BUSINESS CYCLE ASYMMETRIES: LOSS AVERSION, STICKY PRICES, AND WAGES

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SERIE DOCUMENTOS DE TRABAJO

No. 160

Junio de 2014

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June 13, 2014

Abstract

In this chapter, the Smets-Wouters (2003) New Kenesian model is reformulated by introducing the loss aversion utility function developed in chapter two. The purpose of this is to understand how asymmetric real business cycles are linked to asymmetric behavior of agents in a price and wage rigidities set up. The simulations of the model reveal not only that the loss aversion in consumption and leisure is a good mechanism channel for explaining business cycle asymmetries, but also is a good mechanism channel for explaining asymmetric adjustment of prices and wages. Therefore the existence of asymmetries in Phillips Curve. Moreover, loss aversion makes downward rigidities in prices and wages stronger and also reproduces a more severe and persistent fall of the employment. All in all, this model generates asymmetrical real business cycles, asymmetric price and wage adjustment as well as hysteresis.

1 Introduction

In this chapter I continue to explore in depth asymmetries in business cycles and their links to the micro-foundations of agents' behavior. To this end, the prospects utility model developed in the second chapter of this dissertation is now used to build the central block of agents' decision-making in an environment characterized by nominal wages and prices rigidities. Besides the asymmetries treated (and documented) in chapters 1 and 2, there is evidence of asymmetries in Phillips Curve, asymmetric adjustment of prices (and wages), and asymmetries in the response of economies to fiscal and monetary policies.

Regarding nonlinear Phillips curve, there is some empirical evidence. Indeed, Ilmo (1999), for country specific and pooled data, has found that the Phillips Curve is asymmetric for Germany, Finland, Italy, the Netherlands and Spain, Austria and France. The asymmetry detected in the Phillips Curve is such that given a positive output gap (observed income greater than potential income), it has a positive effect on inflation; differently, when the gap is negative, the effect on deflation is very slight and is not significant. This is a signal of price and wages asymmetric

*This is the third chapter of my Ph.D dissertation intended to explain asymmetries in the business cycle. In am very thankful to my advisor Professor Fernando Jaramillo for his guidance and to Universidad de Antioquia and Colciencias for financing my Ph.D. studies. I would like to thank comments and suggestions by Alejandro Torres, John Lopera, Andrés González, Jesus Bejarano, Frans Hamann, Jair Ojeda. All errors and omissions are my own.

adjustment as well. Eliasson (1999) has tested the linearity of Phillips curve for Austria, Sweden and the United States for a sample of quarterly seasonally unadjusted data from 1977 (1) to 1997 (4) for Austria, 1979 (3) - 1997 (4) for Sweden, and 1978 (1) - 1997 (4) for the United States. By using smooth transition regression, she found that the null of linearity is rejected for Austria and Sweden, but not rejected for the United States Phillips curve. Huh (date) uses LSTAR (Logistic Smooth Threshlod Auto Regression) to model several specifications of nonlinear Phillips curve for the U.S. economy, which is used later to derive both the NAIRU and an optimal monetary policy rule that inherits nonlinearity of Phillips curve. For the case of Colombia, Gómez and Julio (2000), by using unobserved components, have found empirical evidence that supports the existence of a nonlinear Phillips curve and a non-constant NAIRU. They also noted that non-linearity of Phillips curve implies non-linearity in sacrifice ratio: the higher the decrease in inflation, the higher the unemployment rate. López and Misas (1999) also encountered evidence of non-linearity and asymmetry in the Phillips curve for Colombia. Flaschel, Gong, and Semmler (2003) studied the implications of a kinked Phillips curve in a Keynesian macro-econometric monetary model. Their simulations of the model using estimated parameters have revealed instability of its steady state and the fact that several optimal policy rules help to stabilize the system.

Among the Neo-Keynesian (NK) DSGE models as proposed by Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), Baxter and Farr (2005), the original aim is to study business cycles in the presence of wage and price rigidities, and how the system evolves facing several shocks. As most DSGE models, these are somewhat successful in explaining business cycles with rigidities. However, neither can they give explanation of business cycle asymmetries, nor they can give explanation of asymmetric stickiness of prices and wages.

With the goal of knowing the link between the agents' behavior and the asymmetric adjustment of prices and wages, and consequently the nonlinear Phillips curve, this chapter presents a modified version of the neokeynesian model presented in Smets and Wouters (2003) including a prospects-utility function, which serves the purpose of modeling asymmetries in consumption and labor choice. This utility function is intra-temporal additive and inter-temporal separable as the reference point is supposed to enter into the utility function as an externality. In the second chapter of this dissertation, it has been used, and succesfully developed, a general prospects theory, which is neither intra-temporal additive nor inter-temporal separable. It means that the utility function is a Cobb-Douglas aggregator of consumption and leisure has been nested into a prospect theory utility function, wherein the reference point is endogenously determined by the choice of consumption and leisure in the previous period. In the first and second chapters of this dissertation, asymmetric business cycles were successfully reproduced by modeling asymmetric investment cost adjustment and prospects theory utility respectively.

Smets and Wouters (2003) has introduced shocks in preferences (on consumption and leisure), mark-ups on wages and goods market, technology, investment, fiscal policy, inflation and monetary rule. This strategy leads to identify and estimate parameters. In this chapter, however, stochastic processes are modeled as autorregresive log normal processes, while shock to goods market mark-up and shock to wage mark-up are modeled as a median plus a perturbation, considering that in Smets and Wouters (2003) all processes have been originally modeled as a mean plus a perturbation. Whereas shocks to policy interest rate are modeled as IID-normal, this shock hereby presented is modeled as the exponential of the IID-normal. In papers such as Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), Baxter and Farr (2005), the model is solved and simulated by log-linearizing the dynamic equations. However, this chapter presents a model solved and simulated by using a third-order perturbations method to guarantee the preservation of utility function asymmetries.

In the second chapter, the model could not only reproduce asymmetries in business cycles, but also generate real rigidities in wage and interest rate, being accompanied by a severe reaction of productive factors during recessions. The model presented here and the simulations performed with it, by using the parameters similar to those estimated in Smets and Wouters (2003), can reproduce asymmetries in business cycles and asymmetries in stickiness of prices and wages. Moreover in this framework, downward rigidity in prices and wages is amplified. Consequently, an asymmetric Phillips curve can be obtained and theoretically explained.

2 The model

As in Smets and Wouters (2003) the inter-temporal utility function has the following compact form:

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} U_{t+i}^{\tau} \tag{1}$$

However, the instantaneous utility function will take a form which is very similar to that one in the consumption article, although not as a time separable function. That is, the utility will be asymmetric and additive; this variation with respect to the utility function in chapter two is done in order to obtain results comparable to the ones in Smets and Wouters (2003):

$$\begin{aligned} U_t^{\tau} &= W^c(\bar{c}_t) + \varepsilon_t^b \left(\phi_{ct} (z_{ct})^{\underline{\theta}} + (1-\phi_{ct}) (z_{ct})^{\overline{\theta}} \right) \\ &+ W^l\left(\bar{l}_t\right) + \varepsilon_t^b \varepsilon_t^L \left(\phi_{lt} (z_{lt})^{\underline{\mu}} + (1-\phi_{lt}) (z_{lt})^{\overline{\mu}} \right) \end{aligned} \tag{2}$$

Where $\bar{c}_t$ and $\bar{l}_t$ are reference points for consumption and leisure, $W^c(\bar{c}_t)$ and $W^l\left(\bar{l}_t\right)$ are utilities delivered by consumption and leisure in the steady state. As in chapter two,$\overline{\theta}$ and $\overline{\mu}$ are such that the utility function is concave (risk-aversion); $\underline{\theta}$ and $\underline{\mu}$ are such that the utility function is convex (loss-aversion).

The reference points for consumption C_t^{τ} and leisure $\mathcal{L}_t$ are defined as

$$\overline{C}_t = (1-\chi)\overline{C}_{t-1} + \chi C_{t-1}^{\tau} \tag{3}$$

$$\overline{\mathcal{L}}_t = (1-\chi_l)\overline{\mathcal{L}}_{t-1} + \chi_l(\mathcal{L}_{t-1}^{\tau}) \tag{4}$$

Then, the comparisons between the current levels and the reference points for consumption and leisure are defined respectively as:

$$z_{ct} = \frac{C_t^{\tau}}{\overline{C}_t} \tag{5}$$

$$z_{lt} = \frac{\mathcal{L}_t^{\tau}}{\overline{\mathcal{L}}_t} \tag{6}$$

Being l_t^τ the time, the agent is then willing to offer in the labor market, so the time constraint is:

$$\mathcal{L}_t^\tau = 1 - l_t^\tau \tag{7}$$

The smooth transition functions for consumption ϕ_{ct}, and for leisure $\phi_{\mathcal{L}t}$ are given by

$$\phi_{ct} = \frac{1}{1+\exp\gamma_c\left(z_{ct}-1\right)} \tag{8}$$

$$\phi_{\mathcal{L}t} = \frac{1}{1+\exp\gamma_{\mathcal{L}}(z_{\mathcal{L}t}-1)} \tag{9}$$

Their first derivatives of the smooth transition functions with respect to z_{ct} and z_{lt} are given by

$$\frac{\partial\phi_{ct}}{\partial z_{ct}} = \frac{-\gamma_c\exp\gamma_c\left(z_{ct}-1\right)}{\left[1+\exp\gamma_c\left(z_{ct}-1\right)\right]^2} \tag{10}$$

$$\frac{\partial\phi_{\mathcal{L}t}}{\partial z_{lt}} = \frac{-\gamma_{\mathcal{L}}\exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t}-1\right)}{\left[1+\exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t}-1\right)\right]^2} \tag{11}$$

ThThe stochastic process for shocks to preferences are log-normal:

$$\ln\varepsilon_t^b = \rho_b\ln\varepsilon_{t-1}^b + \eta_t^b \tag{12}$$

$$\ln\varepsilon_t^L = \rho_L\ln\varepsilon_{t-1}^L + \eta_t^L \tag{13}$$

$\eta_t^b$and η_t^L are homoscedastic and mean zero stochastic shocks.
The labor income is given by $w_t^\tau l_t^\tau$; $r_t^k z_t^\tau K_{t-1}^\tau$ is income of capital; $\Psi(z_t^\tau)K_{t-1}^\tau$ is the cost of adjusting capital, Div_t^τ dividends from firms and A_t^τ are state-dependent securities, $B_t^\tau$is the financial wealth represented in bonds, and I_t^τ is physical capital investment. Thus, the real terms inter-temporal budget constraint becomes:

$$b_t\frac{B_t^\tau}{P_t} = \frac{B_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau \tag{14}$$

where $b_t = \frac{1}{1+i_t}$ is the nominal bond price and Y_t^τ is given by:

$$Y_t^\tau = \left(w_t^\tau l_t^\tau + A_t^\tau\right) + \left(r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau)K_{t-1}^\tau\right) + Div_t^\tau \tag{15}$$

The Lagrangian for this decentralized economy is:

$$\begin{aligned}\Lambda_t = {} & E_t\sum_{i=0}^{\infty}\beta^{t+i}\left[\begin{array}{c} W^c\left(\bar{c}_{t+i}\right) + \varepsilon_{t+i}^b\left(\phi_{ct+i}\left(z_{ct+i}\right)^{\underline{\theta}} + \left(1-\phi_{ct+i}\right)\left(z_{ct+i}\right)^{\overline{\theta}}\right) \\ +W^l\left(\bar{l}_{t+i}\right) + \varepsilon_{t+i}^b\varepsilon_{t+i}^L\left(\phi_{lt+i}\left(z_{lt+i}\right)^{\underline{\mu}} + \left(1-\phi_{lt+i}\right)\left(z_{lt+i}\right)^{\overline{\mu}}\right)\end{array}\right] \\ & + E_t\sum_{i=0}^{\infty}\beta^{t+i}\lambda_{t+i}\left[\begin{array}{c} -b_{t+i}\frac{B_{t+i}^\tau}{P_{t+i}} + \frac{B_{t+i-1}^\tau}{P_{t+i}} + \left(\frac{W_{t+i}^\tau l_{t+i}^\tau}{P_{t+i}} + A_{t+i}^\tau\right) \\ +\left(r_{t+i}^k z_{t+i}^\tau K_{t+i-1}^\tau - \Psi(z_{t+i}^\tau)K_{t+i-1}^\tau\right) + Div_{t+i}^\tau - C_{t+i}^\tau - I_{t+i}^\tau\end{array}\right] \\ & + E_t\sum_{i=0}^{\infty}\beta^{t+i}\mu_{t+i}\left[K_{t+i}(1-\tau) + \left[1 - S\left(\varepsilon_{t+i}^I I_{t+i}/I_{t+i-1}\right)\right]I_{t+i} - K_{t+i+1}\right]\end{aligned} \tag{16}$$

$$\max_{C_{t+i},\widetilde{W}_{t+i},K_{t+i},I_{t+i},B^{\tau}_{t+i},z^{\tau}_{t+i}} \Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[... + \varepsilon^b_{t+i} \left(\phi_{ct+i} \left(z_{ct+i}\right)^{\underline{\theta}} + \left(1-\phi_{ct+i}\right)\left(z_{ct+i}\right)^{\overline{\theta}} \right) \right] \tag{17}$$

$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... - C^{\tau}_{t+i} ... \right]$$

The first order condition for consumption is:

$$\lambda_t = \varepsilon^b_t \frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t} + \varepsilon^b_t \phi_{ct} \left[\frac{\underline{\theta}\left(z_{ct}\right)^{\underline{\theta}-1}}{\overline{C}_t} - \frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t} \right] + \varepsilon^b_t \frac{-\gamma_c \exp \gamma_c \left(z_{ct}-1\right)}{\left[1+\exp \gamma_c \left(z_{ct}-1\right)\right]^2} \frac{\left[\left(z_{ct}\right)^{\underline{\theta}} - \left(z_{ct}\right)^{\overline{\theta}}\right]}{\overline{C}_t} \tag{18}$$

The left side of the equation 18 is the Lagrange multiplier for the inter-temporal restriction, and the right side has three terms: the first and second terms sum up the derivative of the prospect utility function. In the traditional (risk-averse) symmetric model, the marginal utility of consumption would be $\varepsilon^b_t \frac{\overline{\theta}(z_{ct})^{\overline{\theta}-1}}{\overline{C}_t}$. But, in this more general set up, the global marginal utility comprises the marginal utility of consumption for both regimes: under recession (loss-averse) and under boom (risk-averse), and the respective smooth transition between them. The third term on the left is the change in the transition function multiplied by the difference of the utilities level in both regimes. Thus, if we were supposed to model the regime switching marginal utility by only modeling the transition between them, we would be mistakenly specifying the regime switching model and, therefore, its predictions. This particularity is pointed out in the first chapter of this dissertation when explaining that most of the regime switching RBC works are rather imprecise because they impose a transition matrix (with fixed transition probabilities) on the canonical system of dynamic equations.

$$\max_{C_{t+i},\widetilde{W}_{t+i},K_{t+i},I_{t+i},B^{\tau}_{t+i},z^{\tau}_{t+i}} \Lambda_t = ... + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[-b_{t+i} \frac{B^{\tau}_{t+i}}{P_{t+i}} + \frac{B^{\tau}_{t+i-1}}{P_{t+i}} ... \right]$$

$$...$$

The first order condition for bonds is:

$$\beta^{t+i} \lambda_{t+i} b_{t+i} \frac{1}{P_{t+i}} = E_t \left[\beta^{t+i+1} \lambda_{t+i+1} \frac{1}{P_{t+i+1}} \right] \tag{19}$$

$$b_t \lambda_t = \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{20}$$

Equation 20 sets the condition for the equalization between the real interest rate and the marginal rate of substitution (we must keep in mind that λ_t is the marginal utility of consumption in period t). Because marginal utilities are asymmetrical, so it is the marginal rate of substitution and so will be its reactions to movements in real interest rates originated by inflation or by interest rate policy.

2.1 Labor supply decisions and wage setting equation

Under the assumption that wages can be adjusted with probability $1-\xi_w$, households choose a new optimal wage $\widetilde{w}_t^\tau$ taking into account that in the future wages will unlikely be adjusted once again. Thus, there will be a partial wage indexation for those who will not be able to re-optimize:

$$W_t^\tau = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{t-1}^\tau \tag{21}$$

Households set their nominal wage to maximize their inter-temporal utility function subject to the budget constraint. Similarly, the labor demand will be determined as follows. If the aggregate labor demanded by firms is given as:

$$L_t = \left[\int_0^1 (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}} \, d\tau\right]^{1+\lambda_{w,t}} \tag{22}$$

Each unit of labor l_t^τ is paid a nominal wage W_t^τ, then the total labor expenditure is$\int_0^1 l_t^\tau W_t^\tau d\tau$. Thus, the problem of the firm is to minimise the total labor expenditure given it need a quantity of aggregate labor L_t. Minimizing in l_t^τ the first order condition for this problem is

$$W_t^\tau - \mu \left[\int_0^1 (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}} \, d\tau\right]^{\lambda_{w,t}} (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}-1} = 0$$

and solvin for l_t^τ we have:

$$l_t^\tau = \left(\frac{W_t^\tau}{\mu}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{23}$$

by replacing 23 into 22, we have; then, solving for μ:

$$\mu = \left[\int_0^1 (W_t^\tau)^{-1/\lambda_{w,t}} \, d\tau\right]^{-\lambda_{w,t}} = W_t \tag{24}$$

Which is a Dixit-Stiglitz wage aggregator. Finally, after replacing 24 into 23 we have the demand for labor:

$$l_t^\tau = \left(\frac{W_t^\tau}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{25}$$

Then, if we define ξ_w^i as the probability that in time i the wage cannot change; thereby, it is necessary to index it based on the one re-optimized in t; hence, the indexed wage will be:

$$W_{t+i}^\tau = (X_{t,t+i})^{\gamma_w} \widetilde{W}_t \tag{26}$$

Thus, the demand for labor will be:

$$l^{\tau}_{t+i} = \left(\frac{W^{\tau}_{t,t+i}}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{27}$$

$$X_{t,t+i} = \frac{P_{t+i-1}}{P_{t-1}} \tag{28}$$

Thus, $1 - \xi^i_w$ is the probability of having changed optimally the price between t and $t+i$. Now, we will suppose that $t < t^* < t+i$ if households can optimize wages in t^*. Then, the wage W_{t+i} depends on the optimal wage $\widetilde{W}^{\tau}_{t^*}$ set in t^*, but not on the optimal wage $\widetilde{W}^{\tau}_{t}$ set in t. Therefore, we may claim that

$$W^{\tau}_{t+i} = \widetilde{W}_{t^*} \left(X_{t^*,t+i} \right)^{\gamma_w} \tag{29}$$

where

$$X_{t^*,t+i} = \frac{P_{t+i-1}}{P_{t^*-1}} \tag{30}$$

and we define $W^{\tau}_{t^*>t,t+i}$ as the wage $t+i$ given that the last time it was changed optimally was in time $t^* > t$. $W^{\tau}_{t,t+i}$ as the wage in $t+i$ given that the last time it was changed optimally was in time t.

According to this, the wage can be written as:

$$\xi^i_w W^{\tau}_{t,t+i} + (1-\xi^i_w) W^{\tau}_{t^*>t,t+i} = \xi^i_w \left(X_{t,t+i} \right)^{\gamma_w} \widetilde{W}_t + (1-\xi^i_w) W^{\tau}_{t^*>t,t+i} \tag{31}$$

And the demands for labor will become:

$$l^{\tau}_{t+i} = \left(\frac{\left(X_{t,t+i} \right)^{\gamma_w} \widetilde{W}_t}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{32}$$

$$l^{\tau}_{t^*>t,t+i} = \left(\frac{W^{\tau}_{t^*>t,t+i}}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{33}$$

Then, the Lagrangian function for the inter-temporal utility function will be:

$$\max_{C_{t+i},\widetilde{W}_{t+i},K_{t+i},I_{t+i},B^{\tau}_{t+i},z^{\tau}_{t+i}} \Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\begin{array}{c} W^c \left(\overline{c}_{t+i} \right) + W^l \left(\overline{l}_{t+i} \right) + \varepsilon^b_{t+i} \left(\phi_{ct+i} \left(z_{ct+i} \right)^{\underline{\theta}} + (1-\phi_{ct+i}) \left(z_{ct+i} \right) \right. \\ +\varepsilon^b_{t+i}\varepsilon^L_{t+i} \left[\begin{array}{c} \xi^i_w \left(\phi_{lt+i} \left(z_{lt+i} \right)^{\underline{\mu}} + (1-\phi_{lt+i}) \left(z_{lt+i} \right)^{\overline{\mu}} \right) \\ +(1-\xi^i_w) \left(\phi_{l,t^*>t,t+i} \left(z_{l,t^*>t,t+i} \right)^{\underline{\mu}} + (1-\phi_{l,t^*>t,t+i}) \left(z_{l,t^*>} \right. \right. \end{array} \right. \end{array} \right.$$

$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[\begin{array}{c} -b_{t+i} \frac{B^{\tau}_{t+i}}{P_{t+i}} + \frac{B^{\tau}_{t+i-1}}{P_{t+i}} + \left(\frac{\xi^i_w W^{\tau}_{t,t+i} l^{\tau}_{t,t+i} + (1-\xi^i_w) W^{\tau}_{t^*>t,t+i} l^{\tau}_{t^*>t,t+i}}{P_{t+i}} + A^{\tau}_{t+i} \right) \\ +(r^k_{t+i} z^{\tau}_{t+i} K^{\tau}_{t+i-1} - \Psi(z^{\tau}_{t+i}) K^{\tau}_{t+i-1}) + Div^{\tau}_{t+i} - C^{\tau}_{t+i} - I^{\tau}_{t+i} \end{array} \right]$$

$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} \left[K_{t+i-1} \left[1-\tau \right] + \left[1 - S \left(\varepsilon^I_{t+i} I_{t+i} / I_{t+i-1} \right) \right] I_{t+i} - K_{t+i} \right]$$

The previous equation can be expressed compactly leaving explicit only those parts that depend on labor choice and wages:

$$\Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[... + \varepsilon^b_{t+i} \varepsilon^L_{t+i} \left[\begin{array}{c} \xi^i_w \left(\phi_{lt+i} \left(z_{lt+i} \right)^{\underline{\mu}} + \left(1 - \phi_{lt+i} \right) \left(z_{lt+i} \right)^{\overline{\mu}} \right) \\ +(1-\xi^i_w) \left(\phi_{l,t^*>t,t+i} \left(z_{l,t^*>t,t+i} \right)^{\underline{\mu}} + \left(1 - \phi_{l,t^*>t,t+i} \right) \left(z_{l,t^*>t,t+i} \right)^{\overline{\mu}} \right) \end{array} \right] \right]$$

$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... + \frac{\xi^i_w W^\tau_{t,t+i} l^\tau_{t,t+i}}{P_{t+i}} + \frac{(1-\xi^i_w) W^\tau_{t^*>t,t+i} l^\tau_{t^*>t,t+i}}{P_{t+i}} + ... \right]$$

As the maximization of Λ_t on $\widetilde{W}_t$ does only affect the expectations related to the probability ξ^i_w , the previous maximization problem is equal to maximizing the following function:

$$\Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[... + \varepsilon^b_{t+i} \varepsilon^L_{t+i} \left[\xi^i_w \left(\phi_{lt+i} \left(z_{lt+i} \right)^{\underline{\mu}} + \left(1 - \phi_{lt+i} \right) \left(z_{lt+i} \right)^{\overline{\mu}} \right) \right] \right] \tag{34}$$

$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... + \frac{\xi^i_w W^\tau_{t,t+i} l^\tau_{t,t+i}}{P_{t+i}} + ... \right]$$

recalling that

$$l^\tau_{t+i} = \left(\frac{\left(X_{t,t+i} \right)^{\gamma_w} \widetilde{W}_t}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{35}$$

and

$$W^\tau_{t+i} = \widetilde{W}_{t^*} \left(X_{t^*,t+i} \right)^{\gamma_w} \tag{36}$$

The first-order condition with respect to the re-optimized wage will be:

$$\frac{\partial \Lambda_t}{\partial \widetilde{W}_t} = E_t \sum_{i=0}^{\infty} \beta^{t+i} \xi^i_w \frac{\partial U^\tau_{t+i}}{\partial \mathcal{L}_{t+i}} \frac{\partial \mathcal{L}_{t+i}}{\partial l_{t+i}} \frac{\partial l_{t+i}}{\partial \widetilde{W}_t} + E_0 \sum_{i=0}^{\infty} \beta^{t+i} \frac{\lambda_{t+i} \xi^i_w}{P_{t+i}} \left(\frac{\partial W^\tau_{t+i}}{\partial \widetilde{W}_t} l^\tau_{t,t+i} + \frac{\partial l^\tau_{t,t+i}}{\partial \widetilde{W}_t} W^\tau_{t+i} \right) = 0 \tag{37}$$

$$-E_t \sum_{i=0}^{\infty} \beta^{t+i} \xi^i_w \frac{\partial U^\tau_{t+i}}{\partial \mathcal{L}_{t+i}} \frac{\partial \mathcal{L}_{t+i}}{\partial l_{t+i}} \frac{\partial l_{t+i}}{\partial \widetilde{W}_t} = E_0 \sum_{i=0}^{\infty} \beta^{t+i} \frac{\lambda_{t+i} \xi^i_w}{P_{t+i}} \left(\frac{\partial W^\tau_{t+i}}{\partial \widetilde{W}_t} l^\tau_{t,t+i} + \frac{\partial l^\tau_{t,t+i}}{\partial \widetilde{W}_t} W^\tau_{t+i} \right). \tag{38}$$

Recall that $\frac{\partial \mathcal{L}_t}{\partial l_t} < 0$. Hence, the right side of this equation will be always positive as well as the left one. Taking into account that $\lambda_{t+1} = \frac{\partial U^\tau_{t+1}}{\partial C_{t+1}} = U^C_{t+1}$, we will have:

$$-E_t \sum_{i=0}^{\infty} \beta^{t+i} \xi^i_w \frac{\partial U^\tau_{t+i}}{\partial \mathcal{L}_{t+i}} \frac{\partial \mathcal{L}_{t+i}}{\partial l_{t+i}} \frac{\partial l_{t+i}}{\partial \widetilde{W}_t} = E_t \sum_{i=0}^{\infty} \beta^t \frac{U^C_{t+i} \xi^i_w}{P_{t+i}} \left(\frac{\partial W^\tau_{t+i}}{\partial \widetilde{W}_t} l^\tau_{t,t+i} + \frac{\partial l^\tau_{t,t+i}}{\partial \widetilde{W}_t} W^\tau_{t+i} \right) \tag{39}$$

where $\frac{\partial W^\tau_{t+i}}{\partial \widetilde{W}_t} = \left(X_{t^*,t+i} \right)^{\gamma_w}$ and

$$\frac{\partial l^\tau_{t,t+i}}{\partial \widetilde{W}_t} = -\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}} l^\tau_{t+i} \left(\widetilde{W}_t \right)^{-1} \tag{40}$$

thus we will have finally:

$$E_0 \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}} l_{t+i}^\tau \right) = \frac{\widetilde{W}_t}{P_t} E_0 \sum_{i=0}^{\infty} \beta^i U_{t+i}^C \xi_w^i l_{t,t+i}^\tau \frac{P_t}{P_{t+i}} \left(\frac{P_{t+i-1}}{P_{t-1}} \right)^{\gamma_w} \frac{1}{\lambda_{w,t+i}} \quad (41)$$

If $\gamma_w = 1$ and if $\lambda_{w,t+i} = \lambda_w$, the inter-temporal utility maximization problem, since the point of view of the optimal wage choice, will have as solution :

$$\frac{\widetilde{W}_t}{P_t} E_0 \sum_{i=0}^{\infty} \beta^i U_{t+i}^C \xi_w^i l_{t,t+i}^\tau \left(\frac{P_t / P_{t-1}}{P_{t+i}/P_{t+i-1}} \right) \frac{1}{1+\lambda_{w,t+i}} = E_0 \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(l_{t+i}^\tau \right) \quad (42)$$

This equation is very similar to the one derived in Smets and Wouters (2003). However, given the asymmetric nature of our utility function, this re-optimized wage equation also inherits such asymmetry from the marginal utility function of consumption and labor[1]. Because our utility function is asymmetric around the reference point, so are the marginal utilities of labor and consumption. Thus, nominal wage setting will be asymmetric.

If we suppose the wage mark-up shocks $\lambda_{w,t} = \lambda_w + \eta_t^w$ as normal-IID around a constant, considering the aggregate wage equation, the movement law of the aggregate wage index will be given as:

$$\left(W^t\right)^{-1/\lambda_{w,t}} = \xi \left(W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \right)^{-1/\lambda_{w,t}} + (1-\xi)(\widetilde{w}_t)^{-1/\lambda_{w,t}} \quad (43)$$

2.2 Investment and capital accumulation

he law of capital accumulation goes as follows: being τ the depreciation rate, $S(.)$ a positive adjustment cost function of investment changes, this function is zero in the steady state as investment is constant. The stochastic process of shocks to investment is:

$$\ln \varepsilon_t^I = \rho_I \ln \varepsilon_{t-1}^I + \eta_t^I \quad (44)$$

The first-order conditions give rise to the following dynamic equations of the real value of capital, the investment and the utilization rate of capital:

Maximizing on capital utilization:

$$\max_{\ldots z_{t+i}^\tau \ldots} \Lambda_t = \ldots + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[+(r_{t+i}^k z_{t+i}^\tau K_{t+i-1}^\tau - \Psi(z_{t+i}^\tau) K_{t+i-1}^\tau) + Div_{t+i}^\tau - C_{t+i}^\tau - I_{t+i}^\tau \right]$$

$$\ldots$$

$$r_t^k = \Psi'(z_t^\tau) \quad (45)$$

Maximizing on physical capital:

[1] As a matter of fact, because the utility function is intratemporal separable, this wage setting equation has two sources of asymmetry: marginal utility of consumption and marginal (dis)utility of working

$$\max_{...K_{t+i},...} \Lambda_t = ... + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... + (r_{t+i}^k z_{t+i}^\tau K_{t+i-1}^\tau - \Psi(z_{t+i}^\tau) K_{t+i-1}^\tau) + Div_{t+i}^\tau - C_{t+i}^\tau - I_{t+i}^\tau \right]$$
$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} \left[K_{t+i-1} \left[1 - \tau\right] + \left[1 - S\left(\varepsilon_{t+i}^I I_{t+i}/I_{t+i-1}\right)\right] I_{t+i} - K_{t+i} \right]$$

Produces the first order condition:

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[Q_{t+1} \left(1 - \tau\right) + \left(r_{t+1}^k z_{t+1}^\tau - \Psi(z_{t+1}^\tau)\right) \right] \right] = Q_t \tag{46}$$

Being $\frac{\mu_t}{\lambda_t} = Q_t$
Maximizing on investment:

$$\max_{...I_{t+i}...} \Lambda_t = ... + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... - I_{t+i}^\tau \right]$$
$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} \left[... + \left[1 - S\left(\varepsilon_{t+i}^I I_{t+i}/I_{t+i-1}\right)\right] I_{t+i} ... \right]$$

Produces the first order condition:

$$1 = Q_t \left[1 - \frac{\partial S\left(\varepsilon_t^I I_t / I_{t-1}\right)}{\partial I_t} \left(\frac{\varepsilon_t^I}{I_{t-1}} \right) I_t - S\left(\varepsilon_t^I I_t / I_{t+1}\right) \right]$$
$$+ \beta E_t \left\{ Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\partial S\left(\varepsilon_{t+1}^I I_{t+1} / I_t\right)}{\partial I_t} \varepsilon_{t+1}^I I_{t+1}^2 \frac{1}{I_t^2} \right] \right\} \tag{47}$$

2.3 Technologies and firms

There is a continuum of monopolistic firms that produce intermediate goods, which are indexed by $j, j \in [0,1]$. In the final goods sector , there is a competitive firm that purchases intermediate goods from a continuum of firms in a competitive way. Thus, the final goods supply, which is used for consumption and investment, is given by:

$$Y_t = \left[\int_0^1 \left(y_t^j \right)^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}} \tag{48}$$

y_t^j denoting the quantity of domestic intermediate goods type, j is used for the final good in time t, and $\lambda_{p,t}$ is a stochastic parameter that determines a variable margin in the goods market. It is supposed that $\lambda_{p,t} = \lambda_p + \eta_t^p$, being η_t^p IID-normal. As in the case of labor demand, the firm minimizes the cost of buying intermediate goods y_t^j by paying nominal prices P_t^j. -Consequently, the demand for intermediate good j is

$$y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{49}$$

And the price aggregator for this problem is

$$P_t = \left[\int_0^1 \left(P_t^j\right)^{-1/(\lambda_{p,t})} dj\right]^{-\lambda_{p,t}} \tag{50}$$

For the intermediate goods producer, the technology of production is:

$$y_t^j = \varepsilon_t^a \widetilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi \tag{51}$$

$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a \tag{52}$$

$\widetilde{K}_{j,t}$ is the effective capital utilization $\widetilde{K}_{j,t} = z_t K_{j,t-1}$, $L_{j,t}$ is the index of different types of labor used by the firms. Φ is a fixed cost. Cost minimization implies:

$$\min \frac{WL_{j,t}}{P_t} + r_t^k \widetilde{K}_{j,t} + MC_t \left[y_t^j - \varepsilon_t^a \widetilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} + \Phi\right]$$

where the first-order conditions for labor and physical capital lead us to:

$$\frac{W_t}{P_t} = MC_t(1-\alpha)\varepsilon_t^a \widetilde{K}_{j,t}^{\alpha} L_{j,t}^{-\alpha}$$

$$r_t^k = MC_t \alpha \varepsilon_t^a \widetilde{K}_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha}$$

Solving for $\frac{W_t}{P_t}$ we have:

$$\frac{W_t}{P_t}\frac{L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha} \tag{53}$$

which means that capital and labor are the same for every firm j. Then, solving for real marginal costs of firms:

$$MC_t = \frac{1}{\varepsilon_t^a}\left(\frac{W_t}{P_t}\right)^{1-\alpha}\left(r_t^k\right)^{\alpha}\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right) \tag{54}$$

Thus, nominal profits of firm j would be given by:

$$\pi_t^j = (P_t^j - P_t MC_t)\left(\frac{P_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t - P_t MC_t \Phi \tag{55}$$

Given that the stochastic discount rate of firms is $\beta^i \frac{\lambda_{t+i}}{\lambda_t P_{t+i}}$, the inter-temporal profits of producers that can re-optimize their prices in time t are:

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t P_{t+i}} \left((P_{t+i}^j - P_{t+i} MC_{t+i}) \left(\frac{P_{t+i}^j}{P_{t+i}} \right)^{-\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}} Y_{t+i} - P_{t+i} MC_{t+i} \Phi \right)$$

and because of indexation of prices $P_{t+i}^j = \widetilde{P}_t^j \left(X_{t,t+i} \right)^{\gamma_p}$

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t P_{t+i}} \left(P_{t+i}^j \left(\frac{P_{t+i}^j}{P_{t+i}} \right)^{-\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}} Y_{t+i} - P_{t+i} MC_{t+i} \left(\frac{P_{t+i}^j}{P_{t+i}} \right)^{-\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}} Y_{t+i} - P_{t+i} MC_{t+i} \Phi \right)$$

the firm maximizes its inter-temporal profits by choosing $\widetilde{P}_t^j$:, which leads us to the following firt order condition for optimal price setting:

$$\widetilde{P}_t^j E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t} \frac{\left(X_{t,t+i} \right)^{\gamma_p}}{P_{t+i}} y_{t+1}^j \frac{1}{\lambda_{p,t+i}} = E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t} y_{t+1}^j \left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}} MC_{t+i} \right)$$

and after some few algebra, finally, the first order condition for re-optimizing firms is:

$$\frac{\widetilde{P}_t^j}{P_t} \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_p} E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t} \frac{\left(P_{t+i-1}/P_t \right)^{\gamma_p}}{P_{t+i}/P_t} y_{t+1}^j \frac{1}{\lambda_{p,t+i}} = E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t} y_{t+1}^j \left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}} MC_{t+i} \right)$$

Given the definition of prices, the motion law for them is:

$$\left(P_t \right)^{-1/\lambda_{p,t}} = \xi_p \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \right)^{-1/\lambda_{p,t}} + (1-\xi_p) \left(\widetilde{P}_t^j \right)^{-1/\lambda_{p,t}} \tag{56}$$

The market clearing of the economy is given by:

$$Y_t = C_t + G_t + I_t + \Psi(z_t) K_{t-1} \tag{57}$$

$G_t = Y_{ss} gy_t$, and $gy_t = gy \varepsilon_t^g$, $\ln(\varepsilon_t^g) = \rho_g \ln(\varepsilon_{t-1}^g) + \eta_t^g$

2.4 First order conditions and asymmetry

Given the utility function, the marginal utility of consumption and leisure will be given by 58 and 59 respectively:

$$\begin{aligned} \frac{\partial U_{t+1}^{\tau}}{\partial C_{t+1}} = U_t^{C\tau} = \varepsilon_t^b \frac{\overline{\theta} \left(z_{ct} \right)^{\overline{\theta}-1}}{\overline{C}_t} + \varepsilon_t^b \frac{\partial \phi_{ct}}{\partial z_{ct}} \frac{\left[\left(z_{ct} \right)^{\underline{\theta}} - \left(z_{ct} \right)^{\overline{\theta}} \right]}{\overline{C}_t} \\ + \varepsilon_t^b \phi_{ct} \left[\frac{\underline{\theta} \left(z_{ct} \right)^{\underline{\theta}-1}}{\overline{C}_t} - \frac{\overline{\theta} \left(z_{ct} \right)^{\overline{\theta}-1}}{\overline{C}_t} \right] \end{aligned} \tag{58}$$

$$U_t^{\mathcal{L}\tau} = \frac{\partial U_t^\tau}{\partial \mathcal{L}_t}\frac{\partial \mathcal{L}_t}{\partial l_t} = \varepsilon_t^b \varepsilon_t^L \frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t} + \varepsilon_t^b \varepsilon_t^L \frac{\partial \phi_{\mathcal{L}t}}{\partial z_{lt}} \frac{\left[(z_{\mathcal{L}t})^{\underline{\mu}} - (z_{\mathcal{L}t})^{\overline{\mu}}\right]}{\overline{\mathcal{L}}_t} + \varepsilon_t^b \varepsilon_t^L \phi_{\mathcal{L}t}\left[\frac{\underline{\mu}\left(z_{\mathcal{L}t}\right)^{\underline{\mu}-1}}{\overline{\mathcal{L}}_t} - \frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t}\right] \tag{59}$$

Then, the first order conditions for consumption will be expressed as:

$$E_t\left[\beta \frac{\lambda_t}{\lambda_{t+1}}\frac{R_t P_t}{P_{t+1}}\right] = 1 \tag{60}$$

$$\lambda_t = \varepsilon_t^b \frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t} + \varepsilon_t^b \frac{-\gamma\left(z_{ct}\right)^{\gamma-1}}{\left[1+(z_{ct})^\gamma\right]^2} \frac{\left[(z_{ct})^{\underline{\theta}} - (z_{ct})^{\overline{\theta}}\right]}{\overline{C}_t} + \varepsilon_t^b \phi_{ct}\left[\frac{\underline{\theta}\left(z_{ct}\right)^{\underline{\theta}-1}}{\overline{C}_t} - \frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}\right] \tag{61}$$

updating up to time $t+1$

$$\lambda_{t+1} = \varepsilon_{t+1}^b \frac{\overline{\theta}\left(z_{ct+1}\right)^{\overline{\theta}-1}}{\overline{C}_{t+1}} + \varepsilon_{t+1}^b \frac{-\gamma\left(z_{ct+1}\right)^{\gamma-1}}{\left[1+(z_{ct+1})^\gamma\right]^2} \frac{\left[(z_{ct+1})^{\underline{\theta}} - (z_{ct+1})^{\overline{\theta}}\right]}{\overline{C}_{t+1}} + \varepsilon_{t+1}^b \phi_{ct+1}\left[\frac{\underline{\theta}\left(z_{ct+1}\right)^{\underline{\theta}-1}}{\overline{C}_{t+1}} - \frac{\overline{\theta}\left(z_{ct+1}\right)^{\overline{\theta}-1}}{\overline{C}_{t+1}}\right] \tag{62}$$

As it can be seen, by replacing 61 and 62 in 60 it is possible to obtain an asymmetric Euler Equation for consumption.

The policy rule or reaction function of the monetary authority is expressed as in Smets-Wouters (2003) as deviations from the log-linearized steady state:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)\{\overline{\pi}_t + r_\pi(\widehat{\pi}_{t-1} - \overline{\pi}_t) + r_y(\widehat{Y}_t - \widehat{Y}_t^p)\} + r_{\Delta\pi}(\widehat{\pi}_t - \widehat{\pi}_{t-1}) + r_{\Delta y}(\widehat{Y}_t - \widehat{Y}_t^p - (\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^p)) + \eta_t^R \tag{63}$$

where $\widehat{x}$ denotes logarithmic deviations of x from the steady state. Inflation target is represented by $\overline{\pi}_t$. It follows an auto-regressive process $\overline{\pi}_t = \rho\overline{\pi}_{t-1} + \eta_t^\pi$. Finally, η_t^R is a transitory IID-normal shock on the interest rate, which is denoted as a monetary policy shock. However, we will use the Taylor rule expressed in levels because we are to use Perturbations Method implemented in Dynere in order to solve this model:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{r_R}\left[\left(\frac{\Pi_t}{\Pi}\right)^{r_\Pi}\left(\frac{Y_t}{Y}\right)^{r_Y}\right]^{1-r_R}\varepsilon_t^R \tag{64}$$

being $\varepsilon_t^R = \exp(\eta_t^R\ )$.

The first-order conditions, the closures, the equilibrium market and the monetary policy rule represent the dynamic system of this hypothetical economy.

3 Calibration and simulation

We need some concrete functional forms to make the model operative. Also in this dissertation, the solution of the dynamic system is made by using third-order approximation in order to preserve the asymmetric nature of:

for the adjustment cost of investment, we use the following equation:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{\varepsilon_t I_t}{I_{t-1}} - 1\right)^2 \tag{65}$$

This functional function fulfills the properties stated by Smets and Wouters (2003): $S(1) = S'(1) = 0$, and $S''(I) > 0$.

For the utilization cost of capital utilization rate, we will follow Baxter and Farr (2005), but we will introduce a slight variation: whereas they model a convex function for capital depreciation depending on capital utilization rate, we will introduce the same functional form to express the cost of choosing z_t in terms of consumption goods. This function must fulfill $\Psi(1) = 0$ and $\Psi''(1)/\Psi'(1) = \varsigma$, as specified by Baxter and Farr (2005), Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003).

$$\Psi(z_t) = \Psi_0 + \Psi_1 \frac{z_t^{1+\zeta}}{1+\varsigma} \tag{66}$$

Thus, it is required that $\Rightarrow \Psi_0 = \frac{-\Psi_1}{1+\varsigma}$. Therefore, and with no loss of generality in order to facilitate calibration, we set $\Psi(z_t) = -\frac{r^k}{1+\varsigma} + r^k \frac{z_t^{1+\zeta}}{1+\varsigma}$, which is an increasing convex function as suggested by Christiano, Eichenbaum, and Evans (2005).[2] The full dynamic system is summarized in annex 1. In the steady state, this must take place: $z_c = 1$, $z_{\mathcal{L}} = 1$, $\overline{C}_t = C^{\tau}$, $\overline{\mathcal{L}}_t = \mathcal{L}$, $\mathcal{L}^{\tau} = 1 - l^{\tau}$, $\phi_{ct} = 0.5$, $\phi_{\mathcal{L}t} = 0.5$, $\frac{\partial \phi_{ct}}{\partial z_{ct}} = -0.25\gamma_c$, $\frac{\partial \phi_{\mathcal{L}t}}{\partial z_{lt}} = -0.25\gamma_{\mathcal{L}}$, $\beta R_t = \frac{P_{t+1}}{P_t} = (1+\Pi)$, $\lambda_t = \frac{\overline{\theta}}{\overline{C}_t} + 0.5\left[\frac{\underline{\theta}}{\underline{\overline{C}}_t} - \frac{\overline{\theta}}{\overline{C}_t}\right] = \frac{0.5}{\overline{C}_t}\left[\underline{\theta} + \overline{\theta}\right]$, $U_t^{\mathcal{L}\tau} = \frac{\partial U_t^{\tau}}{\partial \mathcal{L}_t}\frac{\partial \mathcal{L}_t}{\partial l_t} = -\frac{\overline{\mu}}{\overline{\mathcal{L}}_t} - 0.5\left[\frac{\underline{\mu}}{\underline{\overline{\mathcal{L}}}_t} - \frac{\overline{\mu}}{\overline{\mathcal{L}}_t}\right] = -\frac{0.5}{\overline{\mathcal{L}}_t}\left[\underline{\mu} + \overline{\mu}\right]$, $K = K[1-\tau] + I$, $S(1) = \frac{\kappa}{2}\left(\frac{I}{I} - 1\right)^2 I^2 = 0$, $\Psi(1) = -\frac{r^k}{1+\varsigma} + r^k \frac{z_t^{1+\zeta}}{1+\varsigma} = 0$, $\frac{1}{\beta} - (1-\tau) = r^k$, $1 = Q, r_t^k = \Psi'(z_t) = r^k$, $Y_t = C_t + G_t + I_t$, $1 = \varepsilon^R$, $\ln \varepsilon^b = 0$, $\ln \varepsilon^L = 0$, $\ln \varepsilon^I = 0$, $\ln \varepsilon^a = 0$, $\ln(\varepsilon^g) = 0$, $\overline{\pi}_t = \rho \overline{\pi}_{t-1} + \eta_t^{\pi}$, $\varepsilon^R = 1$.

In a non-deflationary economy, from the euler equation, it must be satisfied that$\beta R = 1 + \Pi > 1$. Thus, we need to calibrate values for β, R and Π, so that this inequality is accomplished. Besides we also need to guarantee that $\frac{1}{\frac{1+\Pi}{R}} - (1-\tau) = r^k$. Also we need to guarantee that $r^k = MC\alpha K^{\alpha-1}L^{1-\alpha} = \frac{R}{1+\Pi} - (1-\tau)$.

As in CEE (2005), monopolistic rents are eliminated in the the long run; thus, $\pi_t^j = 0$, and we will have[3]:

[2]Because CEE(2005) and Smets-Wouters (2003) use log-linearisation method to simulate and solve the model, they only need to specify some properties of these equations in terms of derivatives and values in the steady state. In this chapter however the nonlinear model is simulated by means of a k-order perturbations method, thus we need concrete functional forms accomplishing the properties just claimed.

[3]Because there is no entry barriers, in the long run, the positive benefits derived from monopolistic competition will be exhausted

$$\pi_t^j = (P_t^j - P_t MC_t)\left(\frac{P_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t - P_t MC_t \Phi = 0 \tag{67}$$

$$(1 - MC_t)Y_t = MC_t \Phi \tag{68}$$

$$\Phi = \frac{(1 - MC_t)Y_t}{MC_t} \tag{69}$$

$$\varphi Y_t = \Phi = \frac{(1 - MC_t)Y_t}{MC_t} \tag{70}$$

$$\varphi = \frac{(1 - MC_t)}{MC_t} \tag{71}$$

Thus, if $\Phi = \varphi Y_t$, it is possible to write:

$$Y_t = \varepsilon_t^a K^\alpha L^{1-\alpha} - \Phi$$
$$Y_t = \varepsilon_t^a K^\alpha L^{1-\alpha} - \varphi Y_t$$

And after some algebra in the steady state, the total income will be:

$$Y = \left[\frac{\varepsilon_t^a (K/Y)^\alpha L^{1-\alpha}}{(1+\varphi)}\right]^{1/(1-\alpha)}$$

A different strategy can be:

$$\Phi = \frac{(1 - MC_t)Y_t}{MC_t}$$

Again, after some algebra:

$$Y = \left\{\left[1 + \frac{(1 - MC_t)}{MC_t}\right]\left(\frac{K}{Y_t}\right)^{-\alpha} L^{-(1-\alpha)}\right\}^{-1/1-\alpha}$$

It must be also accomplished that:

$$\frac{W_t}{P_t} = \frac{1-\alpha}{\alpha}\frac{r^k K}{L} \tag{72}$$

$$1 = (1 + \lambda_p) MC_t \tag{73}$$

$$\frac{W_t}{P_t} = \frac{\overline{C}_t}{\overline{\mathcal{L}}_t}\left(\frac{\underline{\mu} + \overline{\mu}}{\underline{\theta} + \overline{\theta}}\right)(1 + \lambda_w) \tag{74}$$

additionally:

$$1 = (1 + \lambda_p) MC_t \tag{75}$$

$$MC_t = \frac{1}{1+\varphi}$$

$$1 = (1+\lambda_p)\frac{1}{1+\varphi}$$

$$1+\varphi = 1+\lambda_p$$

$$\varphi = \lambda_p$$

Because there is inflation different from zero in the steady state, it is necessary to transform nominal quantities into relative quantities as: $\frac{\widetilde{W}_t}{P_t}$, $\frac{\widetilde{P}_t^j}{P_t}$. Thus, some key equations involving nominal prices and wages can be written as:

$$\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p} E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}\frac{(P_{t+i-1}/P_t)^{\gamma_p}}{P_{t+i}/P_t}y_{t+i}^j\frac{1}{\lambda_{p,t+i}} = E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+i}^j\left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}\right)$$

To make this expression operative for simulation effects we re write it as

$$E_0\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p} S1_t = E_0 S2_t \tag{76}$$

where $S1_t = \sum_{i=0}^{\infty}\frac{1}{\lambda_{p,t+i}}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+i}^j\left(\frac{(P_{t+i-1}/P_t)^{\gamma_p}}{(P_{t+i}/P_t)}\right)$ and $S2_t = \sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+1}^j\frac{(1+\lambda_{p,t+i})}{\lambda_{p,t+i}}MC_{t+i}$, which given their recursive nature are up dated as follows:

$$S1_t = \frac{y_t^j}{\lambda_{p,t}}\left(1/(1+Pit)\right)^{\gamma_p} + (1+Pit(+1))^{\gamma_p-1}\frac{\lambda_{t+1}}{\lambda_t}\beta\xi_p S1_{t+1} \tag{77}$$

$$S2_t = y_t^j\frac{(1+\lambda_{p,t})}{\lambda_{p,t}}MC_t + \beta\xi_p\frac{\lambda_{t+1}}{\lambda_t}S2_{t+1} \tag{78}$$

Then, the Calvo style infinite summation for prices can be written as the respective laws of motion for $S1_t$ and $S2_t$.
Similarly for the summations of wages we have:41, we can obtain the following expression:

$$E_0 S3_t = \frac{\widetilde{W}_t}{P_t}E_0 S4_t \tag{79}$$

and analogously we define $S3_t = \sum_{i=0}^{\infty}\beta^i\xi_w^i\frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}\left(\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}l_{t+i}^\tau\right)$ and $S4_t = \sum_{i=0}^{\infty}\beta^i U_{t+i}^C\xi_w^i l_{t,t+i}^\tau\frac{P_t}{P_{t+i}}\left(\frac{P_{t+i-1}}{P_t}\right)^{\gamma_w}\frac{1}{\lambda_{w,t+i}}$, each with its respective law of motion: $S3_t = \frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}\left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}}\right)l_t^\tau + \beta\xi_w S3_{t+1}$ and $S4_t = U_t^C l_t^\tau\frac{1}{\lambda_{w,t}}\left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} + (1+Pit(+1))^{\gamma_w-1}\beta\xi_w S4_{t+1}$

in the steady state:

$$S1 = \frac{y_t^j}{\lambda_{p,t}} \left(1/(1+Pit)\right)^{\gamma_p} + (1+Pit)^{\gamma_p - 1} \beta \xi_p S1$$
$$S1 = \frac{1}{1-(1+Pit)^{\gamma_p - 1} \beta \xi_p} \frac{y_t^j}{\lambda_{p,t}} \left(1/(1+Pit)\right)^{\gamma_p}$$

If Pit was equal to zero in the steady state, $S1$ would be

$$S1_t = \frac{y_t^j}{\lambda_{p,t}} \left(P_{t-1}/P_t\right)^{\gamma_p} + \beta \xi_p S1_t$$
$$S1_t = \frac{1}{1-\beta \xi_p} \frac{\lambda_t y_t^j}{\lambda_{p,t}} \left(P_{t-1}/P_t\right)^{\gamma_p}$$

For $S2$ we have

$$S2_t = y_t^j \frac{(1+\lambda_{p,t})}{\lambda_{p,t}} MC_t + \beta \xi_p S2_t$$
$$S2_t = \frac{1}{1-\beta \xi_p} \frac{y_t^j (1+\lambda_{p,t})}{\lambda_{p,t}} MC_t$$

For $S4$

$$S4 = U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} + (1+Pit(+1))^{\gamma_w - 1} \beta \xi_w S4$$
$$S4 = \frac{1}{1-(1+Pit(+1))^{\gamma_w - 1} \beta \xi_w} U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w}$$

Similarly If Pit was equal to zero in the steady state, $S4$ would be

$$S4_t = U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} + \beta \xi_w S4_t$$
$$S4_t = \frac{1}{1-\beta \xi_w} U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w}$$

Finally for $S3$

$$S3_t = \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}}\right) l_t^\tau + \beta \xi_w S3_t$$
$$S3_t = \frac{1}{1-\beta \xi_w} \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}} l_t^\tau\right)$$

The summations for prices and wages in the steady state will be

$$\frac{\widetilde{P}_t^j}{P_t}\frac{1}{1-(1+Pit)^{\gamma_p-1}\beta\xi_p}\frac{y_t^j}{\lambda_{p,t}} = \frac{1}{1-\beta\xi_p}\frac{y_t^j(1+\lambda_{p,t})}{\lambda_{p,t}}MC_t$$

$$\frac{\widetilde{P}_t^j}{P_t} = \frac{1-(1+Pit)^{\gamma_p-1}\beta\xi_p}{1-\beta\xi_p}(1+\lambda_{p,t})MC_t$$

$$\frac{\widetilde{W}_t}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}\frac{1}{1-(1+Pit(+1))^{\gamma_w-1}\beta\xi_w}U_t^C l_t^\tau\frac{1}{\lambda_{w,t}}\left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} = -\frac{1}{1-\beta\xi_w}\frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}\left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}}l_t^\tau\right)$$

$$\left(\frac{\widetilde{P}_t}{P_t}\right) = (1+\lambda_{p,t})MC_t$$

$$\widetilde{P}_t = P_t(1+\lambda_p)MC_t$$

$$\frac{\widetilde{W}_t}{P_t}U_t^C = -\frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}(1+\lambda_{w,t})$$

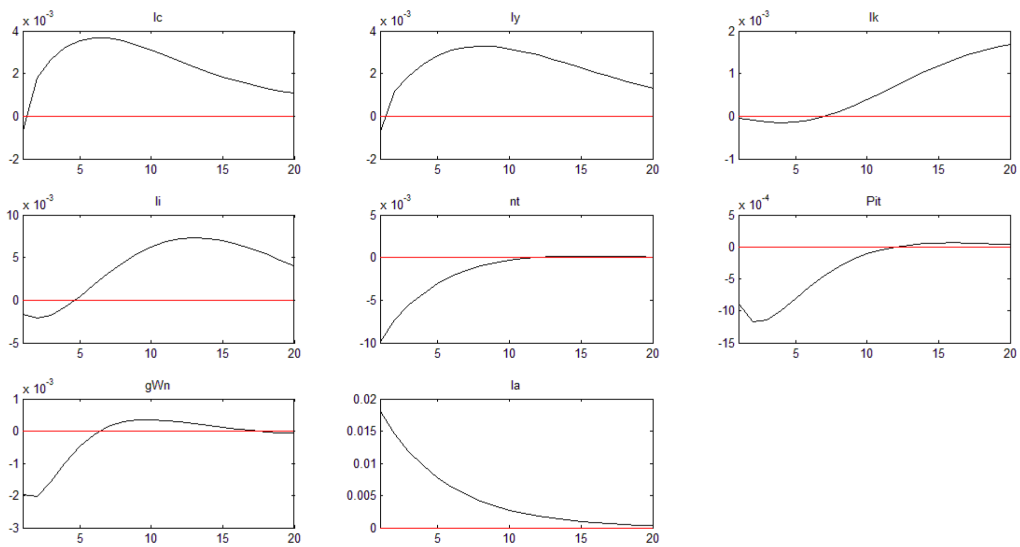
$$\frac{\widetilde{W}_t}{P_t} = \frac{\overline{C}_t}{\overline{\mathcal{L}}_t}\left(\frac{\underline{\mu}+\overline{\mu}}{\underline{\theta}+\overline{\theta}}\right)(1+\lambda_w)$$

$$\widetilde{W}_t = P_t\frac{\overline{C}_t}{\overline{\mathcal{L}}_t}\left(\frac{\underline{\mu}+\overline{\mu}}{\underline{\theta}+\overline{\theta}}\right)(1+\lambda_w)$$

3.1 Comparing the symmetric model with the asymmetric model

In these simulation exercises, we suppose that all of the physical capital is used in production, which means that $\Psi(Z) = 0$. The parameters used for calibration and simulations are those of Smets-Wouters (2003). Since this model has several stochastic processes that may disturb the hypothetical economy, price and wage rigidities, and a non-traditional utility function, it seems necessary to perform some basic experiments in order to know the dynamics of the model under traditional assumptions. In order to check whether our general asymmetrical model under the asumption of symmetry is able to generate comparable results with those of Smets-Wouters (2003), we have firstly run a simulation imposing symmetry (the agent is risk-averse). Some variables are expressed in logarithms. lc, ly, lk, li, la and lW are consumption, income, physical capital, investment, technology, and real aggregate wage logarithms respectively; nt is labor, Pit is the inflation rate of aggregate price, and gWn is the growth of nominal aggregate wage. Figure 1 shows the impulse responses for this first experiment. The results were qualitatively similar to those of Smets-Wouters (2003) under a symmetric model. As seen, a positive shock to technology produces a fall in consumption (in the first period and almost zero) and income. Differently, for capital, investment, labor, inflation and wages, the fall has been different from zero and more-lasting. This behavior can be explained by price rigidity. As a matter of fact, when the model is simulated imposing $\xi_P = 0$, the response was positive for consumption, income, capital, investment, labor, wages, rent of capital and price of capital; response of inflation was negative as expected (Figure 2).

Figure 1: GIRF, symmetry in the general model with price rigidities



Thus, it is possible to conclude that rigidity of prices forces part of the adjustment after the shock by inducing a reduction in real variables (bearing in mind that the value estimated by Smets-Wouters (2003) for the probability of not re-optimizing prices is 0.905).

Table 1 Parameters

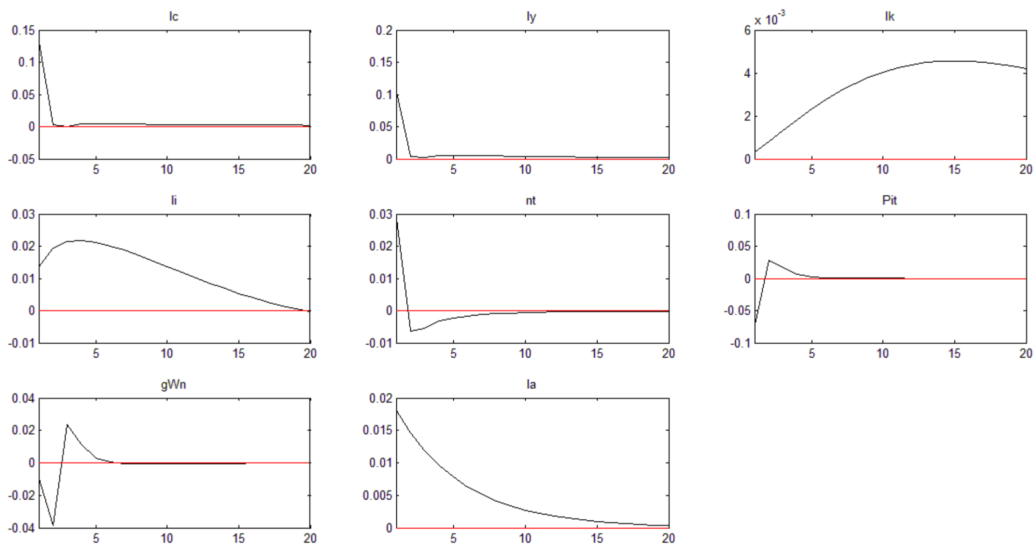
$\overline{\theta}$	$\underline{\theta}$	$\overline{\mu}$	$\underline{\mu}$	χ_c	χ_l	γ_c	γ_l	ξ_W	ξ_P	γ_W	γ_P	κ	ξ_1
0.5	0.5	0.5	0.5	0.5	0.5	50	50	0.742	0.905	0.728	0.477	6.962	0.201

τ	α	λ_W	λ_P	r_R	r_π	r_Y	ρ_b	ρ_l	ρ_I	ρ_a	ρ_π	ρ_g	Φ/Y
0.025	0.3	0.5	0.477	0.961	1.688	0.098	0.838	0.881	0.91	0.811	0.855	0.943	0.417

Source, Except for those of the utility function, all of them are taken from S-W (2003)

In this apart, we will compare the impulse responses of the technological shock in the asymmetric-sticky model with those of the symmetric-sticky model. For this exercise, asymmetry was imposed in the utility function by setting $\underline{\theta} = \underline{\mu} = 1.2$. The difference between the impulse response paths is overwhelming (Figures 3 to 5). Both models receive a positive one-standard deviation shock on technology, variables $xsim$ stand for the variable in the symmetric model and $xasim$ stands for the variable in the asymmetric model. In the first period, consumption and income in the symmetric model react negatively, while in the following periods they are greater than consumption and income respectively, within the asymmetric model. In fact, as it has been shown, price rigidities impose a stronger adjustment on real quantities in the symmetric model, but the loss-averse behavior of the agents (in consumption and leisure) induce a smoother reaction in consumption and income in the asymmetric model. The same occurs for capital, investment, and labor (for only one period). The

Figure 2: GIRF,symmetry in the general model with no price rigidities



reaction of inflation in the asymmetric model is also overwhelming: in this model, inflation of prices has a smaller reaction than in the symmetric model, which means a greater (or additional) stickiness of prices. The fall in inflation, as the wages are indexed, induces a fall in nominal wage inflation rate (gWn) (real wage also falls) in both symmetric and asymmetric models, but the decrease in gWn is greater in the asymmetric model. This can be explained by the loss-aversion in leisure. Evidently, the fall in labor on shock is almost the same in both models. The results in these simulations do not coincide with those from the neoclassical model at least for the moment of the shock. While the neoclassical model predicts, as soon has the economy is shocked, that an increase in technology will produce an increase in labor, capital, consumption and income, simulations show an increase in technology that produces a reduction in labor (for both models), a fall in physical capital (very slight), consumption and income (in the symmetric model), and a decrease in real wage (for both models). Figure 5 display impulse-responses simulations for Tobin´s Q, Q, policy interest rate, R and physical capital interest rate , rk. This behavior is explained by the rigidities explicitly modeled y this Noe-Keynesian model and the intensification suffered by them in the presence of loss-aversion.

3.2 Comparing negative and positive shocks in the asymmetric model

3.2.1 Asymmetry in consumption and leisure

For this exercise, asymmetry was imposed in the utility function by setting $\underline{\theta} = \underline{\mu} = 1.2$. Later, two simulations have been performed: a positive shock and a negative shock on technology. As in the previous exercise, the results showed asymmetries. When the (asymmetric) economy is disturbed by a positive shock on technology, the reaction in consumption, on shock, is almost the same for

Figure 3: GIRF, symmetry and asymmetric sticky models

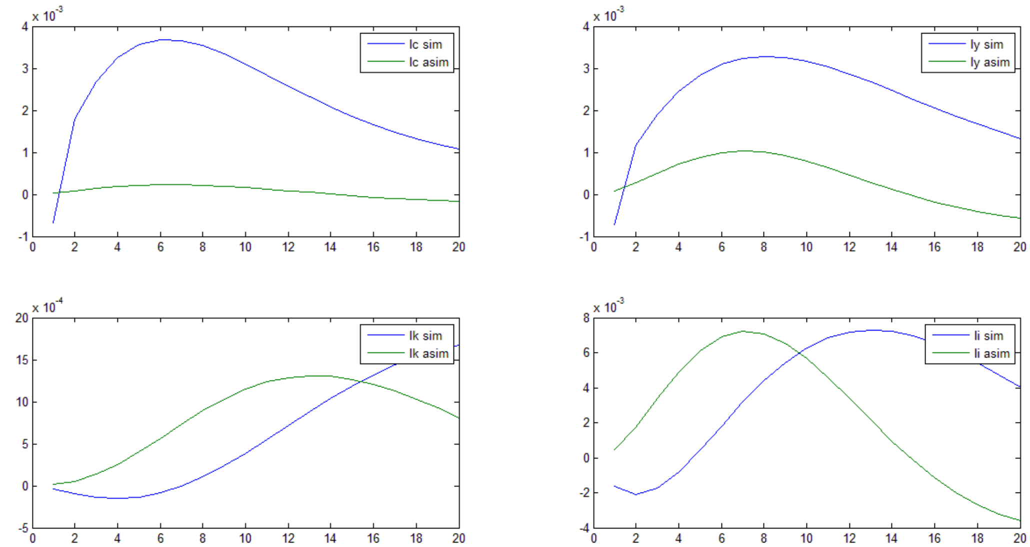


Figure 4: GIRF,symmetry and asymmetric sticky models

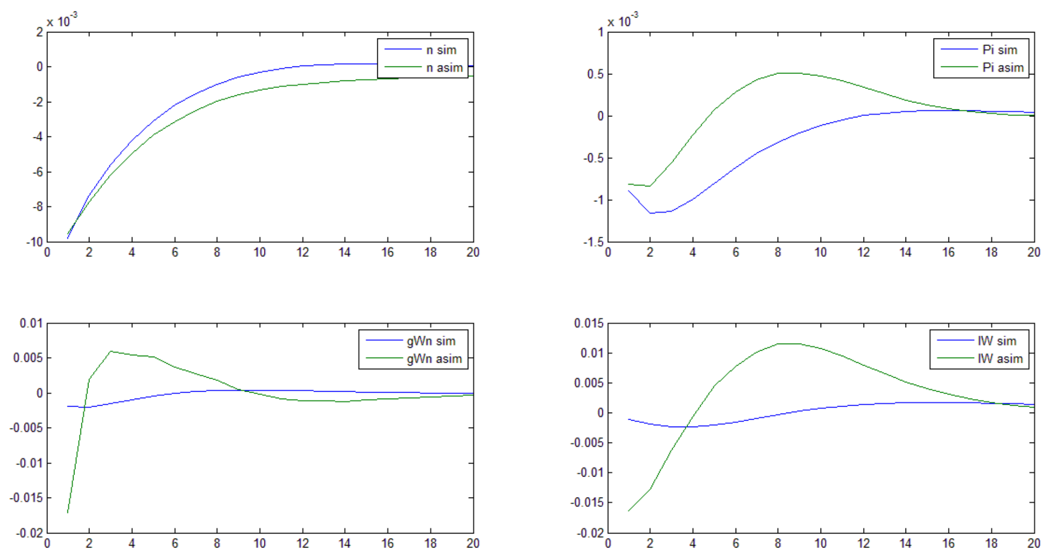
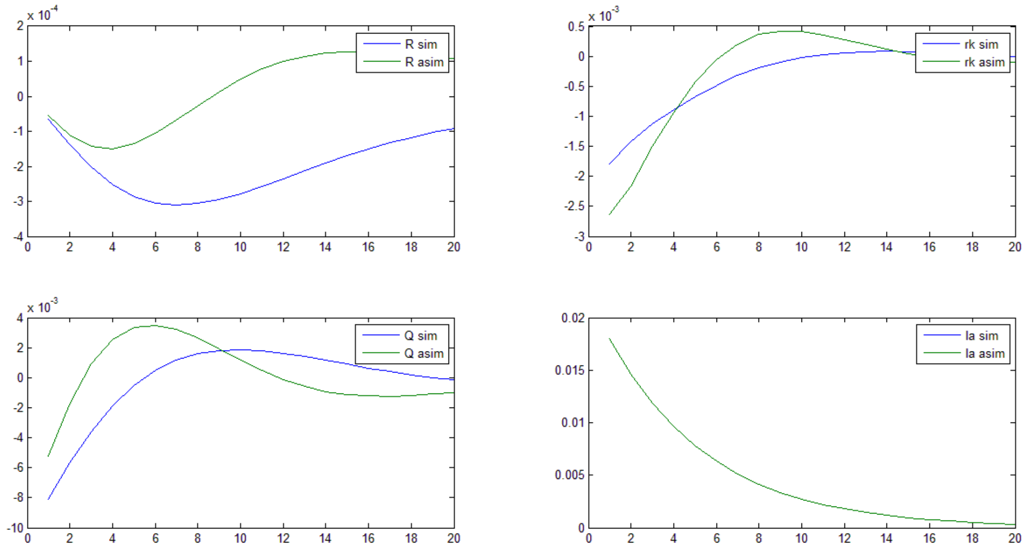


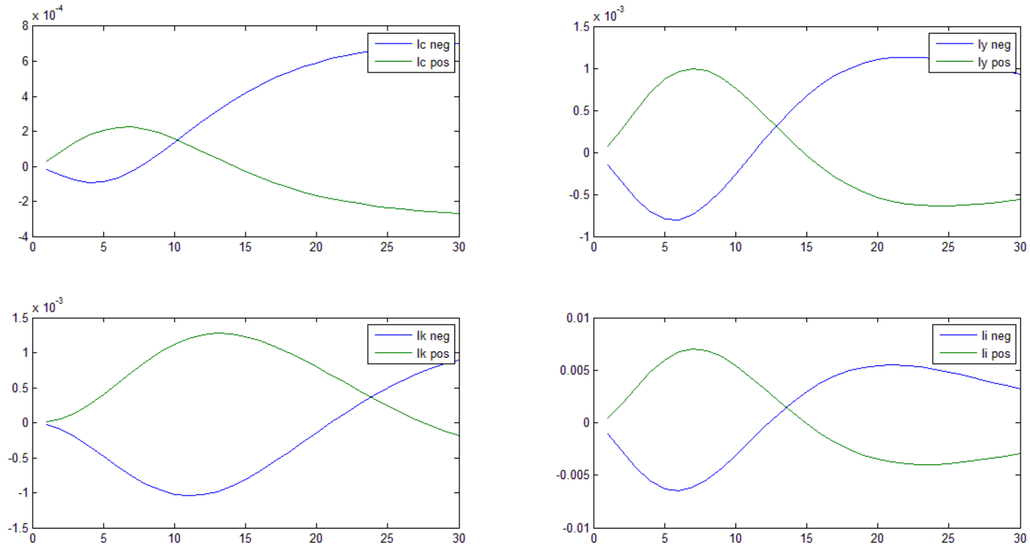
Figure 5: GIRF, symmetry and asymmetric sticky models



negative and positive shocks, but the boom induced in consumption is more long-lasting than the recession.The reaction of income is quite different: recession (on shock and for the following 4 or 5 periods) is stronger than boom, but boom is more long lasting than recession. For capital, the recession phase is stronger (on shock and for 8 or 9 following periods), but in period 9 the boom starts to be greater than recession and consequently more long-lasting. The investment response is also interesting: the recession is more intense on shock for the next 5 or 6 periods; but for period 7, investment boom begins to be higher than recession and more long-lasting (Figures 9 to 11 show absolute values of impulse response paths when the economy receives a positive shock (green line) and when the shock is negative (blue line), figures 6 to 8 show impulse response values with their original sign). The shape of figures 9 to 11 are unusual. They can be explained by the fact that the adjustment in each time path is not asymptotic monotonic but asymptotic harmonic or oscillatory. In other words, when a variable receives a positive shock, in the case of consumption, it initially experiences a boom, but, later in the process of adjustment, it will transit towards a recession. Moreover, since we are taking absolute values, a negative value jumps to a positive value when we take the absolute value operator. That is why the time paths in these figures look similar to the trajectory of a ball falling onto the ground. To be more concrete, the trajectories of variables in figures 6 to 8 are the trajectories' absolute values of variables in figures 9 to 11.

Figure 7 shows the responses of labor, wages, and inflation. For labor, the impact of the recessive shock is stronger and seems to be more long-lasting than the effect of a booming shock (this strongly suggests that this model would be able to explain the hysteresis in unemployment). Again, the positive shock on technology produces a fall in labor and the opposite is true for the negative shock (Figure 10). The most interesting path is the inflation rate one. In fact, whereas inflation in boom has a greater increase and a lasts only one more period, the reaction of inflation is lower

Figure 6: GIRF, comparing positive with negative shock in the asymmetric sticky model



in recession than in boom and has a shorter duration. Thus, prices in recession are more reluctant to decrease than to increase in boom. Wages, general and reoptimized, present a mixed behavior. Either for a general or an aggregate wage, there is a smaller reaction during recession (stronger in boom). Differently, the re-optimized wage shows a stronger movement in recession. From period 2 to period 4, general wages seem to behave similarly. However, from period 4 onwards, wage during recession shows a greater deviation from the steady state.

The response of nominal interest rate is stronger and more long-lasting during a boom, which is a very predictable behavior, although the Taylor rule employed to model monetary policy is not an optimal one. Tobin's Q (real price of capital) shows a stronger reaction in a boom and more long-lasting at least from period 1 to period 3 and from period 7 onwards. Real rent of capital (rk) has a higher reaction in recession from period 1 to 3, and seems to be more long-lasting in general.

3.2.2 Asymmetry in consumption and symmetry in leisure

In order to know the importance of the different sources of asymmetry in this model, we proceed now by imposing symmetry in the utility part delivered by leisure and allow for asymmetry in the utility function part associated with consumption, which means that the parameters of the utility function are these[4]: $\underline{\theta} = 1.2$ and $\underline{\mu} = \overline{\mu} = 0.5$. Once again, two simulations are performed: a positive shock and a negative shock on technology. The direction of the impulse responses is the same as in the previous exercise (figures 3.2.2 to 17). However, what is surprising is the fact that asymmetry is not as overwhelming as in the previous exercise, although it displays the same behavior in qualitative terms.

[4] Recall: the utility function is intratemporal additive separable.

Figure 7: GIRF, comparing positive with negative shock in the asymmetric sticky model

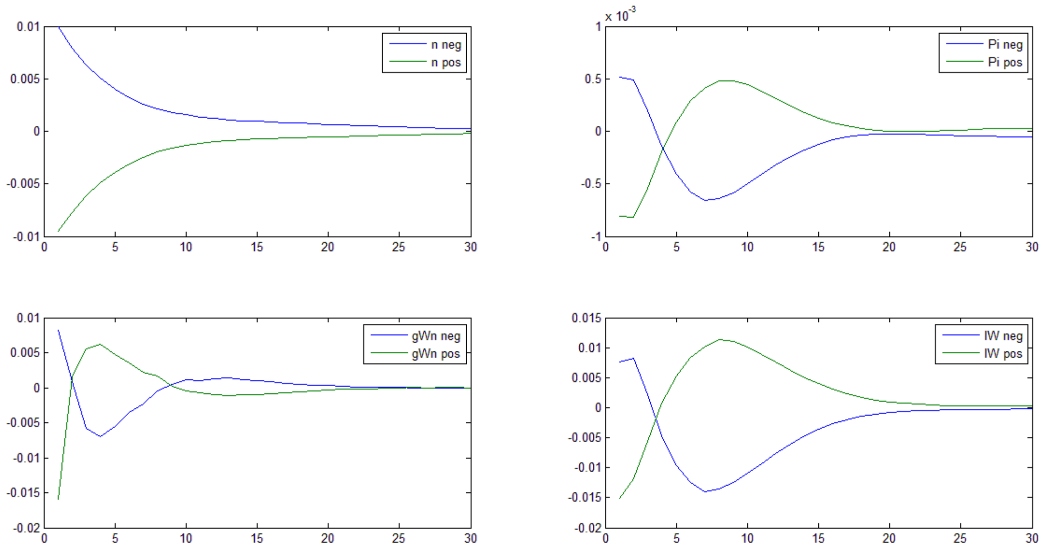


Figure 8: GIRF, comparing positive with negative shock in the asymmetric sticky model

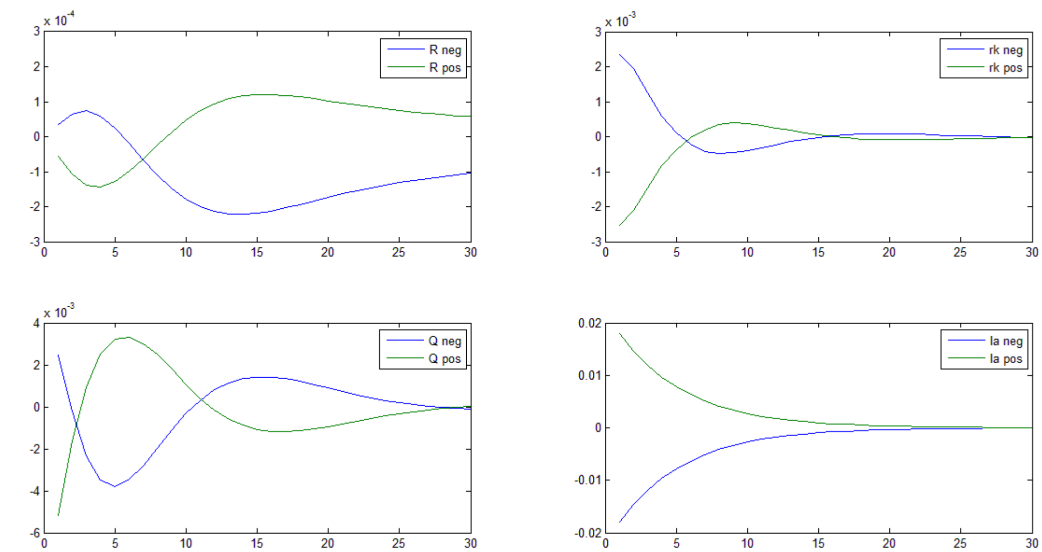


Figure 9: GIRF, comparing positive with negative shock in the asymmetric sticky model

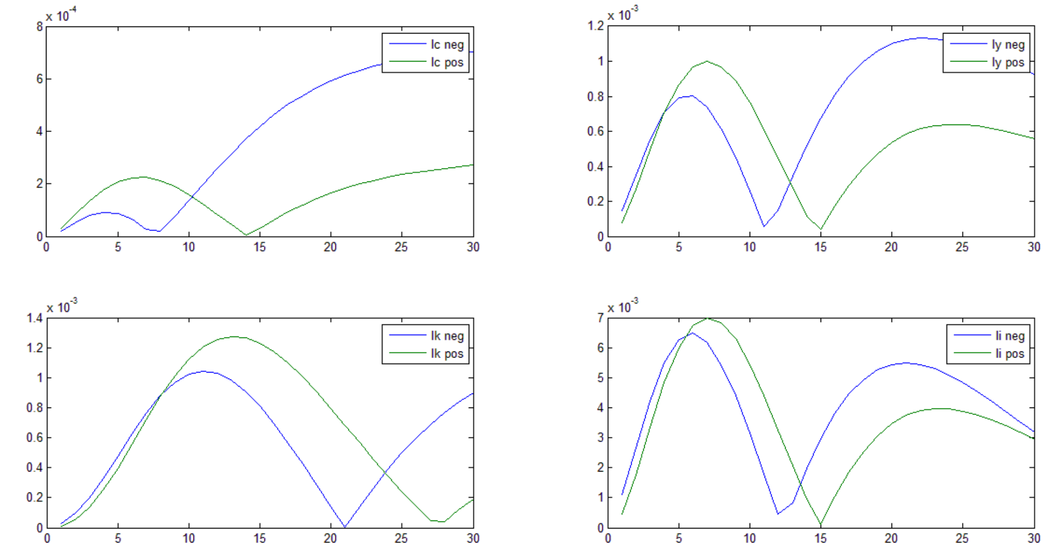


Figure 10: GIRF, comparing positive with negative shock in the asymmetric sticky model

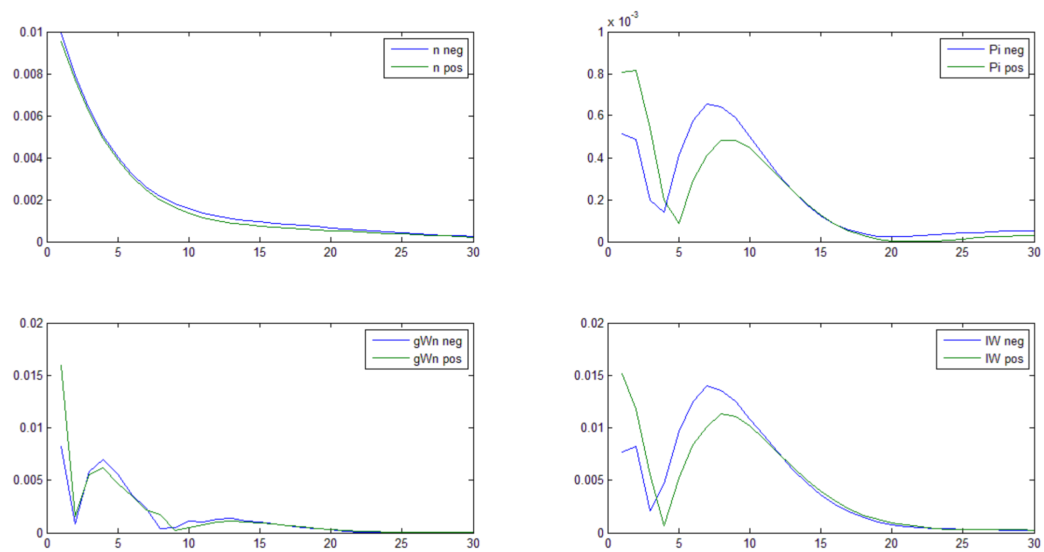
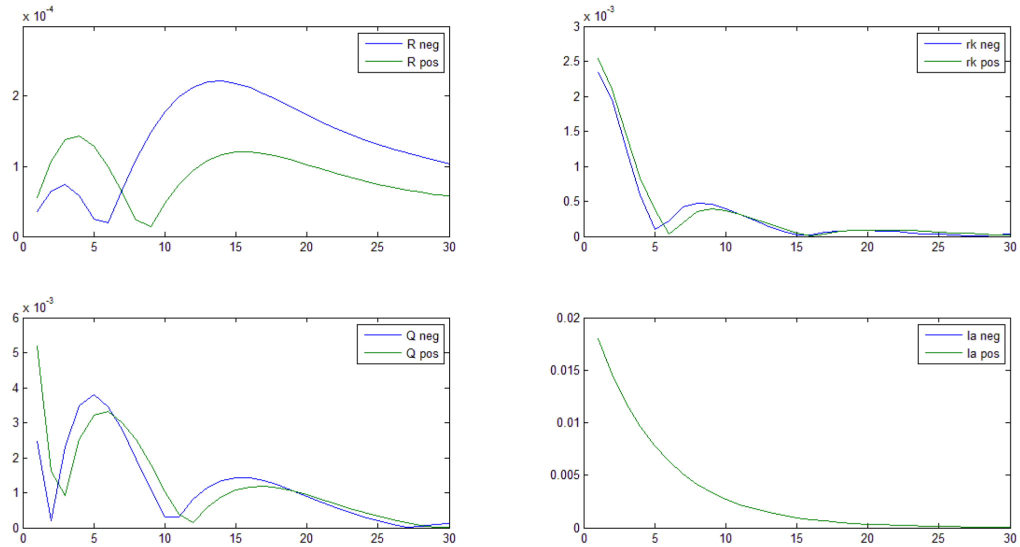


Figure 11: GIRF, comparing positive with negative shock in the asymmetric sticky model



3.2.3 Symmetry in consumption and asymmetry in leisure

In this exercise, we set $\underline{\mu} = 1.2$ and $\underline{\theta} = \overline{\theta} = 0.5$. Two simulations are performed: a positive shock and a negative shock on technology. As in previous exercises, qualitative results remain the same. A positive shock on technology causes increases in consumption, income, investment, capital, fall in labor, inflation, nominal wages inflation, real wages, rent of capital, policy interest rate, and fall in Tobin's Q. However, quantitative results are surprising in the sense that asymmetry seems to be more important under this parametrization than under the previous one. In other words, when we suppose asymmetry only in consumption, the asymmetry is very slight. However, when we suppose asymmetry in leisure alone, the asymmetry is notorious and the trajectories of variables are very similar to those of the simulations when supposing asymmetry in consumption and leisure. We can explain this by the fact that as income falls, the agent needs to dampen the fall in welfare by demanding more leisure (employment will decrease indeed), which means that the agent will demand a higher wage or will reject a huge decrease in wage. (figures 18to 23).

3.3 Comparing negative and positive shocks in the asymmetric model with moderate price stickiness

Previous exercises were performed under extreme price stickiness ($\xi_P = 0.905$). Here, I will present the results of the same technological shock under moderate price stickiness by setting $\xi_P = 0.4525$. Figures 24 to 26 show the impulse response path of variables after a positive shock and a negative shock. Again, a positive shock to technology generates a decrease in labor (the opposite for a negative shock), but, unlike to the case of extreme stickiness, the increase in consumption and

Figure 12: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

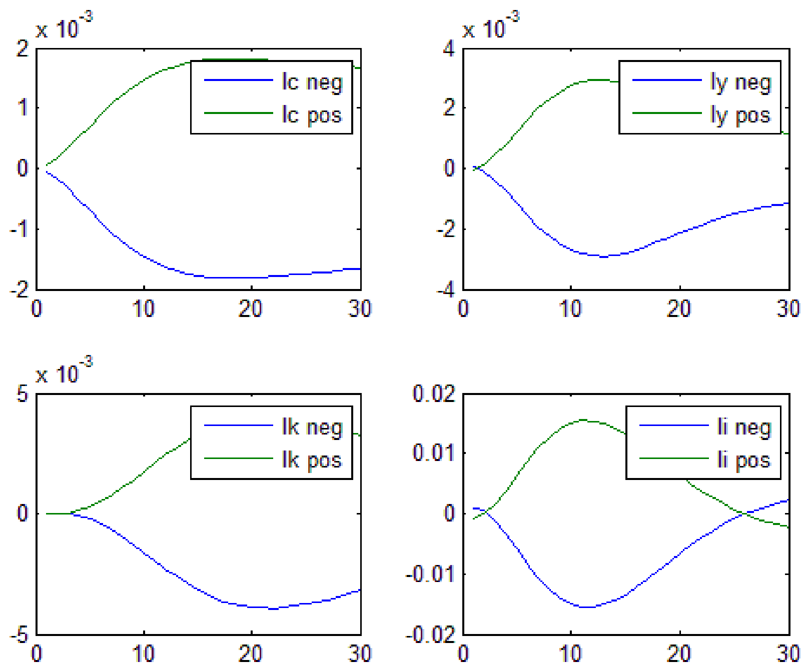


Figure 13: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

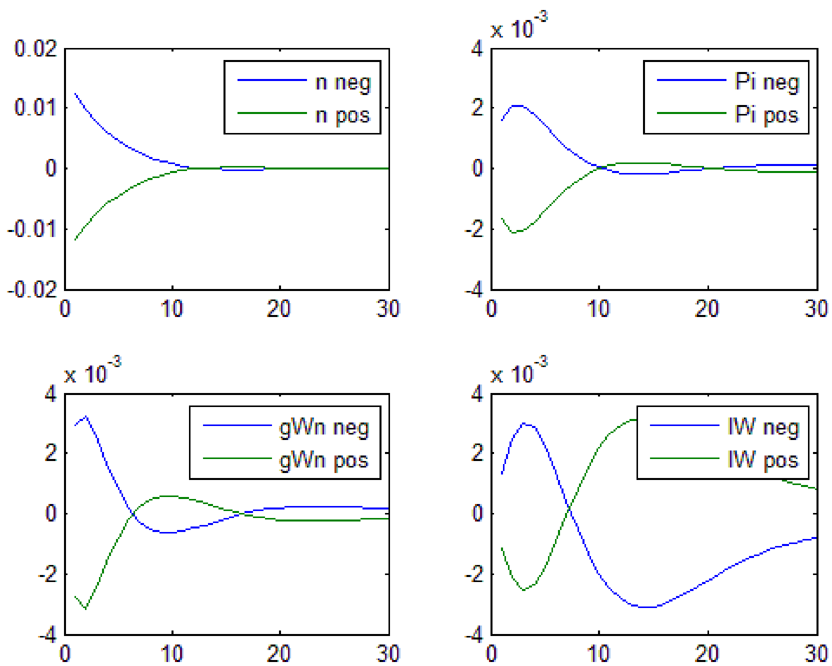


Figure 14: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

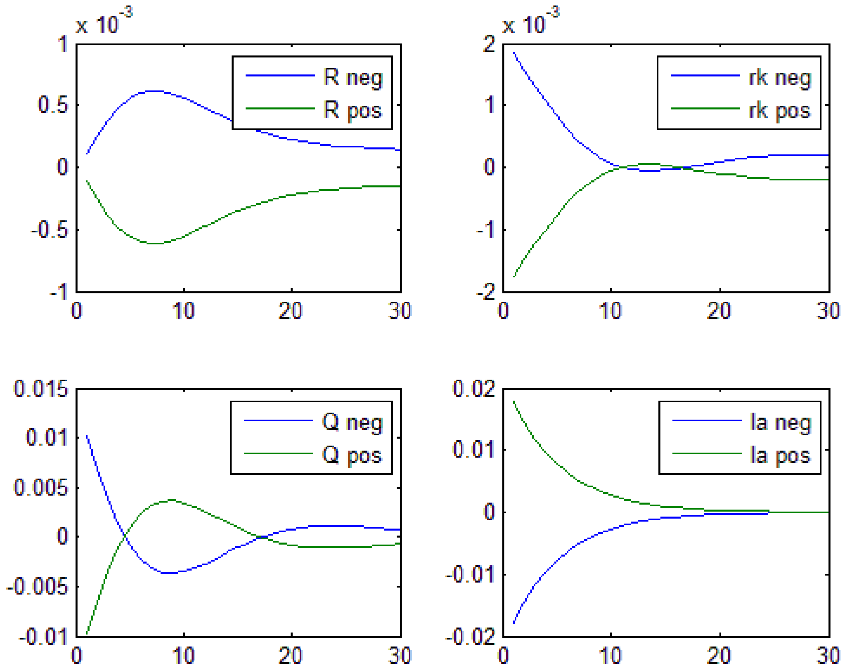


Figure 15: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

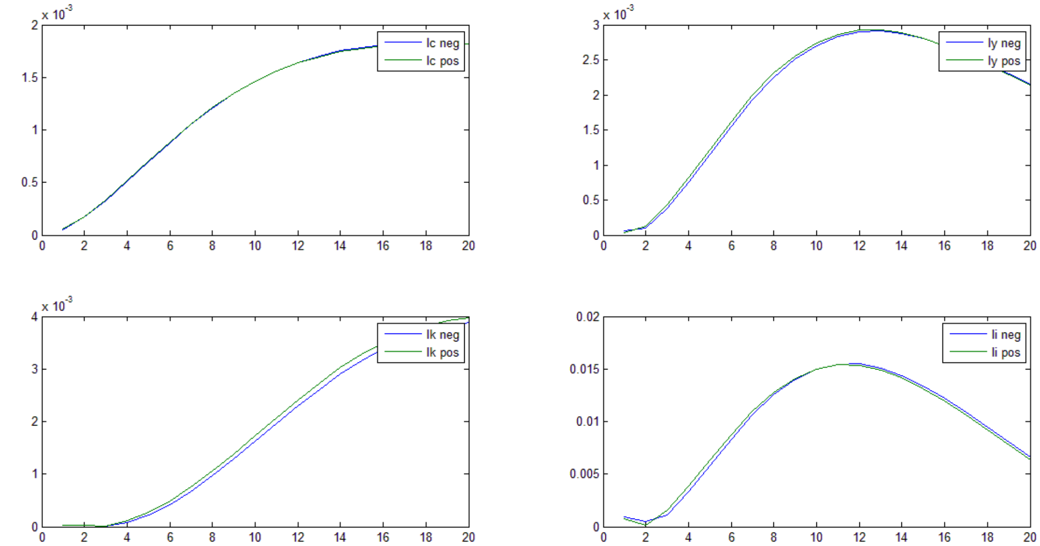


Figure 16: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

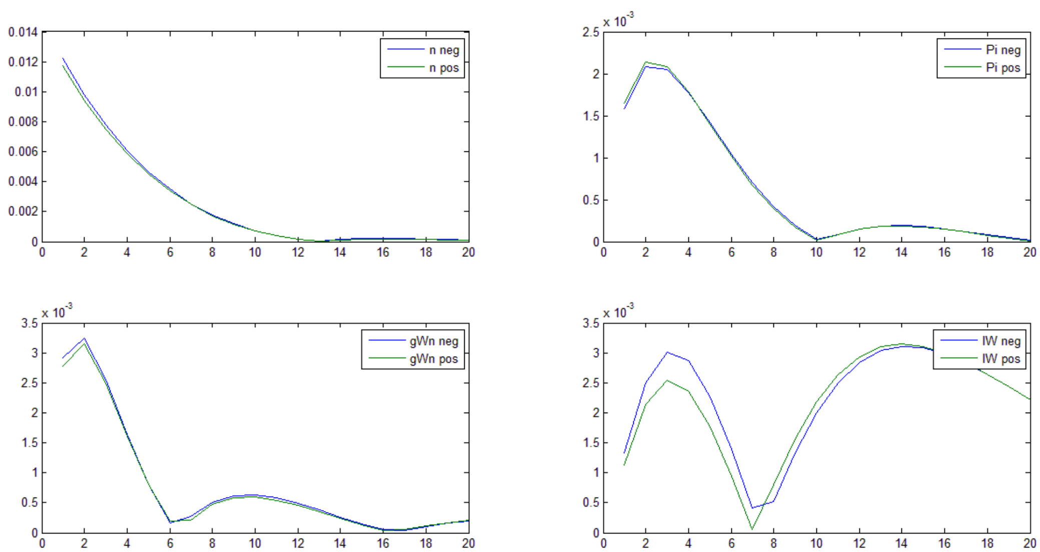


Figure 17: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

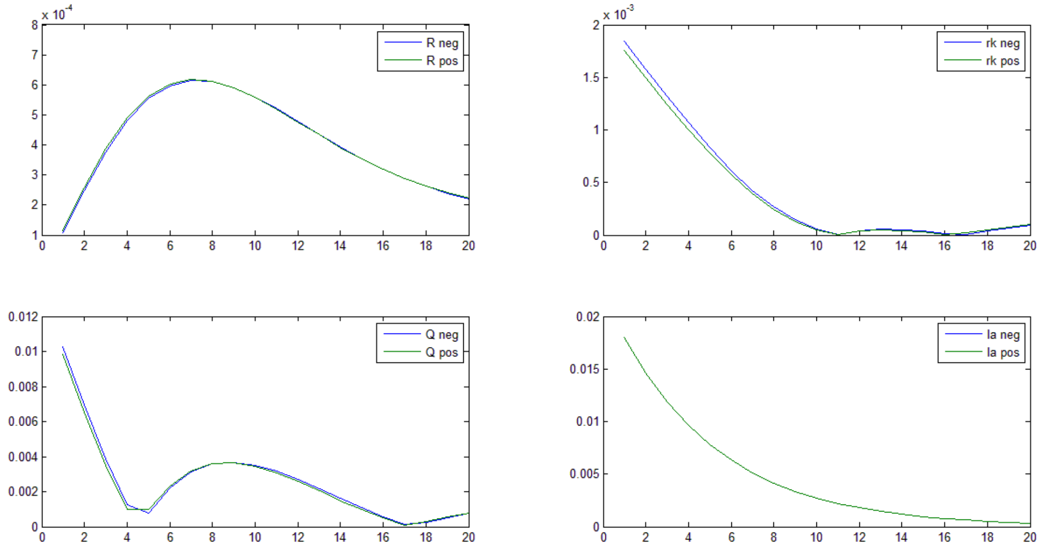


Figure 18: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

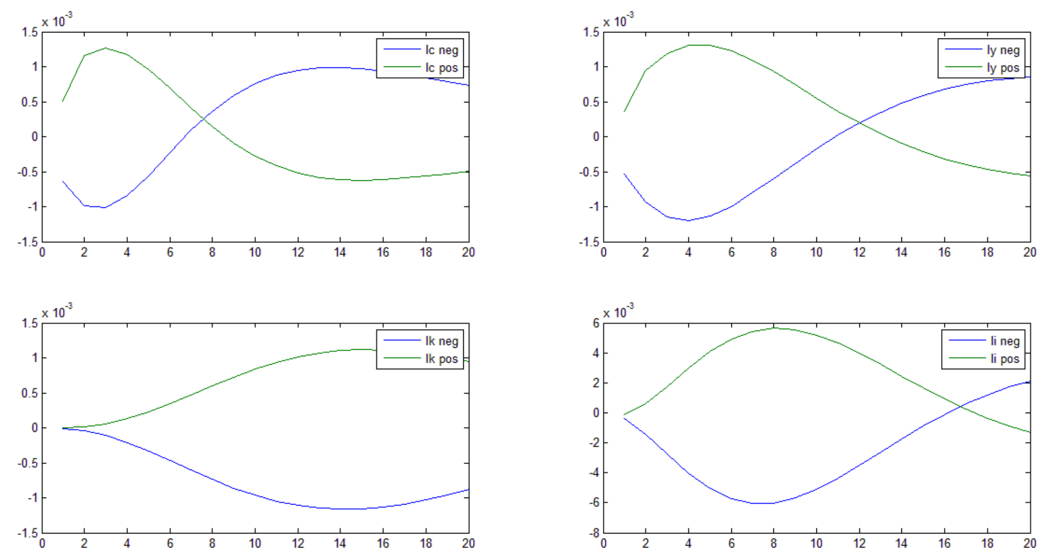


Figure 19: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

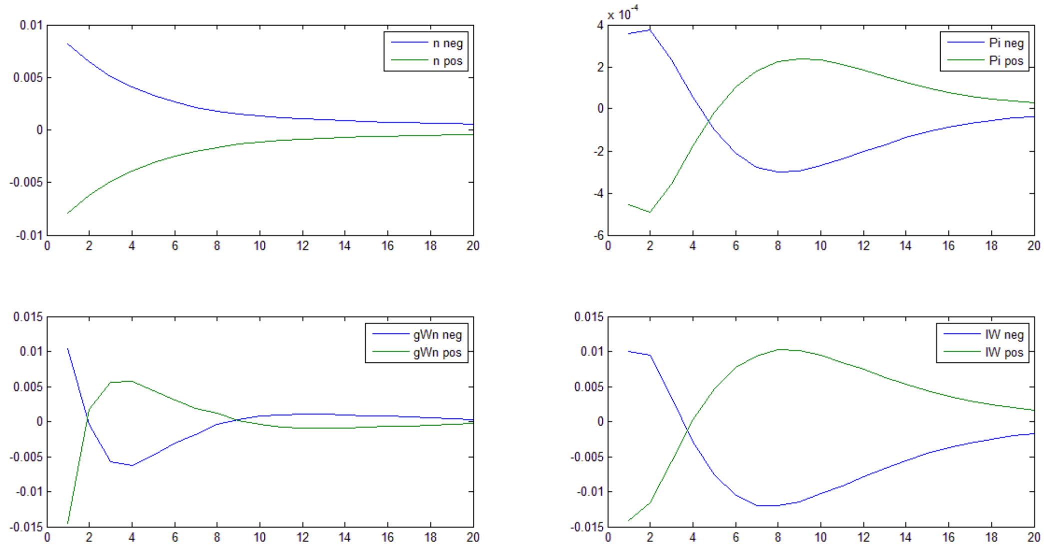


Figure 20: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

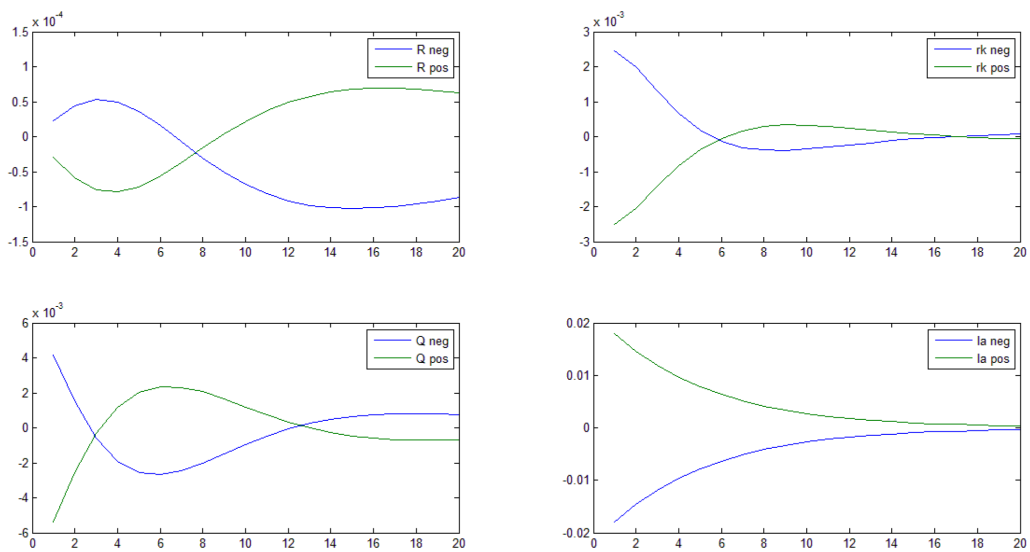


Figure 21: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

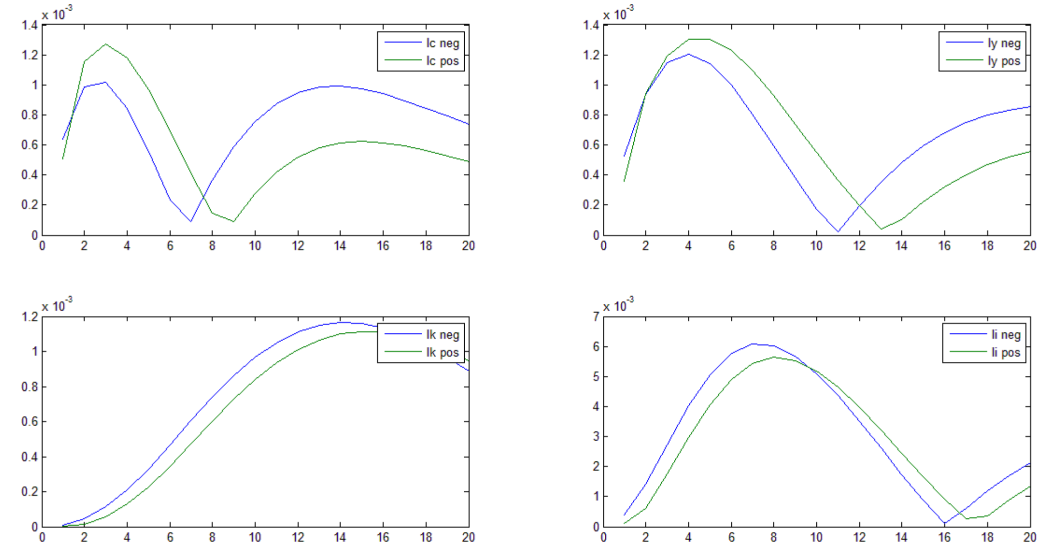


Figure 22: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

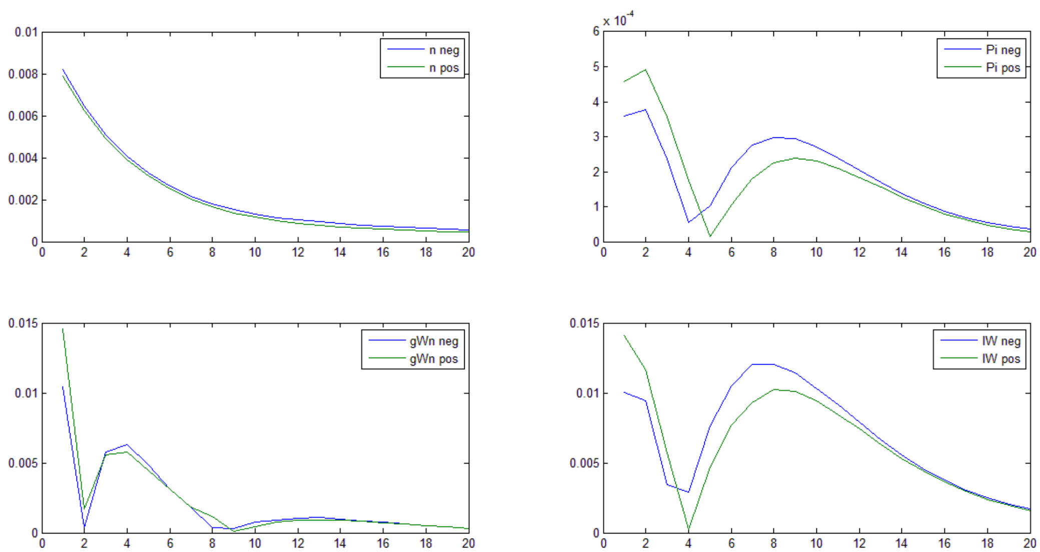
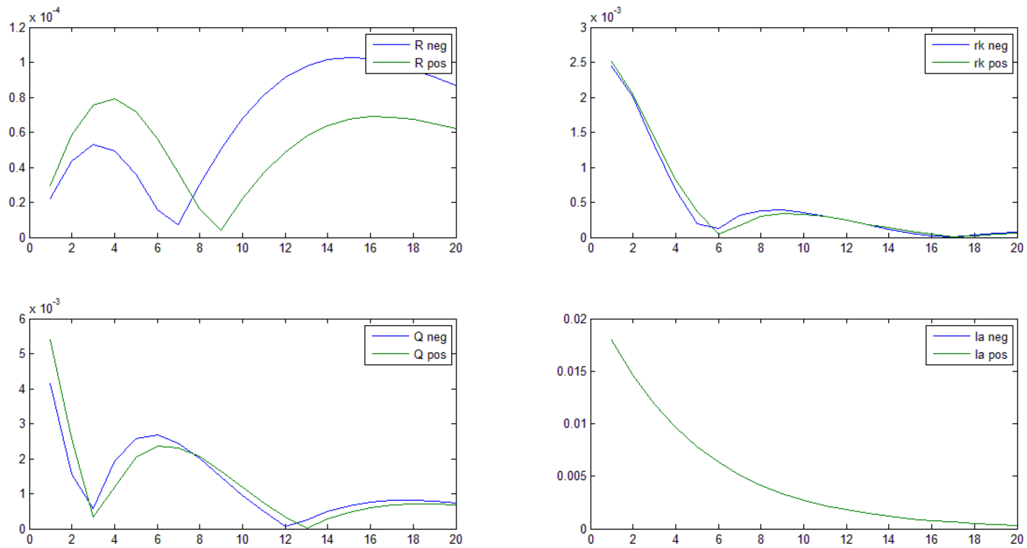


Figure 23: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure



income look important and seem to be different. As in previous exercises, booms in consumption, income, capital and investment seem to be more long-lasting than recessions. The fall in labor (caused by the positive shock) is smaller than its increase (caused by the negative shock). The decrease in inflation and in nominal wages growth are again smaller for recession than for boom. Real wages also present a slight fall during recession. Policy interest rate, rent of capital and Tobin's Q also show smaller reactions during recession, which means that there is also rigidity in real prices (figures 27 to 29 show impulse response in absolute values).

4 Conclusions

As in the case of business cycle asymmetries detected in real macroeconomic aggregates, asymmetries have been detected in nominal macroeconomic variables such as prices and wages. More precisely, their adjustment speed is asymmetric. This fact lies behind the asymmetric (or even kinked) Phillips curve which has been detected and modeled empirically in order to better understand the implications of optimal monetary policies. However, there is no theoretical modelization, at the best of our knowledge, that links these phenomenon to elemental behavior of agents.

In this chapter, the Smets-Wouters (2003) new Keynesian model is modified to build asymmetric DSGE models by introducing an intra-temporal additive prospects utility function in consumption and leisure. The parametrization of the model is quite similar to the one estimated in Smets and Wouters (2003), although the stochastic processes are log normal in our version. In order to verify whether our model looks like the Smets-Wouters (2003) model, we performed a first simulation

Figure 24: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

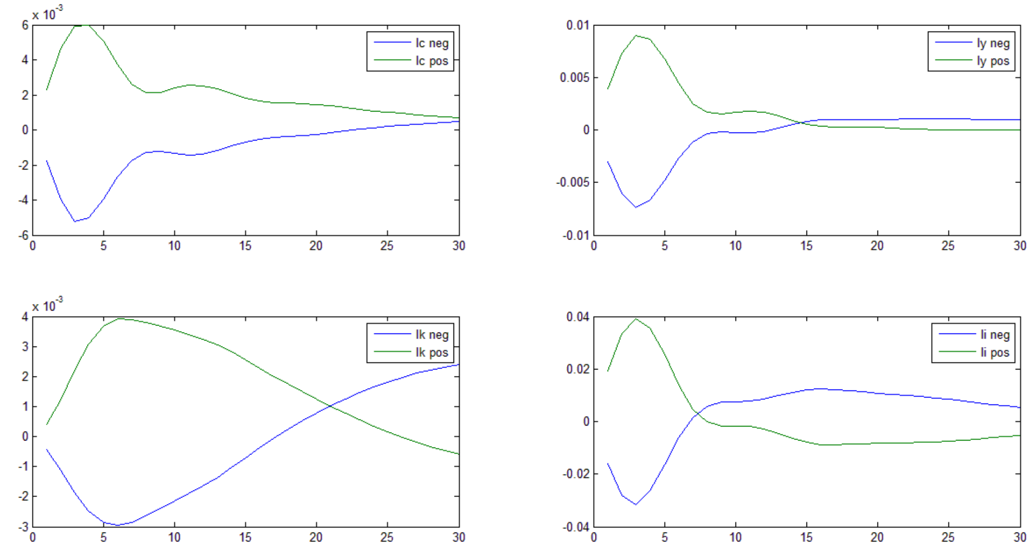


Figure 25: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

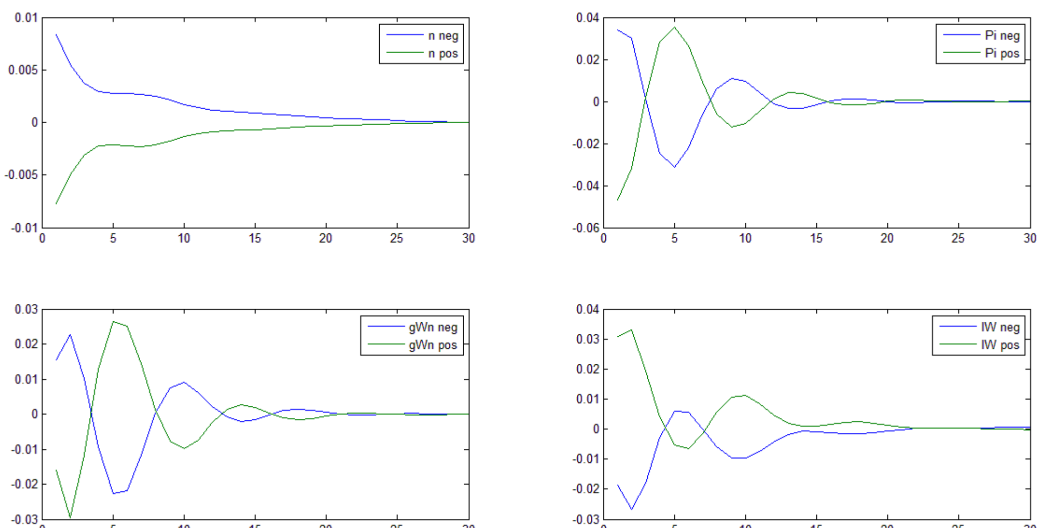


Figure 26: GIRF,comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

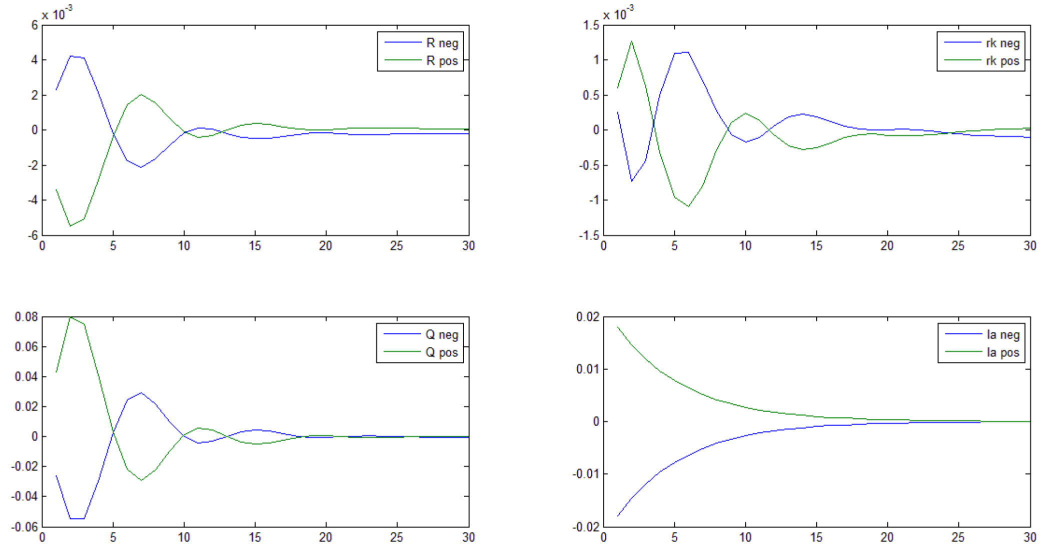


Figure 27: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

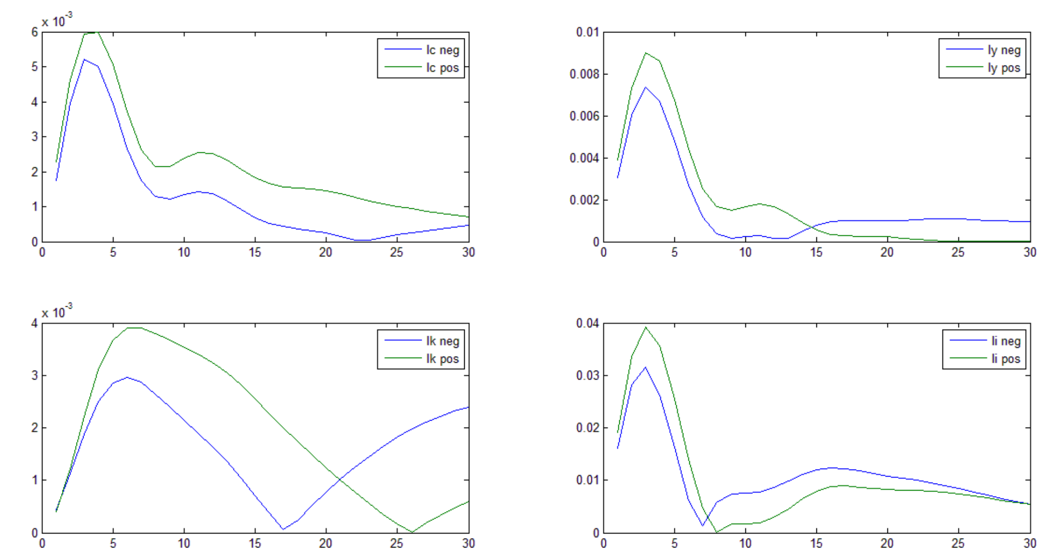


Figure 28: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

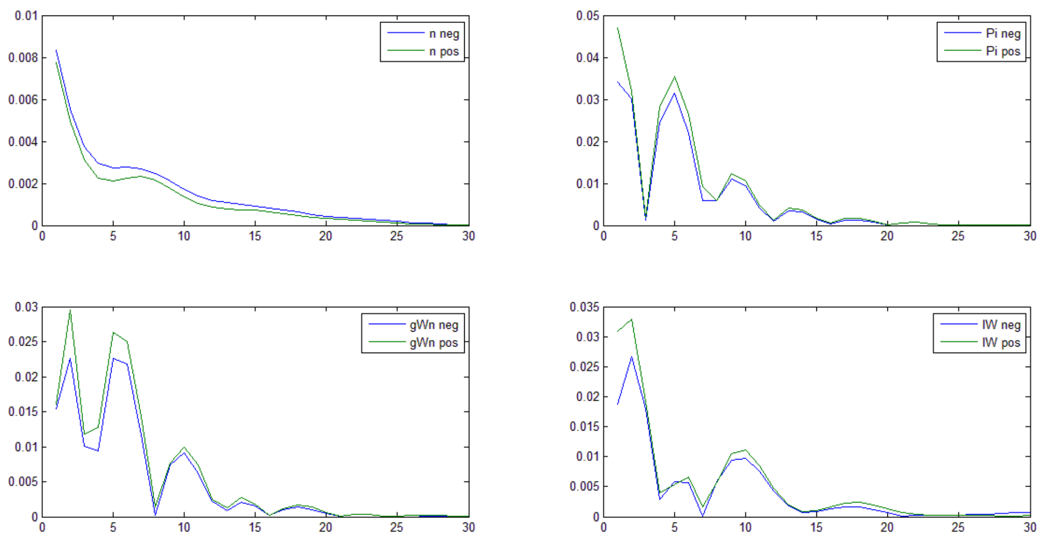
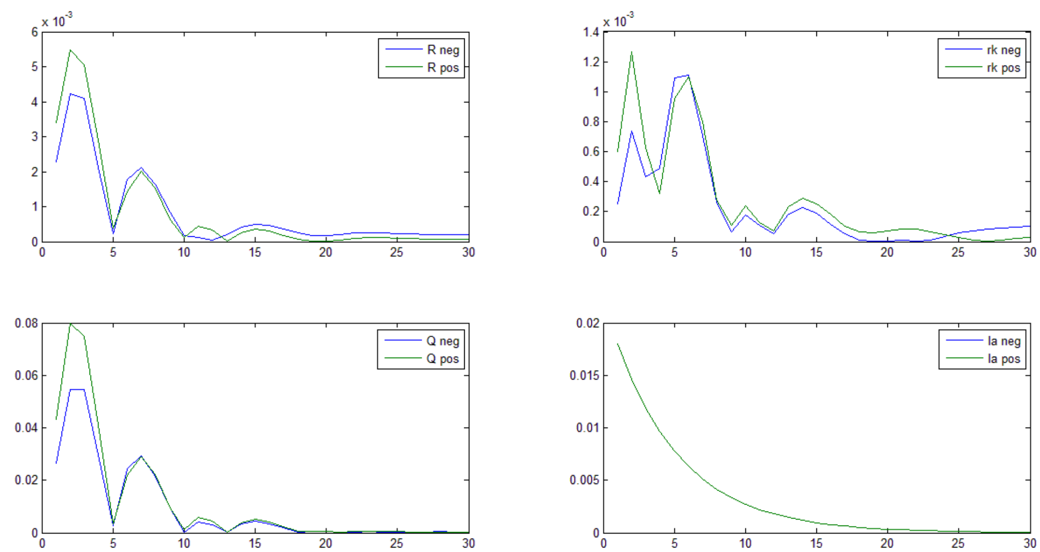


Figure 29: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness



supposing that the symmetric utility function in consumption and leisure maintained price and wage rigidities. As expected, the symmetric version of our model is rather similar (qualitatively) to Smets-Wouters'. In both models, since labor is determined by indexation of wages and inflation falls as technology is positively shocked, the increase in technology causes a fall in labor. This is not a neoclassical effect. However, when price rigidity is removed from the model, the positive shock on technology induces an increase in labor as expected for flexible price models.

When symmetry assumption was removed in our DSGE model, it was possible to generate asymmetric business cycles. The transmission channel in this case was the asymmetry in consumption and leisure. This was modeled by means of a prospects utility function additive separable considering habits as an externality. Impulse response in this exercise shows that price rigidities impose a stronger adjustment on real quantities in the symmetric model. But in the asymmetric model, the loss- averse behavior of the agents (in consumption and leisure) induce a smoother reaction in consumption and income. The reaction of inflation in the asymmetrical model is overwhelming: in this model, prices inflation have a smaller reaction than in the symmetric model, which means a greater (or additional) stickiness of prices. The fall in inflation, as the wages are indexed, induces a fall in nominal wage inflation rate (gWn) (real wage also falls) in both, symmetric and asymmetric models. However, the decrease in gWn is greater in the asymmetric model, which can be explained by the loss aversion in leisure. On shock, the fall in labor is almost the same in both models. The results in these simulations do not coincide with those from the neoclassical model. While the neoclassical model predicts that an increase in technology will produce an increase in labor, capital, consumption and income, this exercise shows that an increase in technology produces a reduction in labor (for both models), a fall in physical capital (slight), consumption and income (in the symmetric model), and a decrease in real wage (for both models).

The simulation of both a negative shock and a positive shock on the asymmetric models revealed that for consumption, income, capital and investment on shock, a recession is more intense than a boom, and a boom is more long-lasting than a recession. Besides, and perhaps one of the most interesting findings in our simulations, the impact of the recessive shock for labor is stronger and seems to be more long-lasting than the effect of a booming shock, which resembles the hysteresis phenomenon in unemployment. Whereas inflation in boom has a greater increase and lasts only one more period, the reaction of inflation is lower in recession than in boom and has a shorter duration. Thus, prices in recession are more reluctant to decrease than to increase in a boom. General or aggregate wage shows a smaller reaction during recession (stronger in boom). To sum up, the stickiness of prices and wages in a prospect utility framework are exacerbated.

Finally, we must highlight that another really interesting finding in our simulations is that when asymmetry is removed from the leisure choice, the asymmetry in business cycles is almost removed as well. However, when asymmetry is removed from the consumption choice, there is not any significant change in the asymmetric pattern of impulse responses. This suggests that the main channel of transmission of asymmetry is the loss aversion in leisure because when income falls, the agent needs to dampen the fall in welfare by demanding more leisure (employment will decrease). This means that the agent will demand a higher wage or will reject a huge decrease in wage.

In general, the model built in this chapter is able to generate not only asymmetric business cycles, but also generate an asymmetric (nonlinear) Phillips curve and asymmetric stickiness of prices and wages. This demonstrates that the asymmetric behavior of consumers modeled by prospects utility function is a suitable transmision mechanism. Moreover, the model reproduces exacerbate (downward) rigidities in wages and prices, and hysteresis in unemployment. As exposed in the first two chapters of this dissertation, it is worth pointing out that asymmetries in RBC models could be more adequately captured by General Impulse Response Functions than by higher-order moments. However, a more rigorous test for the properties of the asymmetric model proposed here would entail the application of nonlinear econometric tools, which might be useful for the purposes hereby exposed. Several issues remain for the research agenda: i) Structural parameters of the model need to be estimated; ii) the ways how my loss aversion DSGE model can be employed to study issues in policy-making, asset-pricing, risk premia puzzle, international asymmetric business cycles, risk-sharing, home bias, among other areas of knowledge.

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6 Annex 1: the full dynamic system

$$\Psi(z_t) = -\frac{\overline{r}^k}{1+\varsigma} + \overline{r}^k \frac{z_t^{1+\zeta}}{1+\varsigma}$$

$$z_{ct} = \frac{C_t^\tau}{\overline{C}_t}$$

$$\overline{C}_t = (1-\chi)\overline{C}_{t-1} + \chi C_t^\tau$$

$$z_{\mathcal{L}t} = \frac{\mathcal{L}_t^\tau}{\overline{\mathcal{L}}_t}$$

$$\overline{\mathcal{L}}_t = (1-\chi_{\mathcal{L}})\overline{\mathcal{L}}_{t-1} + \chi_{\mathcal{L}}(\mathcal{L}_{t-1})$$

$$\mathcal{L}_t^\tau = 1 - l_t^\tau$$

$$\phi_{ct} = \frac{1}{1+\exp\gamma_c\left(z_{ct}-1\right)}$$

$$\phi_{\mathcal{L}t} = \frac{1}{1+\exp\gamma_{\mathcal{L}}(z_{\mathcal{L}t}-1)}$$

$$\frac{\partial\phi_{ct}}{\partial z_{ct}} = \frac{-\gamma_c\exp\gamma_c\left(z_{ct}-1\right)}{\left[1+\exp\gamma_c\left(z_{ct}-1\right)\right]^2}$$

$$\frac{\partial\phi_{\mathcal{L}t}}{\partial z_{lt}} = \frac{-\gamma_{\mathcal{L}}\exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t}-1\right)}{\left[1+\exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t}-1\right)\right]^2}$$

$$E_t\left[\beta\frac{\lambda_t}{\lambda_{t+1}}\frac{R_tP_t}{P_{t+1}}\right] = 1$$

$$\lambda_t = \varepsilon_t^b\frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t} + \varepsilon_t^b\frac{-\gamma\left(z_{ct}\right)^{\gamma-1}}{\left[1+\left(z_{ct}\right)^{\gamma}\right]^2}\frac{\left[\left(z_{ct}\right)^{\underline{\theta}}-\left(z_{ct}\right)^{\overline{\theta}}\right]}{\overline{C}_t} + \varepsilon_t^b\phi_{ct}\left[\frac{\underline{\theta}\left(z_{ct}\right)^{\underline{\theta}-1}}{\overline{C}_t} - \frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}\right]$$

$$\begin{aligned} U_t^{\mathcal{L}\tau} = \frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t} &= \varepsilon_t^b\varepsilon_t^L\frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t} + \varepsilon_t^b\varepsilon_t^L\frac{\partial\phi_{\mathcal{L}t}}{\partial z_{lt}}\frac{\left[\left(z_{\mathcal{L}t}\right)^{\underline{\mu}}-\left(z_{\mathcal{L}t}\right)^{\overline{\mu}}\right]}{\overline{\mathcal{L}}_t} \\ &+ \varepsilon_t^b\varepsilon_t^L\phi_{\mathcal{L}t}\left[\frac{\underline{\mu}\left(z_{\mathcal{L}t}\right)^{\underline{\mu}-1}}{\overline{\mathcal{L}}_t} - \frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t}\right] \end{aligned}$$

$$\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}\frac{\left(P_{t+i-1}/P_t\right)^{\gamma_p}}{P_{t+i}/P_t}y_{t+1}^j\frac{1}{\lambda_{p,t+i}} = E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+1}^j\left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}\right)$$

$$\left(P_t\right)^{-1/\lambda_{p,t}} = \xi_p\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}\right)^{-1/\lambda_{p,t}} + \left(1-\xi_p\right)\left(\widetilde{P}_t^j\right)^{-1/\lambda_{p,t}}$$

$$E_0\sum_{i=0}^{\infty}\beta^i\xi_w^i\frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}\left(\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}l_{t+i}^\tau\right) = \frac{\widetilde{W}_t}{P_t}E_0\sum_{i=0}^{\infty}\beta^iU_{t+i}^C\xi_w^il_{t,t+i}^\tau\frac{P_t}{P_{t+i}}\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{\gamma_w}\frac{1}{\lambda_{w,t+i}} \quad (80)$$

$$y_t^j = \varepsilon_t^a\widetilde{K}_{j,t}^\alpha L_{j,t}^{1-\alpha} - \Phi$$

$$\frac{W_t}{P_t}\frac{L_{j,t}}{r_t^k\widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha}$$

$$MC_t = \frac{1}{\varepsilon_t^a}\left(\frac{W_t}{P_t}\right)^{1-\alpha}\left(r_t^k\right)^\alpha\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right)$$

$$(W_t)^{-1/\lambda_{w,t}} = \xi \left(W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \right)^{-1/\lambda_{w,t}} + (1-\xi)(\widetilde{w}_t)^{-1/\lambda_{w,t}}$$

$$K_t = K_{t-1}\left[1-\tau\right] + \left[1 - S\left(\varepsilon_t^I I_t / I_{t-1}\right)\right] I_t$$

$$S(I) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_{t-1}^2$$

$$\Psi(z_t) = -\frac{r^k}{1+\varsigma} + r^k \frac{z_t^{1+\zeta}}{1+\varsigma}$$

$$Q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[Q_{t+1}\left(1-\tau\right) + \left(r_{t+1}^k z_{t+1}^\tau - \Psi(z_{t+1}^\tau)\right)\right]\right]$$

$$1 = Q_t \left[1 - \frac{\partial S\left(\varepsilon_t^I I_t / I_{t-1}\right)}{\partial I_t} \left(\frac{\varepsilon_t^I}{I_{t-1}} \right) I_t - S\left(\varepsilon_t^I I_t / I_{t+1}\right) \right] + \beta E_t \left\{ Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\partial S\left(\varepsilon_{t+1}^I I_{t+1} / I_t\right)}{\partial I_t} \varepsilon_{t+1}^I I_{t+1}^2 \frac{1}{I_t^2} \right] \right\}$$

$$r_t^k = \Psi'(z_t)$$

$$Y_t = C_t + G_t + I_t + \Psi(z_t) K_{t-1}$$

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R} \right)^{r_R} \left[\left(\frac{1+\Pi_t}{1+\Pi} \right)^{r_\Pi} \left(\frac{Y_t}{Y} \right)^{r_Y} \right]^{1-r_R} \varepsilon_t^R$$

Stochastic processes are such that:

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b$$
$$\ln \varepsilon_t^L = \rho_L \ln \varepsilon_{t-1}^L + \eta_t^L$$
$$\ln \varepsilon_t^I = \rho_I \ln \varepsilon_{t-1}^I + \eta_t^I$$
$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a$$
$$\ln(\varepsilon_t^g) = \rho_g \ln(\varepsilon_{t-1}^g) + \eta_t^g$$
$$\overline{\pi}_t = \rho \overline{\pi}_{t-1} + \eta_t^\pi$$
$$\varepsilon_t^R = \exp(\eta_t^R)$$
$$\lambda_{p,t} = \lambda_p + \eta_t^p$$
$$\lambda_{w,t} = \lambda_w + \eta_t^w$$

$$P_t = \left[\int_0^1 \left(\overline{P}_t\right)^{-1/(\lambda_p)} dj \right]^{-\lambda_p} = \overline{P}_t \text{ , therefore } Y_t = y_t^j = \varepsilon_t^a K_j^\alpha L_j^{1-\alpha} - \Phi = \varepsilon_t^a K^\alpha L^{1-\alpha} - \Phi$$