A Fuzzy Linear Fractional Programming Approach to Design of Distribution Networks

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Abstract. This paper studies the distribution network design problem considering the uncertain information in the demand, capacities, costs and prices in a multi-product environment and multiple periods. We consider a fractional objective function that consist in maximize the ratio between total profit and total cost. We use a model that integrates a facility location problem with a distribution network problem with fuzzy constraints, technological coefficients, and costs. To solve the problem, we use a method that transform the fuzzy linear fractional programming model in an equivalent multi-objective linear fractional programming problem to calculate the upper, middle and lower bounds of the original problem.

Keywords: Fuzzy linear fractional programming · Linear fractional programming · Distribution network · Fuzzy sets

1 Introduction

The distribution network design (DND) problem seeks to meet the demand for a set of customers using a set of available resources from a set of warehouses and supply these warehouses from a set of available factories. The problem consists in selecting the best raw according the distance, time, costs of freight, among other aspects, [1].

In general, the literature about this problem assumes that the information of costs, capacities and demands are known and deterministic, however, the uncertainty of the information is a real-world feature in many situations and consider it could be help to deal with better decisions [2].

In this paper, we study the DNP problem considering the uncertainty in the costs, prices, capacities and demand. In addition, we consider the maximization of the ratio between profits and costs as the objective function. Thus, we have a fuzzy linear fractional programming (FLFP) problem and to solve we use an equivalent multi-objective linear fractional programming problem to calculate the upper, middle and lower bounds of the FLFP problem.

The reminder of this paper is organized as follows: Sect. 2 presents a background and the problem statement. Section 3 describe a brief description of FLFP model.

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Section 4 state the FLFP model to DND and an example is presented in Sect. 5. Finally, Sect. 6 shows the conclusions and states some research lines for future works.

2 Background and Problem Statement

In the DND problem is given a set of possible locations of production factories and warehouses (or distribution centers) with production and storage capacities, respectively. Then, the problem consists in to determine the best location of a given number of factories and warehouses to meet the demand of a set of customers maximizing the ration between the profits and costs associated with the opening factories and warehouses and the distribution of products from factories to end customer's sites. On the other hand, it determines how many products should be and store in factories and warehouses, respectively. In most the problems studied are assumed to be the information of costs, profits, capacities, and demands are known and deterministic. We consider that this information is imprecise and could be modeled as triangular fuzzy numbers.

The DNP problem has been widely studied in the literature [3–7]. This seeks how to set up a distribution network that consists of an integrated system of localization and distribution considering different facilities (stages) like factories, warehouses, centers of consumption and multiple products and periods Within a planning horizon. in a distribution network, warehouses act as intermediary nodes between suppliers and their points of sale, therefore in these you can store products in inventory to supply future demands.

Similar problems have been analyzed in works such as [8–12]. The simplest methods address the problem of logistical networks where the parameters used in the model are deterministic and it is possible to formulate problems of whole linear programming, given the conditions of opening of plants and warehouses. A solution method for the discrete DC-based problem was published in 2007 [3], in the problem the configuration of the network is made for emerging markets or new markets. To use the DC technique Programming, is reformulated a linear problem and then, it solves multiple linear problems until you get the right solution.

López-Santana et al. [1] developed a model that involves multiple products and demand under uncertainty for the DND problem. They propose a fuzzy linear programming model to solve the problem. There are several models have been proposed for optimization under uncertainty and a variety of algorithms have been developed and used successfully in many applications [13–16].

Similar problems with demand under uncertainty have been worked by [17] in which the demand has a stochastic behavior described by a probability density function and solved with stochastic programming. Bread and Nagi [18] use an approximation by stochastic programming and a solution heuristic for this problem.

Finally there are models that involve uncertainty by using fuzzy logic given the difficulty of programming stochastic, by applying fuzzy sets and numbers to handle uncertainty in some parameters of a linear programming model [19].

3 Fuzzy Linear Fractional Programming (FLFP)

We consider a linear fractional programming model with fuzzy constraints and fuzzy objective function based in [19]. The FLFP problem may be written as:

$$\max \widetilde{Z} = \frac{\sum \widetilde{c}_j x_j + \widetilde{\alpha}_j}{\sum \widetilde{d}_j x_j + \widetilde{\beta}_j} \tag{1}$$

Subject to

$$\sum \widetilde{a}_{ij} x_j \leq \widetilde{b}_i,\ i = 1, 2, \ldots, m, \tag{2}$$

$$x_j \geq 0,\ j = 1, 2, \ldots, n. \tag{3}$$

We assume that, $\tilde{c}, \tilde{\alpha}, \tilde{d}, \tilde{\beta}, \tilde{a}$, and $\widetilde{b}$ are triangular fuzzy numbers. Let $r \in (0, 1]$ be the grade of satisfaction associated with the fuzzy constraints of the problem. According with [19], the fuzzy constraints (2) are to be understood with respect to the ranking relation. Thus, for r $\in (0, 1]$, the feasible set of the FLFP problem can be described as $S_r = \left\{x | x \in R^n, x > 0,\ \sum a^L_{ijr} x_{jr} \leq b^L_{ir},\ \sum a^M_{ijr} x_{jr} \leq b^M_{ir},\ \sum a^U_{ijr} x_{jr} \leq b^U_{ir}, \forall i, j\right\}$.

Then, a vector $x \in S_r$ is called r–feasible solution of FLFP problem and is an optimal solution of the FLFP problem, if there does not exist any $x \in \mathrm{S_r}$, such that $\widetilde{Z}(x^*) \leq_r \widetilde{Z}(x)$.

Let x be the acceptable optimal solution of the FLFP problem. Then the corresponding objective value $\widetilde{Z}(x) = [Z^L(x), Z^M(x), Z^U(x)]$ is called acceptable optimal value of the FLFP problem. The method proposed by [19] are stated as follows:

1. Formulate the real-life problem as a FLFP problem as (1) to (3). We assume that all fuzzy numbers are triangular. Any triangular fuzzy numbers can be represented by $\tilde{a}_{ij} = (s_{ij}, l_{ij}, r_{ij})$, $\widetilde{b} = (t_i, u_i, v_i)$, $\tilde{c}_j = (k_j, m_j, n_j)$, $\tilde{d}_j = (f_j, g_j, p_j)$, $\tilde{\alpha}_j = (q_{j1}, q_{j2}, q_{j3})$, $\widetilde{\beta}_j = (r_{j1}, r_{j2}, r_{j3})$ for all i, j. Then the problem obtained in Step 1 may be written as:

$$\max \tilde{Z} = \frac{\sum (k_j, m_j n_j) x_j + (q_{j1}, q_{j2}, q_{j3})}{\sum (f_j, g_j, p_j) x_j + (r_{j1}, r_{j2}, r_{j3})} \tag{4}$$

Subject to

$$\sum (s_{ij}, l_{ij}, r_{ij}) x_j \leq (t_i, u_i, v_i),\ i = 1, 2 \ldots, m, \tag{5}$$

$$x_j \geq 0,\ j = 1, 2, \ldots, n. \tag{6}$$

2. To transform both the objective function and constraints into its equivalent crisp problem (a crisp problem could be stated as a mathematical programming model without the fuzzy information). Then the problem may be written as follows:

$$\max \frac{\sum k_j x_j + q_1,\ \sum (m_j - k_j)x_j + (q_2 - q_1),\ \sum (n_j + k_j)x_j + (q_1 + q_3)}{\sum f_j x_j + r_1,\ \sum (g_j - f_j)x_j + (r_2 - r_1),\ \sum (p_j + f_j)x_j + (r_1 + r_3)} \tag{7}$$

Subject to

$$\sum s_{ij} x_j \le t_i,\ i = 1, 2\ldots, m, \tag{8}$$

$$\sum (l_{ij} - s_{ij})x_j \le (u_i - t_i), i = 1, 2, \ldots, m, \tag{9}$$

$$\sum (r_{ij} + s_{ij})x_j \le (v_i + t_i), i = 1, 2, \ldots, m, \tag{10}$$

$$x_j \ge 0,\ j = 1, 2, \ldots\ldots, n. \tag{11}$$

3. The FLFP problem is reduced into an equivalent tri-objective programming (TOP) problem as:

$$\max\left\{\frac{\sum k_j x_j + q_1}{\sum f_j x_j + r_1}, \frac{\sum (m_j - k_j)x_j + (q_2 - q_1)}{\sum (g_j - f_j)x_j + (r_2 - r_1)}, \frac{\sum (n_j + k_j)x_j + (q_1 + q_3)}{\sum (p_j + f_j)x_j + (r_1 + r_3)}\right\} \tag{12}$$

Subject to constrains (8) to (11).

4. Formulate the following model for obtaining lower bounds $Z^L(\mathrm{x}), \mathrm{Z}^M(x)$ and $Z^U(x)$ of the objective value as follows:

$$(\mathrm{LFP}) \max Z^L = \frac{\sum k_j x_j + q_1}{\sum f_j x_j + r_1} \tag{13}$$

$$(\mathrm{MFP}) \max Z^M = \frac{\sum (m_j - k_j)x_j + (q_2 - q_1)}{\sum (g_j - f_j)x_j + (r_2 - r_1)} \tag{14}$$

$$(\mathrm{UFP}) \max Z^U = \frac{\sum (n_j + k_j)x_j + (q_1 + q_3)}{\sum (p_j + f_j)x_j + (r_1 + r_3)} \tag{15}$$

5. The above problems (LFP), (MFP), (UFP) are crisp linear fractional programming problem subject to constrains (8) to (11), which can be solved by transformation of Charnes and Cooper [20] that consists in given a linear fraction as:

$$\max z = \frac{\sum c_j x_j + \alpha}{\sum d_j x_j + \beta} \tag{16}$$

Subject to

$$\sum a_{ij} x_j \le b_i, i = 1, 2, \ldots, m, \tag{17}$$

$$x_j \ge 0,\ j = 1, 2, \ldots, n. \tag{18}$$

The transformation $x_j^{'} = tx_j$ and $\sum d_j x_j + \beta = t$, with a variable $t \geq 0$ transform the problem in a linear programming model as:

$$\max z = \sum c_j x_j^{'} + \alpha t \tag{19}$$

Subject to

$$\sum a_{ij} x_j' - b_i t \leq 0, i = 1, 2, \ldots, m, \tag{20}$$

$$\sum d_j x_j^{'} + \beta t = 1 \tag{21}$$

$$\begin{gathered} x_j \geq 0, j = 1, 2, \ldots, n. \\ t \geq 0 \end{gathered} \tag{22}$$

This equivalent linear programming model is solved with traditional methods.

6. The solutions could be stated and obtain the optimal solutions of $\widetilde{Z} = (Z^L, Z^M, Z^U)$.

4 A FFLP Model to Design Distribution Networks

Our model for the DND problem is based on [1] where a dedicated system to manufacturing of different products for which is considered some possible locations of your production factories and a series of distribution centers where your products will be stored before being transported to the final customers. On the other hand, you should evaluate the type of technology for factory, which has different consumptions and costs. Also, in the warehouse it is necessary to select the most appropriate technology that ensures adequate storage, since the products must be subject to certain conditions of storage. As main assumption of this problem consists in defining as initial condition the selected factories and stores since the model proposed by [1] use binary variables, but our approach does not consider this feature.

The sets are defined as follows:

- L is the product set, indexed in $l = 1 \ldots L$
- I is the set of factories, indexed in $i = 1 \ldots I$
- J is the set of warehouses, indexed in $j = 1 \ldots J$
- K Is the set of sale points, Indexed in $k = 1 \ldots K$
- P is the type of technology of each factory, Indexed in $p = 1 \ldots P$
- Q is the type of technology of each store, indexed in $q = 1 \ldots Q$
- T is the set of periods, indexed in $t = 1 \ldots T$

The parameters are stated as follows:

- M_{ipt}: Maximum capacity matrix (available hours) for the factory i with technology p in the period t.
- V_{jqt}: Maximum Capacity matrix (Available hours) for the distribution center j with technology q in the period t.

- r_{lp}: portion of technology consuming the product lp roduced in a factory with one of the technology p (in hours per unit of product).
- s_{lq}: portion of technology consuming the product l stored in a distribution center with one of the technology q (in hours per unit of product).
- g_{ip}: fixed cost associated with factory opening i with the type of technology p.
- f_{jq}: fixed cost associated with the opening of the distribution center j with the type of technology q.
- c_{ijlp}: Unit cost associated with product type l produced and shipped from a factory i with a technology Q to a distribution center j.
- b_{jklq}: Unit cost associated with product type l stored and shipped from the distribution center j with a technology q to a point of demand k.
- h_{jl}: unit cost of inventory for the product l in the Warehouse j.
- ρ_l: Unit profit associated with a product type l.
- D_{klt}: demand for a point of sale k of the type of product l in the period t.

The decision variables are:

- X_{ijlpt}: Quantity in units shipped from the factory i with technology p to the warehouse j of the product l in period t.
- Y_{jklqt}: Quantity in units shipped from warehouse j with technology q to the point of sale k of the product l in the period t.
- W_{jlt}: Inventory at the end of the period t of the product l in the Warehouse j.

The linear fractional programming model is stated as follows:

$$\max Z = \frac{\mathcal{J}}{\mathcal{C}} \tag{23}$$

$$\mathcal{J} = \sum_{t\in T}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{q\in Q}\rho_l Y_{jklqt} \tag{24}$$

$$\begin{aligned}\mathcal{C} = &\sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{p\in P} c_{ijlp}X_{ijlpt} + \sum_{t\in T}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{q\in Q} b_{jklq}Y_{jklqt}\\ &+ \sum_{t\in T}\sum_{j\in J}\sum_{l\in L} h_{jl}W_{jlt} + \sum_{i\in I}\sum_{p\in P} g_{ip} + \sum_{j\in Q}\sum_{q\in Q} f_{jq}\end{aligned} \tag{25}$$

subject to:

$$\sum_{j\in J}\sum_{l\in L} r_{lp} * X_{ijlpt} \le M_{ip} \quad \forall i \in I, \forall p \in P, \forall t \in T \tag{26}$$

$$\sum_{k\in K}\sum_{l\in L} s_{lq} * Y_{jklqt} \le V_{jqt} \quad \forall j \in J; \forall q \in Q; \forall t \in T \tag{27}$$

$$W_{jlt} = W_{j,l,t-1} + \sum_{i\in I}\sum_{p\in P} X_{ijlpt} - \sum_{k\in K}\sum_{q\in Q} Y_{jklqt} \quad \forall j \in J; \forall l \in L; \forall t \in T \tag{28}$$

$$W_{jlt} = W_{j,l0} + \sum_{i \in I} \sum_{p \in P} X_{ijlpt} - \sum_{k \in K} \sum_{q \in Q} Y_{jklqt} \quad \forall j \in J; \forall l \in L; t = 1 \tag{29}$$

$$\sum_{j \in J} \sum_{q \in Q} Y_{jklqt} \leq D_{klt} \quad \forall k \in K; \forall l \in L; \forall t \in T \tag{30}$$

$$\begin{array}{ll} X_{ijlpt} \geq 0 & \forall i \in I; \forall j \in J; \forall l \in L; \forall p \in P; \forall t \in T \\ Y_{jklqt} \geq 0 & \forall j \in J; \forall k \in K; \forall l \in L; \forall q \in Q; \forall t \in T \\ W_{jlt} \geq 0 & \forall j \in J; \forall l \in L; \forall t \in T \end{array} \tag{31}$$

The objective function (23) seeks to maximize the ration between the profit (24) and the total costs (25) of the distribution network. The constraints (26) and (27) refer to the fulfillment of the capacity of factories and distribution centers in each of the periods, respectively. Constraints (28) are product balance and inventories in each distribution center, for each product and each period. Constraints (29) are special case of (28) when $t = 1$, then this consider the initial inventory denoted by $W_{j,l0}$. Constraints (30) ensure that the demand for each customer, product and each period must meet. Constraints (31) refers to the nature of the decision variables.

For the model described in (23) to (31), all parameters are modeled as triangular fuzzy numbers in similar way of the parameters defined in Sect. 3. To solve the problem, we apply the steps defined in Sect. 3 to obtain the optimal lower bounds $Z^L(\mathrm{x}), Z^M(x)$ and $Z^U(x)$.

5 Example of Application

The following example is presented to show the application of the proposed model and the solution approach. We consider 5 factories, 4 warehouses, 4 customers, 4 and 3 types of technology for factories and warehouses, respectively, 5 planning horizon periods. A single Type of product (although it can be extended to multiple products). The information of the parameters is described in the Tables 1, 2, 3, 4, 5, 6 and 7. The facility is taken the capacity as constant throughout the planning horizon. In addition, the profit is $\rho_{1t} = (100, 200, 250)$ for periods in the planning horizon.

Table 1. Consumption portion of technology (Hours/unit)

Factory				Store			
l, p	r_{lp}^M	r_{lp}^L	r_{lp}^U	l, q	s_{lq}^M	s_{lq}^L	s_{lq}^U
1, 1	0.969085	0.7462	1.15321	1, 1	0.918330	0.78058	1.02853
1, 2	0.389046	0.34625	0.50576	1, 2	0.913122	0.74876	1.08662
1, 3	0.858348	0.65234	1.04718	1, 3	0.994356	0.74577	1.13357
1, 4	0.928971	0.74318	1.11477				

Table 2. Capacities and fixed costs of stores

j, q	V_{jq}^M	V_{jq}^L	V_{jq}^U	f_{jq}^M	f_{jq}^L	f_{jq}^U
1, 1	4500	3735	5625	3650000	3029500	4015000
2, 1	3825	2677.5	4245.75	5600000	4480000	6160000
3, 1	4005	2963.7	4405.5	5600000	4424000	6552000
4, 1	4140	3270.6	5216.4	3400000	2958000	3978000
1, 2	3735	2950.65	4108.5	4300000	3569000	4988000
2, 2	4140	2898	4968	4200000	3276000	5166000
3, 2	3915	3484.35	4580.55	5150000	4532000	5716500
4, 2	3825	3327.75	4590	3650000	2591500	4343500
1, 3	4725	3543.75	5292	4450000	3827000	5740500
2, 3	4185	2971.35	4938.3	4900000	3724000	6125000
3, 3	4545	3636	5726.7	4800000	3792000	5568000

Table 3. Capacity and fixed cost of factories

i, p	M_{ip}^M	M_{ip}^L	M_{ip}^U	g_{ip}^M	g_{ip}^L	g_{ip}^U
1, 1	2000	1460	2500	3090000	2193900	3615300
2, 1	2800	2184	3388	3060000	2601000	3916800
3, 1	2250	2025	2812.5	3480000	2610000	4036800
4, 1	2475	2153.25	2846.25	3480000	2888400	4489200
5, 1	3000	2100	3810	3000000	2490000	3750000
1, 2	2000	1700	2400	2850000	2451000	3192000
2, 2	2175	1653	2414.25	3180000	2226000	3847800
3, 2	2125	1827.5	2741.25	2850000	2536500	3192000
4, 2	2400	1896	3048	3360000	2956800	4368000
5, 2	2275	1774.5	2752.75	3030000	2393700	3393600
1, 3	2225	1780	2692.25	3000000	2340000	3570000
2, 3	2000	1540	2360	3330000	2331000	3796200
3, 3	2675	1872.5	3263.5	3540000	2761200	4566600
4, 3	2250	1980	2902.5	3450000	2794500	4450500
5, 3	2900	2523	3451	3360000	2788800	4368000
1, 4	2200	1760	2596	3120000	2776800	3525600
2, 4	2975	2380	3599.75	2880000	2476800	3571200
3, 4	2475	2004.75	2747.25	3510000	2843100	4457700
4, 4	2900	2552	3654	2880000	2448000	3283200
5, 4	2225	1691	2536.5	3570000	2570400	4533900

Table 4. Cost of shipping from factory to warehouse

i, j, l	c^M_{ijlp}				c^L_{ijlp}				c^U_{ijlp}			
	1	2	3	4	1	2	3	4	1	2	3	4
1, 1, 1	47	59	48	57	40.42	44.25	38.88	49.59	55.93	74.93	55.68	74.1
2, 1, 1	47	50	55	34	39.01	40	47.3	28.9	59.69	55.5	61.05	39.44
3, 1, 1	50	57	38	41	41.5	49.59	33.44	32.39	57	69.54	42.56	47.15
4, 1, 1	29	41	27	59	25.23	32.8	21.87	43.66	37.7	52.48	30.24	65.49
5, 1, 1	43	49	39	56	35.26	43.12	30.03	39.2	53.75	55.86	47.97	68.32
1, 2, 1	31	40	55	55	26.35	31.2	44	47.85	37.2	52	60.5	62.15
2, 2, 1	35	27	30	35	30.8	19.44	24.9	26.6	44.45	34.56	38.7	45.15
3, 2, 1	55	53	43	57	42.35	47.17	31.82	42.18	64.35	60.42	49.02	73.53
4, 2, 1	39	44	37	37	28.86	38.72	31.08	32.56	45.63	49.28	46.62	46.62
5, 2, 1	38	41	43	32	30.78	33.62	34.83	25.6	49.4	52.07	48.16	38.4
1, 3, 1	25	46	58	53	21.5	34.96	49.3	39.22	29.75	57.5	66.7	58.3
2, 1, 3	46	43	43	39	40.94	38.27	36.98	30.42	55.66	54.18	49.88	46.02
3, 3, 1	60	33	52	35	42	27.39	38.48	27.3	69	37.95	58.24	40.95
4, 1, 3	57	44	55	45	41.61	31.24	40.15	40.05	70.11	48.4	69.85	56.25
5, 1, 3	50	26	39	40	44	23.14	31.2	33.6	56	31.98	43.68	47.2
1, 4, 1	52	43	58	31	36.92	36.98	46.98	26.66	62.92	48.16	73.08	35.65
2, 4, 1	33	35	56	52	24.09	30.45	50.4	39	36.96	39.55	69.44	58.24
3, 4, 1	59	55	27	44	43.07	48.95	22.68	31.24	70.21	65.45	32.67	56.76
4, 4, 1	30	40	48	25	21.6	33.2	34.56	18.5	35.1	47.2	60.96	30.75
5, 4, 1	47	30	54	27	33.37	23.1	48.6	20.25	61.1	38.1	59.4	29.97

Table 5. Cost of shipping from warehouse to client

j, k, l	t^M_{jkl}			t^L_{jkl}			t^U_{jkl}		
	1	2	3	1	2	3	1	2	3
1, 1, 1	27	23	27	21.87	20.24	20.25	30.78	25.53	34.02
2, 1, 1	38	25	40	30.4	21.75	28	41.8	31	46.8
3, 1, 1	44	46	33	36.08	40.48	27.72	50.6	59.34	39.93
4, 1, 1	20	36	32	15	27.36	28.48	24.2	43.92	36.8
1, 2, 1	21	26	38	18.27	18.98	26.98	27.3	33.28	45.98
2, 2, 1	42	46	39	33.18	33.12	30.03	54.6	58.88	47.97
3, 2, 1	28	23	24	23.8	18.86	20.88	35.56	29.67	30.24
4, 2, 1	37	46	32	28.86	32.66	23.68	44.4	58.42	36.48
1, 3, 1	40	37	23	35.6	27.75	20.47	45.6	45.88	28.52
2, 1, 3	36	27	45	27.36	22.68	31.5	39.6	33.75	52.65
3, 3, 1	22	41	27	17.6	29.52	19.17	24.42	51.25	34.29
4, 1, 3	48	25	25	39.84	20	18	54.72	31.25	27.75
1, 4, 1	36	38	40	25.92	27.36	35.2	42.84	44.46	47.6
2, 4, 1	42	36	46	37.38	25.2	37.26	52.92	42.12	57.5
3, 4, 1	30	22	46	25.5	17.38	41.4	33.6	27.06	51.06
4, 4, 1	36	47	36	29.16	40.42	27	42.48	52.17	44.64

Table 6. Demand in the planning horizon

k, l, t	D^M_{klt}	D^L_{klt}	D^U_{klt}	k, l, t	D^M_{klt}	D^L_{klt}	D^U_{klt}
1, 1, 1	931	765	1210	3, 1, 3	1149	989	1498
2, 1, 1	987	832	1273	4, 1, 3	1468	1367	1772
3, 1, 1	806	666	1175	1, 1, 4	1933	1831	2158
4, 1, 1	830	699	1200	2, 1, 4	1115	941	1401
1, 1, 2	1155	1001	1379	3, 1, 4	1089	947	1457
2, 1, 2	1270	1121	1525	4, 1, 4	1110	967	1436
3, 1, 2	760	572	1055	1, 1, 5	2287	2148	2582
4, 1, 2	1052	909	1363	2, 1, 5	2425	2240	2686
1, 1, 3	1323	1191	1582	3, 1, 5	951	771	1270
2, 1, 3	1670	1484	2040	4, 1, 5	1572	1390	1873

Table 7. Cost of inventory

j, l	h^M_{jl}	h^L_{jl}	h^U_{jl}
1, 1	11	9.02	12.65
2, 1	10	8.4	11.9
3, 1	10	8	11.7
4, 1	11	9.79	12.32

This problem was executed in Xpress-MP .8.4 on Windows 10 64-bit, with a processor Intel i5 3337 (2 × 1.8 GHz) y 6 GB of RAM. Figure 1 shows the result of the membership function of $\widetilde{Z}(x)$. The fuzzy optimal solution is $\widetilde{Z} = (Z^L, Z^M, Z^u) = (0.37382, 0.75764, 0.93456)$. This result gives a lower bound of the fuzzy optimal solutions for the DND problem and helps the decision makers to involve the uncertain information.

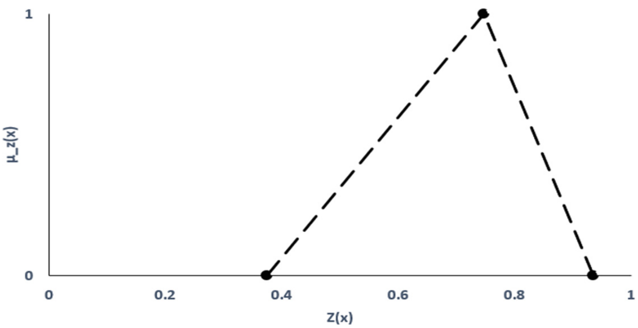

Fig. 1. Membership function of $\widetilde{Z}(x)$

6 Conclusions

This paper reviews the DND problem under uncertain information of costs, profits, capacities and demands. We consider a fractional objective function and solve the fuzzy linear fractional objective function with a method to transform the problem in an equivalent multi-objective linear fractional programming to obtain a lower bound of the fuzzy optimal solution. In addition, our proposed approach is easy to implement than other iterative methods that solve the similar problems in the literature.

This work generates possible futures as extended the approach to multi-objective linear fractional programming. Moreover, it is possible to apply the approach in real-world instances and compare with another approach likes stochastic programming.

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