Testing for unit roots in three-dimensional heterogeneous panels in the presence of cross-sectional dependence

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Introduction

Panel unit root tests combine information from the time-series dimension with that from the cross-section dimension, such that fewer time observations are required for these tests to have power.

- Levin and Lin (1992)
- Maddala and Wu (1999)
- Im, Pesaran and Shin (2003) (IPS).

A critical assumption underlying these tests is that of cross sectional independence. Failing to account for potential cross-sectional dependence leads to size distortions.

Introduction

This paper examines the time series properties of a 3D panel, which were first looked at by Ghosh (1976), and subsequently by Baltagi (1987).

Within the context of a 3D panel, N might denote countries (or industries) and M might be regions (or firms within that country or industry).

With such panels there is the possibility of an error covariance matrix in which there is different correlation across the N units compared to across the M units.

Introduction

Examples of 3D panels are still relatively rare, but are starting to appear in the literature.

- Matyas (1997), Baltagi et al. (2003). Econometric gravity models that account for time, exporter and importer effects.
- Goldberger and Verboven (2005). Market integration and convergence in the European car market using a large panel of make of car, countries and time.
- Davies and Lahiri (1995). Inflation forecasts available for different individuals, across varying time horizons over the period from 1977 through to 1992.

Introduction

This paper uses Monte Carlo simulation to investigate the small sample properties of the IPS test, for a 3D panel of data that allows for cross-sectional correlation over both the N and M dimensions.

The simulation results show a severe size distortion of the IPS test, and we consider two alternative procedures to account for potential cross-sectional dependency:

- An extended version of the cross-sectionally augmented IPS (CIPS) test statistic; see Pesaran (2007). This involves tabulating a new set of critical values to apply the CIPS test in a 3D setup.
- A procedure based on a bootstrap of the residuals of the IPS test.

Introduction

The plan of the paper is as follows.

- Brief review of the IPS approach to unit root testing.
- Design of the Monte Carlo simulations.
- Main simulation results.
- Empirical application.

IPS test

Let us assume that $y_{i,t}$ is generated by a first-order autoregressive process:

$$\Delta y_{i,t} = a_i + b_i y_{i,t-1} + \sum_{r=1}^{p} c_{ir} \triangle y_{i,t-r} + \varepsilon_{i,t}, \quad \begin{array}{l} i = 1, \ldots, N, \\ t = 1, \ldots, T. \end{array} \tag{1}$$

The null hypothesis to test the presence of a unit root is $H_0 : b_i = 0$ for all i, against the alternative that at least one of the individual series in the panel is stationary, that is $H_1 : b_i < 0$ for at least one i.

IPS test

The IPS test averages the ADF statistics obtained in equation (1) across the N cross-sectional units of the panel, denoted as:

$$\widetilde{tbar}_{NT} = (N)^{-1} \sum_{i=1}^{N} \tilde{t}_{i,T},$$

where $\tilde{t}_{i,T}$ is the ADF test for the i^{th} cross-sectional unit.

IPS show that a suitable standardisation of the $\widetilde{tbar}_{NT}$ statistic, denoted as $Z_{\widetilde{tbar}}$, follows a standard normal distribution.

A critical assumption underlying the IPS test is that of cross sectional independence. Failing to account for potential cross-sectional dependence leads to over-rejection of the unit root test statistics, the extent of which is related to the degree of this dependence.

CIPS test

Pesaran (2007) suggests modifying the IPS test by augmenting equation (1) with the cross section averages of lagged level and lagged first-differences of the individual series in the panel.

Thus, the test would be based on the following p^{th} order cross-sectionally augmented ADF regressions:

$$\Delta y_{it} = a_i + b_i y_{i,t-1} + \sum_{r=1}^{p} c_{ir} \triangle y_{it-r} + d_i \bar{y}_{t-1} + \sum_{r=0}^{p} f_{ir} \Delta \bar{y}_{t-r} + \varepsilon_{it}, \qquad (2)$$

where $\bar{y}_t$ is the cross section mean of y_{it}, defined as $\bar{y}_t = (N)^{-1} \sum_{i=1}^{N} y_{it}$.

CIPS test

The corresponding cross-sectionally augmented version of the IPS (CIPS) test statistic is:

$$\text{CIPS} = (N)^{-1} \sum_{i=1}^{N} t_i,$$

where t_i is the cross-sectionally ADF statistic for the i^{th} unit.

Critical values of the CIPS statistic are tabulated by Pesaran (2007) for:

- Case I: No intercepts or trends,
- Case II: Intercepts only, and
- Case III: Intercepts and trends.

3D CIPS test

Pesaran (2007) (p.27) suggests the CIPS test could be generalised "... for a richer pattern of cross dependence ...(which) can be dealt with by augmenting the individual ADF regressions with additional cross-section averages formed over subgroups, such as regions, sectors or industries."

For a 3D panel the CIPS test would be based on the following p^{th} order cross-sectionally augmented ADF regressions:

$$\begin{aligned} \Delta y_{ij,t} &= a_{ij} + b_{ij} y_{ij,t-1} + \sum_{r=1}^{p} c_{ijr} \triangle y_{ij,t-r} + d_{ij} \bar{y}_{t-1} + \sum_{r=0}^{p} f_{ijr} \Delta \bar{y}_{t-r} \quad (3) \\ &\quad + g_{ij} \bar{y}_{it-1} + \sum_{r=0}^{p} h_{ijr} \Delta \bar{y}_{it-r} + k_{ij} \bar{y}_{jt-1} + \sum_{r=0}^{p} l_{ijr} \Delta \bar{y}_{jt-r} + \varepsilon_{ij,t}, \end{aligned}$$

where $\bar{y}_t$ is the cross-sectional mean of $y_{ij,t}$, $\bar{y}_{i,t}$ is the cross-sectional mean formed over j and $\bar{y}_{j,t}$ is the cross-sectional mean formed over i.

3D CIPS test

The corresponding 3D-CIPS test, would then be:

$$\text{3D-CIPS} = (NM)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{M} t_{ij},$$

where t_{ij} is the cross-sectionally ADF statistic for the ij^{th} cross-sectional unit.

Table 1 presents critical values of the 3D-CIPS (based on 20,000 Monte Carlo replications) for different values of N, M and T, and deterministic components.

These critical values are larger in absolute terms than the corresponding values tabulated by Pesaran (2007).

Table 1. Critical values of the 3D-CIPS test

N	M	T	nc, nt		c		c, t	
			5%	10%	5%	10%	5%	10%
5	2	20	-2.51	-2.30	-3.01	-2.78	-3.43	-3.20
5	5	20	-2.06	-1.96	-2.54	-2.43	-2.94	-2.83
5	10	20	-1.94	-1.86	-2.41	-2.33	-2.80	-2.73
10	2	20	-2.25	-2.10	-2.72	-2.57	-3.13	-2.98
10	5	20	-1.93	-1.86	-2.41	-2.33	-2.80	-2.73
10	10	20	-1.86	-1.80	-2.32	-2.26	-2.72	-2.66
20	2	20	-2.10	-1.99	-2.56	-2.45	-2.98	-2.85
20	5	20	-1.87	-1.81	-2.33	-2.27	-2.73	-2.67
20	10	20	-1.82	-1.77	-2.28	-2.23	-2.67	-2.63
25	2	20	-2.05	-1.95	-2.51	-2.42	-2.93	-2.82
25	5	20	-1.86	-1.80	-2.31	-2.26	-2.72	-2.66
25	10	20	-1.81	-1.76	-2.27	-2.22	-2.67	-2.62

Table 1 (cont'd). Critical values of the 3D-CIPS test

N	M	T	nc, nt		c		c, t	
			5%	10%	5%	10%	5%	10%
5	2	25	-2.51	-2.31	-2.99	-2.79	-3.41	-3.21
5	5	25	-2.08	-1.97	-2.55	-2.45	-2.97	-2.86
5	10	25	-1.96	-1.89	-2.43	-2.36	-2.84	-2.77
10	2	25	-2.26	-2.12	-2.73	-2.59	-3.14	-3.00
10	5	25	-1.96	-1.89	-2.44	-2.36	-2.84	-2.77
10	10	25	-1.89	-1.83	-2.36	-2.30	-2.77	-2.71
20	2	25	-2.10	-2.00	-2.57	-2.46	-2.98	-2.87
20	5	25	-1.90	-1.84	-2.36	-2.31	-2.77	-2.71
20	10	25	-1.85	-1.80	-2.32	-2.27	-2.72	-2.68
25	2	25	-2.07	-1.98	-2.53	-2.44	-2.94	-2.85
25	5	25	-1.88	-1.83	-2.35	-2.30	-2.76	-2.70
25	10	25	-1.84	-1.79	-2.31	-2.26	-2.71	-2.67

Table 1 (cont'd). Critical values of the 3D-CIPS test

N	M	T	nc, nt		c		c, t	
			5%	10%	5%	10%	5%	10%
5	2	40	-2.51	-2.33	-2.98	-2.80	-3.41	-3.22
5	5	40	-2.12	-2.02	-2.59	-2.50	-3.01	-2.92
5	10	40	-2.00	-1.93	-2.48	-2.41	-2.90	-2.83
10	2	40	-2.28	-2.14	-2.75	-2.62	-3.16	-3.03
10	5	40	-1.99	-1.92	-2.47	-2.41	-2.90	-2.83
10	10	40	-1.93	-1.88	-2.41	-2.36	-2.82	-2.77
20	2	40	-2.14	-2.04	-2.60	-2.51	-3.02	-2.93
20	5	40	-1.94	-1.88	-2.42	-2.36	-2.83	-2.78
20	10	40	-1.89	-1.85	-2.37	-2.33	-2.78	-2.74
25	2	40	-2.11	-2.02	-2.57	-2.48	-2.98	-2.90
25	5	40	-1.92	-1.87	-2.40	-2.35	-2.82	-2.77
25	10	40	-1.89	-1.84	-2.36	-2.32	-2.78	-2.74

Bootstrap approach (BIPS)

As an alternative procedure to handle cross-sectional dependency, Maddala and Wu (1999) and Chan (2004) have considered bootstrapping unit root tests. This involves:

1. Resample the restricted residuals $\Delta y_{ij,t} = y_{ij,t} - y_{ij,t-1} = \varepsilon_{ij,t}$ after centring, since $y_{ij,t}$ has a unit root under the null hypothesis. Let $\varepsilon^*_{ij,t}$ be the bootstrap samples of $\varepsilon_{ij,t}$.
2. Preserve the cross-correlation structure of the error term within each cross section i, j, for which we resample the restricted residuals with the cross-section index fixed.
3. Generate the bootstrap samples of y^*_{ijt} by taking partial sums of $\varepsilon^*_{ij,t}$, that is $y^*_{ij,t} = y^*_{ij,0} + \sum_{k=1}^{t} \varepsilon^*_{ij,k}$, where $y^*_{ij,0}$ is set equal to zero.

These Monte Carlo simulations are based on 2,000 replications each of which uses 100 bootstrap repetitions. This bootstrapped IPS test will be denoted as BIPS.

Monte Carlo simulation design

Let us assume that $y_{ij,t}$ is generated by a first-order autoregressive process:

$$y_{ij,t} = \left(1 - \phi_{ij}\right)\mu_{ij} + \phi_{ij} y_{ij,t-1} + \varepsilon_{ij,t}, \quad \begin{array}{l} \mu_{ij} \sim N(0,1), \\ \varepsilon_{ij,t} \sim N\left(0, \sigma^2_{ij}\right), \\ \sigma^2_{ij} \sim U[0.5, 1.5] \end{array}$$

Under the null hypothesis $\phi_{ij} = 1$ for all i, j. Under the alternative hypothesis $\phi_{ij} = 0.9$ for all i, j.

- $N = (5, 10, 20, 25)$; $M = (2, 5, 10)$; $T = (10, 20, 25, 40)$.
- Replications: 2,000 (the first 50 observations are discarded).
- μ_{ij} and σ^2_{ij} are generated independently of ε_{ijt} once, and then fixed throughout replications.

Monte Carlo simulation design

Further, we allow the cross-sectional dependence to differ across the i, j cross-sectional units, of the form:

$$E(\varepsilon_{ikt}, \varepsilon_{ilt}) = \begin{cases} \sigma^2 & k = l \\ \omega_{kl} & k \neq l \end{cases}, E(\varepsilon_{pjt}, \varepsilon_{qjt}) = \begin{cases} \sigma^2 & p = q \\ \omega_{pq} & p \neq q \end{cases},$$

where, following O'Connell (1998), we set $\omega_{kl} = 0.3, 0.5, 0.7, 0.9$, and restrict the cross-sectional correlation over i to be a constant proportion of that over j, that is, $\omega_{pq} = \theta\omega_{kl}$, where $\theta = 0.0, 0.25, 0.5, 0.75, 1.0$.

Main results: Size

Both the 3D-CIPS and BIPS tests are approximately correctly sized, although the empirical size of the BIPS test tends to be slightly low when ω is small and N and/or M are large.

Main results: Power

The power of both the BIPS and the 3D-CIPS tests increases with increases in either the number of cross-sectional units, N or M, or with increases in T.

However, while the power of the BIPS test falls as the degree of cross-sectional correlation (in either ω_{kl} or θ) increases, the power of the 3D-CIPS test is largely invariant to the degree of cross-sectional correlation.

Main results: Power when there is cross-correlation over j, but none over i

(in other words $\theta = 0.0$)

For example, consider $i = 2$, $j = 3$, $\theta = 0.0$, $\omega_{kl} = 0.5$

$$\begin{array}{cccccc}
1 \\
0.5 & 1 \\
0.5 & 0.5 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0.5 & 1 \\
0 & 0 & 0 & 0.5 & 0.5 & 1
\end{array}$$

Main results: Power when there is cross-correlation over j, but none over i

The BIPS test unambiguously dominates the 3D-CIPS test for all values of ω_{kl}.

As an alternative procedure, aggregating the data over j to produce a 2D panel removes the cross-sectional dependency, although reduces the cross-sectional units from NM to N. In this case, the standard IPS test applied over the 2D aggregated data is correctly sized, and its power is better than that of the 3D-CIPS test, but remains inferior to that of the BIPS test.

Note, however, that even for relatively small values of θ, aggregation over j yields incorrectly sized tests, as correlation remains over i and is therefore inappropriate.

Main results: Power when there is cross-correlation over j, but none over i

N	M	T	$3D-CIPS$ $(\theta=0.0)$ ω_{kl} 0.3	0.5	0.7	0.9	$BIPS$ $(\theta=0.0)$ ω_{kl} 0.3	0.5	0.7	0.9
5	2	40	8.8	9.0	9.7	10.1	55.2	51.8	46.3	41.2
5	5	40	18.6	18.3	17.8	18.7	86.5	76.0	61.4	45.1
5	10	40	31.9	31.6	31.4	31.8	95.3	83.6	64.5	44.1
10	2	40	13.4	13.6	14.0	14.3	82.5	79.3	73.9	65.2
10	5	40	31.3	31.4	31.8	31.0	98.2	92.5	82.0	67.3
10	10	40	47.0	46.8	46.1	46.2	99.8	97.3	87.7	70.3
20	2	40	18.4	18.6	18.9	19.0	98.5	97.5	94.9	89.8
20	5	40	44.2	44.8	44.9	43.5	100	99.8	97.7	91.0
20	10	40	59.8	60.0	59.8	60.0	100	100	99.0	92.7
25	2	40	20.9	20.6	20.5	21.4	99.4	99.0	97.7	94.9
25	5	40	47.6	48.1	47.6	47.5	100	99.9	98.7	94.9
25	10	40	63.9	63.5	62.6	61.9	100	100	99.9	96.9

Main results: Power when there is cross-correlation over j, and little over i

(in other words $0 < \theta < 1$)

For example, consider $i = 2$, $j = 3$, $\theta = 0.25$, $\omega_{kl} = 0.5$

1					
0.5	1				
0.5	0.5	1			
0.125	0.125	0.125	1		
0.125	0.125	0.125	0.5	1	
0.125	0.125	0.125	0.5	0.5	1

Main results: Power when there is cross-correlation over j, and little over i

In general the BIPS test has greater power compared to the 3D-CIPS test for all values of θ considered, when $\omega_{kl} < 0.5$.

For $\omega_{kl} \geq 0.5$ the power of the 3D-CIPS test is occasionally greater than that of the BIPS test, with the dominance of the 3D-CIPS over the BIPS test more likely when θ is bigger and N, M or T are larger. For example, for $\theta = 0.75$, $\omega_{kl} = 0.9$, $N = 25$, $M = 10$ and $T = 40$ the power of the 3D-CIPS test is 76.2% compared to 30.1% for the BIPS test.

Main results: Power when there is cross-correlation over j, and little over i

N	M	T	$3D - CIPS$ $(\theta = 0.75)$				$BIPS$ $(\theta = 0.75)$			
			ω_{kl}				ω_{kl}			
			0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9
5	2	40	9.0	9.3	9.9	9.9	49.6	39.7	31.7	24.0
5	5	40	18.3	19.0	18.6	19.0	72.0	52.2	35.7	23.4
5	10	40	31.4	31.5	30.8	31.7	81.3	58.1	37.2	23.6
10	2	40	13.4	13.1	12.6	13.7	66.2	49.4	34.5	24.7
10	5	40	31.5	32.2	31.5	31.7	82.7	58.5	40.2	26.3
10	10	40	45.4	44.0	45.0	46.0	87.1	62.4	40.8	26.6
20	2	40	18.6	18.8	19.1	18.5	79.6	58.2	40.2	27.6
20	5	40	43.0	43.1	43.6	43.4	87.4	61.8	40.1	27.8
20	10	40	58.9	58.9	60.0	59.3	91.6	67.8	45.4	29.2
25	2	40	20.5	20.8	20.7	20.3	83.5	60.7	40.3	25.9
25	5	40	47.5	47.1	47.5	48.1	89.4	64.8	44.6	31.0
25	10	40	64.9	63.1	63.0	63.2	91.8	67.9	45.9	30.1

Main results: Power when there is identical correlation over i and j

(in other words $\theta = 1$)

For example, consider $N = 2$, $M = 3$, $\theta = 1$, $\omega_{kl} = 0.5$

1					
0.5	1				
0.5	0.5	1			
0.5	0.5	0.5	1		
0.5	0.5	0.5	0.5	1	
0.5	0.5	0.5	0.5	0.5	1

Main results: Power when there is identical correlation over i and j

The CIPS test as outlined in equation (2) can be applied. The size of the CIPS test is correct and the power is greater than that of the 3D-CIPS test. For example, in the case of $\theta = 1$ reported above, the CIPS test has power of 82.0%.

Main results: Power when there is identical correlation over i and j

N	M	T	$3D-CIPS$ $(\theta = 1.0)$ ω_{kl} 0.3	0.5	0.7	0.9	$BIPS$ $(\theta = 1.0)$ ω_{kl} 0.3	0.5	0.7	0.9
5	2	40	9.2	9.2	9.1	9.0	45.3	34.3	25.0	17.2
5	5	40	19.2	19.3	19.7	20.1	64.5	42.2	25.3	16.2
5	10	40	32.0	31.7	31.1	31.4	73.4	44.1	25.9	14.7
10	2	40	13.2	12.9	12.4	12.7	58.4	37.7	24.1	15.4
10	5	40	31.7	32.3	32.1	31.6	73.4	44.1	25.9	14.7
10	10	40	45.0	44.9	44.1	42.9	76.3	44.8	26.9	15.2
20	2	40	18.7	18.5	18.8	18.1	69.6	43.4	25.7	15.8
20	5	40	42.3	42.3	43.2	42.8	76.3	44.8	26.9	15.2
20	10	40	59.0	58.8	57.4	56.8	81.5	49.6	28.4	16.1
25	2	40	20.1	20.6	20.5	20.8	73.4	44.1	25.9	14.7
25	5	40	47.6	47.5	47.4	47.8	78.3	48.9	30.0	16.6
25	10	40	64.4	64.3	65.0	63.4	80.2	50.6	29.6	16.0

Empirical application

Consider an international macro data set consisting of the log of real bilateral exports (imports) from (to) country i to (from) country j at year t.

- i includes Belgium-Luxembourg, Canada, France, Germany, Italy, Japan, The Netherlands, Spain, Switzerland, United Kingdom and United States;
- j includes Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, Mexico, Paraguay, Peru, Uruguay and Venezuela;
- Sample period is 1963 to 2006 ($T = 44$).

Trade data were collected from the UN Commodity Trade Statistics Database, while export and import deflators were taken from the IFS produced by the IMF. The total number of observations used for estimation is 5,676.

Empirical application

The individual ADF(p) regressions (with and without cross-section augmentations) are estimated for $p = 0$, as determined by the Schwartz information criterion.

For the BIPS test, the percentiles of the bootstrap distribution were calculated as described in Maddala and Wu (1999), using 2,000 replications and a block size equal to 1.

Empirical application

Under the unit root hypothesis and in the absence of cross-section dependence, the IPS statistic indicates very strong rejection of the null hypothesis.

The 3D-CIPS and BIPS test statistics, both of which account for differing degrees of potential cross-sectional dependencies over i and j, also indicate a strong rejection of the null hypothesis of a unit root, although there is a marked reduction in the absolutely size of the test statistic (qualitatively similar results are obtained when choosing $p = 1$ or when the block size is set equal to 2).

Empirical application. Panel unit root tests include intercept and trend

Variables	IPS	3D-CIPS	BIPS
Real bilateral exports	-10.369	-4.290	-2.901
1% critical value	(-2.330)	(-2.920)	(-2.560)
5% critical value	(-1.650)	(-2.820)	(-2.450)
10% critical value	(-1.280)	(-2.770)	(-2.383)
Real bilateral imports	-7.730	-3.442	-2.716
1% critical value	(-2.330)	(-2.920)	(-2.510)
5% critical value	(-1.650)	(-2.820)	(-2.408)
10% critical value	(-1.280)	(-2.770)	(-2.361)