ASYMMETRIES IN BUSINESS CYCLES

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Declaration

I hereby declare that this dissertation entitled "Asymmetries in business cycles" is the result of my own research except as cited in the references. This dissertation has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

For Gabriel, Gregorio and Alejandra

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Abstract

This dissertation intends to study the transmission mechanisms that link the behavior of agents and firms with asymmetries present in business cycles. Asymmetries in business cycles have been well documented in the field of nonlinear econometrics. However, it seems that few efforts have been devoted to understanding and explaining such phenomena. In order to achieve this goal, three DSGE models, one for each chapter of this dissertation, were built by removing the assumption of symmetric behavior of firms and individuals along the different phases of business cycles. In the first chapter, the assumption of a quadratic-symmetric cost adjustment of investment has been removed. In the context of a closed economy, the canonical RBC model was reformulated supposing that dis-investing is costlier than investing one unit of physical capital. This can be explained by irreversibilities, sunk costs, microeconomic rigidities, and asymmetric costs embodied in the decision of adjusting other productive factors such as labor. The simualation of the model by using deterministic and stochastic impulse-response exercises revealed that it is possible to adequately reproduce asymmetric business cycles by modeling this kind of asymmetry. In the second chapter, the most important contribution of this dissertation is presented: the construction of a general utility function which nests loss aversion, risk aversion and habits formation by means of a smooth transition function. The reason for doing so is the fact that individuals are loss-averse in recessions and they are risk-averse in booms. The simulations of the model show the possibility to reproduce asymmetric business cycles. In this model real wages display downward stickiness and as a consequence, the fall of employment in recessions is more persistent than in booms. Thus, the model reproduces not only asymmetrical business cycles but also real stickiness and hysteresis. Finally, in the third chapter, asymmetries in real business cycles are analyzed along with asymmetric adjustment of prices and wages in a Neo-Keynesian framework pursuing a theoretical explanation for the well-documented asymmetries found in the Phillips Curve. The purpose then is to understand how asymmetric real business cycles are linked to the asymmetric behavior of agents in a price and wage rigidities set up. Simulation results show that loss aversion makes downward rigidities in prices and wages stronger and also reproduces a more severe and persistent fall of employment. All in all, this model generates asymmetric real business cycles, asymmetric price and wage adjustments as well as hysteresis.

Table 1: Kurtosis and Skewness for a sample of countries

		Kurtosis			Skewness	
		HP			HP	
Variable	Full sample	Negative values	Positive values	Full sample	Negative values	Positive values
	Colombia					
GDP	2.0899	1.8927	1.8858	0.017	-0.462	0.2024
C	2.1659	3.3298	2.1108	0.193	-0.5571	0.3256
I	4.9049	6.0205	5.232	-0.3166	-1.8673	1.6237
	Germany					
GDP	2.4663	3.2255	1.9461	0.2942	-0.8865	0.2278
C	2.7961	1.8534	2.3599	0.2871	-0.5974	0.7277
I	1.8434	3.3246	1.7065	0.1644	-0.255	-0.0722
	USA					
GDP	3.0601	3.2327	2.0607	-0.559	-1.1154	0.3173
C	2.2794	4.3438	2.3978	-0.3902	-1.3491	0.3064
I	2.8452	3.118	2.1146	-0.625	-0.8702	0.1003
	United Kingdom					
GDP	3.1953	4.7287	2.5273	-0.2233	-1.3995	0.8305
C	2.8111	3.8048	3.0562	0.453	-0.5264	1.0482
I	3.6498	3.3733	3.9337	-0.0489	-1.0363	1.3929
	France					
GDP	2.2161	3.75	1.8382	-0.1168	-0.8477	0.269
C	2.2506	2.483	3.8269	-0.3778	-0.3094	0.7721
I	2.1549	1.6976	2.2693	0.0998	-0.1145	0.5048

Source: World Bank web page. Annual per capita real series, logarithms of data filtered with Hodrik-Prescott filter. For France and USA sample is from 1970 to 2009 and for the other countries from 1960.

Introduction

Traditional analysis on economic fluctuations have achieved certain consensus regarding business cycle causes with somewhat predictive and explicative power. Yet uncertain remain some relevant facts such as the one of the asymmetric behavior present in the GDP components along the business cycles. Asymmetries and nonlinearities can be seen through stylized facts, time varying amplitude in cyclical components of macroeconomic variables, for instance. A simple way to easily identify such asymmetries is by calculating higher order moments for the distribution of cyclical components. Table 1 shows kurtosis and skewness for GDP, consumption (C) and investment (I) for several countries frequently studied and whose asymmetry has been detected: Germany, USA, United Kingdom and France. As it can be seen, full sample Kurtosis and skewness are not those corresponding to normal distributions; moreover, these higher order moments also computed for positive and negative values of the cycle gap show that booms and recessions have different properties, which is a symptom that the distribution of the full business cycle is a mixture of the distributions of boom and recessions.

Overall, a very important challenge for economic models lies on data particularities, namely nonlinearities and asymmetries, particularly for the case of DSGE models. Despite being highly nonlinear, they seem to have symmetric behavior and symmetric transmission mechanisms as well as symmetric technology shocks. Models with these features are unable to adequately reproduce third and fourth moments of the empirical distributions of cyclical components of macroeconomic variables (Valderrama, 2007). On the basis of empirical analysis, business cycles asymmetries have been treated by Nonlinear Econometrics, and mostly through Switching

Regime Econometrics. For example, Neftci (1984) uses Swithching Markov Estimation in order to study whether correlations of economic variables differ throughout the phases of business cycles. Supported on basic intuition, Neftci states that if a times series is symmetric along the business cycles and two regimes or states exist, the probability of remaining in state 1 is the same of remaining in state 2. Based on maximum likelihood and a Bayesian refinement of this, Neftci discovered that for unemployment series of the US economy the probability of remaining in a consecutive decrease state is higher than the probability of remaining in a consecutive increase state.

Supported on concepts developed by Sichel (1993) and on the study by Clements and Krolzig (2003), Belaire-Franch and Contreras (2003) attempted to detect and estimate three kinds of asymmetries by means of a parametric test: deepness, steepness, and sharpness. Under the supposal that a time series is generated by a Markov Switching-Autorresive model with M regimes in the mean (MS-AR(p)), it was found that most of the countries sampled have certain asymmetry, except for the US and Germany. In turn, Gefang and Strachan (2010) employed a smooth transition VAR to measure the impact of international business cycles on the UK economy. The estimations were performed on the GDP growth rate. The countries involved in the analysis were the US, France, and Germany. It was found that the UK economy is influenced asymmetrically by other countries in the sample.

On DSGE modeling, there are Pytelarczyk (2005), Eo (2009), and Davig and Leeper (2005). These works have developed DSGE models with *ad hoc* switching regimes on the linearized dynamic equations of the model. Parameter estimations are used to perform impulse-response exercises. Other works such as Tristani and Amisano (2010), Karagikli, Matheson, Smith and Vahey (2007), and Bullard and Singh (2009) have developed DSGE models that introduce exogenous regime switching disturbances. Recent applications of Bayesian Econometrics have contributed to estimate parameters for DSGE models that include explicit regime switching for the impulse-response matrix of coefficients as well as for time process of disturbances. Differently from these DSGE models, Li and Dressler (2011) develop a modified version of a traditional small open economy RBC model by including an occasionally international borrowing constraint. When the economy is negatively shocked and has a high stock of foreign debt, the borrowing constraint binds and the recovering is slower than it should be if the restriction would not bind. Thus this model is successful in producing business cycle asymmetries. However, in this model, the basic set up of utility, production and adjustment of capital costs remain symmetrical. In the same way of introducing a quantitative restriction on the economy, Knüppel (2014) in a closed economy RBC model includes capacity utilization restrictions as a source of asymmetry. The model can replicate asymmetry of most of variables except that of labor productivity. It is also found that more capital is accumulated and utilization is lower when capacity constraints are introduced.

Modern Econometrics and, up to some extent, DSGE modeling have been concerned with nonlinearities and asymmetry of data generating processes. However, there is a further task for economists regarding the construction of models that take into account asymmetries as the result of endogenous optimal decision-making or, at least, include them in the basic behavioral equations of the models. Thus, in spite of the sophisticated tools used by the the authors aforementioned, a question remains unanswered: *Where do asymmetries come from?* The answer to this question might lie in modeling the behavior of firms and agents, considering that during booms they may behave differently than during recessions. That is to say, it is necessary to study the transmission mechanisms and behaviors that cause differences between phases of business cycles. Thus, the intention of this dissertation is to study the transmission mechanisms that link the behavior of agents and firms with asymmetries present in business cycles.

In the first chapter, investment cost asymmetry is introduced in order to test whether this kind of asymmetry can account for asymmetries in business cycles. By using a smooth transition function, asymmetric investment cost is modeled and introduced in a canonical RBC model. Simulations of the model with Perturbations Method (PM) are very close to simulations through Parametrized Expectations Algorithm (PEA), which allows the use of the former for the sake of time reduction and computational costs. Both symmetric and asymmetric models were simulated and compared. Deterministic and stochastic impulse-response excercises revealed that it is possible to adequately reproduce asymmetric business cycles by modeling asymmetric investment costs. Simulations also showed that higher order moments are insufficient to detect asymmetries. Instead, methods such as Generalized Impulse Response Analysis (GIRA) and Nonlinear Econometrics (NE) prove to be more efficient diagnostic tools.

One of the most important contributions of this dissertation is the construction of a general utility function which nests loss aversion, risk aversion and habits formation by means of a smooth transition function, presented in chapter two. The main idea behind this asymmetric utility function is that under recession the agents over-smooth consumption and leisure choices in order to prevent a huge deviation of them from the reference level of the utility; while under boom, the agents simply smooth consumption and leisure, but try to be as far as possible from the reference level of utility. The simulations of this model by means of PM show that it is possible to reproduce asymmetrical business cycles whereas recessions (on shock) are stronger than booms and booms are more long-lasting than recessions. One additional and unexpected result is a downward stickiness displayed by real wages. As a consequence of this, there is a more persistent fall of employment in recessions than in booms. Thus, the model reproduces not only asymmetrical business cycles but also real stickiness and hysteresis.

Besides the asymmetries present in the cycle of real variables, nominal prices and wages exhibit downward adjustment rigidity during recessions while during booms they seem to be more quickly upward adjustable, this is, there is a nonlinear and asymmetric Phillips Curve whose existence is also well documented. Thus, in the third chapter, the Smets-Wouters (2003) New Kenesian model is reformulated by introducing the loss aversion utility function developed in chapter two. The purpose of this is to understand how asymmetric real business cycles are linked to asymmetric behavior of agents in a price and wage rigidities set up. The simulations of the model reveal not only that the loss aversion in consumption and leisure is a good mechanism channel for explaining business cycle asymmetries, but also is a good mechanism channel for explaining asymmetric adjustment of prices and wages, therefore the existence of asymmetries in Phillips Curve. Moreover, loss aversion makes downward rigidities in prices and wages stronger and also reproduces a more severe and persistent fall of the employment. All in all, this model generates asymmetrical real business cycles, asymmetric price and wage adjustment as well as hysteresis.

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Chapter 1

An Investment Cost Approach

1.1 Introduction

Asymmetries in production and productive factor utilization can be found in the literature. Nonetheless, some of them present controversial findings. Partial equilibrium models of representative firms and convex (symmetric) adjustment functions have been criticized as they ignore diverse features of firms, idiosyncratic shocks, and microeconomic rigidities. These aspects have drawn more attention with their possible links to aggregate investment dynamics: fixed adjustment costs, irreversibilities, (S,s) dynamics, and lumpy investment. In this sense, Doms and Dunne (1998) found in a sample of firms that they adjust capital in lumpy ways, and fixed costs explain a significant part of firms' total investment expenditure, aggregate investment itself. A similar result has been obtained by Caballero and Engel (1994) through the estimation of a nonlinear model. Caballero and Engel (1991) with an extension of a (S,s) model also found that lumpy investment affects aggregate investment dynamics, thus showing analytically that the cross section distribution of firms' investment converges towards a long-run distribution. Caballero, Engel, and Haltiwanger (1995) also observe the asynchronicities of firms. By joining micro elements and aggregation, they deduce an inverse aggregate investment equation. Their estimations indicate that investment elasticities of shocks vary throughout time, which means that firms are willing to adjust capital when facing a high scarcity of it. With respect to the existence of micro rigidities, Cooper and Haltiwanger (2000) used an indirect inference method for a sample of firms.They found evidence supporting the joint existence of convex and non-convex costs, and irreversibilities.

In opposite direction, there are DSGE with micro rigidities, which have not encountered relative consensus of those works in partial equilibrium. Veracierto (2002) concludes that investment irreversibilities generate a small difference compared to a canonical RBC model. In a similar fashion, Thomas (2002) claims that lumpy investment does not have significant effects on aggregate investment. Khan and Thomas (2003) discovered that when fixing prices, it is possible to produce non-linear dynamics in aggregate investment, which then disappears by allowing price adjustment. Differently, Bachman, Caballero and Engel (2006a), and Bachman, Caballero and Engel (2006b) pointed out two existing smoothing mechanisms: pre-general equilibrium smoothing (which explains 60% of investment variance) and general equilibrium smoothing (which explains the remaining 40%). They have also demonstrated that the particular specification used by Khan and Thomas (2003) involves a small partial equilibrium effect which is reproduced in general equilibrium. A more realistic specification entails a big partial equilibrium effect that, as a consequence, implies an important aggregate effect on a general equilibrium model.

Since consensus between those studies has been unmet, this paper addresses a different and more "aggregate" modeling strategy. In this paper, asymmetric investment cost is introduced in order to test whether this asymmetry can account for asymmetries in business cycles. Among some works on asymmetries in factor demand and factor adjustment costs, an excellent contribution in this line, and roughly close to the present work, has been made by Palm and Pfann (1997). Their work addresses sources of asymmetry in production factors dynamics[1].They have indicated that linear-quadratic models and the implications of their symmetry is unable to pass statistical tests. Although they are not interested in the study of business cycles in a general equilibrium framework, their proposal poses two questions also addressed in the present paper: *What are the sources of the asymmetries?* and *Why do all tests for the underlying structures of adjustment costs are important for the aggregate production factors dynamics?*[2]. Their model for asymmetric production factor dynamics is built on the assumption that "*(...) firms, when making contingency plans on the use of factor inputs, account for differences in adjustment costs during different phases of business cycles*" .[3] A generalization of adjustment functions is proposed for both capital and labor. Given specific functional forms for production functions and adjustment costs (which nests the symmetric cost function), the model is estimated for first order conditions of profit maximization, and the null of symmetric cost function is rejected. Next, the estimated model is solved and simulated by means of Parametrized Expectations Algorithm (PEA) given the real prices of factors and productivity shocks. The aim of the simulation is to test whether the existence of external non-linearity has some impact on dynamic factor input asymmetry of data. External non-linearity is introduced by modeling real prices of factors as a nonlinear (quadratic) bivariate AR(1,1) process. A linear bivariate AR(1,1) is also modeled to serve the purpose of control framework. The main conclusion reveals that 50% of the dynamic factor demand asymmetry in the manufacturing sector of the Netherlands is explained by *behavioral or internal asymmetries* caused by asymmetric adjustment costs, while the remaining 50% is caused by *external nonlinearities* in real price factors.

Other studies have dealt with asymmetries in factor adjustment costs. Jaramillo, Shciantarelli and Sembenelli (1993) have worked on asymmetries for labor of the Italian industry, with firing costs being different to hiring costs. Their hypothesis was tested by a general model that nested symmetric costs, thus rejecting the null of symmetry. Pfann and Palm (1993) make a distinction between skilled and unskilled labor [4] for manufacturing sectors in the UK and the Netherlands. They found that data rejected the null of symmetric costs. Moreover, their results revealed a very interesting fact: hiring costs are higher than firing costs for unskilled labor, whereas the opposite is also true for skilled labor. About adjusting labor costs, Hamermesh and Pfann (1995) used a generalized cost function including gross and net changes in labor. Their estimations have revealed that this modeling is necessary to track down correctly labor demand dynamics of the US manufacturing sector.

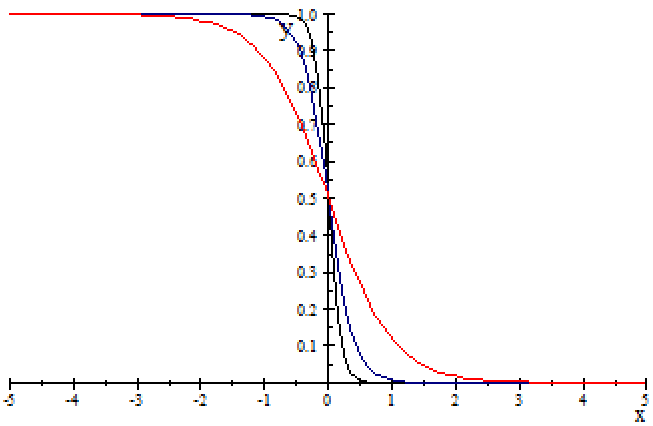
The contribution of this chapter is to show that it is possible to reproduce asymmetric business cycles as an endogenous outcome of the economy when asymmetries in adjustment costs are taken into account. Asymmetric investment cost is modeled and introduced in a canonical RBC model by means of a smooth transition function. The model is simulated with Perturbations Method (PM) as well as with Parametrized Expectations Algorithm (PEA). A comparison between those simulations shows that they are very similar, which allows the use of the former for the sake of time reduction and computational costs. Then, both symmetric and asymmetric models were simulated and compared. Finally, Deterministic and stochastic impulse-response excersices revealed that it is possible to adequately reproduce asymmetric business cycles by modeling asymmetric investment costs

1.2 A simple model with asymmetric investment costs

In this chapter, the model to be used is the Basic Neoclassical Model as presented in King, Plosser and Rebelo (1988) with two simplifications: neither technology nor population growth. This simplified model is modified to take into account the fact that investment is costly, as in Obstfeld and Rogoff (1996), but the contribution in chapter is the recognition and explicit modeling of asymmetries in the cost of investment: disinvesting one unit is costlier than investing the same quantity. In the symmetric case, the investment cost assumes the very traditional form $\frac{\psi}{2}(\Delta k - \overline{in})^2$, being $\overline{in}$ net investment in steady state and $\Delta k = k_{+1} - k$ net investment. When $\Delta k = \overline{in}$, there is not any investment cost, i.e. the cost function reaches a threshold at that point. It

[1]This is exactly the title of their paper
[2]*I quote textually from Palm and Pfann (1997) pag. 362*
[3]*I quote textually from Palm and Pfann (1997) pag. 364*
[4]More precisely, they distinguish production and non-production workers.

Figure 1.1: Smooth transition function



is important to bear in mind that in an economy with neither population growth nor technological progress, net investment equals zero in the steady state.

Furthermore, it is known that the investment cost around that point, for instance $\Delta k = \epsilon$ or $\Delta k = -\epsilon$ ($\epsilon > 0$), is not the same in such a case where the economy is in recession, $\Delta k = -\epsilon$, and when it is in expansion, $\Delta k = \epsilon$. Thus, if we suppose that decreasing the investment is more costly than increasing it, the investment cost in each state is:

$$C(\Delta k) = \left\{ \begin{array}{c} \frac{\psi_1}{2}(k_{+1} - k - \overline{in})^2, \text{if } \Delta k < 0 \\ \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2, \text{ if } \Delta k > 0 \end{array} \right\}, \psi_1 > \psi_2 \tag{1.1}$$

Let us suppose ϕ_t a smooth transition function between the states. If we define such transition function as an indicator function (or as a probability function), the regime switching cost function will be[5]:

$$C(\Delta k) = \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2 + \phi_t(\frac{\psi_1}{2}(k_{+1} - k - \overline{in})^2 - \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2) \tag{1.2}$$

where ϕ_t is a logistic one:

$$\phi_t = b/(1 + exp(\gamma(k_{+1} - k - \overline{in}))) \tag{1.3}$$

$$b = \left\{ \begin{array}{c} 1, \text{ if asymmetric beharior} \\ 0, \text{ if symmetric behavior} \end{array} \right\} \tag{1.4}$$

If $\gamma \to \infty, \phi$ has an almost instanteneuos change,if $\gamma \to 0$, $\phi \to 0.5$. if $k_{t+1} - k_t - \overline{in} < 0, \phi_t \to 1$, if $k_{t+1} - k_t - \overline{in} > 0, \phi_t \to 0$. figure 1.1 shows the transition function for $\gamma = 10, 5, 2.5$, and figure 1.2 shows symmetric and asymmetric (black line) cost functions for $\psi_2 = 1$ (red line), $\psi_1 = 4$ (green line) and $\gamma = 0.5$.

Thus, if we rewrite the capital cost adjustment we have:

$$C(\Delta k) = \varphi_t = \varphi_{2t} + \phi_t\left(\varphi_{1t} - \varphi_{2t}\right)$$

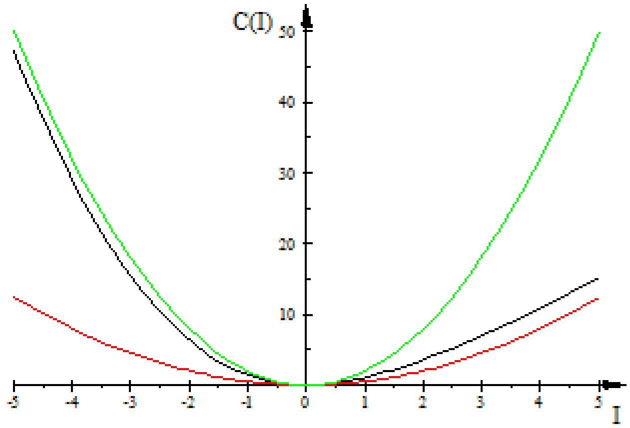
being

$$\varphi_{1t} = \frac{\psi_1}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{1.5}$$

$$\varphi_{2t} = \frac{\psi_2}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{1.6}$$

[5]Pfann and Palm (1997) proppose a quadratic-exponential function to model asymmeties in cost functions $C(\Delta k) = \exp(\beta_k \Delta k) - 1 - \beta_k \Delta k + \frac{1}{2}\gamma_k(\beta \Delta k)^2$ and $C(\Delta n) = \exp(\beta_n \Delta n) - 1 - \beta_n \Delta n + \frac{1}{2}\gamma_n(\beta \Delta n)^2$ for capital and labor respectively.

Figure 1.2: Symmetric and asymmetric adjustment cost functions



The marginal cost of adjusting capital in periods t and $t+1$ respectively will be

$$\frac{\partial \varphi_t}{\partial k_{t+1}} = \frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}} (\varphi_{1t} - \varphi_{2t}) + \phi_t \left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}} \right) \tag{1.7}$$

$$\frac{\partial \varphi_{t+1}}{\partial k_{t+1}} = \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}} (\varphi_{1t+1} - \varphi_{2t+1}) + \phi_{t+1} \left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} \right) \tag{1.8}$$

Suppose that capital evolves as:[6]

$$k_{t+1} = (1-\delta)k + y - c - C(\Delta k) \tag{1.9}$$

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha} \tag{1.10}$$

$$1 = n_t + l_t \tag{1.11}$$

In other words, we assume neither population nor technological growth. The problem of the family, supposing a central planner perspective, is the standard one: choose consumption, leisure, and capital sequences to maximize the intertemporal utility function.

$$U(c) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\eta} l_t^{1-\eta} \right]^{1-\theta}}{1-\theta} \tag{1.12}$$

Subject to equations (1.2), (1.3) and (1.4). The lagrangean function for this problem is:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\eta} l_t^{1-\eta} \right]^{1-\theta}}{1-\theta} + \sum_{t=0}^{\infty} \lambda_t \beta^t \left[(1-\delta) k_t + y_t - c_t - C(i_t) - k_{t+1} \right] \right\} \tag{1.13}$$

First order conditions are:

$$\frac{\partial \mathcal{L}}{\delta c} = \left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} \eta c_t^{\eta-1} l_t^{1-\eta} - \lambda_t = 0 \tag{1.14}$$

[6]This model is as simple as possible, the standard way to model costs of investment is to includ them into the entertemporal profit function of firms and then solve for the descentralised equilibrium. However altough is possible to do this so, is preferable to first solve and simulate this simple model and introduce more complex elements later.

$$\frac{\partial \mathcal{L}}{\delta k_{t+1}} = -\lambda_t - \lambda_t \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}} + \beta E_t \left\{ \lambda_{t+1} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}} \right] \right\} = 0 \quad (1.15)$$

$$\frac{\partial \mathcal{L}}{\delta n_t} = -\left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta} + \lambda_t (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} = 0 \quad (1.16)$$

$$\frac{\partial \mathcal{L}}{\delta \lambda_t} = -k_{+1} + (1-\delta)k + y - i - C(\Delta k_{t+1}) = 0 \quad (1.17)$$

By using the functional forms we have that:

$$\left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} c_t^{\eta-1} l_t^{1-\eta} \left\{ 1 + \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}} \right\} - \beta E_t \left\{ \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta} \right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}} \right] \right\} = 0 \quad (1.18)$$

$$-\left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta} + \left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} \eta c_t^{\eta-1} l_t^{1-\eta} (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} = 0 \quad (1.19)$$

$$k_{+1} - (1-\delta)k - y + c + \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2 + \phi_t \left(\frac{\psi_1}{2}(k_{+1} - k - \overline{in})^2 - \frac{\psi_2}{2}(k_{+1} - k - \overline{in})^2 \right) = 0 \quad (1.20)$$

As we can see (1.18) is the Euler equation for consumption which seems to be quite similar to the traditional one. However, by taking into account that $\frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}}$ and $\frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}}$ are no longer linear expressions and, in fact, depend on the sign of Δk, if we replace the expressions corresponding to these derivatives within the Euler equation for consumption, we will have:

$$\begin{aligned} 0 = \eta \left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} c_t^{\eta-1} l_t^{1-\eta} \left\{ 1 + \left(\frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}} (\varphi_{1t} - \varphi_{2t}) + \phi_t \left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}} \right) \right) \right\} \quad (1.21) \\ - \beta E_t \left\{ \eta \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta} \right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \left(\begin{array}{c} \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}} (\varphi_{1t+1} - \varphi_{2t+1}) \\ + \phi_{t+1} \left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} \right) \end{array} \right) \right] \right\} \end{aligned}$$

In this expression, it is possible to see that the transition probability between regimes ϕ_t does appear on both sides of the equation for t and $t+1$, and so does the change on this probability in interaction with the difference of adjustment costs $\frac{\partial \phi_t}{\partial k_{t+1}} (\varphi_{1t} - \varphi_{2t})$ [7] . In this line of reasoning, the equation for intratemporal optimality condition in the canonical RBC will also be miss specified. By transforming the equivalent of $\left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta}$ from(1.18) into (1.19) we will have:

$$\begin{aligned} \left[c_t^{\eta} l_t^{1-\eta} \right]^{-\theta} (1-\eta) c_t^{\eta} l_t^{-\eta} \\ = \eta \left[\begin{array}{c} \beta \left\{ 1 + \left(\frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}} (\varphi_{1t} - \varphi_{2t}) + \phi_t \left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}} \right) \right) \right\}^{-1} \\ E_t \left\{ \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta} \right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \left(\begin{array}{c} \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}} (\varphi_{1t+1} - \varphi_{2t+1}) \\ + \phi_{t+1} \left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} \right) \end{array} \right) \right] \right\} \end{array} \right] \\ \times (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} \quad (1.22) \end{aligned}$$

Thus (1.22) shows that regime change probability and the interaction between probability derivative and adjustment costs difference also induce asymmetries.

[7] If we admit that including,\textit{ad hoc},transition probabilities matrices into the dynamic systemofacanonical RBC model, there is still remaining a mispecification error, which is the expression related $\frac{\partial \phi_t}{\partial k_{t+1}} (\varphi_{1t} - \varphi_{2t})$

1.3 Dynamics, calibration and simulation

Since the Euler equation of this model is nonlinear, as a regular DSGE's Euler equation, and asymmetric, it is necessary to use numerical methods to simulate it and solve it. Two alternative methods are addressed hereafter to show the inconvenience of using traditional log-linearization: Parametrized Expectations Approach (PEA) and Perturbations Method (PM). PEA was formalized by Marcet and Marshall (1994) and is a global method consisting of approaching the expectations equations[8]. PM is a local procedure based on k-order Taylor approximations around a particular point (the steady state for the case of DSGE and RBC models). A very useful and powerful tool for this method is Dynare, which allows up to third-order approximations[9].

1.3.1 Log-linearisation

Lets suppose that $\eta = 0$ and that $n_t = 1$, we will rewrite the system as:

$$\varphi_{1t} = \frac{\psi_1}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{1.23}$$

$$\varphi_{2t} = \frac{\psi_2}{2}\left(k_{t+1} - k_t - \overline{in}\right)^2 \tag{1.24}$$

$$C(\Delta k) = \varphi_t = \varphi_{2t} + \phi_t\left(\varphi_{1t} - \varphi_{2t}\right) \tag{1.25}$$

$$k_{+1} = (1-\delta)k + y - c - C(\Delta k) \tag{1.26}$$

$$y_t = f(k_t) = A_t k_t^{\alpha} \tag{1.27}$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_{t,}\varepsilon \backsim (0, \sigma_\varepsilon^2) \tag{1.28}$$

first order conditions are:

$$c_t^{-\theta} = \lambda_t \tag{1.29}$$

$$\lambda_t + \lambda_t \frac{\partial \varphi_t}{\partial k_{t+1}} = \lambda_{t+1}\beta\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial \varphi_{t+1}}{\partial k_{t+1}}\right] \tag{1.30}$$

$$\frac{\partial \varphi_t}{\partial k_{t+1}} = \frac{\partial \varphi_{2t}}{\partial k_{t+1}} + \frac{\partial \phi_t}{\partial k_{t+1}}\left(\varphi_{1t} - \varphi_{2t}\right) + \phi_t\left(\frac{\partial \varphi_{1t}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t}}{\partial k_{t+1}}\right) \tag{1.31}$$

$$\frac{\partial \varphi_{t+1}}{\partial k_{t+1}} = \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}} + \frac{\partial \phi_{t+1}}{\partial k_{t+1}}\left(\varphi_{1t+1} - \varphi_{2t+1}\right) + \phi_{t+1}\left(\frac{\partial \varphi_{1t+1}}{\partial k_{t+1}} - \frac{\partial \varphi_{2t+1}}{\partial k_{t+1}}\right) \tag{1.32}$$

Now, we consider the first order Taylor approximation around the log of the steady state for each regime, this is for $\Delta k > 0$ and for $\Delta k < 0$:

In the first regime or during a recession as$\Delta k < 0$, the log-linearized model is:

$$-\theta \hat{c}_t = \hat{\lambda}_t \tag{1.33}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t - \varphi\varphi_t \tag{1.34}$$

$$y\hat{y}_t = Ak^{\alpha}\hat{A}_t + \alpha Ak\hat{k}_t^{\alpha} \tag{1.35}$$

[8] An excelente and didactic reference about this method and its practical applications is Marcet and Lorenzoni (2001).

[9] The package also includes a Dynare++ module which allows up to seven-order approximation.

$$\varphi\hat{\varphi}_t = \varphi_1\hat{\varphi}_{1t} \tag{1.36}$$

$$\varphi_1\hat{\varphi}_{1t} = \psi_1 \left(k_{t+1} - k_t\right) k\hat{k}_{t+1} - \psi_1 \left(k_{t+1} - k_t\right) k\hat{k}_t \tag{1.37}$$

We will now take advantage of the fact that if we have a function $g(x)$, its log-linearisation becomes $g(X_t) \simeq g(X)(1 + \eta x_t)$, being $x_t = \ln(X_t/X), \eta = \frac{\partial f(X)}{\partial X}\frac{X}{f(X)}$.

$$\begin{aligned} &\lambda\hat{\lambda}_t + \lambda\frac{\partial\varphi(x)}{\partial k_{t+1}}\left(1 + \eta_{11}\hat{k}_{t+1}\right) + \lambda\frac{\partial\varphi_t(x)}{\partial k_{t+1}}\left(1 + \eta_{21}\hat{k}_{t+1}\right) \\ &= \beta\lambda\hat{\lambda}_{t+1}\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\right] \\ &+ \beta\lambda\left[\begin{array}{c} f'(k)\left(1 + \eta_{31}\hat{k}_{t+1}\right) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1 + \eta_{41}\hat{k}_{t+1}\right) \\ -\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1 + \eta_{51}\hat{k}_{t+2}\right) \end{array}\right] + \beta\lambda f'(k)\left(1 + \eta_{61}\hat{A}_{t+1}\right) \end{aligned} \tag{1.38}$$

$$\eta_{11} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_t}\frac{k_t}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{21} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{31} = \frac{\partial^2 f(k_{t+1})}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{f'(k_{t+1})},$$

$$\eta_{41} = \frac{\partial^2\varphi_{t+1}}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}}, \eta_{51} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_{t+2}}\frac{k_{t+2}}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{61}\frac{\partial^2 f(k_{t+1})}{\partial k_{t+1}\partial A_{t+1}}\frac{A_{t+1}}{f'(k_{t+1})}$$

Thus, for the previous equations (evaluated in the steady state which implies $k_{t+1} = k_t = \bar{k}$,) we have:

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{1.39}$$

$$\bar{k}\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \bar{y}\hat{y}_t - \bar{c}\hat{c}_t \tag{1.40}$$

$$\hat{\lambda}_t = \beta\hat{\lambda}_{t+1}\left[(1-\delta) + f'(\bar{k})\right] + \beta f'(\bar{k})\left(1 + \eta_{31}\hat{k}_{t+1}\right) + \beta\lambda f'(\bar{k})\left(1 + \eta_{61}\hat{A}_{t+1}\right) \tag{1.41}$$

$$\bar{y}\hat{y}_t = \bar{A}\bar{k}\hat{A}_t + \alpha\bar{A}\bar{k}^{\alpha}\hat{k}_t \tag{1.42}$$

$$\ln A_t = \rho\ln A_{t-1} + \varepsilon_t, \varepsilon_t \backsim (0, \sigma_\varepsilon^2) \tag{1.43}$$

Notice that this linearized model for the recession regime is formed by linear equations.
In the second regime or during a boom as $\Delta k > 0$, the linearized model becomes:

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{1.44}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t - \varphi\varphi_t \tag{1.45}$$

$$y\hat{y}_t = Ak^{\alpha}\hat{A}_t + \alpha Ak\hat{k}_t^{\alpha} \tag{1.46}$$

$$\varphi\hat{\varphi}_t = \varphi_2\hat{\varphi}_{2t} \tag{1.47}$$

$$\varphi_2\hat{\varphi}_{2t} = \psi_2 \left(k_{t+1} - k_t\right) k\hat{k}_{t+1} - \psi_2 \left(k_{t+1} - k_t\right) k\hat{k}_t \tag{1.48}$$

Because this approximation is evaluated in the steady state, which implies $k_{t+1} = k_t = \bar{k}$,

$$\begin{aligned}&\lambda\hat{\lambda}_t + \lambda\frac{\partial\varphi(x)}{\partial k_{t+1}}\left(1+\eta_{12}\hat{k}_{t+1}\right) + \lambda\frac{\partial\varphi_t(x)}{\partial k_{t+1}}\left(1+\eta_{22}\hat{k}_{t+1}\right) \\ &= \beta\lambda\hat{\lambda}_{t+1}\left[(1-\delta)+f'(k_{t+1})-\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\right] \\ &+ \beta\lambda\left[\begin{array}{c} f'(k)\left(1+\eta_{32}\hat{k}_{t+1}\right)-\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_{42}\hat{k}_{t+1}\right) \\ -\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1+\eta_{52}\hat{k}_{t+2}\right)\end{array}\right] + \beta\lambda f'(k)\left(1+\eta_{62}\hat{A}_{t+1}\right)\end{aligned} \tag{1.49}$$

$$\begin{aligned}\eta_{12} &= \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_t}\frac{k_t}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{22} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{32} = \frac{\partial^2 f(k_{t+1})}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{f'(k_{t+1})}, \\ \eta_{42} &= \frac{\partial^2\varphi_{t+1}}{\partial k_{t+1}\partial k_{t+1}}\frac{k_{t+1}}{\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}}, \eta_{52} = \frac{\partial^2\varphi_t}{\partial k_{t+1}\partial k_{t+2}}\frac{k_{t+2}}{\frac{\partial\varphi_t(x)}{\partial k_{t+1}}}, \eta_{62}\frac{\partial^2 f(k_{t+1})}{\partial k_{t+1}\partial A_{t+1}}\frac{A_{t+1}}{f'(k_{t+1})}\end{aligned}$$

Thus, for the previous equation (evaluated in the steady state implying $k_{t+1} = k_t = \bar{k}$), we have:

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{1.50}$$

$$\bar{k}\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \bar{y}\hat{y}_t - \bar{c}\hat{c}_t \tag{1.51}$$

$$\hat{\lambda}_t = \beta\hat{\lambda}_{t+1}\left[(1-\delta)+f'(\bar{k})\right] + \beta f'(\bar{k})\left(1+\eta_{32}\hat{k}_{t+1}\right) + \beta\lambda f'(\bar{k})\left(1+\eta_{62}\hat{A}_{t+1}\right) \tag{1.52}$$

$$\bar{y}\hat{y}_t = \bar{A}\bar{k}\hat{A}_t + \alpha\bar{A}\bar{k}^{\alpha}\hat{k}_t \tag{1.53}$$

$$\ln A_t = \rho\ln A_{t-1} + \varepsilon_t, \varepsilon_t \backsim (0, \sigma_\varepsilon^2) \tag{1.54}$$

As seen, even for the recession regime, the linearization of the model in the boom regime leads us evidently to a set of linear equations with different coefficients. Then, in order to simulate the full model including the possibility of moving from one regime to the other, it would be necessary to model a transition probability matrix for all the equations in the system, which has the inconvenience of being *ad hoc*. Therefore, it imposes transitions on the dynamics of all the equations, which is not modeled as the model's internal mechanisms of transmission. This makes it asymmetric by itself (Belaygoned & Michel, 2006; Davig & Leeper, 2005; Eo, 2009; and Pytelarczyk, 2005).

1.3.2 PEA algorithm

Now, PEA will be used in order to preserve the nonlinear features of the model.[10] With the goal of mapping the general form of PEA, the Euler equation and the capital transition equations are written as in (1.18), (1.19) and (1.20), these conform the system as:

$$g(E_t\left[\Phi(z_{t+1}, z_t)\right], z_{t+1}, z_t, u_t) = 0$$

in this seting,

$$\begin{aligned}\Phi(z_{t+1}, z_t) = l_t^{-(1-\eta)(1-\theta)} E_t\left\{\left[c_{t+1}^{\eta} l_{t+1}^{1-\eta}\right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta}\left[(1-\delta)+f'(k_{t+1})-\frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}}\right]\right\} \\ \times\left\{1+\frac{\partial C(\Delta k_t)}{\partial k_{t+1}}\right\}^{-1}\end{aligned}$$

[10] convergence results, and algorithm basics are found in Marcet and Marshall (1994) and Marcet and Lorenzoni (1998).

thus

$$c_t^{\eta(1-\theta)-1} = \beta\Phi(z_{t+1}, z_t)$$

$$z_t = (c_t, k_t, k_{t-1}, A_t)$$

$$z_{t+1} = (c_{t+1}, k_{t+1}, k_{t+2}, A_{t+1})$$

$$x_t = (k_{t-1}, A_t)$$

The complete PEA algorithm is as follows:

1. According to Marcet and Marshall (1994), it seems necessary to choose an adequate function $\Psi\left(\widetilde{\beta}, x_t\right)$ to approximate arbitrarily close to $\Phi(z_{t+1}, z_t)$. This will represent almost any function, except for a neural network. z_t is the vector of endogenous and exogenous variables as shown in the expectations function; x_t is a subset of variables used as regressor in the function Ψ; and $\widetilde{\beta}$ is a parameter vector in the approximation function Ψ.

2. Choose an initial $\widetilde{\beta}$,and for both initial values of state variables and a sequence of stochastic shocks compute

$$c_t = \left[\beta\Psi\left(\widetilde{\beta}, x_t\right)\right]^{1/(\eta(1-\theta)-1)} \tag{1.55}$$

β, η and θ are parameters of the utility function.

3. From step 2 we have series for c_t, and with k_t, and z_t; we are now to use Newton-Raphson (N-R) in order to approximate l_t, from the equilibrium equation.

$$l_t = \frac{(1-\eta)c_t}{\eta(1-\alpha)z_t k_t^{\alpha}(1-l_t)^{-\alpha}} \tag{1.56}$$

4. From series obtained in steps 1 and 2 , obtain k_{t+1} from the motion equation of capital:

$$k_{t+1} - (1-\delta)k_t - y_t + c_t + \frac{\psi_2}{2}(k_{t+1} - k_t - \overline{in})^2 + \phi_t(\frac{\psi_1}{2}(k_{t+1} - k_t - \overline{in})^2 - \frac{\psi_2}{2}(k_{t+1} - k_t - \overline{in})^2) = 0 \tag{1.57}$$

then we have time series for c_{t+1}, $k_{t+1}, k_{t+2}, z_{t+1}$and l_t.

5. Define and compute:

$$c_t^{RE} = \left\{ \begin{array}{c} \beta l_t^{-(1-\eta)(1-\theta)} E_t \left\{ \left[c_{t+1}^{\eta} l_{t+1}^{1-\eta} \right]^{-\theta} c_{t+1}^{\eta-1} l_{t+1}^{1-\eta} \left[(1-\delta) + f'(k_{t+1}) - \frac{\partial C(\Delta k_{t+2})}{\partial k_{t+1}} \right] \right\} \\ \times \left\{ 1 + \frac{\partial C(\Delta k_{t+1})}{\partial k_{t+1}} \right\}^{-1} \end{array} \right\}^{1/(\eta(1-\theta)-1)} \tag{1.58}$$

6. Regress $\frac{1}{\beta}(c_t^{RE})^{\eta(1-\theta)-1}$ on $\Psi\left(\widetilde{\beta}, x_t\right)$ and obtain new estimated values for $\widetilde{\beta}$.stop when you find a fixed point for $\widetilde{\beta}$ such that $\widetilde{\beta}_f = G(\widetilde{\beta}_f)$

being

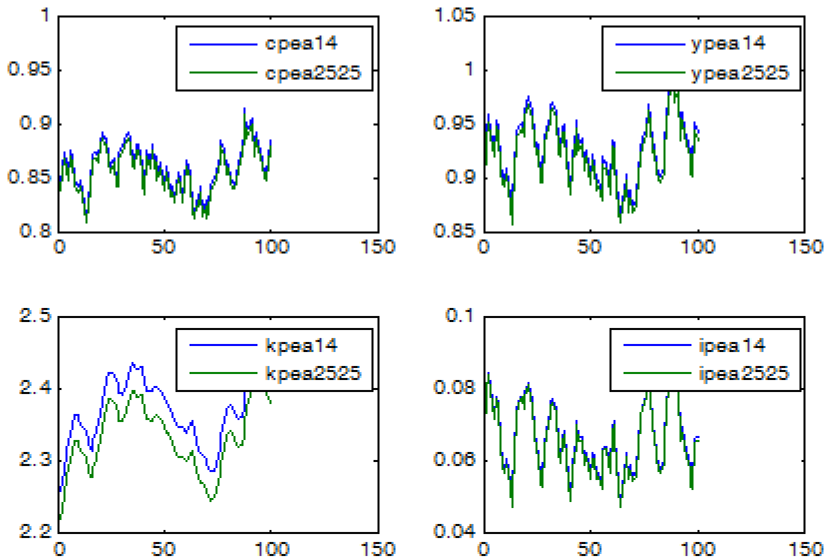
$$G(\widetilde{\beta}) = \arg\min_{\zeta} \frac{1}{T}\sum_{t=0}^{T} \left\| \Phi(z_{t+1}\left(\widetilde{\beta}\right), z_t\left(\widetilde{\beta}\right)) - \Psi\left(\zeta, x_t\left(\widetilde{\beta}\right)\right) \right\|^2 \tag{1.59}$$

In order to capture nonlinearities and asymmetries from this model set up, it is needed a more flexible functional form:

$$\Psi\left(\widetilde{\beta}, x_t\right) = \exp\left(\Omega(\widetilde{\beta})\right) \tag{1.60}$$

$$\Omega(\widetilde{\beta}) = \widetilde{\beta}_1 + \widetilde{\beta}_2 \ln k_{t-1} + \widetilde{\beta}_3 \ln z_t \tag{1.61}$$

Figure 1.3: Asymmetric and symmetric models simulations



To stabilize the algorithm and to assist for convergence, steps 3 and 4 must be modified by imposing moving bands as suggested by Maliar and Maliar (2003).

Note that if we impose $b = 0$, or $\psi_1 = \psi_2$, we will obtain a standard DSGE model with symmetric adjustment costs. In this way, we can simulate both models for the the same time series of shocks and compare their time path as well as their higher order moments.Table 1.1 displays calibration parameters and steady state values in order to compare an asymmetric model with a symmetric one. Prameters are chosen to get as close as possible to the colombian economy. Parameter cost for the symmetric model is calibrated as $\psi = 0.5\ (\psi_1 + \psi_2)$. Thus, the symmetric adjustment cost will be an intermediate case of low and high costs regime.

Figure 1.4: Asymmetric and symmetric models simulations

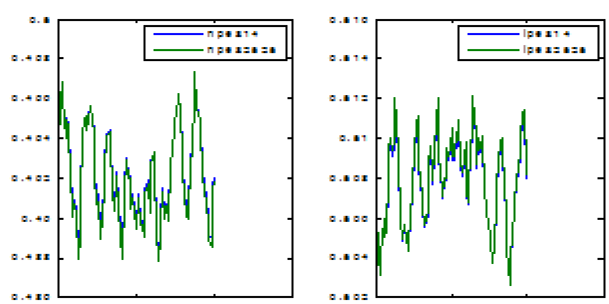


Table 1.1: Calibration

Parameter		Variable	
α	0.4	kss	2,329
ψ_1	4	yss	0,916
ψ_2	1	css	0,852
$\psi = 0.5(\psi_1 + \psi_2)$	2,5	$inss$	0
ρ	0.9	kss/yss	2.543
θ	2	rss	0.013
δ	0.0273	nss	0,492
σ_ε^2	0.018	$\beta = \frac{1}{1+rss}$	0.885
γ	500	$ibss$	0.0636

Table 1.2: Differences in moments of raw data from PEA for symmetric and asymmetric models simulations

	Difautocorr			Difsigrel		
variable	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	0.0005	0.0012	0.0024	-0.787	-0.1805	0.3864
'Yt'	-0.0004	0.0001	0.0009	0	0	0
'Kt'	-0.0005	-0.0001	0.0003	-0.8998	0.2043	2.3846
'Ibt'	-0.0031	-0.0018	-0.0009	-1.0718	0.3293	3.4855
'nt'	-0.0034	-0.0021	-0.0009	-1.6144	0.42	4.9376
'It'	-0.0034	-0.0021	-0.0009	-2.356	0.5032	5.22
Zr'	0	0	0	-2.3524	0.5942	5.9742
	DifsigrelHP			DifsigrelBK		
	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0125	-0.0119	-0.0112	-0.013	-0.0126	-0.0122
'Yt'	0	0	0	0	0	0
'Kt'	0.0067	0.0078	0.0087	0.006	0.0069	0.0076
'Ibt'	0.1368	0.158	0.1774	0.1423	0.164	0.1809
'nt'	0.0062	0.0065	0.0068	0.0066	0.0067	0.0069
'It'	0.0058	0.0062	0.0065	0.0062	0.0064	0.0066
Zr'	-0.0041	-0.0039	-0.0036	-0.0039	-0.0037	-0.0035

Note: for each variable, autocorrelation (for the raw data), relative standard deviations (for the raw data, HP filtered and BK filtered time series), are computed on the time series simulated by using PEA for the symmetric and the asymmetric versions of the model, then differendes were taken as follows: $\rho_x^{sim} - \rho_x^{asim}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{sim} - \left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{asim}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{sim} - \left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{asim}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{sim} - \left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{asim}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.3: Differences in Kurtosis and Skewness of HP filtered data from PEA for symmetric and asymmetric models simulations

	DifkurtHP			DifkurtHPneg			DifkurtHPpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0083	0.0011	0.0096	-0.0331	0.0029	0.0369	-0.0379	0.0019	0.0428
'Yt'	-0.0023	0.0005	0.0031	-0.0112	0.0011	0.0165	-0.0151	0.0009	0.0158
'Kt'	-0.0481	-0.0124	0.009	-0.1557	-0.0248	0.0508	-0.136	-0.0217	0.0407
'Ibt'	-0.2382	-0.0533	0.0127	-1.022	-0.1843	0.0485	-0.1751	-0.0267	0.0506
'nt'	-0.0223	-0.002	0.013	-0.069	-0.0019	0.0497	-0.0629	-0.0048	0.0409
'lt'	-0.0214	-0.002	0.0136	-0.0627	-0.0035	0.0435	-0.0703	-0.003	0.0516
Zr'	0	0	0	0	0	0	0	0	0
	DifasimHP			DifasimHPneg			DifasimHPpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.2169	-0.0044	0.2121	-0.0111	-0.0009	0.0088	-0.0108	0.0007	0.0114
'Yt'	-0.2175	-0.0012	0.221	-0.0044	-0.0003	0.0039	-0.0045	0.0003	0.0046
'Kt'	-0.3106	-0.0086	0.2827	-0.0125	0.0068	0.0362	-0.0298	-0.0066	0.0123
'Ibt'	-0.7117	-0.3374	-0.0705	-0.0096	0.0297	0.1179	-0.036	-0.0091	0.0121
'nt'	-0.2282	0.0002	0.2665	-0.0145	0.0005	0.0183	-0.0181	-0.0011	0.0123
'lt'	-0.2877	-0.0213	0.2044	-0.0128	0.0007	0.0162	-0.0187	-0.0009	0.0144
Zr'	-0.2085	0.0008	0.2183	0	0	0	0	0	0

Note: for each variable (simulated by using symmetric and asymmetric versions of the model), cyclical components were computed using HP filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)^{sim}_{HP} - (Kutosis)^{asim}_{HP}$, $(Skewness)^{sim}_{HP} - (Skewness)^{asim}_{HP}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.4: Differences in Kurtosis and Skewness of BK filtered data from PEA for symmetric and asymmetric models simulations

	DifcurktBK			DifkurtBKneg			DifkurtBKpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0064	0.0013	0.0078	-0.0275	0.0036	0.0353	-0.0346	0.0006	0.0331
'Yt'	-0.0017	0.0003	0.0024	-0.0104	0.0006	0.0135	-0.0126	0.0006	0.0184
'Kt'	-0.05	-0.0132	0.0093	-0.1721	-0.0247	0.0496	-0.1521	-0.0257	0.0445
'Ibt'	-0.2005	-0.0424	0.0199	-0.7958	-0.1522	0.0703	-0.1758	-0.0183	0.0607
'nt'	-0.0166	-0.0037	0.0042	-0.0591	-0.0063	0.0256	-0.0567	-0.0081	0.0212
'lt'	-0.017	-0.0037	0.0046	-0.0584	-0.0073	0.0254	-0.0534	-0.0072	0.0241
Zr'	0	0	0	0	0	0	0	0	0
	DifasimBK			DifasimBKneg			DifasimBKpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0044	-0.0002	0.0042	-0.0091	-0.0009	0.0078	-0.0101	0.0002	0.0094
'Yt'	-0.0013	0	0.0013	-0.0039	-0.0002	0.0033	-0.0043	0.0002	0.005
'Kt'	-0.0114	-0.0008	0.0101	-0.0151	0.0068	0.0358	-0.0323	-0.0071	0.0117
'Ibt'	-0.0127	0.0016	0.0259	-0.0117	0.0259	0.1041	-0.0353	-0.0056	0.0146
'nt'	-0.0051	-0.0006	0.0037	-0.0076	0.0017	0.0117	-0.0136	-0.0022	0.0067
'lt'	-0.0043	0	0.0046	-0.0066	0.0019	0.014	-0.0132	-0.002	0.0074
Zr'	0	0	0	0	0	0	0	0	0

Note: for each variable (simulated by using symmetric and asymmetric versions of the model), cyclical components were computed using BK filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)^{sim}_{BK} - (Kutosis)^{asim}_{BK}$, $(Skewness)^{sim}_{BK} - (Skewness)^{asim}_{BK}$. Sample periods were 500, replicated 500 times. Thus, estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.5: Correlations of raw data simulated by PM with data simulated by PEA, symmetric model

Correlations	
Variables	PEA/PM
'Ct'	0.9943
'Yt'	0.9953
'Kt'	0.9762
'Ibt'	0.9764
'nt'	0.9682
'lt'	0.9682
Zr'	0.999

Note: each variable was simulated (in the symmetric model) by using both PEA and PM algorithms, then the correlations (of the raw data) are computed as
follows: $corr(x_t^{PEA}, x_t^{PM})$. Sample periods were 500, replicated 500 times.

1.3.3 Perturbations algorithm

Although PEA algorithm and projections algorithm are generally time expensive, they are more precise as they are global approximation methods. However, it is possible to use a higher order [11] PM, which approximates the steady state and is less expensive than PEA algorithm. Through simulations carried out on Dynare, this latter method uses higher order derivatives of the dynamic system evaluated in the steady state.

1.3.4 Comparing PEA and Perturbations algorithms

It is known that global approximation methods such as PEA are costly in terms of time and computation. However, local approximations such as log-linearization and perturbations are less expensive. Notwithstanding, the issue of accuracy is yet a matter of concern. In this section, simulated time series with both algorithms are compared in order to asses accuracy and get an idea about how similar these algorithms are, making it possible to decide whether, without loss of accuracy, to use perturbations algorithm instead of a PEA algorithm. This experiment is performed by simulating pseudo-data for both methods in the following fashion: i) imposing symmetry in adjustment costs ($\psi_1 = \psi_2 = \overline{\psi} = 2.5$), and ii) imposing asymmetry ($\psi_1 = 4, \psi_2 = 1$). Other parameters remain the same as shown on table1.1.

1.3.4.1 Simulating the symmetric model

Table 1.3.4.1 shows the correlations of macro variables simulated by using PEA and PM (for the symmetric model). It can be seen that time series simulated by PEA move quite close to those simulated by PM.

Table 1.6 displays differences in autocorrelations, relative variances (compared to σ_x/σ_{GDP}) computed on the raw data in the upper panels, and differences in relative variances computed on both HP and BK filtered data. Lower and upper bounds for a 95% confidence interval are also reported. In general, gross investment, labor, and leisure seem to have more persistence in the PEA than in the PM algorithm, whereas consumption and capital present lower persistence. Relative standard deviations seem to be quite similar for the two algorithms. For the case of relative standard deviations, they seem to be very similar because the mean value of their differences lies inside the confidence interval. The results for the differences of relative standard deviations are mixed: While both HP and BK filtered data of labor and leisure seem to have the same relative standard deviation, consumption seems to decrease and be higher for capital and investment. Tables1.7 and 1.3.4.1 show the differences in kurtosis and skewness of PEA data and PM data for HP and BK filtered data respectively. In these tables, an unabiguous result is evident: mean of differences in kurtosis and

[11] Log-linearisation is a first order Taylor approximation. Thus, higher-order approximation refers to second order, third order and so on.

Table 1.6: Differences in moments of raw data from PEA and PM methods for the symmetric model simulations

	Difautocorr			Difsigrel		
variable	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0039	0	0.004	-1.8246	-0.225	0.1406
'Yt'	-0.0019	0.0025	0.0083	0	0	0
'Kt'	-0.0028	-0.0015	-0.0001	-0.9011	0.1371	1.5828
'Ibt'	0.0182	0.0339	0.0531	-1.0656	0.1881	3.7804
'nt'	0.0339	0.0527	0.0758	-1.169	0.2183	3.0755
'It'	0.0339	0.0527	0.0758	-1.4631	0.2813	3.7006
Zr'	-0.0021	0	0.0027	-1.5636	0.3724	3.563
	DifsigrelHP			DifsigrelBK		
	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0375	-0.0298	-0.022	-0.0449	-0.039	-0.033
'Yt'	0	0	0	0	0	0
'Kt'	0.0408	0.0469	0.0535	0.0331	0.0387	0.0444
'Ibt'	0.2834	0.4052	0.5398	0.3993	0.5034	0.6308
'nt'	-0.0065	-0.0028	0.0009	-0.0028	0.0005	0.0035
'It'	-0.0063	-0.0026	0.0009	-0.0025	0.0005	0.0034
Zr'	0.0069	0.0097	0.0122	0.0088	0.0108	0.0128

Note: for each variable in the symmetric model, autocorrelation (for the raw data), relative standard deviations (for the raw data, HP filtered and BK filtered time series), are computed on the time series simulated by using PEA and Perturbations Method, then differendes were taken as follows: $\rho_x^{PEA} - \rho_x^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: PEA simulations are very close to PM simulations.

asymmetries are containend within a 95% confidence interval. Under the light of these results, it is possible to think of the Perturbations algorithm as one very close to PEA.

1.3.4.2 Simulating the asymmetric model:

Tables 1.9-1.3.4.2 show the same statistics as tables 1.3.4.1-1.3.4.1, but computed on the simulations of the asymmetric model ($\psi_1 = 4, \psi_2 = 1$). Several simulations were made for different values of γ (500, 100, 50, and 25), because for $\gamma \to \infty$ the smooth transition function becomes a step function; its derivative tends to infinite as well, and the model will lose its differentiability which is the corner stone of the PM algorithm. However, 100, 50 or 25 are still high values, thus the results reported in tables 1.9-1.3.4.2 are those for the simulations using $\gamma = 25$. In general, the means of the differences between moments of PEA and PM are contained within the 95% confidence interval as well as for the case of the results in the symmetric model, which means that PEA and PM are very close to each other.

1.3.4.3 Moments of data vs. Moments of simulated data

In order to see how good the model is to reproduce higher order moments, table 1.13 shows kurtosis and skewness o simulated data and the same moments for the sample of countries as in table 1. Highlighted numbers in yellow show that simulated data can approach the behavior in relative magnitud and sign for kurtosis and skewnes of full sample, and negative-positive values of ciclical components of macroeconomic variables. More over the model is successful in reproducing positive and negative skewness. However, because

Table 1.7: Differences in moments of HP filtered data from PEA and PM methods for the symmetric model simulations

	DifkurtHP			DifkurtHPneg			DifkurtHPpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0737	0.0021	0.0679	-0.2185	0.0247	0.2708	-0.2901	-0.0171	0.2035
'Yt'	-0.0236	-0.0011	0.0262	-0.1066	-0.0048	0.0855	-0.0892	-0.0012	0.1033
'Kt'	-0.1262	-0.0114	0.1178	-0.4507	-0.0439	0.3859	-0.4954	-0.0229	0.461
'Ibt'	-0.3172	0.0129	0.5085	-1.3133	0.0401	1.9176	-0.6787	-0.0415	0.6984
'nt'	-0.2352	-0.0206	0.2434	-0.9184	-0.1123	0.8092	-0.7168	0.0084	0.8834
'It'	-0.2277	-0.0162	0.2463	-0.7171	0.013	0.935	-0.895	-0.1051	0.7757
Zr'	-0.0028	0	0.0025	-0.0073	0	0.0084	-0.0077	0	0.0081
	DifasimHP			DifasimHPneg			DifasimHPpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.2094	-0.0041	0.2436	-0.0736	-0.0078	0.0552	-0.0684	-0.0054	0.0607
'Yt'	-0.2054	-0.001	0.2412	-0.0218	0.0011	0.0251	-0.024	0	0.0257
'Kt'	-0.2843	-0.0032	0.2766	-0.082	0.0148	0.11	-0.1203	-0.0025	0.1062
'Ibt'	-0.5988	-0.2793	0.0226	-0.2935	-0.0045	0.2472	-0.176	-0.0149	0.1611
'nt'	-0.214	0.0025	0.2627	-0.161	0.035	0.2265	-0.1854	0.0025	0.2122
'It'	-0.2788	-0.0211	0.1995	-0.2113	-0.0039	0.1846	-0.2256	-0.0334	0.1609
Zr'	-0.2001	0.0007	0.2352	-0.0027	0	0.0028	-0.0026	-0.0001	0.0025

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using HP filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)^{PEA}_{HP} - (Kutosis)^{PM}_{HP}, (Skewness)^{PEA}_{HP} - (Skewness)^{PM}_{HP}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.8: Differences in moments of BK filtered data from PEA and PM methods for the symmetric model simulations

	DifcurktBK			DifkurtBKneg			DifkurtBKpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0506	-0.0004	0.0474	-0.1811	0.0242	0.2005	-0.2164	-0.0239	0.1631
'Yt'	-0.016	-0.0005	0.0176	-0.0803	-0.0049	0.0659	-0.0769	-0.0005	0.0821
'Kt'	-0.1404	-0.0078	0.1366	-0.5528	-0.0831	0.371	-0.3734	0.0365	0.5468
'Ibt'	-0.2135	0.0421	0.4355	-0.959	0.133	1.6097	-0.6609	-0.0706	0.4636
'nt'	-0.1429	-0.0116	0.1476	-0.5909	-0.0781	0.4252	-0.469	0.0069	0.5053
'It'	-0.1311	-0.0075	0.1474	-0.4671	0.017	0.5497	-0.5678	-0.0761	0.4143
Zr'	-0.004	-0.0001	0.002	-0.0128	-0.0001	0.0088	-0.0106	-0.0004	0.0085
	DifasimBK			DifasimBKneg			DifasimBKpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0329	-0.0099	0.0119	-0.0537	-0.0073	0.0421	-0.0584	-0.0072	0.0397
'Yt'	-0.0086	0.0017	0.012	-0.0194	0.0012	0.0207	-0.0208	0.0003	0.021
'Kt'	-0.0368	0.0332	0.1052	-0.0917	0.0249	0.1408	-0.107	0.0124	0.1441
'Ibt'	-0.1191	-0.0385	0.0175	-0.261	-0.029	0.175	-0.1387	-0.0198	0.1132
'nt'	-0.0206	0.0256	0.0708	-0.1224	0.023	0.1512	-0.1168	0.0064	0.1393
'It'	-0.0759	-0.0281	0.0182	-0.1404	-0.0093	0.1112	-0.1497	-0.0228	0.1185
Zr'	-0.0016	0	0.0016	-0.0031	0	0.0032	-0.0036	-0.0002	0.0029

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using BK filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)^{PEA}_{BK} - (Kutosis)^{PM}_{BK}, (Skewness)^{PEA}_{BK} - (Skewness)^{PM}_{BK}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.9: Correlations of raw data simulated by PM with data simulated by PEA, asymmetric model

Correlations	
Variables	PEA/PM
'Ct'	0.9941
'Yt'	0.9952
'Kt'	0.9754
'Ibt'	0.9726
'nt'	0.9648
'lt'	0.9648
Zr'	0.999

Note: each variable was simulated (in the symmetric model) by using both PEA and PM algorithms, then the correlations (of the raw data) are computed as
follows: $corr(x_t^{PEA}, x_t^{PM})$. Sample periods were 500, replicated 500 times.

Table 1.10: Differences in moments of the raw data from PEA and PM methods for the asymmetric model simulations

	Difautocorr			Difsigrel		
variable	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0043	-0.0003	0.0036	-2.0118	-0.2431	0.0924
'Yt'	-0.0023	0.0024	0.0079	0	0	0
'Kt'	-0.0028	-0.0014	0	-0.8857	0.1131	1.4397
'Ibt'	0.0196	0.0351	0.0537	-1.1679	0.1482	2.0336
'nt'	0.036	0.0543	0.076	-0.9365	0.2625	1.9434
'lt'	0.036	0.0543	0.076	-1.1301	0.3046	2.8407
Zr'	-0.0021	0	0.0027	-1.207	0.3755	2.5604
	DifsigrelHP			DifsigrelBK		
	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0368	-0.0285	-0.0195	-0.0451	-0.0376	-0.0299
'Yt'	0	0	0	0	0	0
'Kt'	0.0379	0.0454	0.0539	0.0311	0.0372	0.0437
'Ibt'	2.1646	2.3645	2.5613	2.274	2.4673	2.6492
'nt'	-0.0089	-0.0045	-0.0001	-0.0056	-0.0013	0.0026
'lt'	-0.009	-0.0047	-0.0005	-0.0058	-0.0016	0.0021
Zr'	0.0073	0.0104	0.0134	0.0089	0.0115	0.0139

Note: for each variable in the symmetric model, autocorrelation (for the raw data), relative standard deviations (for the raw data, HP filtered and BK filtered time series), are computed on the time series simulated by using PEA and Perturbations Method, then differendes were taken as follows: $\rho_x^{PEA} - \rho_x^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{rawdata}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{HP}^{PM}$, $\left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PEA} - \left(\frac{\sigma_x}{\sigma_y}\right)_{BK}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: PEA simulations are very close to PM simulations.

Table 1.11: Differences in moments of the HP filtered data from PEA and PM methods for the asymmetric model simulations

	DifkurtHP			DifkurtHPneg			DifkurtHPpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0783	0.0106	0.0986	-0.357	-0.0531	0.2102	-0.1892	0.1037	0.4462
'Yt'	-0.0423	-0.007	0.0236	-0.0897	0.0177	0.1484	-0.194	-0.0465	0.0648
'Kt'	-0.5809	-0.1319	0.1502	-1.8606	-0.2768	0.6609	-1.6199	-0.2642	0.5352
'Ibt'	-0.4188	0.2669	1.6097	-0.3533	1.4383	5.991	-2.6023	-0.6715	0.7754
'nt'	-0.8713	-0.2699	0.164	-0.8629	0.1674	1.2799	-3.1599	-0.9723	0.284
'It'	-0.9264	-0.3085	0.1205	-3.3604	-1.0421	0.2741	-0.9337	0.1363	1.221
Zr'	-0.0027	0	0.0024	-0.0073	0	0.0077	-0.008	-0.0002	0.0081
	DifasimHP			DifasimHPneg			DifasimHPpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.2546	-0.0414	0.1885	-0.0544	0.0166	0.0885	-0.0429	0.031	0.1016
'Yt'	-0.1875	0.0149	0.2553	-0.0341	-0.0061	0.0205	-0.0424	-0.0134	0.015
'Kt'	-0.3123	-0.0098	0.2768	-0.1345	0.0743	0.3698	-0.2999	-0.0674	0.1161
'Ibt'	-0.0782	0.1349	0.3726	-1.0093	-0.3452	0.0448	-0.5171	-0.2029	0.122
'nt'	0.0168	0.2444	0.5063	-0.3063	-0.0579	0.2113	-0.5941	-0.248	0.0438
'It'	-0.5415	-0.2661	-0.035	-0.0263	0.2604	0.6197	-0.2156	0.0498	0.296
Zr'	-0.2002	0.0011	0.2338	-0.0026	0	0.0027	-0.0026	-0.0001	0.0025

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using HP filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{HP}^{PEA} - (Kutosis)_{HP}^{PM}$, $(Skewness)_{HP}^{PEA} - (Skewness)_{HP}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.12: Differences in moments of the BK filtered data from PEA and PM methods for the asymmetric model simulations

	DifkurktBK			DifkurtBKneg			DifkurtBKpos		
variable	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0756	0.0028	0.0751	-0.3304	-0.0329	0.1987	-0.163	0.0553	0.3464
'Yt'	-0.035	-0.0043	0.0246	-0.093	0.0151	0.1503	-0.1455	-0.0345	0.0471
'Kt'	-0.5868	-0.1501	0.1114	-2.1725	-0.3584	0.6327	-1.5992	-0.2695	0.351
'Ibt'	-0.4611	0.2389	1.4123	-0.7601	1.1683	5.0841	-2.4337	-0.47	1.065
'nt'	-0.7568	-0.2021	0.1379	-1.0124	0.0641	1.2095	-2.4905	-0.7405	0.189
'It'	-0.8197	-0.2317	0.109	-2.6465	-0.7921	0.1782	-1.1107	0.0253	1.0716
Zr'	-0.003	-0.0001	0.002	-0.0124	-0.0002	0.0088	-0.0081	-0.0003	0.0085
	DifasimBK			DifasimBKneg			DifasimBKpos		
	Lb	Mean	Ub	Lb	Mean	Ub	Lb	Mean	Ub
'Ct'	-0.0012	0.0255	0.0543	-0.0465	0.011	0.0745	-0.0405	0.017	0.0788
'Yt'	-0.0237	-0.0114	-0.0001	-0.0323	-0.0051	0.0193	-0.0344	-0.0098	0.0121
'Kt'	-0.0725	0.0267	0.1188	-0.1342	0.0958	0.3973	-0.2817	-0.0675	0.0969
'Ibt'	-0.5756	-0.3421	-0.1955	-0.9432	-0.2919	0.097	-0.5102	-0.1423	0.1939
'nt'	-0.2996	-0.1749	-0.0867	-0.2617	-0.0279	0.2059	-0.4831	-0.1922	0.0365
'It'	0.0843	0.1743	0.3002	-0.0272	0.2023	0.5119	-0.2175	0.0173	0.2493
Zr'	-0.0016	0	0.0016	-0.0029	0	0.0031	-0.0033	-0.0001	0.0029

Note: for each variable in the symmetric model (simulated by PEA and PM), cyclical components were computed using BK filter. Then, Kurtosis and skewness were calculated on the full sample and on the negative and positive of the cyclical components. Then differendes were taken as follows: $(Kurtosis)_{BK}^{PEA} - (Kutosis)_{BK}^{PM}$, $(Skewness)_{BK}^{PEA} - (Skewness)_{BK}^{PM}$. Sample periods were 500, replicated 500 times. Thus estatistics reported are means of differences and critical values for 95% confidence interval. Null hypothesis: symmetric model is different than asymmetric model.

Table 1.13:

	Kurtosis			Skewness		
	HP			HP		
Variable	Full sample	Negative values	Positive values	Full sample	Negative values	Positive values
	Model economy (500 replications)					
GDP	2.874	3.0508	3.2269	0.0429	-0.773	0.8473
C	2.742	2.9783	2.9727	-0.1018	-0.7529	0.7298
I	3.965	3.399	4.6117	0.4151	-0.8723	1.2896
	Colombia					
GDP	2.0899	1.8927	1.8858	0.017	-0.462	0.2024
C	2.1659	3.3298	2.1108	0.193	-0.5571	0.3256
I	4.9049	6.0205	5.232	-0.3166	-1.8673	1.6237
	Germany					
GDP	2.4663	3.2255	1.9461	0.2942	-0.8865	0.2278
C	2.7961	1.8534	2.3599	0.2871	-0.5974	0.7277
I	1.8434	3.3246	1.7065	0.1644	-0.255	-0.0722
	USA					
GDP	3.0601	3.2327	2.0607	-0.559	-1.1154	0.3173
C	2.2794	4.3438	2.3978	-0.3902	-1.3491	0.3064
I	2.8452	3.118	2.1146	-0.625	-0.8702	0.1003
	United Kingdom					
GDP	3.1953	4.7287	2.5273	-0.2233	-1.3995	0.8305
C	2.8111	3.8048	3.0562	0.453	-0.5264	1.0482
I	3.6498	3.3733	3.9337	-0.0489	-1.0363	1.3929
	France					
GDP	2.2161	3.75	1.8382	-0.1168	-0.8477	0.269
C	2.2506	2.483	3.8269	-0.3778	-0.3094	0.7721
I	2.1549	1.6976	2.2693	0.0998	-0.1145	0.5048

Source: Autor calculations and World Bank web page. Annual per capita real series, logarithms of data filtered with Hodrik-Prescott filter. For France and USA sample is from 1970 to 2009 and for the other countries from 1960.

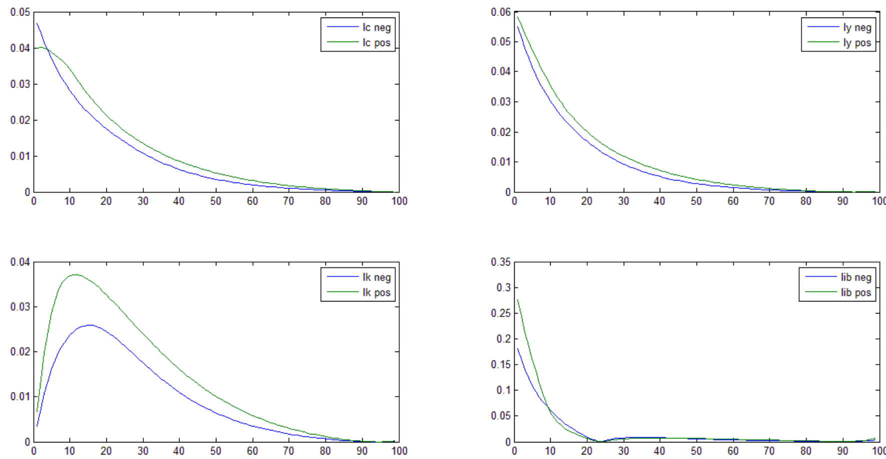
de calibracition of the model does not refer to a particular country in the sample, this results can not be taken as conclusive about the goodnes of the model to reproduce asymmetries of business cycles. Sections 1.3.5 and 1.3.6 contain impulse response exercises performed to see how different is a recession from a boom.

1.3.5 Deterministic simulation

Moreover, in order to test the the model's construction consistency, deterministic simulations were performed imposing a deviation (negative and positive) of the technology process when simulating a one time shock. The model was solved by using the extended path algorithm implemented in Dynare, by imposing $a = 1.06$ and $a = 0.94$, which is equivalent to having $e = 0.058268908$ and $e = -0.061875404$ respectively.[12] Figures1.5 and 1.6 show a path time of key macro variables $c_t, y_t, k_t, in_t, n_t, ib_t, a_t, \phi_t$ (consumption, income, capital, net investment, labor, gross investment, technology, and transition function).

[12] It would be also possible to impose a symmetric e (this is, the same size of the shock in absolute value) but there would not be a great difference in the results.

Figure 1.5: Re-scaled variables



Figures 1.5 and 1.6 show re-scaled variables [13]. Given the calibration, several interesting behaviors were observed. For instance, the reaction of consumption towards a negative perturbation is stronger than when a positive shock occurs. This can be explained by the fact that disinvestment costs are higher than investment costs. It is important to note that the investment reaction during recession is lower that that during a boom. Consistently, income reaction during a recession is lower than during booms. Not only can this be explained by the investment decrease, but also by the labor decrease. The size of the adjustment in labor during recession is higher than during boom due to the fact that the decrease in wage during recession is not as big as the increase during boom. Thus, the models reproduces labor as well as wages increases during booms, and labor decreases and smaller wage reductions, which is all a signal of real rigidities in wages. Thus, most of the adjustment in this economy is led by consumption and labor. Obviously, in expansion periods, capital increases are bigger than capital decreases during recessions. It can also be seen that expansions are longer than recessions. This is also seemingly true for income and consumption. Figures 1.7 and 1.8 show time path deviations from the steady state.

1.3.6 Impulse response

Impulse Response (IR) is one of the most used analysis tools in macroeconometrics. However, it must be used carefully. Because the DSGE model studied in this paper is non-linear and asymmetric, IR analysis should not be performed as usual, assuming that the DGP is linear-multivariate. Moreover, it could be mistaken to simply shock technology once and then follow the whole system's adjustment. Therefore, in order to gauge asymmetric effects of shocks in this hypothetical economy, General Impulse Response Function (Koop et al., 1996) (GIRF hereafter) is to be adopted.[14].

Because of asymmetric DGP of this DSGE model, multivariate data simulated by using this very model lacks the following properties: symmetry property, linearity property, and history-independence property. Thus, linear impulse response functions (VAR-based) are not appropriate tools for analyzing the dynamics of such DSGE model. The GIRF, as defined by Koop et al. (1996), is conditioned by shocks and/or history:

$$GI_Y(n, v_t, \omega_{t-1}) = E[Y_{t+n}|v_t, \omega_{t-1}] - E[Y_{t+n}|\omega_{t-1}], \quad for \quad n = 0, 1...$$

[13]Rescalation is necesary for comparison of the variables in a single plane. For simulated variables with a negative shock the computation is $abs(x_t) - max(x_t)$ ad for variables with a positive shock $x_t - \min(x_t)$.

[14]Local Projections Impulse Response (Jordá, 2005) could also be used, but this technique is susceptible of symmetry, thus it would be not possible to detect asymmetry in data of this hipothetical model.

Figure 1.6: Re-scaled variables

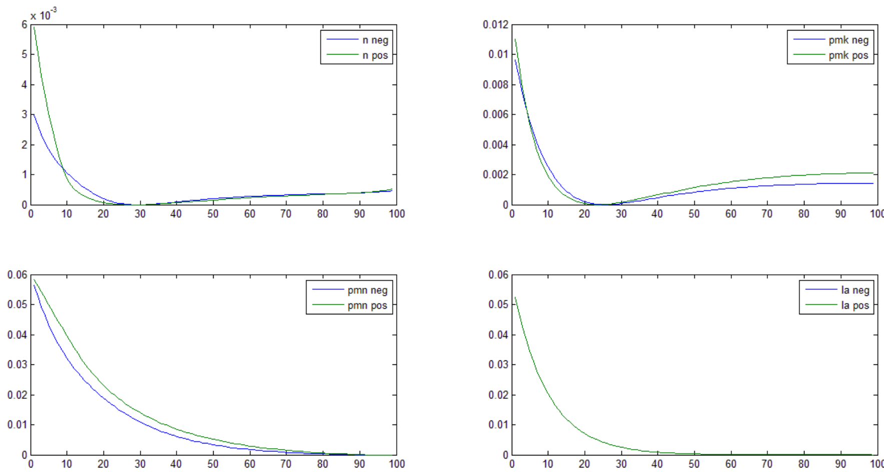


Figure 1.7: Deviations from the steady state

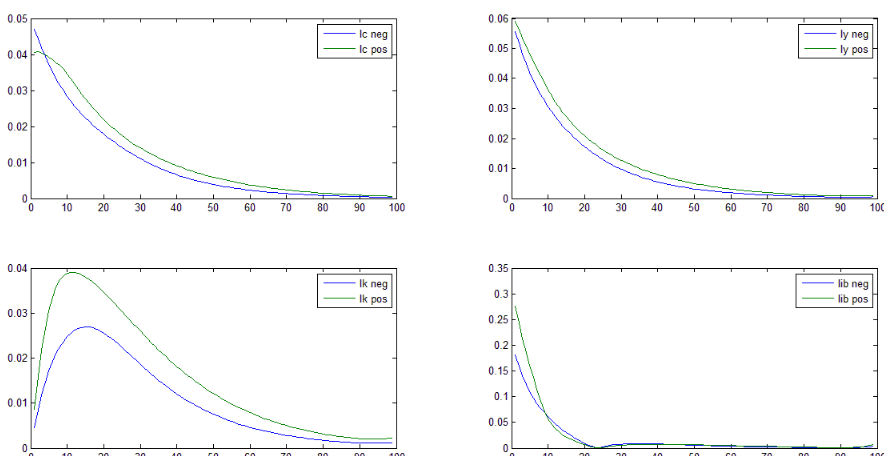
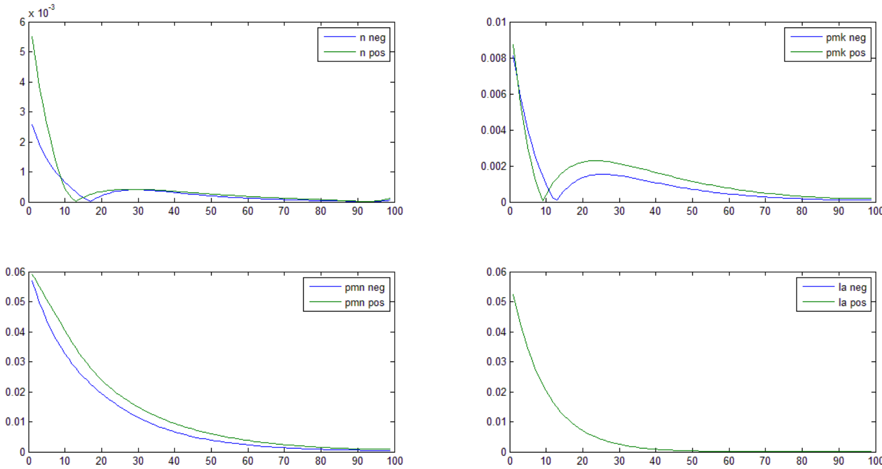


Figure 1.8: Deviations from the steady state



Wherein Y_t is a vector of variables, v_t, a current shock ω_{t-1} is the history, and n is the forecast horizon. Koop et al. (1996) also describe a simple algorithm to compute these conditional expectations by means of Monte Carlo integration. According to this method, GIRF could be considered as a distribution of impulse responses for each period in the forecast horizon. Impulse responses computed in this way are calculated and reported by Dynare. By default, Dynare throws the first 100 observations and reports GIRF for a horizon of 40 periods ahead.Figures 1.9 and 1.10 show impulse response (50 draws) for one standard deviation shock (positive and negative) on the perturbation term of the technology process; all variables except for labor and marginal products are considered as logarithms.

1.3.7 Conditioning on a particular shock

The first simulation exercise consisted of giving one standard deviation shock (positive and negative) to the technology process in the asymmetric investment cost model. The parametrization for this version of the model is the same as in table 1 fixing $\psi_1 = 4, \psi_2 = 1$ and $\gamma = 100$. The simulation was performed once (one replication); the response of macroeconomic variables in this hypothetical economy to negative shocks (in average) are asymmetric with respect to positive shocks (graphs 11 and 12 show the absolute values of responses of variables to negative shocks (blue) and to possible time shocks (green)). Replications of this exercise consisted in simulating 500 time series for the history of the model; this isω_{t-1} simulated 500 times. The economy was given the same standard deviation shock. Thus, the GIRF was computed as $GI_Y(n, v_t, \Omega_{t-1}) = E[Y_{t+n}|v_t, \Omega_{t-1}] - E[Y_{t+n}|\Omega_{t-1}]$ being Ω_{t-1} an information set of the previous history, and v_t a particular negative and positive standard deviation shock. Figures1.11 and 1.12 show these IR functions. The time paths for these impulse responses look softer, but this fact in no way affects the nature of the results. Figure 1.13 shows a Relative Intensity Indicator (RII), which means the ratio of impulse responses as shown in Figure 1.9; this is: impulse-response after positive shock divided by impulse-response after negative shock for each variable. If this indicator is greater than -1 and smaller than 0, negative shock is greater than the positive one; and the opposite occurs if the indicator is smaller than -1. On shock, negative impact on consumption, income, and labor are more intense than the positive impact, which is, however, more long-lasting than the negative, at least for consumption and income. For labor, negative effect is more intense and long-lasting. On the other hand, for capital positive shock, it is always more intense and long-lasting. This means that at short-term the adjustment is spread all over the variables, whereas at medium-term the adjustment is shared only between capital and labor.

Figure 1.9: GIRF (50 replicas)

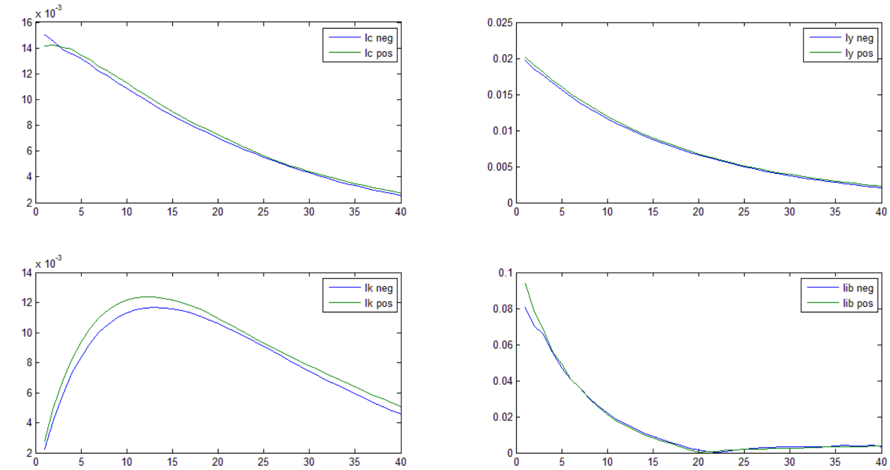


Figure 1.10: GIRF (50 replicas)

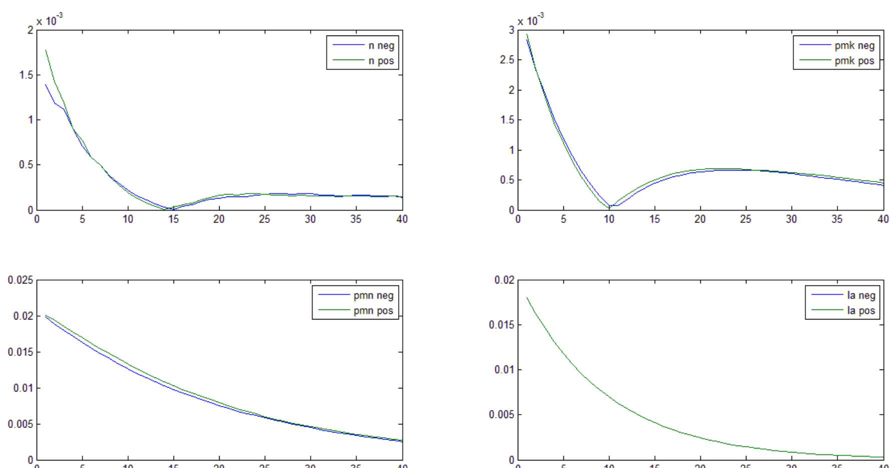


Figure 1.11: General Impulse Response Function (500 replicas)

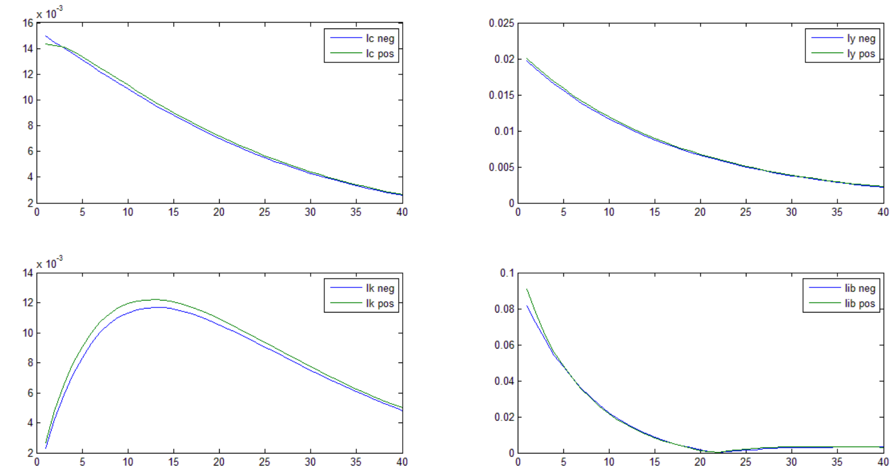


Figure 1.12: General Impulse Response Function (500 replicas)

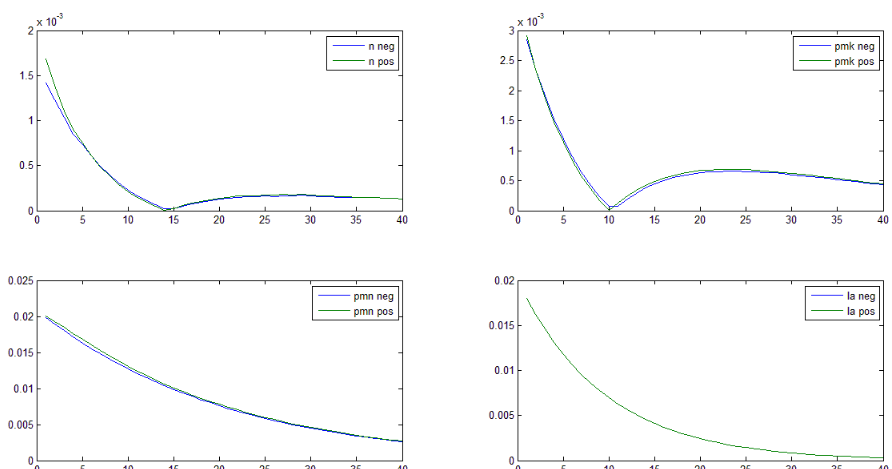
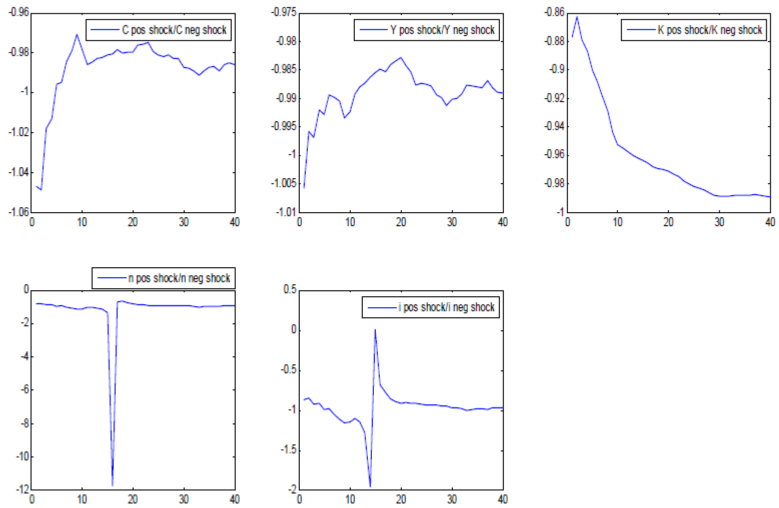


Figure 1.13: Relative Intensity Indicator



1.3.8 Conditioning on a particular history

Due to the fact that asymmetric models are history-dependent, it is necessary to ask ourselves the question on what the time path of the economy would be when in a boom that is positively or negatively shocked, or when in a recession that is positively or negatively shocked. The results of simulating a positive shock as the economy undergoes a boom or simulating a positive shock as the economy undergoes a recession are trivial: a recession deepening and boom sharpening. However, because of business cycles asymmetry, it would be necessary to perform the simulation in order to know the quantitative effects. In fact, it would be interesting to know the quantitative effects of a negative shock during a boom and a positive shock during a recession. To perform the exercise here proposed, it must be supposed that the economy is initially shocked (positively or negatively) in period one, and in period four it will receive a shock in the opposite direction to the one received in period one. Thus, the exercise deals with computing $GI_Y(n, v_t, \tilde{\Omega}_{t-1}) = E[Y_{t+n}|v_t, \tilde{\Omega}_{t-1}] - E[Y_{t+n}|\tilde{\Omega}_{t-1}]$ being $\tilde{\Omega}_{t-1}$ the state of the economy (either in boom or in recession) and v_t a positive or negative shock.

There is another important detail to consider: this exercise is time-dependent. This implies that the new position of the economy after the second shock would depend directly on how far it is from the steady state. That is to say, the longer the horizon of GIRF, the closer the economy will be to the steady state. Therefore, depending on the size of the shock (and on the economy's asymmetric structure) the economy could jump (suddenly perhaps) from a boom onto a recession, and vice versa. In order to standardize the timing problem, the exercise was performed as follows: the second (positive or negative) shock was introduced in a time $t0$ so that the technology gap were half of its initial value on shock. In this section, all variables have been measured in logarithms. In such a way, gaps between variables can be interpreted as log-deviations from the steady state.

1.3.8.1 A second shock in the opposite direction of the first shock

Figure 1.14 shows the GIRF of the economy after receiving a positive shock during a recession and a negative shock during a boom. In this exercise, it was very clear that a shock in the opposite direction pushes the economy to the next phase of the cycle, making it fall from a boom to a recession or making it jump from a recession to a boom. For the case of capital, it slowly reverses, nonetheless, the accumulation (deaccumulation) process induced by a positive (negative shock).

Figure 1.14: GIRF for the first and the second shock

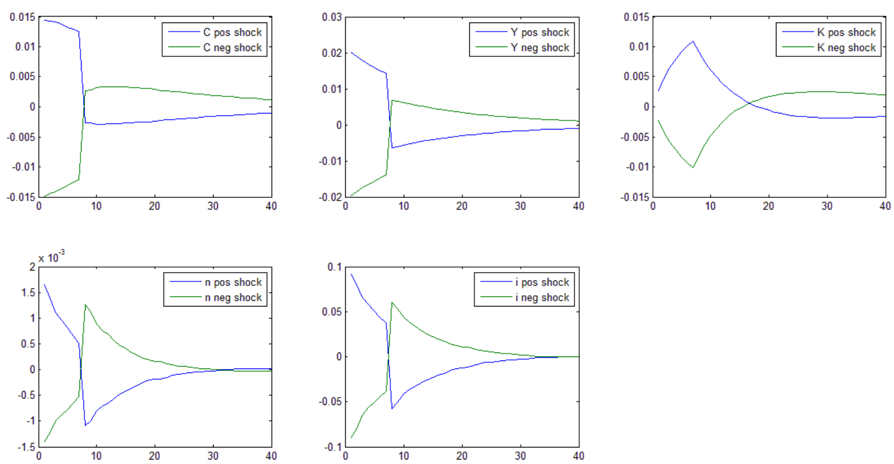


Table 1.14: Variation of the gap from the steady state after shocks

	Consumption		GDP		Capital		Investment		Labour		Leisure	
Period	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos	pos/neg	neg/pos
0	**1.443%**	**-1.498%**	**2.001%**	**-1.973%**	**0.262%**	**-0.224%**	**9.073%**	**-8.845%**	**0.338%**	**-0.285%**	**-0.330%**	**0.275%**
1	-0.012%	0.045%	-0.111%	0.116%	0.214%	-0.190%	-1.225%	1.134%	-0.060%	0.042%	0.059%	-0.041%
2	-0.034%	0.059%	-0.100%	0.102%	0.181%	-0.167%	-0.936%	0.786%	-0.040%	0.027%	0.039%	-0.025%
3	-0.024%	0.042%	-0.101%	0.101%	0.145%	-0.138%	-1.037%	0.996%	-0.046%	0.036%	0.046%	-0.035%
4	-0.032%	0.043%	-0.097%	0.095%	0.114%	-0.113%	-0.880%	0.806%	-0.039%	0.031%	0.038%	-0.030%
5	-0.043%	0.047%	-0.090%	0.088%	0.091%	-0.093%	-0.723%	0.747%	-0.029%	0.025%	0.028%	-0.024%
6	-0.042%	0.044%	-0.085%	0.084%	0.071%	-0.073%	-0.635%	0.632%	-0.026%	0.024%	0.026%	-0.023%
7	**-1.539%**	**1.505%**	**-2.055%**	**2.071%**	**-0.174%**	**0.194%**	**-9.299%**	**9.556%**	**-0.311%**	**0.341%**	**0.301%**	**-0.332%**
8	-0.003%	0.021%	0.041%	-0.038%	-0.152%	0.165%	0.641%	-0.724%	0.026%	-0.036%	-0.026%	0.035%
9	-0.006%	0.022%	0.041%	-0.041%	-0.129%	0.135%	0.724%	-0.792%	0.028%	-0.038%	-0.028%	0.038%
10	0.004%	0.004%	0.036%	-0.037%	-0.113%	0.115%	0.480%	-0.533%	0.020%	-0.025%	-0.019%	0.024%

Because the model is asymmetric, the intensity of the fall will be different from the intensity of the jump. Then, it is necessary to compare the time paths after the second shock. Figure 1.15 shows the path of the economy after the second shock. Nevertheless, it is not conclusive about the asymmetries and the intensity of the shock. Figure 1.16 shows absolute deviation values from the steady state after the second shock. Also, figure 1.17 shows the intensity indicator (absolute values). This reveals that, on shock, the negative shock effect during a boom is more intense than the one for consumption and income. Differently, the opposite takes place for labor investment and capital; besides, for medium-term effects of positive shock during recession, it seems to be more long-lasting.

Table 1.14 shows the variation of the gap after each shock. Gap variations when the economy is disturbed by a negative (positive) shock during a boom (recession), in absolute values, are greater only for consumption, whereas they are smaller for other variables; i.e., the pos/neg column is greater than the neg/pos column for consumption in period seven, while the opposite occurs for other variables. This takes place as investment decreases are more expensive than investment increases. As a consequence, consumption will suffer the major part of the adjustment on a negative shock. Hitherto, it could be concluded that during a recession the effect of a positive shock on the economy is more intense than the effect of a negative shock during a boom and this probably occurs because booms are more long-lasting than recessions.

Figure 1.15: GIRF for the second shock

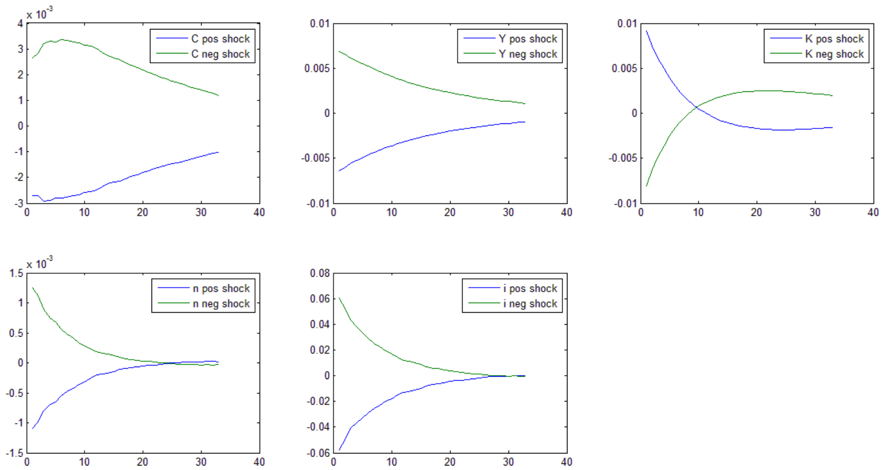


Figure 1.16: GIRF for the second shock (absolute values)

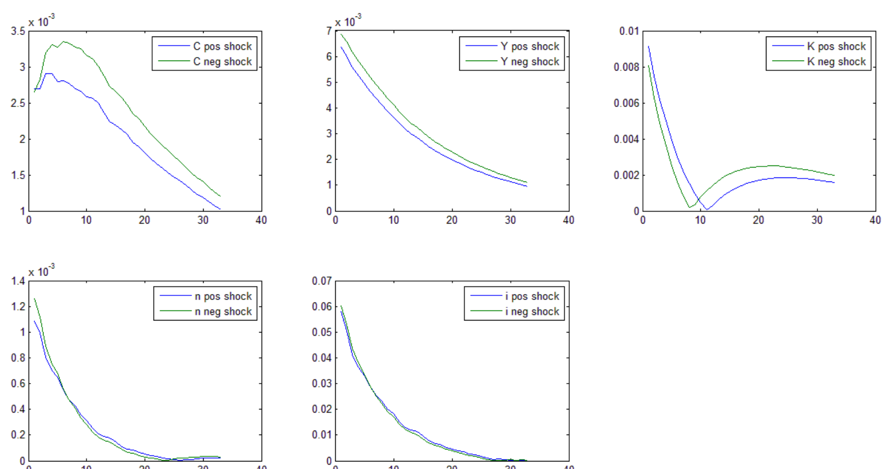


Figure 1.17: Relative Intensity Indicator for the second shock (absolute values)

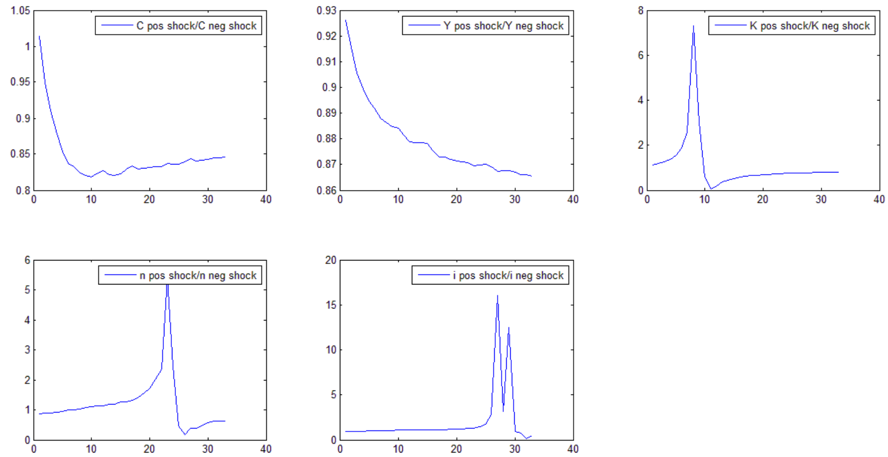


Table 1.15: Variation of the gap from the steady state after shocks

	Consumption		GDP		Capital		Investment		Labour		Leisure	
Period	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg	pos/pos	neg/neg
0	**1.428%**	**-1.477%**	**2.009%**	**-1.984%**	**0.270%**	**-0.236%**	**9.433%**	**-10.045%**	**0.351%**	**-0.302%**	**-0.343%**	**0.292%**
1	-0.004%	0.033%	-0.110%	0.116%	0.220%	-0.197%	-1.373%	1.935%	-0.064%	0.049%	0.063%	-0.047%
2	-0.015%	0.039%	-0.107%	0.109%	0.177%	-0.164%	-1.174%	1.170%	-0.056%	0.042%	0.055%	-0.041%
3	-0.027%	0.042%	-0.102%	0.102%	0.141%	-0.135%	-1.105%	1.120%	-0.045%	0.036%	0.044%	-0.034%
4	-0.038%	0.047%	-0.096%	0.094%	0.113%	-0.112%	-0.806%	0.750%	-0.035%	0.028%	0.035%	-0.028%
5	-0.042%	0.046%	-0.090%	0.089%	0.090%	-0.091%	-0.699%	0.694%	-0.030%	0.026%	0.029%	-0.026%
6	-0.047%	0.048%	-0.084%	0.083%	0.071%	-0.074%	-0.608%	0.611%	-0.023%	0.021%	0.022%	-0.020%
7	**1.380%**	**-1.449%**	**1.931%**	**-1.897%**	**0.327%**	**-0.283%**	**8.721%**	**-8.811%**	**0.333%**	**-0.265%**	**-0.327%**	**0.256%**
8	-0.053%	0.093%	-0.182%	0.188%	0.264%	-0.237%	-1.694%	1.781%	-0.079%	0.056%	0.077%	-0.054%
9	-0.049%	0.087%	-0.181%	0.178%	0.202%	-0.193%	-1.605%	1.255%	-0.080%	0.055%	0.079%	-0.054%
10	-0.076%	0.092%	-0.168%	0.165%	0.158%	-0.157%	-1.299%	1.388%	-0.055%	0.044%	0.054%	-0.042%

1.3.8.2 A second shock in the same direction of the first shock

We might also wonder about the effect of a positive shock during a boom or about the effect of a negative shock during recession. To answer these questions, we have performed an excercise similar to the previous one. But, instead of giving a negative shock after a positive one, we give both a first and a second positive shocks. A first negative shock and a second negative shock are also simulated. Figures 1.18 to 1.20 show that when the economy is disturbed by a second positive shock, the boom regime protracts and the recession regime exacerbates.

This qualitative effect is expected, but what really concerns us here is its magnitude. Table 1.15 shows the size of the increase (decrease) of the gaps after the second positive (negative) shock. When the economy is in a boom and receives a positive perturbation (columns pos/pos), the variation value of the consumption gap in period 7 (in absolute values) is smaller than that when the economy is in a recession and receives a negative perturbation (columns neg/neg). For the other variables, the completely opposite case takes place. One more time, the explanation for this behavior is that the booms are more long-lasting than recessions. Also, because decreasing investment is more expensive than increasing it, the most part of the adjustment on shock relies on consumption.

Figure 1.18: GIRF for the first and the second shocks (absoloute values)

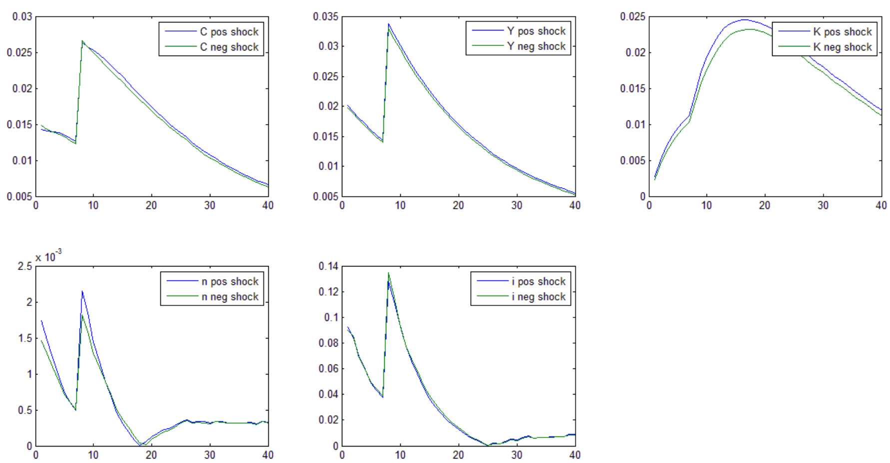


Figure 1.19: RII for the first and the second shocks

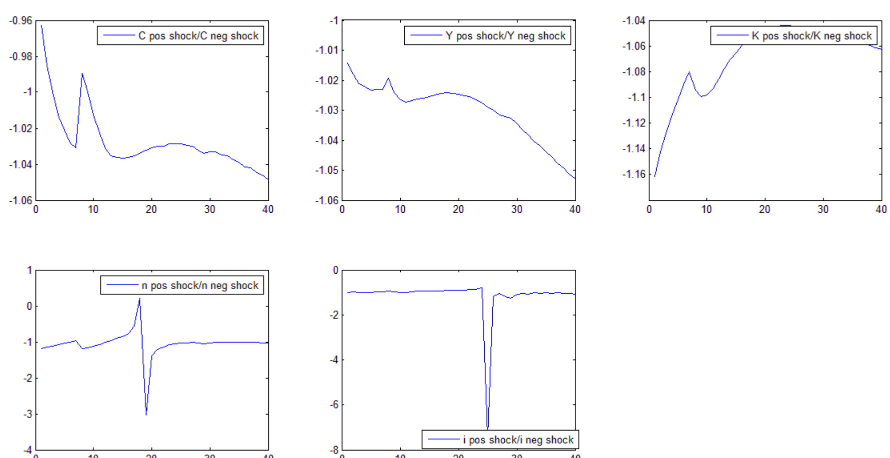
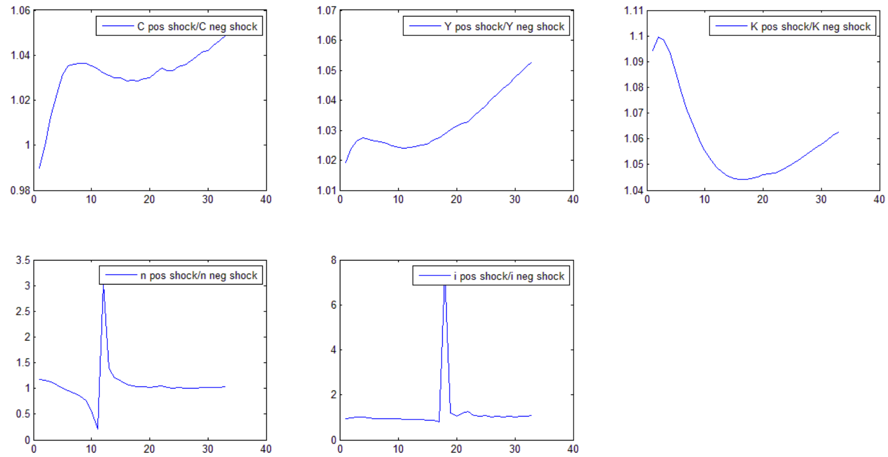


Figure 1.20: RII for second shock (absolute values)



1.4 Conclusions

The DSGE model proposed here with asymmetric investment costs is able to generate asymmetric business cycles.

In general, recessions seem to be deeper (for consumption) than expansions and expansions seem to be more long-lasting than recessions. Thus, deepness and sharpness would be captured by this model's dynamics.

The adjustment intensity suffered by consumption and labor, with a smaller reaction in wages during recession and a greater increase in wage during booms, is an indicator that there is real rigidity on wages.

Asymmetries in RBC models could be more adequately captured by General Impulse Response Functions than by higher order moments. However, a more rigorous test for the properties of the asymmetric model proposed here could include the application of nonlinear econometric tools that could serve indeed a powerful tool for this purpose.

For the agenda: i) estimate parameters of the model for the real economy; ii) test for asymmetries in the time series and simulate with these nonlinear econometric models.

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1.5 Appendix

1.5.1 Log-linearizing the model

When we linearize the full model including the asymmetric cost function of investment, we have as well as in the linearization regime by regime, a set of linear equations and the nonlinear and asymmetric dynamics of the theoretical model originally constructed disappears.

$$-\theta\hat{c}_t = \hat{\lambda}_t \tag{1.62}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t - \varphi\varphi_t \tag{1.63}$$

$$y\hat{y}_t = Ak^{\alpha}\hat{A}_t + \alpha Ak\hat{k}_t^{\alpha} \tag{1.64}$$

$$\varphi\hat{\varphi}_t = \varphi_2\hat{\varphi}_{2t} + \phi\hat{\phi}_t\left(\varphi_1 - \varphi_2\right) + \phi\left(\varphi_1\hat{\varphi}_{1t} - \varphi_2\hat{\varphi}_{2t}\right) \tag{1.65}$$

$$\varphi_1\hat{\varphi}_{1t} = \psi_1\left(k_{t+1} - k_t\right)k\hat{k}_{t+1} - \psi_1\left(k_{t+1} - k_t\right)k\hat{k}_t \tag{1.66}$$

$$\varphi_2\hat{\varphi}_{2t} = \psi_2\left(k_{t+1} - k_t\right)k\hat{k}_{t+1} - \psi_2\left(k_{t+1} - k_t\right)k\hat{k}_t \tag{1.67}$$

$$\phi\hat{\phi}_t = \frac{\gamma exp(-\gamma(k-k))}{\left[1 + exp(-\gamma(k-k))\right]^2}k\hat{k}_{t+1} - \frac{\gamma exp(-\gamma(k-k))}{\left[1 + exp(-\gamma(k-k))\right]^2}k\hat{k}_t \tag{1.68}$$

$$\begin{aligned}
&\lambda\hat{\lambda}_t + \lambda\frac{\partial\varphi(x)}{\partial k_{t+1}}\left(1 + \eta_1\hat{k}_{t+1}\right) + \lambda\frac{\partial\varphi_t(x)}{\partial k_{t+1}}\left(1 + \eta_2\hat{k}_{t+1}\right) \qquad (1.69)\\
&= \beta\lambda\hat{\lambda}_{t+1}\left[(1-\delta) + f'(k_{t+1}) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\right] + \beta\lambda\left[\begin{array}{c} f'(k)\left(1 + \eta_3\hat{k}_{t+1}\right) - \frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1 + \eta_4\hat{k}_{t+1}\right) \\ -\frac{\partial\varphi_{t+1}(x)}{\partial k_{t+1}}\left(1 + \eta_5\hat{k}_{t+2}\right) \end{array}\right]\\
&+ \beta\lambda f'(k)\left(1 + \eta_6\hat{A}_{t+1}\right)
\end{aligned}$$

This system is the same as the one we have when there is no asymmetries in the cost of investment. thus, nonlinear behavior of investment is not captured when using first order Taylor approximations. Thus we need a different numerical method to simulate and test this model.

Chapter 2

The Role of Loss Aversion

2.1 Introduction

Empirical evidence has cast doubts about the relevance of Life Cycle/Permanent Income Hypothesis (LCH/PIH) to explain the dynamics of consumption. According to this theory, the only variables determining variations in consumption are interest rate and shifts in preferences, which means that consumption should not respond to changes and expected income. In econometric terms, the previous affirmation means that in a regression of interest rate and with variables related to expectations of future income on consumption growth, the null of LCH/PIH will imply a zero vector for the expected income variables. However, empirical evidence rejects most of the times LCH/PIH. In this line, the most important and inspiring paper to uphold this chapter is Shea's (1995). He cites some papers that find rejections of the LCH/PIH: Campbell and Mankiw (1990) found a statistically significant relationship between predictable income and increases in consumption; in Zeldes (1989), the empirical evidence rejects LCH/PIH, which is attributed to liquidity constraints; Flaving (1991) also rejects LCH/PIH stating that this is caused by myopic behavior of agents. By using data of unionized family heads (for the U.S), Shea (1995) has found three possible problems leading to the rejection of LCH/PIH: i) Panel Study of Income Dynamic (PSID) is mostly used to cover food consumption, ii) PSID is mostly interested in labor-market behavior rather than in consumption behavior, and iii) it is difficult to find variables in households information sets as good predictors of future income growth[1]. He also proposes to find correct instruments for expectations of future income to test the null hypothesis of LCH/PIH and the null of liquidity constraints. His hypothesis tests lead to rejecting the permanent income hypothesis as well as the liquidity constraint hypothesis. Moreover, not only do his findings imply that increases in consumption are related to expected income, but also to an asymmetric reaction of consumption growth: consumption reacts more strongly to expected decreases than to income increases, which is more explainable, at least qualitatively, by loss aversion. In table 1, I will show here table 4 as in Shea (1995).

Previous results by Shea (1995) for the U.S. economy were confirmed in 1995 and 1999 by Bowman, Minehart and Rabin (1995; 1999). In 1995, they formally showed the properties of more general utility functions with loss aversion for two periods. In 1999 and based on their analytical results, they tested econometrically for five OECD countries (United States, Japan, Germany and France) the existence of loss aversion in consumption by using a version of the growth consumption equation proposed and estimated by Shea (1995). Bowman, Minehart and Rabin (1995; 1999) included dummy variables to capture differences in slope for negative and positive expected income variations. This means that before an expected income decrease, there is a stronger reaction of individuals than facing an expected income increase. The upper panel of table 2 is table 1, as reproduced from Bowman et al. (1999), shows that expected income changes are correlated to consumption growth, and the lower panel shows the estimation of the model taking into account the inclusion of dummy variables for increases and decreases in expected income. The estimated parameters (and hypothesis testing of equality between) λ_1 and λ_2[2] reveal that consumption reacts more

[1] Shea (1995) pag. 187.

[2] These are the coefficients for expected increase and expected decrease in income repectively, in an ordinary least squares

Table 2.1: Evidens of asymmetry in consupmtion, Shea (1995)

GC_t = Time Dummies + $\gamma GAFN_t + \sigma \log(1+r_t) + \beta(EDWAGE) + \varepsilon_t$					
Variable	(i)	(ii)	(iii)	(iv)	(v)
GAFN	0.154 (0.192) [0.802]	0.486 (0.121) [4.017]	0.226 (0.187) [1.209]	0.388 (0.093) [4.172]	0.294 (0.143) [2.056]
$\log(1+r_t)$	0.631 (2.098) [0.301]	2.702 (2.007) [1.346]	0.756 (1.675) [0.451]	4.748 (2.876) [1.651]	1.720 (1.407) [1.222]
EDWAGE	0.997 (0.703) [1.418]	0.765 (0.539) [1.419]	0.961 (0.614) [1.565]	0.867 (0.593) [1.462]	—
Positive EDWAGE	—	—	—	—	0.063 (0.785) [0.080]
Negative EDWAGE	—	—	—	—	2.242 (0.951) [2.358]
R^2:	0.010	0.067	0.014	0.121	0.011
Number of observations:	372	275	475	172	647
Split:	Zero wealth	Positive wealth	Low ratio	High ratio	Full sample

Source: Taken from Shea (1995), pp. 195.

strongly to predictable income decreases than to predictable increases in it.

Shea (1995) and Bowman et al. (1999) agree that prospect theory can provide a powerful explanation for the asymmetry response of consumption to expected variations in income. They strongly point out the importance and need for developing other research lines by using loss aversion and by taking into account dynamic models of more than two periods. Bowman et al. (1999) express that "loss aversion can usefully be incorporated into areas of economic research other than consumption and saving" (p. 168) and also noted: "Formal modeling along the lines developed in this paper may help researchers begin to systematically investigate the implications of loss aversion in a wider array of economic situations" (p. 168). On the same order, Shea (1995) states that "further research should investigate the implications of loss aversion for the dynamic behavior of consumption in more general settings and should attempt to derive additional testable implications of loss aversion beyond asymmetric rejection of the LCH/PIH" (p. 199).

Prospect theory and functional forms including loss aversion have been hardly used in DSGE research, although there are some interesting papers intended to explicitly model loss aversion and test its presence in macroeconomic time series. Rosenblatt-Wisch (2005) had introduced loss aversion in a traditional Ramsey model calibrated for the steady state. This author also estimated parameters by GMM and has tested the null hypothesis of loss aversion in the macroeconomic time series (on U.S data) (2008). Her results have been for loss aversion. Foellmi, Rosenblatt-wisch and Schenk Hoppé (2010) have found that when agents have loss aversion, their consumption path is smoother and the economy could stay in a poverty trap which can be explained by a sub-accumulation of physical capital. **However, these works use linear utility functions, exclude labor, and do not perform impulse response exercises either simulation or comparison of theoretical and sample moments.** Gaffeo et al. (2010) in a DSGE framework, employ loss aversion to study the asymmetrical responses of output and prices for the monetary policy. However, their aim was different from explaining the asymmetries of macroeconomic time series along the business cycle. The utility function in their work had additive separable labor decisions in a concave function, while the part of consumption was a convex combination of a neoclassical utility function and an exponential

regression, as the one in table 1

Table 2.2: Evidence of asymmetry in consumption, Bowman, Minehart and Rabin (1999)

Table 1
Estimates of the model

Country	Sample period	$\bar{R}_c^2$	$\bar{R}_y^2$	λ
Canada	1972 : 4–94 : 1	0.271	0.161	0.497 (5.31)
France	1971 : 2–93 : 2	0.117	0.065	0.293 (1.97)
West Germany	1961 : 2–90 : 2	0.005	0.048	0.592 (2.48)
Japan	1971 : 2–93 : 1	−0.007	−0.037	−0.269 (−0.89)
United Kingdom	1956 : 2–93 : 3	0.109	0.012	0.423 (2.63)

Table 2
Estimates of the model

Country	Obs.	Quarters			p-value
		$\Delta \ln\hat{y}_t < 0$	λ_1	λ_2	$\lambda_1 = \lambda_2$
Canada	86	21	0.270 (1.59)	1.128 (3.14)	0.067
France	78	13	0.046 (0.22)	1.045 (2.33)	0.080
West Germany	114	5	0.412 (1.91)	3.805 (2.12)	0.074
Japan	75	0	−0.269 (−0.89)	–	–
United Kingdom	118	13	0.356 (1.58)	0.651 (1.24)	0.649
Panel	471	52	0.155 (3.79)	1.136 (5.21)	0.003

Source: taken from Bowman et al (1999), pages 166 and 167.

gain-loss function, which satisfies concavity for gains and convexity for losses, as proposed by KT (1979). The set up of their utility functions excludes smooth transition, which means that the authors must perform separated simulations for each of the regimes of the model, and need to model a two-state Markov chain stochastic process for simulations. Notwithstanding, the model succeeds at reproducing the documented empirical regularities of output responses more strongly to monetary policy during recessions than during booms. But for the case of inflation responses, it does not seem to present differences during recessions and booms.

Up to now, to the best of my knowledge, prospect theory has been mostly applied in consumption-based asset pricing models and has been applied to finance. Andries (2011) has redefined preferences by using loss aversion in order to study asset pricing. Her model, which includes loss aversion, performs more efficiently than the recursive utility model as it explains the excess of returns varying with skewness of returns distribution. This model also captures an effect level on the risk-free returns assets. Han and Hsu (2004) have documented the work of researchers by using prospect theory in financial theory. Regarding the disposition effect, they cite Odean (1998), Grinblatt and Keloharju (2001), Heath, Huddart and Lang (1999), Shapita and Venezia (2001), Garvey and Murphy (2004), thus finding a tendency to sell papers experiencing gains while not selling papers experiencing losses. Loss aversion explains this behavior: if the agents sell losing papers they will realize such a loss, which is not desirable for agents; thus the utility is convex for losses, implying that they take the risk of keeping those papers until experiencing gains. Home bias is another fact explainable through prospect theory. Home bias is a tendency to hold domestic stock in a higher share proportion than the international stock share. This contradicts the results implied by the mean-variance framework (Stracca, 2002). Equity premium puzzle, discovered by Mehra and Prescott (1985), is also analyzed by means of prospect theory. In neoclassical models, risk aversion coefficient should be 30 in order to explain such a phenomenon, while the empirical evidence suggests a value of 1 for this parameter. According to Benartzi and Thaler (1995), loss aversion helps to explain equity premium by means of the attractiveness of a risky asset. This will depend on the planning horizon of the investor: the more frequently the investor evaluates his portfolio, the more likely he experiences losses, and this will imply loss aversion in such a way the investor will demand a higher return in order to hold riskier assets.

Given the state of art of RBC models and the cited application of prospects theory, the goal of this chapter is to build a DSGE model whose core is the inclusion of prospects theory utility function in order to capture the asymmetric behavior of agents along the different phases of a business cycle. The original expression of the prospects utility function is kinked in the reference point (Zero for the original proposal of T-K (1979)), which makes it non-differentiable at that point. Additionally, the prospects utility is originally defined by monetary gains and losses rather than consumption and leisure as it is commonly defined. Thus, I propose three modifications of the prospect utility function. Firstly, I defined it on an aggregator of consumption and leisure. Secondly, I redefined the reference point in such a way that for consumption there is a weighed average of the reference point for consumption in the previous period and consumption in the previous period as well. For leisure, the reference point is analogously defined. Thus, the utility function argument is a consumption and leisure aggregator and its reference point is defined as an aggregator of reference points for consumption and leisure respectively. The utility function is then defined as the consumption-leisure bundle divided by the bundle of reference points for consumption-leisure. Then, when this ratio is greater than one, the agent has gains (and is risk averse); when it is lower than one, the agent has losses (and is loss averse). Thirdly, in order to get differentiability of the utility function, I defined a smooth transition function (by using a logistic function) whose threshold is one. The importance and contribution of this work is extending knowledge of prospect theory utility function, which is a general form that nests loss aversion, risk aversion and habits formation. The simulation of the DSGE model proposed here reveals that loss aversion is a mechanism of transmission suitable to explain asymmetries in business cycles. In Section 2, I present the basic properties of prospect theory utility function. Section 3 presents the construction of prospects utility for consumption and leisure. In section 4, I discuss the reference point formulation. In Section 5, I present the a Prospect Theory-DSGE model. Section 6 has the first order and equilibrium conditions. Section 7 discuses the uniqueness of equilibrium in a model with Prospects Theory utility. Section 8 presents the calibration. Section 9 displays both deterministic and stochastic simulations results. Section 10 deals with conclusions.

2.2 Prospect theory utility function: Basic properties.

The properties and deduction of the "kinked" utility function were initially derived by T-K (1979) y T-K (1992) who, based on experimental data, discovered the violation of expected utility assumptions. Bowman et al. (1999) have formalized the properties of a utility function based on loss aversion, have developed a consumption-savings model, and have made estimations to test the existence of loss aversion in consumption savings for Canada, France, West Germany, Japan, United States and for the panel. The results were supportive for loss aversion. They also proposed how to model the reference point. Köbberling and Wakker (2005) formalized an index of loss aversion and derived implications for parametric forms of utility functions based on itself. Koszegi and Rabin (2006) developed a model of reference dependent preferences focused on determining the reference point, which has been a very controversial element of Prospect theory. Insofar, they found that the reference point must be rational expectations determined.

According to T-K (1979) and T-K (1992), when agents face the possibility of random losses, expected utility is an inappropriate descriptor of consumer behavior. Moreover, T-K (1979; 1992) argue that the agent is more sensitive to losses than to gains. This means that losing a quantity of x generates a disutility greater than the utility of wining x. Thus, while for gains the agent will prefer the certainty equivalent of the uncertain bundle (concavity), the certainty equivalence is not preferred for losses. This is, the agent behaves as if he were risk-loving (convexity). Thus, T-K (date) derived the basic properties of Prospect theory utility function, which are summarized as follows:

Let suppose that $u(x)$ is a concave function and $v(-x)$ is a convex function; if $x > 0$, is a gain, $-x < 0$ is a loss, then:

1. The utility of a gain is positive $u(x) > 0$;the utility (disutility) of a loss is negative $v(-x) < 0$. $u(0) = v(0) = 0$

2. The magnitude of the disutility for lossing x is greater than the magnitude of the utitility for gaining $x : |v(-x)| > u(x) \Rightarrow \frac{|v(-x)|}{u(x)} > 1$

3.$v'(-x) > 0, u'(x) > 0$, and $v'(-x) > u'(x) \Rightarrow \frac{v'(-x)}{u'(x)} > 1$

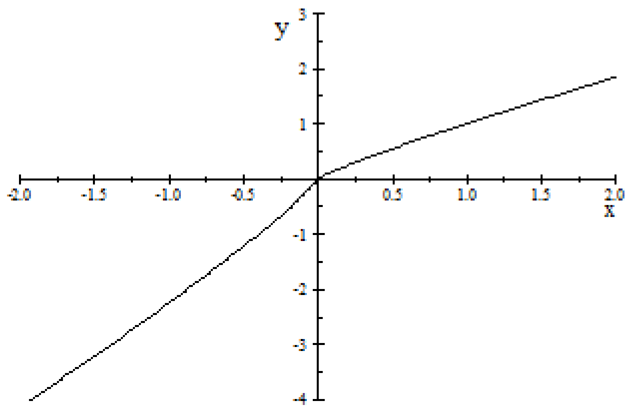
4.$v''(-x) > 0, u''(x) < 0$

Based on these properties, T-K (1992) has formally proposed the following utility function:

$$U(x) = \begin{Bmatrix} u(x) = x^{\alpha}, if\ x \geq 0 \\ v(x) = -\lambda(-x)^{\beta}, if\ x < 0 \end{Bmatrix} \tag{2.1}$$

In the figure 2.1 below, I use the parameters $\alpha = 0.88, \beta = 0.88$ and $\lambda = 2.25$ as estimated by T-K (1992).

Figure 2.1: Prospects utility function, Kahneman and Tversky (1979)



There are two additional characteristics of this utility function that are defined in this way: i) it is kinked (and consequently non-differentiable) in $x = 0$; and ii) as it is defined on gains and losses, its reference point is always the same: $x = 0$. I will deepen into the controversial problem of the reference point in section 4, and I also will deal further with the kink of the function exposed in section 3.

In accordance with Köbberling and Wakker (2005), KW (hereafter), the original formulation by TK (1992) has an implicit scaling convention: because the reference point is $x = 0$, their function implies that $u(1) = v(-1) = 1$, which also would imply $\lambda = 1$. Thus, they propose a scaling untied to the unit of payments, which would also imply that the utility function may be differentiable at the reference point. They define the *loss aversion index* as: $\lambda = \frac{v'_{\uparrow}(0)}{u'_{\downarrow}(0)}$. $v'_{\uparrow}(0)$ and $u'_{\downarrow}(0)$ are left and right derivatives respectively supposing that those derivatives do exist as positives and finites. As this loss aversion index is independent from the unit or/of? payments, it is the same for different countries and needs no adjustment (Köbberling and Wakker, 2005, p. 125). Theorem 1 of KW (2005) applies for any index of loss aversion for $\frac{v(\tau)}{u(\tau)}$ for $\tau > 0$ fixed (p. 125).

2.3 Prospect theory utility function for consumption and leisure

Because the original prospect theory was at first built on losses and gains of wealth, it is centered in zero. However, it is possible to re-define the utility function so that basic properties are fulfilled in order to change the reference point, the payment units, thus conserving a loss aversion utility function with desirable properties as stated by KT (1992) and KW (2005).

The purpose of this section is to build a "general" prospect utility function for consumption and leisure which can be used in a DSGE framework. Thus, we are looking for a utility function that fulfills both loss aversion properties as well as desirable properties for RBC models such as those proposed by King, Plosser, and Rebelo (2001) (see technical appendix for **"Production, growth and business cycles"**). But first, we must deal with the issue of the kink of the utility function when evaluated in the reference point. First, let us assume that the utility function satisfies the properties discussed so far. Then, we will define $U(c, l, r^c, r^l)$ as the utility function derived from consumption and leisure (c, l) and (r^c, r^l) as the reference points for them. Moreover, we will assume for the moment that utility is only derived from consumption:

$$U(c_t, r_t^c) = \left\{ \begin{array}{l} \overline{u}(c_t, r_t^c), \text{ for gains respect to } r_t^c \\ \underline{u}(c_t, r_t^c), \text{ for losses respect to } r_t^c \end{array} \right\} \tag{2.2}$$

$\overline{u}(c_t, r_t^c)$ and $\underline{u}(c_t, r_t^c)$ are such that:

$$\frac{\partial \overline{u}(.)}{\partial c_t} > 0, \frac{\partial^2 \overline{u}(.)}{\partial c_t^2} < 0, \text{ for gains} \qquad (2.3)$$
$$\frac{\partial \underline{u}(.)}{\partial c_t} > 0, \frac{\partial^2 \underline{u}(.)}{\partial c_t^2} > 0, \text{ for losses}$$

In order to build a stable DSGE model, the utility function must be consistent with a balanced growth path as demonstrated by King, Plosser and Rebelo (2001), which means that it must have a constant relative risk aversion:

$$R(c) = -c\frac{\overline{u}''(.)}{\overline{u}'(.)} = \overline{\sigma} \qquad (2.4)$$

And for the loss-averse part of the utility function, we can also require and define analogously the constant relative loss aversion coefficient:

$$L(c) = c\frac{\underline{u}''(.)}{\underline{u}'(.)} = \underline{\sigma} \qquad (2.5)$$

Power functions like CRRA fulfill the properties requested for risk averse behavior when agent experiences gains:

$$\overline{u}(c_t, r_t^c) = \frac{(\psi(c_t, r_t^c))^{\overline{\theta}}}{\overline{\theta}} \qquad (2.6)$$

When the agent experiences loss, the utility function will be:

$$\underline{u}(c_t, r_t^c) = \frac{\lambda\,(\psi(c_t, r_t^c))^{\underline{\theta}}}{\underline{\theta}} \qquad (2.7)$$

Being $\psi(c_t, r_t^c)$ a function in c_t and r_t^c. I will visit this point in the discussion of the reference point. Parameters values for $\underline{\theta}$, $\overline{\theta}$, and λ are such that the conditions 1-4 for loss aversion and the definition loss aversion index are fulfilled.

One of the most important features of prospect theory utility function, originally proposed by TK (1979), is its kink in the reference point, which makes the function non-differentiable at such point. To deal with the problem of non-differentiability, one of my contributions in this chapter is to include a smooth transition function between two states or regimes, for instance, boom and recession. Smooth transition functions are mostly used in nonlinear econometrics to model transitions between regimes (references here). Let $\phi_t \in [0, 1]$ be such that (figure 2.2,$\gamma = 5$):

$$\phi_t = \frac{1}{1 + \exp(\gamma(x))} \qquad (2.8)$$
$$x \to -\infty \Rightarrow \phi_t \to 1$$
$$x \to \infty \Rightarrow \phi_t \to 0$$
$$x \to 0 \Rightarrow \phi_t \to 0.5$$
$$\gamma \to \infty \Rightarrow \phi_t \ step\ function$$
$$\gamma \to 0 \Rightarrow \phi_t \to 0.5$$

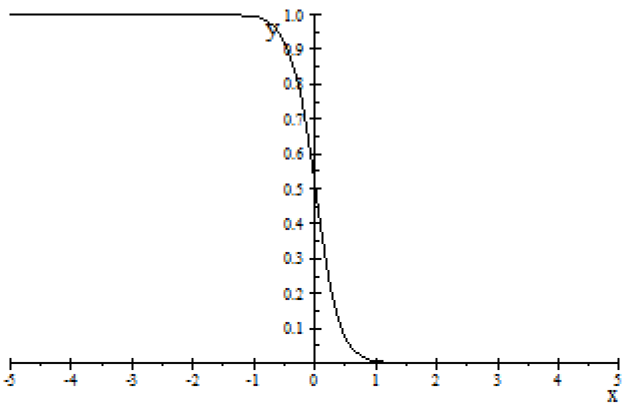
Thus, if we define

$$\phi_{ct} = \phi(c_t, r_t^c) = \frac{1}{1 + \exp(\gamma(\tau(c_t, r_t^c)))} \qquad (2.9)$$

The utility function with loss aversion and smooth transition is:

$$U_{L,S}(c_t, r_t^c) = \phi_{ct}\underline{u}(c_t, r_t^c) + (1 - \phi_{ct})\overline{u}(c_t, r_t^c) = \phi_{ct}\frac{\lambda\,(\psi(c_t, r_t^c))^{\underline{\theta}}}{\underline{\theta}} + (1 - \phi_{ct})\frac{(\psi(c_t, r_t^c))^{\overline{\theta}}}{\overline{\theta}} \qquad (2.10)$$

Figure 2.2: Smooth transition function



In the limit, if γ is large enough, ϕ_{ct} becomes a step function and $U_{L,S}(c_t, r_t^c)$ will be kinked, but still differentiable at the reference point. Similarly, I will define a smooth transition loss aversion utility function only for leisure:

$$\phi_{lt} = \phi(l_t, r_t^l) = \frac{1}{1 + \exp(\gamma(\tau(l_t, r_t^l)))} \tag{2.11}$$

$$U_{L,S}(l_t, r_t^l) = \phi_{lt}\underline{u}(l_t, r_t^l) + (1 - \phi_{lt})\overline{u}(l_t, r_t^l) = \phi_{lt}\frac{\lambda\left(\psi(l_t, r_t^l)\right)^{\underline{\mu}}}{\underline{\mu}} + (1 - \phi_{lt})\frac{\left(\psi(l_t, r_t^l)\right)^{\overline{\mu}}}{\overline{\mu}} \tag{2.12}$$

In a more general form, if we have an additive separable utility function of leisure and consumption, we will have:

$$U(c, l, r^c, r^l) = \phi_{ct}\underline{u}(c_t, r_t^c) + (1 - \phi_{ct})\overline{u}(c_t, r_t^c) + \phi_{lt}\underline{u}(l_t, r_t^l) + (1 - \phi_{lt})\overline{u}(l_t, r_t^l) \tag{2.13}$$

If there was not any additive separability, the utility function would be:

$$U(c, l, r^c, r^l) = \phi_{ct}(\underline{u}(c_t, r_t^c))^{\omega}(\underline{u}(l_t, r_t^l))^{(1-\omega)} + (1 - \phi_{ct})(\overline{u}(c_t, r_t^c))^{\upsilon}(\overline{u}(l_t, r_t^l))^{(1-\upsilon)} \tag{2.14}$$

2.4 The reference point

The reference point is perhaps the most controversial element of prospect theory and reference-dependent preferences. The initial proposal by TK (1979) focused on gains and losses of wealth indicate that the reference point was zero. Bowman et al. (1999) originally proposed that the reference point should be a convex combination of both lagged reference point and past consumption. However, if I define the reference point in this same way, and given that I have power functions (CRRA and CRLA), the immediate consequence is that in the steady state the argument of the utility function will be zero and the marginal utilities will be not defined at that point. This is a problem because the steady state will not be defined as well. To deal with this problem, Bowman et al. (1999) propose for a two-period model a utility function as follows:

$$U(c, r) = \begin{Bmatrix} wr + \frac{(b_g + c - r)^{1-\gamma}}{1-\gamma}, if\ c > r \\ wr - \frac{(b_l + c - r)^{1-\lambda}}{1-\lambda}, if\ c \leq r \end{Bmatrix} \tag{2.15}$$

$$r_2 = (1 - \alpha)r_1 + \alpha c_1 \tag{2.16}$$

Thus, for a multi-period model, if we write $r_t = (1 - \alpha)r_{t-1} + \alpha c_{t-1}$ and replace into the utility function:

$$U(c_t, r_t) = \begin{Bmatrix} wr_t + \frac{(b_g + c_t - (1-\alpha)r_{t-1} - \alpha c_{t-1})^{1-\gamma}}{1-\gamma}, if\ c > r \\ wr_t - \frac{(b_l + c_t - (1-\alpha)r_{t-1} - \alpha c_{t-1})^{1-\lambda}}{1-\lambda}, if\ c \leq r \end{Bmatrix} \tag{2.17}$$

Which is a prospect theory utility function; it also generalizes a utility function with habits formation and wr_t is the utility derived from consuming the reference point. In this model of multiple periods by imposing the steady state condition, it results: $r = c$ and

$$U(c,r) = \begin{Bmatrix} wc + \frac{(b_g)^{1-\gamma}}{1-\gamma}, if\ c > r \\ wc - \frac{(b_l)^{1-\lambda}}{1-\lambda}, if\ c \leq r \end{Bmatrix} \tag{2.18}$$

if we impose that (which is not a condition required for Bowman's model) $wc + \frac{(b_g)^{1-\gamma}}{1-\gamma} = wc - \frac{(b_l)^{1-\lambda}}{1-\lambda} \Rightarrow \frac{(b_g)^{1-\gamma}}{1-\gamma} = -\frac{(b_l)^{1-\lambda}}{1-\lambda} \Rightarrow b_g = \left[\left[-\frac{(b_l)^{1-\lambda}}{1-\lambda}\right](1-\gamma)\right]^{\frac{1}{1-\gamma}}, 0 < b_l$

2.4.1 The alternatives of modeling the utility

2.4.1.1 In the way of Bowman et al.

In a similar way, by defining

$$\widetilde{c_t} = c_t - \overline{c}_t \tag{2.19}$$

Where

$$\overline{c}_t = \vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - c^*), \text{ is the reference point} \tag{2.20}$$

and c^**is not necessarily the consumption in the steady state.*

$$U(c_t, \overline{c}_t) = \begin{Bmatrix} \frac{(c-\overline{c})^{\overline{\theta}}}{\overline{\theta}}, if\ c_t > \overline{c}_t \\ \frac{\lambda(c-\overline{c})^{\underline{\theta}}}{\underline{\theta}}, if\ c_t \leq \overline{c}_t \end{Bmatrix} \tag{2.21}$$

The existence of a kinked point in the utility function means that at the reference point, the risk aversion as part of the utility function intersects the loss aversion part of it; thus, they are equal at that point:

$$\frac{\lambda\,(c-\overline{c})^{\underline{\theta}}}{\underline{\theta}} = \frac{(c-\overline{c})^{\overline{\theta}}}{\overline{\theta}} \tag{2.22}$$

Being $c - \overline{c} = \widetilde{c}$, at some moment $\widetilde{c}$ will reach a level where the agent neither loses nor wins. The previous equation can be solved analytically to get:

$$c^* = \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}} \tag{2.23}$$

Thus the utility function becomes

$$U(c_t, \overline{c}_t) = \begin{Bmatrix} \frac{\left(c-(\vartheta\overline{c}_{t-1}+(1-\vartheta)(c_{t-1}-\left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\overline{\theta}}}{\overline{\theta}}, if\ c_t > \overline{c}_t \\ \frac{\lambda\left(c-(\vartheta\overline{c}_{t-1}+(1-\vartheta)(c_{t-1}-\left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\underline{\theta}}}{\underline{\theta}}, if\ c_t \leq \overline{c}_t \end{Bmatrix} \tag{2.24}$$

For the case of consumption, these definitions imply that, in the steady state $\widetilde{c_t} = c_t - \overline{c}_t \to c^*$, this is a desirable property. However, the most important thing is the dynamics of the steady state. By means of backward induction, I found:

$$\widetilde{c_t} = c_t - \overline{c}_t = c_t - \vartheta\overline{c}_{t-1} - (1-\vartheta)(c_{t-1} - c^*) \tag{2.25}$$

$$\widetilde{c_t} = c_t - \vartheta^m\overline{c}_{t-m} - (1-\vartheta)\sum_{j=1}^{m}\vartheta^{j-1}(c_{t-j} - c^*)$$

if $m \to \infty$, $\widetilde{c_t}$ an infinite moving average process centered in c^* :

$$\widetilde{c_t} = c^* + c_t - (1-\vartheta)\sum_{j=1}^{\infty}\vartheta^{j-1}c_{t-j} \tag{2.26}$$

Thereby, I can express this equation as:

$$\widetilde{c_t} = c_t - \left[(1-\vartheta)\sum_{j=1}^{\infty}\vartheta^{j-1}c_{t-j} - c^*\right] \tag{2.27}$$

Then, I can define the reference point as an infinite moving average of the past deviations of consumptions with respect to its steady state (a moving average of consumption gaps with respect to c^*). This definition is rather general because the reference point is a dynamic one and also includes habits formation.

By defining $\tau(c_t, r_t^c) = c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))$ and $\psi(c_t, r_t^c) = c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))$, the smooth transition loss aversion (STLA) utility function will be as follows:

$$\begin{aligned} U_{L,S}(c_t, r_t^c) &= \phi_{ct}\frac{\lambda\left(c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\underline{\theta}}}{\underline{\theta}} \\ &+ (1-\phi_{ct})\frac{\left(c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\overline{\theta}}}{\overline{\theta}} \end{aligned} \tag{2.28}$$

$$\phi_{ct} = \phi(c_t, r_t^l) = \frac{1}{1 + \exp(\gamma(c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}})))} \tag{2.29}$$

2.4.1.2 In the way of habits formation as defined by Carrol (2000)

It is also possible to define the STLA utility function in a more intuitive way: if we define

$$\overline{c}_t = (1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1}) \Rightarrow \tag{2.30}$$

$$\overline{c}_t - \overline{c}_{t-1} = \vartheta\left(c_{t-1} - \overline{c}_{t-1}\right) \tag{2.31}$$

This equation is equivalent to equation (2) in Carrol (2000). In the steady state,

$$\overline{c}_t = \overline{c}_{t-1} = \overline{c}_\infty, \Rightarrow c_\infty = \overline{c}_\infty. \tag{2.32}$$

This is, the reference point in the steady state equals per capita consumption (things will change in the presence of population growth, see Carrol (2000)). However, out of the steady state, $c_t \neq \overline{c}_t$.

$$z_y = \frac{c_t}{\overline{c}_t} = \frac{c_t}{(1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1})} \tag{2.33}$$

$$U(c_t, \overline{c}_t) = \begin{cases} (z_t)^\beta & if\ z_t > 1,\ c > c_\infty,\ 0 < \beta < 1,\ concavity \\ \lambda\,(z_t)^\alpha & z_t < 1,\ c < c_\infty,\ \alpha > 1, convexity \end{cases}, \tag{2.34}$$

Note that $\frac{c_t}{\overline{c}_t} = z_t \in (0, +\infty)$. Defined in this way, the utility function fulfills the loss aversion index proposed by Köbberling and Wakker (2005): $\frac{v'_\uparrow(1)}{u'_\downarrow(1)} = \frac{\alpha\lambda}{\beta}$. Thus, the STLA utility function becomes:

$$\phi_{ct} = \frac{1}{1 + \exp\left(\gamma(z-1)\right)} \tag{2.35}$$

$$U(z_t) = \left(\frac{1}{1+\exp\left(\gamma(z-1)\right)}\right)\lambda\,(z)^{\alpha} + \left(\frac{\exp\left(\gamma(z-1)\right)}{1+\exp\left(\gamma(z-1)\right)}\right)(z)^{\beta} \tag{2.36}$$

In this chapter of the dissertation, I will use this utility function as it turns out to be more intuitive, easier to deal with, and permits to generalize habits formation, which is already a conventional form to induce persistence in consumption and reference-dependence modeling.

2.4.1.3 Defining percentage deviations from the reference point

It is also possible to define losses and gains in percentage deviations with respect to the reference level: $x = \frac{c-\bar{c}}{\bar{c}}$, $x \in [-1, \infty)$, and the utility function can be written as:

$$U(c) = \begin{cases} (1+x)^{\beta} & if\ x > 0,\ 0 < \beta < 1,\ concavity \\ (1+x)^{\alpha} & if\ x < 0,\ \alpha > 1, convexity \end{cases} \tag{2.37}$$

Using the definition by Booij and van de Kuilen (2006), and Köbberling and Wakker (2005), the loss aversion coefficient will take place

$$loss\ aversion\ coefficient = \frac{U'_{\uparrow}(0)}{U'_{\downarrow}(0)} = \frac{\alpha\,(1+x)^{\alpha-1}}{\beta\,(1+x)^{\beta-1}} = \frac{\alpha}{\beta}, \tag{2.38}$$

where is the left derivative and $U'_{\downarrow}(c)$ the right derivative. Thus, the STLA will become:

$$U(x) = \phi_{ct}\,(1+x)^{\alpha} + (1-\phi_{ct})\,(1+x)^{\beta} \tag{2.39}$$

$$\phi_{ct} = \left(\frac{1}{1+\exp\left(\gamma x\right)}\right) \tag{2.40}$$

2.5 Uniqueness of the equilibrium

Our prospects utility function is re-defined around the reference point z_{ct} and is given by:

$$U(c_t, \bar{c}_t) = \begin{cases} (z_{ct})^{\overline{\theta}} & if\ z_t > 1,\ c > c_{\infty},\ 0 < \overline{\theta} < 1,\ concavity \\ \lambda\,(z_{ct})^{\underline{\theta}} & z_t < 1,\ c < c_{\infty},\ \underline{\theta} > 1, convexity \end{cases}, \tag{2.41}$$

Proposition: In a two period economy, the prospects utility function has only one optimum and therefore a unique equilibrium.

Proof: Note that $u(z_{ct}) = (z_{ct})^{\overline{\theta}}$ would be also defined for any value of z_t greater than zero and not only for values greater than 1. similarly, $v(z_{ct}) = (z_{ct})^{\underline{\theta}}$ would be also defined for values greater than 1. This function is kinked but its respective approximated function with smooth transition is not kinked.

Lets suppose the function $y = x$ which is the 45 degrees line with slope equal to 1.

1. By construction, $u(z_{ct}) = (z_{ct})^{\overline{\theta}}$ is concave in S such that $S = [0, \infty)$ and particularly for $S' \subset S$. y $S' = [1, \infty)$.Thus, $\forall\ x \in [1, \infty)$, $u'(x) = \overline{\theta}\,(x)^{\overline{\theta}-1} < 1$, and $\nexists x_o \in [1, \infty) : u'(x_o) = \overline{\theta}\,(x_o)^{\overline{\theta}-1} = 1$.

2. Also by construction, $v(z_{ct}) = (z_{ct})^{\underline{\theta}}$ is convex in $S'' = [0, 1]$, and is also convex in $S = [0, \infty)$. Thus,

a. $\exists x_1 \in [0, 1] : v'(x_1) = \overline{\theta}\,(z_{ct})^{\underline{\theta}-1} = 1$

b. $\forall x \in (x_1, 1], v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} > 1$

c. $\forall x \in [0, x_1), v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} < 1$

From (a) and (b) it is deduced that $\nexists x_2 \in [x_1, 1] : v'(x_2) = \overline{\theta}\,(x_2)^{\underline{\theta}-1} = u'(x_3) = \overline{\theta}\,(x_3)^{\overline{\theta}-1}, x_3 \in [1, \infty)$. Thus, there does not exist a straight line touching more than one point of the function $U(c_t, \bar{c}_t)$ in the interval $[x_1, \infty)$.

What about the interval $[0, x_1)$ where $v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} < 1$?

Given that $u'(x) = \overline{\theta}\,(x)^{\overline{\theta}-1} < 1, \forall\, x \in [1, \infty)$ and $v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} < 1, \forall x \in [0, x_1)$, hence $\exists x_4 \in [0, x_1)$,$\exists\, x_5 \in [1, \infty)$ such that $v'(x_4) = \overline{\theta}\,(x_4)^{\underline{\theta}-1} = u'(x_5) = \overline{\theta}\,(x_5)^{\overline{\theta}-1} < 1$.

But given $x_4 < x_5$ and $v(x_4) < u(x_5)$ there does not exist a straight line simultaneously touching the utility function in $v(x_4)$ and $u(x_5)$.

3. Suppose now, that in this two period economy the agent has an income M such that $M = P_1C_1 + P_2C_2$. The agent seeks maximizing her inter-temporal utility $U(c_1, \overline{c}_1) + \beta U(c_2, \overline{c}_2)$, subject to $M = P_1C_1 + P_2C_2$.

Thus, $U'(c_1, \overline{c}_1) = \lambda P_1$ and $U'(c_2, \overline{c}_2)\beta = \lambda P_2$. Then, if the price for consumption in the first period is one and the price for consumption in period 2 is $1/1 + r$, first order conditions will be $U'(c_1, \overline{c}_1) = \lambda$ and $U'(c_2, \overline{c}_2)\beta = \lambda \frac{1}{1+r}$. Thus, $U'(c_2, \overline{c}_2)\beta = U'(c_1, \overline{c}_1)\frac{1}{1+r}$. Note that it is possible to have four cases:

3.1. $u'(c_2, \overline{c}_2)\beta = u'(c_1, \overline{c}_1)\frac{1}{1+r}$

3.2. $v'(c_2, \overline{c}_2)\beta = v'(c_1, \overline{c}_1)\frac{1}{1+r}$

3.3. $u'(c_2, \overline{c}_2)\beta = v'(c_1, \overline{c}_1)\frac{1}{1+r}$

3.4. $v'(c_2, \overline{c}_2)\beta = u'(c_1, \overline{c}_1)\frac{1}{1+r}$

Thus, $U(c_t, \overline{c}_t)$ has one and only one constrained maximum and therefore, only one equilibrium (QED).

2.6 A DSGE model with loss aversion

In this chapter, one of the departure points of the mainstream literature on RBC is the use of prospect theory to micro-found decisions of agents in an uncertain environment. As in chapter 1, the model developed here is based on the Basic Neoclassical Model exposed in King, Plosser and Rebelo (1988). The purpose of this section is to build a model of a representative agent in a closed economy; this agent owns the firms, chooses consumption and leisure, and accumulates physical capital; there is a neoclassical production function and a stochastic technology shock introduces uncertainty into the model.

Thus, considering the utility function specification of section 2.4.1.2, defining

$$z_{ct} = \frac{c_t}{\overline{c}_t} = \frac{c_t}{(1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1})} \tag{2.42}$$

In the steady state:

$$\overline{c}_t = \overline{c}_{t-1} = \overline{c}_\infty, \Rightarrow c_\infty = \overline{c}_\infty \Rightarrow z_{c\infty} = 1 \tag{2.43}$$

When the economy is not in the steady state $c_t \neq \overline{c}_t$

$$z_{ct} = \frac{c_t}{\overline{c}_t} = \frac{c_t}{(1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1})} \neq 1 \tag{2.44}$$

$$U(c_t, \overline{c}_t) = \begin{cases} (z_{ct})^{\overline{\theta}} & if\ z_t > 1,\ c > c_\infty,\ 0 < \overline{\theta} < 1,\ concavity \\ \lambda\,(z_{ct})^{\underline{\theta}} & z_t < 1,\ c < c_\infty,\ \underline{\theta} > 1, convexity \end{cases}, \tag{2.45}$$

Note that $\frac{c_t}{\overline{c}_t} = z_{ct} \in (0, +\infty)$ and, when defined in this way, the utility function fulfills the loss aversion index proposed by Köbberling and Wakker (2005): $\frac{v'_{\uparrow}(1)}{u'_{\downarrow}(1)} = \frac{\underline{\theta}\lambda}{\overline{\theta}}$. Thus, the STLA utility function becomes:

$$U(z_{ct}) = \phi_{ct}\lambda\,(z_{ct})^{\underline{\theta}} + (1-\phi_{ct})\,(z_{ct})^{\overline{\theta}} \tag{2.46}$$

$$\phi_{ct} = \frac{1}{1 + \exp\left(\gamma(z_{ct} - 1)\right)} \tag{2.47}$$

$$U(z_{ct}) = \left(\frac{1}{1 + \exp\left(\gamma(z_{ct} - 1)\right)}\right)\lambda\,(z_{ct})^{\underline{\theta}} + \left(\frac{\exp\left(\gamma(z_{ct} - 1)\right)}{1 + \exp\left(\gamma(z_{ct} - 1)\right)}\right)(z_{ct})^{\overline{\theta}} \tag{2.48}$$

For leisure, it is also possible to define a loss averse utility function and also a reference point:

$$z_{lt} = \frac{l_t}{\bar{l}_t} = \frac{l_t}{(1-\chi)\bar{l}_{t-1} + \chi(l_{t-1})} \tag{2.49}$$

In the steady state:

$$\bar{l}_t = \bar{l}_{t-1} = \bar{l}_\infty, \Rightarrow l_\infty = \bar{l}_\infty \Rightarrow z_{l\infty} = 1 \tag{2.50}$$

When the economy is not in the steady state $l_t \neq \bar{l}_t$

$$z_{lt} = \frac{l_t}{\bar{l}_t} = \frac{l_t}{(1-\chi)\bar{l}_{t-1} + \chi(l_{t-1})} \neq 1 \tag{2.51}$$

$$U(l_t, \bar{l}_t) = \begin{cases} (z_{lt})^{\overline{\mu}} & if\ z_{lt} > 1,\ l > l_\infty,\ 0 < \overline{\mu} < 1,\ concavity \\ \lambda\,(z_{lt})^{\underline{\mu}} & z_t < 1,\ l < l_\infty,\ \underline{\mu} > 1, convexity \end{cases}, \tag{2.52}$$

Note that $\frac{l_t}{\bar{l}_t} = z_{lt} \in (0, +\infty)$ and, when defined in this way, the utility function fulfills the loss aversion index proposed by Köbberling and Wakker (2005): $\frac{v'_{\uparrow}(1)}{u'_{\downarrow}(1)} = \frac{\underline{\mu}\lambda}{\overline{\mu}}$. Thus, if the only argument of the utility were leisure, the STLA utility function would be:

$$U(z_{lt}) = \phi_{lt}\lambda\,(z_{lt})^{\underline{\mu}} + (1-\phi_{lt})\,(z_{lt})^{\overline{\mu}} \tag{2.53}$$

$$\phi_{lt} = \frac{1}{1+\exp\left(\gamma(z_{lt}-1)\right)} \tag{2.54}$$

$$U(z_{lt}) = \left(\frac{1}{1+\exp\left(\gamma(z_{lt}-1)\right)}\right)\lambda\,(z_{lt})^{\underline{\mu}} + \left(\frac{\exp\left(\gamma(z_{lt}-1)\right)}{1+\exp\left(\gamma(z_{lt}-1)\right)}\right)(z_{lt})^{\overline{\mu}} \tag{2.55}$$

Another contribution in this chapter is the inclusion of leisure into the utility function, given that consumption is not the only good delivering utility to individuals. In this respect, I invoke Veblen from his seminal work "The theory of the leisure class". He argues that manual work or industrious run puts the individual as one belonging to a lower social and economic class. Thus, leisure demand is not due only to the fact that offers utility by itself, but also due to an intention of emulation by those who truly want to look for boasts in a higher social and economic class. In this sense, it is supposed that leisure and consumption are not additive separable in the instantaneous utility function. Thus, the prospect theory utility function for consumption and leisure would be:

$$U(c,l,r^c,r^l) = \phi_{clt}(\lambda\,(z_{ct})^{\underline{\theta}}))^{\omega}(\lambda\,(z_{lt})^{\underline{\mu}})^{(1-\omega)} + (1-\phi_{clt})((z_{ct})^{\overline{\theta}}))^{\upsilon}((z_{lt})^{\overline{\mu}})^{(1-\upsilon)} \tag{2.56}$$

Note also that for the non-additive separable function, if $\theta_i = \mu_i$, and the aggregation parameters of consumption and labor do not change between states (i.e. $\omega = \upsilon$)

$$U(c,l,r^c,r^l) = \phi_{clt}\lambda\left(z_{ct}^{\omega} z_{lt}^{(1-\omega)}\right)^{\underline{\theta}} + (1-\phi_{clt})\left(z_{ct}^{\omega} z_{lt}^{(1-\omega)}\right)^{\overline{\theta}} \tag{2.57}$$

thus, the reference point for the agent is $agr_t = z_{ct}^{\omega} z_{lt}^{(1-\omega)}$, which I will call the *"aggregator"*; thus, the transition function in this case will be:

$$\phi_{clt} = \frac{1}{1+\exp\left(\gamma(z_{ct}^{\omega} z_{lt}^{(1-\omega)}-1)\right)} = \frac{1}{1+\exp\left(\gamma(agr_t-1)\right)} \tag{2.58}$$

Regarding the values of $agr_t = z_{ct}^{\omega} z_{lt}^{(1-\omega)}$, it is obvious that when z_{ct} and $z_{lt} > 1$, $agr_t > 1$, and when z_{ct} and $z_{lt} < 1$, $agr_t < 1$. However, it is not so evident what happens when $z_{ct} > 1$ and $z_{lt} < 1$ or vice versa. In a situation where $z_{ct} > 1$ and $z_{lt} < 1$, we take logs of agr_t and ask ourselves under what cases it would be greater than one when we have $\frac{lnz_{ct}}{-lnz_{lt}} > \frac{(1-\omega)}{\omega} = \varrho$. Thus, the size of agr_t will depend directly on the size of $\frac{lnz_{ct}}{-lnz_{ct}}$ and the size of ϱ will be the size of . The same line of reasoning applies for the situation where $z_{ct} < 1$ and $z_{lt} > 1$ and the relevant expression is $\frac{lnz_{lt}}{-lnz_{ct}} > \frac{\omega}{1-\omega} = \varpi$. In the calibration proposed for the simulation exercises, $\omega = 0.5$, which is a capricious choice in order to free the results from bias. However, empirical work is yet needed in order to know the model parameters suggested by the data.

2.7 The consumer problem, first order conditions and equilibrium

The production function, physical capital accumulation, and technological shocks are the same as those of a canonical RBC model:

$$Y_t = A_t K_t^{\alpha} n_t^{(1-\alpha)} \tag{2.59}$$

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t \tag{2.60}$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t \tag{2.61}$$

$$\varepsilon_t \backsim N(0,\sigma) \tag{2.62}$$

$$l_t = 1 - n_t \tag{2.63}$$

The Lagrangian function for this central planer problem will be:

$$\pounds_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[U(c,l,r^c,r^l) = \phi_{clt}\lambda \left(z_{ct}^{\omega} z_{lt}^{(1-\omega)} \right)^{\underline{\theta}} + (1-\phi_{clt}) \left(z_{ct}^{\upsilon} z_{lt}^{(1-\upsilon)} \right)^{\overline{\theta}} \right] \\ +\mu_t \left[-K_{t+1} + (1-\delta)K_t + Y_t - C_t \right] \end{array} \right\} \tag{2.64}$$

Where μ_t is the Lagrange multiplier, and I define:

$$\overline{u}_t(.) = \left(z_{ct}^{\upsilon} z_{lt}^{(1-\upsilon)} \right)^{\overline{\theta}} \tag{2.65}$$

$$\underline{u}_t(.) = \lambda \left(z_{ct}^{\omega} z_{lt}^{(1-\omega)} \right)^{\underline{\theta}} \tag{2.66}$$

It is possible to write in compact form:

$$\pounds_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\phi_{clt}\underline{u}_t(.) + (1-\phi_{clt})\overline{u}_t(.) \right] + \mu_t \left[-K_{t+1} + (1-\delta)K_t + Y_t - C_t \right] \right\} \tag{2.67}$$

$$\pounds_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\overline{u}_t(.) + \phi_{clt} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right] + \mu_t \left[-K_{t+1} + (1-\delta)K_t + Y_t - C_t \right] \right\} \tag{2.68}$$

control variables are c_t, K_{t+1}, n_t. and the first order conditions are:

$$\begin{aligned} \frac{\partial \pounds_t}{\partial c_t} = 0 = & \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{ct}} - \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} \right] + \frac{\partial \phi_{clt}}{\partial z_{ct}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{ct}}{\partial c_t} \\ & - \mu_t + \beta E_t \left\{ \begin{array}{c} \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{ct+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} \right] \\ + \frac{\partial \phi_{clt+1}}{\partial z_{ct+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \end{array} \right\} \frac{\partial z_{ct+1}}{\partial c_t} \end{aligned} \tag{2.69}$$

$$\frac{\partial \pounds_t}{\partial l_t} = 0 = \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{lt}} - \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} \right] + \frac{\partial \phi_{clt}}{\partial z_{lt}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{lt}}{\partial l_t} \tag{2.70}$$

$$- \mu_t \frac{\partial Y_t}{\partial n_t} + \beta E_t \left\{ \begin{array}{c} \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{lt+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} \right] \\ + \frac{\partial \phi_{clt+1}}{\partial z_{lt+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \end{array} \right\} \frac{\partial z_{lt+1}}{\partial l_t}$$

$$\frac{\partial \pounds_t}{\partial K_{t+1}} = 0 = -\mu_t + \beta E_t \left\{ \mu_{t+1} \left[(1-\delta) + \frac{\partial Y_{t+1}}{\partial K_{t+1}} \right] \right\} \tag{2.71}$$

$$\frac{\partial \pounds_t}{\partial \mu_t} = 0 = -K_{t+1} + (1-\delta) K_t + Y_t - C_t \tag{2.72}$$

We define the regime switching marginal utility of consumption and leisure respectively as:

$$\varphi_{ct} = \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{ct}} - \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} \right] + \frac{\partial \phi_{clt}}{\partial z_{ct}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{ct}}{\partial c_t} \tag{2.73}$$

$$\varphi_{lt} = \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{lt}} - \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} \right] + \frac{\partial \phi_{clt}}{\partial z_{lt}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{lt}}{\partial l_t} \tag{2.74}$$

We define the switching marginal disutily of consumption and leisure of time t in the period $t+1$, caused by the efect of habits on the reference point.

$$\xi_{ct+1} = \left\{ \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{ct+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} \right] + \frac{\partial \phi_{clt+1}}{\partial z_{ct+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \right\} \frac{\partial z_{ct+1}}{\partial c_t} \tag{2.75}$$

$$\xi_{lt+1} = \left\{ \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{lt+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} \right] + \frac{\partial \phi_{clt+1}}{\partial z_{lt+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \right\} \frac{\partial z_{lt+1}}{\partial l_t} \tag{2.76}$$

thus the first order conditions for consumption and leisure can be written in a compact form as:

$$\varphi_{ct} - \mu_t + \beta E_t \{\xi_{ct+1}\} = 0 \tag{2.77}$$

$$\varphi_{lt} - \mu_t \frac{\partial Y_t}{\partial n_t} + \beta E_t \{\xi_{lt+1}\} = 0 \tag{2.78}$$

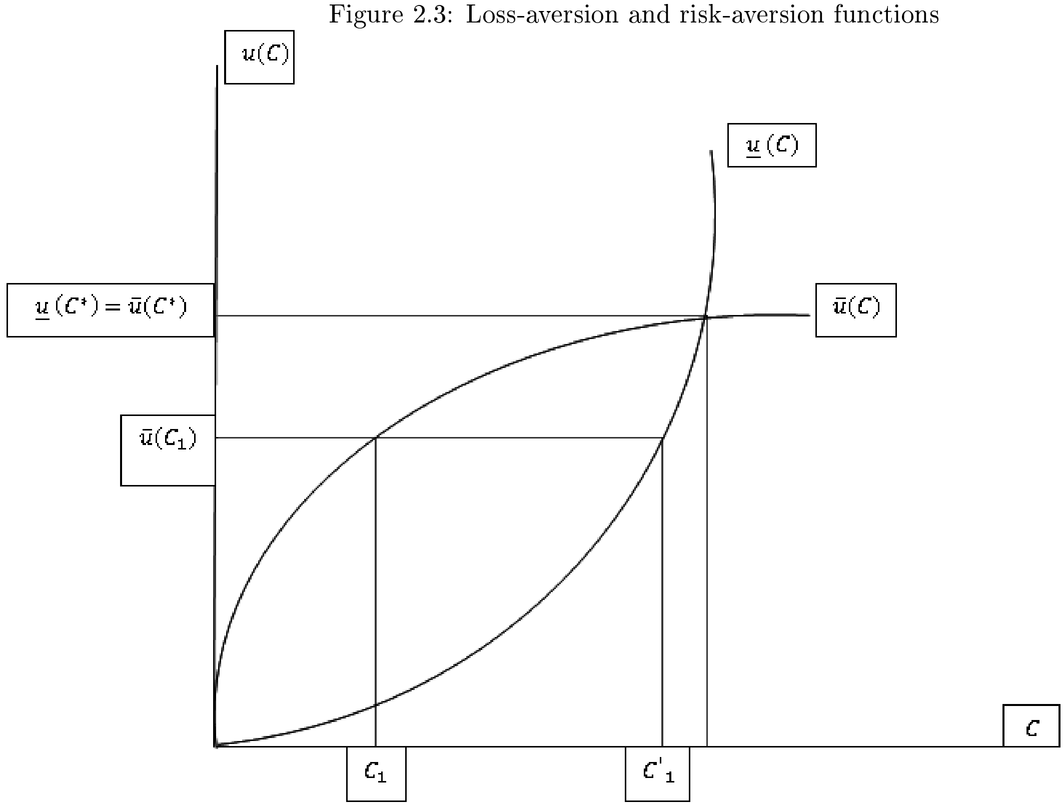
Thus the dynamic equilibrium equations for consumption and leisure respectively become:

$$\varphi_{ct} = \beta E_t \left\{ \left[\varphi_{ct+1} + \beta E_t \{\xi_{ct+2}\} \right] \left[(1-\delta) + \frac{\partial Y_{t+1}}{\partial K_{t+1}} \right] - E_t \{\xi_{ct+1}\} \right\} \tag{2.79}$$

$$\varphi_{lt} = \{\varphi_{ct} + \beta E_t \{\xi_{ct+1}\}\} \frac{\partial Y_t}{\partial n_t} - \beta E_t \{\xi_{lt+1}\} \tag{2.80}$$

These equations jointly with transition equation for physical capital and the stochastic process for technology shocks conform the dynamical equilibrium of this ecnomiy populated by the representative prospect theory agent.

Figure 2.3: Loss-aversion and risk-aversion functions



2.7.1 On why loss aversion could be a good explanation for business cycles asymmetries

Let us suppose that the economy is in a situation such that the steady state coincides with the reference point. Thus, the representative agent has a utility equal to "reference utility": $C_t = C^*, K_t = K^*, Y_t = Y^*, \underline{u}(C^*) = \overline{u}(C^*)$. Now, let us suppose that the economy is negatively shocked. If the agent were risk averse, he would choose C_1 and would enjoy $\overline{u}(C_1)$. But if he were loss averse he would choose a consumption level such that the departure from the reference level (C^*) would not be too large, minimizing therefore their loss of welfare. Note that by choosing C_1, the loss averse agent would have $\underline{u}(C_1) < \overline{u}(C_1)$. Thus, a loss averse agent needs to choose a consumption level such that he can, at least, enjoy a utility equivalent to $\underline{u}(C_1)$; this is, she has to choose $C_1^{'}$ such that $\underline{u}(C_1^{'}) = \overline{u}(C_1)$, which means that $C_1^{'} = \underline{u}^{-1}(\overline{u}(C_1)) > C_1$. Thus, in face of a negative shock, the loss averse agent would choose a consumption level lower than the one before the shock, but greater than the one the agent would choose if he were risk averse (figure 2.3).

What are the consequences on saving and investment? Let us suppose now that $Y_1 = A_1 K_1^{\alpha} n_1^{(1-\alpha)}$, the outcome in steady state, and $Y_1 = A_1 K_0^{\alpha} n_0^{(1-\alpha)}$, the outcome on shock, being $A_1 < A_0$. Thus, the capital accumulation will be $K_1^{LA} = (1-\delta)K_0 + Y_1 - C_1^{'}$ and $K_1^{RA} = (1-\delta)K_1 + Y_1 - C_1$ for the loss-averse and for the risk-averse agents respectively, and $K_0 = (1-\delta)K_0 + Y_0 - C_0$ would correspond to the capital accumulation in the steady state. As the economy was negatively shocked, savings and investment will fall below their steady state levels; thus, $I_1^{LA} < I_0$, $I_1^{RA} < I_0$. After substracting $I_1^{LA} - I_0$ from $I_1^{RA} - I_0$, we will have $I_1^{RA} - I_1^{LA} = -C_1 + C_1^{'} > 0$, which is equivalent to stating that $I_1^{LA} < I_1^{RA}$. Thus, loss aversion amplifies the effect of a negative shock to the economy on capital accumulation and increases the variability of investment during recessions. Moreover, as the technology suffers a negative shock, marginal product of capital decreases. This means that savers will require a premium on the return of savings; otherwise, they will consume more while saving less. We need to keep in mind that the marginal product equals the interest rate in equilibrium; thus, $U'(c_2, \overline{c}_2)\beta = U'(c_1, \overline{c}_1)\frac{1}{1+MPK_2}$ and $MPK_2 = \alpha A_2 K^{\alpha-1} n^{(1-\alpha)}$. Thus, when marginal product of capital falls marginal, utility and consumption fall as well because consumers save less and consume more. This reaction in consumption is more severe when the agent is loss-averse than when the agent is risk-averse.

This result is, however, a very particular case of a more general one where $\underline{u}$ could be not only a convex but also a concave function with less curvature than that of $\overline{u}$.

2.8 Steady state and calibration

The steady state for this economy is a situation such that: $\overline{c}_t \to c^*, \overline{l}_t \to l^*, k_t \to k^*$ and thus $\overline{u} = \underline{u} = 1, \phi_t \to \phi^* = 0.5$:

$$\varphi_c^* = \left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\} \omega(c^*)^{-1} \tag{2.81}$$

$$\varphi_l^* = \left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\} (1-\omega)(l^*)^{-1} \tag{2.82}$$

$$\xi_{ct+1} = -\left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\} \vartheta\omega(c^*)^{-1} \tag{2.83}$$

$$\xi_{lt+1} = -\left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\} \chi(1-\omega)(l^*)^{-1} \tag{2.84}$$

Thus, from the Euler equation for consumption we have:

$$\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \omega(c^*)^{-1} = \beta E_t \left\{ \begin{bmatrix} \left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \omega(c^*)^{-1} \\ +\beta E_t \left\{-\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \vartheta\omega(c^*)^{-1}\right\} \\ -E_t\left\{-\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \vartheta\omega(c^*)^{-1}\right\} \end{bmatrix} \left[(1-\delta) + \frac{\partial Y_{t+1}}{\partial K_{t+1}}\right] \right\} \tag{2.85}$$

$$1 = \beta\left\{\left[(1-\delta) + \frac{\partial Y}{\partial K}\right]\right\} \tag{2.86}$$

This is the well-known equation for the stochastic (symmetric) growth model without population or technological long-run growth.

For the Euler equation for leisure, we have:

$$\begin{aligned} \left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} (1-\omega)(l^*)^{-1} = &\left\{\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \omega(c^*)^{-1} + \beta E_t\left\{-\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \vartheta\omega(c^*)^{-1}\right\}\right\} \frac{\partial Y_t}{\partial n_t} \\ &- \beta E_t\left\{-\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\} \chi(1-\omega)(l^*)^{-1}\right\} \end{aligned} \tag{2.87}$$

$$\{1 - \beta\chi\}(1-\omega)(l^*)^{-1} = \{1 - \beta\vartheta\}\omega(c^*)^{-1}\frac{\partial Y_t}{\partial n_t} \tag{2.88}$$

thus for calibration purposes, the key equations equations will be:

$$1 = \beta\left\{\left[(1-\delta) + \alpha K^{\alpha-1} n^{1-\alpha}\right]\right\} \tag{2.89}$$

$$\frac{c^*}{l^*} = \frac{\{1-\beta\vartheta\}\omega}{\{1-\beta\chi\}(1-\omega)}(1-\alpha)K^{\alpha}n^{-\alpha} \tag{2.90}$$

$$\delta k^* = y^* - c^* \tag{2.91}$$

Note that since there is neither population nor technological growth in the long run, the concavity-convexity of the utility function does not play any role in the steady state determination (this seems to be very useful because it helps to compare steady state results with those of other models). Also, it has $\lambda = 1$ by construction, disappearing either in the long run equations, and in the transitional dynamics.

Table 2.3: Calibration

parameter	value	variable	value
risk averse		c^*	1.69
$\overline{\theta}$	0.7	y^*	2.41
ω	0.5	i^*	0.725
loss averse		n^*	0.46
$\underline{\theta}$	1.5	l^*	0.54
ω	0.5	k^*	29.03
λ	1	c/y^*	0.7
production		n/y^*	0.19
α	0.4	k/y^*	12.04
δ	0.025	i/y^*	0.3
A	1	$Mgpk$	0.033
ρ	0.9	$Mgpn$	3.144
σ_ε	0.0018		
smooth transition			
γ	100		
reference points			
ϑ	0.5		
χ	0.5		
discount factor			
β	0.9917		

2.9 Simulations and results

2.9.1 Moments of data Vs. Moments of simulated data

As usual in the literature of DSGE and RBC models, to see the goodness of the model to replicate empirical regularities, table 2.4 shows kurtosis and skewness o simulated data and the same moments for the sample of countries as in table 1. Highlighted numbers in yellow show how simulated data can mimic the behavior in relative magnitud and sign for kurtosis and skewnes of full sample, and negative-positive values of cyclical components of macroeconomic variables. More over the model is successful in reproducing positive and negative skewness. However, because de calibracition of the model does not refer to a particular country in the sample, this results can not be taken as conclusive about the goodness of the model to reproduce asymmetries of business cycles. Sections 2.9.2 and 2.9.3 contain impulse-response exercises performed to see how different is a recession from a boom.

2.9.2 Deterministic simulation

In order to test the consistency of the model construction, deterministic simulations have been run initially. To this end, technology is shocked one time (negative and positive). However, instead of solving it by any approximation algorithm, I have used the extended path method implemented in Dynare by imposing that $a = 1.06$ (positive shock) and $a = 0.9433$, which is equivalent to having $e = 0.058268908$ and $e = -0.0566$ respectively.[3] Figures 2.4 and 2.5 show path time of key macro variables: logarithms of consumption, income, capital, investment, labor and technology $(lc_t, ly_t, lk_t, li_t, ln_t, la_t)$, and marginal products of labor and capital $(pml_t,\ pmk_t)$.

[3] It would be also possible to impose a symmetric e (this is, the same size of the shock in absolute value) but there would not be a great difference in the results.

Table 2.4: Kurtosis and Skewness for Sample of countries vs. Kurtosis and Skewness for simulated data

		Kurtosis			Skewness	
		HP			HP	
Variable	Full sample	Negative values	Positive values	Full sample	Negative values	Positive values
	Model Economy (replications)					
GDP	2.8019	3.0315	3.063	0.0171	-0.7722	0.7791
C	2.7764	2.9852	3.0337	0.0264	-0.7509	0.7587
I	2.8508	3.1931	2.9898	-0.086	-0.8416	0.7478
	Colombia					
GDP	2.0899	1.8927	1.8858	0.017	-0.462	0.2024
C	2.1659	3.3298	2.1108	0.193	-0.5571	0.3256
I	4.9049	6.0205	5.232	-0.3166	-1.8673	1.6237
	Germany					
GDP	2.4663	3.2255	1.9461	0.2942	-0.8865	0.2278
C	2.7961	1.8534	2.3599	0.2871	-0.5974	0.7277
I	1.8434	3.3246	1.7065	0.1644	-0.255	-0.0722
	USA					
GDP	3.0601	3.2327	2.0607	-0.559	-1.1154	0.3173
C	2.2794	4.3438	2.3978	-0.3902	-1.3491	0.3064
I	2.8452	3.118	2.1146	-0.625	-0.8702	0.1003
	United Kingdom					
GDP	3.1953	4.7287	2.5273	-0.2233	-1.3995	0.8305
C	2.8111	3.8048	3.0562	0.453	-0.5264	1.0482
I	3.6498	3.3733	3.9337	-0.0489	-1.0363	1.3929
	France					
GDP	2.2161	3.75	1.8382	-0.1168	-0.8477	0.269
C	2.2506	2.483	3.8269	-0.3778	-0.3094	0.7721
I	2.1549	1.6976	2.2693	0.0998	-0.1145	0.5048

Source: Autor calculations and World Bank web page. Annual per capita real series, logarithms of data filtered with Hodrik-Prescott filter. For France and USA sample is from 1970 to 2009 and for the other countries from 1960.

Figure 2.4: Deterministic simulation (absolute values of deviations from the steady state)

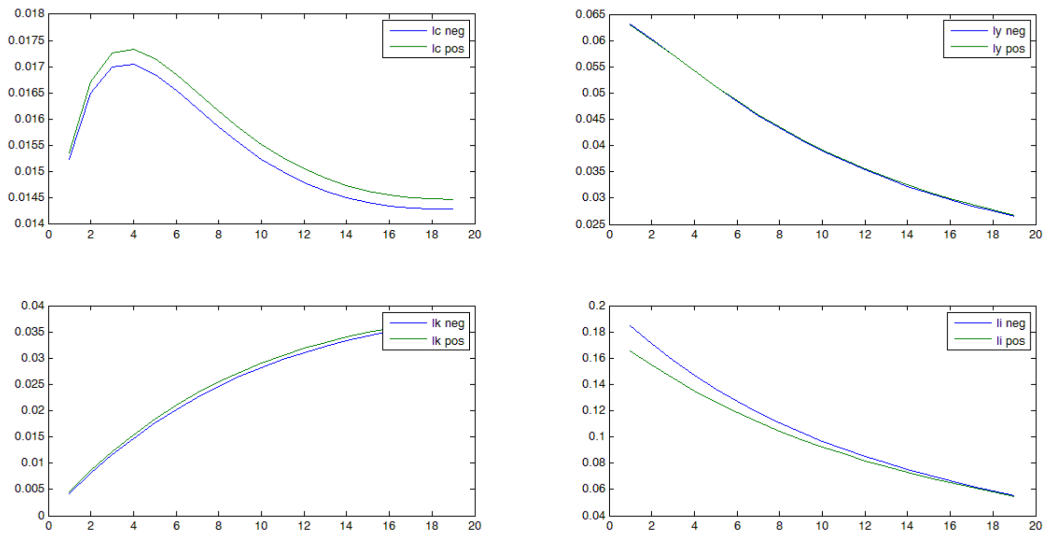
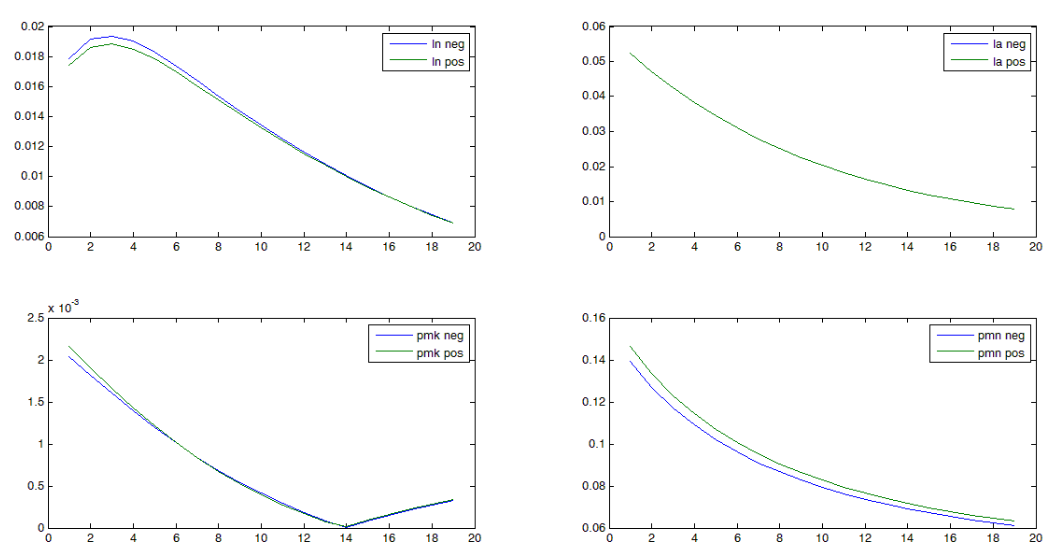


Figure 2.5: Deterministic simulation (absolute values of deviations from the steady state)



2.9.3 Stochastic simulation: Impulse response

Impulse response is one of the most used analysis tools in macro-econometrics. However, it must be used carefully. Because the DSGE model studied in this chapter is non-linear and asymmetric, impulse response analysis should not be performed as usual assuming that the DGP is linear-multivariate. Moreover, it would be a mistake to simply shock technology once and then follow the adjustment of the whole system. Therefore, in order to gauge asymmetric effects of shocks in this hypothetical economics General Impulse Response Function (Koop et al., 1996) (GIRF hereafter) are to be adopted.[4]

Because asymmetric DGP of this DSGE model, multivariate data therein simulated lack the following properties: Symmetry property, linearity property and history independence property. Thus, linear impulse response functions (VAR-based) are inappropriate tools for analyzing the dynamics of such a DSGE model. The GIRF as defined by Koop et al. (1996) is conditioned on shocks and/or history:

$$GI_Y(n, v_t, \omega_{t-1}) = E[Y_{t+n}|v_t, \omega_{t-1}] - E[Y_{t+n}|\omega_{t-1}], \quad for \quad n = 0, 1...$$

Being Y_t a vector of variables, v_t a current shock, ω_{t-1} the history, and n the forecasting horizon. Koop et al. (1996) also describes a simple algorithm to compute these conditional expectations through Monte Carlo integration. According to this method, GIRF could resemble a distribution of impulse-responses for each period in the forecast horizon. Impulse responses computed in this fashion are calculated and reported by Dynare. By default, Dynare drops the first 100 observations and then reports GIRF for a horizon of 40 periods ahead. Figures 2.9.3.1 and 2.9.3.1 show impulse responses (50 draws) for one standard deviation shock (positive and negative) on the perturbation term of the technology process.

2.9.3.1 Conditioning on a particular shock

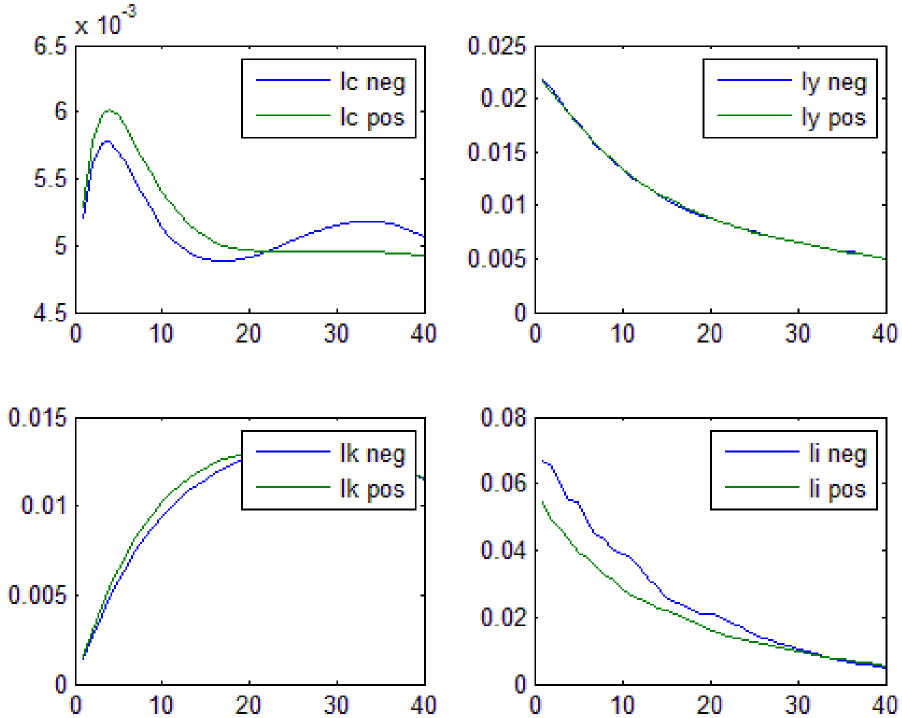
The first simulation exercise consisted in giving a once standard deviation shock (positive and negative) to the technology process in the asymmetric model. The simulation was performed for fifty replications; the response of macroeconomic variables in this hypothetical economy to negative shocks (in average) are asymmetric with respect to positive shocks. Thus, the GIRF computed was $GI_Y(n, v_t, \Omega_{t-1}) = E[Y_{t+n}|v_t, \Omega_{t-1}] - E[Y_{t+n}|\Omega_{t-1}]$, being Ω_{t-1} an information set of the previous history, and v_t a particular negative and positive shock. Figures 2.9.3.1 and 2.9.3.1 show these impulse response functions.

For consumption, capital, income, investment, and technology, the graphs show log-deviations while labor and marginal products of capital and labor are in levels. As it can be seen, the reaction of consumption to a negative shock is stronger than the reaction to a positive shock. However, the fall in consumption during recession is less deep and less long-lasting than the increase during boom. This can be explained by the loss-averse nature of the agents in this model. When the agent suffers a fall in income which deviates him from the reference point, he minimizes the loss induced by such deviation. Thus, his fall in consumption will be as small as possible. To this end, the agent reduces savings which brings about reduction in investment and consequently in physical capital. For the case of income, the reaction to a negative shock seems to be greater than the reaction to the positive shock, although the difference between them is almost imperceptible. For capital, the responses to perturbations on shock are very similar. Nevertheless, for positive shock, capital increase during the boom seems to be deeper and more long-lasting than the decrease during the recession. For investment, the fall in recession is very severe compared with the increase in the boom. Investment boom is less deep than during recession, but lasts longer.

For labor, interesting results were also found: the negative shock generates a stronger reaction than the positive shock and is matched by a significant smaller fall in wage (compared with the increase of wage induced by the positive shock). This means that although this model does not have either involuntary unemployment or (explicitly modeled) rigidities, a greater negative reaction of labor during a recession is accompanied by a smaller reaction in real wage; certainly, the opposite does occur after a positive technological shock. Because the utility function also includes leisure, the mechanics is the same as for consumption: a fall in income and consumption will induce an increase in leisure (as big as possible) in order to dampen the utility loss. For

[4] Local Projections Impulse Response (Jordá, 2005) could also be used, but this technique is susceptible of symmetry, thus it would not be possible to detect asymmetry in data of this hipothetical model.

Figure 2.6: GIRF for positive and negative shocks (absolute values of deviations from steady state)



the physical capital, its marginal product does also seem to show some rigidity during recessions compared to booms.

2.9.3.2 Conditioning on a particular history

Because asymmetric models are history-dependent, it is necessary to ask what the time path of the economy in boom would be, or ask what the time path of the economy in recession would be, either positively or negatively shocked.. The results of simulating a positive shock as the economy undergoes a boom or simulating a negative shock as the economy undergoes a recession are trivial: a recession deepening and boom sharpening take place. However, since business cycles are asymmetric, it would be necessary to perform the simulation in order to know the quantitative effects. Nonetheless, it would be more interesting to know the quantitative effects of a negative shock during boom and a positive shock during recession. To perform the exercise above proposed, it must be supposed that the economy is initially shocked (positively or negatively) in period one, and in period four it receives a shock in the opposite direction to the one received in period one. Thus, the exercise consisted in computing $GI_Y(n, v_t, \tilde{\Omega}_{t-1}) = E[Y_{t+n}|v_t, \tilde{\Omega}_{t-1}] - E[Y_{t+n}|\tilde{\Omega}_{t-1}]$, being $\tilde{\Omega}_{t-1}$ the state of the economy (being in boom or in recession) and v_t a positive or negative shock.

There is another important detail to take into account: this exercise is time-dependent. This implies that the new position of the economy after the second shock would depend on how far it is from the steady state. That is to say, the longer the horizon of GIRF, the closer the economy will be to the steady state and, therefore, depending on the size of the shock (and on the asymmetric structure of the economy), the economy could jump (suddenly perhaps) from a boom into a recession and vice versa. In order to standardize the problem of timing, the exercise was performed as follows: the second (positive or negative) shock was introduced in a time $t0$ in such a way that the technology gap were a half of its initial value on shock. In this section, all variables are measured in logarithms. Then, a gap of variables can be interpreted as log-deviations

Figure 2.7: GIRF for positive and negative shocks (absolute values of deviations from steady state)

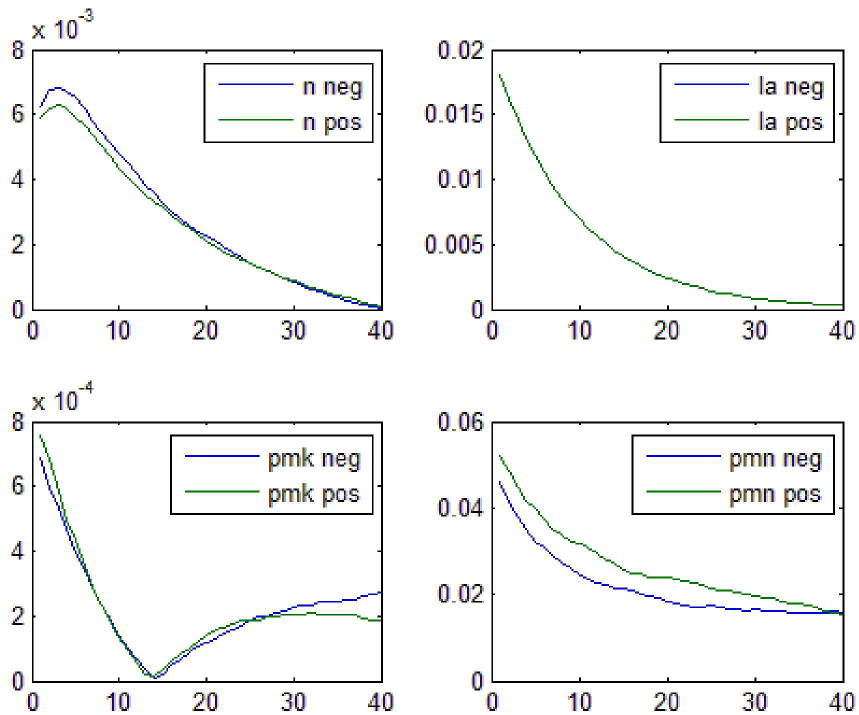
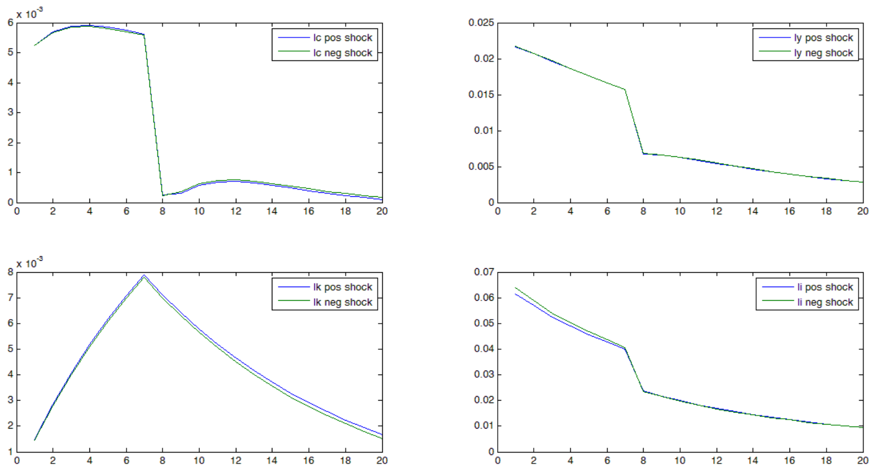


Figure 2.8: GIRF for the first and the second shocks (absolute value of deviations from steady state)



from the steady state.

2.9.3.3 A second shock in the opposite direction of the first shock

Figures 2.8 and 2.9 show the adjustment path of the economy after receiving a positive shock during a recession and a negative shock during a boom. In this exercise, it seems clear that a shock in the opposite direction pushes the economy to the other phase of the cycle, this is, making it fall from a boom into a recession or makes it jump from a recession into a boom. In the case of capital, it reverses, however, the accumulation slowly (deaccumulation) process induced by a positive (negative shock).

In figures 2.10 and 2.11 (absolute values) the path of the economy is shown from the period it receives the second negative (positive) during a boom (recession).

When the economy is perturbed by a negative shock while in a boom, the asymmetrical nature of this model can be seen again. The reaction of consumption on shock when the economy is negatively shocked is greater than the reaction when the shock is positive, but this is only for the first period. However, in general terms, the recession in consumption induced by the negative shock during the boom is less deep and less long-lasting than the boom induced by the positive shock during a recession. For income, the previous result holds even since the period when the economy receives the second shock. In the case of capital, the negative shock during a boom induces a more severe capital deaccumulation than the accumulation induced by the positive shock during the recession. The dynamics of investment is consistent with what happens in capital: the reaction of investment to the second negative shock is stronger and more long-lasting than the reaction to the second positive shock. Why do these facts result like that? When the economy is in a boom, the risk aversion households makes them to desire being as far as possible from the reference point. But when the economy is negatively shocked and income falls and the household needs to adjust its consumption level, it wants to stay as close as possible to the reference point because of its loss aversion.

Figure 2.11 shows what happens to labor, wages, and interest rate. The reaction of labor is very similar for both shocks, although there is an important difference between the wage reactions. When the economy is shocked by a negative perturbation during a boom, the marginal product of labor shows a reaction weaker than the one shown when the economy receives a positive shock during a recession. Once again, this model seems to exhibit some rigidity in wage: in recession, the fall in wage is smaller than the increase in boom. For the case of physical capital, when the economy is negatively shocked during the boom, the interest rate fall is smaller (during five periods) than the increase when positively shocked during a recession, which is consistent with the greater fall in capital when the negative shocks occur during boom.

Figure 2.9: GIRF for the first and the second shocks (absolute value of deviations from steady state)

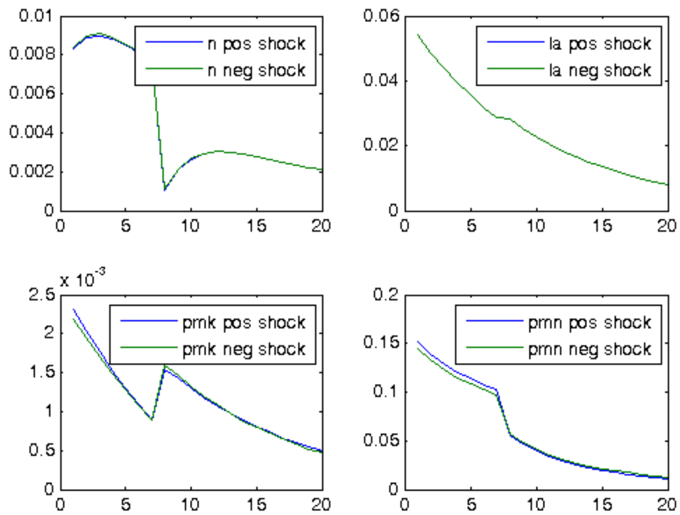


Figure 2.10: GIRF for the second shock (absolute value of deviations from steady state)

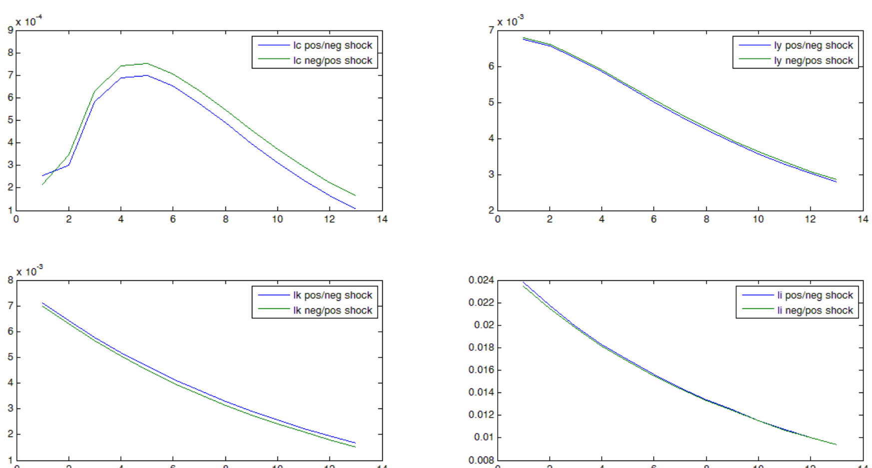
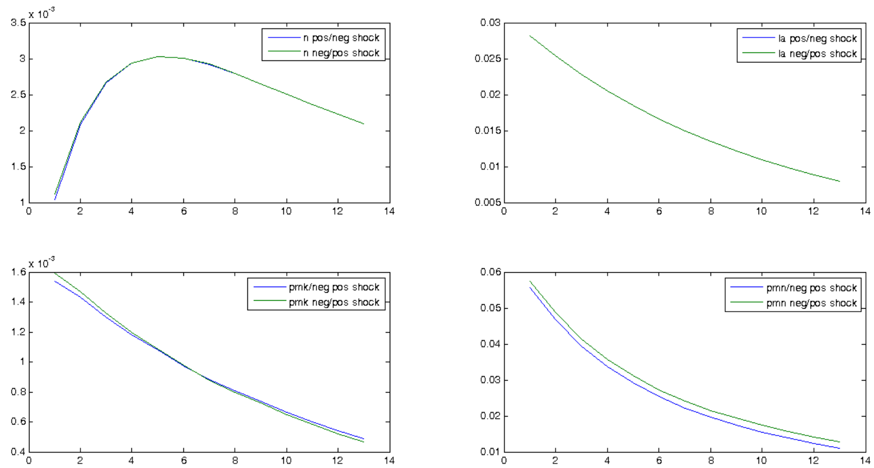


Figure 2.11: GIRF for the second shock (absolute value of deviations from steady state)



2.9.3.4 A second shock in the same direction of the first shock

We would also feel eager to ask about the effect of a positive shock during a boom or about the effect of a negative shock during recession. To answer these questions, we have performed an exercise as the previous one, but instead of giving a negative shock after a positive one, we give both: a first and a second positive shocks. First negative shock and second negative shock are also simulated.

Figures 2.12 and 2.13 show the trajectories of the economy when it is shocked by a second positive (negative) technological perturbation. The reactions observed after the first positive (negative) shock exacerbate with the second positive (negative) shock, although the asymmetrical nature of the model is revealed once again. For consumption, positive shock induces a higher reaction than the negative shock. While for income, the asymmetry is almost imperceptible. For capital, the positive shock accelerates the accumulation process while the negative shock accelerates its deaccumulation. Asymmetry in investment reaction is also evident, The effect of the negative shock is larger than the one of the positive shock. This is also consequence of the loss aversion of households. Labor paths are very similar, but the wage paths show the same asymmetry as in the first shock. Again, some rigidity is shown by wage after a negative shock.

Figures 2.14 and 2.15 shows more clearly that consumption reacts strongly and deeper to positive shocks. They also display that the boom induced in consumption by the positive shock lasts longer than the recession. The reactions in income are, again, almost the same while the reaction in capital to the positive shock is stronger than its reaction to the negative shock. The fall in investment caused by the negative shock is higher than the increase induced by the positive shock; thus, there is a severe and apparently long-lasting fall of investment. Labor has a stronger reaction to the negative shock than to the positive, which explains why income reactions are not very different as noted above. The smaller fall in wage is also evident here in recession. Furthermore, while in boom there is a higher increase in wage, and the same for marginal product of capital.

2.9.3.5 Shocks in the same direction during different phases of the cycle

At this point, it is necessary to compare shocks in the same direction, but in a different phase of the cycle. This means comparing the reaction of the economy receiving a negative shock during a boom with the reaction of the economy receiving a negative shock during recession, and the same comparison is pursued for the case of positive shocks.

Figures 2.16 and 2.17 show the time path for the experiment of perturbing the economy with a positive shock, both during boom and during recession. It is obvious that positive shocks exacerbate booms in

Figure 2.12: GIRF, second shock in the same direction of the first shock (absolute value of deviations from steady state)

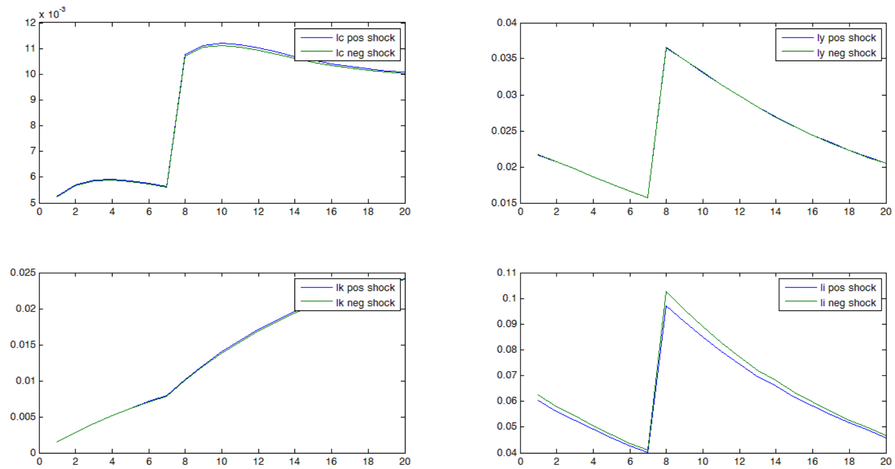


Figure 2.13: GIRF, second shock in the same direction of the first shock (absolute value of deviations from steady state)

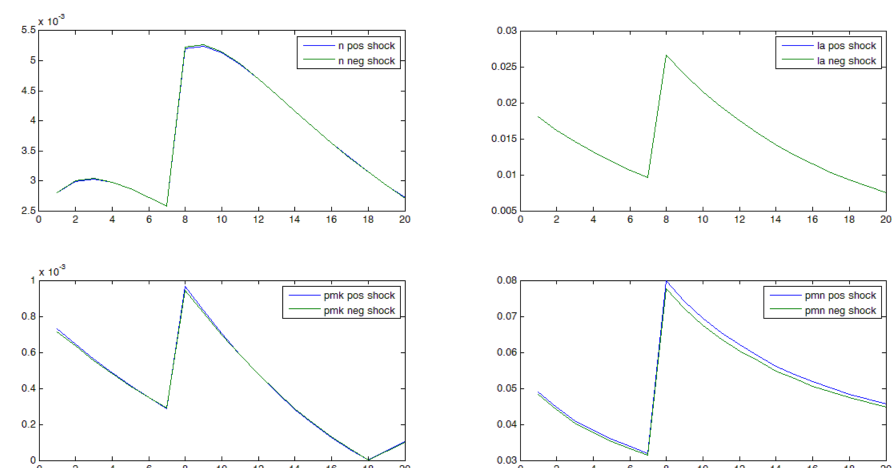


Figure 2.14: GIRF for the second shock in the same direction of the first shock (absolute value of deviations from steady state)

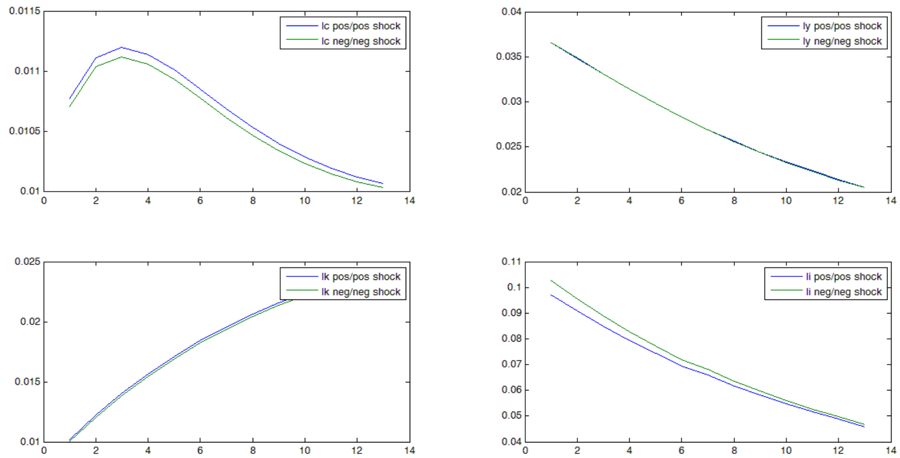


Figure 2.15: GIRF for the second shock in the same direction of the first shock (absolute value of deviations from steady state)

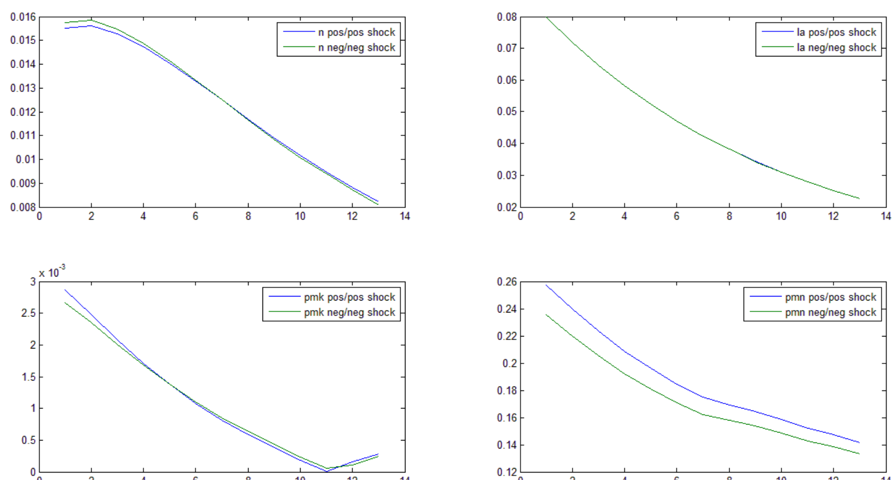
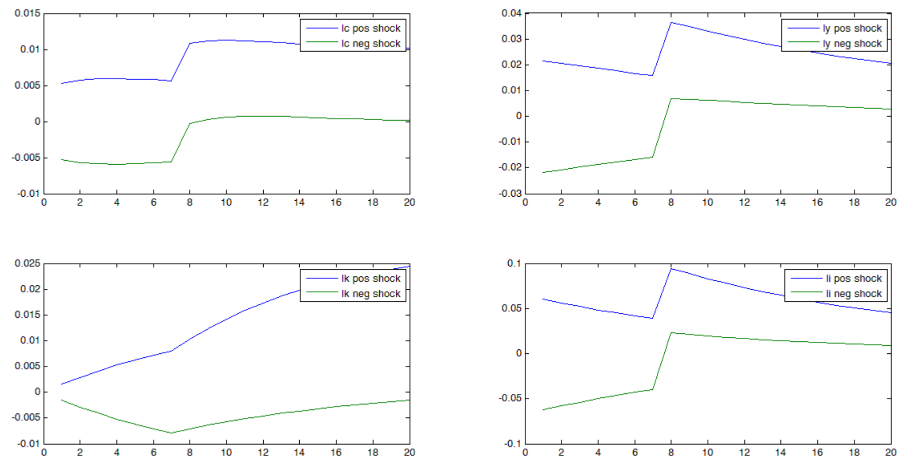


Figure 2.16: GIRF, positive shocks during different phases of the cycle (absolute value of deviations from steady state)



consumption, income, investment, labor, real wage, and real interest rate, inducing an increased process of capital accumulation. The effect of the positive shock during recession is more interesting because it induces a rapid recovering of the whole system: for consumption, income, investment, labor, real wage, and, real interest rate, the trajectories go above the steady state (not too far from the steady state, though). For the case of physical capital, the positive shock during recession reverses the deaccumulation process induced initially by the negative shock.

Figures 2.18 and 2.19 show the time path for the experiment of perturbing the economy with a positive shock, both during boom and during recession. The negative shock during recession, as in the case of positive shock during boom, exacerbates the bad situation of the economy and deepens the deaccumulation process of capital. For the case of the negative shock during boom, the effect is somewhat catastrophic: the fall of the economy is mostly severe for labor, investment, and capital because of the loss-averse behavior of agents, which helps the agents maintain their consumption as close as possible to the steady state.

2.10 Conclusions

In this chapter it was possible to build a DSGE model including loss aversion and risk aversion in a more general functional form known as prospects theory utility function following TK (1979) and KT (1992). I call my model Prospects Theory-DSGE Model (PT-DSGE). The main contribution of my work is extending the original (and rather simple) prospect theory utility function, developed by TK(1979, 1992), into a general form that nests loss aversion, risk aversion, and habits formation. In order to achieve this, I proposed three modifications of the prospect utility function. First, I have defined it on an aggregator of consumption and leisure. Second, I have redefined the reference point. For consumption, it is a weighted average of its reference point in the previous period and consumption in the previous period as well. Consequently, I used the same definition for the leisure reference point. Thus, the utility function argument is an aggregator of both consumption and leisure, and its reference point is defined as an aggregator of reference points for consumption and leisure respectively. The utility function is defined as the consumption-leisure bundle divided by the bundle of reference points for consumption-leisure. Then, when this ratio is greater than one, the agent has gains (and is risk-averse); and, when it is lower than one, he has losses (and is loss-averse). Third, in order to get differentiability of the utility function (in the kinked point), I defined a smooth transition function (by using a logistic function) whose threshold is 1. Given that my PT-DSGE model has a utility

Figure 2.17: GIRF, positive shocks during different phases of the cycle (absolute value of deviations from steady state)

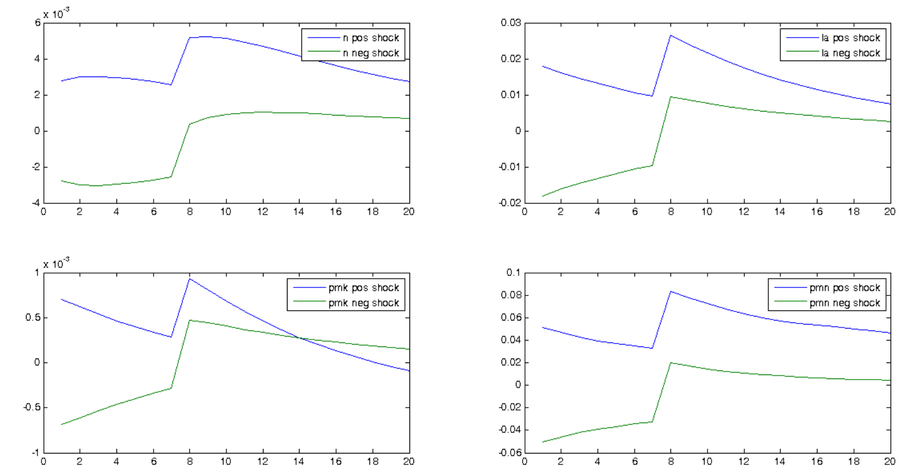


Figure 2.18: GIRF, negative shocks during different phases of the cycle (absolute value of deviations from steady state)

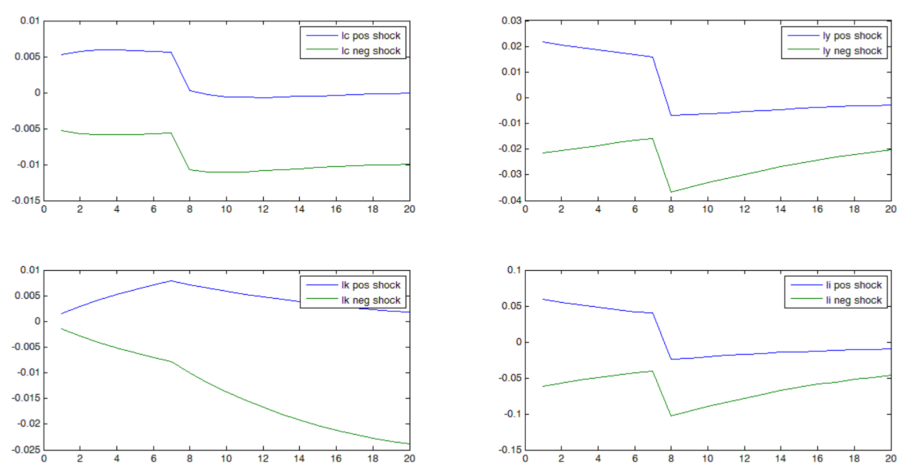
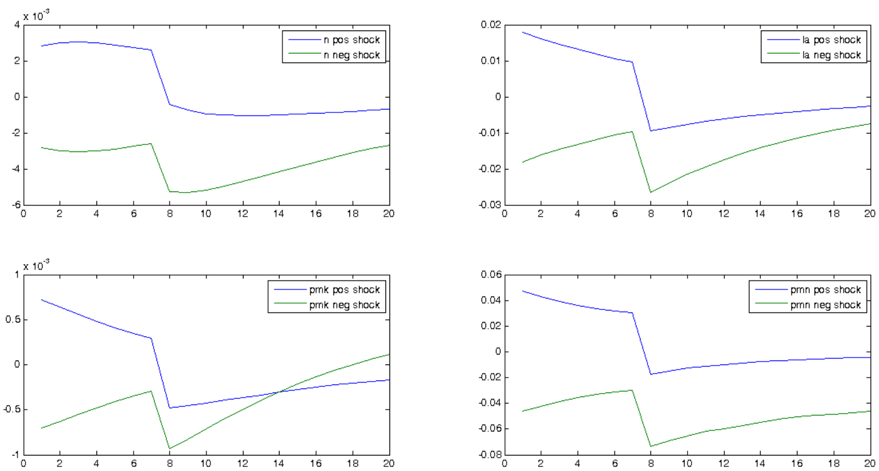


Figure 2.19: GIRF, negative shocks during different phases of the cycle (absolute value of deviations from steady state)



function which is convex below the reference point and concave above, it was necessary to establish a result about optimum uniqueness of the (restricted) optimum for such a utility function.

The aim of building this PT-DSGE model was to track the link between the dynamics of the business cycle asymmetries and the asymmetric behavior of agents along the phases of the cycle. In order to evaluate the effectiveness of the model to generate asymmetrical business cycles, it is necessary to simulate the model deterministically and stochastically using extended path (or exact solution) and perturbation method (third-order approximation) respectively by means of Dynare. In the deterministic simulation, two exercises were performed: i) positive shock to technology and ii) negative shock to technology. In the stochastic simulation, General Impulse Response Functions were calculated for positive and negative shocks as well, so that the shocks were equivalent to those in the deterministic case. Results for stochastic simulations were qualitatively the same and quantitatively similar to the ones in the deterministic procedure. On shock, the reaction of consumption to a positive shock is stronger than the reaction to a negative shock, but the fall in consumption in recession is less deep and less long-lasting than the increase in the boom phase. For physical capital, the responses to perturbations on shock are similar. However, for positive shock, capital increases during the boom seem to be deeper and more long-lasting than the decrease during the recession. For investment, the fall in recession is greater as compared with the increase in the boom phase. A boom of investment is less deep than during recession, but it lasts longer. For labor, interesting results were also found: the negative shock generates a stronger reaction than the positive shock and is matched by a significant smaller fall in wage (compared with the increase of wage induced by the positive shock), which means that although this model has neither involuntary unemployment nor (explicitly modeled) rigidities, a greater negative reaction of labor during a recession is accompanied by a smaller reaction in real wage. Meanwhile, the opposite does occur after a positive technological shock. This seems to be consistent with the stylized facts of business cycles. For the physical capital, its marginal product also seems to show some rigidity during recessions as compared to booms. For the case of income, the results have been almost trivial. The reaction to a negative shock seems to be greater than the reaction to the positive shock, although the difference between them is almost imperceptible. This can be explained by the fact that the production function is symmetrical, the shocks are also symmetrical, and the movement in capital is compensated by a move in labor in the opposite way.

Since the model proposed here is state-dependent, it is necessary to simulate shocks (positive and negative) while the economy is in boom or in recession. Then, the first exercise was based on the supposal that the economy receives a second shock in the same direction of the first shock. That is, during a boom phase induced by a positive shock, the economy is shocked one more time by a positive shock; during a recession, a new negative shock is received by the economy. Indeed, consumption reacts strongly and deeper to positive shocks, and the boom induced in consumption by the positive shock lasts longer than the recession. The reactions in income are again almost the same, given that the reaction in capital to the positive shock is stronger than its reaction to the negative shock. The fall in investment caused by the negative shock is higher than the increase induced by the positive shock. Thus, there is a severe and apparently long-lasting investment fall. Labor has a stronger reaction to the negative shock than to the positive, which explains why income reactions are not very different as noted above. The smaller fall in wage reveals wage stickiness as an endogenous result in this model. In recession, it is also evident here that, while in boom, there is a higher increase in wage, and the same occurs for marginal capital product.

It was also necessary to simulate positive (negative) shocks while the economy is in recession (expansion). This means a simulation of shocks in the opposite direction of the first shock. In this exercise, it has been clear that a shock in the opposite direction pushes the economy to the other phase of the cycle, thus making it fall from a boom into a recession or jump from a recession into a boom. In the case of capital, it reverses (however) slowly the accumulation (deaccumulation) process induced by a positive (negative shock).

We have seen again the asymmetrical nature of this model when the economy is perturbed by a negative shock while being in a boom. The reaction of consumption on shock when the economy is negatively shocked is greater than the reaction when the shock is positive, but this is only for the first period. Despite that, in general terms, the recession in consumption induced by the negative shock during the boom is less deep and less long-lasting than the boom induced by the positive shock during a recession. For income, the previous result holds even since the period when the economy receives the second shock. In the case of capital, the negative shock during a boom induces a more severe capital deaccumulation than the accumulation induced by the positive shock during the recession. The dynamics of investment is consistent with what happens in capital: the reaction of investment to the second negative shock is stronger and more long-lasting than the reaction to the second positive shock. The explantion for this is the fact that as the economy goes through a boom risk aversion of households, they are led to choose a consumption level as far as possible from the reference point. However, when the economy is in recession, the income fall raises the need of households to adjust their consumption level in such a way that the consumption level is as close as possible to the reference point, which can be explained by loss aversion. For wages and interest rate, the reaction of labor is similar for both shocks, although there is an important difference between the reactions of wage. When the economy is shocked by a negative perturbation during a boom the marginal product of labor shows a reaction weaker than the one showed when the economy receives a positive shock during a recession. Once again this model seems to exhibit rigidity in wage: in recession, the fall in wage is smaller than the increase in boom. For the case of physical capital, when the economy is negatively shocked during the boom, interest rate fall is smaller (during five periods) than the increase when positively shocked during a recession, which is consistent with the greater fall in capital when the negative shocks occurs during a boom.

In general, the model built in this chapter is able to generate asymmetrical business cycles, which proves that asymmetrical behavior of consumers modeled by prospects utility function is a suitable transmission mechanism. In the model, expansions are deeper and more long-lasting for consumption and capital than contractions, while the over smoothing of consumption, facing a negative shock, causes (on shock) a more severe, deeper, and more long-lasting reaction of investment than facing a positive shock. The fall in employment in a recession is severe, deeper, and more long-lasting than in expansion, and is accompanied by a fall in wages less intense than the increase shown by them during the expansion. Thus, this model also reproduces real rigidities in wages and some hysteresis in unemployment. According to Bowman et al. (1999) and with Shea (1995), Loss Aversion implies that the reaction of consumption facing a reduction in expected income is stronger than the reaction facing increase in expected income. This is not inconsistent with the smoother reaction in consumption, because their results are based on a growth rate regression of consumption on interest rate and instruments for expected income, whereas the results in this chapter are derived from GIRF. Moreover, as it can be seen that as expected income decreases, consumption reacts stronger from period 4 on, while as expected income increases, consumption reacts less intensely.

Asymmetries in RBC models could be more adequately captured by General Impulse Response Functions than by higher-order moments. However, a more rigorous test for the properties of the asymmetric model proposed here would entail the application of nonlinear econometric tools. Even though nonlinear econometrics could be a useful tool for this purpose, several issues remain open for the research agenda: i) Structural parameters of the model need to be estimated; ii) the Loss Aversion DSGE model I propose can be used to study issues in policy making, asset pricing, risk premium puzzle, international asymmetric business cycles, risk sharing, home bias, among other areas and disciplines.

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Chapter 3

Loss Aversion, Sticky Prices, and Wages

3.1 Introduction

In this chapter I continue to explore in depth asymmetries in business cycles and their links to the micro-foundations of agents' behavior. To this end, the prospects utility model developed in the second chapter of this dissertation is now used to build the central block of agents' decision-making in an environment characterized by nominal wages and prices rigidities. Besides the asymmetries treated (and documented) in chapters 1 and 2, there is evidence of asymmetries in Phillips Curve, asymmetric adjustment of prices (and wages), and asymmetries in the response of economies to fiscal and monetary policies.

Regarding nonlinear Phillips curve, there is some empirical evidence. Indeed, Pyyhtiä (1999), for country specific and pooled data, has found that the Phillips Curve is asymmetric for Germany, Finland, Italy, the Netherlands, Spain, Austria and France. The asymmetry detected in the Phillips Curve is such that given a positive output gap (observed income greater than potential income), it has a positive effect on inflation; differently, when the gap is negative, the effect on disinflation is very slight and is not significant. This is a signal of price and wages asymmetric adjustment as well. Eliasson (1999) has tested the linearity of Phillips curve for Austria, Sweden and the United States for a sample of quarterly seasonally unadjusted data from 1977 (1) to 1997 (4) for Austria, 1979 (3) - 1997 (4) for Sweden, and 1978 (1) - 1997 (4) for the United States. By using smooth transition regression, she found that the null of linearity is rejected for Austria and Sweden, but not rejected for the United States Phillips curve. Huh (date) uses LSTAR (Logistic Smooth Threshlod Auto Regression) to model several specifications of nonlinear Phillips curve for the U.S. economy, which is used later to derive both the NAIRU and an optimal monetary policy rule that inherits nonlinearity of Phillips curve. For the case of Colombia, Gómez and Julio (2000), by using unobserved components, have found empirical evidence that supports the existence of a nonlinear Phillips curve and a non-constant NAIRU. They also noted that non-linearity of Phillips curve implies non-linearity in sacrifice ratio: the higher the decrease in inflation, the higher the unemployment rate. López and Misas (1999) also encountered evidence of non-linearity and asymmetry in the Phillips curve for Colombia. Flaschel, Gong, and Semmler (2003) studied the implications of a kinked Phillips curve in a Keynesian macro-econometric monetary model. Their simulations of the model using estimated parameters have revealed instability of its steady state and the fact that several optimal policy rules help to stabilize the system.

Among the Neo-Keynesian (NK) DSGE models as proposed by Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), Baxter and Farr (2005), the original aim is to study business cycles in the presence of wage and price rigidities, and how the system evolves facing several shocks. As most DSGE models, these are somewhat successful in explaining business cycles with rigidities. However, neither can they give explanation of business cycle asymmetries, nor they can give explanation of asymmetric stickiness of prices and wages.

With the goal of knowing the link between the agents' behavior and the asymmetric adjustment of prices and wages, and consequently the nonlinear Phillips curve, this chapter presents a modified version of the neokeynesian model presented in Smets and Wouters (2003) including a prospects-utility function, which serves the purpose of modeling asymmetries in consumption and labor choice. This utility function is intra-temporal additive and inter-temporal separable as the reference point is supposed to enter into the utility function as an externality. In the second chapter of this dissertation, it has been used, and succesfully developed, a general prospects theory, which is neither intra-temporal additive nor inter-temporal separable. It means that the utility function is a Cobb-Douglas aggregator of consumption and leisure has been nested into a prospect theory utility function, wherein the reference point is endogenously determined by the choice of consumption and leisure in the previous period. In the first and second chapters of this dissertation, asymmetric business cycles were successfully reproduced by modeling asymmetric investment cost adjustment and prospects theory utility respectively.

Smets and Wouters (2003) has introduced shocks in preferences (on consumption and leisure), mark-ups on wages and goods market, technology, investment, fiscal policy, inflation and monetary rule. This strategy leads to identify and estimate parameters. In this chapter, however, stochastic processes are modeled as autorregresive log normal processes, while shock to goods market mark-up and shock to wage mark-up are modeled as a median plus a perturbation, considering that in Smets and Wouters (2003) all processes have been originally modeled as a mean plus a perturbation. Whereas shocks to policy interest rate are modeled as IID-normal, this shock hereby presented is modeled as the exponential of the IID-normal. In papers such as Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), Baxter and Farr (2005), the model is solved and simulated by log-linearizing the dynamic equations. However, this chapter presents a model solved and simulated by using a third-order perturbations method to guarantee the preservation of utility function asymmetries.

In the second chapter, the model could not only reproduce asymmetries in business cycles, but also generate real rigidities in wage and interest rate, being accompanied by a severe reaction of productive factors during recessions. The model presented here and the simulations performed with it, by using the parameters similar to those estimated in Smets and Wouters (2003), can reproduce asymmetries in business cycles and asymmetries in stickiness of prices and wages. Moreover in this framework, downward rigidity in prices and wages is amplified. Consequently, an asymmetric Phillips curve can be obtained and theoretically explained.

3.2 The model

As in Smets and Wouters (2003) the inter-temporal utility function has the following compact form:

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} U_{t+i}^{\tau} \tag{3.1}$$

However, the instantaneous utility function will take a form which is very similar to that one in the consumption article, although not as a time separable function. That is, the utility will be asymmetric and additive; this variation with respect to the utility function in chapter two is done in order to obtain results comparable to the ones in Smets and Wouters (2003):

$$\begin{aligned} U_t^{\tau} &= W^c\left(\bar{c}_t\right) + \varepsilon_t^b \left(\phi_{ct}\left(z_{ct}\right)^{\underline{\theta}} + \left(1-\phi_{ct}\right)\left(z_{ct}\right)^{\overline{\theta}}\right) \\ &+ W^l\left(\bar{l}_t\right) + \varepsilon_t^b \varepsilon_t^L \left(\phi_{lt}\left(z_{lt}\right)^{\underline{\mu}} + \left(1-\phi_{lt}\right)\left(z_{lt}\right)^{\overline{\mu}}\right) \end{aligned} \tag{3.2}$$

Where $\bar{c}_t$ and $\bar{l}_t$ are reference points for consumption and leisure, $W^c\left(\bar{c}_t\right)$ and $W^l\left(\bar{l}_t\right)$ are utilities delivered by consumption and leisure in the steady state. As in chapter two,$\overline{\theta}$ and $\overline{\mu}$ are such that the utility function is concave (risk-aversion); $\underline{\theta}$ and $\underline{\mu}$ are such that the utility function is convex (loss-aversion).

The reference points for consumption C_t^{τ} and leisure $\mathcal{L}_t$ are defined as

$$\overline{C}_t = (1-\chi)\overline{C}_{t-1} + \chi C_{t-1}^{\tau} \tag{3.3}$$

$$\overline{\mathcal{L}}_t = (1-\chi_l)\overline{\mathcal{L}}_{t-1} + \chi_l(\mathcal{L}_{t-1}^{\tau}) \tag{3.4}$$

Then, the comparisons between the current levels and the reference points for consumption and leisure are defined respectively as:

$$z_{ct} = \frac{C_t^{\tau}}{\overline{C}_t} \tag{3.5}$$

$$z_{lt} = \frac{\mathcal{L}_t^{\tau}}{\overline{\mathcal{L}}_t} \tag{3.6}$$

Being l_t^{τ} the time, the agent is then willing to offer in the labor market, so the time constraint is:

$$\mathcal{L}_t^{\tau} = 1 - l_t^{\tau} \tag{3.7}$$

The smooth transition functions for consumption ϕ_{ct}, and for leisure $\phi_{\mathcal{L}t}$ are given by

$$\phi_{ct} = \frac{1}{1 + \exp\gamma_c\left(z_{ct} - 1\right)} \tag{3.8}$$

$$\phi_{\mathcal{L}t} = \frac{1}{1 + \exp\gamma_{\mathcal{L}}(z_{\mathcal{L}t} - 1)} \tag{3.9}$$

Their first derivatives of the smooth transition functions with respect to z_{ct} and z_{lt} are given by

$$\frac{\partial \phi_{ct}}{\partial z_{ct}} = \frac{-\gamma_c \exp\gamma_c\left(z_{ct} - 1\right)}{\left[1 + \exp\gamma_c\left(z_{ct} - 1\right)\right]^2} \tag{3.10}$$

$$\frac{\partial \phi_{\mathcal{L}t}}{\partial z_{lt}} = \frac{-\gamma_{\mathcal{L}} \exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t} - 1\right)}{\left[1 + \exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t} - 1\right)\right]^2} \tag{3.11}$$

ThThe stochastic process for shocks to preferences are log-normal:

$$\ln\varepsilon_t^b = \rho_b \ln\varepsilon_{t-1}^b + \eta_t^b \tag{3.12}$$

$$\ln\varepsilon_t^L = \rho_L \ln\varepsilon_{t-1}^L + \eta_t^L \tag{3.13}$$

$\eta_t^b$and η_t^L are homoscedastic and mean zero stochastic shocks.

The labor income is given by $w_t^{\tau} l_t^{\tau}$; $r_t^k z_t^{\tau} K_{t-1}^{\tau}$ is income of capital; $\Psi(z_t^{\tau})K_{t-1}^{\tau}$ is the cost of adjusting capital, Div_t^{τ} dividends from firms and A_t^{τ} are state-dependent securities, $B_t^{\tau}$is the financial wealth represented in bonds, and I_t^{τ} is physical capital investment. Thus, the real terms inter-temporal budget constraint becomes:

$$b_t \frac{B_t^{\tau}}{P_t} = \frac{B_{t-1}^{\tau}}{P_t} + Y_t^{\tau} - C_t^{\tau} - I_t^{\tau} \tag{3.14}$$

where $b_t = \frac{1}{1+i_t}$ is the nominal bond price and Y_t^{τ} is given by:

$$Y_t^{\tau} = (w_t^{\tau} l_t^{\tau} + A_t^{\tau}) + (r_t^k z_t^{\tau} K_{t-1}^{\tau} - \Psi(z_t^{\tau})K_{t-1}^{\tau}) + Div_t^{\tau} \tag{3.15}$$

The Lagrangian for this decentralized economy is:

$$\begin{aligned}
\Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} & \left[\begin{array}{c} W^c\left(\overline{c}_{t+i}\right) + \varepsilon_{t+i}^b \left(\phi_{ct+i}\left(z_{ct+i}\right)^{\underline{\theta}} + \left(1 - \phi_{ct+i}\right)\left(z_{ct+i}\right)^{\overline{\theta}}\right) \\ +W^l\left(\overline{l}_{t+i}\right) + \varepsilon_{t+i}^b \varepsilon_{t+i}^L \left(\phi_{lt+i}\left(z_{lt+i}\right)^{\underline{\mu}} + \left(1 - \phi_{lt+i}\right)\left(z_{lt+i}\right)^{\overline{\mu}}\right) \end{array} \right] \\
+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} & \left[\begin{array}{c} -b_{t+i}\frac{B_{t+i}^{\tau}}{P_{t+i}} + \frac{B_{t+i-1}^{\tau}}{P_{t+i}} + \left(\frac{W_{t+i}^{\tau} l_{t+i}^{\tau}}{P_{t+i}} + A_{t+i}^{\tau}\right) \\ +(r_{t+i}^k z_{t+i}^{\tau} K_{t+i-1}^{\tau} - \Psi(z_{t+i}^{\tau})K_{t+i-1}^{\tau}) + Div_{t+i}^{\tau} - C_{t+i}^{\tau} - I_{t+i}^{\tau} \end{array} \right] \\
+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} & \left[K_{t+i}(1-\tau) + \left[1 - S\left(\varepsilon_{t+i}^I I_{t+i}/I_{t+i-1}\right)\right] I_{t+i} - K_{t+i+1}\right]
\end{aligned} \tag{3.16}$$

$$\max_{C_{t+i},\widetilde{W}_{t+i},K_{t+i},I_{t+i},B^{\tau}_{t+i},z^{\tau}_{t+i}} \Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[... + \varepsilon^b_{t+i} \left(\phi_{ct+i} \left(z_{ct+i} \right)^{\underline{\theta}} + \left(1 - \phi_{ct+i} \right) \left(z_{ct+i} \right)^{\overline{\theta}} \right) \right] + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... - C^{\tau}_{t+i} ... \right] \tag{3.17}$$

The first order condition for consumption is:

$$\lambda_t = \varepsilon^b_t \frac{\overline{\theta} \left(z_{ct} \right)^{\overline{\theta}-1}}{\overline{C}_t} + \varepsilon^b_t \phi_{ct} \left[\frac{\underline{\theta} \left(z_{ct} \right)^{\underline{\theta}-1}}{\overline{C}_t} - \frac{\overline{\theta} \left(z_{ct} \right)^{\overline{\theta}-1}}{\overline{C}_t} \right] + \varepsilon^b_t \frac{-\gamma_c \exp \gamma_c \left(z_{ct} - 1 \right)}{\left[1 + \exp \gamma_c \left(z_{ct} - 1 \right) \right]^2} \frac{\left[\left(z_{ct} \right)^{\underline{\theta}} - \left(z_{ct} \right)^{\overline{\theta}} \right]}{\overline{C}_t} \tag{3.18}$$

The left side of the equation 3.18 is the Lagrange multiplier for the inter-temporal restriction, and the right side has three terms: the first and second terms sum up the derivative of the prospect utility function. In the traditional (risk-averse) symmetric model, the marginal utility of consumption would be $\varepsilon^b_t \frac{\overline{\theta}(z_{ct})^{\overline{\theta}-1}}{\overline{C}_t}$. But, in this more general set up, the global marginal utility comprises the marginal utility of consumption for both regimes: under recession (loss-averse) and under boom (risk-averse), and the respective smooth transition between them. The third term on the left is the change in the transition function multiplied by the difference of the utilities level in both regimes. Thus, if we were supposed to model the regime switching marginal utility by only modeling the transition between them, we would be mistakenly specifying the regime switching model and, therefore, its predictions. This particularity is pointed out in the first chapter of this dissertation when explaining that most of the regime switching RBC works are rather imprecise because they impose a transition matrix (with fixed transition probabilities) on the canonical system of dynamic equations.

$$\max_{C_{t+i},\widetilde{W}_{t+i},K_{t+i},I_{t+i},B^{\tau}_{t+i},z^{\tau}_{t+i}} \Lambda_t = ... + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[-b_{t+i} \frac{B^{\tau}_{t+i}}{P_{t+i}} + \frac{B^{\tau}_{t+i-1}}{P_{t+i}} ... \right]$$

$$...$$

The first order condition for bonds is:

$$\beta^{t+i} \lambda_{t+i} b_{t+i} \frac{1}{P_{t+i}} = E_t \left[\beta^{t+i+1} \lambda_{t+i+1} \frac{1}{P_{t+i+1}} \right] \tag{3.19}$$

$$b_t \lambda_t = \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{3.20}$$

Equation 3.20 sets the condition for the equalization between the real interest rate and the marginal rate of substitution (we must keep in mind that λ_t is the marginal utility of consumption in period t). Because marginal utilities are asymmetrical, so it is the marginal rate of substitution and so will be its reactions to movements in real interest rates originated by inflation or by interest rate policy.

3.2.1 Labor supply decisions and wage setting equation

Under the assumption that wages can be adjusted with probability $1 - \xi_w$, households choose a new optimal wage $\widetilde{w}^{\tau}_t$ taking into account that in the future wages will unlikely be adjusted once again. Thus, there will be a partial wage indexation for those who will not be able to re-optimize:

$$W^{\tau}_t = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W^{\tau}_{t-1} \tag{3.21}$$

Households set their nominal wage to maximize their inter-temporal utility function subject to the budget constraint. Similarly, the labor demand will be determined as follows. If the aggregate labor demanded by firms is given as:

$$L_t = \left[\int_0^1 (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}}\, d\tau\right]^{1+\lambda_{w,t}} \tag{3.22}$$

Each unit of labor l_t^τ is paid a nominal wage W_t^τ, then the total labor expenditure is$\int_0^1 l_t^\tau W_t^\tau d\tau$. Thus, the problem of the firm is to minimise the total labor expenditure given it need a quantity of aggregate labor L_t. Minimizing in l_t^τ the first order condition for this problem is

$$W_t^\tau - \mu\left[\int_0^1 (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}}\, d\tau\right]^{\lambda_{w,t}} (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}-1} = 0$$

and solvin for l_t^τ we have:

$$l_t^\tau = \left(\frac{W_t^\tau}{\mu}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{3.23}$$

by replacing 3.23 into 3.22, we have; then, solving for μ:

$$\mu = \left[\int_0^1 (W_t^\tau)^{-1/\lambda_{w,t}}\, d\tau\right]^{-\lambda_{w,t}} = W_t \tag{3.24}$$

Which is a Dixit-Stiglitz wage aggregator. Finally, after replacing 3.24 into 3.23 we have the demand for labor:

$$l_t^\tau = \left(\frac{W_t^\tau}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{3.25}$$

Then, if we define ξ_w^i as the probability that in time i the wage cannot change; thereby, it is necessary to index it based on the one re-optimized in t; hence, the indexed wage will be:

$$W_{t+i}^\tau = (X_{t,t+i})^{\gamma_w}\, \widetilde{W}_t \tag{3.26}$$

Thus, the demand for labor will be:

$$l_{t+i}^\tau = \left(\frac{W_{t,t+i}^\tau}{W_{t+i}}\right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{3.27}$$

$$X_{t,t+i} = \frac{P_{t+i-1}}{P_{t-1}} \tag{3.28}$$

Thus, $1-\xi_w^i$ is the probability of having changed optimally the price between t and $t+i$. Now, we will suppose that $t < t^* < t+i$ if households can optimize wages in t^*. Then, the wage W_{t+i} depends on the optimal wage $\widetilde{W}_{t^*}^\tau$ set in t^*, but not on the optimal wage $\widetilde{W}_t^\tau$ set in t. Therefore, we may claim that

$$W_{t+i}^\tau = \widetilde{W}_{t^*}\, (X_{t^*,t+i})^{\gamma_w} \tag{3.29}$$

where

$$X_{t^*,t+i} = \frac{P_{t+i-1}}{P_{t^*-1}} \tag{3.30}$$

and we define $W_{t^*>t,t+i}^\tau$ as the wage $t+i$ given that the last time it was changed optimally was in time $t^* > t$. $W_{t,t+i}^\tau$ as the wage in $t+i$ given that the last time it was changed optimally was in time t.

According to this, the wage can be written as:

$$\xi_w^i W_{t,t+i}^{\tau} + (1-\xi_w^i) W_{t^*>t,t+i}^{\tau} = \xi_w^i \left(X_{t,t+i}\right)^{\gamma_w} \widetilde{W}_t + (1-\xi_w^i) W_{t^*>t,t+i}^{\tau} \tag{3.31}$$

And the demands for labor will become:

$$l_{t+i}^{\tau} = \left(\frac{\left(X_{t,t+i}\right)^{\gamma_w} \widetilde{W}_t}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{3.32}$$

$$l_{t^*>t,t+i}^{\tau} = \left(\frac{W_{t^*>t,t+i}^{\tau}}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{3.33}$$

Then, the Lagrangian function for the inter-temporal utility function will be:

$$\max_{C_{t+i},\widetilde{W}_{t+i},K_{t+i},I_{t+i},B_{t+i}^{\tau},z_{t+i}^{\tau}} \Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\begin{array}{c} W^c\left(\bar{c}_{t+i}\right) + W^l\left(\bar{l}_{t+i}\right) + \varepsilon_{t+i}^b \left(\phi_{ct+i}\left(z_{ct+i}\right)^{\underline{\theta}} + \left(1-\phi_{ct+i}\right)\left(z_{ct+i}\right)^{\overline{\theta}} \right) \\ +\varepsilon_{t+i}^b \varepsilon_{t+i}^L \left[\begin{array}{c} \xi_w^i \left(\phi_{lt+i}\left(z_{lt+i}\right)^{\underline{\mu}} + \left(1-\phi_{lt+i}\right)\left(z_{lt+i}\right)^{\overline{\mu}} \right) \\ +(1-\xi_w^i)\left(\phi_{l,t^*>t,t+i}\left(z_{l,t^*>t,t+i}\right)^{\underline{\mu}} + \left(1-\phi_{l,t^*>t,t+i}\right)\left(z_{l,t^*>t,t+i}\right)^{\overline{\mu}} \right) \end{array} \right] \end{array} \right]$$
$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[\begin{array}{c} -b_{t+i}\frac{B_{t+i}^{\tau}}{P_{t+i}} + \frac{B_{t+i-1}^{\tau}}{P_{t+i}} + \left(\frac{\xi_w^i W_{t,t+i}^{\tau} l_{t,t+i}^{\tau} + (1-\xi_w^i) W_{t^*>t,t+i}^{\tau} l_{t^*>t,t+i}^{\tau}}{P_{t+i}} + A_{t+i}^{\tau} \right) \\ +(r_{t+i}^k z_{t+i}^{\tau} K_{t+i-1}^{\tau} - \Psi(z_{t+i}^{\tau}) K_{t+i-1}^{\tau}) + Div_{t+i}^{\tau} - C_{t+i}^{\tau} - I_{t+i}^{\tau} \end{array} \right]$$
$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} \left[K_{t+i-1}\left[1-\tau\right] + \left[1 - S\left(\varepsilon_{t+i}^I I_{t+i}/I_{t+i-1}\right)\right] I_{t+i} - K_{t+i} \right]$$

The previous equation can be expressed compactly leaving explicit only those parts that depend on labor choice and wages:

$$\Lambda_t = E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[... + \varepsilon_{t+i}^b \varepsilon_{t+i}^L \left[\begin{array}{c} \xi_w^i \left(\phi_{lt+i}\left(z_{lt+i}\right)^{\underline{\mu}} + \left(1-\phi_{lt+i}\right)\left(z_{lt+i}\right)^{\overline{\mu}} \right) \\ +(1-\xi_w^i)\left(\phi_{l,t^*>t,t+i}\left(z_{l,t^*>t,t+i}\right)^{\underline{\mu}} + \left(1-\phi_{l,t^*>t,t+i}\right)\left(z_{l,t^*>t,t+i}\right)^{\overline{\mu}} \right) \end{array} \right] \right]$$
$$+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... + \frac{\xi_w^i W_{t,t+i}^{\tau} l_{t,t+i}^{\tau}}{P_{t+i}} + \frac{(1-\xi_w^i) W_{t^*>t,t+i}^{\tau} l_{t^*>t,t+i}^{\tau}}{P_{t+i}} + ... \right]$$

As the maximization of Λ_t on $\widetilde{W}_t$ does only affect the expectations related to the probability ξ_w^i , the previous maximization problem is equal to maximizing the following function:

$$\begin{aligned} \Lambda_t = {} & E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[... + \varepsilon_{t+i}^b \varepsilon_{t+i}^L \left[\xi_w^i \left(\phi_{lt+i}\left(z_{lt+i}\right)^{\underline{\mu}} + \left(1-\phi_{lt+i}\right)\left(z_{lt+i}\right)^{\overline{\mu}} \right) \right] \right] \\ & + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... + \frac{\xi_w^i W_{t,t+i}^{\tau} l_{t,t+i}^{\tau}}{P_{t+i}} + ... \right] \end{aligned} \tag{3.34}$$

recalling that

$$l_{t+i}^{\tau} = \left(\frac{\left(X_{t,t+i}\right)^{\gamma_w} \widetilde{W}_t}{W_{t+i}} \right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_t \tag{3.35}$$

and

$$W_{t+i}^{\tau} = \widetilde{W}_{t^*} \left(X_{t^*,t+i}\right)^{\gamma_w} \tag{3.36}$$

The first-order condition with respect to the re-optimized wage will be:

$$\frac{\partial \Lambda_t}{\partial \widetilde{W}_t} = E_t \sum_{i=0}^{\infty} \beta^{t+i} \xi_w^i \frac{\partial U_{t+i}^{\tau}}{\partial \mathcal{L}_{t+i}} \frac{\partial \mathcal{L}_{t+i}}{\partial l_{t+i}} \frac{\partial l_{t+i}}{\partial \widetilde{W}_t} + E_0 \sum_{i=0}^{\infty} \beta^{t+i} \frac{\lambda_{t+i} \xi_w^i}{P_{t+i}} \left(\frac{\partial W_{t+i}^{\tau}}{\partial \widetilde{W}_t} l_{t,t+i}^{\tau} + \frac{\partial l_{t,t+i}^{\tau}}{\partial \widetilde{W}_t} W_{t+i}^{\tau} \right) = 0 \tag{3.37}$$

$$-E_t \sum_{i=0}^{\infty} \beta^{t+i} \xi_w^i \frac{\partial U_{t+i}^{\tau}}{\partial \mathcal{L}_{t+i}} \frac{\partial \mathcal{L}_{t+i}}{\partial l_{t+i}} \frac{\partial l_{t+i}}{\partial \widetilde{W}_t} = E_0 \sum_{i=0}^{\infty} \beta^{t+i} \frac{\lambda_{t+i} \xi_w^i}{P_{t+i}} \left(\frac{\partial W_{t+i}^{\tau}}{\partial \widetilde{W}_t} l_{t,t+i}^{\tau} + \frac{\partial l_{t,t+i}^{\tau}}{\partial \widetilde{W}_t} W_{t+i}^{\tau} \right). \tag{3.38}$$

Recall that $\frac{\partial \mathcal{L}_t}{\partial l_t} < 0$. Hence, the right side of this equation will be always positive as well as the left one. Taking into account that $\lambda_{t+1} = \frac{\partial U_{t+1}^{\tau}}{\partial C_{t+1}} = U_{t+1}^C$, we will have:

$$-E_t \sum_{i=0}^{\infty} \beta^{t+i} \xi_w^i \frac{\partial U_{t+i}^{\tau}}{\partial \mathcal{L}_{t+i}} \frac{\partial \mathcal{L}_{t+i}}{\partial l_{t+i}} \frac{\partial l_{t+i}}{\partial \widetilde{W}_t} = E_t \sum_{i=0}^{\infty} \beta^{t} \frac{U_{t+i}^C \xi_w^i}{P_{t+i}} \left(\frac{\partial W_{t+i}^{\tau}}{\partial \widetilde{W}_t} l_{t,t+i}^{\tau} + \frac{\partial l_{t,t+i}^{\tau}}{\partial \widetilde{W}_t} W_{t+i}^{\tau} \right) \tag{3.39}$$

where $\frac{\partial W_{t+i}^{\tau}}{\partial \widetilde{W}_t} = (X_{t^*,t+i})^{\gamma_w}$ and

$$\frac{\partial l_{t,t+i}^{\tau}}{\partial \widetilde{W}_t} = -\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}} l_{t+i}^{\tau} \left(\widetilde{W}_t \right)^{-1} \tag{3.40}$$

thus we will have finally:

$$E_0 \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\partial U_t^{\tau}}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}} l_{t+i}^{\tau} \right) = \frac{\widetilde{W}_t}{P_t} E_0 \sum_{i=0}^{\infty} \beta^i U_{t+i}^C \xi_w^i l_{t,t+i}^{\tau} \frac{P_t}{P_{t+i}} \left(\frac{P_{t+i-1}}{P_{t-1}} \right)^{\gamma_w} \frac{1}{\lambda_{w,t+i}} \tag{3.41}$$

If $\gamma_w = 1$ and if $\lambda_{w,t+i} = \lambda_w$, the inter-temporal utility maximization problem, since the point of view of the optimal wage choice, will have as solution :

$$\frac{\widetilde{W}_t}{P_t} E_0 \sum_{i=0}^{\infty} \beta^i U_{t+i}^C \xi_w^i l_{t,t+i}^{\tau} \left(\frac{P_t/P_{t-1}}{P_{t+i}/P_{t+i-1}} \right) \frac{1}{1+\lambda_{w,t+i}} = E_0 \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\partial U_t^{\tau}}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(l_{t+i}^{\tau} \right) \tag{3.42}$$

This equation is very similar to the one derived in Smets and Wouters (2003). However, given the asymmetric nature of our utility function, this re-optimized wage equation also inherits such asymmetry from the marginal utility function of consumption and labor[1]. Because our utility function is asymmetric around the reference point, so are the marginal utilities of labor and consumption. Thus, nominal wage setting will be asymmetric.

If we suppose the wage mark-up shocks $\lambda_{w,t} = \lambda_w + \eta_t^w$ as normal-IID around a constant, considering the aggregate wage equation, the movement law of the aggregate wage index will be given as:

$$\left(W^t \right)^{-1/\lambda_{w,t}} = \xi \left(W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \right)^{-1/\lambda_{w,t}} + (1-\xi)(\widetilde{w}_t)^{-1/\lambda_{w,t}} \tag{3.43}$$

3.2.2 Investment and capital accumulation

he law of capital accumulation goes as follows: being τ the depreciation rate, $S(.)$ a positive adjustment cost function of investment changes, this function is zero in the steady state as investment is constant. The stochastic process of shocks to investment is:

$$\ln \varepsilon_t^I = \rho_I \ln \varepsilon_{t-1}^I + \eta_t^I \tag{3.44}$$

[1] As a matter of fact, because the utility function is intratemporal separable, this wage setting equation has two sources of asymmetry: marginal utility of consumption and marginal (dis)utility of working

The first-order conditions give rise to the following dynamic equations of the real value of capital, the investment and the utilization rate of capital:

Maximizing on capital utilization:

$$\max_{...z^{\tau}_{t+i}...} \Lambda_t = ... + E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[+(r^k_{t+i} z^{\tau}_{t+i} K^{\tau}_{t+i-1} - \Psi(z^{\tau}_{t+i}) K^{\tau}_{t+i-1}) + Div^{\tau}_{t+i} - C^{\tau}_{t+i} - I^{\tau}_{t+i} \right]$$

...

$$r^k_t = \Psi'(z^{\tau}_t) \tag{3.45}$$

Maximizing on physical capital:

$$\begin{aligned} \max_{...K_{t+i},...} \Lambda_t = ... &+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... + (r^k_{t+i} z^{\tau}_{t+i} K^{\tau}_{t+i-1} - \Psi(z^{\tau}_{t+i}) K^{\tau}_{t+i-1}) + Div^{\tau}_{t+i} - C^{\tau}_{t+i} - I^{\tau}_{t+i} \right] \\ &+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} \left[K_{t+i-1} \left[1 - \tau \right] + \left[1 - S\left(\varepsilon^I_{t+i} I_{t+i} / I_{t+i-1} \right) \right] I_{t+i} - K_{t+i} \right] \end{aligned}$$

Produces the first order condition:

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[Q_{t+1} \left(1 - \tau \right) + (r^k_{t+1} z^{\tau}_{t+1} - \Psi(z^{\tau}_{t+1})) \right] \right] = Q_t \tag{3.46}$$

Being $\frac{\mu_t}{\lambda_t} = Q_t$

Maximizing on investment:

$$\begin{aligned} \max_{...I_{t+i}...} \Lambda_t = ... &+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left[... - I^{\tau}_{t+i} \right] \\ &+ E_t \sum_{i=0}^{\infty} \beta^{t+i} \mu_{t+i} \left[... + \left[1 - S\left(\varepsilon^I_{t+i} I_{t+i} / I_{t+i-1} \right) \right] I_{t+i} ... \right] \end{aligned}$$

Produces the first order condition:

$$\begin{aligned} 1 = & Q_t \left[1 - \frac{\partial S\left(\varepsilon^I_t I_t / I_{t-1} \right)}{\partial I_t} \left(\frac{\varepsilon^I_t}{I_{t-1}} \right) I_t - S\left(\varepsilon^I_t I_t / I_{t+1} \right) \right] \\ & + \beta E_t \left\{ Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\partial S\left(\varepsilon^I_{t+1} I_{t+1} / I_t \right)}{\partial I_t} \varepsilon^I_{t+1} I^2_{t+1} \frac{1}{I^2_t} \right] \right\} \end{aligned} \tag{3.47}$$

3.2.3 Technologies and firms

There is a continuum of monopolistic firms that produce intermediate goods, which are indexed by $j, j \in [0,1]$. In the final goods sector , there is a competitive firm that purchases intermediate goods from a continuum of firms in a competitive way. Thus, the final goods supply, which is used for consumption and investment, is given by:

$$Y_t = \left[\int_0^1 \left(y^j_t \right)^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}} \tag{3.48}$$

y_t^j denoting the quantity of domestic intermediate goods type, j is used for the final good in time t, and $\lambda_{p,t}$ is a stochastic parameter that determines a variable margin in the goods market. It is supposed that $\lambda_{p,t} = \lambda_p + \eta_t^p$, being η_t^p IID-normal. As in the case of labor demand, the firm minimizes the cost of buying intermediate goods y_t^j by paying nominal prices P_t^j. -Consequently, the demand for intermediate good j is

$$y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{3.49}$$

And the price aggregator for this problem is

$$P_t = \left[\int_0^1 \left(P_t^j\right)^{-1/(\lambda_{p,t})} dj\right]^{-\lambda_{p,t}} \tag{3.50}$$

For the intermediate goods producer, the technology of production is:

$$y_t^j = \varepsilon_t^a \widetilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi \tag{3.51}$$

$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a \tag{3.52}$$

$\widetilde{K}_{j,t}$ is the effective capital utilization $\widetilde{K}_{j,t} = z_t K_{j,t-1}$, $L_{j,t}$ is the index of different types of labor used by the firms. Φ is a fixed cost. Cost minimization implies:

$$\min \frac{W L_{j,t}}{P_t} + r_t^k \widetilde{K}_{j,t} + MC_t \left[y_t^j - \varepsilon_t^a \widetilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} + \Phi\right]$$

where the first-order conditions for labor and physical capital lead us to:

$$\frac{W_t}{P_t} = MC_t(1-\alpha)\varepsilon_t^a \widetilde{K}_{j,t}^{\alpha} L_{j,t}^{-\alpha}$$

$$r_t^k = MC_t \alpha \varepsilon_t^a \widetilde{K}_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha}$$

Solving for $\frac{W_t}{P_t}$ we have:

$$\frac{W_t}{P_t}\frac{L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha} \tag{3.53}$$

which means that capital and labor are the same for every firm j. Then, solving for real marginal costs of firms:

$$MC_t = \frac{1}{\varepsilon_t^a}\left(\frac{W_t}{P_t}\right)^{1-\alpha} \left(r_t^k\right)^{\alpha} \left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right) \tag{3.54}$$

Thus, nominal profits of firm j would be given by:

$$\pi_t^j = (P_t^j - P_t MC_t)\left(\frac{P_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t - P_t MC_t \Phi \tag{3.55}$$

Given that the stochastic discount rate of firms is $\beta^i \frac{\lambda_{t+i}}{\lambda_t P_{t+i}}$, the inter-temporal profits of producers that can re-optimize their prices in time t are:

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t P_{t+i}} \left((P_{t+i}^j - P_{t+i} MC_{t+i})\left(\frac{P_{t+i}^j}{P_{t+i}}\right)^{-\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}} Y_{t+i} - P_{t+i} MC_{t+i} \Phi\right)$$

and because of indexation of prices $P_{t+i}^{j}=\widetilde{P}_{t}^{j}\left(X_{t,t+i}\right)^{\gamma_{p}}$

$$E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t P_{t+i}}\left(P_{t+i}^j\left(\frac{P_{t+i}^j}{P_{t+i}}\right)^{-\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}}Y_{t+i}-P_{t+i}MC_{t+i}\left(\frac{P_{t+i}^j}{P_{t+i}}\right)^{-\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}}Y_{t+i}-P_{t+i}MC_{t+i}\Phi\right)$$

the firm maximizes its inter-temporal profits by choosing $\widetilde{P}_t^j$:, which leads us to the following firt order condition for optimal price setting:

$$\widetilde{P}_t^j E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}\frac{\left(X_{t,t+i}\right)^{\gamma_p}}{P_{t+i}}y_{t+1}^j\frac{1}{\lambda_{p,t+i}}=E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+1}^j\left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}\right)$$

and after some few algebra, finally, the first order condition for re-optimizing firms is:

$$\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}\frac{\left(P_{t+i-1}/P_t\right)^{\gamma_p}}{P_{t+i}/P_t}y_{t+1}^j\frac{1}{\lambda_{p,t+i}}=E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+1}^j\left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}\right)$$

Given the definition of prices, the motion law for them is:

$$\left(P_t\right)^{-1/\lambda_{p,t}}=\xi_p\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}\right)^{-1/\lambda_{p,t}}+\left(1-\xi_p\right)\left(\widetilde{P}_t^j\right)^{-1/\lambda_{p,t}} \tag{3.56}$$

The market clearing of the economy is given by:

$$Y_t=C_t+G_t+I_t+\Psi(z_t)K_{t-1} \tag{3.57}$$

$G_t=Y_{ss}gy_t$, and $gy_t=gy\varepsilon_t^g$, $\ln(\varepsilon_t^g)=\rho_g\ln(\varepsilon_{t-1}^g)+\eta_t^g$

3.2.4 First order conditions and asymmetry

Given the utility function, the marginal utility of consumption and leisure will be given by 3.58 and 3.59 respectively:

$$\begin{aligned}\frac{\partial U_{t+1}^{\tau}}{\partial C_{t+1}}=U_t^{C\tau}&=\varepsilon_t^b\frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}+\varepsilon_t^b\frac{\partial\phi_{ct}}{\partial z_{ct}}\frac{\left[\left(z_{ct}\right)^{\underline{\theta}}-\left(z_{ct}\right)^{\overline{\theta}}\right]}{\overline{C}_t}\\&+\varepsilon_t^b\phi_{ct}\left[\frac{\underline{\theta}\left(z_{ct}\right)^{\underline{\theta}-1}}{\overline{C}_t}-\frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}\right]\end{aligned} \tag{3.58}$$

$$\begin{aligned}U_t^{\mathcal{L}\tau}=\frac{\partial U_t^{\tau}}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}&=\varepsilon_t^b\varepsilon_t^L\frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t}+\varepsilon_t^b\varepsilon_t^L\frac{\partial\phi_{\mathcal{L}t}}{\partial z_{lt}}\frac{\left[\left(z_{\mathcal{L}t}\right)^{\underline{\mu}}-\left(z_{\mathcal{L}t}\right)^{\overline{\mu}}\right]}{\overline{\mathcal{L}}_t}\\&+\varepsilon_t^b\varepsilon_t^L\phi_{\mathcal{L}t}\left[\frac{\underline{\mu}\left(z_{\mathcal{L}t}\right)^{\underline{\mu}-1}}{\overline{\mathcal{L}}_t}-\frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t}\right]\end{aligned} \tag{3.59}$$

Then, the first order conditions for consumption will be expressed as:

$$E_t\left[\beta\frac{\lambda_t}{\lambda_{t+1}}\frac{R_tP_t}{P_{t+1}}\right]=1 \tag{3.60}$$

$$\lambda_t=\varepsilon_t^b\frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}+\varepsilon_t^b\frac{-\gamma\left(z_{ct}\right)^{\gamma-1}}{\left[1+\left(z_{ct}\right)^{\gamma}\right]^2}\frac{\left[\left(z_{ct}\right)^{\underline{\theta}}-\left(z_{ct}\right)^{\overline{\theta}}\right]}{\overline{C}_t}+\varepsilon_t^b\phi_{ct}\left[\frac{\underline{\theta}\left(z_{ct}\right)^{\underline{\theta}-1}}{\overline{C}_t}-\frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}\right] \tag{3.61}$$

updating up to time $t+1$

$$\lambda_{t+1} = \varepsilon_{t+1}^{b}\frac{\overline{\theta}\left(z_{ct+1}\right)^{\overline{\theta}-1}}{\overline{C}_{t+1}} + \varepsilon_{t+1}^{b}\frac{-\gamma\left(z_{ct+1}\right)^{\gamma-1}}{\left[1+\left(z_{ct+1}\right)^{\gamma}\right]^{2}}\frac{\left[\left(z_{ct+1}\right)^{\underline{\theta}}-\left(z_{ct+1}\right)^{\overline{\theta}}\right]}{\overline{C}_{t+1}} + \varepsilon_{t+1}^{b}\phi_{ct+1}\left[\frac{\underline{\theta}\left(z_{ct+1}\right)^{\underline{\theta}-1}}{\overline{C}_{t+1}} - \frac{\overline{\theta}\left(z_{ct+1}\right)^{\overline{\theta}-1}}{\overline{C}_{t+1}}\right] \tag{3.62}$$

As it can be seen, by replacing 3.61 and 3.62 in 3.60 it is possible to obtain an asymmetric Euler Equation for consumption.

The policy rule or reaction function of the monetary authority is expressed as in Smets-Wouters (2003) as deviations from the log-linearized steady state:

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\{\overline{\pi}_t + r_{\pi}(\widehat{\pi}_{t-1} - \overline{\pi}_t) + r_y(\widehat{Y}_t - \widehat{Y}_t^p)\} + r_{\Delta\pi}(\widehat{\pi}_t - \widehat{\pi}_{t-1}) + r_{\Delta y}(\widehat{Y}_t - \widehat{Y}_t^p - (\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^p)) + \eta_t^R \tag{3.63}$$

where $\widehat{x}$ denotes logarithmic deviations of x from the steady state. Inflation target is represented by $\overline{\pi}_t$. It follows an auto-regressive process $\overline{\pi}_t = \rho\overline{\pi}_{t-1} + \eta_t^{\pi}$. Finally, η_t^R is a transitory IID-normal shock on the interest rate, which is denoted as a monetary policy shock. However, we will use the Taylor rule expressed in levels because we are to use Perturbations Method implemented in Dynare in order to solve this model:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{r_R}\left[\left(\frac{\Pi_t}{\Pi}\right)^{r_{\Pi}}\left(\frac{Y_t}{Y}\right)^{r_Y}\right]^{1-r_R}\varepsilon_t^R \tag{3.64}$$

being $\varepsilon_t^R = \exp(\eta_t^R)$.

The first-order conditions, the closures, the equilibrium market and the monetary policy rule represent the dynamic system of this hypothetical economy.

3.3 Calibration and simulation

We need some concrete functional forms to make the model operative. Also in this dissertation, the solution of the dynamic system is made by using third-order approximation in order to preserve the asymmetric nature of:

for the adjustment cost of investment, we use the following equation:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{\varepsilon_t I_t}{I_{t-1}} - 1\right)^2 \tag{3.65}$$

This functional function fulfills the properties stated by Smets and Wouters (2003): $S(1) = S'(1) = 0$, and $S''(I) > 0$.

For the utilization cost of capital utilization rate, we will follow Baxter and Farr (2005), but we will introduce a slight variation: whereas they model a convex function for capital depreciation depending on capital utilization rate, we will introduce the same functional form to express the cost of choosing z_t in terms of consumption goods. This function must fulfill $\Psi(1) = 0$ and $\Psi''(1)/\Psi'(1) = \varsigma$, as specified by Baxter and Farr (2005), Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003).

$$\Psi(z_t) = \Psi_0 + \Psi_1\frac{z_t^{1+\varsigma}}{1+\varsigma} \tag{3.66}$$

Thus, it is required that $\Rightarrow \Psi_0 = \frac{-\Psi_1}{1+\varsigma}$. Therefore, and with no loss of generality in order to facilitate calibration, we set $\Psi(z_t) = -\frac{r^k}{1+\varsigma} + r^k\frac{z_t^{1+\varsigma}}{1+\varsigma}$, which is an increasing convex function as suggested by Christiano, Eichenbaum, and Evans (2005).[2] The full dynamic system is summarized in annex 1. In the steady state, this must take place: $z_c = 1$, $z_{\mathcal{L}} = 1$, $\overline{C}_t = C^{\tau}$, $\overline{\mathcal{L}}_t = \mathcal{L}$, $\mathcal{L}^{\tau} = 1 - l^{\tau}$, $\phi_{ct} = 0.5$, $\phi_{\mathcal{L}t} = 0.5$, $\frac{\partial \phi_{ct}}{\partial z_{ct}} = -0.25\gamma_c$, $\frac{\partial \phi_{\mathcal{L}t}}{\partial z_{lt}} = -0.25\gamma_{\mathcal{L}}$, $\beta R_t = \frac{P_{t+1}}{P_t} = (1+\Pi)$, $\lambda_t = \frac{\overline{\theta}}{\overline{C}_t} + 0.5\left[\frac{\underline{\theta}}{\overline{C}_t} - \frac{\overline{\theta}}{\overline{C}_t}\right] = \frac{0.5}{C_t}\left[\underline{\theta} + \overline{\theta}\right]$, $U_t^{\mathcal{L}\tau} = \frac{\partial U_t^{\tau}}{\partial \mathcal{L}_t}\frac{\partial \mathcal{L}_t}{\partial l_t} = -\frac{\overline{\mu}}{\overline{\mathcal{L}}_t} - 0.5\left[\frac{\underline{\mu}}{\overline{\mathcal{L}}_t} - \frac{\overline{\mu}}{\overline{\mathcal{L}}_t}\right] = -\frac{0.5}{\overline{\mathcal{L}}_t}\left[\underline{\mu} + \overline{\mu}\right]$, $K = K\left[1-\tau\right] + I$, $S(1) = \frac{\kappa}{2}\left(\frac{I}{I} - 1\right)^2 I^2 = 0$, $\Psi(1) = -\frac{r^k}{1+\varsigma} + r^k \frac{z_t^{1+\varsigma}}{1+\varsigma} = 0$, $\frac{1}{\beta} - (1-\tau) = r^k$, $1 = Q, r_t^k = \Psi'(z_t) = r^k$, $Y_t = C_t + G_t + I_t$, $1 = \varepsilon^R$, $\ln \varepsilon^b = 0$, $\ln \varepsilon^L = 0$, $\ln \varepsilon^I = 0$, $\ln \varepsilon^a = 0$, $\ln(\varepsilon^g) = 0$, $\overline{\pi}_t = \rho \overline{\pi}_{t-1} + \eta_t^{\pi}$, $\varepsilon^R = 1$.

In a non-deflationary economy, from the euler equation, it must be satisfied that$\beta R = 1 + \Pi > 1$. Thus, we need to calibrate values for β, R and Π, so that this inequality is accomplished. Besides we also need to guarantee that $\frac{1}{\frac{1+\Pi}{R}} - (1-\tau) = r^k$. Also we need to guarantee that $r^k = MC\alpha K^{\alpha-1}L^{1-\alpha} = \frac{R}{1+\Pi} - (1-\tau)$.

As in CEE (2005), monopolistic rents are eliminated in the the long run; thus, $\pi_t^j = 0$, and we will have[3]:

$$\pi_t^j = (P_t^j - P_t MC_t)\left(\frac{P_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t - P_t MC_t \Phi = 0 \tag{3.67}$$

$$(1 - MC_t)Y_t = MC_t\Phi \tag{3.68}$$

$$\Phi = \frac{(1 - MC_t)Y_t}{MC_t} \tag{3.69}$$

$$\varphi Y_t = \Phi = \frac{(1 - MC_t)Y_t}{MC_t} \tag{3.70}$$

$$\varphi = \frac{(1 - MC_t)}{MC_t} \tag{3.71}$$

Thus, if $\Phi = \varphi Y_t$, it is possible to write:

$$Y_t = \varepsilon_t^a K^{\alpha} L^{1-\alpha} - \Phi$$
$$Y_t = \varepsilon_t^a K^{\alpha} L^{1-\alpha} - \varphi Y_t$$

And after some algebra in the steady state, the total income will be:

$$Y = \left[\frac{\varepsilon_t^a (K/Y)^{\alpha} L^{1-\alpha}}{(1+\varphi)}\right]^{1/(1-\alpha)}$$

A different strategy can be:

$$\Phi = \frac{(1 - MC_t)Y_t}{MC_t}$$

Again, after some algebra:

$$Y = \left\{\left[1 + \frac{(1 - MC_t)}{MC_t}\right]\left(\frac{K}{Y_t}\right)^{-\alpha} L^{-(1-\alpha)}\right\}^{-1/1-\alpha}$$

It must be also accomplished that:

$$\frac{W_t}{P_t} = \frac{1-\alpha}{\alpha}\frac{r^k K}{L} \tag{3.72}$$

[2]Because CEE(2005) and Smets-Wouters (2003) use log-linearisation method to simulate and solve the model, they only need to specify some properties of these equations in terms of derivatives and values in the steady state. In this chapter however the nonlinear model is simulated by means of a k-order perturbations method, thus we need concrete functional forms accomplishing the properties just claimed.

[3]Because there is no entry barriers, in the long run, the positive benefits derived from monopolistic competition will be exhausted

$$1 = (1+\lambda_p)MC_t \tag{3.73}$$

$$\frac{W_t}{P_t} = \frac{\overline{C}_t}{\overline{\mathcal{L}}_t}\left(\frac{\underline{\mu}+\overline{\mu}}{\underline{\theta}+\overline{\theta}}\right)(1+\lambda_w) \tag{3.74}$$

additionally:

$$1 = (1+\lambda_p)MC_t \tag{3.75}$$

$$MC_t = \frac{1}{1+\varphi}$$

$$1 = (1+\lambda_p)\frac{1}{1+\varphi}$$

$$1+\varphi = 1+\lambda_p$$

$$\varphi = \lambda_p$$

Because there is inflation different from zero in the steady state, it is necessary to transform nominal quantities into relative quantities as: $\frac{\widetilde{W}_t}{P_t}$, $\frac{\widetilde{P}_t^j}{P_t}$. Thus, some key equations involving nominal prices and wages can be written as:

$$\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p} E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}\frac{(P_{t+i-1}/P_t)^{\gamma_p}}{P_{t+i}/P_t}y_{t+i}^j\frac{1}{\lambda_{p,t+i}} = E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+i}^j\left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}\right)$$

To make this expression operative for simulation effects we re write it as

$$E_0\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p} S1_t = E_0 S2_t \tag{3.76}$$

where $S1_t = \sum_{i=0}^{\infty}\frac{1}{\lambda_{p,t+i}}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+i}^j\left(\frac{(P_{t+i-1}/P_t)^{\gamma_p}}{(P_{t+i}/P_t)}\right)$ and $S2_t = \sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+1}^j\frac{(1+\lambda_{p,t+i})}{\lambda_{p,t+i}}MC_{t+i}$, which given their recursive nature are up dated as follows:

$$S1_t = \frac{y_t^j}{\lambda_{p,t}}\left(1/(1+Pit)\right)^{\gamma_p} + (1+Pit(+1))^{\gamma_p-1}\frac{\lambda_{t+1}}{\lambda_t}\beta\xi_p S1_{t+1} \tag{3.77}$$

$$S2_t = y_t^j\frac{(1+\lambda_{p,t})}{\lambda_{p,t}}MC_t + \beta\xi_p\frac{\lambda_{t+1}}{\lambda_t}S2_{t+1} \tag{3.78}$$

Then, the Calvo style infinite summation for prices can be written as the respective laws of motion for $S1_t$ and $S2_t$.

Similarly for the summations of wages we have:3.41, we can obtain the following expression:

$$E_0 S3_t = \frac{\widetilde{W}_t}{P_t}E_0 S4_t \tag{3.79}$$

and analogously we define $S3_t = \sum_{i=0}^{\infty}\beta^i\xi_w^i\frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}\left(\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}l_{t+i}^\tau\right)$ and $S4_t = \sum_{i=0}^{\infty}\beta^i U_{t+i}^C\xi_w^i l_{t,t+i}^\tau\frac{P_t}{P_{t+i}}\left(\frac{P_{t+i-1}}{P_t}\right)^{\gamma_w}\frac{1}{\lambda_{w,t+i}}$, each with its respective law of motion: $S3_t = \frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t}\left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}}\right)l_t^\tau + \beta\xi_w S3_{t+1}$ and $S4_t = U_t^C l_t^\tau\frac{1}{\lambda_{w,t}}\left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} + (1+Pit(+1))^{\gamma_w-1}\beta\xi_w S4_{t+1}$

in the steady state:

$$S1 = \frac{y_t^j}{\lambda_{p,t}} \left(1/(1+Pit)\right)^{\gamma_p} + (1+Pit)^{\gamma_p - 1} \beta \xi_p S1$$
$$S1 = \frac{1}{1-(1+Pit)^{\gamma_p - 1} \beta \xi_p} \frac{y_t^j}{\lambda_{p,t}} \left(1/(1+Pit)\right)^{\gamma_p}$$

If Pit was equal to zero in the steady state, $S1$ would be

$$S1_t = \frac{y_t^j}{\lambda_{p,t}} \left(P_{t-1}/P_t\right)^{\gamma_p} + \beta \xi_p S1_t$$
$$S1_t = \frac{1}{1-\beta\xi_p} \frac{\lambda_t y_t^j}{\lambda_{p,t}} \left(P_{t-1}/P_t\right)^{\gamma_p}$$

For $S2$ we have

$$S2_t = y_t^j \frac{(1+\lambda_{p,t})}{\lambda_{p,t}} MC_t + \beta \xi_p S2_t$$
$$S2_t = \frac{1}{1-\beta\xi_p} \frac{y_t^j (1+\lambda_{p,t})}{\lambda_{p,t}} MC_t$$

For $S4$

$$S4 = U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} + (1+Pit(+1))^{\gamma_w - 1} \beta \xi_w S4$$
$$S4 = \frac{1}{1-(1+Pit(+1))^{\gamma_w - 1} \beta \xi_w} U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w}$$

Similarly If Pit was equal to zero in the steady state, $S4$ would be

$$S4_t = U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} + \beta \xi_w S4_t$$
$$S4_t = \frac{1}{1-\beta\xi_w} U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w}$$

Finally for $S3$

$$S3_t = \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}}\right) l_t^\tau + \beta \xi_w S3_t$$
$$S3_t = \frac{1}{1-\beta\xi_w} \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}} l_t^\tau\right)$$

The summations for prices and wages in the steady state will be

$$\frac{\widetilde{P}_t^j}{P_t} \frac{1}{1-(1+Pit)^{\gamma_p - 1} \beta \xi_p} \frac{y_t^j}{\lambda_{p,t}} = \frac{1}{1-\beta\xi_p} \frac{y_t^j (1+\lambda_{p,t})}{\lambda_{p,t}} MC_t$$
$$\frac{\widetilde{P}_t^j}{P_t} = \frac{1-(1+Pit)^{\gamma_p - 1} \beta \xi_p}{1-\beta\xi_p} (1+\lambda_{p,t}) MC_t$$

$$\frac{\widetilde{W}_t}{P_t} \left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w} \frac{1}{1-(1+Pit(+1))^{\gamma_w - 1} \beta \xi_w} U_t^C l_t^\tau \frac{1}{\lambda_{w,t}} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma_w} = -\frac{1}{1-\beta\xi_w} \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t}}{\lambda_{w,t}} l_t^\tau\right)$$

$$\left(\frac{\widetilde{P}_t}{P_t}\right) = (1+\lambda_{p,t})MC_t$$
$$\widetilde{P}_t = P_t(1+\lambda_p)MC_t$$

$$\frac{\widetilde{W}_t}{P_t}U_t^C = -\frac{\partial U_t^\tau}{\partial \mathcal{L}_t}\frac{\partial \mathcal{L}_t}{\partial l_t}(1+\lambda_{w,t})$$
$$\frac{\widetilde{W}_t}{P_t} = \frac{\overline{C}_t}{\overline{\mathcal{L}}_t}\left(\frac{\underline{\mu}+\overline{\mu}}{\underline{\theta}+\overline{\theta}}\right)(1+\lambda_w)$$
$$\widetilde{W}_t = P_t\frac{\overline{C}_t}{\overline{\mathcal{L}}_t}\left(\frac{\underline{\mu}+\overline{\mu}}{\underline{\theta}+\overline{\theta}}\right)(1+\lambda_w)$$

3.3.1 Comparing the symmetric model with the asymmetric model

In these simulation exercises, we suppose that all of the physical capital is used in production, which means that $\Psi(Z) = 0$. The parameters used for calibration and simulations are those of Smets-Wouters (2003). Since this model has several stochastic processes that may disturb the hypothetical economy, price and wage rigidities, and a non-traditional utility function, it seems necessary to perform some basic experiments in order to know the dynamics of the model under traditional assumptions. In order to check whether our general asymmetrical model under the asumption of symmetry is able to generate comparable results with those of Smets-Wouters (2003), we have firstly run a simulation imposing symmetry (the agent is risk-averse). Some variables are expressed in logarithms. lc, ly, lk, li, la and lW are consumption, income, physical capital, investment, technology, and real aggregate wage logarithms respectively; nt is labor, Pit is the inflation rate of aggregate price, and gWn is the growth of nominal aggregate wage. Figure 3.1 shows the impulse responses for this first experiment. The results were qualitatively similar to those of Smets-Wouters (2003) under a symmetric model. As seen, a positive shock to technology produces a fall in consumption (in the first period and almost zero) and income. Differently, for capital, investment, labor, inflation and wages, the fall has been different from zero and more-lasting. This behavior can be explained by price rigidity. As a matter of fact, when the model is simulated imposing $\xi_P = 0$, the response was positive for consumption, income, capital, investment, labor, wages, rent of capital and price of capital; response of inflation was negative as expected (Figure 3.2). Thus, it is possible to conclude that rigidity of prices forces part of the adjustment after the shock by inducing a reduction in real variables (bearing in mind that the value estimated by Smets-Wouters (2003) for the probability of not re-optimizing prices is 0.905).

Table 1 Parameters

$\overline{\theta}$	$\underline{\theta}$	$\overline{\mu}$	$\underline{\mu}$	χ_c	χ_l	γ_c	γ_l	ξ_W	ξ_P	γ_W	γ_P	κ	ξ_1
0.5	0.5	0.5	0.5	0.5	0.5	50	50	0.742	0.905	0.728	0.477	6.962	0.201

τ	α	λ_W	λ_P	r_R	r_π	r_Y	ρ_b	ρ_l	ρ_I	ρ_a	ρ_π	ρ_g	Φ/Y
0.025	0.3	0.5	0.477	0.961	1.688	0.098	0.838	0.881	0.91	0.811	0.855	0.943	0.417

Source, Except for those of the utility function, all of them are taken from S-W (2003)

In this apart, we will compare the impulse responses of the technological shock in the asymmetric-sticky model with those of the symmetric-sticky model. For this exercise, asymmetry was imposed in the utility function by setting $\underline{\theta} = \underline{\mu} = 1.2$. The difference between the impulse response paths is overwhelming (Figures 3.3 to 3.5). Both models receive a positive one-standard deviation shock on technology, variables $xsim$ stand for the variable in the symmetric model and $xasim$ stands for the variable in the asymmetric model. In the first period, consumption and income in the symmetric model react negatively, while in the following periods

Figure 3.1: GIRF, symmetry in the general model with price rigidities

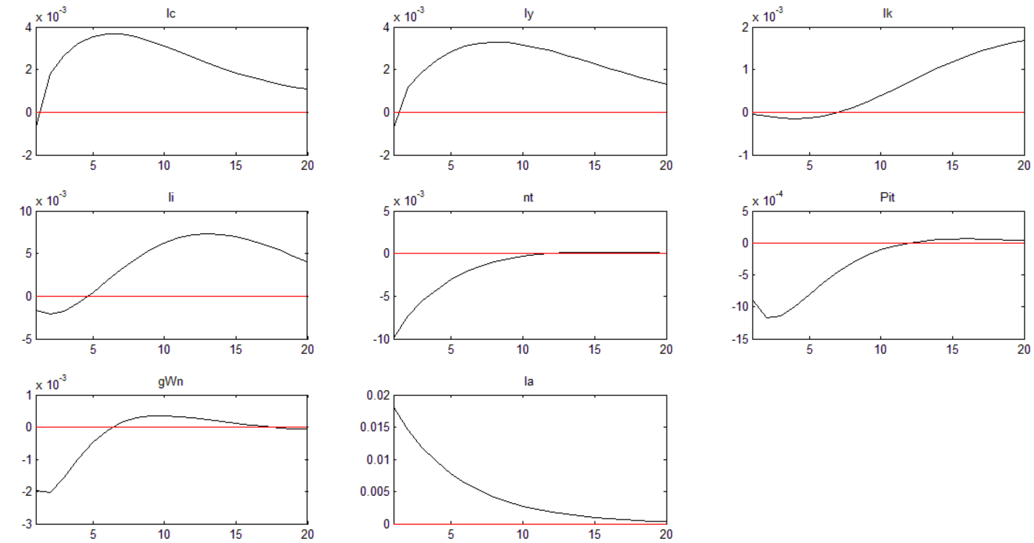


Figure 3.2: GIRF,symmetry in the general model with no price rigidities

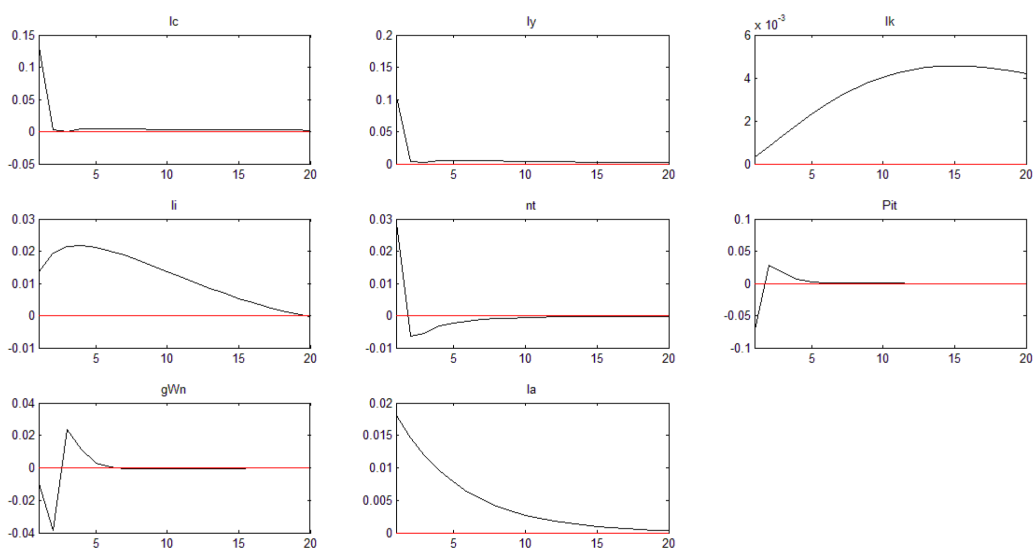
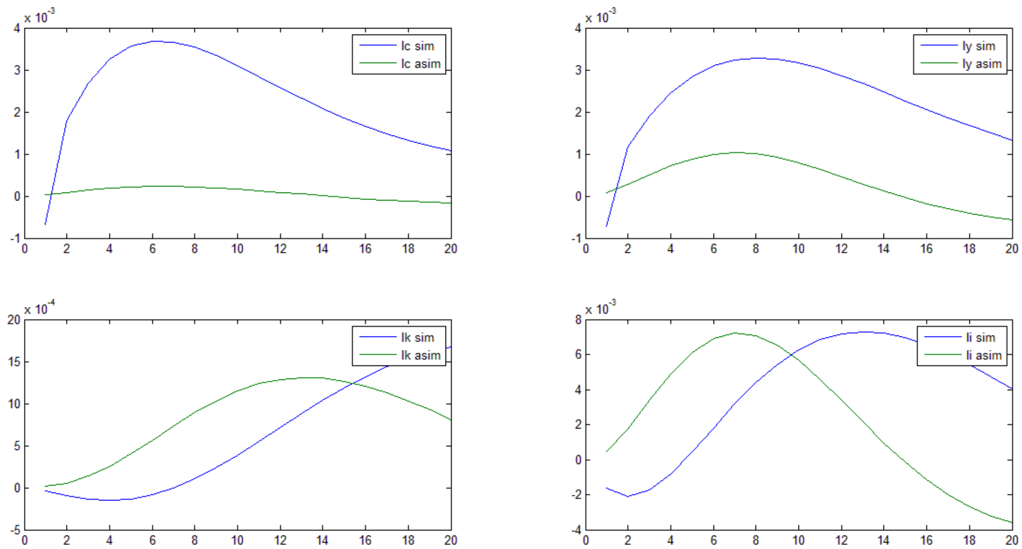


Figure 3.3: GIRF, symmetry and asymmetric sticky models



they are greater than consumption and income respectively, within the asymmetric model. In fact, as it has been shown, price rigidities impose a stronger adjustment on real quantities in the symmetric model, but the loss-averse behavior of the agents (in consumption and leisure) induce a smoother reaction in consumption and income in the asymmetric model. As shown in section 2.7.1 when households are loss averse, they are reluctant to experience big departures from the reference utility level, and consequently, they choose a consumption level as close as possible to its reference level during a recession. However, during a boom, when the households are risk averse, they want to choose a consumption level as high as possible, but taking into account that if this consumption level is very high and a sudden recession comes, they will experience a very high loss of welfare. The same occurs for capital, investment, and labor (for only one period). The reaction of inflation in the asymmetric model is also overwhelming: in this model, inflation of prices has a smaller reaction than in the symmetric model, which means a greater (or additional) stickiness of prices. The fall in inflation, as the wages are indexed, induces a fall in nominal wage inflation rate (gWn) (real wage also falls) in both symmetric and asymmetric models, but the decrease in gWn is greater in the asymmetric model. This can be explained by the loss-aversion in leisure. Evidently, the fall in labor on shock is almost the same in both models. The results in these simulations do not coincide with those from the neoclassical model at least for the moment of the shock. While the neoclassical model predicts, as soon has the economy is shocked, that an increase in technology will produce an increase in labor, capital, consumption and income, simulations show an increase in technology that produces a reduction in labor (for both models), a fall in physical capital (very slight), consumption and income (in the symmetric model), and a decrease in real wage (for both models). Figure 3.5 display impulse-responses simulations for Tobin´s Q, Q, policy interest rate, R and physical capital interest rate , rk. This behavior is explained by the rigidities explicitly modeled y this Noe-Keynesian model and the intensification suffered by them in the presence of loss-aversion.

3.3.2 Comparing negative and positive shocks in the asymmetric model

3.3.2.1 Asymmetry in consumption and leisure

For this exercise, asymmetry was imposed in the utility function by setting $\underline{\theta} = \underline{\mu} = 1.2$. Later, two simulations have been performed: a positive shock and a negative shock on technology. As in the previous exercise, the results showed asymmetries. When the (asymmetric) economy is disturbed by a positive shock

Figure 3.4: GIRF,symmetry and asymmetric sticky models

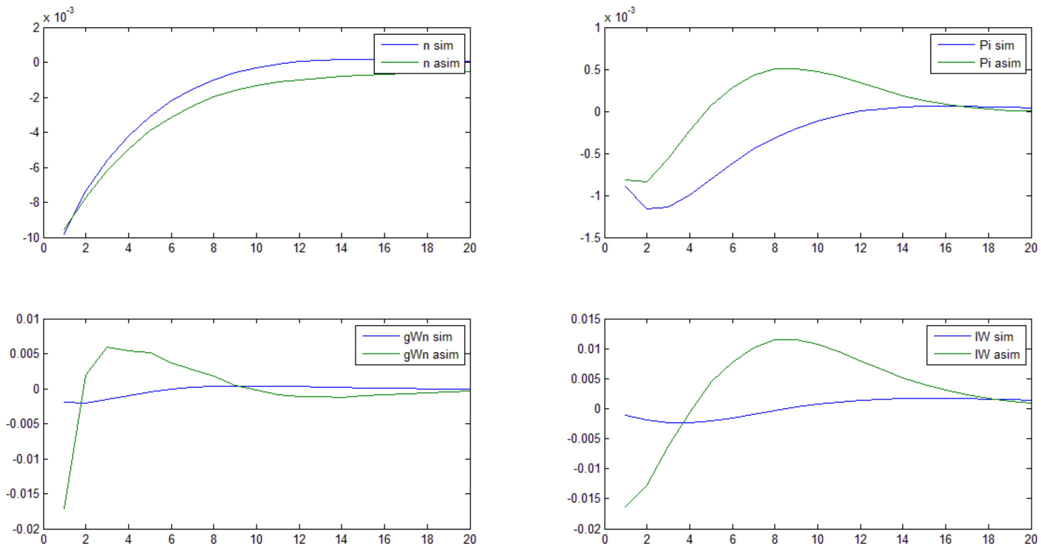


Figure 3.5: GIRF, symmetry and asymmetric sticky models

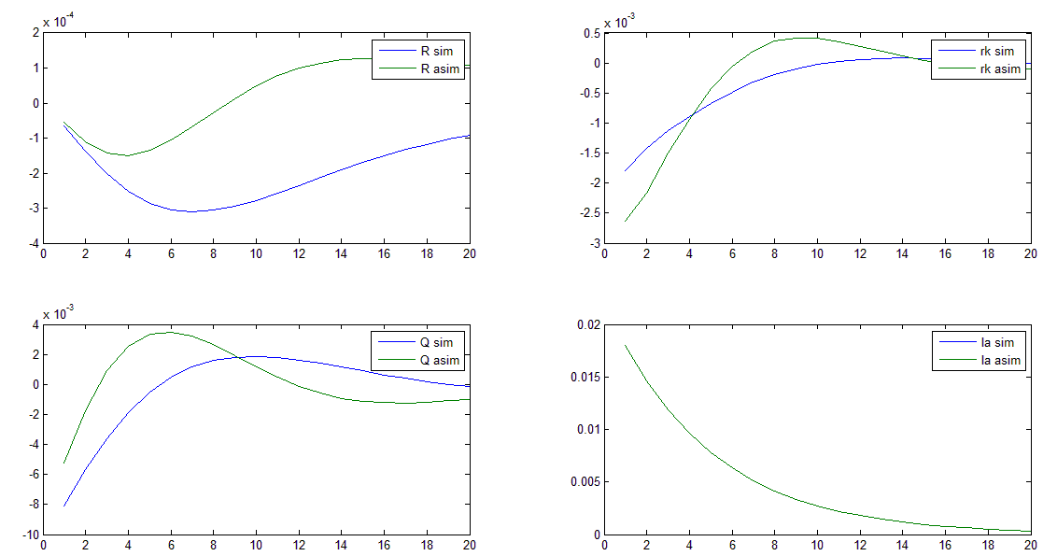
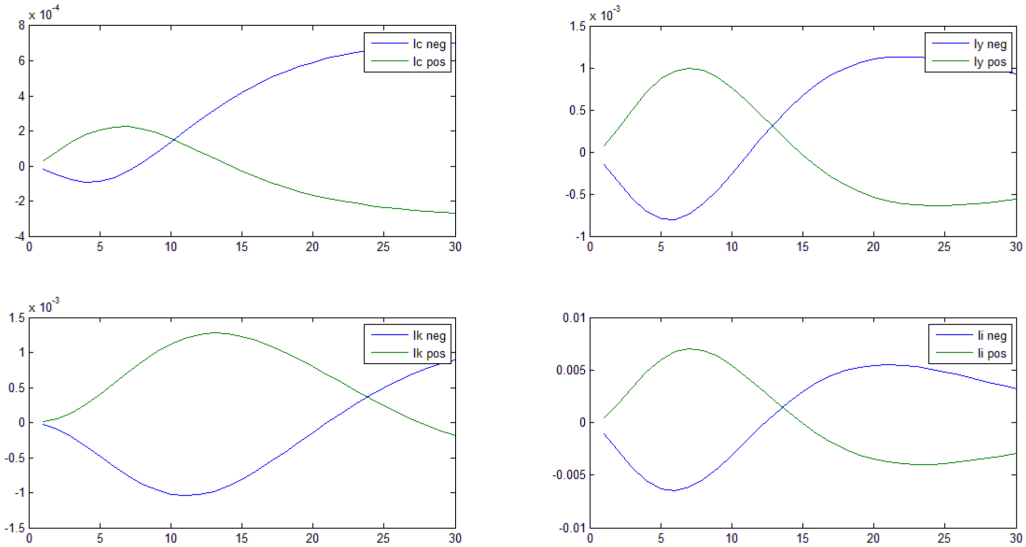


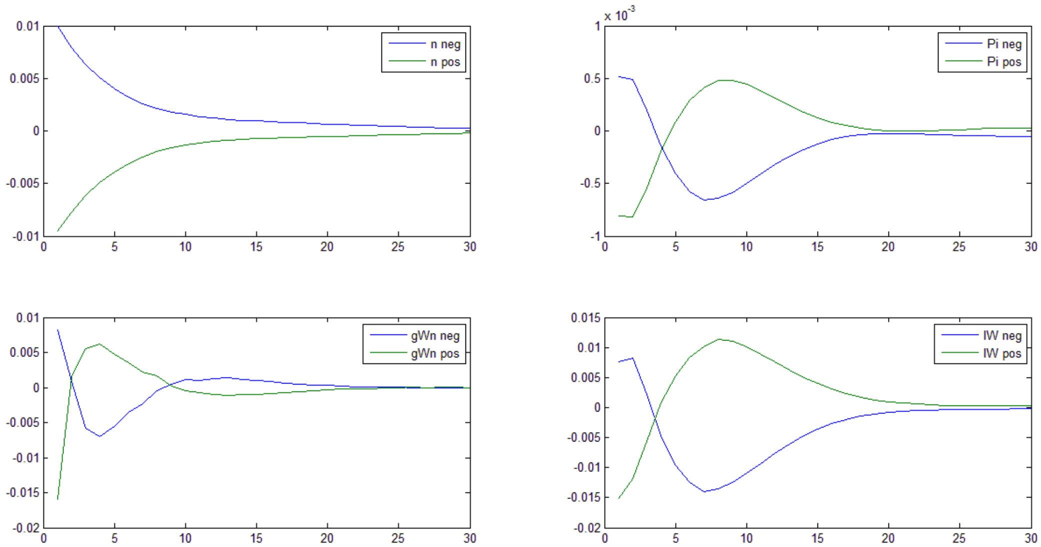
Figure 3.6: GIRF, comparing positive with negative shock in the asymmetric sticky model



on technology, the reaction in consumption, on shock, is almost the same for negative and positive shocks, but the boom induced in consumption is more long-lasting than the recession.The reaction of income is quite different: recession (on shock and for the following 4 or 5 periods) is stronger than boom, but boom is more long lasting than recession. For capital, the recession phase is stronger (on shock and for 8 or 9 following periods), but in period 9 the boom starts to be greater than recession and consequently more long-lasting. The investment response is also interesting: the recession is more intense on shock for the next 5 or 6 periods; but for period 7, investment boom begins to be higher than recession and more long-lasting (Figures 3.9 to 3.11 show absolute values of impulse response paths when the economy receives a positive shock (green line) and when the shock is negative (blue line), figures 3.6 to 3.8 show impulse response values with their original sign). The shape of figures 3.9 to 3.11 are unusual. They can be explained by the fact that the adjustment in each time path is not asymptotic monotonic but asymptotic harmonic or oscillatory. In other words, when a variable receives a positive shock, in the case of consumption, it initially experiences a boom, but, later in the process of adjustment, it will transit towards a recession. Moreover, since we are taking absolute values, a negative value jumps to a positive value when we take the absolute value operator. That is why the time paths in these figures look similar to the trajectory of a ball falling onto the ground. To be more concrete, the trajectories of variables in figures 3.6 to 3.8 are the trajectories' absolute values of variables in figures 3.9 to 3.11.

Figure 3.7 shows the responses of labor, wages, and inflation. For labor, the impact of the recessive shock is stronger and seems to be more long-lasting than the effect of a booming shock (this strongly suggests that this model would be able to explain the hysteresis in unemployment). Again, the positive shock on technology produces a fall in labor and the opposite is true for the negative shock (Figure 3.10). The most interesting path is the inflation rate one. In fact, whereas inflation in boom has a greater increase and a lasts only one more period, the reaction of inflation is lower in recession than in boom and has a shorter duration. Thus, prices in recession are more reluctant to decrease than to increase in boom. The explanation for this result is the loss aversion in consumption. When agents income depends on profits of competitive monopolistic firms and consequently depends on good prices variations, loss-aversion of agents implies that a reduction in (reoptimized) prices will be as small as possible to reduce the fall in income and consumption, as the economy is going through a recession. The opposite happens when the economy is in a boom and the agents behavior is risk-averse: as demand and income improve, (reoptimized) prices can be upwards adjusted

Figure 3.7: GIRF, comparing positive with negative shock in the asymmetric sticky model



in a higher proportion. Wages, general and reoptimized, present a mixed behavior. Either for a general or an aggregate wage, there is a smaller reaction during recession (stronger in boom). Differently, the re-optimized wage shows a stronger movement in recession. From period 2 to period 4, general wages seem to behave similarly. However, from period 4 onwards, wage during recession shows a greater deviation from the steady state. Accordingly with this, in absence of indexation schems, loss aversion would enough to explain the existence of rigidities in prices and wages.

The response of nominal interest rate is stronger and more long-lasting during a boom, which is a very predictable behavior, although the Taylor rule employed to model monetary policy is not an optimal one. Tobin's Q (real price of capital) shows a stronger reaction in a boom and more long-lasting at least from period 1 to period 3 and from period 7 onwards. Real rent of capital (rk) has a higher reaction in recession from period 1 to 3, and seems to be more long-lasting in general.

3.3.2.2 Asymmetry in consumption and symmetry in leisure

In order to know the importance of the different sources of asymmetry in this model, we proceed now by imposing symmetry in the utility part delivered by leisure and allow for asymmetry in the utility function part associated with consumption, which means that the parameters of the utility function are these[4]: $\underline{\theta} = 1.2$ and $\underline{\mu} = \overline{\mu} = 0.5$. Once again, two simulations are performed: a positive shock and a negative shock on technology. The direction of the impulse responses is the same as in the previous exercise (figures 3.3.2.2 to 3.17). However, what is surprising is the fact that asymmetry is not as overwhelming as in the previous exercise, although it displays the same behavior in qualitative terms.

3.3.2.3 Symmetry in consumption and asymmetry in leisure

In this exercise, we set $\underline{\mu} = 1.2$ and $\underline{\theta} = \overline{\theta} = 0.5$. Two simulations are performed: a positive shock and a negative shock on technology. As in previous exercises, qualitative results remain the same. A positive shock on technology causes increases in consumption, income, investment, capital, fall in labor, inflation, nominal wages inflation, real wages, rent of capital, policy interest rate, and fall in Tobin's Q. However, quantitative

[4]Recall: the utility function is intratemporal additive separable.

Figure 3.8: GIRF, comparing positive with negative shock in the asymmetric sticky model

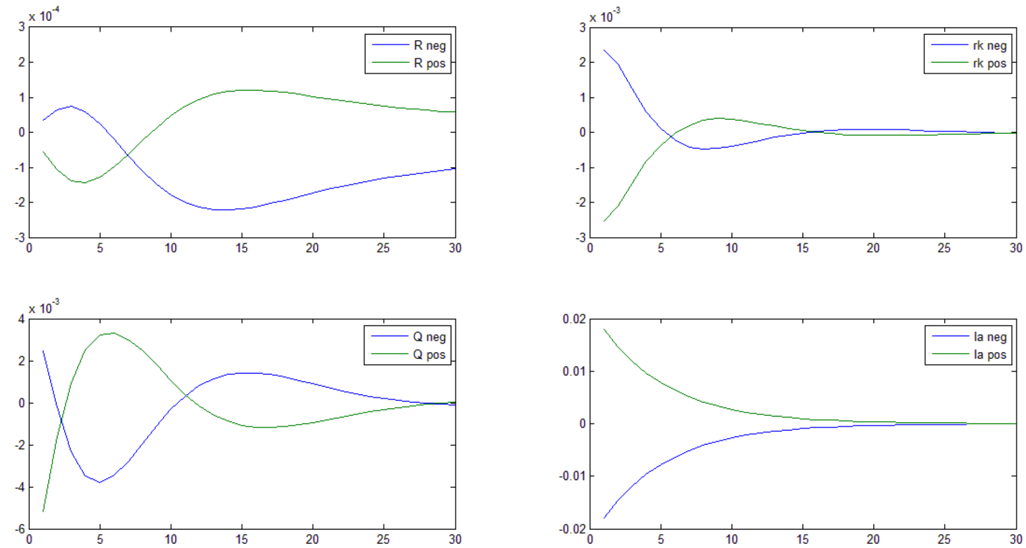


Figure 3.9: GIRF, comparing positive with negative shock in the asymmetric sticky model

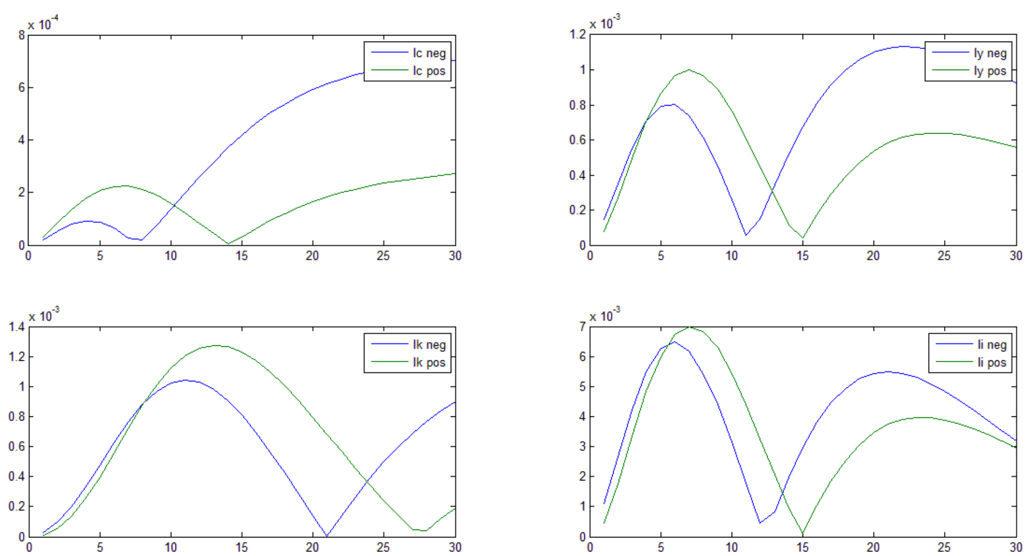


Figure 3.10: GIRF, comparing positive with negative shock in the asymmetric sticky model

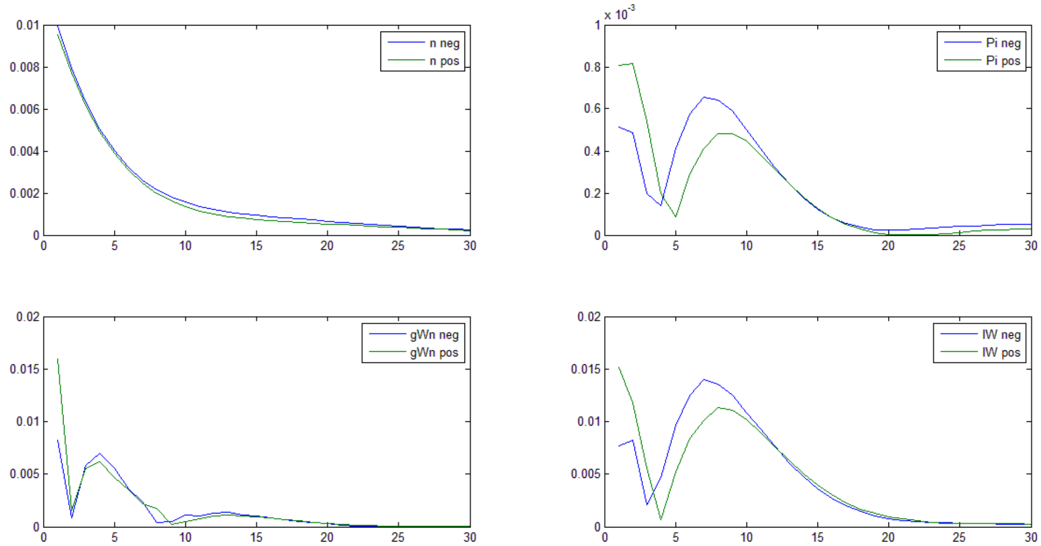


Figure 3.11: GIRF, comparing positive with negative shock in the asymmetric sticky model

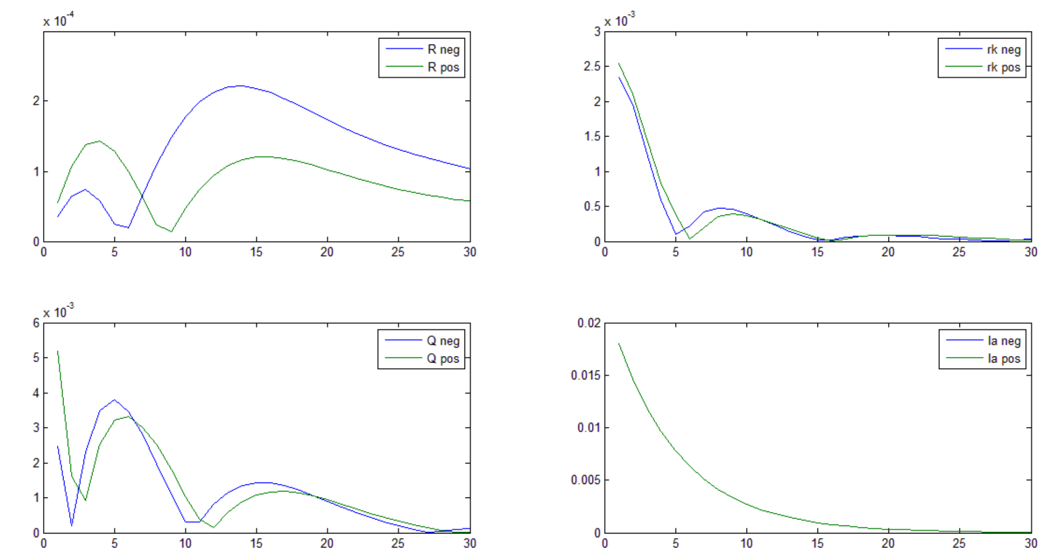


Figure 3.12: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

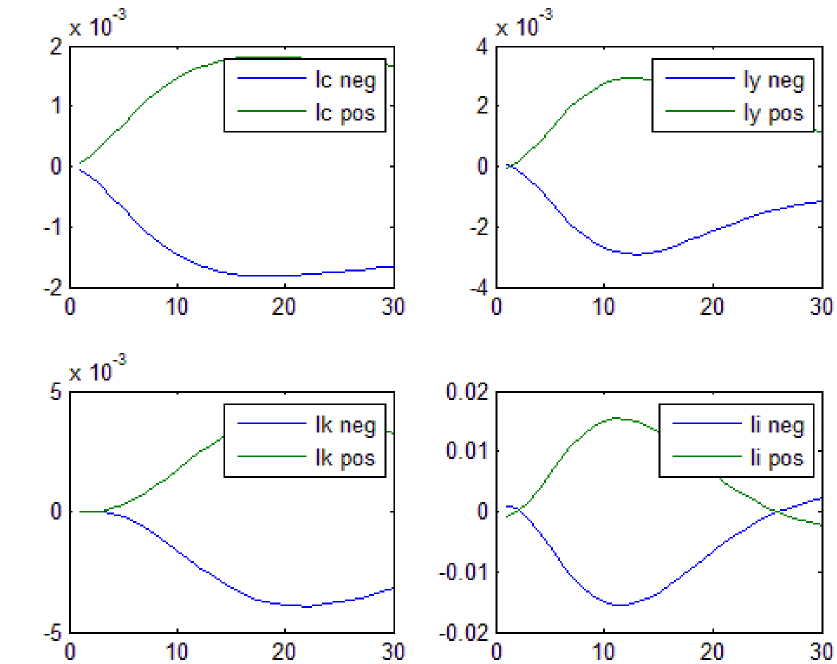


Figure 3.13: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

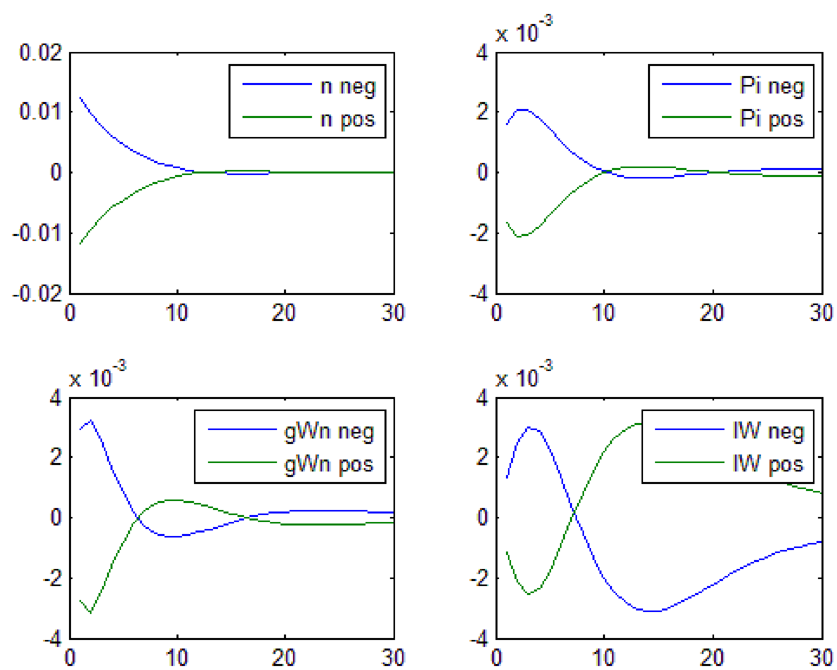


Figure 3.14: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

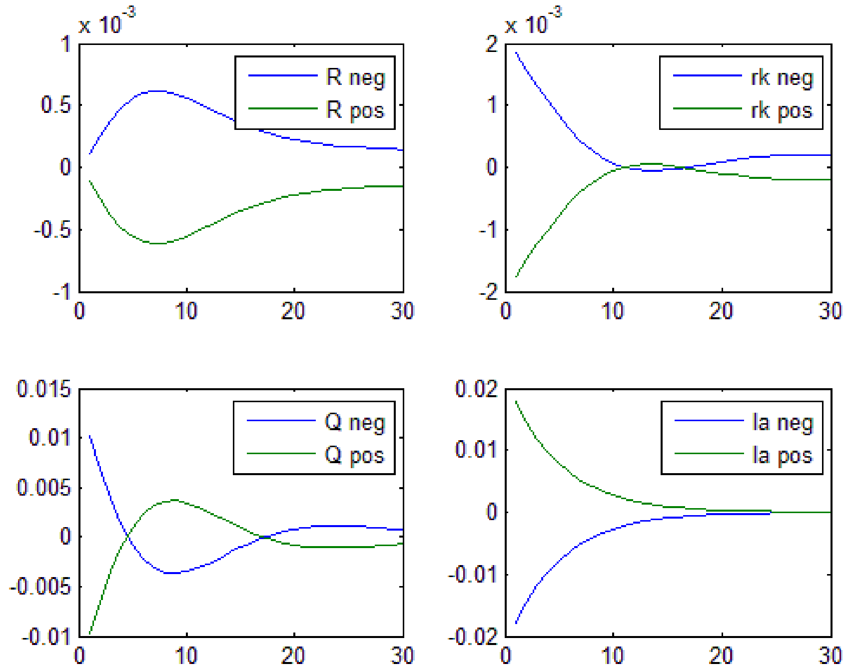


Figure 3.15: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

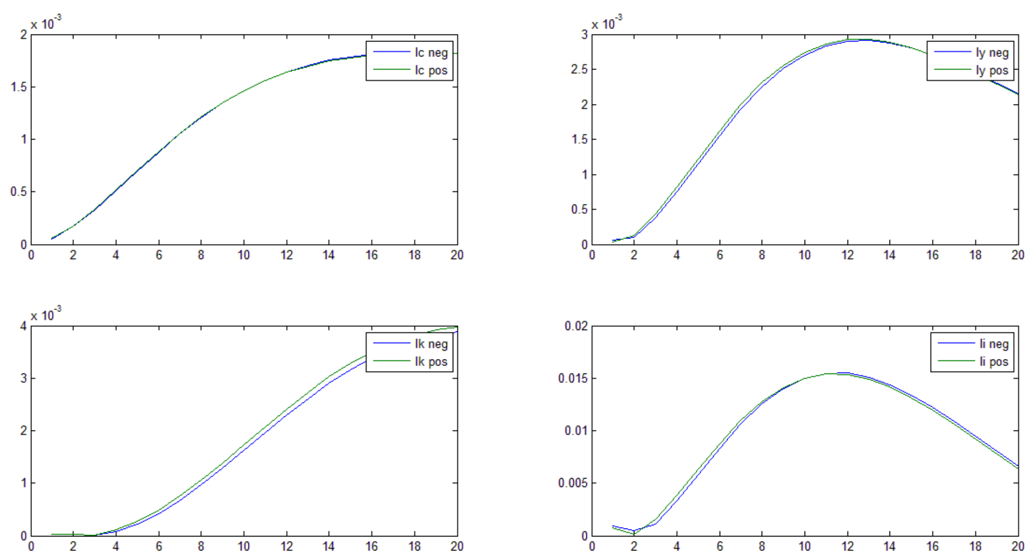


Figure 3.16: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

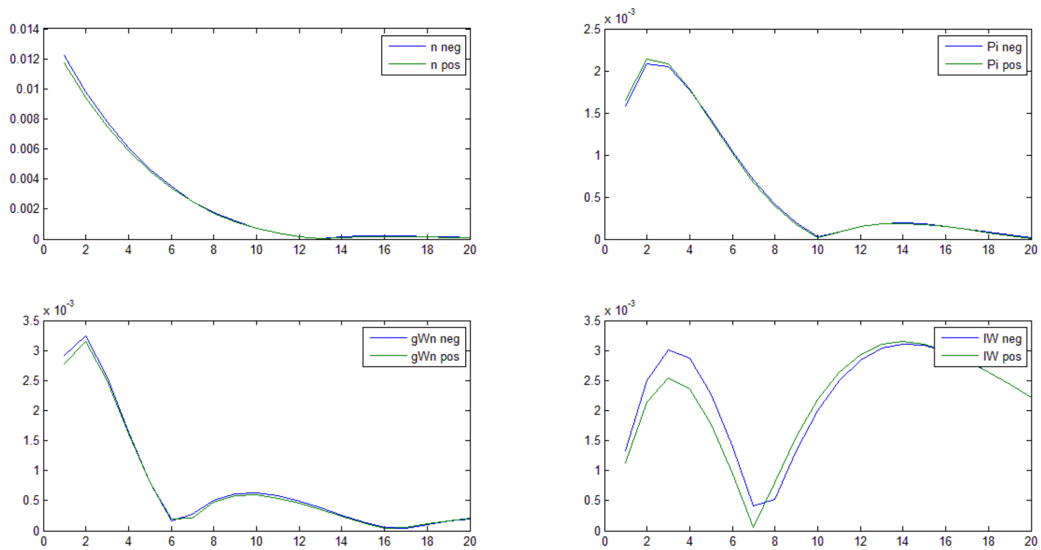


Figure 3.17: GIRF, comparing positive with negative shock in the asymmetric sticky model. Asymmetry in consumption and symmetry in leisure

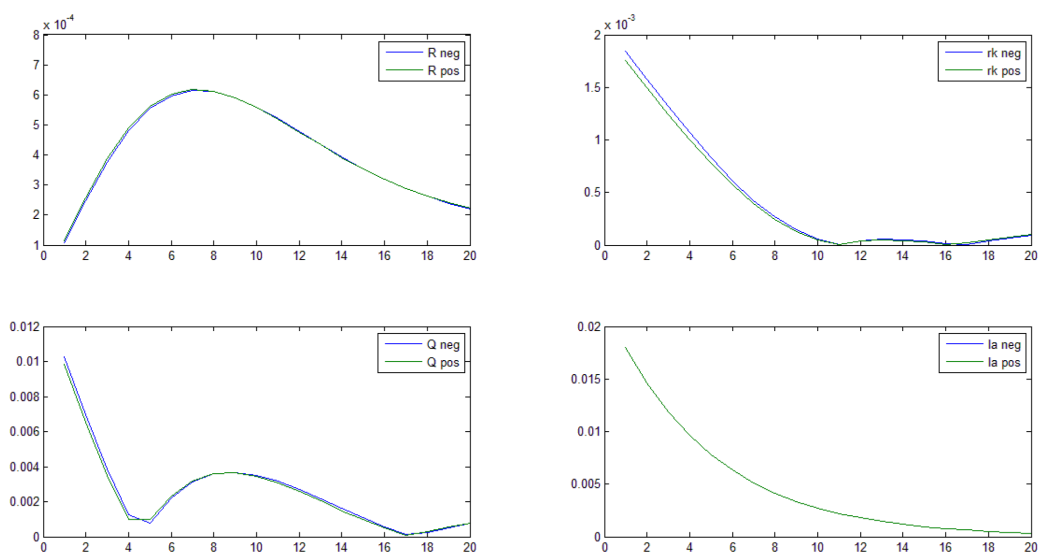
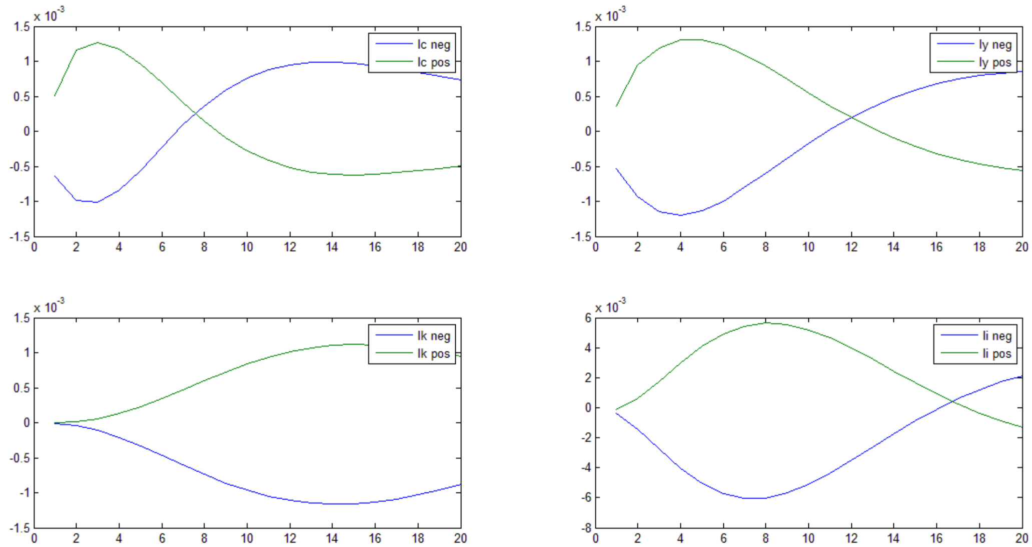


Figure 3.18: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure



results are surprising in the sense that asymmetry seems to be more important under this parametrization than under the previous one. In other words, when we suppose asymmetry only in consumption, the asymmetry is very slight. However, when we suppose asymmetry in leisure alone, the asymmetry is notorious and the trajectories of variables are very similar to those of the simulations when supposing asymmetry in consumption and leisure. We can explain this by the fact that as income falls, the agent needs to dampen the fall in welfare by demanding more leisure (employment will decrease indeed), which means that the agent will demand a higher wage or will reject a huge decrease in wage. (figures 3.18to 3.23).

3.3.3 Comparing negative and positive shocks in the asymmetric model with moderate price stickiness

Previous exercises were performed under extreme price stickiness ($\xi_P = 0.905$). Here, I will present the results of the same technological shock under moderate price stickiness by setting $\xi_P = 0.4525$. Figures 3.24 to 3.26 show the impulse response path of variables after a positive shock and a negative shock. Again, a positive shock to technology generates a decrease in labor (the opposite for a negative shock), but, unlike to the case of extreme stickiness, the increase in consumption and income look important and seem to be different. As in previous exercises, booms in consumption, income, capital and investment seem to be more long-lasting than recessions. The fall in labor (caused by the positive shock) is smaller than its increase (caused by the negative shock). The decrease in inflation and in nominal wages growth are again smaller for recession than for boom. Real wages also present a slight fall during recession. Policy interest rate, rent of capital and Tobin's Q also show smaller reactions during recession, which means that there is also rigidity in real prices (figures 3.27 to 3.29 show impulse response in absolute values).

3.4 Conclusions

As in the case of business cycle asymmetries detected in real macroeconomic aggregates, asymmetries have been detected in nominal macroeconomic variables such as prices and wages. More precisely, their adjustment speed is asymmetric. This fact lies behind the asymmetric (or even kinked) Phillips curve which has been

Figure 3.19: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

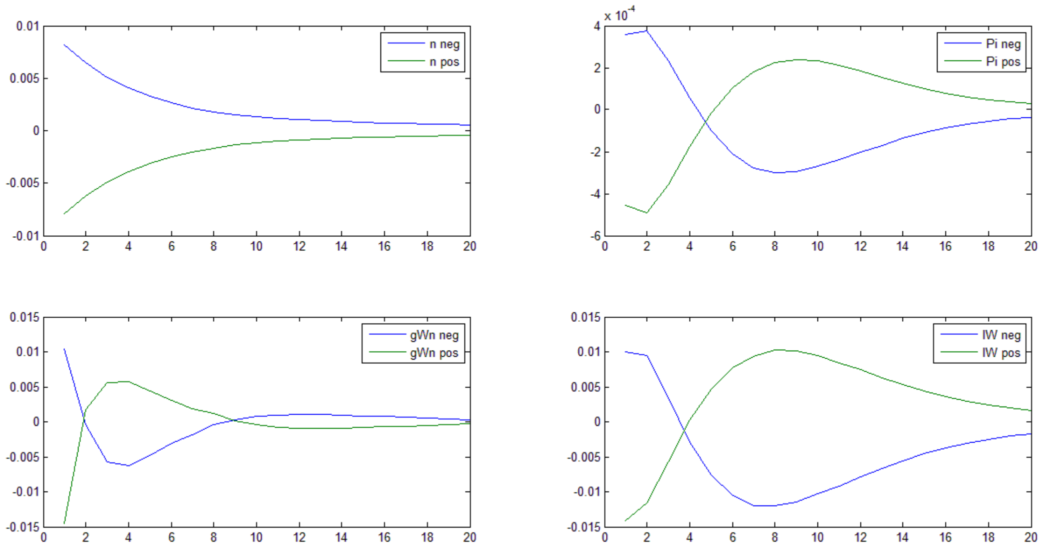


Figure 3.20: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

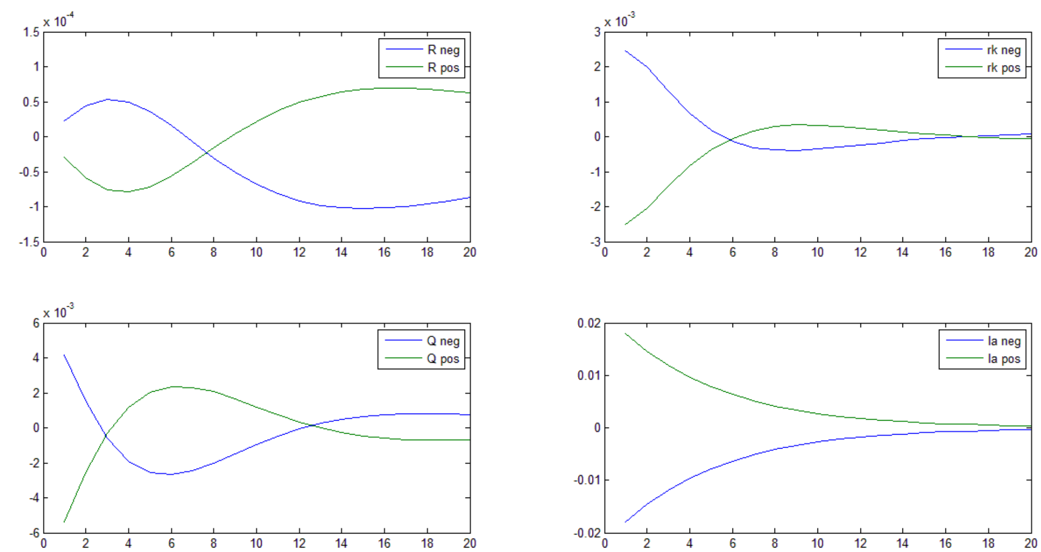


Figure 3.21: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

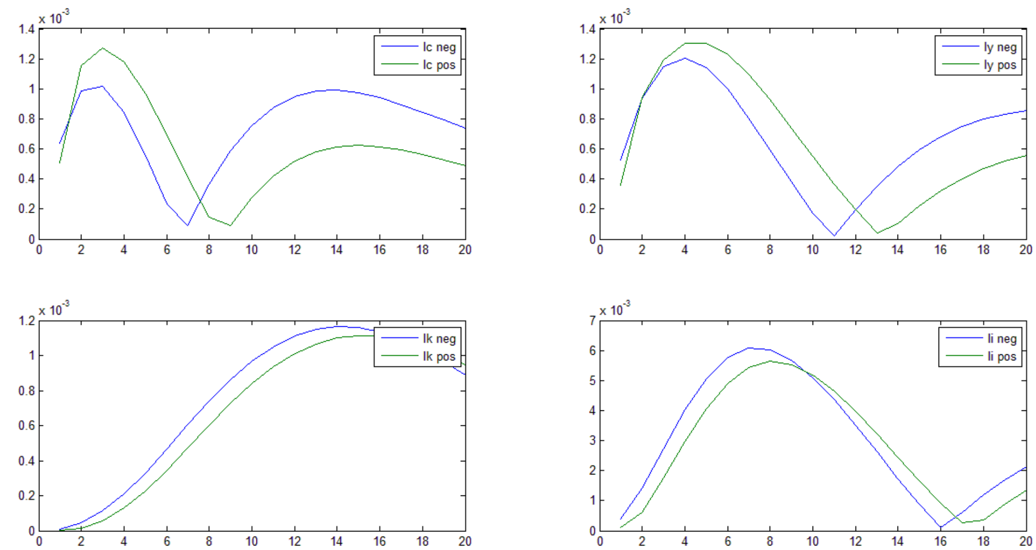


Figure 3.22: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

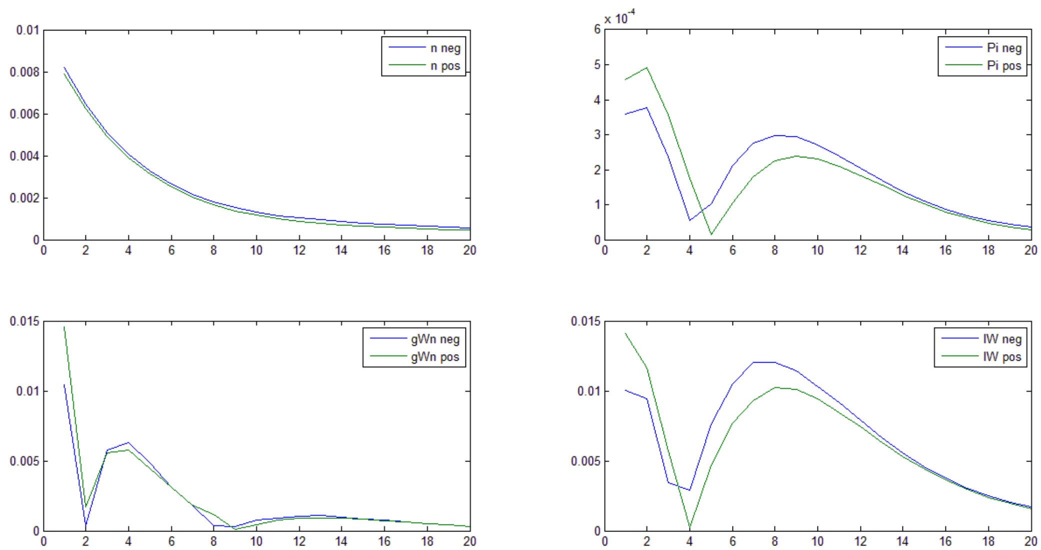


Figure 3.23: GIRF, comparing positive with negative shock in the asymmetric sticky model. Symmetry in consumption and asymmetry in leisure

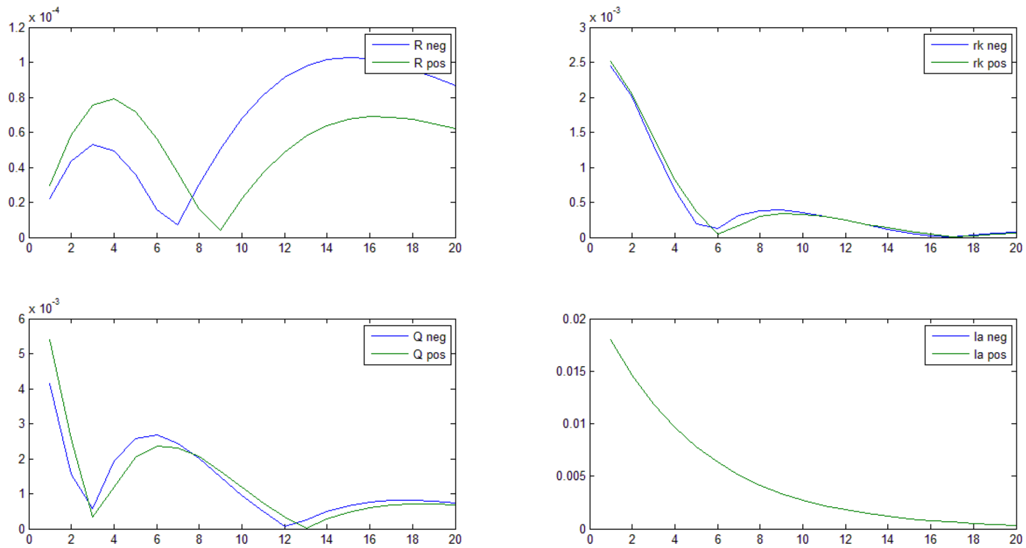


Figure 3.24: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

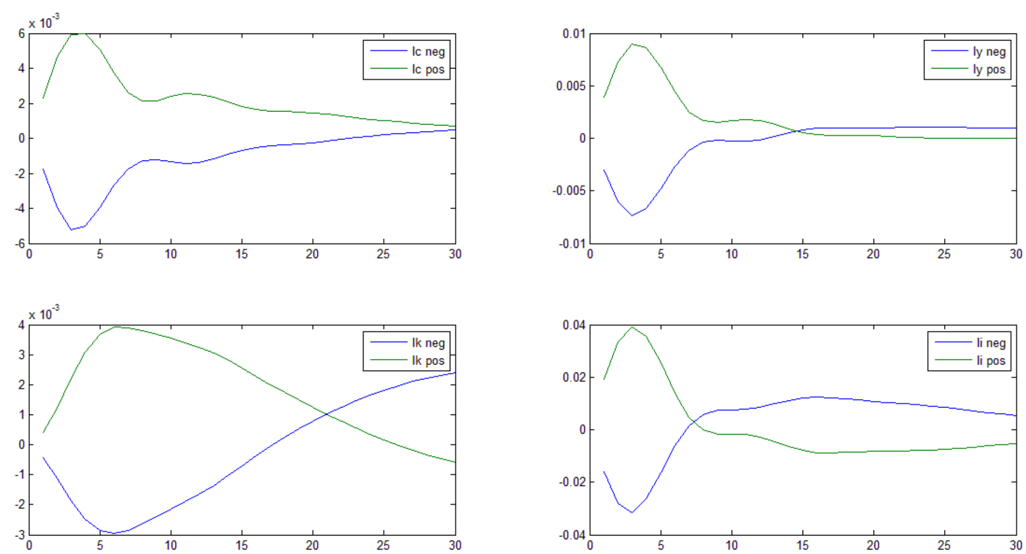


Figure 3.25: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

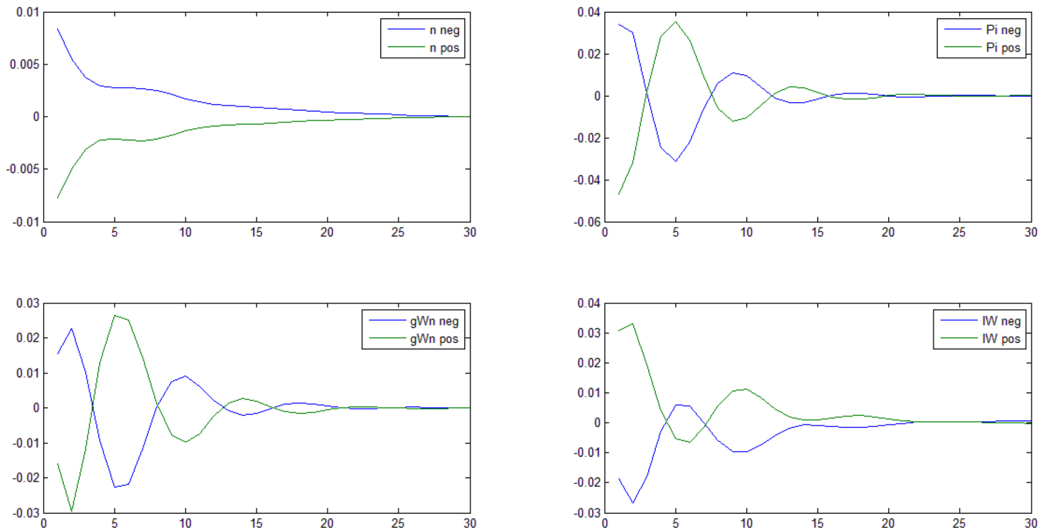


Figure 3.26: GIRF,comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

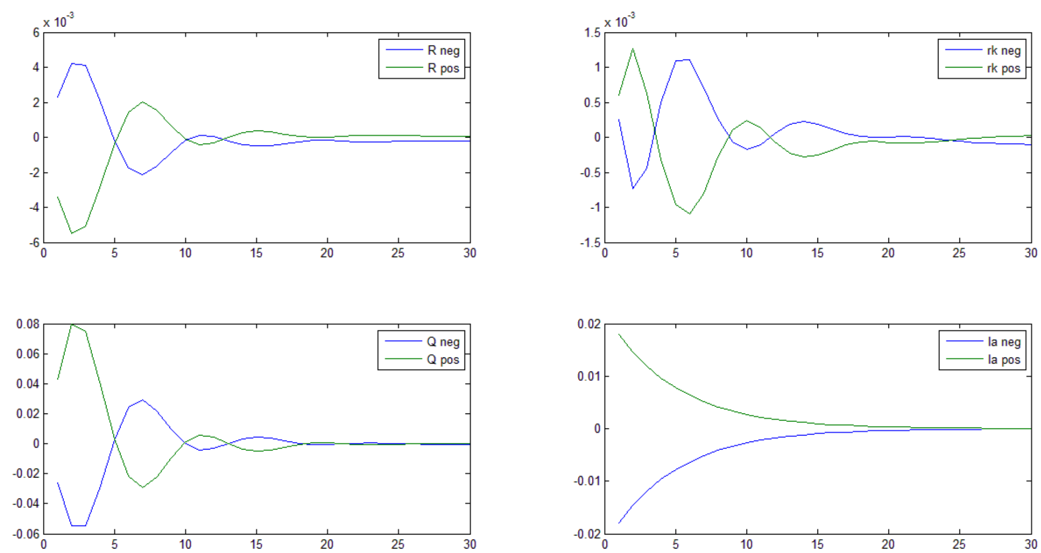


Figure 3.27: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

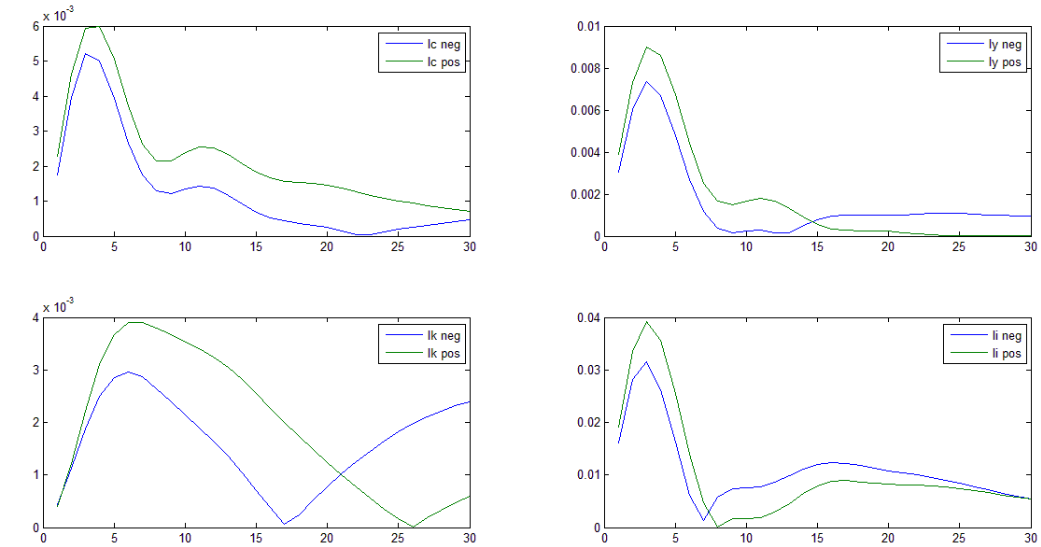


Figure 3.28: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness

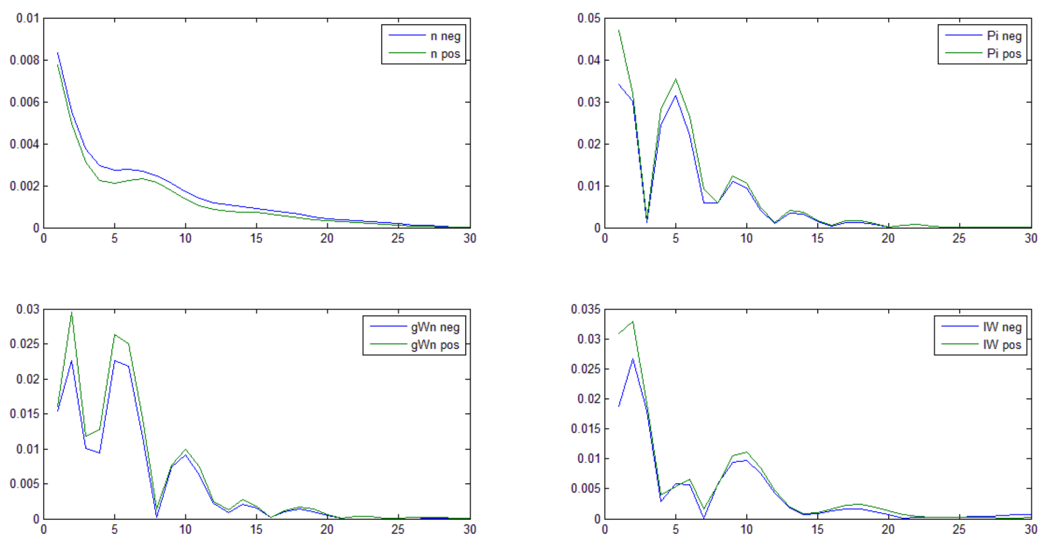
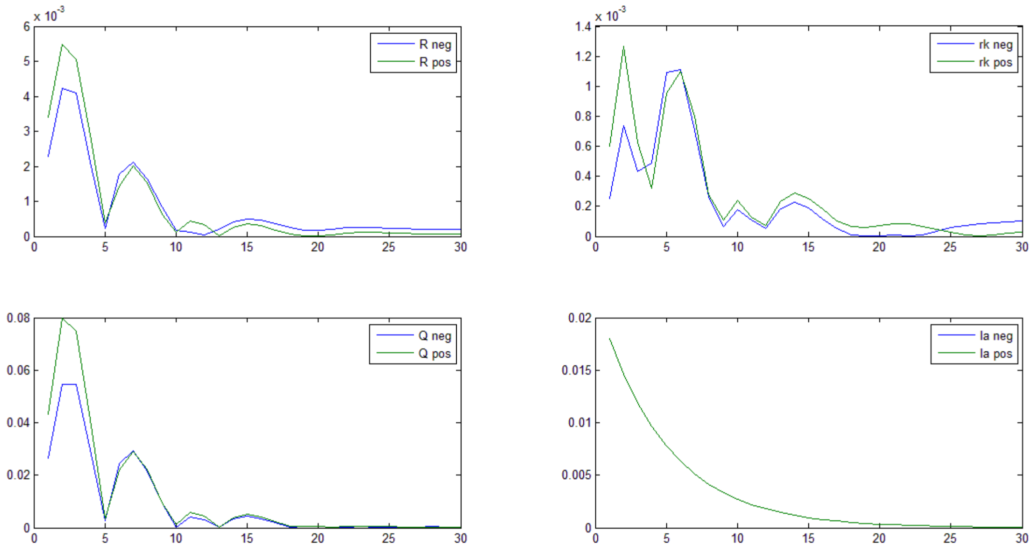


Figure 3.29: GIRF, comparing positive with negative shock in the asymmetric sticky model. Moderate price stickiness



detected and modeled empirically in order to better understand the implications of optimal monetary policies. However, there is no theoretical modelization, at the best of our knowledge, that links these phenomenon to elemental behavior of agents.

In this chapter, the Smets-Wouters (2003) New-Keynesian model is modified to build asymmetric DSGE models by introducing an intra-temporal additive prospects utility function in consumption and leisure. The parametrization of the model is quite similar to the one estimated in Smets and Wouters (2003), although the stochastic processes are log normal in our version. In order to verify whether our model looks like the Smets-Wouters (2003) model, we performed a first simulation supposing that the symmetric utility function in consumption and leisure maintained price and wage rigidities. As expected, the symmetric version of our model is rather similar (qualitatively) to Smets-Wouters'. In both models, since labor is determined by indexation of wages and inflation falls as technology is positively shocked, the increase in technology causes a fall in labor. This is not a neoclassical effect. However, when price rigidity is removed from the model, the positive shock on technology induces an increase in labor as expected for flexible price models.

When symmetry assumption was removed in our DSGE model, it was possible to generate asymmetric business cycles. The transmission channel in this case was the asymmetry in consumption and leisure. This was modeled by means of a prospects utility function additive separable considering habits as an externality. Impulse response in this exercise shows that price rigidities impose a stronger adjustment on real quantities in the symmetric model. But in the asymmetric model, the loss- averse behavior of the agents (in consumption and leisure) induce a smoother reaction in consumption and income. The reaction of inflation in the asymmetrical model is overwhelming: in this model, prices inflation have a smaller reaction than in the symmetric model, which means a greater (or additional) stickiness of prices. The fall in inflation, as the wages are indexed, induces a fall in nominal wage inflation rate (gWn) (real wage also falls) in both, symmetric and asymmetric models. However, the decrease in gWn is greater in the asymmetric model, which can be explained by the loss aversion in leisure. On shock, the fall in labor is almost the same in both models. The results in these simulations do not coincide with those from the neoclassical model. While the neoclassical model predicts that an increase in technology will produce an increase in labor, capital, consumption and income, this exercise shows that an increase in technology produces a reduction in labor (for both models), a fall in physical capital (slight), consumption and income (in the symmetric model), and a decrease in real wage (for both models). This result is counterintuitive in the light of a model with full flexible prices and wages as the neoclasical one. However, it is explainable as a consequence of the asumptions of Neo Keynesian model with high rigidity in prices and wages, given that $\xi_P = 0.905$ and$\xi_W = 0.742$ in the estimations of SW (2003). Thus, in front of a positive shock to technology, this highly sticky economy needs a (very small and temporary) reduction in real quantities in order to preserve the equilibrium in goods market. Consequently, because the increased technology helps to produce at lower cost with the same quantity of factors (on shock), it would be possible to reduce prices, but as prices are highly sticky and firms and households are reluctant to reduce prices, equilibrium labor is lower (as well as equilibrium capital), then consumption and output are lower because the fall in labor income and the fall in labor.

The simulation of both a negative shock and a positive shock on the asymmetric models revealed that for consumption, income, capital and investment on shock, a recession is more intense than a boom, and a boom is more long-lasting than a recession. Besides, and perhaps one of the most interesting findings in our simulations, the impact of the recessive shock for labor is stronger and seems to be more long-lasting than the effect of a booming shock, which resembles the hysteresis phenomenon in unemployment. Whereas inflation in boom has a greater increase and lasts only one more period, the reaction of inflation is lower in recession than in boom and has a shorter duration. Thus, prices in recession are more reluctant to decrease than to increase in a boom. General or aggregate wage shows a smaller reaction during recession (stronger in boom). To sum up, the stickiness of prices and wages in a prospect utility framework are exacerbated.

Finally, we must highlight that another really interesting finding in our simulations is that when asymmetry is removed from the leisure choice, the asymmetry in business cycles is almost removed as well. However, when asymmetry is removed from the consumption choice, there is not any significant change in the asymmetric pattern of impulse responses. This suggests that the main channel of transmission of asymmetry is the loss aversion in leisure because when income falls, the agent needs to dampen the fall in welfare by demanding more leisure (employment will decrease). This means that the agent will demand a higher wage or will reject a huge decrease in wage.

In general, the model built in this chapter is able to generate not only asymmetric business cycles, but also generate an asymmetric (nonlinear) Phillips curve and asymmetric stickiness of prices and wages. This demonstrates that the asymmetric behavior of consumers modeled by prospects utility function is a suitable transmision mechanism. Moreover, the model reproduces exacerbate (downward) rigidities in wages and prices, and hysteresis in unemployment; and more interestingly, these results show that in absence of indexation schems, loss aversion would enough to explain the existence of rigidities in prices and wages. As exposed in the first two chapters of this dissertation, it is worth pointing out that asymmetries in RBC models could be more adequately captured by General Impulse Response Functions than by higher-order moments. However, a more rigorous test for the properties of the asymmetric model proposed here would entail the application of nonlinear econometric tools, which might be useful for the purposes hereby exposed. Several issues remain for the research agenda: i) Structural parameters of the model need to be estimated; ii) the ways how my loss aversion DSGE model can be employed to study issues in policy-making, asset-pricing, risk premia puzzle, international asymmetric business cycles, risk-sharing, home bias, among other areas of knowledge.

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3.5 Annex 1: the full dynamic system

$$\Psi(z_t) = -\frac{\overline{r}^k}{1+\varsigma} + \overline{r}^k \frac{z_t^{1+\zeta}}{1+\varsigma}$$

$$z_{ct} = \frac{C_t^\tau}{\overline{C}_t}$$

$$\overline{C}_t = (1-\chi)\overline{C}_{t-1} + \chi C_t^\tau$$

$$z_{\mathcal{L}t} = \frac{\mathcal{L}_t^\tau}{\overline{\mathcal{L}}_t}$$

$$\overline{\mathcal{L}}_t = (1-\chi_{\mathcal{L}})\overline{\mathcal{L}}_{t-1} + \chi_{\mathcal{L}}(\mathcal{L}_{t-1})$$

$$\mathcal{L}_t^\tau = 1 - l_t^\tau$$

$$\phi_{ct} = \frac{1}{1+\exp\gamma_c\left(z_{ct}-1\right)}$$

$$\phi_{\mathcal{L}t} = \frac{1}{1+\exp\gamma_{\mathcal{L}}(z_{\mathcal{L}t}-1)}$$

$$\frac{\partial\phi_{ct}}{\partial z_{ct}} = \frac{-\gamma_c\exp\gamma_c\left(z_{ct}-1\right)}{\left[1+\exp\gamma_c\left(z_{ct}-1\right)\right]^2}$$

$$\frac{\partial\phi_{\mathcal{L}t}}{\partial z_{lt}} = \frac{-\gamma_{\mathcal{L}}\exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t}-1\right)}{\left[1+\exp\gamma_{\mathcal{L}}\left(z_{\mathcal{L}t}-1\right)\right]^2}$$

$$E_t\left[\beta\frac{\lambda_t}{\lambda_{t+1}}\frac{R_tP_t}{P_{t+1}}\right] = 1$$

$$\lambda_t = \varepsilon_t^b\frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t} + \varepsilon_t^b\frac{-\gamma\left(z_{ct}\right)^{\gamma-1}}{\left[1+\left(z_{ct}\right)^{\gamma}\right]^2}\frac{\left[\left(z_{ct}\right)^{\underline{\theta}} - \left(z_{ct}\right)^{\overline{\theta}}\right]}{\overline{C}_t} + \varepsilon_t^b\phi_{ct}\left[\frac{\underline{\theta}\left(z_{ct}\right)^{\underline{\theta}-1}}{\overline{C}_t} - \frac{\overline{\theta}\left(z_{ct}\right)^{\overline{\theta}-1}}{\overline{C}_t}\right]$$

$$\begin{aligned} U_t^{\mathcal{L}\tau} = \frac{\partial U_t^\tau}{\partial\mathcal{L}_t}\frac{\partial\mathcal{L}_t}{\partial l_t} &= \varepsilon_t^b\varepsilon_t^L\frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t} + \varepsilon_t^b\varepsilon_t^L\frac{\partial\phi_{\mathcal{L}t}}{\partial z_{lt}}\frac{\left[\left(z_{\mathcal{L}t}\right)^{\underline{\mu}} - \left(z_{\mathcal{L}t}\right)^{\overline{\mu}}\right]}{\overline{\mathcal{L}}_t} \\ &+ \varepsilon_t^b\varepsilon_t^L\phi_{\mathcal{L}t}\left[\frac{\underline{\mu}\left(z_{\mathcal{L}t}\right)^{\underline{\mu}-1}}{\overline{\mathcal{L}}_t} - \frac{\overline{\mu}\left(z_{\mathcal{L}t}\right)^{\overline{\mu}-1}}{\overline{\mathcal{L}}_t}\right] \end{aligned}$$

$$\frac{\widetilde{P}_t^j}{P_t}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p} E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}\frac{\left(P_{t+i-1}/P_t\right)^{\gamma_p}}{P_{t+i}/P_t}y_{t+1}^j\frac{1}{\lambda_{p,t+i}} = E_t\sum_{i=0}^{\infty}\beta^i\xi_p^i\frac{\lambda_{t+i}}{\lambda_t}y_{t+1}^j\left(\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}\right)$$

$$(P_t)^{-1/\lambda_{p,t}} = \xi_p \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \right)^{-1/\lambda_{p,t}} + (1-\xi_p) \left(\widetilde{P}_t^j \right)^{-1/\lambda_{p,t}}$$

$$E_0 \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\partial U_t^\tau}{\partial \mathcal{L}_t} \frac{\partial \mathcal{L}_t}{\partial l_t} \left(\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}} l_{t+i}^\tau \right) = \frac{\widetilde{W}_t}{P_t} E_0 \sum_{i=0}^{\infty} \beta^i U_{t+i}^C \xi_w^i l_{t,t+i}^\tau \frac{P_t}{P_{t+i}} \left(\frac{P_{t+i-1}}{P_{t-1}} \right)^{\gamma_w} \frac{1}{\lambda_{w,t+i}} \tag{3.80}$$

$$y_t^j = \varepsilon_t^a \widetilde{K}_{j,t}^\alpha L_{j,t}^{1-\alpha} - \Phi$$

$$\frac{W_t}{P_t} \frac{L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha}$$

$$MC_t = \frac{1}{\varepsilon_t^a} \left(\frac{W_t}{P_t} \right)^{1-\alpha} \left(r_t^k \right)^\alpha \left(\alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \right)$$

$$(W_t)^{-1/\lambda_{w,t}} = \xi \left(W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \right)^{-1/\lambda_{w,t}} + (1-\xi)(\widetilde{w}_t)^{-1/\lambda_{w,t}}$$

$$K_t = K_{t-1}\left[1-\tau\right] + \left[1 - S\left(\varepsilon_t^I I_t / I_{t-1}\right)\right] I_t$$

$$S(I) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_{t-1}^2$$

$$\Psi(z_t) = -\frac{r^k}{1+\varsigma} + r^k \frac{z_t^{1+\zeta}}{1+\varsigma}$$

$$Q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[Q_{t+1}(1-\tau) + \left(r_{t+1}^k z_{t+1}^\tau - \Psi(z_{t+1}^\tau) \right) \right] \right]$$

$$1 = Q_t \left[1 - \frac{\partial S\left(\varepsilon_t^I I_t / I_{t-1}\right)}{\partial I_t} \left(\frac{\varepsilon_t^I}{I_{t-1}} \right) I_t - S\left(\varepsilon_t^I I_t / I_{t+1}\right) \right] + \beta E_t \left\{ Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\partial S\left(\varepsilon_{t+1}^I I_{t+1} / I_t\right)}{\partial I_t} \varepsilon_{t+1}^I I_{t+1}^2 \frac{1}{I_t^2} \right] \right\}$$

$$r_t^k = \Psi'(z_t)$$

$$Y_t = C_t + G_t + I_t + \Psi(z_t) K_{t-1}$$

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R} \right)^{r_R} \left[\left(\frac{1+\Pi_t}{1+\Pi} \right)^{r_\Pi} \left(\frac{Y_t}{Y} \right)^{r_Y} \right]^{1-r_R} \varepsilon_t^R$$

Stochastic processes are such that:

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b$$

$$\ln \varepsilon_t^L = \rho_L \ln \varepsilon_{t-1}^L + \eta_t^L$$

$$\ln \varepsilon_t^I = \rho_I \ln \varepsilon_{t-1}^I + \eta_t^I$$

$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a$$

$$\ln(\varepsilon_t^g) = \rho_g \ln(\varepsilon_{t-1}^g) + \eta_t^g$$

$$\overline{\pi}_t = \rho \overline{\pi}_{t-1} + \eta_t^\pi$$

$$\varepsilon_t^R = \exp(\eta_t^R)$$

$$\lambda_{p,t} = \lambda_p + \eta_t^p$$

$$\lambda_{w,t} = \lambda_w + \eta_t^w$$

$$P_t = \left[\int_0^1 \left(\overline{P}_t \right)^{-1/(\lambda_p)} dj \right]^{-\lambda_p} = \overline{P}_t \text{ , therefore } Y_t = y_t^j = \varepsilon_t^a K_j^\alpha L_j^{1-\alpha} - \Phi = \varepsilon_t^a K^\alpha L^{1-\alpha} - \Phi$$