Commitment in prices: case for volatility averse consumers

Juan José Galeano Hermosa *
Guillem Roig Roig †

Masters in Economics
Department of Economics
Universidad del Rosario

August 24, 2022

Abstract

This document considers a two period market where a monopoly faces an heterogeneous mass of consumers with a proportion of "savvy" consumers that are averse to price changes in a context where the demand changes in the second period. With this structure, I find that the monopolist sees their benefits diminished by these consumers even when they are only a proportion of the total demand. This is explained because the consumer internalizes both period prices and minimizes the loses caused by the adjustment cost. The main finding of the model is that: when markets have a sufficiently large proportion of consumers with volatility aversion, the monopoly is constrained by the consumer "savvyness" to the changed price. Therefore, a competition authority should not be worried in this market context because the monopoly will not be able to exercise complete market power.

1 Introduction

This article studies the interaction between one monopoly and a mass of consumers. In the model considered, the monopoly faces "savvy" consumers with a key characteristic which is

*Economist, Universidad del Rosario. juanj.galeanohe@urosario.edu.co. I would like to acknowledge my supervisor Guillem Roig who made this work possible. His guidance and advice made possible the completion if this project. I would also like to give special thanks to my family as a whole for their continuous support and understanding when undertaking my research and writing my project.

†Professor, Faculty of Economics, Universidad del Rosario

the aversion to the change in prices, in this model, this article studies two possible strategies of price decision: contingent prices and commitment in prices. An example of this type of market is a retailer who perceives constant shocks in his demand and must set a price for a big set of consumers. So, in markets where a monopoly faces a heterogenous mass of consumers with a proportion of consumers with aversion to changes in prices, how the monopoly sees it's market power affected? Is it profitable to set a commitment in prices to attract the "savvy" consumers that have an aversion to the volatility in prices?

This article contributes to the existing discussion and implementation of behavioral agents in Industrial organization (IO) and how the firms can change their prices decisions by the existence of the behavioral elements in the models. In this article it is considered a Hotelling (1929) classical model of spatial competition. I propose a variant of the Hotelling model in which a monopoly and a mass of consumers interact for two periods, this mass of consumers is expanded considering the article of Armstrong (2015) where the market has a proportion of "savvy" consumers. To make this model interesting, the consumers have to choose between the provider of the good (or service) in every period, but the consumers will have a shock in their valuation for the good. The firms know it and have some expectations or believes about it, and try to take the shock into account in their price decisions. This shock is set with the intention to model the high volatility that firms faces in some markets, like an agricultural market, in the short run. One of the results of the model proposed is that the proportion of "savvy" consumers constrain the market power of the monopoly and those consumers make the monopoly put more similar prices between the two periods.

A growing literature in Behavioral Industrial Organization (BIO) relaxes the assumptions of standard models by focusing on the existence of behavioral consumers. This behavioral consumers have non-standard preferences, in the case of this model, by having a variant of loss-averse consumers. In the literature (Rosato 2014), the loss aversion of the consumers is generated by a reference point that can be modified by the seller. This article incorporates that concept by taking a mass of consumers that have the first period as a reference point. In the case of this article, the consumers do not have only an aversion for the losses, but to any deviation of the reference point.

Another recurring theme in the behavioral IO literature is the consumer heterogeneity, the presence of various types of consumers and how this multiplicity have an impacts in the market outcomes. For this paper the core concept can be related to that of the heterogeneity present in the Armstrong paper (Armstrong, 2015). In the paper of Armstrong: "savvy and non-savvy consumers interact in the marketplace". The current article relates to Armstrong by setting two type of consumers, one averse to the changes in prices, and one that does not.

My article broadly relates also to articles on switching costs and commitement in prices, some papers like Fudenberg and Tirole (2000) where they establish a situation where monopolies can take advantage from their strategic pricing in a situation where there are switching cost between firms. Chen and Pearcy (2010) use the commitment approach to state cases in which loyalty from the consumer can be rewarded. In essence, I relate to those papers by using other costs that are not related to the production process per se, instead those costs are phenomena of the environment/market in which the firms and consumers exist. Papers such as Suslow (1986) and more recently Jullien and Rey (2007) in general focus on the use of switching costs as a force that leads firms to stabilize the price of their goods. By internalizing switching costs, firms can maintain an attractive price for consumers and thus can gain market power. Some examples shown by the previous papers are goods such as durable goods and also strategies in retail prices for big stores.

The rest of the article is structured as follows. Section II contains the structure of the model. Section III analyzes the optimal strategy of the monopoly when: (i) it takes a period-contingent strategy and how the optimal strategy changes with changes in the key parameters (λ, γ) , (ii) it takes a commitment strategy in prices. Section IV presents the comparison between the strategies that the monopoly can take and which benefit is greater according to the parameters. Section V concludes. The proofs of the propositions are in Appendix A and the codes of the simulations in Appendix B.

2 Model

Consider a market composed by a monopoly that sets prices for two periods, and produce a non-durable good. This market is also composed by a mass of consumers that have a preference for the good the monopoly produces. This monopoly and these consumers are located in a Hotelling line, the monopoly will be placed in the origin (0) of the line and the consumers are placed in a position x_i along this line. The consumers also face a transport cost θ when they want to consume the good of the monopoly. The model representation can be seen in Figure 1.

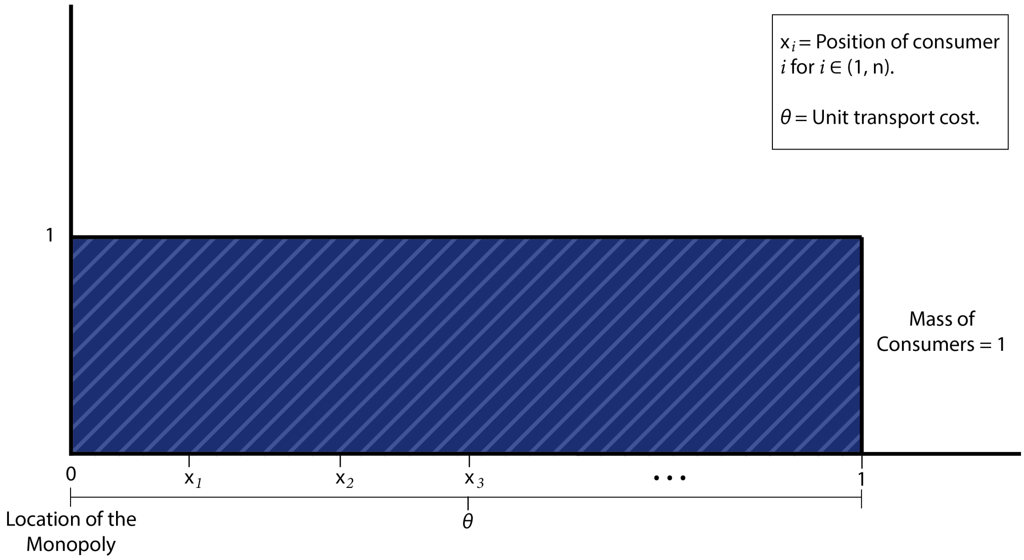


Figure 1: Representation of the Model

The strategies for the agents in the two-period game can be set as follows:

- The consumers can buy or not the product, this action can be made in any of the two periods.
- The firm can set a spot price p_t^M for $t = 1, 2$. The alternative is to set the prices of the two periods from the start of the game (p_1^M, p_2^M) in $t = 0$.

In the model, the behavioral component that is key is that the consumers have another characteristic, a proportion λ of "savvy" consumers that have an aversion for volatility in prices and a proportion of "non-savvy" consumers that do not have it and therefore do not care for the change in prices.

In the first period the consumers decide whether to consume from the firm or to not consume, given their preferences for the good and locations x_i. The utility of not consuming is normalized to 0.

- $U_1^{x_i}$: $\begin{cases} V - p_1^M - \theta x_i & if \;\; Buy \\ \underline{U} = 0 & if \;\; Not \;\; Buy \end{cases}$

In the second period the valuation for the good changes with some probability α to a level $\overline{V}$ and with probability $(1-\alpha)$ to $\underline{V}$, where $\overline{V} > V > \underline{V}$.

In the second period is where the volatility aversion represented by a parameter (γ) has an effect on the consumers utility. This parameter acts upon the absolute difference between the price in the first period and the second period. The monopoly knows then that it will face different situations with certain probability, so this shock causes that the second period price can be $\overline{p}_2^M$ or $\underline{p}_2^M$ with probability α and $(1-\alpha)$ respectively. The utility for the consumers in the second period is:

- U_2^{λ,x_i}: for the "savvy" consumers that buys in the second period:
$$\begin{cases} \overline{V} - \overline{p}_2^M - \theta x_i - \gamma|p_1^M - \overline{p}_2^M| & if \;\; V \;\; changes \;\; to \;\; \overline{V} \\ \underline{V} - \underline{p}_2^M - \theta x_i - \gamma|p_1^M - \underline{p}_2^M| & if \;\; V \;\; changes \;\; to \;\; \underline{V} \end{cases}$$
- $U_2^{1-\lambda,x_i}$,: for the "non-savvy" consumers that buys in the second period:
$$\begin{cases} \overline{V} - \overline{p}_2^M - \theta x_i & if \;\; V \;\; changes \;\; to \;\; \overline{V} \\ \underline{V} - \underline{p}_2^M - \theta x_i & if \;\; V \;\; changes \;\; to \;\; \underline{V} \end{cases}$$

The total benefits of the monopoly are then equal to:

$$\Pi = p_1 D_1 + \alpha * \overline{p}_2 \overline{D}_2 + (1-\alpha) * \underline{p}_2 \underline{D}_2 \tag{1}$$

where $(p_1^M, \overline{p}_2^M, \underline{p}_2^M)$ are the variables the firm will decide and D_t will be the demand it has in the period t: $(1,2)$. This demand is going to depend on the prices that the firms set, so the firm will have a demand each period $D(p_t^M)$ and will decide over p_t^M.

3 Analysis

In the first period, the consumers will decide if they consume or not, given the first period price and their transport cost θ. In the second period, the consumers and the monopoly will know in which state of the world they are $(\overline{V}, \underline{V})$. Given the state of the world, the consumers will also decide if they consume, but now, some proportion λ of "savvy" consumers are going to take into account the first period price, and they will suffer a penalty in their utility if the price between periods differ. This penalization will change then the utility of the consumer by a factor $\gamma \in (0, 1)$ that acts over the absolute difference of the first and second period prices. By reducing the utility, this penalization γ will reduce the demand for the good.

3.1 Equilbrium with contingent prices

In this set up, in the first period exists a consumer who is indifferent between buying and not buying, for this reason, he compares the utility that both options generate. The indifference is implicit in the equation: $V - p_1^M - \theta \hat{x}_i = 0$, where $\hat{x}$ is the location of the indifferent consumer. We solve the equation to know where the indifferent consumer $\hat{x}$ is located, the monopoly can know the amount of consumers that will buy the good and therefore know its demand. In the first period the demand will not depend of the proportion of consumers with volatility aversion λ nor the probability α of changing to the state $\overline{V}$. The demand then for the first period will be:

$$D_1 = \frac{V - p_1}{\theta} \tag{2}$$

Similarly to the process in the first period, in the second period for each state of the shock $(\overline{V}, \underline{V})$ and for each type of consumer ("savvy" or "non-savvy") that exist with probability λ, there is an indifferent consumer located in some point $\hat{x}$. For the "non-savvy" consumers, similarly to the first period, the indifferent consumer will equate the utilities that both options generate: for the positive shock the indifferent consumer will equate $\overline{V} - \overline{p}_2 - \theta \hat{x}_i = 0$ and for the negative shock the indifferent consumer will equate $\underline{V} - \underline{p}_2 - \theta \hat{x}_i = 0$.

For the "savvy" consumers, the penalization γ upon the utility of the consumer will be active. The indifferent consumer for the "savvy" consumers will equate the utility of buying the good and not buying it. For the positive shock the indifferent "savvy" consumer will equate $\overline{V} - \overline{p}_2 - \theta \hat{x}_i - \gamma |p_1^M - \overline{p}_2^M| = 0$ and for the negative shock the indifferent consumer will equate $\underline{V} - \underline{p}_2 - \theta \hat{x}_i - \gamma |p_1^M - \underline{p}_2^M| = 0$.

By solving for the location of the indifferent consumer, the demand generated by the "savvy" and "non-savvy" consumers when there is a positive shock in the valuation $\overline{V}$ will be:

$$D_2^{\lambda} = \frac{\overline{V} - \overline{p}_2 - \gamma|p_1 - \overline{p}_2|}{\theta} \ \ and \ \ D_2^{1-\lambda} = \frac{\overline{V} - \overline{p}_2}{\theta} \tag{3}$$

By solving for the location of the indifferent consumer, the demand generated by the "savvy" and "non-savvy" consumers when there is a positive shock in the valuation $\underline{V}$ will be:

$$D_2^{\lambda} = \frac{\underline{V} - \underline{p}_2 - \gamma|p_1 - \underline{p}_2|}{\theta} \ \ and \ \ D_2^{1-\lambda} = \frac{\underline{V} - \underline{p}_2}{\theta} \tag{4}$$

With the set of demands that are generated, the monopoly finds their best prices, contingent on the demand for the first period maximizing its profits, as described by:

$$\hat{\Pi} = p_1 \left(\frac{V - p_1}{\theta} \right) \tag{5}$$

And for the second period:

$$\begin{aligned} \Pi_2(p_1, p_2) = \alpha & \left[\lambda \overline{p}_2 * \left(\frac{\overline{V} - \overline{p}_2 - \gamma|p_1 - \overline{p}_2|}{\theta} \right) + (1-\lambda)\overline{p}_2 * \left(\frac{\overline{V} - \overline{p}_2}{\theta} \right) \right] \\ + (1-\alpha) & \left[\lambda \underline{p}_2 * \left(\frac{\underline{V} - \underline{p}_2 - \gamma|p_1 - \underline{p}_2|}{\theta} \right) + (1-\lambda) * \left(\frac{\underline{V} - \underline{p}_2}{\theta} \right) \right] \end{aligned} \tag{6}$$

Proposition 3.1. *With a proportion of consumers $\lambda \in (0,1)$ who have volatility aversion, if:*

$$\frac{\overline{V}}{V} > \frac{2(2 + 3\gamma\lambda + (\gamma\lambda)^2)}{(4 + 2\gamma\lambda - 2(\gamma\lambda)^2)} \ \ and \ \ \frac{V}{\underline{V}} > \frac{(6 - 2\gamma\lambda - (\gamma\lambda)^2)}{2(2 - 3\gamma\lambda + (\gamma\lambda)^2)} \tag{7}$$

when the monopoly sets spot prices the Perfect Bayesian Nash equilibrium is:

$$p_1^* = \frac{2V + \alpha\dfrac{\overline{V}\gamma\lambda}{1+\gamma\lambda} - (1-\alpha)\dfrac{\underline{V}(\gamma\lambda)}{1-\gamma\lambda}}{4 - \alpha\dfrac{(\gamma\lambda)^2}{1+\gamma\lambda} - (1-\alpha)\dfrac{(\gamma\lambda)^2}{1-\gamma\lambda}} \tag{8}$$

$$\overline{p}_2^* = \frac{1}{2(1+\gamma\lambda)}\left[\overline{V} + \frac{\gamma\lambda\left(2V + \alpha\dfrac{\overline{V}\gamma\lambda}{1+\gamma\lambda} - (1-\alpha)\dfrac{\underline{V}(\gamma\lambda)}{1-\gamma\lambda}\right)}{4 - \alpha\dfrac{(\gamma\lambda)^2}{1+\gamma\lambda} - (1-\alpha)\dfrac{(\gamma\lambda)^2}{1-\gamma\lambda}}\right] \tag{9}$$

$$\underline{p}_2^* = \frac{1}{2(1-\gamma\lambda)}\left[\underline{V} - \frac{\gamma\lambda\left(2V + \alpha\dfrac{\overline{V}\gamma\lambda}{1+\gamma\lambda} - (1-\alpha)\dfrac{\underline{V}(\gamma\lambda)}{1-\gamma\lambda}\right)}{4 - \alpha\dfrac{(\gamma\lambda)^2}{1+\gamma\lambda} - (1-\alpha)\dfrac{(\gamma\lambda)^2}{1-\gamma\lambda}}\right] \tag{10}$$

Proof. See Appendix 1 □

The equilibrium described in the first proposition shows that if the monopoly knows that there is a shock in the demand in the next period and a proportion of consumers that are averse to changes in prices, the monopoly is going to internalize the change $(\overline{V}, \underline{V})$ and the proportion of the changes. Notice that, when the proportion of "savvy" consumers λ or the penalization in the utility γ equal 0 the price on the first period do not depend on the valuation the consumers will have in the future, and likewise, the second period prices will not depend upon the first period valuation. This intuition is in line with the literature of heterogeneous consumers. It is important to show that for the situation without "savvy" consumers, the first period price if $\lambda = 0$ is $p_1^* = \frac{V}{2}$, the second period price for the positive shock in demand is $\overline{p}_2^* = \frac{\overline{V}}{2}$ and the second period price for the negative shock would be $\underline{p}_2^* = \frac{\underline{V}}{2}$.

Now, the presence of consumers averse to the change in prices $(1 > \lambda > 0)$ makes the monopoly realize that not only the prices of the second period, where the aversion of the "savvy" consumers is active, will have to change. In order to reduce the variance that exists in prices, the monopoly will have to change the prices of the first period so that the losses generated by consumers averse to the change in prices are reduced, implicitly meaning that the valuations of the second period and the probability α is internalized in the first period price p_1.

I find the equilibrium in a three step process of backward induction, first the monopoly finds the best response prices given p_1 and states the conjectures $\overline{p}_2 > p_1$ and $\underline{p}_2 < p_1$, then given the best responses, the monopoly finds the first period price, and then I find if the conjectures hold and in which case te equilibrium holds. In this case it is also necessary to provide the region where the equilibrium holds, this region is found by testing where the assumptions to find the equilibrium hold. This assumptions are: $\overline{p}_2 > p_1$ and $p_1 > \underline{p}_2$, in other words, the shocks in the initial valuation V, have a direct relation with the prices. As it can be resumed in the next plot, the sets where the equilibrium exists for different values of γ and λ are above the blue line and below the orange one. The next plot represents the different values of $(\overline{V}, \underline{V})$ where the equilibrium exists when the value of $\lambda = 0.1$ & $\alpha = 0.5$ and a changing value of γ.

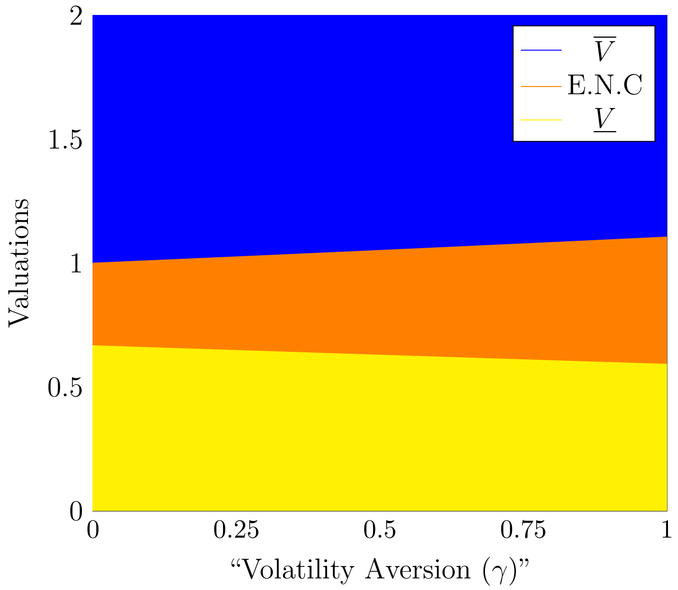


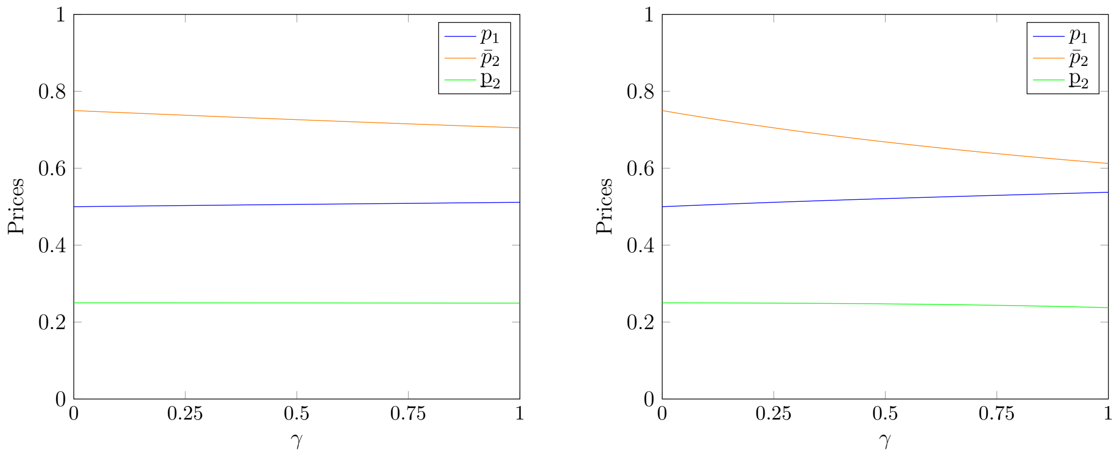
Figure 2: Feasible regions with values of $\lambda = 0.1$ & $\alpha = 0.5$ and a changing value of γ (Blue and yellow region). E.N.C= Equilibrium not characterized.)

Figure 2 shows which are the relative values of the valuation $(\overline{V}, \underline{V})$ when the valuation for the first period V is normalized to 1. The figure shows the values that the pair of valuations $(\overline{V}, \underline{V})$ can have for the equilibrium to exist, essentially for the equilibrium to be characterized the consumers have to have a $\overline{V}$ in the blue region and a $\underline{V}$ in the yellow region. Figure 2 shows that with a greater aversion to volatility, the area in which the pair of valuations $(\overline{V}, \underline{V})$ have an equilibrium like the one established in proposition 3.1 is lower. The reason that $\overline{V}$ has to be higher with values higher than γ is that volatility aversion takes away profits from the company if the price is too high in the first place. To offset the profits for the company, the monopoly needs values higher than $\overline{V}$ which increases the expected value of the profits. Also the negative shock in valuation $\underline{V}$ has to be lower so that the price of the first period p_1 remains low enough so that the volatility is lower.

3.1.1 Comparative static

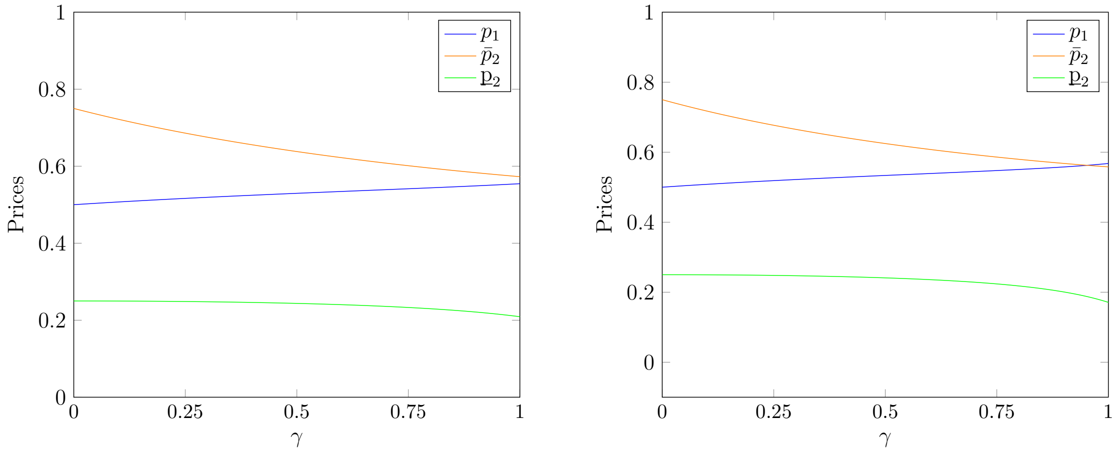
In this section, we compare the changes that the parameters of volatility aversion and proportion of consumers with volatility aversion, respectively, λ and γ do to the equilibrium prices. The following graphics are set with an increasing value of λ, the first represents a $\lambda = 0.1$ and the subsequent graphics an increase in λ. As the parameters γ and λ increase, the demand decreases and so the prices in the second period also decrease. These changes restrict the monopoly, because with more consumers with aversion to volatility, the prices of the second period will be lower. Additionally, we can see that the intensive margin γ does not have a big effect in the prices if the extensive margin λ is low, but the bigger the proportion of consumers is in the market, the bigger the effect the penalization γ exert on the prices.

Another effect seen in the graphs is that the change in the price in the second period where there is a negative shock, $\underline{p}_2$, is less as the aversion to volatility grows. In other words, it is as if the price in the second period with a negative shock $\underline{p}_2$ did not change, but the price does indeed change and gradually falls as aversion grew, and thus the change became more pronounced as the proportion of averse consumers grew. This effect can be explained because as both parameters grow, the tradeoff between earning more profits in the second period and the first period becomes more important. As can be seen in the graph as both parameters grow, it is better to put a higher price in the first period to earn more benefits in the first period, sacrificing the price of the second high period, to keep the second price stable.



(a) When $\lambda = 0.1$ (b) When $\lambda = 0.4$

Figure 3: Changes in prices for different value of the parameters λ and γ. Part A.



(a) When $\lambda = 0.6$

(b) When $\lambda = 0.75$

Figure 4: Changes in prices for different value of the parameters λ and γ. Part B.

3.2 Equilibrium with price commitment

In this section, I am going to consider the case in which the monopoly establishes a price commitment where prices are fixed for both periods. The monopoly does the commitment strategy with the idea of generating less variance for "savvy" consumers, since in the previous contingent case, consumers reacted to a price that was established in the second period.

When setting the prices, the monopoly will maximize the next profit function that takes into account both periods.

$$\hat{\Pi}(p_1, p_2) = \Pi_1(p_1) + \alpha * \overline{\Pi}_2(p_1, p_2, \alpha) + (1 - \alpha) * \underline{\Pi}_2(p_1, p_2, \alpha) \tag{11}$$

As in the previous section the demands will be aggregated between "savvy" and "non-savvy" consumers resulting in:

$$\begin{aligned}\hat{\Pi} = p_1 \left(\frac{V - p_1}{\theta}\right) + \alpha \left[\lambda p_2 * \left(\frac{\overline{V} - p_2 - \gamma|p_1 - p_2|}{\theta}\right) + (1 - \lambda) p_2 * \left(\frac{\overline{V} - p_2}{\theta}\right)\right] \\ + (1 - \alpha) \left[\lambda p_2 * \left(\frac{\underline{V} - p_2 - \gamma|p_1 - p_2|}{\theta}\right) + (1 - \lambda) * \left(\frac{\underline{V} - p_2}{\theta}\right)\right]\end{aligned} \tag{12}$$

By having this profit function for the two periods, the monopoly can decide the optimal prices that maximize its benefits.

Proposition 3.2. *With a proportion of consumers $\lambda \in (0, 1)$ who have volatility aversion, if:*

$p_2 > p_1$, *the Perfect Bayesian Nash equilibrium is:*

$$p_1^* = \frac{2V(1+\lambda\gamma) + \lambda\gamma(\alpha\overline{V} - (1-\alpha)\underline{V})}{4(1+\lambda\gamma) - (\gamma\lambda)^2}; \tag{13}$$

$$p_2^* = \frac{1}{2(1+\lambda\gamma)}\left[\alpha\overline{V} + (1-\alpha)\underline{V} + \lambda\gamma\frac{\left(2V(1+\lambda\gamma) + \lambda\gamma(\alpha\overline{V} - (1-\alpha)\underline{V})\right)}{4(1+\lambda\gamma) - (\gamma\lambda)^2}\right] \tag{14}$$

Proof. See Appendix 2 □

The results in the proposition 2, show that if the monopoly take the strategy of commitment in prices, it will have to take into account a medium point as a linear combination of the valuation shocks $(\underline{V}, \overline{V})$ seen in the prices as: $(\alpha\overline{V} - (1-\alpha)\underline{V})$. As the majority of models that uses commitment in prices is set (i.e Fundenberg & Tirole, 2000), this price strategy will have an intuitive result related to the mechanism in which the monopoly commits itself to prices. Since prices are decided ex-ante, the price commitment strategy will bring the prices of the two periods closer together naturally. But, in addition to this effect given by the price strategy, prices will have an additional effect provided by savvy consumers. In essence, the existence of consumers increases the price of the first period, seeing the effects of γ and λ, and generates a proportional decrease in the price of the second period.

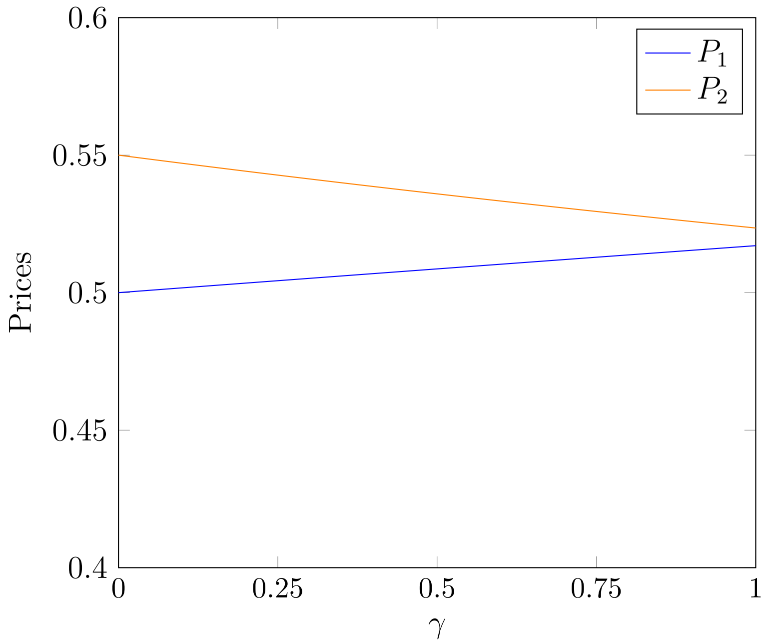


Figure 5: Price Comparison

The figure 6 shows the changes in prices for a $\lambda = 0.1$ of consumers with the volatility aversion and a $\alpha = 0.6$ chance of changing to the $\overline{V}$ state. The figure then shows that the existence of the penalty (γ) generates that consumers reduce their demand drastically, and since the price strategy of the monopoly sets a lower period 2 price (p_2), the prices are fairly constant between the two periods, even with a relatively low proportion of "savvy" consumers.

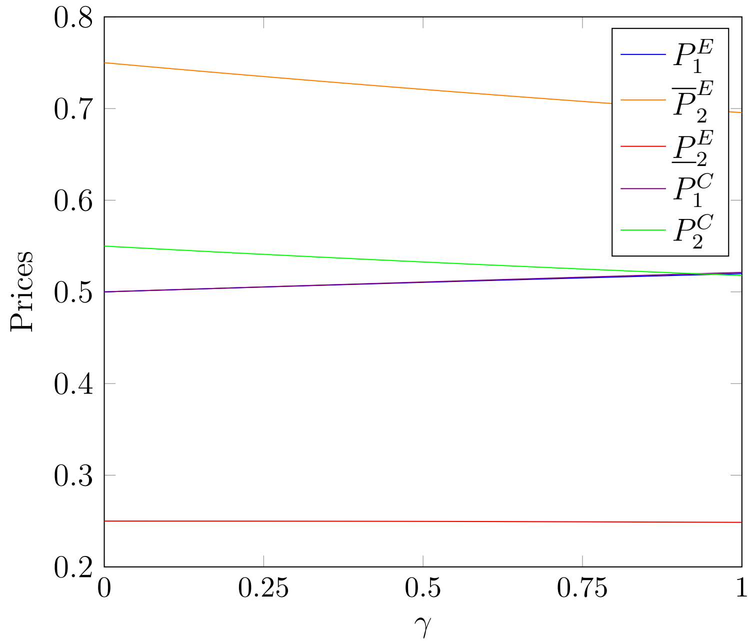


Figure 6: Price Comparison

To fully characterize the differences in the strategies, the figure 7 shows price changes for a $\lambda = 0.6$ volatility-averse consumer and a $\alpha = 0.6$ chance to switch to $\overline{V}$ status. This figure allows us to explain that the second price with price commitment, represented by the green line (P_2^C) has the same direction as the second price of the contingent equilibrium ($\overline{P}_2^E$). Also, the first prices between strategies P_1^E and P_1^C are close to each other with the commitment price being slihgtly higher, but this change is almost negligible. I can infer then that the contingent prices will be a preferable strategy for the monopoly as it will be discused in the next section.

4 Monoply profits with and without price commitment

For the final analysis I will show that is not a better strategy for the monopoly to commit in prices than to maintain a contingent strategy. By replacing the equilibrium prices with a contingent strategy in the equation (1) and the commitment prices in the equation (11) I construct figure 7. This figure represent the profits for a probability $\alpha = 0.5$ and a proportion of consumers of $\lambda = 0.1$. Figure 7 shows that, for every gamma, the contingent benefits are always greater than the commitment benefits. This implies then, that the monopoly will not have any incentives to deviate from the contingent strategy, not even when the penalization (γ) that the "savvy" consumers have is close to 1. To simplify the figure 7, the benefits are shown just with one value of λ, but the effect of the proportion is that the benefit curves are even closer.

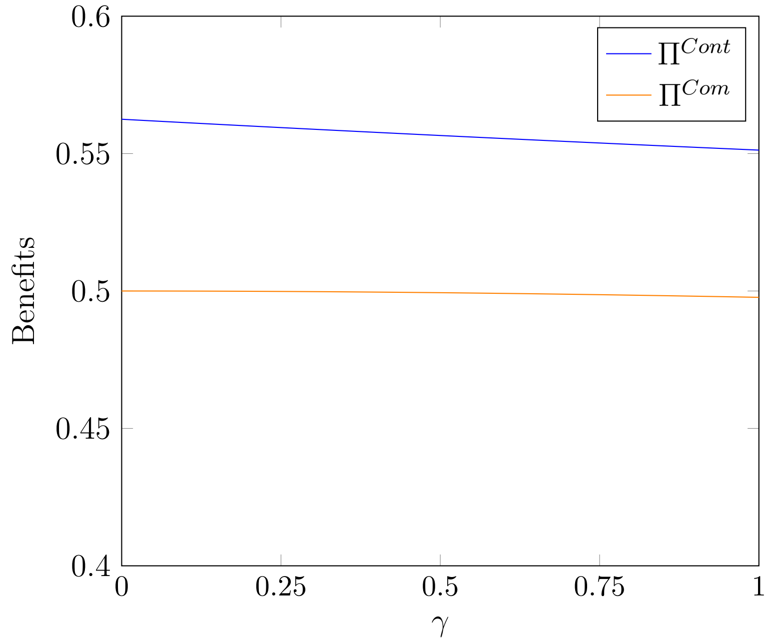


Figure 7: Benefit Comparison

5 Conclusions

Taking into account the results, in line with the model described by the literature (Armstrong, 2015) it can be seen that for a competition authority the existence of these "savvy" consumers can represent a way in which the monopoly can be disciplined, where the same consumers can take away monopoly power from the firm if it establishes very volatile prices. This discipline will depend on the proportion of consumers that in this market are averse to volatility and in the same way the cost that they inflict over this volatility.

The model and the results then show that in a case where there is only a monopoly and a mass with "savvy" and "non-savvy" consumers, the monopoly will not have any incentives to commit in prices for a specific case where the prcies changes between periods. Now let us imagine a similar set up, but with competition. The existence of two or more firms could show different results because a firm may like to commit in prices in order to attract consumers in later periods. This set up can allow us to understand other processes that the firms could internalize when competing in prices, and also understand why retail markets can have fairly constant prices, as shown by Jullien and Rey (2007).

I recommended that the parameter λ could be a parameter that can be promoted with information by the regulation agency. Also, taking into account that these consumers react to the price that is presented to them, that is, they tend to be shortsighted with respect to the future, it would be advisable to study this model in the light of "forward-looking" consumers.

6 Appendix

6.1 Proof of Proposition 3.1

The proof for the two-stage equilibrium goes as it follows:

Proof.

$$\pi_2(p_1, p_2) = p_2[\lambda D_2^{\lambda} + (1-\lambda)D_2^{1-\lambda}]$$

First, lets see the demand when the valuation changes to $\overline{V}$:

$$\begin{aligned}
D^{\lambda} &= \frac{\overline{V} - \overline{p}_2 - \gamma|p_1 - \overline{p}_2|}{\theta} \\
D^{1-\lambda} &= \frac{\overline{V} - \overline{p}_2}{\theta} \\
\overline{\pi}_2(p_1, \overline{p}_2) &= \overline{p}_2\left[\lambda\left(\frac{\overline{V} - \overline{p}_2 - \gamma|p_1 - \overline{p}_2|}{\theta}\right) + (1-\lambda)\left(\frac{\overline{V} - \overline{p}_2}{\theta}\right)\right]
\end{aligned}$$

Then, the firm will maximize with respect of p_2:

$$\frac{\partial \overline{\pi}_2}{\partial \overline{p}_2} : 0$$

$$\lambda\big(\overline{V} - 2\overline{p}_2 + \frac{\gamma\overline{p}_2(p_1 - \overline{p}_2)}{|p_1 - \overline{p}_2|} - \gamma|p_1 - \overline{p}_2|\big) + (1-\lambda)\left(\overline{V} - 2\overline{p}_2\right) = 0$$

After taking derivatives, I make a key assumption $\overline{p}_2 > p_1$

$$\lambda(\overline{V} - 2\overline{p}_2 - \gamma\overline{p_2} - \gamma(\overline{p}_2 - p_1)) + (1-\lambda)(\overline{V} - 2\overline{p}_2) = 0$$
$$\overline{p}_2(p_1) = \frac{\overline{V} + \lambda\gamma p_1}{2(1+\lambda\gamma)}$$

Then I can find the Profit for the monopoly in the second period if the valuation changes to $\overline{V}$:

$$\overline{\Pi}_2 = \frac{\overline{V} + \lambda\gamma p_1}{2(1+\lambda\gamma)} \left[\lambda \left(\frac{\overline{V} - \frac{\overline{V} + \lambda\gamma p_1}{2(1+\lambda\gamma)} - \gamma(p_1 - \frac{\overline{V} + \lambda\gamma p_1}{2(1+\lambda\gamma)})}{\theta} \right) + (1-\lambda) \left(\frac{\overline{V} - \frac{\overline{V} + \lambda\gamma p_1}{2(1+\lambda\gamma)}}{\theta} \right) \right]$$
$$\overline{\Pi}_2 = \frac{\overline{V} + \lambda\gamma p_1}{4\theta(1+\lambda\gamma)^2} \left[(\overline{V} + p_1\lambda\gamma)(1+\lambda\gamma) \right]$$

Second, if the valuation changes to $\underline{V}$:

$$D^{\lambda} = \frac{\underline{V} - \underline{p}_2 - \gamma|p_1 - \underline{p}_2|}{\theta}$$
$$D^{1-\lambda} = \frac{\underline{V} - \underline{p}_2}{\theta}$$
$$\underline{\Pi}_2(p_1, p_2) = \underline{p}_2 \left[\lambda \left(\frac{\underline{V} - \underline{p}_2 - \gamma|p_1 - \underline{p}_2|}{\theta} \right) + (1-\lambda) \left(\frac{\underline{V} - \underline{p}_2}{\theta} \right) \right]$$

Finding the the best response function in terms of p_1:

$$\frac{\partial \underline{\Pi}}{\partial p_2} : 0$$
$$\underline{V} - 2\overline{p}_2 + \frac{\gamma \underline{p}_2(p_1 - \underline{p}_2)}{|p_1 - \underline{p}_2|} - \gamma|p_1 - \underline{p}_2| + (1-\lambda)\left(\underline{V} - \underline{p}_2\right) = 0$$

The other key assumption for the model is $\underline{p}_2 < p_1$:

$$\underline{V} - 2\underline{p}_2 + \gamma\underline{p}_2 - \gamma(-\underline{p}_2 + p_1) + (1-\lambda)\left(\underline{V} - \underline{p}_2\right) = 0$$
$$\underline{p}_2 = \frac{\underline{V} - \lambda\gamma p_1}{2(1-\lambda\gamma)}$$

The profits for this case are:

$$\underline{\Pi}_2(p_1, p_2) = \frac{\underline{V} - \lambda\gamma p_1}{2(1-\lambda\gamma)}\left[\lambda\left(\frac{\underline{V} - \frac{\underline{V} - \lambda\gamma p_1}{2(1-\lambda\gamma)} - \gamma|p_1 - \frac{\underline{V} - \lambda\gamma p_1}{2(1-\lambda\gamma)}|}{\theta}\right) + (1-\lambda)\left(\frac{\underline{V} - \frac{\underline{V} - \lambda\gamma p_1}{2(1-\lambda\gamma)}}{\theta}\right)\right]$$
$$\underline{\Pi}_2(p_1, p_2) = \frac{\underline{V} - \lambda\gamma p_1}{\theta(2(1-\lambda\gamma))^2}\left[(\underline{V} - p_1\lambda\gamma)(1+\lambda\gamma)\right]$$

Then I define $\hat{\Pi}$ as the profit function for the combine two periods:

$$\hat{\Pi} = p_1\left(\frac{V - p_1}{\theta}\right) + \alpha\left[\frac{\overline{V} + \lambda\gamma p_1}{4\theta(1+\lambda\gamma)^2}\left[(\overline{V} + p_1\lambda\gamma)(1+\lambda\gamma)\right]\right] + (1-\alpha)\left[\frac{\underline{V} - \lambda\gamma p_1}{4\theta(1-\lambda\gamma)^2}\left[(\underline{V} - p_1\lambda\gamma)(1+\lambda\gamma)\right]\right]$$

Canceling terms:

$$\hat{\Pi} = p_1\left(\frac{V - p_1}{\theta}\right) + \frac{\alpha}{\theta}\left[\frac{(\overline{V} + \lambda\gamma p_1)^2}{4(1+\lambda\gamma)}\right] + \frac{(1-\alpha)}{\theta}\left[\frac{(\underline{V} - \lambda\gamma p_1)^2}{4(1-\lambda\gamma)}\right]$$

Deriving respect to p_1 to find the price that maximizes the function:

$$\frac{\partial\hat{\Pi}}{\partial p_1} = 0$$

$$\frac{V - 2p_1}{\theta} + \frac{\alpha}{\theta}\left[\frac{2(\overline{V} + \lambda\gamma p_1) * \lambda\gamma}{4(1+\lambda\gamma)}\right] + \frac{(1-\alpha)}{\theta}\left[\frac{2(\underline{V} - \lambda\gamma p_1) * (-\lambda\gamma)}{4(1-\lambda\gamma)}\right] = 0$$

Cancelling θ

$$
\begin{aligned}
V - 2p_1 + \alpha\left[\frac{(\overline{V} + \lambda\gamma p_1) * \lambda\gamma}{2(1+\lambda\gamma)}\right] + (1-\alpha)\left[\frac{(\underline{V} - \lambda\gamma p_1) * (-\lambda\gamma)}{2(1-\lambda\gamma)}\right] &= 0 \\
2V + \frac{\alpha\lambda\gamma}{(1+\lambda\gamma)}\overline{V} - \frac{(1-\alpha)\lambda\gamma}{(1-\lambda\gamma)}\underline{V} = 4p_1 - \frac{\alpha\lambda\gamma}{(1+\gamma)}(\lambda\gamma p_1) + \frac{(1-\alpha)\lambda\gamma}{2(1-\lambda\gamma)}(-\lambda\gamma p_1) \\
2V + \frac{\alpha\lambda\gamma}{(1+\lambda\gamma)}(\overline{V}) - \frac{(1-\alpha)\lambda\gamma}{(1-\lambda\gamma)}(\underline{V}) = p_1\left(4 - \frac{\alpha(\lambda\gamma)^2}{(1+\lambda\gamma)} - \frac{(1-\alpha)(\lambda\gamma)^2}{(1-\lambda\gamma)}\right)
\end{aligned}
$$

So the optimal p_1 is

$$
p_1^* = \frac{2V + \alpha\frac{\overline{V}\lambda\gamma}{1+\lambda\gamma} - (1-\alpha)\frac{(\lambda\gamma)\underline{V}}{1-\lambda\gamma}}{4 - \alpha\frac{(\lambda\gamma)^2}{1+\lambda\gamma} - (1-\alpha)\frac{(\lambda\gamma)^2}{1-\lambda\gamma}}
$$

To satisfy the first assumption: $\overline{p}_2 > p_1$. I use $\overline{p}_1$ as the highest possible p_1 that can be set, achieved by $\alpha = 1$, meaning the valuation will change to $\overline{V}$ with certainty. This p_1 is useful because if this price is lower than p_2 it means, that all other possible prices in the first period are also lower.

$\overline{p}_2 > p_1$ results then to:

$$
\begin{aligned}
\overline{V} &> (2+\lambda\gamma)p_1 \\
\overline{V} &> (2+\lambda\gamma)\left[\frac{2V + \frac{\overline{V}\lambda\gamma}{1+\lambda\gamma}}{4 - \frac{(\lambda\gamma)^2}{1+\lambda\gamma}}\right] \\
\overline{V}(4(1+\lambda\gamma) - 2(\lambda\gamma)^2 - (2+\lambda\gamma)\lambda\gamma) &> 2V(1+\lambda\gamma)(2+\lambda\gamma) \\
\overline{V}(4 + 2\lambda\gamma - 2\lambda\gamma^2) &> 2V(2 + 3\lambda\gamma + (\lambda\gamma^2)) \\
\frac{\overline{V}}{V} &> \frac{2(2+3\lambda\gamma+(\lambda\gamma)^2)}{(4+2\lambda\gamma-2(\lambda\gamma)^2)}
\end{aligned}
$$

Similarly to satisfy the assumption $p_1 > \underline{p}_2$

$$
\begin{aligned}
\underline{V} &< (2-\lambda\gamma)p_1 \\
\underline{V} &< (2-\lambda\gamma)\left[\frac{2V - \frac{(\lambda\gamma)\underline{V}}{1-\lambda\gamma}}{4 - \frac{(\lambda\gamma)^2}{1-\lambda\gamma}}\right] \\
\underline{V}(4(1-\lambda\gamma) - 2\lambda\gamma^2 + (2+\lambda\gamma)\lambda\gamma) &< 2V(1-\lambda\gamma)(2-\lambda\gamma) \\
\overline{V}(6 - 2\lambda\gamma - 2(\lambda\gamma)^2) &< 2V(2 - 3\lambda\gamma + (\lambda\gamma)^2) \\
\frac{V}{\underline{V}} &> \frac{(6 - 2\lambda\gamma - 2(\lambda\gamma)^2)}{2(2 - 3\lambda\gamma + (\lambda\gamma)^2)}
\end{aligned}
$$

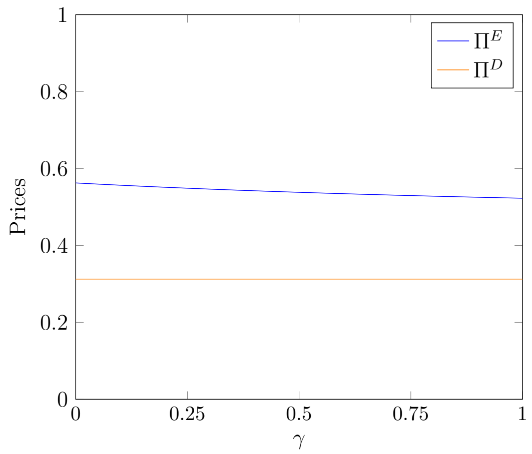
The values for the valuations where the equilibrium exists are where $\overline{V}$ is above the $\overline{V}$ line, and $\underline{V}$ is below $\underline{V}$ line.

Having into account the equilibrium stated in propostion 3.1, is it possible for the firm to deviate from this equilibirum and expect a better outcome by not setting a price in the first period, to acquire the total market in the second period?
Let's compare then the situation of equilibrium:

$$\Pi^E = p_1^E D_1 + \alpha * \overline{p}_2^E \overline{D}_2 + (1-\alpha) * \underline{p}_2^E \underline{D}_2 \tag{15}$$

against the deviation where the monopoly only offers its good in the second period to eliminate any possible price aversion by the consumers in the second period:

$$\Pi^E = p_2^D D_2 \tag{16}$$



(a) Comparison Benefits

Figure 8: Changes in Benefits for different value of the parameters λ and γ.

As whe can see in the figure 3, the benefits from the equilibrium described in the proposition 3.1 are always higher for any given values of the proportion of consumers with volatility aversion and penalization of the prices. □

6.2 Proof of Proposition 3.2

In this case the firm will face a profit function given by the prices of the the two periods.

Proof.

$$\hat{\Pi} = p_1 \left(\frac{V - p_1}{\theta} \right) + \frac{\alpha}{\theta} \left[\lambda p_2(\overline{V} - p_2 - \gamma |p_1 - p_2|) + (1-\lambda) p_2 (\overline{V} - p_2) \right] \tag{17}$$

$$+ \frac{(1-\alpha)}{\theta} \left[\lambda p_2(\underline{V} - p_2 - \gamma |p_1 - p_2|) + (1-\lambda) p_2 (\underline{V} - p_2) \right] \tag{18}$$

This function depends on two parameters (p_1, p_2) for which the monopoly is going to decide. To maximize this profit function, the derivatives $\frac{\partial \hat{\Pi}}{\partial p_1}$ and $\frac{\partial \hat{\Pi}}{\partial p_2}$ have to meet the maximum conditions and be equal to 0. So:

$$\frac{\partial \hat{\Pi}}{\partial p_1} = \left(\frac{V - 2p_1}{\theta} \right) + \frac{\alpha \lambda \gamma}{\theta} \left[p_2 \frac{p_2 - p_1}{|p_2 - p_1|} \right] + \frac{(1-\alpha)\lambda\gamma}{\theta} \left[p_2 \frac{p_2 - p_1}{|p_2 - p_1|} \right] = 0 \tag{19}$$

And

$$\frac{\partial \hat{\Pi}}{\partial p_2} = \frac{\alpha}{\theta}\left[\lambda\left(\overline{V} - 2p_2 + \gamma p_2\left(\frac{p_1 - p_2}{|p_2 - p_1|} - |p_2 - p_1|\right)\right) + (1-\lambda)(\overline{V} - 2p_2)\right]$$
$$+ \frac{(1-\alpha)}{\theta}\left[\lambda\left(\underline{V} - 2p_2 + \gamma p_2\left(\frac{p_1 - p_2}{|p_2 - p_1|} - |p_2 - p_1|\right)\right) + (1-\lambda)(\underline{V} - 2p_2)\right] = 0$$

Assuming $p_2 > p_1$ in the second derivative and cancelling θ:

$$\frac{\partial \hat{\Pi}}{\partial p_2} = \alpha\left[\lambda\left(\overline{V} - 2p_2 - \gamma\left(p_2 + (p_2 - p_1)\right)\right) + (1-\lambda)(\overline{V} - 2p_2)\right]$$
$$+ (1-\alpha)\left[\lambda\left(\underline{V} - 2p_2 - \gamma\left(p_2 + (p_2 - p_1)\right)\right) + (1-\lambda)(\underline{V} - 2p_2)\right] = 0$$

Grouping simlar terms and cancelling the terms that have λ and $(1-\lambda)$:

$$\frac{\partial \hat{\Pi}}{\partial p_2} = \alpha\left[\overline{V} - 2p_2 - 2\gamma\lambda p_2 + \lambda\gamma p_1\right]$$
$$+ (1-\alpha)\left[\lambda\left(\underline{V} - 2p_2 - \gamma\left(p_2 + (p_2 - p_1)\right)\right) + (1-\lambda)(\underline{V} - 2p_2)\right] = 0$$

□

References

[1] Armstrong, M., & Chen, Y. (2009). Inattentive consumers and product quality. *Journal of the European Economic Association*, 7(2-3), 411-422.

[2] Armstrong, M. (2015). Search and ripoff externalities.*Review of Industrial Organization*. doi:10.1007/ s11151-015-9480-1.

[3] Beladi, H., & Oladi, R. (2006). Strategic advertising: The fat-cat effect and stability. *Mathematical Social Sciences*, 51(2), 153-161.

[4] Colombo, S. (2011). Taxation and predatory prices in a spatial model. *Papers in Regional Science*, 90(3), 603-612.

[5] Fudenberg, D., & Tirole, J. (2000). Customer poaching and brand switching. *RAND Journal of Economics*, 634-657.

[6] Jullien, B., & Rey, P. (2007). Resale price maintenance and collusion. *The RAND Journal of Economics*, 38(4), 983-1001.

[7] Hotelling, H. (1929). Stability in Competition. *The Economic Journal*, 39(153), 41-57.

[8] Rosato, A. (2016). Selling substitute goods to loss-averse consumers: Limited availability, bargains, and rip-offs. *The RAND Journal of Economics*, 47(3), 709-733.

[9] Suslow, V. Y. (1986). Commitment and monopoly pricing in durable goods models. *International Journal of Industrial Organization*, 4(4), 451-460.

[10] Yongmin Chen, & Jason Pearcy. (2010). Dynamic pricing: when to entice brand switching and when to reward consumer loyalty. *The RAND Journal of Economics*, 41(4), 674.

7 Code Annex

7.1 To construct de difference in benefits

```
%Naming the parameters of the benefits
syms ped lamda ganma apha V V2 V3 pe1 pe2 theta;
V=1;
tetta=1;
gama = [0:0.005:1];
apha=0.5;
lamda=0.1;
V2=1.5;
V3=0.5;
%Establishing the prices in commitment and contingent strategies.
ped =(2.*V+(apha.*V2.*lamda.*gama)./(1+lamda.*gama)-((1-apha).*
    V3.*lamda.*gama)./(1-lamda*gama))./(4-apha.*(lamda.*gama).^2./
    (1+lamda.*gama)-((1-apha).*(lamda.*gama).^2./(1-lamda.*gama)));
pedC=(2.*V.*(1+lamda.*gama)+apha.*lamda.*gama.*
```

```
    V2-(1-apha).*gama.*lamda.*V3)./
    (4.*(1+lamda.*gama)-(lamda.*gama).^2);
pe2C=((1)./(2.*(1+lamda.*gama))).*
    (apha.*V2+(1-apha).*V3+lamda.*gama.*pedC);
plot(gama,ped);hold on; plot(gama,pedC);
hleg = legend('$$primero$$','$$segundo$$');
set(hleg,'Interpreter','latex','Location','Southwest','Fontsize',10)
%Establishing the benefits
PiE= ped.*((V-ped)./tetta)+(apha./tetta).*
(((V2+lamda.*gama.*ped).^2)./(4.*(1+gama.*lamda))+(1-apha./tetta).
*(((V3-lamda.*gama.*ped).^2)./(4.*(1-gama.*lamda)));
PiC=pedC.*((V-pedC)./tetta)+apha.*
    (lamda.*pe2C.*(V2-pe2C-gama.*abs(pedC-pe2C))./
    (tetta)+(1-lamda).*pe2C.*(V2-pe2C)./(tetta))+(1-apha).*(lamda.*pe2C.*
    (V3-pe2C-gama.*abs(pedC-pe2C))./
    (tetta)+(1-lamda).*pe2C.*(V3-pe2C)./(tetta));
plot(gama,PiE);hold on; plot(gama,PiC);
writematrix([gama' PiE' PiC'],'DiferenciaB.csv');
hleg = legend('$$primero$$','$$segundo$$');
set(hleg,'Interpreter','latex','Location','Southwest','Fontsize',10)
```