, when consumers heavily place calls to destinations in their calling cicles.

Finally, the model is extended to analyze how retail price regulation for the large network shapes the pricing stretegic behavior of the firm. The regulated price is always higher than the discriminating on-net price, while it can be higher or lower than off-net price. When markets are more concentrated and consumers display a moderate to high valuation for incoming calls together with moderate probability of making calls to destinations in their circle, the uniform price will be lower than off-net price set under discrimination. This means that price regulation should be avoided in markets that do not display these features, since consumers will always face higher prices for calls to any destination.

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A Proof of Proposition 1

To find optimal p_{ii} both firms perform the following maximization program: $\max_{p_{ii}} \bar{\Pi}_i(\hat{x})$, which can be written as:

$$\max_{p_{ii}} \bar{\Pi}_i(\hat{x}) = \max_{p_{ii}} \int \Pi_i(x, \hat{x})\, dx = \frac{\partial}{\partial p_{ii}} \left\{ \int \Pi_i(x, \hat{x})\, dx \right\}.$$

A.1 Network 1 problem to decide on-net price

Solving the problem of firm 1 to set prices, network decides on-net price p_{11} performing the maximization program:

$$\begin{aligned}
\frac{\partial}{\partial p_{11}} \left\{ \bar{\Pi}_1(\hat{x}) \right\} &= \frac{\partial}{\partial p_{11}} \left\{ \int_{-\hat{x}}^{\hat{x}} \Pi_1(x, \hat{x})\, dx \right\} \\
&= \frac{\partial}{\partial p_{11}} \left\{ \int_{-\hat{x}}^{\hat{x}} \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f\, dx \right\} = 0 \\
&= \frac{\partial}{\partial p_{11}} \left\{ \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)(p_{11} - c_{11})q(p_{11}) + \left[1 - G(\hat{x}|x)\right](p_{12} - c_{12})q(p_{12}) \right. \\
&\left. + F_1 + \left[1 - G(\hat{x}|x)\right](a - c_T)q_{p21} - f_1\, dx \right\} = 0 \\
&= \frac{\partial}{\partial p_{11}} \left\{ \hat{x}(F_1 - f_1) + L_{11}(\hat{x})(p_{11} - c_{11})q(p_{11}) + \right. \\
&\left. L_{12}(\hat{x})(p_{12} - c_{12})q(p_{12}) + L_{21}(\hat{x})(a - c_0)q(p_{21}) \right\} = 0
\end{aligned} \tag{18}$$

where I have defined:

$$L_{11}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)\, dx \tag{19}$$

$$L_{12}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} \left[1 - G(\hat{x}|x)\right] dx \tag{20}$$

$$L_{21}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} \left[1 - G(\hat{x}|x)\right] dx. \tag{21}$$

Performing the derivative stated at equation (18), it is possible to establish the condition from which p_{11} can be found:

$$\frac{\partial \overline{\Pi}_1}{\partial p_{11}} = \hat{x}\frac{\partial F_1}{\partial p_{11}} + L_{11}(\hat{x})q(p_{11}) + L_{11}(\hat{x})(p_{11} - c_{11})q'(p_{11}) = 0 \tag{22}$$

$$\begin{aligned}
\frac{\partial F_1}{\partial p_{11}} &= \frac{\partial}{\partial p_{11}} \left\{ F_2 + u_1(\hat{x}, \hat{x}) - u_2(\hat{x}, \hat{x}) - \tau\hat{x} \right\} \\
&= \frac{\partial u_1(\hat{x}, \hat{x})}{\partial p_{11}} \\
&= G(\hat{x}|\hat{x}) \left[v'(p_{11}) + \gamma u'(q_{11})q'(p_{11}) \right].
\end{aligned}$$

Substituting $\partial u_1(\hat{x},\hat{x})/\partial p_{11}$ into (22):

$$\frac{\partial\overline{\Pi}_1}{\partial p_{11}} = \hat{x}G(\hat{x}|\hat{x})\left[-q(p_{11})+\gamma p_{11}q'(p_{11})\right] + L_{11}(\hat{x})q(p_{11}) + L_{11}(\hat{x})(p_{11}-c_{11})q'(p_{11}) = 0$$

and defining

$$\hat{L}_{11} = \hat{x}G(\hat{x}|\hat{x}) \tag{23}$$

first order condition follows:

$$q(p_{11})\left[-\hat{L}_{11}+L_{11}(\hat{x})\right] + q'(p_{11})\left[\hat{x}G(\hat{x}|\hat{x})\gamma p_{11} + L_{11}(\hat{x})(p_{11}-c_{11})\right] = 0$$

$$\begin{aligned}
q'(p_{11})\left[\hat{L}_{11}(\hat{x})\gamma p_{11} + L_{11}(\hat{x})(p_{11}-c_{11})\right] &= q(p_{11})\left[\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})\right]\\
q'(p_{11})\left[\hat{L}_{11}(\hat{x})\gamma + \frac{L_{11}(\hat{x})(p_{11}-c_{11})}{p_{11}}\right] &= \frac{q(p_{11})}{p_{11}}\left[\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})\right]\\
\frac{L_{11}(\hat{x})(p_{11}-c_{11})}{p_{11}} &= \frac{q(p_{11})}{q'(p_{11})\,p_{11}}\left[\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})\right] - \gamma\hat{L}_{11}(\hat{x})\\
\frac{p_{11}-c_{11}}{p_{11}} &= \frac{q(p_{11})}{q'(p_{11})\,p_{11}}\left[\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})} - 1\right] - \gamma\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\\
\frac{p_{11}-c_{11}}{p_{11}} &= \frac{1}{\eta}\left[1 - \frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\right] - \gamma\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}.
\end{aligned} \tag{24}$$

The latest expression is the equation (7) stated in Proposition 1 and can be easily written as follows:

$$p_{11} = \frac{\eta c_{11}L_{11}(\hat{x})}{\hat{L}_{11}(\hat{x}) - L_{11}(\hat{x})(1-\eta) + \eta\gamma\hat{L}_{11}(\hat{x})}, \tag{25}$$

or in a closed form:

$$p_{11} = \frac{\eta\, c_{11}\,\left((1-\lambda)\hat{x}^2 + \lambda\left(\hat{x}-\frac{\epsilon}{4}\right)\right)}{(\eta-1)\left[(1-\lambda)\hat{x}^2 + \lambda\left(\hat{x}-\frac{\epsilon}{4}\right)\right] + (1+\gamma\eta)\left[(1-\lambda)\hat{x}^2 + \frac{\lambda\hat{x}}{2}\right]}. \tag{26}$$

A.2 Network 2 problem to decide on-net price

Solving the problem of firm 2 to set prices, network decides on-net price p_{22} performing the maximization program:

$$\begin{aligned}
\frac{\partial}{\partial p_{22}}\left\{\bar{\Pi}_2(\hat{x})\right\} &= \frac{\partial}{\partial p_{22}}\left\{\int \Pi_2(x,\hat{x})\,dx\right\}\\
&= \frac{\partial}{\partial p_{22}}\left\{\int_{\hat{x}}^{1} \pi_2(x,\hat{x}) + F_2 + R_{21}(x,\hat{x}) - f_2 dx + \right.\\
&\left.\int_{-1}^{-\hat{x}} \pi_2(x,\hat{x}) + F_2 + R_{21}(x,\hat{x}) - f_2 dx\right\}\\
&= \frac{\partial}{\partial p_{22}}\left\{(1-\hat{x})(F_2-f_2) + L_{22}(\hat{x})(p_{22}-c_{22})q(p_{22})\right.\\
&\left.+L_{21}(\hat{x})(p_{21}-c_{21})q(p_{21}) + L_{12}(\hat{x})(a_2-c_0)q(p_{12})\right\} = 0.
\end{aligned} \tag{27}$$

Performing the derivative stated at equation (27), it is possible to establish the condition from which p_{22} can be found:

$$\begin{aligned}\frac{\partial \overline{\Pi}_2}{\partial p_{22}} &= (1-\hat{x})\frac{\partial F_2}{\partial p_{22}} + L_{22}(\hat{x})\left[q(p_{22}) + (p_{22}-c_{22})q'(p_{22})\right] = 0 \\ \frac{\partial F_2}{\partial p_{22}} &= \frac{\partial}{\partial p_{22}}\left\{F_1 + u_2(\hat{x},\hat{x}) - u_1(\hat{x},\hat{x}) - \tau(1-2\hat{x})\right\} \\ &= \left[1 - G(\hat{x}|\hat{x})\right]\left(v'(p_{22}) + \gamma u'(q_{22})q'(p_{22})\right)\end{aligned}$$

defining

$$\hat{L}_{22} = (1-\hat{x})[1 - G(\hat{x}|\hat{x})] \tag{28}$$

and substituting $\partial F_2/\partial p_{22}$ into derivative equation, first order condition is written as follows:

$$\frac{\partial \overline{\Pi}_2}{\partial p_{22}} = (1-\hat{x})\left[1 - G(\hat{x}|\hat{x})\right]\left[-q(p_{22}) \right.$$
$$\left. + \gamma p_{22} q'(p_{22})\right] + L_{22}(\hat{x})\left[q(p_{22}) + (p_{22}-c_{22})q'(p_{22})\right] = 0$$

$$\begin{aligned}L_{22}(\hat{x})(p_{22}-c_{22})q'(p_{22}) &= \hat{L}_{22}(\hat{x})q(p_{22}) - \hat{L}_{22}(\hat{x})\gamma p_{22}q'(p_{22}) - L_{22}(\hat{x})q(p_{22}) \\ L_{22}(\hat{x})(p_{22}-c_{22}) &= \hat{L}_{22}(\hat{x})\frac{q(p_{22})}{q'(p_{22})} - \hat{L}_{22}(\hat{x})\gamma p_{22} - L_{22}(\hat{x})\frac{q(p_{22})}{q'(p_{22})} \\ \frac{p_{22}-c_{22}}{p_{22}} &= \frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\frac{q(p_{22})}{q'(p_{22})p_{22}} - \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})} - \frac{q(p_{22})}{q'(p_{22})p_{22}} \\ \frac{p_{22}-c_{22}}{p_{22}} &= -\frac{1}{\eta}\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\frac{q(p_{22})}{q'(p_{22})p_{22}} + \frac{1}{\eta} - \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})} \\ \frac{p_{22}-c_{22}}{p_{22}} &= \frac{1}{\eta}\left[1 - \frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}\right] - \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{22}(\hat{x})}.\end{aligned} \tag{29}$$

The latest expression is the equation (8) stated in Proposition 1 and can be easily written as follows:

$$p_{22} = \frac{\eta\, c_{22}\, L_{22}}{(\eta-1)L_{22} + (1+\gamma\eta)\hat{L}_{22}}. \tag{30}$$

or in a closed form:

$$p_{22} = \frac{\eta\, c_{22}\,\left((1-\lambda)(1-\hat{x})^2 + \lambda\left(1-\hat{x}-\frac{\epsilon}{4}\right)\right)}{(\eta-1)\left[(1-\lambda)(1-\hat{x})^2 + \lambda\left(1-\hat{x}-\frac{\epsilon}{4}\right)\right] + (1+\eta\gamma)\left[(1-\hat{x})\left(1-\hat{x}-\lambda\hat{x}-\frac{\lambda}{2}\right)\right]}. \tag{31}$$

B Proof of Corollary 1.1

When calling circle weight λ increases, it is of interest to establish the magnitude of change in the factor accompanying call externality in the on-net Lerner indices, thus the next derivative is of interest:

$$\frac{\partial}{\partial \lambda}\left[\frac{\hat{L}_{ii}}{L_{ii}}\right] = \frac{1}{L_{ii}^2}\left(\frac{\partial \hat{L}_{ii}}{\partial \lambda}L_{ii} - \frac{\partial L_{ii}}{\partial \lambda}\hat{L}_{ii}\right) \tag{32}$$

where expression inside parenthesis defines the sign of the result. This expression can be proved to be negative using definitions for $\hat{L}_{ii}$ and L_{ii}, as follows:

$$\frac{\partial \hat{L}_{11}}{\partial \lambda} L_{11} - \frac{\partial L_{11}}{\partial \lambda} \hat{L}_{11} = \frac{\hat{x}^2}{4}(\varepsilon - 2\hat{x}) < 0$$
$$\frac{\partial \hat{L}_{22}}{\partial \lambda} L_{22} - \frac{\partial L_{22}}{\partial \lambda} \hat{L}_{22} = \frac{(1-\hat{x})^2}{4}(\varepsilon + 2\hat{x} - 2) < 0$$

where inequalities follow using Assumption 1.

C Proof of Proposition 2

To find optimal p_{ij} both firms perform the following maximization program: $\max_{p_{ij}} \bar{\Pi}_i(\hat{x})$, which can be written as:

$$\max_{p_{ij}} \bar{\Pi}_i(\hat{x}) = \max_{p_{ij}} \int \Pi_i(x, \hat{x})\, dx = \frac{\partial}{\partial p_{ij}} \left\{ \int \Pi_i(x, \hat{x})\, dx \right\}.$$

C.1 Network 1 problem to decide off-net price

Solving the problem of firm 1 to set prices, network decides off-net price p_{12} performing the maximization program:

$$\begin{aligned}
\frac{\partial}{\partial p_{12}} \left\{ \bar{\Pi}_1(\hat{x}) \right\} &= \frac{\partial}{\partial p_{12}} \left\{ \int_{-\hat{x}}^{\hat{x}} \Pi_1(x, \hat{x})\, dx \right\} \\
&= \frac{\partial}{\partial p_{12}} \left\{ \int_{-\hat{x}}^{\hat{x}} \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f\, dx \right\} = 0 \\
&= \frac{\partial}{\partial p_{12}} \left\{ \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)(p_{11} - c_{11})q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c_{12})q(p_{12}) \right. \\
&\left. + F_1 + [1 - G(\hat{x}|x)]\,(a - c_T)q_{p21} - f_1\, dx \right\} = 0 \\
&= \frac{\partial}{\partial p_{12}} \left\{ \hat{x}(F_1 - f_1) + L_{11}(\hat{x})(p_{11} - c_{11})q(p_{11}) + \right. \\
&\left. L_{12}(\hat{x})(p_{12} - c_{12})q(p_{12}) + L_{21}(\hat{x})(a - c_0)q(p_{21}) \right\} = 0
\end{aligned} \tag{33}$$

where $L_{11}(\hat{x})$, $L_{12}(\hat{x})$ and $L_{21}(\hat{x})$ follow the definitions given in (19), (20) and (21).

Performing the derivative stated at equation (33), it is possible to establish the condition from which p_{12} can be found:

$$\frac{\partial \bar{\Pi}_1}{\partial p_{12}} = \hat{x}\frac{\partial F_1}{\partial p_{12}} + L_{12}(\hat{x})\left[q(p_{12}) + (p_{12} - c_{12})q'(p_{12})\right] = 0 \tag{34}$$

$$\begin{aligned}
\frac{\partial F_1}{\partial p_{12}} &= \frac{\partial}{\partial p_{12}} \left\{ F_2 + u_1(\hat{x}, \hat{x}) - u_2(\hat{x}, \hat{x}) + \tau(1 - 2\hat{x}) \right\} \\
&= \frac{\partial u_1(\hat{x}, \hat{x})}{\partial p_{12}} - \frac{\partial u_2(\hat{x}, \hat{x})}{\partial p_{12}} \\
&= [1 - G(\hat{x}, \hat{x})]v'(p_{12}) - G(\hat{x}, \hat{x})\gamma u'(q_{12})q'(p_{12}).
\end{aligned}$$

Substituting $\partial u_1(\hat{x},\hat{x})/\partial p_{12} - \partial u_2(\hat{x},\hat{x})/\partial p_{12}$ into (34):

$$\begin{aligned}\frac{\partial \overline{\Pi}_1}{\partial p_{12}} &= \hat{x}\left\{\left[1-G(\hat{x},\hat{x})\right]v'(p_{12}) - G(\hat{x},\hat{x})\gamma u'(q_{12})q'(p_{12})\right\}\\ &+ L_{12}(\hat{x})\left[q(p_{12}) + (p_{12}-c_{12})q'(p_{12})\right] = 0 \Leftrightarrow\\ &- q(p_{12})\hat{x}[1-G(\hat{x}|\hat{x})] - \hat{x}G(\hat{x}|\hat{x})\gamma u'(q_{12})q'(p_{12}) + L_{12}(\hat{x})q(p_{12})\\ &+ L_{12}(\hat{x})(p_{12}-c_{12})q'(p_{12}) = 0.\end{aligned}$$

Defining:

$$\hat{L}_{12} = \hat{x}[1-G(\hat{x}|\hat{x})], \tag{35}$$

former equation can be written as:

$$\begin{aligned}q(p_{12})\hat{L}_{12} + \hat{L}_{11}\gamma u'(q_{12})q'(p_{12}) &= L_{12}(\hat{x})q(p_{12}) + L_{12}(\hat{x})(p_{12}-c_{12})q'(p_{12})\\ q(p_{12})[\hat{L}_{12} - L_{12}] + \hat{L}_{11}\gamma p_{12}q'(p_{12}) &= L_{12}(\hat{x})(p_{12}-c_{12})q'(p_{12})\\ \frac{q(p_{12})}{p_{12}}[\hat{L}_{12} - L_{12}] + \hat{L}_{11}\gamma q'(p_{12}) &= L_{12}(\hat{x})q'(p_{12})\frac{p_{12}-c_{12}}{p_{12}}\\ \frac{q(p_{12})}{q'(p_{12})\,p_{12}}\left[\frac{\hat{L}_{12}}{L_{12}} - 1\right] + \gamma\frac{\hat{L}_{11}}{L_{12}} &= \frac{p_{12}-c_{12}}{p_{12}}\\ \frac{1}{\eta}\left[1 - \frac{\hat{L}_{12}}{L_{12}}\right] + \gamma\frac{\hat{L}_{11}}{L_{12}} &= \frac{p_{12}-c_{12}}{p_{12}}.\end{aligned} \tag{36}$$

The latest expression is the equation (9) stated in Proposition 2 and can be easily written as follows:

$$p_{12} = \frac{\eta\, c_{12}\, L_{12}}{(\eta-1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}} \tag{37}$$

or in a closed form:

$$p_{12} = \frac{\eta\, c_{11}\,\left(\hat{x} - (1-\lambda)\hat{x}^2 - \lambda\left(\frac{\varepsilon}{4} - \hat{x}\right)\right)}{(\eta-1)\left[\hat{x} - (1-\lambda)\hat{x}^2 - \lambda\left(\frac{\varepsilon}{4} - \hat{x}\right)\right] + \hat{x} - (1-\lambda)\hat{x}^2 - \frac{\lambda\hat{x}}{2} - \gamma\eta\left((1-\lambda)\hat{x}^2 + \frac{\lambda\hat{x}}{2}\right)}. \tag{38}$$

C.2 Network 2 problem to decide off-net price

Solving the problem of firm 2 to set prices, network decides off-net price p_{21} performing the maximization program:

$$\begin{aligned}\frac{\partial}{\partial p_{21}}\left\{\int \Pi_2(x,\hat{x})\,dx\right\} &= \frac{\partial}{\partial p_{21}}\left\{\bar{\Pi}_2(\hat{x})\right\}\\ &= \frac{\partial}{\partial p_{21}}\left\{(1-\hat{x})(F_2 - f_2) + L_{22}(\hat{x})(p_{22}-c_{22})q(p_{22})\right.\\ &\left.+ L_{21}(\hat{x})(p_{21}-c_{21})q(p_{21}) + L_{12}(\hat{x})(a_2 - c_0)q(p_{12})\right\} = 0.\end{aligned} \tag{39}$$

Performing the derivative stated at equation (27), it is possible to establish the condition from which p_{21} can be found:

$$\frac{\partial \overline{\Pi}_2}{\partial p_{21}} = (1-\hat{x})\frac{\partial F_2}{\partial p_{21}} + L_{22}(\hat{x})\left[q(p_{22}) + (p_{22}-c_{22})q'(p_{22})\right] = 0$$

$$\begin{aligned}\frac{\partial F_2}{\partial p_{21}} &= \frac{\partial}{\partial p_{21}}\left\{F_1 + u_2(\hat{x},\hat{x}) - u_1(\hat{x},\hat{x}) - \tau(1-2\hat{x})\right\} \\ &= G(\hat{x}|\hat{x})v'(p_{21}) - [1 - G(\hat{x}|\hat{x})]\left[\gamma u'(q_{21})q'(p_{21})\right]\end{aligned}$$

defining

$$\hat{L}_{21} = (1-\hat{x})G(\hat{x}|\hat{x}) \tag{40}$$

substituting $\partial F_2/\partial p_{21}$ into derivative equation, and using definition given in equation (28), first order condition is written as follows:

$$-\hat{L}_{21}(\hat{x})q(p_{21}) - \hat{L}_{22}(\hat{x})\gamma p_{21}q'(p_{21}) + L_{21}(\hat{x})q(p_{21}) + L_{21}(\hat{x})\left[(p_{21}-c_{21})q'(p_{21})\right] = 0$$

$$\begin{aligned}L_{21}(\hat{x})(p_{21}-c_{21})q'(p_{21}) &= \hat{L}_{21}(\hat{x})q(p_{21}) + \hat{L}_{22}(\hat{x})\gamma p_{21}q'(p_{21}) - L_{21}(\hat{x})q(p_{21}) \\ L_{21}(\hat{x})\frac{p_{21}-c_{21}}{p_{21}} &= \hat{L}_{21}(\hat{x})\frac{q(p_{21})}{q'(p_{21})p_{21}} + \gamma\hat{L}_{22}(\hat{x}) - L_{21}(\hat{x})\frac{q(p_{21})}{q'(p_{21})p_{21}} \\ \frac{p_{21}-c_{21}}{p_{21}} &= -\frac{1}{\eta}\frac{\hat{L}_{21}(\hat{x})}{L_{21}(\hat{x})} + \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{21}(\hat{x})} + \frac{1}{\eta} \\ \frac{p_{21}-c_{21}}{p_{21}} &= \frac{1}{\eta}\left[1 - \frac{\hat{L}_{21}(\hat{x})}{L_{21}(\hat{x})}\right] + \gamma\frac{\hat{L}_{22}(\hat{x})}{L_{21}(\hat{x})}.\end{aligned} \tag{41}$$

The latest expression is the equation (10) stated in Proposition 2 and can be easily written as:

$$p_{21} = \frac{\eta\, c_{21}\, L_{21}}{(\eta-1)L_{21} + \hat{L}_{21} - \gamma\eta\hat{L}_{22}} \tag{42}$$

or in a closed form:

$$p_{21} = \frac{\eta\, c_{21}\,\left[1-\hat{x}+(1-\lambda)(1-\hat{x})^2 - \lambda\left(1-\hat{x}-\frac{\varepsilon}{4}\right)\right]}{\begin{array}{l}(\eta-1)\left[1-\hat{x}+(1-\lambda)(1-\hat{x})^2-\lambda\left(1-\hat{x}-\frac{\varepsilon}{4}\right)\right] + \\ \quad (1-\lambda)\hat{x}-(1-\lambda)\hat{x}^2+\frac{\lambda}{2}(1-\hat{x})-\gamma\eta\left[1-2\hat{x}+(1-\lambda)\hat{x}^2+\frac{\lambda\hat{x}}{2}+\lambda\left(\hat{x}-\frac{1}{2}\right)\right]\end{array}}. \tag{43}$$

D Proof of Corollary 2.1

When calling circle weight (λ) increases, it is of interest to establish the magnitude of change in the factor accompanying call externality in the off-net Lerner index, thus the next derivative is of interest:

$$\frac{\partial}{\partial\lambda}\left[\frac{\hat{L}_{ii}}{L_{ij}}\right] = \frac{1}{L_{ij}^2}\left(\frac{\partial\hat{L}_{ii}}{\partial\lambda}L_{ij} - \frac{\partial L_{ij}}{\partial\lambda}\hat{L}_{ii}\right)$$

where expression inside parenthesis defines the sign of the result. This expression can be proved to be positive using definitions for $\hat{L}_{ii}$ and L_{ij}, as follows:

$$\begin{aligned}\frac{\partial\hat{L}_{11}}{\partial\lambda}L_{12} - \frac{\partial L_{12}}{\partial\lambda}\hat{L}_{11} &= \frac{1}{4}\hat{x}^2(2(1-\hat{x})-\varepsilon) > 0 \\ \frac{\partial\hat{L}_{22}}{\partial\lambda}L_{21} - \frac{\partial L_{21}}{\partial\lambda}\hat{L}_{22} &= \frac{1}{4}(1-\hat{x})^2(2\hat{x}-\varepsilon) > 0\end{aligned}$$

where inequalities follow using Assumption 1 .

To establish whether call externality effect is counterbalanced, inequality that follows is of interest:

$$\gamma\left(\frac{\partial\hat{L}_{ii}}{\partial\lambda}L_{ij}-\frac{\partial L_{ij}}{\partial\lambda}\hat{L}_{ii}\right)<\frac{1}{\eta}\left(\frac{\partial\hat{L}_{ij}}{\partial\lambda}L_{ij}-\frac{\partial L_{ij}}{\partial\lambda}\hat{L}_{ij}\right)$$

which turns into

$$\gamma\hat{x}(2-2\hat{x}-\varepsilon)\leq\frac{1}{\eta}(1-\hat{x})(2\hat{x}-\varepsilon) \tag{44}$$

for the case of network 1, and

$$\gamma(1-\hat{x})(2\hat{x}-\varepsilon)<\frac{1}{\eta}\hat{x}(2-2\hat{x}-\varepsilon) \tag{45}$$

for the case of network 2.

The previous expression will hold depending upon values of the parameters of the model. In particular, as long as call externality is low enough or price elasticity of demand is low enough, firms will set lower off-net prices in response to a stronger calling circle weight. However, the effect might be reversed for very strong call externality or very elastic markets, being the firm 2 the most likely to react with off-net price increase withstanding a strong calling circle weight.

E Proof of Proposition 3

Recalling closed form for equations (7) and (8) -(25) and (30)-:

$$\begin{aligned}\frac{p_{11}}{p_{22}}&=\frac{\frac{\eta c_{11}L_{11}}{(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}}}{\frac{\eta c_{22}L_{22}}{(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}}}\\&=\frac{\eta c_{11}L_{11}\left[(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}\right]}{\eta c_{22}L_{22}\left[(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}\right]}\end{aligned}$$

assuming symmetric costs of the firms ($c_{ii}=c_O+c_T$), thus:

$$\frac{p_{11}}{p_{22}}=\frac{L_{11}\left[(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}\right]}{L_{22}\left[(\eta-1)L_{11}+(1+\gamma\eta)\hat{L}_{11}\right]}.$$

$\hat{L}_{11}$, L_{22} and $\hat{L}_{22}$ can be written in terms of L_{11}:

$$\frac{p_{11}}{p_{22}}=\frac{L_{11}\left[(\eta-1)(L_{11}+1-2\hat{x})+(1+\gamma\eta)\left(1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right)\right]}{(L_{11}+1-2\hat{x})\left[(\eta-1)L_{11}+(1+\gamma\eta)\left(L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right)\right]}. \tag{46}$$

Ratio (46) must be compared against 1, which means that when prices are positive, the condition $p_{11}\geq p_{22}$ will hold when (46) is equivalent to

$$\begin{aligned}L_{11}\left[(\eta-1)(L_{11}+1-2\hat{x})+(1+\gamma\eta)\left(1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right)\right]\geq\\(L_{11}+1-2\hat{x})\left[(\eta-1)L_{11}+(1+\gamma\eta)\left(L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right)\right]\end{aligned}$$

$$L_{11}(1+\gamma\eta)\left[1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right]\geq$$

$$(L_{11}+1-2\hat{x})\left[(1+\gamma\eta)\left(L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right)\right]$$

$$\begin{aligned}
L_{11}\left[1-2\hat{x}+\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right)+L_{11}\right] &\geq (L_{11}+1-2\hat{x})\left[L_{11}-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right]\\
L_{11}\frac{\lambda}{2}\left(\hat{x}-1+\frac{\varepsilon}{2}\right) &\geq (L_{11}+1-2\hat{x})\left[-\frac{\lambda}{2}\left(\hat{x}-\frac{\varepsilon}{2}\right)\right]\\
L_{11}(2\hat{x}-1) &\geq \hat{x}(2\hat{x}-1)+\varepsilon\left(\frac{1}{2}-\hat{x}\right)\\
L_{11} &\geq \hat{x}-\frac{\varepsilon}{2}. \qquad (47)
\end{aligned}$$

After procedural algebra on (47), next expression is the relationship of magnitude between on-net prices:

$$\lambda \geq \frac{\hat{x}(1-\hat{x})-\frac{\varepsilon}{2}}{\hat{x}(1-\hat{x})-\frac{\varepsilon}{4}}=\hat{\lambda}_{\text{on}}(\hat{x},\varepsilon) \qquad (48)$$

which is the result of the Proposition.

F Proof of Proposition 4

Recalling closed form for equations (8) and (10) -(37) and (42)-, p_{12}/p_{21} ratio is posed using expressions:

$$\begin{aligned}
\frac{p_{12}}{p_{21}} &= \frac{\frac{\eta\, c_{12}\, L_{12}}{(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}}}{\frac{\eta\, c_{21}\, L_{21}}{(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}}}\\
&= \frac{\eta\, c_{12}\, L_{12}\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right]}{\eta\, c_{21}\, L_{21}\left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right]}\\
&= \frac{c_{12}\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right]}{c_{21}\left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right]} \qquad (49)
\end{aligned}$$

where the third equality follows since $L_{12}=L_{21}$. Ratio (49) must be compared against 1 which means that when prices are positive, the condition $p_{12}\geq p_{21}$ will hold when (49) is equivalent to:

$$(c_O+a)\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right]\geq(c_O+a)\left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right] \qquad (50)$$

where $c_{12}=c_{21}=c_O+a$, given we asume firms are equally cost-efficient.

$$\begin{aligned}
\left[(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right] &\geq \left[(\eta-1)L_{12}+\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right]\\
\left[\hat{L}_{21}-\gamma\eta\hat{L}_{22}\right] &\geq \left[\hat{L}_{12}-\gamma\eta\hat{L}_{11}\right]\\
\hat{L}_{21}-\hat{L}_{12} &\geq \gamma\eta\left[\hat{L}_{22}-\hat{L}_{11}\right]
\end{aligned}$$

Using simplified expresions for $\hat{L}_{21} - \hat{L}_{12}$ as well as $\hat{L}_{22} - \hat{L}_{11}$, comparison under discussion turns out to be

$$\gamma\eta(2\hat{x} - 1) \;\geq\; \lambda \left(\hat{x} - \frac{1}{2}\right)(1 + \gamma\eta) \tag{51}$$

that finally leads to

$$\hat{\lambda}_{\text{off}}(\gamma, \eta) = \frac{2}{1 + \frac{1}{\gamma\eta}} \;\geq\; \lambda \tag{52}$$

which is the result stated in the Proposition.

G Proof of Proposition 5

G.1 Network 1

Recalling closed form for equations (7) and (9) -(25) and (37)-, p_{11}/p_{12} ratio is posed:

$$\begin{aligned}
\frac{p_{11}}{p_{12}} &= \frac{\frac{\eta\, c_{11}\, L_{11}}{(\eta-1)L_{11} + (1+\gamma\eta)\hat{L}_{11}}}{\frac{\eta\, c_{12}\, L_{12}}{(\eta-1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}}} \\
&= \frac{\eta\, c_{11}\, L_{11}\left[(\eta-1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right]}{\eta\, c_{12}\, L_{12}\left[(\eta-1)L_{11} + (1+\gamma\eta)\hat{L}_{11}\right]} \\
&= \frac{(c_O + c_T)\, L_{11}\left[(\eta-1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right]}{(c_O + a)\, L_{12}\left[(\eta-1)L_{11} + (1+\gamma\eta)\hat{L}_{11}\right]}.
\end{aligned} \tag{53}$$

Ratio (53) must be compared against 1 which means that when prices are positive, the condition $p_{11} \geq p_{12}$ will hold when (53) is equivalent to:

$$(c_O + c_T)\, L_{11}\left[(\eta-1)L_{12} + \hat{L}_{12} - \gamma\eta\hat{L}_{11}\right] \;\geq\; (c_O + a)\, L_{12}\left[(\eta-1)L_{11} + (1+\gamma\eta)\hat{L}_{11}\right]. \tag{54}$$

If symmetric costs are considered and $a = c_T$, (54) can be written as:

$$\begin{aligned}
L_{11}\left[\hat{L}_{12} - \gamma\eta\hat{L}_{11}\right] &\;\geq\; L_{12}\left[(1+\gamma\eta)\hat{L}_{11}\right] \\
L_{11}\hat{L}_{12} - \gamma\eta L_{11}\hat{L}_{11} &\;\geq\; L_{12}\hat{L}_{11} + \gamma\eta L_{12}\hat{L}_{11} \\
L_{11}\hat{L}_{12} - (\hat{x} - L_{11})\hat{L}_{11} &\;\geq\; \gamma\eta\left[L_{11}\hat{L}_{11} + (\hat{x} - L_{11})\hat{L}_{11}\right] \\
L_{11} &\;\geq\; \hat{L}_{11}(1+\gamma\eta).
\end{aligned} \tag{55}$$

Simplifying $L_{11} - \hat{L}_{11}$ and clearing for λ, (55) can be written as:

$$\lambda \;\geq\; \frac{\hat{x}^2}{\hat{x}\left(\hat{x} - \frac{1}{2}\right) + \frac{1}{2\gamma\eta}\left(\hat{x} - \frac{\varepsilon}{2}\right)} = \hat{\lambda}^1_{\text{dif}}(\hat{x}, \gamma, \eta, \varepsilon) \tag{56}$$

which is the result stated in the Proposition.

G.2 Network 2

Recalling closed form for equations (8) and (10) -(30) and (42)-, p_{22}/p_{21} ratio is posed:

$$\frac{p_{22}}{p_{21}} = \frac{\frac{\eta\, c_{22}\, L_{22}}{(\eta-1)L_{22}+(1+\gamma\eta)\hat{L}_{22}}}{\frac{\eta\, c_{21}\, L_{21}}{(\eta-1)L_{21}+\hat{L}_{21}-\gamma\eta\hat{L}_{22}}}$$
$$= \frac{(c_O + c_T)\, L_{22}\left[(\eta-1)L_{21} + \hat{L}_{21} - \gamma\eta\hat{L}_{22}\right]}{(c_O + a)\, L_{21}\left[(\eta-1)L_{22} + (1+\gamma\eta)\hat{L}_{22}\right]}. \tag{57}$$

Ratio (57) must be compared against 1 which means that when prices are positive, the condition $p_{22} \geq p_{21}$ will hold when (57) is equivalent to:

$$(c_O + c_T)\, L_{22}\left[(\eta-1)L_{21} + \hat{L}_{21} - \gamma\eta\hat{L}_{22}\right] \;\geq\; (c_O + a)\, L_{21}\left[(\eta-1)L_{22} + (1+\gamma\eta)\hat{L}_{22}\right]. \tag{58}$$

If symmetric costs are considered and $a = c_T$, (54) can be written as:

$$\begin{aligned}
L_{22}\left[\hat{L}_{21} - \gamma\eta\hat{L}_{22}\right] \;&\geq\; L_{21}\left[(1+\gamma\eta)\hat{L}_{22}\right]\\
L_{22}\hat{L}_{21} - \gamma\eta L_{22}\hat{L}_{22} \;&\geq\; L_{21}\hat{L}_{22} + \gamma\eta L_{21}\hat{L}_{22}\\
L_{22}\hat{L}_{21} - L_{21}\hat{L}_{22} \;&\geq\; \gamma\eta\left[L_{22}\hat{L}_{22} + L_{21}\hat{L}_{22}\right]\\
L_{22}\hat{L}_{21} - (1-\hat{x}-L_{22})\hat{L}_{22} \;&\geq\; \gamma\eta\left[L_{22} + (1-\hat{x}-L_{22})\right]\hat{L}_{22}\\
L_{22} \;&\geq\; \hat{L}_{22}(1+\gamma\eta).
\end{aligned} \tag{59}$$

Equation (59) can be written as:

$$\frac{\lambda}{2}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)\left(1+\frac{1}{\gamma\eta}\right) \;\geq\; L_{22}. \tag{60}$$

An intermediate important step to establish the relationship under consideration is the following: using the expresio for L_{22} in the right-hand side of (60)

$$\begin{aligned}
\frac{\lambda}{2}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)\left(1+\frac{1}{\gamma\eta}\right) \;&\geq\; (1-\lambda)(1-\hat{x})^2 + \lambda\left(1-\hat{x}-\frac{\varepsilon}{4}\right)\\
\frac{\lambda}{2}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) + \frac{\lambda}{2\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) \;&\geq\; (1-\lambda)(1-2\hat{x}+\hat{x}^2) + \lambda - \lambda\hat{x} - \frac{\lambda\varepsilon}{4}\\
\frac{\lambda}{2}\left[1-3\hat{x}+2\hat{x}^2 + \frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)\right] \;&\geq\; 1-2\hat{x}+\hat{x}^2.
\end{aligned}$$

Before solving for λ, term in square brackets $[\cdot]$ should be verified to be positive, so that both sides of the inequality can be multiplied by $1/[\cdot]$.

$$
\begin{aligned}
1-3\hat{x}+2\hat{x}^2+\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &> 0\\
(1-\hat{x})(1-2\hat{x})+\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &> 0\\
\frac{1}{\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right) &> (1-\hat{x})(2\hat{x}-1)\\
1-\hat{x}-\frac{\varepsilon}{2} &> \gamma\eta\,(1-\hat{x})(2\hat{x}-1)\\
1-\frac{\varepsilon}{2(1-\hat{x})} &> \gamma\eta\,(2\hat{x}-1)\\
\frac{1-\frac{\varepsilon}{2(1-\hat{x})}}{2\hat{x}-1} &> \gamma\eta. \qquad (61)
\end{aligned}
$$

When $\gamma = 0$, both sides of (60) can be multiplied by $1/[\cdot]$. The most stringent case would be $\gamma = 1$, that leads (61) into:

$$
\frac{1-\frac{\varepsilon}{2(1-\hat{x})}}{2\hat{x}-1} > \eta. \qquad (62)
$$

Inequality (62) states a condition on η to be able to multiply both sides of (60) by $1/[\cdot]$ and solve for λ. However, it must also be taken into account that since the model is considering $\eta > 1$ given the functional forms for $\upsilon(\cdot)$ and $q(p)$, then left hand side of unequality (62) must be greater than one. This is satisfied if

$$
\varepsilon < 4(1-\hat{x})^2. \qquad (63)
$$

Inequality (63) holds as long as $\hat{x} < 2/3$. This is obtained by ussing Assumption 1 and setting $\varepsilon = 2\hat{x}(1-\hat{x})$ as the most stringent case in the latest inequality. Using the expresion for L_{22} and assuming (62) and (63) hold, (60) is written as follows:

$$
\lambda \geq \frac{(1-\hat{x})^2}{(1-\hat{x})\left(\frac{1}{2}-\hat{x}\right)+\frac{1}{2\gamma\eta}\left(1-\hat{x}-\frac{\varepsilon}{2}\right)} = \hat{\lambda}^2_{\text{dif}}(\hat{x},\gamma,\eta,\varepsilon) \qquad (64)
$$

which is the result stated in the Proposition.

H Proof of Proposition 6

Recalling equation (18) and substituting off-net price for $p_{12} = p_{11} + \Delta$:

$$
\begin{aligned}
\frac{\partial\overline{\Pi}_1}{\partial p_{11}} =& \frac{\partial}{\partial p_{11}}\{\hat{x}(F_1-f_1)+L_{11}(\hat{x})(p_{11}-c_{11})q(p_{11})+\\
&L_{12}(\hat{x})(p_{11}+\Delta-c_{12})q(p_{11}+\Delta)+L_{21}(\hat{x})(a-c_0)q(p_{21})\} = 0
\end{aligned}
$$

$$
\frac{\partial\overline{\Pi}_1}{\partial p_{11}} = \hat{x}\frac{\partial F_1}{\partial p_{11}} + L_{11}(\hat{x})\left[q(p_{11})+(p_{11}-c_{11})q'(p_{11})\right] + L_{12}(\hat{x})\left[\frac{dq_{12}}{dp_{11}}(p_{11}+\Delta-c_{12})+q_{12}\right] = 0
$$

$$
\begin{aligned}
\frac{\partial F_1}{\partial p_{11}} &= \frac{\partial}{\partial p_{11}}\left\{F_2 + u_1(\hat{x},\hat{x}) - u_2(\hat{x},\hat{x}) - \tau\hat{x}\right\} \\
&= \frac{\partial u_1(\hat{x},\hat{x})}{\partial p_{11}} - \frac{\partial u_2(\hat{x},\hat{x})}{\partial p_{11}} \\
&= G(\hat{x}|\hat{x})\left[\upsilon'(p_{11}) + \gamma u'(q_{11})\right] + [1 - G(\hat{x}|\hat{x})]\left[\upsilon'(p_{11}+\Delta)\right] - G(\hat{x}|\hat{x})[\gamma u'(q_{12})] \\
&= G(\hat{x}|\hat{x})\left[-q_{11} + \gamma(-\eta q_{11})\right] + [1 - G(\hat{x}|\hat{x})][-q'_{12}] - G(\hat{x}|\hat{x})[\gamma(-\eta q'_{12})]
\end{aligned}
$$

where for the latest equality I have used:

$$
\begin{aligned}
\frac{du(q_{ii})}{dp_{ii}} &= \frac{du}{dq_{ii}}\frac{dq_{ii}}{dp_{ii}} \\
&= q_{ii}(-\eta)p_{ii}^{-\eta-1} \\
&= -\eta p_{ii}^{-\eta} \\
&= -\eta q_{ii}.
\end{aligned}
$$

Substituting $\partial F_1/\partial p_{11}$ into the maximization problem:

$$
\begin{aligned}
\frac{\partial\overline{\Pi}_1}{\partial p_{11}} =& \hat{x}\left\{G(\hat{x}|\hat{x})\left[-q_{11} + \gamma(-\eta q_{11})\right] + [1 - G(\hat{x}|\hat{x})][-q'_{12}] - G(\hat{x}|\hat{x})[\gamma(-\eta q'_{12})]\right\} \\
&+ L_{11}(\hat{x})\left[q_{11} + (p_{11} - c_{11})q'_{11}\right] + L_{12}(\hat{x})\left[q'_{12}(p_{11} + \Delta - c_{12}) + q_{12}\right] = 0
\end{aligned}
$$

$$
\begin{aligned}
L_{11}(\hat{x})(p_{11} - c_{11})q'_{11} + L_{12}(\hat{x})q'_{12}(p_{11} + \Delta - c_{12}) =& \hat{x}G(\hat{x}|\hat{x})\left[q_{11} + \gamma\eta q_{11}\right] + \hat{x}[1 - G(\hat{x}|\hat{x})]q'_{12} \\
&- \hat{x}G(\hat{x}|\hat{x})[\gamma\eta q_{12}] - L_{11}(\hat{x})q(p_{11}) - L_{12}(\hat{x})q_{12}
\end{aligned}
$$

Rearranging terms:

$$
\begin{aligned}
p_{11}\left[L_{11}(\hat{x})q'_{11} + L_{12}(\hat{x})q'_{12}\right] =& L_{11}(\hat{x})c_{11}q'_{11} - L_{12}(\hat{x})(\Delta - c_{12})q'_{12} + p_{11}\left\{-\hat{x}G(\hat{x}|\hat{x})\frac{1+\gamma\eta}{\eta}q'_{11}\right. \\
&\left. + \gamma\hat{x}G(\hat{x}|\hat{x})q'_{12} + \frac{L_{11}(\hat{x})}{\eta}q'_{11} + \frac{L_{12}(\hat{x})}{\eta}q'_{12}\right\} + \hat{x}[1 - G(\hat{x}|\hat{x})]q'_{12} + \gamma\hat{x}G(\hat{x}|\hat{x})q'_{12}\Delta \\
&+ \frac{L_{12}(\hat{x})}{\eta}q'_{12}\Delta.
\end{aligned}
$$

$$
\begin{aligned}
p_{11}&\left\{q'_{11}\left[L_{11}(\hat{x}) + \hat{x}G(\hat{x}|\hat{x})\frac{1+\gamma\eta}{\eta} - \frac{L_{11}(\hat{x})}{\eta}\right] + q'_{12}\left[L_{12}(\hat{x}) - \gamma\hat{x}G(\hat{x}|\hat{x}) - \frac{L_{12}(\hat{x})}{\eta}\right]\right\} = L_{11}(\hat{x})c_{11}q'_{11} \\
&+ L_{12}(\hat{x})c_{12}q'_{12} + \hat{x}[1 - G(\hat{x}|\hat{x})]q'_{12} + \left(\gamma\hat{x}G(\hat{x}|\hat{x}) + \frac{L_{12}(\hat{x})}{\eta} - L_{12}(\hat{x})\right)q'_{12}\Delta.
\end{aligned}
$$

Dividing both sides of the previous equality by $L_{11}(\hat{x})q'_{11}$ and using definitions (23) and (35):

$$
\begin{aligned}
p_{11}&\left\{1 - \frac{1}{\eta} + \frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\frac{1+\gamma\eta}{\eta} + \frac{q'_{12}}{L_{11}(\hat{x})q'_{11}}\left[L_{12}(\hat{x}) - \gamma\hat{L}_{11}(\hat{x}) - \frac{L_{12}(\hat{x})}{\eta}\right]\right\} = c_{11} \\
&+ c_{12}\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}} + \frac{\hat{L}_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}} + \frac{q'_{12}}{L_{11}(\hat{x})q'_{11}}\left(\gamma\hat{L}_{11}(\hat{x}) + \frac{L_{12}(\hat{x})}{\eta} - L_{12}(\hat{x})\right)\Delta.
\end{aligned}
$$

$$p_{11}\left\{1-\frac{1}{\eta}+\frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\frac{1+\gamma\eta}{\eta}+\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}}\left[1-\frac{1}{\eta}-\gamma\frac{\hat{L}_{11}(\hat{x})}{L_{12}(\hat{x})}\right]\right\}=c_{11}$$
$$+\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\frac{q'_{12}}{q'_{11}}\left\{c_{12}+\frac{\hat{L}_{12}(\hat{x})}{L_{12}(\hat{x})}-\Delta\left[1-\frac{1}{\eta}-\gamma\frac{\hat{L}_{11}(\hat{x})}{L_{12}(\hat{x})}\right]\right\}.$$

Clearing for p_{11} leads to result in Proposition 6.

I Proof of Corollary 6.1

I am concerned about the case when the differential is set to zero, i.e. $\Delta = 0$. In this case, $q'_{11} = q'_{12}$, leading the the expression for uniform price stated in Corollary 6.1:

$$p_1^{\Delta}=\frac{\eta c_{11}L_{11}^2(\hat{x})+\eta c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+\eta L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}{L_{11}(\hat{x})\left\{(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x}\right\}}.$$

J Proof of Corollary 6.2

Using the results of the previous section, I develop the algebra to establish how p_1^{Δ} behaves against p_{11}. I look for conditions when expression (16) is larger than (25):

$$\frac{\eta c_{11}L_{11}(\hat{x})+\eta c_{12}L_{12}(\hat{x})+\eta\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\hat{L}_{12}(\hat{x})}{(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x})}>\frac{\eta c_{11}L_{11}(\hat{x})}{(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})}$$

$$\begin{aligned}\eta c_{11}L_{11}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]&+\eta c_{12}L_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]\\&+\eta\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\hat{L}_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]\\&>\eta c_{11}L_{11}(\hat{x})\left[(\eta-1)\left[L_{11}(\hat{x})+L_{12}(\hat{x})\right]+\hat{L}_{11}(\hat{x})\right]\end{aligned}$$

$$\begin{aligned}\gamma\eta^2c_{11}L_{11}(\hat{x})\hat{L}_{11}(\hat{x})&+\eta c_{12}L_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]\\&+\eta\frac{L_{12}(\hat{x})}{L_{11}(\hat{x})}\hat{L}_{12}(\hat{x})\left[(\eta-1)L_{11}(\hat{x})+\hat{L}_{11}(\hat{x})+\gamma\eta\hat{L}_{11}(\hat{x})\right]>\eta(\eta-1)c_{11}L_{11}(\hat{x})L_{12}(\hat{x}).\end{aligned}$$

Since the right hand side of the latest expression contains a single term $\eta(\eta-1)$, then I can just compare this with the coorresponding terms in the left hand side, given the rest of the left hand side expression is positive:

$$\begin{aligned}\eta(\eta-1)\left[c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]&>\eta(\eta-1)c_{11}L_{11}(\hat{x})L_{12}(\hat{x})\\c_{12}L_{11}(\hat{x})L_{12}(\hat{x})+L_{12}(\hat{x})\hat{L}_{12}(\hat{x})&>c_{11}L_{11}(\hat{x})L_{12}(\hat{x})\\c_{12}L_{11}(\hat{x})+\hat{L}_{12}(\hat{x})&>c_{11}L_{11}(\hat{x})\\\hat{L}_{12}(\hat{x})&>0\end{aligned}$$

where I have assumed symmetric costs and cost-based access charges. Therefore, under these assumptions, $p_1^{\Delta} > p_{11}$.

K Proof of Corollary 6.3

Using the results of Corollary 6.1, I now develop the algebra to establish how $p_1^{\triangle}$ behaves against p_{12}. I look for conditions when expression (16) is larger than (37):

$$\frac{\eta c_{11}L_{11}^2(\hat{x}) + \eta c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + \eta L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}{L_{11}(\hat{x})\left\{(\eta-1)\left[L_{11}(\hat{x}) + L_{12}(\hat{x})\right] + \hat{L}_{11}(\hat{x})\right\}} > \frac{\eta c_{12}L_{12}(\hat{x})}{(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \gamma\eta\hat{L}_{11}(\hat{x})}$$

$$\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]\left[(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \gamma\eta\hat{L}_{11}(\hat{x})\right] >$$
$$c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left\{(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right\}$$
$$\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]\left[(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x})\right]$$
$$-\gamma\eta\hat{L}_{11}(\hat{x})\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right] >$$
$$c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right]$$
$$\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]\left[(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x})\right]$$
$$-c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right] >$$
$$\gamma\eta\hat{L}_{11}(\hat{x})\left[c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})\right]$$
$$(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \frac{c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[(\eta-1)\hat{x} + \hat{L}_{11}(\hat{x})\right]}{c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})} >$$
$$\gamma\eta\hat{L}_{11}(\hat{x})$$
$$\frac{1}{\eta\hat{L}_{11}(\hat{x})}\left\{(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \frac{c_{12}L_{11}(\hat{x})L_{12}(\hat{x})\left[\eta\hat{x} - \hat{x} + \hat{L}_{11}(\hat{x})\right]}{c_{11}L_{11}^2(\hat{x}) + c_{12}L_{11}(\hat{x})L_{12}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}\right\} > \gamma$$
$$\frac{1}{\eta\hat{L}_{11}(\hat{x})}\left\{(\eta-1)L_{12}(\hat{x}) + \hat{L}_{12}(\hat{x}) - \frac{c_{11}L_{11}(\hat{x})L_{12}(\hat{x})\left[\eta\hat{x} - \hat{L}_{12}(\hat{x})\right]}{c_{11}\hat{x}L_{11}(\hat{x}) + L_{12}(\hat{x})\hat{L}_{12}(\hat{x})}\right\} > \gamma.$$

In the latest inequality I have assumed termination rate is set equal to cost and firms are equally efficient. This is the result stated in Corollary 6.3.

L Supporting figures

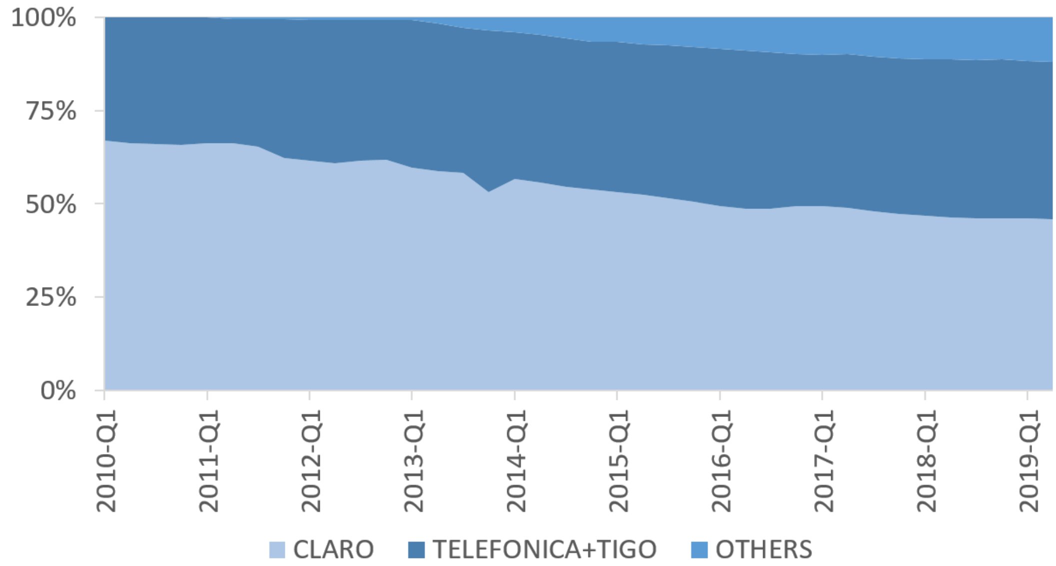


Figure 6: Market shares in the Colombian mobile market.

Source: MINTIC web portal and author's calculations.

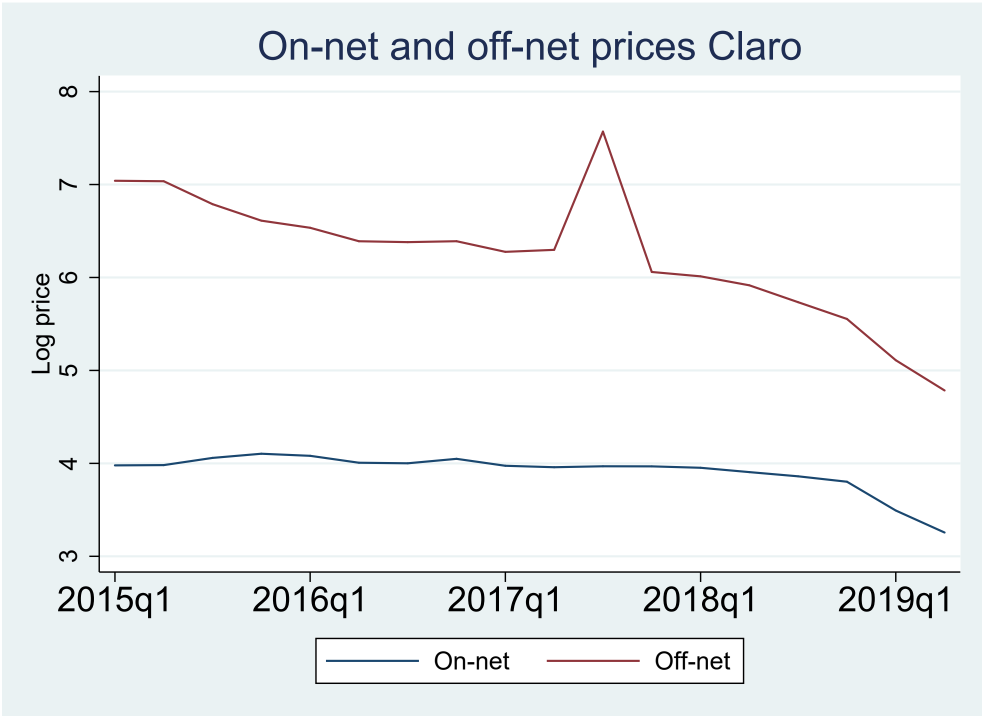


Figure 7: Mobile voice prices set by Comcel network in Colombia.
Source: MINTIC web portal and author's calculations.

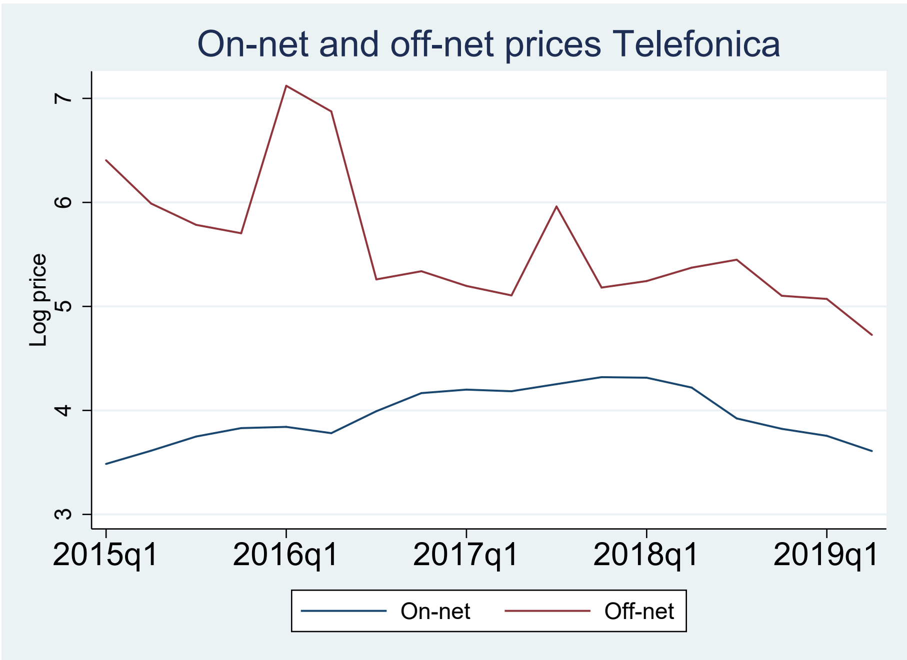


Figure 8: Mobile voice prices set by Telefonica network in Colombia.
Source: MINTIC web portal and author's calculations.

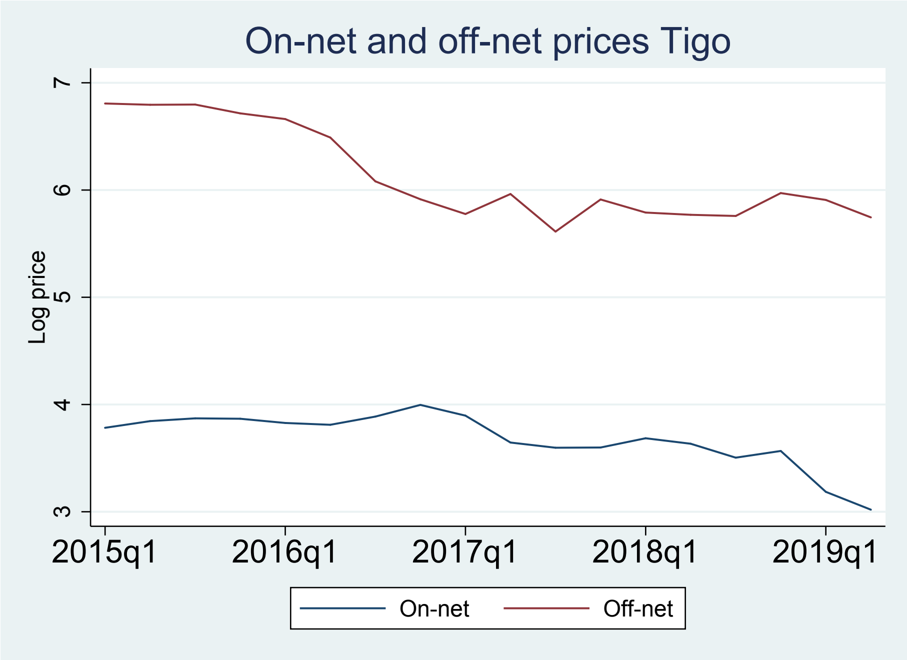


Figure 9: Mobile voice prices set by Tigo network in Colombia.
Source: MINTIC web portal and author's calculations.

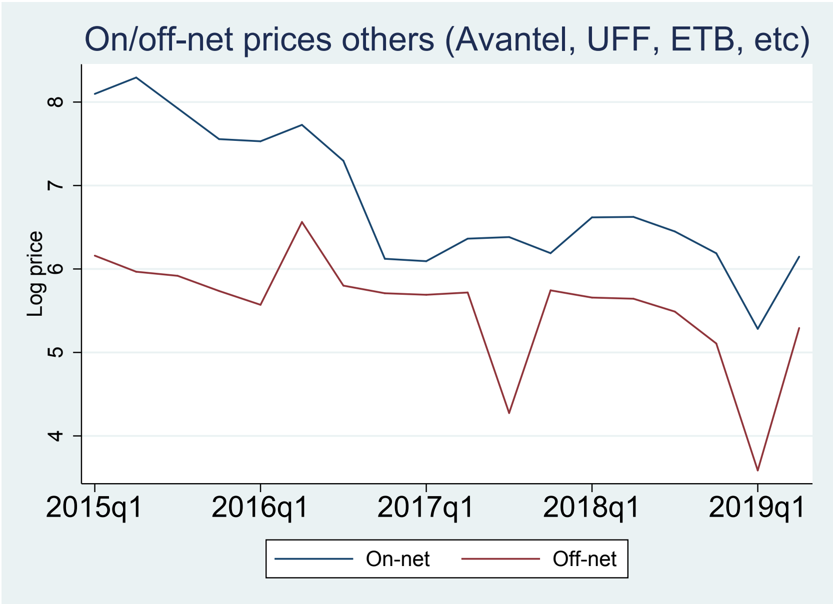


Figure 10: Mobile voice prices set by other networks in Colombia.
Source: MINTIC web portal and author's calculations.

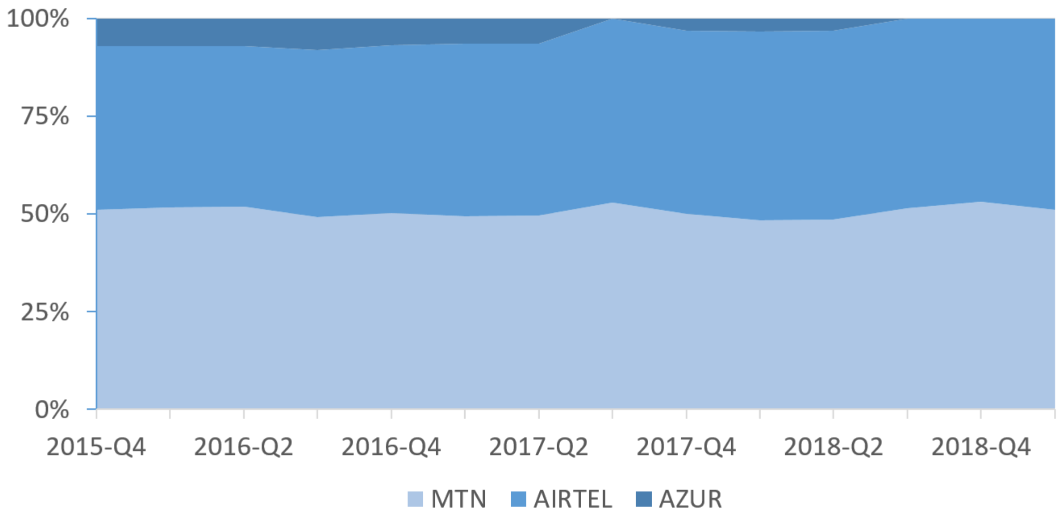
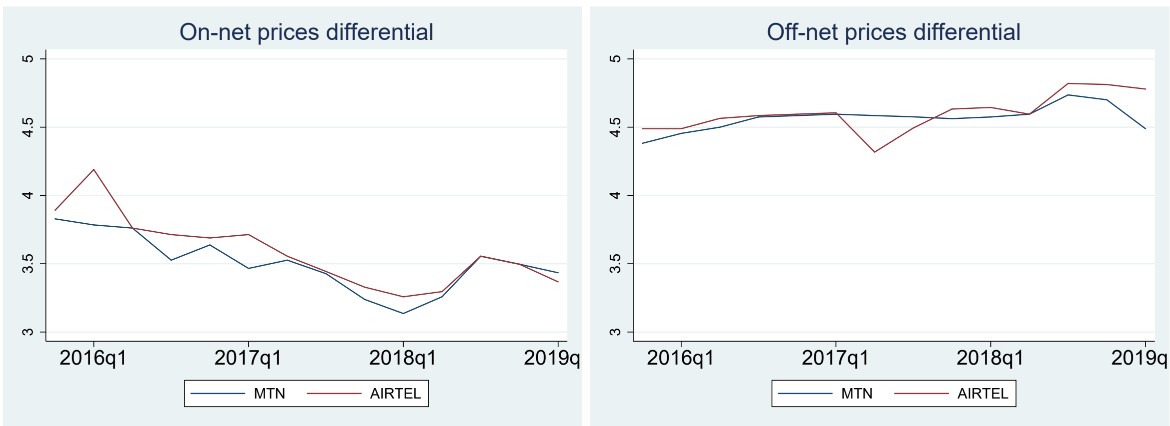


Figure 11: Mobile market shares in Republic of Congo.
Source: ARPCE and author's calculations.



(a) On-net prices paths.

(b) Off-net prices paths.

Figure 12: On-net and off-net price paths in Republic of Congo.
Source: ARPCE and author's calculations.