## PERMISSIONED BLOCKCHAIN IN A SUPPLY CHAIN PROBLEM

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## TABLE OF CONTENTS

List of Figures ..... vi
Chapter 1: Introduction ..... 1
1.1 What is blockchain? ..... 5
1.1.1 Permissioned vs Non-permissioned blockchain ..... 7
Chapter 2: Methodology ..... 10
2.1 Basic Model ..... 10
2.1.1 Consumers ..... 13
2.1.2 Signal Matrices and admissibility ..... 13
2.2 Model with blockchain ..... 15
2.2.1 Manufacturer Problem ..... 17
2.2.2 Potential demands ..... 17
Chapter 3: Results ..... 29
3.1 Welfare Analysis ..... 29
3.1.1 Consumer Welfare ..... 29
3.1.2 Manufacturer welfare ..... 33
Chapter 4: The Platform construction ..... 41
4.0.1 Why blockchain and not other technology? ..... 41
4.0.2 Two Sided markets ..... 46
Chapter 5: Conclusions ..... 50
Appendices ..... 52
References ..... 53

## SUMMARY

This thesis shows the importance of considering technologies such as blockchain to identify an information improvement process in a supply chain problem. We propose an extension of the paper "Economics of Permissioned Blockchain Adoption" (Iyengar et al. 2020) where the number of consumers is discrete. This extension provides comparative statics of the consumer's and manufacturer's welfare under the adoption or non-adoption of the technology. We confirm the results of the original paper regarding the challenges of successful adoption, but with specific results respecting the importance of the size of the market (number of consumers) and the distribution of manufacturers among low and highquality types. Furthermore, we show the advantages that this technology can offer through the Practical Byzantine Fault Tolerance consensus (PBFT) that guarantees the correctness of the information. Finally, using a two-sided markets model we provide a preliminary setup of a fee system that creates incentives to enter the platform: the blockchain network. Our results suggest that implementing blockchain improves the consumer's welfare, while the manufacturer's welfare varies depending on the quality of the product they offer in the market. In addition, we analyze a consensus mechanism to reduce information asymmetries and the restructuring of the problem as a model of two-sided markets.

## LIST OF FIGURES

1.1 Client-Server vs P2P architecture ..... 6
1.2 Example of a hash function with a small excerpt from the book "the foun- dation" of Isaac Asimov. ..... 7
2.1 Detection probability: Plot of the function $1-e^{-\lambda s_{i}}$, when $\lambda=0.1$ and $s_{i}$ is changing. ..... 12
2.2 Graph of the Binomial distribution given the number of manufacturers and the probability of a defective product fixed $(m=20 ; p=0.5)$ and the number of the high-quality manufacturers changing. ..... 25
2.3 Potential demands simulation: Values of the high and low potential de- mands for the manufacturers varying the number of high-quality manufac- turers in the market ( $d$ ). ..... 26
2.4 Potential demands simulation: Values of the high and low potential de- mands for the manufacturers varying the probability of send a correct signal ( $\alpha$ ). ..... 27
2.5 Potential demands simulation: Values of the high and low potential de- mands for the manufacturers varying the number of manufacturers in the market, ..... 28
3.1 Welfare simulation varying the number of manufacturers in the blockchain adoption case. ..... 32
3.2 Plots of the function $y=e^{-x * \lambda} x^{2}$, given different values of $\lambda$. ..... 34
3.3 Cost analysis: Manufacturer cost Varying $\alpha$ given different values of $\lambda$ ..... 35
3.4 $T_{1}-T_{2}$ : Varying $A$. Difference between the cost of the manufacturers when is non-adoption and when there is, given different number of consumers in the market ..... 39
3.5 $T_{1}-T_{2}$ : Varying $m$. Difference between the cost of the manufacturers when is non-adoption and when there is, given different number of manufacturers in the market. ..... 40
4.1 Blockchain roles construction ..... 42
4.2 PBFT consensus protocol ..... 43
4.3 PBFT consensus protocol in the theorical model ..... 44
4.4 PBFT in the theorical model, changing the $\chi$ value parameter ..... 45
4.5 Blockchain network as two sided platform ..... 47

## CHAPTER 1

## INTRODUCTION

Questions regarding competition among firms are answered using the tools of industrial organization. Firms compete with each other by providing information about their product in the hope that these signals will give them a greater share of the demand for that product in a particular market. Information management between business to consumer and business to business is key to understand how firms are able to hold on or increase their share of the market (Demary 2015).

The information that is provided from the business to the consumer is an aspect to consider to understand a firm's performance. An empirical study (Yoon, Guffey, and Kijewski 1993) shows that the information on the sale of services plays a fundamental role in consumers' purchase expectations. In addition, the reputation of a company has an effect on consumers and hence firms need to develop a communication strategy. Therefore, the information that a company provides to consumers in the form of a particular signal, for example, the quality of the product, will affect the potential demand.

New technologies provide different mechanisms to supply information to consumers and different preference shocks affect how consumers receive information. Some new technologies, like blockchain and quantum computing, are considered to be potential gamechangers in terms of their effects on socioeconomic relations. The Covid-19 pandemic is an example of a recent shock that quickly changed established habits and affects consumer demand for some goods and services.

For instance, (Halaburda et al. 2020) states the importance of cryptocurrencies as new asset class with effects on the supply, demand and trading of these and other financial instruments. Furthermore, the technology behind cryptocurrencies has the potential to change financial markets and socioeconomic relationships beyond the mere existence of
a new asset class. (Catalini and Gans 2020) mentions that the appearance of distributed ledger technology (DLT) as exemplified by the bitcoin network allows the reliable transfer of digitally created assets between two parts without the need of an intermediary.

Currently, different forms of DLTs are being implemented for tracking assets that are traded along a supply chain. The most well-known example is everledger for diamonds implemented by Fred Meyer Jewelers company and other similar products, TradeLens for international shipping, and Walmart for food safety. The supply chain is one of the most famous case studies for the implementation of DLTs (Zheng, Zhang, and Gauthier 2020). Understanding how this technology affects economic relationships has started to attract the attention of academics. For example, the supply chain intelligent factoring business model proposed by (Kangning Zheng and Gauthier 2020) points out the added value that blockchain offers in the information flow of a factoring problem; Also, the economic model proposed by (Iyengar et al. 2020) shows how decentralization works within a supply chain and how a set of firms that distribute goods can adopt a blockchain to share information concerning the quality of the products sold to consumers. This is the economic model counterpart of the prototype developed by Walmart using a permissioned blockchain developed by the Linux Foundation (hyperledger fabric).

Iyengar et al. 2020 provides an economic model where DLTs provide mechanisms to improve information problems for the consumers on the quality of goods. The adoption of a permissioned blockchain (a particular DLT) is intended to provide a system to identify bad quality goods and quickly recall these goods and reduce undesirable situations to consumers and costs to distributors. It is clear that firms desire is to send quality information about the products that they sell, so it's important to make a strategy to reduce the information gap, and blockchain can play a role in that strategy. The objective of the model is to show if a permissioned blockchain network increase the welfare of the agents involved: consumers and manufacturers. The main result of the paper suggests that even in the bestcase scenario, where the technology provides a better signal of product quality, adoption of
the technology might not be feasible. First, not all firms benefited from adoption, because some of them prefer to keep information asymmetries of the quality of goods that they sell. Second, even though consumers benefit from adoption if they do not pay for part of the cost of the technology then it might not be adopted by the manufacturers.

In this paper, we propose an extension to the model proposed by Iyengar et al. 2020. For simplicity, the original model assumes a continuum of consumers. The extension is based on a finite number of consumers.

The extension brings some challenges because the distribution of the signals becomes a problem with more dimensions and possibilities. The modeling problem is solved by considering a signal vector from manufacturers to consumers. The model becomes analytically untraceable for the potential demand but we can still simulate scenarios of potential demand and pin down the original model Iyengar et al. 2020 in the case when blockchain technology is adopted. In addition, the structure of the signal matrix has some elements to characterized the economic effects of the consensus mechanism (byzantine-fault tolerant consensus) used in permissioned blockchain (e.g. hyperledger fabric). The consensus mechanism is overlooked in the original paper and most of the literature on permissioned blockchains.

The extension of the model proposed by Iyengar et al. 2020 provides comparative statistics that are easier to understand given the variation on the most important parameters: the number of consumers (this is not feasible in the original model), manufacturers, and in particular the number of high-quality manufacturers. For example, we confirm that this information-related innovation benefits primarily the high-quality manufacturers with an increase in the potential demand. However, under adoption, the significant increase in the potential demand is very quickly driven down by the potential entrance of new high-quality manufacturers. We also confirm that the consumer always benefits from adoption. Under non-adoption consumers welfare is independent of the number of manufacturers and only depends on the quality dispersion of the goods. Under full adoption of blockchain, con-
sumer welfare only increases for a few number of manufacturers but when there is a large number of manufacturers then welfare behaves as in the no-adoption case.

Our simulations also provide an important conclusion regarding manufacturer welfare. Following Iyengar et al. 2020 we also consider the extreme cases: non-adoption and full adoption. Our results show that costs are the main driver of welfare when comparing these extreme cases. The simulations show that adoption is favorable for a fixed size of the market (number of consumers) because for a very large size the potential demand is large enough to cover the cost of defective products and adoption is not beneficial. In addition, if there is a large number of manufacturers the benefits of the increase in potential demand from a better informational environment are significantly reduced, and adoption is not beneficial. In summary, our results are in line with Iyengar et al. 2020 but the extension provides a more specific cost-benefit analysis in terms of the two main agents in the model: consumers and manufacturers.

There are three contributions from this work to the literature. The first one is based on the implementation of a permissioned blockchain problem into a supply chain problem with a fixed number of agents. This extends the study of potential demands by suppliers and in turn, establishes the basis to understand how the cost mechanism works for the firms to adopt or not blockchain.

The second one is relatedd to the implementation of a consensus mechanism into the blockchain model. Different types of consensus mechanisms affect the level of transparency across the network ( Chod et al. 2019). We show that in the supply chain model the success of the Byzantine Fault Tolerance consensus is related to the number of high-quality manufacturers. This is interesting because it reinforces the idea that product quality will be enhanced under full adoption.

The third contribution is to use the extended model to identify the network externalities and relate the solution to a problem of two-sided markets. We provide an expression for the profit function of the platform operator that is a first step toward finding the optimal
pricing strategies in the platform.
The document is organized as follows. chapter 1 provides the introduction and a brief explanation of blockchain technology. chapter 2 presents the model proposed in Iyengar et al. 2020 and the extension developed for the thesis. chapter 3 uses the extended model to derive welfare analysis and comparative statistics regarding the most important parameters of the model. chapter 4 discusses the technological innovation in the context of the supply chain model and the results in relation to the literature on two-sided markets. chapter 5 concludes.

### 1.1 What is blockchain?

Information technology and in particular the internet has transformed socio-economic relations and provided the opportunity to exchange digital assets. Because of security concerns regarding the exchange of information and digital assets over a public information infrastructure like the world wide web, cybersecurity has become increasingly important for governments, firms, and individuals. Cryptography is a core element in cybersecurity and has become a popular topic because of the increasing numbers of cyberattacks in the world (Jang-Jaccard and Nepal 2014) and how increasingly elaborate they are (Tang et al. 2016).

Blockchain is the result of a group of existing technologies and it is introduced in 2008 (Pilkington 2016) with the creation of the bitcoin network. Blockchain is defined as an impenetrable security system that guarantees the anonymity of users and transparency in the transfers made there (Nakamoto 2009). These transfers are registered in a distributed and decentralized database with some particular properties that we will introduce in the following paragraphs.

The network structure is based on a "peer-to-peer" (P2P) architecture (Milojicic et al. 2002) which compared to the client-server architecture allows nodes in the network to connect with each other without the presence of a regulator that manages the data requests in the network. This means that this architecture is decentralized. Figure 1.1 represents the
difference between both architectures.


Figure 1.1: Client-Server vs P2P architecture

There are different advantages of a decentralized architecture: If a node stops working, the information flow in the network keeps going without problems because the architecture management does not fall to a specific node, this implies that the information is not stored only in one place (Puthal et al. 2018). Given the previous property, the network inherits the immutability property where the information in the network cannot be maliciously tampered with Hofmann et al. 2017, because the data is not concentrated.

Another important advantage of blockchain is the security system. The use of hash functions guarantees network sustainability decreasing the probability of cyberattacks. This is possible because the construction of this function is based on the concept that is called a "one-way function" (Impagliazzo and Luby, n.d.) ${ }^{1}$ and also a property to avoid collisions. Without collision's every element in the image of the function has only one element in the domain of the function; this is important because the elements in the function image, the hash codes, are unique and this allows to identify of every transaction in the network (Rogaway and Shrimpton 2004). Figure 1.2 shows an illustration of the hash function.

Blockchain is also a distributed log system. This means that every node can make transactions into the network but the information will be registered in the blockchain only if "special nodes" accept the transaction, this process is known as consensus. The con-

1. The most relevant characteristic of this type of function is that is easy to encrypt but is difficult to decrypt, meaning that the inverse function is hard to compute


Figure 1.2: Example of a hash function with a small excerpt from the book "the foundation" of Isaac Asimov.
sensus protocols or algorithms in blockchain allow reaching an agreement about the data that will be stored and shared through the network. Furthermore, the consensus is also the mechanism that makes it very difficult to tamper with the information that is stored in the database that is distributed over the network (Bach, Mihaljevic, and Zagar 2018). For example in the Bitcoin network, special nodes known as miners follow the consensus algorithm that allows that network to write information on the database. The rule only allows miners to add new blocks (write onto the database) if they are able to win an open competition to find the proper hash codes to link a new block to the previous block, therefore increasing the size of the blockchain. The miner that solves the puzzle not only gets to incorporate information into the new block but also gets a reward in the form of newly minted bitcoins. The consensus algorithm currently in use in the bitcoin network is known as "Proof-of-Work" (PoW) (Kiayias and Zindros 2018).

PoW is one of many proposed consensus algorithms for blockchain networks. Currently, it is also the mechanism that is used in the most well-known permissionless networks such as Bitcoin and Ethereum. In the next section, we will explore other consensus mechanisms and other types of blockchain.

### 1.1.1 Permissioned vs Non-permissioned blockchain

Two functions exist for the nodes that are part of the blockchain network: read and write data. Both functions can be restricted to public or private access, the information may have restricted access or be read-only. In addition, the managers of a blockchain project may
restrict the users that can change or write information stored in the network.
The two types of blockchain are distinguished by these node functions, in a permissionless blockchain entry is not restricted so every node in the network can read, and if the node desires also can write but this is indirectly restricted for some nodes. This is because the consensus algorithm, in a particular PoW requires a lot of computational power to successfully add new blocks to the network and hence write new information on the database (Zhao, Yang, and Luo 2019). On the other hand, in a permissioned blockchain writing and reading the information can be restricted by the managers of the blockchain, this type of restriction is given because all nodes are identified and anonymity does not exist. In addition, the consensus algorithms that can be established in the network restrict the functions of the nodes, for example, the "Proof-of-Authority" protocol sets the write function only to the nodes that the network managers select (Angelis et al. 2018).

Another important difference between these two types of blockchain is the infrastructure that supports the network. In a permissionless blockchain, the infrastructure is already there, which means that if a node desires to enter into the network he/she can plug into the existing network. While in a permissioned blockchain the infrastructure is nonendogenous, it has to be constituted from "mother block" using some framework like R3, Corda, or Fabric (Polge, Robert, and Le Traon 2021).

Finally, the fixed and variable costs change depending on the blockchain type (Yang et al. 2020). A technical report by EY (Brody et al. 2019) shows the costs of implementation of both types of blockchain through a time frame of 5 years. The conclusion is that in the first two years the permissioned blockchain incurs in higher cost because of the fixed cost to build the network, but in the next years, the variable cost will be equal to or less than the cost of the permissionless blockchain. This result depends on the particular setup of the exercise and cannot be easily adapted to every use case. The difficulty with these initial exercises is that they lack an economic model to understand the decision that determines the adoption of the technology and the motivation behind the use of the technology in a
particular context.
One of the most important consensus mechanisms used in permissioned blockchains is Byzantine-Fault-Tolerant. This type of consensus tries to solve the byzantine-generals problem proposed by (Lamport, Shostak, and Pease 1982) which consists of a coordination problem: the decision that a group of generals of a byzantine army to attack or not a city. The capture of the city is feasible if all generals at different parts of the city's walls attack simultaneously. The problem appears when some generals are traitors (a malicious agents). These traitors send mixed signals: first, send the decision to attack to a group of generals that have a decisive majority to attack. Second, they send a decision to retreat to the group which has a majority decision to retreat. In this way, this consensus has to provide a mechanism so that the decision that will be taken should be the one that has the majority of the non-traitor generals. The generals in the network represent specific nodes of the blockchain, and the correct information can be distributed in different ways, for instance, if each general/node has a unique digital signature.

Although the nodes writing information onto the shared database used in the network are identified there is still the risk that one of them will act maliciously and send the tampered information. The solution to this is quite simple if the consensus requires that the messages or transactions transmitted must be signed by the respective sender. This of course will immediately allow identifying the malicious node in the network. So adoption of blockchain technology in this context forces a truth-revealing mechanism that we will consider explicitly in the economic model. Meaning that if we are considering the signals from firms/distributors to consumers then under the adoption of a permissioned blockchain then distributors are forced to send the same quality signal to all of the consumers. In the sequel, we will show that this imposes an important restriction on the signal matrix that is shared among the agents interacting in the economic model. This is an element that is particularly relevant for the extension of the model proposed by Iyengar et al. 2020.

## CHAPTER 2

## METHODOLOGY

### 2.1 Basic Model

The economic model considers the adoption problem, therefore we must consider the decisions of the agents with and without adoption as in (Iyengar et al. 2020). We have three agents involved in this model: the vendor, the manufacturers, and the consumers, however, the vendor is not an active participant ${ }^{1}$. In an extended supply chain, these agents will abstract different roles: vendor as the producer, manufacturer as the distributor/retailer, and consumer as the customer ${ }^{2}$.

A single vendor is assumed, $m$ manufacturers where $m \geq 2$ and a discrete number of consumers $A \geq 3$ consumers. The vendor can offer two types of products according to their quality: high or low-quality products, these will be defined as H and L respectively where $0<L<H$. The manufacturers sell the products for the vendor but the information of the product quality is only known by them, which means that it is private information. For simplicity, we assume that the manufacturer and vendor are of the same quality as the goods that it supplies to consumers, where $q_{i} \in\{H, L\}$ for $i=1, \ldots, m$ denotes the true quality of the good/manufacturer. The quality of the manufacturer is a fair draw of nature, $P\left(q_{i}=H\right)=P\left(q_{i}=L\right)=\frac{1}{2}$. However, the manufacturer may send a signal of quality to the consumer that is different across consumers and also different from the true quality of the product it distributes. This is important because in the context of blockchain adoption and the type of consensus that we consider (byzantine-fault-tolerant) we will not allow sending different signals of quality across consumers. The consensus mechanism

1. There is no explicit role for the platform provider, rather the active agents choose adoption and must pay the cost of adoption.
2. Participants in extended supply chains
requires that the signals are signed by the sender, therefore it is cost-less to verify that the manufacturer sends different quality signals to the consumers. The manufacturer must be consistent in sending the same signal to consumers but he can still send a high-quality signal event if he is a low-quality manufacturer.

Consumers receive a signal from the manufacturer. This signal is information about the product quality that they are buying. We denote the signal of the manufacturer to the consumer $v$ about manufacture quality $i$ as $\tilde{q}_{i v} \in\{H, L\}$, so the signals of the A consumers for the $m$ manufacturers is denoted by $\left\{\left\{\tilde{q}_{i v}\right\}_{v \geq 3}^{A}\right\}_{i \geq 2}^{m}$

As mentioned previously, the true quality of the product is private information of the manufacturer, however, the consumer may receive a different signal of quality before he acquires the good. Why would it be optimal for a manufacturer to send a false signal of quality? because the manufacturer's demand will depend on the quality perceived (though the signal) by the consumer, hence low-quality manufacturers have an incentive to send high-quality signals and increase their market share.

Let the conditional probability of a correct (incorrect) signal from manufacturer $i$ to consumer $v$ be, $P\left(\tilde{q}_{i v}=H \mid q_{i}=H\right)=\alpha$ with $\alpha \in\left[\frac{1}{2}, 1\right)\left(P\left(\tilde{q}_{i v}=H \mid q_{i}=L\right)=1-\alpha\right)^{3}$. We consider that is always more likely to get a correct signal because of the lower bound on $\alpha$.

Iyengar et al. 2020 proposed a discrete three-period model, $t=0,1,2^{4}$. In $t=0$, the consumers receive the signal from each manufacturer and decide which product to demand, in this way we have the demand vector $\left(s_{1}, s_{2}, \ldots, s_{m}\right)$ where $\sum_{i=1}^{m} s_{i}=A$, this means that all consumers buy in this market, to end this period, the vendor produces the goods and send the products to the manufacturers to later be delivered to consumers ${ }^{5}$. An exogenous probability $p \in(0,1)$ defines if the product can be defective. Then in $t=1$, if the consumer detects the defective product, it can be exchanged immediately by the
3. This is symmetric for the low quality good, $P\left(\tilde{q}_{i v}=L \mid q_{i}=L\right)=\alpha$ and $P\left(\tilde{q}_{i v}=H \mid q_{i}=L\right)=1-\alpha$.
4. Before the first period there is a draw of nature that predetermines the true quality of manufacturers.
5. Product demand for each consumer is normalized to 1 unit so that the sum of the demand of all consumers is equal to the number of consumers A .
manufacturer. We can think that the manufacturer can make an immediate recall of the defective product which generates no additional cost to the manufacturer. Detection by the consumer $v$ occurs with a probability $1-e^{-\lambda s_{i}}$, with $\lambda>0$ (See Figure 2.1).


Figure 2.1: Detection probability: Plot of the function $1-e^{-\lambda s_{i}}$, when $\lambda=0.1$ and $s_{i}$ is changing.

In the final period $t=2$, the product defects become public knowledge and the assumption is that there is a significant cost to the manufacturer to recall the product at this stage; the details regarding this cost will be explained in subsection 2.2.1. It is important to consider that the model does not consider repeated interaction between consumers and manufacturers so there is no reputational cost. This cost reflects that the manufacturer has to give a larger fixed compensation to the consumer for the faulty good if he/she waits too long to recall the product. This is different from the reputation cost faced if there is
repeated interaction.

### 2.1.1 Consumers

In the basic model, the only agent that has to take decisions is the consumer. $\mathrm{He} /$ she decides from which manufacturer to buy from. The information is imperfect so the problem consists in:

$$
\begin{equation*}
\max _{i} E_{v}[\mathbf{q}] \tag{2.1}
\end{equation*}
$$

The $\mathbf{q}$ inside the expected value indicates the quality signal vector that consumer $v$ receives from the $m$ manufacturers. In case of indifference, the consumer selects randomly and uniformly from the manufacturers with the same signal. We are also assuming that consumers do not share information regarding the signals, this is how a manufacturer could send different signals to the consumers.

### 2.1.2 Signal Matrices and admissibility

In the model proposed by Iyengar et al. 2020 there is a continuum of consumers so the parameter $\alpha$ (the probability of receiving the true quality of the good) also represents the percentage of the $A$ consumers that receive a correct signal. We adjust the model to make it consistent with the fact that we consider a discrete number of consumers instead of a continuum. Therefore, the signals do not only vary across the manufacturers but also along the dimension of the consumers ${ }^{6}$.

To illustrate the signals we define a signal matrix which is constructed in the following way.
6. For a continuum of consumers there is no need to worry about the exact distribution of the signals across them but rather the percentage of consumers that receive the signal $H$ and $L$. This is not the case when we have a discrete number of consumers

Consider the consumer $v$, an example of the signals that he/she receives is:

$$
\tilde{\mathbf{q}} \mathbf{v}=\left(\begin{array}{c}
L_{1}  \tag{2.2}\\
H_{2} \\
L_{3} \\
\vdots \\
H_{m}
\end{array}\right)
$$

Now, if we see the problem from the manufacture $i$ view, it sends A signals $\left(q_{i 1}, q_{i 2}, \ldots \ldots q_{i A}\right)$. Finally, we combine the A consumer vectors with the m manufacture vectors, it results in the signal matrix $\tilde{q}$ :

$$
\tilde{\mathbf{q}}=\left(\begin{array}{ccccc}
L & L & H & \ldots & L  \tag{2.3}\\
L & L & H & \ldots & L \\
L & L & L & \ldots & L \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
L & L & H & \ldots & H
\end{array}\right)_{M x A}
$$

According to the notation defined earlier the proportion of consumers receiving a correct signal is at least half of the number of consumers ${ }^{7}$.

Therefore, given this lower bound on $\alpha$, the model does not admit the following cases:

- All of A consumers receive an incorrect signal from manufacture $i$.
- More than $50 \%$ of the A consumers receive an incorrect signal of the manufacture $i$.

A signal matrix that is consistent with the assumptions on the proportion of consumers that receive the correct signal will be called the admissible matrix.

This implies that there is a restriction on the signal vector that consumers receive from
7. Note that $\alpha A$ gives the proportion of consumers that receive a correct signal, for example, if $\alpha=0.7$ and $A=11$, we round up this proportion: $\lceil\alpha A\rceil=\lceil 11 * 0.7\rceil=8$
manufacturers. Let's show an example of an admissible information matrix.

$$
\tilde{\mathbf{q}}=\left(\begin{array}{lll}
L & L & H  \tag{2.4}\\
L & L & H \\
H & H & H
\end{array}\right)_{3 x 3}
$$

In the previous signal matrix, we have the base case of the supply chain problem. We have 3 manufacturers (rows) and 3 consumers (columns). The first manufacturer is of low-quality because he/she sends two low-quality signals and one high-quality signal. The second manufacturer is also of low-quality. Finally, the third manufacturer sends a highquality signal to all consumers, so it has to be a high-quality manufacturer. A signal matrix that we cannot accept would be the case where the third manufacturer (last row) sends two or three low-quality signals while he is a high-quality manufacturer:

$$
\tilde{\mathbf{q}_{\mathbf{E}}}=\left(\begin{array}{ccc}
L & L & H  \tag{2.5}\\
L & L & H \\
L & L & H
\end{array}\right)_{3 x 3},
$$

then $\mathrm{q}_{\mathrm{E}}$ is not an admissible signal matrix.

### 2.2 Model with blockchain

Adoption of this new technology must consider the benefits and the cost. In Iyengar et al. 2020 the benefit is obtained by a better information environment for consumers through two channels and fixed costs that are paid by manufacturers and in the second stage also by consumers. The first channel consists of a better probability of coincidence between the signal and the true quality of the good. The second channel implies that the relevant probability of detection of defective goods is the aggregate detection probability rather than the individual probability because of the shared information through the blockchain.

We go a step further and introduce into the model the type of information environment that is consistent with a consensus protocol that has been successfully implemented in permissionless blockchain and in the context of supply chain management: Byzantine-fault-tolerant consensus that will be explained in the chapter 4.

In Iyengar et al. 2020 blockchain adoption is modeled as follows.
For manufacturers to enter the blockchain network they must pay $\chi_{i} \geq 0^{8}$. On the other hand, consumers receive a better signal from the manufacturers that adopt the technology (enters the blockchain network). With adoption the conditional probability of having a high quality good from a high quality signal always increases: $P\left(\tilde{q}_{i v}=H \mid q_{i}=H\right)=\hat{\alpha}=\alpha+\delta$ with $\alpha \in\left[\frac{1}{2}, 1\right)$ and $\delta \in(0,1-\alpha]$. As in Iyengar et al. (2020) $\tilde{\alpha}$ represents the conditional probability with adoption, $\tilde{\alpha}=\alpha+\delta$ where $\tilde{\alpha} \geq \alpha$.

The decision problem also involves the manufacturers that decide whether to adopt or not blockchain technologies. As we mentioned earlier, they have to pay a cost but they also have benefits to enter into the blockchain network. If there is more demand for the manufacturer, he/she can detect with a better probability the defective products. In comparison to the basic model, the manufacturer detects the lower quality products with the following probability $1-\prod_{i \in I} e^{-\lambda s_{i}}$ where $I$ indicates the manufacturers that adopt blockchain.

The product term inside the probability comes from the public information that is in the blockchain, i.e when the blockchain enters into the equation the private information from the manufacturers that accept to enter into the blockchain becomes public information in the network.

Recall that the manufacturers that do not detect the defective products in the second period $(t=1)$ have to pay a higher recall cost.
8. This cost in the model is a fixed cost. In reality, a firm entering a permissioned network must pay a fixed cost used to build the network and a marginal cost of using the network depending on the intensity.

### 2.2.1 Manufacturer Problem

The manufacturer solves the following problem

$$
\max _{a_{i}} E_{i}\left[s_{i}\left(a_{i}, a_{-i}\right)\right]-E_{i}\left[c\left(s_{i}\left(a_{i}, a_{-i}\right)\right)\right]-\chi_{i} a_{i}
$$

If we develop each term we find that the problem is equal to:

$$
\begin{equation*}
E\left[\max _{s_{i}} E_{i}\left[s_{i}\right]-E_{i}\left[c\left(s_{i} Y_{i}\right)\right]\right]=P\left(q_{i}=L\right) E\left[Z_{\alpha}^{L}-p e^{-\lambda Z_{\alpha}^{L}}\right]+P\left(q_{i}=H\right) E\left[Z_{\alpha}^{H}-p e^{-\lambda Z_{\alpha}^{H}}\right] \tag{2.6}
\end{equation*}
$$

Where $Z_{\alpha}$ is the potential demand from the consumers that receive a correct signal and $Z_{1-\alpha}$ is the potential demand from the consumers that receive a wrong signal ${ }^{9} . p e^{-\lambda Z_{\alpha}^{L}}$ and $p e^{-\lambda Z_{\alpha}^{H}}$ are the expected cost that a manufacturer has to pay when there is a defective product that has been detected by consumers in the last period $t=2$.

From a given realization of signal matrices, the potential demand faced by each manufacturer is estimated.

### 2.2.2 Potential demands

The potential demands are the number of consumers that a manufacturer expects to sell the product. In this way, the potential demand will consist in two terms, the number of consumers and the probability that they buy to him/her, given that, each manufacturer construct him/her potential demand depending on the signal that he/she sends and the signal than the others send. In this case, we are going to assume a generic manufacturer which is going to have two general cases send the correct/incorrect signal and from that, 4 sub-cases are going to appear.

## - First case, Low quality manufacturer

9. We mean by a correct signal situation where the quality of the good and the signal coincide and an incorrect signal is when the signal denotes a quality that is not equivalent to the true quality of the product.

This means that the real quality of the manufacturer $i$ is low, however, he can increase or decrease demand for his product depending on the signal that is sent. We are going to analyze the generic consumer $v$ who receives a signal $L$ or a wrong signal $H$ from the manufacturer $i$. This produces four situations:

$$
-\tilde{q}_{j, v}=L \forall j \neq i
$$

This expression means that the consumer $v$ receives a $L$ signal of all $j$ manufacturers different from $i$ manufacturer. In the signal matrix, the signal vector that the consumer $v$ receives is:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
L  \tag{2.7}\\
L \\
\vdots \\
L
\end{array}\right)
$$

The red element is the signal that the consumer $v$ receives from $i$ manufacturer. In this case, the manufacturer $i$ has potential demand because he/she could at least receive demand from consumer $v$, who is indifferent because he observes the same quality signal from all of the manufacturers. This situation can happen with the following probability:

$$
\begin{equation*}
P\left(\tilde{q}_{i, v}=L, \tilde{q}_{-i, v}=L\right)=\frac{m!}{m!(m-m)!} *\left(\frac{1}{2}\right)^{m} \tag{2.8}
\end{equation*}
$$

Remember that $m$ is the total of manufacturers in the market. Finally, we can see also that the manufacturer $i$ sends a true signal (being of low quality), so the consumers that receive a true signal are given by: $\lceil\alpha A\rceil$.

$$
\text { - } \tilde{q}_{j, v}=L \forall j \neq i \text { and } \tilde{q_{i}^{v}}=H
$$

The second situation appears when manufacturer $i$ sends a wrong signal equal to $H$ and the rest of manufacturers send $L$ as a signal. This looks like this:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
H  \tag{2.9}\\
L \\
\vdots \\
L
\end{array}\right)
$$

In this situation, the consumer $v$ demands only from $i$ with a probability equal to:

$$
\begin{equation*}
P\left(\tilde{q}_{i, v}=H, \tilde{q}_{-i, v}=L\right)=\frac{m!}{(m-1)!(m-(m-1))!} *\left(\frac{1}{2}\right)^{m}\left(\frac{1}{2}\right)^{m-(m-1)}=m\left(\frac{1}{2}\right)^{m} \tag{2.10}
\end{equation*}
$$

So, $i$ is sending a wrong signal, we focus on the $(1-\alpha)$ consumers. The final expression results in:

$$
\begin{equation*}
m *\left(\frac{1}{2}\right)^{m} *\lceil(1-\alpha) A\rceil \tag{2.11}
\end{equation*}
$$

- $\tilde{q_{j, v}}=H$ for some $j \neq i$ and $\tilde{q_{i}^{v}}=H$

The third situation looks in the signal vector of $v$ like this:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
H  \tag{2.12}\\
H \\
\vdots \\
L
\end{array}\right)
$$

In this case, the demand of $v$ could be to $i$ or $j$. Let $d$ denote the number of manufacturers that send a signal $H$ to $v: \tilde{q_{j, v}}=H$, so that demand will be distributed uniformly among the manufacturers sending the high-quality signal. The probability that this happens is given by:

$$
\begin{equation*}
\frac{m!}{d!(m-d)!} *\left(\frac{1}{2}\right)^{d}\left(\frac{1}{2}\right)^{m-d}=\frac{m!}{d!(m-d)!} *\left(\frac{1}{2}\right)^{m} \tag{2.13}
\end{equation*}
$$

Finally, we are analyzing the consumers that receives a wrong signal so the total expression is:

$$
\begin{equation*}
\frac{m!}{d!(m-d)!} *\left(\frac{1}{2}\right)^{m} * \frac{\lceil(1-\alpha) A\rceil}{d} \tag{2.14}
\end{equation*}
$$

- $\tilde{q}_{j}^{v}=H$ for some $j \neq i$

The final situation is the result of the existence of at least one manufacturer $j$ different from $i$ that sends a $H$ signal to the consumer $v$. In the signal matrix, the signal vector of $v$ is given by:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
L  \tag{2.15}\\
H \\
\vdots \\
L
\end{array}\right)
$$

The blue element is the manufacturer that sends a $H$ signal. This situation occurs with a probability equal to $1-r$, where $r$ is equal to the sum of the probability of the previous three sub-cases. In this sub-case, the manufacturer $i$ will not receive demand from $v$ because he/she prefers the manufacturer $j$ or any other with a high-quality signal.

## - Second case, High quality manufacturer

The second case means that the real quality of the manufacturer is high. However, his potential demand will depend on his/her signal and the signal of others. We are going to analyze the demand of a generic consumer $v$ who receives a signal $H$ of the manufacturer $i$. This produces two situations:

$$
-\tilde{q_{j, v}}=H \text { for some } j \neq i
$$

In this situations, at least one manufacturer $j$ different from $i$ is also sending a $H$ signal to $v$. This looks like this:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
H  \tag{2.16}\\
H \\
\vdots \\
L
\end{array}\right)
$$

The manufacturer $j$ in this case given by the blue element is also sending the same signal of $i$ to $v$, so the demand will be distributed equally among the manufacturers sending the high quality signal. To find the probability, let $d$ denote the number of manufacturers that send to $v$ a $H$ signal: $q_{j, v}=H$. From the binomial probability we can see that the probability of this situation is given by: $\frac{m!}{d!(m-d)!} *\left(\frac{1}{2}\right)^{m}$, the complete expression is:

$$
\begin{equation*}
\frac{m!}{d!(m-d)!} *\left(\frac{1}{2}\right)^{m} * \frac{\lceil(\alpha) A\rceil}{d} \tag{2.17}
\end{equation*}
$$

Also, we can see that in this case we are analyzing the consumers that receive a correct signal because $i$ is sending the signal of his/her product quality.

- $q_{j, v}=L$ for all $j \neq i$

In this second situation all manufacturers different from $i$ are sending a $L$ signal. It looks like this:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
H  \tag{2.18}\\
L \\
\vdots \\
L
\end{array}\right)
$$

So, we can conclude that consumer $v$ only demands from manufacturer $i$. This happens with the following probability:

$$
\begin{equation*}
P\left(\tilde{q}_{i, v}=H, \tilde{q}_{-i, v}=L\right)=\frac{m!}{(m-1)!(m-(m-1))!} *\left(\frac{1}{2}\right)^{m}=m *\left(\frac{1}{2}\right)^{m} \tag{2.19}
\end{equation*}
$$

Again, we are focusing that $i$ is sending a correct signal, so the complete expression results in:

$$
\begin{equation*}
m *\left(\frac{1}{2}\right)^{m} *\lceil(\alpha) A\rceil \tag{2.20}
\end{equation*}
$$

- $q_{j, v}=L \forall j \neq i$ and $q_{i}^{v}=L$

The third situation look like this:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
L  \tag{2.21}\\
L \\
\vdots \\
L
\end{array}\right)
$$

In this case, there is a positive probability that $i$ receives demand because consumer $v$ is indifferent among the signals. This occurs with a probability equal
to:

$$
\begin{equation*}
P\left(\tilde{q}_{i, v}=L, \tilde{q}_{-i, v}=L\right)=\frac{m!}{m!(m-m)!} *\left(\frac{1}{2}\right)^{m} \tag{2.22}
\end{equation*}
$$

Finally, remember that the consumer is receiving a wrong signal so the final expression is equal to that probability multiplying by: $\lceil(1-\alpha) A\rceil$.
$-q_{j, v}=H$ for some $j \neq i$ and $q_{i}^{v}=L$

The final situation results when $i$ sends a wrong signal to $v$, this look like this:

$$
\tilde{\mathbf{q}_{\mathbf{v}}}=\left(\begin{array}{c}
L  \tag{2.23}\\
H \\
\vdots \\
L
\end{array}\right)
$$

In this situation, the consumer $v$ will not demand from manufacturer $i$ because he/she prefers to buy to $j$, this occurs with a probability given by: $1-r$, where $r$ is equal to the sum of the probability of the previous three sub-cases.

To understand the different cases for potential demand we summarize them in Table 2.1. The table defines the potential on manufacturer $i$ based on the quality of his own signal $q_{i}=L, H: Z_{\alpha}^{q_{i}=x}\left(\tilde{q}_{i}=x\right)\left(\right.$ match good quality with signal), $Z_{1-\alpha}^{q_{i}=x}\left(\tilde{q}_{i}=y\right)($ no match good quality with signal) and the signal of the other $j$ manufacturers.

The potential demand of manufacturer $i$ we have the following expression for highquality manufacturer (without loss of generality this also occurs to a low-quality manufacturer):

$$
\begin{equation*}
Z_{i}^{H}=Z_{\alpha}^{H}+Z_{1-\alpha}^{H} \tag{2.24}
\end{equation*}
$$

| Signal, $j \neq i$ | $\tilde{q}_{i}=H$ | $\tilde{q}_{i}=L$ |
| ---: | ---: | ---: |
| $\tilde{q}_{j}=H$ | uniform distribution <br> with providers $j$ with | 0 |
|  | same signal (some $\left.\tilde{q}_{j}=H\right)$ |  |
| $\tilde{q}_{j}=L$ | all demand | uniform distribution <br> (all $\left.\tilde{q}_{j}=L\right)$ <br> with providers $j$ with <br> same signal (all $\left.\tilde{q}_{j}=L\right)$ |
|  |  |  |

Table 2.1: Potential demands for the generic manufacturer

Where:

$$
\begin{gather*}
Z_{\alpha}^{H}=\frac{\lceil\alpha A\rceil}{d} *\left[\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}+m\right)\right] \\
Z_{1-\alpha}^{H}=\frac{\lceil(1-\alpha) A\rceil}{m} *\left(\left(\frac{1}{2}\right)^{m}\right) \tag{2.25}
\end{gather*}
$$

While:

$$
\begin{gather*}
Z_{\alpha}^{L}=\frac{\lceil\alpha A\rceil}{m} *\left(\left(\frac{1}{2}\right)^{m}\right) \\
Z_{1-\alpha}^{L}=\frac{\lceil(1-\alpha) A\rceil}{d} *\left[\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}+m\right)\right] \tag{2.26}
\end{gather*}
$$

## How does the parameter d affects the potential demands?

Recall that the parameter $d$ represents the number of manufacturers sending a high-quality signal in the market. Given the construction of the potential demands, this value appears from the binomial distribution generated by the case when the demand is distributed by the manufacturers that send a high-quality signal to the consumers. The binomial distribution that we have, has an equal probability that the manufacturer be low or high quality, as we mention before. For instance, if we have in a market 20 manufacturers then the binomial probability is going to determine how many of them could be high-quality manufacturers. The Figure 2.2 shows the probability distribution of high-quality manufacturers in the
market with 20 manufacturers.


Figure 2.2: Graph of the Binomial distribution given the number of manufacturers and the probability of a defective product fixed $(m=20 ; p=0.5)$ and the number of the highquality manufacturers changing.

Given the previous analysis of the binomial distribution of the manufacturers quality, we can see that the binomial probability equation appears in the potential demand equations, this term is $\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}\right)$, also, from the Figure 2.2 we can see that the expected number of high-quality manufacturers is consistent with the matrix rule that more than $50 \%$ of the total number of manufacturers will send, in a random way, the correct signal to the consumers. Therefore, the probability of a higher or lower potential demand will also depend on $d$, and from now on we are going to assume that $d$ will vary in the interval of $[50 \% \mathrm{~m}, 100 \% \mathrm{~m})$. The case of $100 \% \mathrm{~m}$ is not considered in the simulation, this implies that all manufacturers are high-quality.

The Figure 2.3, shows different potential demands that will have a higher and lower quality manufacturer, holding $A, m$ and $\alpha$ fixed and varying $d$.


Figure 2.3: Potential demands simulation: Values of the high and low potential demands for the manufacturers varying the number of high-quality manufacturers in the market ( $d$ ).

As we can see from the Figure 2.3, when the value of $d$ is higher the potential demand will be reduce for the low and high quality manufacturer.

## Potential demand simulations: Varying $A, m$ and $\alpha$

From the potential demand equations, it can be inferred that a higher value of $A$ (representing the size of the market) gives to the manufacturers a higher value of potential demand, even to a low-quality manufacturer.

$$
\begin{equation*}
\frac{d Z_{i}^{H / L}}{d A} \geq 0 \tag{2.27}
\end{equation*}
$$

The Equation 2.27 is always true because $A$ is multiply by $\alpha$ or $1-\alpha$, and by a probability which is always positive or equal to zero.

Now, the assumption of $\alpha$ variable, allows us to think that higher values of $\alpha$ benefit the high-quality manufacturers and non-benefit the low-quality manufacturers; The Figure 2.4
rectifies this idea, simulating the potential demands holding $A, m$ and $d$ fixed.


Figure 2.4: Potential demands simulation: Values of the high and low potential demands for the manufacturers varying the probability of send a correct signal $(\alpha)$.

The Figure 2.4 shows the potential demands when the value of $\alpha$ is increasing, as we expected, a higher value of $\alpha$ benefits the high-quality manufacturer while the low-quality manufacturer is negatively affected. Remember, that $\alpha$ is the probability to send a correct signal to the consumers. This feature of the model is also the cornerstone of Iyengar et al. 2020 result, where technology improvements such as blockchain that provide a better information environment to consumers predominantly benefit the high-quality manufacturers. The downside is that it will eventually drive out manufacturers from the market with a higher market concentration among the higher quality manufacturers.

Finally, the results of increasing the number of manufacturers is presented in Figure 2.5.
The final simulation Figure 2.5 shows that in a market with more manufacturers, the potential demand of the generic manufacturer will decrease because there are more competitors. However, the result is stronger in relative terms for the high-quality manufacturer because the expected number of high-quality manufacturers is nominally larger.


Figure 2.5: Potential demands simulation: Values of the high and low potential demands for the manufacturers varying the number of manufacturers in the market,

## CHAPTER 3

## RESULTS

### 3.1 Welfare Analysis

### 3.1.1 Consumer Welfare

The expected utility for a generic consumer $v$ will depend on the coincidence between the signal and the true quality of the product. Therefore, we must consider two cases based on the signal that is send by the manufacturer. This result already assumes that blockchain technology has been adopted therefore, following Iyengar et al. 2020, we already incorporate $\tilde{\alpha}$ in the expression and recall that $\tilde{\alpha}>\alpha$ :

- $\tilde{q}_{i}^{v}=H$,

$$
\begin{aligned}
E\left[q_{i} \mid \tilde{q}_{i}=H\right] & =H P\left(q_{i}=H \mid q_{i}=H\right)+L P\left(q_{i}=L \mid q_{i}=H\right) \\
& =H \tilde{\alpha}+L(1-\tilde{\alpha}) \\
& =\tilde{\alpha}(H-L)+L
\end{aligned}
$$

- $\tilde{q}_{i}^{v}=L$,

$$
\begin{aligned}
E\left[q_{i} \mid \tilde{q}_{i}=L\right] & =H P\left(q_{i}=H \mid q_{i}=L\right)+L P\left(q_{i}=L \mid q_{i}=L\right) \\
& =H(1-\tilde{\alpha})+L \tilde{\alpha}+L-L \\
& =(1-\tilde{\alpha})(H-L)+L
\end{aligned}
$$

If the consumer receives $L$ or $H$ signal from all of the manufacturers the probability of this case is given by:

$$
P\left(\tilde{q}_{1}^{v}=L, \ldots, \tilde{q}_{m}^{v}=L\right)=P\left(\tilde{q}_{1}^{v}=L\right)\left(\tilde{q}_{m}^{v}=L\right)=\left(\frac{1}{2}\right)^{m}
$$

In other case we have the probability that at least one manufacturer sends a different signal from the other manufacturers, equal to: $1-\left(\frac{1}{2}\right)^{m}$

So, the welfare of the generic consumer is given by:

$$
\begin{align*}
W^{v} & =\left(\left(1-\left(\frac{1}{2}\right)^{m}\right)(\tilde{\alpha}(H-L)+L)+\left(\left(\frac{1}{2}\right)^{m}\right)((1-\tilde{\alpha})(H-L))+L\right.  \tag{3.1}\\
& =\left(\left(\tilde{\alpha}\left(1-\left(\frac{1}{2}\right)^{m}\right)+(1-\tilde{\alpha})\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L
\end{align*}
$$

Now for the $A$ consumers the total welfare (with blockchain) is equal to:

$$
\begin{equation*}
W^{B}=A\left(\left(\left(\tilde{\alpha}\left(1-\left(\frac{1}{2}\right)^{m}\right)+(1-\tilde{\alpha})\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L\right) \tag{3.2}
\end{equation*}
$$

Without blockchain it is equal to:

$$
\begin{equation*}
W_{N}=A\left(\left(\left(\alpha\left(1-\left(\frac{1}{2}\right)^{m}\right)+(1-\alpha)\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L\right) \tag{3.3}
\end{equation*}
$$

Remember: $\tilde{\alpha}>\alpha \Rightarrow W^{B}>W^{N}$. This implies that consumers always benefit from the adoption of blockchain. As in Iyengar et al. 2020 consumer welfare is also affects by the number of manufacturers and quality dispersion $(H-L)$. We are going to see how these terms change the consumer welfare under two cases: adoption and non-adoption.

## Consumer Welfare: Non-adoption case

In this case, we are going to use the extreme case where $\alpha=\frac{1}{2}$. Developing the Equation 3.3 we have:

$$
\begin{equation*}
W_{N}=A\left(\left(\left(\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{m}\right)+\left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L\right) \tag{3.4}
\end{equation*}
$$

$$
\begin{gather*}
W_{N}=A\left(\left(\left(\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{m}+\left(\frac{1}{2}\right)^{m}\right)\right)(H-L)\right)+L\right)  \tag{3.5}\\
W_{N}=A\left(\left(\frac{1}{2}(H-L)\right)+L\right) \tag{3.6}
\end{gather*}
$$

From the Equation 3.6, we can see that consumer welfare does not depend on the number of manufacturers. On the other hand, we can see that the quality dispersion change has a positive linear effect on welfare. This implies that for goods with similar quality welfare effects are negligible, only when there are is a significant quality dispersion it becomes relevant for the consumers. Quality has to be important to test signal enhancement technology related to the quality of the goods.

## Consumer Welfare: Blockchain full adoption case

For this case, as we said previously, $\alpha=1$ (full adoption). Full adoption considers that all manufacturers use the technology, share the information on the potential defective goods, and in addition given the consensus mechanism there is only one admissible signal matrix where a manufacturer cannot send misleading signals to consumers because they are required to submit signed messages.

Now, developing the Equation 3.2 we have:

$$
\begin{equation*}
W^{B}=A\left(\left(\left(1\left(1-\left(\frac{1}{2}\right)^{m}\right)+(1-1)\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L\right) \tag{3.7}
\end{equation*}
$$

$$
\begin{equation*}
W^{B}=A\left(\left(1-\left(\frac{1}{2}\right)^{m}\right)(H-L)+L\right) \tag{3.8}
\end{equation*}
$$

From Equation 3.8 we can see that the number of manufacturers affects the welfare only if they are below a certain value. The Figure 3.1 shows how while the value of $m$ is getting higher the consumer welfare converges to a fixed value. We can conclude that the welfare in the blockchain adoption is non-decreasing function on the number of manufacturers. Mathematically, we can see that if $m \Rightarrow \infty$ the term $\left(\frac{1}{2}\right)^{m} \Rightarrow 0$.

A:1000 alpha:1 H-L:20


Figure 3.1: Welfare simulation varying the number of manufacturers in the blockchain adoption case.

Given the previous conclusion, in a market with a enough number of manufacturers the term $\left(\frac{1}{2}\right)^{m}$ can be deprecated, so the Equation 3.8 as follows:

$$
\begin{equation*}
W^{B}=A((H-L)+L) \tag{3.9}
\end{equation*}
$$

$$
\begin{equation*}
W^{B}=A(H) \tag{3.10}
\end{equation*}
$$

The Equation 3.9 and Equation 3.10 lets us conclude something similar to the proposal 3.4 in the Iyengar et al. 2020 paper. Under full adoption when the dispersion quality increase consumer welfare also increases; In particular, we find that the increase is higher than the case with no adoption.

### 3.1.2 Manufacturer welfare

The total welfare of the $m$ manufacturers is given by:

$$
\begin{equation*}
W_{m}=m\left(\frac{1}{2} E\left[Z_{\alpha}^{L}-p e^{-Z_{\alpha}^{L} \lambda} c\left(Z_{\alpha}^{L}\right)\right]+\frac{1}{2} E\left[Z_{\alpha}^{H}-p e^{-Z_{\alpha}^{H} \lambda} c\left(Z_{\alpha}^{H}\right)\right]\right) \tag{3.11}
\end{equation*}
$$

Factoring terms:

$$
\begin{gather*}
W_{m}=m\left(\frac{1}{2} E\left[Z_{\alpha}^{L}+Z_{\alpha}^{H}\right]\right)-T  \tag{3.12}\\
T=\frac{1}{2} m p E\left[e^{-Z_{\alpha}^{L} \lambda} c\left(Z_{\alpha}^{L}\right)+e^{-Z_{\alpha}^{H} \lambda} c\left(Z_{\alpha}^{H}\right)\right] \tag{3.13}
\end{gather*}
$$

As we can see, the manufacturers welfare depends on the values of the potential demands. The assumption that (Iyengar et al. 2020) established on the welfare said that the manufacturers, in absence of blockchain, will benefit from higher values from the potential demands.

$$
\begin{equation*}
\frac{d W_{m}}{d Z_{\alpha}^{L / H}} \geq 0 \tag{3.14}
\end{equation*}
$$

Without loss of generality, developing the Equation 3.14 expression for $Z_{\alpha}^{H}$ we obtain:

$$
\begin{equation*}
\pi^{\prime}\left(Z_{\alpha}^{H}\right)-\left[-p \lambda e^{-\lambda Z_{\alpha}^{H}} c\left(Z_{\alpha}^{H}\right)+p e^{-\lambda Z_{\alpha}^{H}} c^{\prime}(x)\right] \geq 0 \tag{3.15}
\end{equation*}
$$

$$
\begin{gather*}
\pi^{\prime}\left(Z_{\alpha}^{H}\right)+p \lambda e^{-\lambda Z_{\alpha}^{H}} c\left(Z_{\alpha}^{H}\right) \geq p e^{-\lambda Z_{\alpha}^{H}} c^{\prime}(x)  \tag{3.16}\\
\frac{\pi^{\prime}\left(Z_{\alpha}^{H}\right) e^{\lambda Z_{\alpha}^{H}}}{p}+\lambda c\left(Z_{\alpha}^{H}\right) \geq c^{\prime}(x) \tag{3.17}
\end{gather*}
$$

## Cost Analysis

Given the result in the Equation 3.17 from the welfare assumption, the use of a $c(*)$ as a convex function, such as $x^{2}$, will not break the inequality.

Whereas, seeing the Equation 3.13 which shows the T expression that has the following parameters: $\lambda, p, m, Z_{\alpha}^{L}$ and $Z_{\alpha}^{H}$, we can analyze the cost equation by parts, first the $e^{-Z_{\alpha}^{L} \lambda} c\left(Z_{\alpha}^{L}\right)+e^{-Z_{\alpha}^{H} \lambda} c\left(Z_{\alpha}^{H}\right)$; In this part of the equation we have an only parameter that is $\lambda$, in the Figure 3.2 we can see different plots of the function $y=e^{-x * \lambda} x^{2}$, where $\lambda$ takes different values.


Figure 3.2: Plots of the function $y=e^{-x * \lambda} x^{2}$, given different values of $\lambda$.

The Figure 3.2 allows us to conclude that the value of $\lambda$ nearest, equal or higher to one gives to the manufacturers a cost that converges to zero. So, we can set $\lambda(0,0.2$. Moreover, if we focus on any of the three plots, for instance, the image of $\lambda=0.1$, we can see that for potential demand values higher than 100 the cost will be near to zero.

To find a good value of $\lambda$ to simulate the cost and welfare function, it is important to
identify that given fixed values for $A, m, d$ and a resulting potential demand varying $\alpha$, which value of $\lambda$ give us a majority values of the cost different from zero. The Figure 3.3 illustrates the total cost, and the higher/low cost with $A=1000, m=20, d=10$ and $p=0.5^{1}$.


Figure 3.3: Cost analysis: Manufacturer cost Varying $\alpha$ given different values of $\lambda$

From the Figure 3.3, we can see the total costs, low-quality costs, and the high-quality costs (colors from the figure), and the conclusion created is that for $\lambda$ values such as 0.2 the cost for the high quality manufacturer is equal or near to zero while $\alpha$ is getting higher; Whilst, for the values 0.1 and 0.15 the cost function has an interesting behavior. This is happening for the values of the potential demands, for example, given the values fixed ( $A=1000, m=20, d=10$ ) we obtain a potential demand equal to 88.10 , that if we see Figure 3.2 for any value of $\lambda$ the cost is almost zero. So, for the welfare analysis we are going to consider the values that give to all manufacturers cost that vary significantly with the increase/decrase of $\alpha$, for instance $\lambda=0.1$.

1. After different simulations from $p$ we conclude that varying this parameter not make significant changes into the cost results, in this way we set $p$ equal to 0.5 .

These simulations results are also consistent with a value of the probability of detection that is not too high so as to make the adoption of a new technology for detection irrelevant.

## Manufacturer Welfare under non-adoption and full adoption

As in the consumer case and in the Iyengar et al. 2020 paper we will consider welfare under the two extreme cases: no adoption when $\alpha=\frac{1}{2}$ and full adoption of the technology $\alpha=1$. The reason is that we want see the behavior of the extension under these conditions.

First, we are going the start with the non-adoption case. From the potential demands equations, we are going to solve the problem when the real quality of the good is low:

$$
Z_{\alpha}^{L}=\frac{\lceil\alpha A\rceil}{m}\left(\left(\frac{1}{2}\right)^{m}\right)+\frac{\lceil(1-\alpha) A\rceil}{d}\left[\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}+m\right)\right]
$$

Replacing $\alpha$ by $\frac{1}{2}$ :

$$
Z_{\alpha}^{L}=\left\lceil\frac{1}{2 m d} A\right\rceil\left(\frac{1}{2}\right)^{m}\left[\left(\frac{m!}{d!(m-d)!}+m\right)\right]
$$

Given this mathematical solution, from the $Z_{\alpha}^{H}$ equation we can see that factoring $\alpha$ and $(1-\alpha)$ the $Z_{\alpha}^{H}$ expression is equal to $Z_{\alpha}^{L}$.

After found $Z_{\frac{1}{2}}^{L}$ and $Z_{\frac{1}{2}}^{H}$, we can solve the sum between those terms. As we found before, we can denote $Z_{\frac{1}{2}}^{L}=Z_{\frac{1}{2}}^{H}=Z^{*}$.

$$
Z_{\frac{1}{2}}^{L}+Z_{\frac{1}{2}}^{H}=2 Z^{*}=2 *\left\lceil\frac{1}{2 m d} A\right\rceil\left(\frac{1}{2}\right)^{m}\left[\left(\frac{m!}{d!(m-d)!}+m\right)\right]
$$

Replacing in the first part of the manufacturer welfare equation we find:

$$
\frac{1}{2} m E\left[Z_{\frac{1}{2}}^{L}+Z_{\frac{1}{2}}^{H}\right]=m\left\lceil\frac{1}{2 m d} A\right\rceil\left(\frac{1}{2}\right)^{m}\left[\left(\left(\frac{m!}{d!(m-d)!}+m\right)\right]=m Z^{*}\right.
$$

Then the benefits of the manufacturers are given by $m Z^{*}$.
Now, we can solve the cost part that we denoted as $T$, in Equation 3.13:

$$
T=\frac{1}{2} m p E\left[e^{-Z_{\frac{1}{2}}^{L} \lambda} c\left(Z_{\frac{1}{2}}^{L}\right)+e^{-Z_{\frac{1}{2}}^{H} \lambda} c\left(Z_{\frac{1}{2}}^{H}\right)\right]=m p\left[e^{-Z^{*} \lambda} c\left(Z^{*}\right)\right]
$$

Finally, the manufacturer welfare is equal to:

$$
W_{m}=m Z^{*}-m p\left[e^{-Z^{*} \lambda} c\left(Z^{*}\right)\right]
$$

So, the welfare to the low and high quality manufacturer is equal in this case.
Now, solving the problem with blockchain adoption, we have:

$$
\begin{equation*}
Z_{1}^{L}=\left\lceil\frac{A}{m}\right\rceil \tag{3.18}
\end{equation*}
$$

and

$$
\begin{equation*}
Z_{1}^{H}=\left\lceil\frac{A}{d}\right\rceil \tag{3.19}
\end{equation*}
$$

$Z_{1}^{L}$ occurs with a probability equal to $\left(\frac{1}{2}\right)^{m}$ and $Z_{1}^{H}$ with $\left(\frac{1}{2}\right)^{m}\left(\binom{m}{d}+m\right)$. Then the manufacturer welfare is equal to:

$$
\begin{gathered}
W_{m, 1}=m\left(\frac{1}{2} E\left[Z_{1}^{L}-p e^{-Z_{1}^{L} \lambda} c\left(Z_{1}^{L}\right)\right]+\frac{1}{2} E\left[Z_{1}^{H}-p e^{-Z_{1}^{H} \lambda} c\left(Z_{1}^{H}\right)\right]\right) \\
\frac{1}{2} m E\left[Z_{1}^{L}+Z_{1}^{H}\right]=\frac{1}{2} m\left(\left\lceil\frac{A}{m}\right\rceil\left(\frac{1}{2}\right)^{m}+\left\lceil\frac{A}{d}\right\rceil\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}+m\right)\right. \\
\frac{1}{2} m E\left[Z_{1}^{L}+Z_{1}^{H}\right]=\frac{1}{2} m\left\lceil\frac{A}{m d}\right\rceil\left(\frac{1}{2}\right)^{m}\left[\left(\frac{m!}{d!(m-d)!}+m\right)\right]
\end{gathered}
$$

Now, solving the $T$ equation in Equation 3.13:

$$
T=\frac{1}{2} m p E\left[e^{-Z_{1}^{L} \lambda} c\left(Z_{1}^{L}\right)+e^{-Z_{1}^{H} \lambda} c\left(Z_{1}^{H}\right)\right]+m \chi
$$

When blockchain is adopted, $\chi$ is that cost of enter into the network.

$$
\begin{gathered}
e^{-Z_{1}^{L} \lambda} c\left(Z_{1}^{L}\right)=e^{-\left(\left\lceil\frac{A}{m}\right\rceil\left(\frac{1}{2}\right)^{m}\right)} c\left(\left\lceil\frac{A}{m}\right\rceil\left(\frac{1}{2}\right)^{m}\right) \\
e^{-Z_{1}^{H} \lambda} c\left(Z_{1}^{H}\right)=e^{\left\lceil\frac{A}{d}\right\rceil\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}+m\right)} c\left(\left\lceil\frac{A}{d}\right\rceil\left(\frac{1}{2}\right)^{m}\left(\frac{m!}{d!(m-d)!}+m\right)\right)
\end{gathered}
$$

## Manufacturer Welfare $\alpha=\frac{1}{2}$ - Manufacturer Welfare $\alpha=1$

Making the difference between $W_{m, \frac{1}{2}}$ and $W_{m, 1}$, we are able to find the case when the adoption could be a bad idea thinking in the manufacturer welfare and when the adoption improves the manufacturer welfare value. Unlike Iyengar et al. 2020, the extension of the model does not give us a simple expression for the welfare difference under non-adoption and full adoption. Therefore to understand and interpret the result we simulate the results.

Focusing first in the profit part , for $W_{m, \frac{1}{2}}$ we have: $m Z^{*}$ where $Z^{*}=\left\lceil\frac{A}{2 m d}\right\rceil\left(\frac{1}{2}\right)^{m}\left[\left(\frac{m!}{d!(m-d)!}+\right.\right.$ $m)]$ and for $W_{m, 1}$ we have $m\left\lceil\frac{A}{2 m d}\right\rceil\left(\frac{1}{2}\right)^{m}\left[\left(\frac{m!}{d!(m-d)!}+m\right)\right]$. From the previous expression we have that the difference between both is equal to zero, therefore the key is in the cost difference.

The cost difference consists of seeing which of both expressions is bigger given different values. Set the following variables:

$$
\begin{gather*}
T_{1}=m p e^{-Z^{*} \lambda} * c\left(Z^{*}\right)  \tag{3.20}\\
T_{2}=\frac{1}{2} m p\left[e^{-Z_{1}^{L} \lambda} c\left(Z_{1}^{L}\right)+e^{-Z_{1}^{H} \lambda} c\left(Z_{1}^{H}\right)\right]+m \chi \tag{3.21}
\end{gather*}
$$

For then following simulations, We set the following parameters $A=5000, m=$ $50, d=25, p=0.5, \lambda=0.15, \chi=1$.

In this case, we are going to assume again $c(*)$ as a quadratic function. The following figures shows the cost difference between non-adoption $T_{1}$ and adoption $T_{2}$. So negative values are in favor of non-adoption and positive values are in favor of adoption. First,
the Figure 3.4 shows how given higher values of $A$, the blockchain adoption could be a convenient decision. In a market with few number of consumers, adopting blockchain is not convenient because the cost represented by $\chi$ is bigger enough than the benefits that blockchain gives to the manufacturer. On the other hand, the manufacturer benefits from the adoption if in the market exist a sufficient but not so large number of consumers; and finally, if there is a big number of consumers in the market, then the blockchain adoption is not necessary because the potential demand of the manufacturer is big enough to cover the cost of the defective products, then these costs become negligible.


Figure 3.4: $T_{1}-T_{2}$ : Varying $A$. Difference between the cost of the manufacturers when is non-adoption and when there is, given different number of consumers in the market.

The Figure 3.5 shows a similar result from the previous graph. There is an interval where adopt blockchain is convenient, because the manufacturer wants to guarantee his/her potential demand, while if there is a big number of manufacturers the costs of adoption are greater than the benefits that would be obtained from it.


Figure 3.5: $T_{1}-T_{2}$ : Varying $m$. Difference between the cost of the manufacturers when is non-adoption and when there is, given different number of manufacturers in the market.

## CHAPTER 4

## THE PLATFORM CONSTRUCTION

### 4.0.1 Why blockchain and not other technology?

As we can see from the previous chapters, the theoretical model shows how the consumer and manufacturer welfare is affected when the consumers receive a correct signal of the product quality, furthermore this affects the potential demand of the manufacturer.

The information of the product quality can be written, stored, and read for instance in a database with a connection to a public web page, however, three questions arise from this approach, what is the cost of running the database? How is the security of the information guaranteed and how do we know that the information is accurate?

We are going to focus on the last question and explain the relevance of blockchain technology and consensus in the supply chain. In this supply chain problem, the manufacturers write the information of the product quality and the consumers read it, therefore, we are going to set all manufacturers as validators of the information. The Figure 4.1 shows how blockchain technology works in this context, and in particular, the role of the smart contracts (Zou et al. 2021).

Thus, we can define the consensus protocol in the permissioned network as a combination of the Byzantine-Fault-Tolerant (BFT) problem (explained in the sub-section 1.1.1) and the assumptions on the potential demand matrix in the model. The first property from BFT problem avoids the corruption of the information in a distributed system (Lamport, Shostak, and Pease 1982). On the other hand, the results of the model show that the highquality manufacturers desire to send the correct signal; that is if more than a half of the manufacturers must send the correct signal, at least, that percentage of manufacturers are of high-quality (observation made where we explain the effects on welfare of parameter $d$, see Figure 2.2).


Figure 4.1: Blockchain roles construction

The Practical Byzantine Fault Tolerance consensus (PBFT) proposed by (Castro and Liskov 1999) introduces an algorithm that tolerates the byzantine faults. There are two kinds of faults in a distributed network a "traitor general" which is a malicious node that corrupts the information and a network problem that disconnects a node from the network. In our case, the "traitor generals" are the manufacturers with bad quality products sending good quality signals and this will be the main byzantine failure.

The nodes on the blockchain are arranged sequentially and the operation of the algorithm starts with a request from a client, in our case, the clients are the consumers which is going to receive the product quality signal. Then a primary node in the blockchain receives the request and execute it to the backup nodes, which are the other validator nodes. After the backup nodes respond to the primary and to the other nodes they expect a response from them to achieve a commitment of new information in the blockchain. Finally, all the validator nodes reply to the client.


Figure 4.2: PBFT consensus protocol

The Figure 4.2 shows the complete operation of the PBFT protocol. In the figure the node "backup 3" is the byzantine failure. In the process the traitor general will not respond to the message that the primary node sends to him/her. In this way, the malicious node will not participate in the protocol operation. One important thing to note is that if the primary node is malicious, the consensus has an algorithm called "view change" that is in charge to change the primary node to the next node in the sequence list after a predefined time of no response by the primary node.

Hence, (Castro and Liskov 1999) established different conclusions, the first one and the most important for our case is that to achieve a consensus into the blockchain is necessary that byzantines failures cannot be more than $\frac{1}{3}$ of the total validator nodes. Applying this conclusion to our model the low-quality manufacturers cannot be more than $\frac{1}{3}$ of the total manufacturers in the market or in other words, is necessary that at least $70 \%$ of the manufacturers must be of high-quality, for the consensus to succeed.

To support the conclusion, we compare the cases where there is non-adoption and full adoption, we assume the parameters: $A=5000, m=50, p=0.5, \lambda=0.15$ and $\chi=1$. In the non-adoption case, we assume the $d$ value equal to $50 \%$ while in the full adoption case we change the percentage between $60 \%$ and $100 \%$ as in the Figure 4.3. The interesting
observation from this figure is that while the number of high-quality manufacturers (d) is increasing the adoption of blockchain is less expensive. However, when $d$ is $67 \%$ of manufacturers the adoption does not produce more benefits. This value almost matches the $70 \%$ that we need to secure the information in all the cases; more than $70 \%$ is not necessary because the network will have already achieve consensus on the information.

## T1-T2



Figure 4.3: PBFT consensus protocol in the theorical model

After the previous result, we increase the value of $\chi$ that goes from 1 to 2 (Figure 4.4). In this case, the $70 \%$ of manufacturers achieve the highest benefits under full adoption. Finally, we can conclude that higher values of the cost of adoption $(\chi)$ need higher values of $d$ to achieve the highest profit to the manufacturers. This is happening because if $\chi$ is increasing fewer manufacturers have incentives to be in the blockchain, so the network prefers that
the manufacturers that stay should be of high-quality, in order to archive consensus.


Figure 4.4: PBFT in the theorical model, changing the $\chi$ value parameter

In summary, the extension of the model guarantees that the information in the blockchain network will be accurate: The Byzantine problem is avoided. The economic model we propose enhances the properties of the permissioned blockchain beyond any other distributed database to be applied to the supply chain case. That is, the validators in the network will guarantee that the consumers will not find inconsistencies with concerning what is written on the database (the blocks).

### 4.0.2 Two Sided markets

The extension to (Iyengar et al. 2020) where we introduce a discrete number of consumers provides a modeling framework to address wether a platform such as blockchain in supply chain problem can be considered as a two-sided platform between manufacturers and consumers sharing information on the quality of the goods. Section 3, in particular, Equation 3.8 shows that consumer welfare is a function of the number of manufacturers, quality dispersion $H-L$ and the conditional probability of quality of the good conditional on the signal.

More importantly, Equation 3.11 shows that manufacturer welfare is a function of the potential demand, contingent recall cost on bad products, and lump-sum cost of adoption of blockchain technology. The most important contribution of the paper is that potential demand $Z_{\alpha}^{H / L}$ is a function of the number of consumers, the number of manufacturers and which of them send a high-quality signal, and the conditional probability of quality of the good conditional on the signal.

Using the definition provided by (Rochet and Tirole 2003) our model setup admits network effects because the number of consumers affects the manufacturer welfare and similarly manner the number of manufacturers affect the consumer welfare. We follow (Armstrong 2006) and setup of a two-sided platform and claim that the extension of the model allows us to provide a first approximation of the current supply chain problem within this context.

The Figure 4.5 provides a way to identify the inter and intra externalities in our problem. From the welfare analysis, we conclude different things, the Equation 3.2 shows us that the consumer is not affected positively or negatively by the number of consumers into the blockchain, while for a value $\alpha \neq \frac{1}{2}$ a higher number of manufacturers ( $m$ ) will benefit the consumers.

Nevertheless, in a more general sense, it is unclear how the number of consumers $(A)$ and manufacturers $(m)$ affect the welfare of the manufacturers. This is because we only


Figure 4.5: Blockchain network as two sided platform
analyze the extreme cases of non-adoption versus full adoption. Therefore, in this context, we find that for a certain number of consumers and manufacturers where the manufacturer benefits (in terms of welfare) from the adoption.

The utility of the consumer to be in the blockchain will be given by:

$$
\begin{equation*}
U=\left(\left(\tilde{\alpha}\left(1-\left(\frac{1}{2}\right)^{m}\right)+(1-\tilde{\alpha})\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L-\kappa \tag{4.1}
\end{equation*}
$$

This utility will reflect the benefits that the consumer always receives when blockchain is adopted, but they have to pay to receive these benefits a fee denoted by $\kappa$.

On the other hand, the profit of the manufacturer will be:

$$
\begin{equation*}
\pi_{m}=\left(\frac{1}{2} E\left[Z_{\alpha}^{L}-p e^{-Z_{\alpha}^{L} \lambda} c\left(Z_{\alpha}^{L}\right)\right]+\frac{1}{2} E\left[Z_{\alpha}^{H}-p e^{-Z_{\alpha}^{H} \lambda} c\left(Z_{\alpha}^{H}\right)\right]\right)-\chi+\tau \tag{4.2}
\end{equation*}
$$

Remember that $\chi$ is the cost of the manufacturer to enter into the blockchain. The variable $\tau$ is a benefit to be in the platform, that is paid by the consumer through the platform
(lump-sum transfer from consumer to the manufacturer). Now, the pemissioned blockchain operation needs an intermediary to adequate the platform to all agents, for example, in the case of food safety in a mango supply chain quality problem (Kamath 2018), IBM was in charge of the creation and maintenance of the network in hyperledger fabric; Hence, we introduce the benefit of the blockchain intermediary where this agent incurs the cost of the maintenance of the network and receives payments made by consumers and manufacturers. The profit function for the operator of the platform is,

$$
\begin{equation*}
\pi_{h}=A \kappa+m(\chi-\tau) \tag{4.3}
\end{equation*}
$$

The cost of the blockchain intermediary is based on the fees provided to the manufacturer, represented by $\tau$ in Equation 4.6. Now, from the Equation 4.5 and Equation 4.4, we rewrite the equation:

$$
\begin{gather*}
\kappa=\left(\left(\tilde{\alpha}\left(1-\left(\frac{1}{2}\right)^{m}\right)+(1-\tilde{\alpha})\left(\frac{1}{2}\right)^{m}\right)(H-L)\right)+L-U  \tag{4.4}\\
\chi=\left(\frac{1}{2} E\left[Z_{\alpha}^{L}-p e^{-Z_{\alpha}^{L} \lambda} c\left(Z_{\alpha}^{L}\right)\right]+\frac{1}{2} E\left[Z_{\alpha}^{H}-p e^{-Z_{\alpha}^{H} \lambda} c\left(Z_{\alpha}^{H}\right)\right]\right)+\tau-\pi_{m} \tag{4.5}
\end{gather*}
$$

To solve this problem Armstrong 2006 defines that the number of consumers $A$ and manufacturers $m$ value depends on an increasing function on the utility of the consumers and the profit from the manufacturers: $A=\phi(U)$ and $m=\phi\left(\pi_{m}\right)$.

This provides a mapping from the utility of the consumer and the profits of the manufacturer in terms of the profits of the platform operator,

$$
\begin{equation*}
\pi_{h}\left(U, \pi_{m}\right)=\phi(U) \kappa+\phi\left(\pi_{m}\right)(\chi-\tau) \tag{4.6}
\end{equation*}
$$

This mapping is the first step to derive the optimal pricing strategies of the platform. How-
ever, this result is beyond the scope of the thesis and we leave it as a subject of further research.

## CHAPTER 5

## CONCLUSIONS

Today technology offers us different ways to solve a problem, in this case, an information asymmetry between consumers and manufacturers. A permissioned blockchain network allows the manufacturers to improve the signal that they send to the consumers about the product quality offered. As we saw, the consumer probability detection improves when the manufacturers adopt blockchain, the reduction of the signal matrix equilibrium allows that consumer welfare always is benefited by the adoption. Moreover, under full adoption, the consumers' welfare will benefit in more proportion by the quality dispersion than in the non-adoption case.

However, if we consider the manufacturer welfare adoption is not necessarily the best option. In both cases, the manufacturer's profit is the same, while the costs are different. With two simulations where the number of consumers changes and the second one where the number of manufacturers changes, we see that in a specific range the adoption works. When the consumer's number is getting higher the adoption reaches a limit, where given the value of $\chi$, the manufacturer prefers to not be in the blockchain, this happens because the main reason for the cost in the manufacturers is that the consumers don't detect the product quality, so as we said when the potential demand is higher the cost will be less, and the potential demand increase by the number of consumers in the market. Likewise, when the number of manufacturers is large enough the adoption is not the best option for the manufacturers, because when $m$ is increasing the potential demand will be less for each manufacturer, and also they are paying the cost of $\chi$ to be in the blockchain, so the faster detection of product quality not give them a cost reduction.

The theoretical analysis of the PBFT consensus protocol allows us to understand that is necessary that at least $70 \%$ of the manufacturers in the network must be high-quality
to achieve an agreement and guarantee the consistency of the signals that the consumers receive. Our results suggest that by finding the exact value of $\chi$ given the other market parameters, we can reach the consensus in the blockchain.

Finally, modeling the problem as a two-sided market platform, we identify the way to understand the network externalities to make the incentives to be in the blockchain. We set up the platform operator profit and given this construction we identify a fee strategy for the consumers and the manufacturers. Our future work is to derive optimal prices.

## Appendices

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