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LABOR SUPPLY, BIASED TECHNOLOGICAL CHANGE AND ECONOMIC GROWTH[1]

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Abstract

We consider a model of factor saving innovations and study the effects of exogenous changes in labor supply. In a biased innovations setting, as economies accumulate capital, labor becomes relatively scarce and expensive. As a consequence, incentives for labor saving and capital using innovations appear. By the same token, exogenous changes in labor supply affect factor prices. In general, a reduction in labor supply decreases current output and generates incentives for labor saving innovations. Therefore, the effect that a change in the supply of labor has on factor prices is mitigated and, depending on the initial conditions, it may be contrasted by the effect of the technological bias. Finally, the movements of the factor prices affect the saving decisions and consequently the dynamics of economic growth. We explore the consequences of an exogenous decrease in labor supply in two different settings: a homogenous agents model with infinite horizon and an overlapping generations model.

JEL Classification: O11, 031, J20, J31.
Key words: Labor supply, factor income shares, economic growth.

Resumen

Se utiliza un modelo de innovaciones sesgadas para estudiar los efectos de cambios exógenos en la oferta laboral. En un contexto de innovaciones sesgadas, a medida que las economías acumulan capital, el trabajo se hace relativamente más escaso y más caro, por este motivo, hay incentivos para adoptar tecnologías ahorradoras de trabajo. Del mismo modo un cambio en la oferta laboral afecta la abundancia de factores y sus precios relativos. En general, una reducción de la oferta laboral, hace que el trabajo sea más caro y genera incentivos para cambio tecnológico ahorrador de trabajo. Así, el efecto inicial que tiene el cambio en la oferta laboral sobre los precios de los factores es mitigado por el cambio tecnológico. Finalmente, los movimientos en la remuneración a los factores afectan las decisiones de ahorro y, por lo tanto, la dinámica del crecimiento. En este trabajo se exploran las consecuencias de una reducción de la oferta laboral en dos contextos teóricos diferentes: un modelo de agentes homogéneos y horizonte infinito y un modelo de generaciones traslapadas.

Clasificación JEL: O11, 031, J20, J31.

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Palabras Clave: Oferta de trabajo, participación de los factores en el ingreso, crecimiento económico.

1. Introduction

We study the effects of changing labor supply on total output in a setting of factor saving innovations where the choice of technologies depends on the relative abundance of factors. The sign and the size of the effect of a technological change on output depend on the relative abundance of factors. Therefore, an exogenous change in the supply of one (or more) factors can have different effects depending on the initial conditions.

Following Zuleta (2006) we assume a continuous set of Cobb-Douglas technologies. Capital owners choose the capital intensity of the technology within a set of technologies differentiated by the elasticity of output with respect to capital. Other models where capital intensity is endogenous those proposed by Jones and Manuelli (1992), Klump and De la Grandeville (2000), and Zuleta (2004) but such endogeneity is independent of the producers' decisions. In our model, any technology can be obtained paying a cost and if the amount of assets is high enough to make technological changes profitable then there exists a positive relation between the amount of assets and the capital intensity of the technology. The gains derived from adopting capital intensive technologies positively depend on the level of assets, so in equilibrium the amount of assets determines both the capital stock K and the capital share α. This implies that savings (changes in assets) determines the change in the average product of capital.

We describe below the direct and indirect effects of a change in labor supply, given the nonlinearity in the model.

1. A reduction in labor supply generates a decrease in current income. In general, the effect of a reduction in the labor supply on aggregate output is small when the technology is capital intensive and big when the technology is labor intensive. Given that capital (labor) abundant economies use capital (labor) intensive technologies, the negative effect of a reduction in labor supply is likely to be stronger in labor abundant economies.

2. A reduction in labor supply increases the relative abundance of capital and generates an increase in the relative price of labor. Now, the change in relative factor prices

constitutes an incentive for capital using or labor saving technological changes. Therefore: (i) If the economy is capital abundant this change may increase output. (ii) It increases capital income share and reduces labor income share. In principle, this redistribution reduces the income of the workers as well as their savings. So, the net effect on savings depends on the marginal propensity to consume capital income and the marginal propensity to consume labor income.

In summary, a decrease in the number of workers has a negative effect on current output but may have a positive effect on the growth rate of the economy. The negative effect of a reduction in labor supply is likely to be higher for labor abundant economies but the dynamic effect depends on the structure of the model. We provide two examples to illustrate this fact. A homogenous agents model with infinite horizon and an overlapping generations model.

The paper is organized in 6 sections. Section 2 presents a review of the literature on labor supply and economic growth. Section 3 presents a review of the literature on factor saving innovations. In section 4 we explain how the firms choose among different technologies and the effects that a technological change has on factor prices. In section 5 we analyze the effect that an exogenous decrease in labor supply has on the growth rate of the economy using two different settings. The last section concludes.

2. Labor Supply and Economic Growth

In the basic neoclassical model the growth rate of population affects negatively both the steady state capital labor ratio and the growth rate of the economy during the transition. Now, a change once and for all in the population size affects the growth rate of the economy but not the steady state capital labor ratio. According to Mankiw, Romer and Weil (1992) the empirical evidence generally supports the Solow model and implies negative relation between labor force growth and economic growth.

The models of endogenous growth that follow the line of Romer (1986) and explain the existence of persistent growth as a consequence of the positive effect of aggregate capital on total factor productivity predict, in general, a positive relation between population or aggregate human capital and economic growth. Some authors model the positive effect of aggregate capital on total factor productivity introducing invention of new goods (or new qualities) containing technology (Romer, 1990 and Grossman and Helpman, 1991). Since technology is a public good whose rate of growth is linear in the number of workers or on the aggregate level of human capital then, the bigger the population, the stronger the effect of an innovation.

There is no much evidence supporting the existence of scale effects (the main exception is Kremer, 1993). In addition, the existence of scale effects is not supported by a number of stylized facts and empirical studies. First, even if rich cities are densely populated, some poor countries are also densely populated and this fact seems to be an obstacle for development. Second, in a large scale study on the determinant of economic growth, Sala-i-Martin (1997) finds that scale effects are not significantly different from zero. Moreover, Brander and Dowrick (1994), using a 107-country panel data (1960-1985) find that reductions in the birthrate have a strong positive medium-term impact on per capita income growth, that is, for their sample there is a reverse-scale effect. Finally, regarding the effect of human capital on growth, the evidence presented by O'Neill (1995) suggest that the contribution of human capital to growth is higher in richer economies and is not significantly different from zero for very poor countries.

Several theoretical models that try to eliminate scale effects has been proposed: Jones (1995), Young (1998), Segerstrom (1998) and Howitt (1999) eliminate the scale effect with models a la Romer. In these papers the scale effect is eliminated assuming either decreasing returns in the production of ideas or increasing costs in the number of ideas discovered. Zuleta (2004) modifies the Romer (1990) model assuming a CES production function and finds that if the elasticity of substitution between factors is higher than one during the transition (short and medium run) rich economies present higher scale effects than poor economies and very poor economies may present negative scale effects.

Here, we develop a growth model where the factor intensity of the technology used by the firms is endogenous and, in equilibrium, is determined by the factor abundance of the economy. In contrast to Zuleta (2004) in our model the change in factor shares is the result of conscience actions by the economic agents.

3. Factor Saving Innovations

The literature on biased innovations is extensive: Kennedy (1964), Zeira (1998 and 2005), Acemoglu (2002), Boldrin and Levine (2002), Peretto and Seater (2005), Zuleta (2006) and Zuleta and Young (2006) among others, present models of endogenous growth with biased technological change. They use this concept to explain differences in productivity across countries, the behavior of wage dispersion, to explain long run growth or the behavior of factor shares.

A capital using and labor saving innovation is a change in the technological parameters such that, holding factors' prices constant, the optimal capital labor ratio is increased. If, in contrast, the optimal capital ratio is decreased, we say that the technical change is capital saving and labor using. Now, the effect of a labor saving innovation on output depends on the relative abundance of labor. If labor is relatively scarce its price is high and firms have incentives to adopt labor saving technologies. Therefore, models of factor saving innovations generally predict that both the elasticity of output with respect to capital and the capital income share must be higher in richer economies. These two predictions are supported by empirical evidence: 1. In richer economies the technology is more intensive in reproducible factors, that is, physical and human capital (Durlauf and Johnson, 1995). 2. The share of reproducible capital is higher in rich countries (Caselli and Feyrer, 2006). 3. For the United States of America, the share of land in Net National Product decreases as the economy grows (Hansen and Prescott (2002)). 4. The share of raw labor in National Income decreases as the economy grows (Krueger 1999, Krusell *et. al*. 2000 and Acemoglu, 2002 among others).

3.1 Technological Change and elasticity of substitution between factors

When we endogenize the capital intensity of the technology using a Cobb-Douglas $Y = AK^{\alpha}L^{1-\alpha}$, the elasticity of substitution becomes a function of capital labor ratio and it is not constant. In the Cobb-Douglas case the technical rate of substitution is given by,

TRS $= \frac{\alpha}{1-\alpha}\frac{L}{K}$ and the elasticity of substitution is given by, $\varepsilon = \left(1 - \frac{\partial\left(\frac{1-\alpha}{\alpha}\right)}{\partial k}\frac{1-\alpha}{\alpha}\right)^{-1}$.

Therefore, if α is endogenous then the elasticity of substitution between factors is also endogenous.

3.2 Labor Supply and Factor Saving Innovations in Continental Europe

An interesting natural experiment of the effects of changes in labor supply can be found in the economies of Continental Europe. According to Blanchard (1997), in the 1970's most continental countries were affected by negative shifts in labor supply and, since the early 1980's their labor markets have been characterized by adverse shifts in labor demand. During these years the average labor share fell form 66 in 1960 to 59 in 1995. "There are two potential explanations for this decrease. The first is a shift in the distribution of rents from workers to firms. The second is technological bias: at a given factor prices, firms have been adopting technologies that use less labor and more capital, thus decreasing the marginal productivity of labor at a given ratio of labor to capital" (Blanchard,1997: pp 1-52.)

The model we present is consistent with the second explanation. The negative shift in labor supply increases the relative abundance of capital and generates an incentive for capital using or labor saving technological changes. On its turn, the technological change reduces the labor income share and increases the capital income share. We explore the possible effects of these changes on income and economic growth.

4. Modeling Technical Change

We assume that for any technology there is a Cobb-Douglas production function and technologies are differentiated by their capital intensity, α, with the restriction that $\alpha \epsilon [0,1)$. Hence, for every technology labor is a necessary input. Any technology has a non negative cost which depends on the desired α. This cost is paid by the capital owners before the production process. So, when firms want to improve technologies, a share of the assets must be devoted to change the technology α. We assume that all technologies exist and are available in the market, so that firms do not pay fixed costs related to R&D. The cost of new technologies is higher for more capital-intensive technologies and there are decreasing returns to scale.

Following Zuleta (2006), we assume that for *B* units of output devoted to build capital goods operating with technology α, the number of capital goods is given by $K = B + \ln(1-\alpha)\Phi$ where *Φ* is a measure of the size of the market. We use population as a measure of the size of the market, so if L_i is the amount of people consuming the good produced by firm *i* then the output produced by a firm *i* using *K* units of capital and technology *α* is given by $A\left(B_i + \ln(1-\alpha)L_i\right)^{\alpha_i} l_i^{1-\alpha_i}$ where *l* is labor.

Finally, we assume a population of 1, so in the aggregate *B* and *K* can be interpreted as assets and capital per capita. However, as we model negative shocks in the labor supply, so that after the shock the labor supply *l* is smaller than one.

4.1 Choosing technology

Firm owners decide the technology they want to use given the amount of assets. The capital intensity of the technology is modified only if the gain derived from the change is positive. We also assume that a primitive technology α_0 exists and is freely available. If assets are devoted to the most labor intensive technology, the production per unit of labor is given by AB^{α_0}.

Given the factor prices firms choose factors in order to maximize profits and given the amount of assets a firm *i* chooses technology to maximize income,

$$\max_{\alpha_i} A\left(B_i + \ln(1-\alpha_i)L_i\right)^{\alpha_i} l_i^{1-\alpha_i} \text{ s.t. } \alpha_i \geq \alpha_0$$

Thus, the optimal level of *α* is given by the First Order Condition:

$$Ak_i^{\alpha_i}\left(\ln\left(\ln B_i + \ln(1-\alpha_i)L_i\right) - \frac{\alpha_i}{1-\alpha_i}\frac{1}{k_i}\right) + \lambda_\alpha = 0$$

Where λ_α is the multiplier of the restriction $\alpha_i \geq \alpha_0$ and k_i is the capital labor ratio.
The wage is given by the marginal productivity of labor, so *α* is equal to capital income share and *(1-α)* is equal to labor income share.

(1) $$w = (1-\alpha_i)Ak_i^{\alpha_i}$$

Firms are also competing for assets and the free entry condition implies $r = \alpha_i A k_i^{\alpha_i - 1}\dfrac{K_i}{B_i}$

Note that in the interior solution, after paying the cost of technology, the capital labor ratio is higher than one, that is, *k>1*. If *k<1* then $\lambda_\alpha > 0$ and $\alpha=\alpha_0$.

Now, in the interior solution $\lambda_\alpha = 0$ and $\alpha_i = \dfrac{\frac{K_i}{L_i}\ln k_i}{1+\frac{K_i}{L_i}\ln k_i}$

Note also that holding the rest constant, any increase in the size of the firm affects K_i and L_i in the same proportions, so the equilibrium level of α is independent of the size of the firm. If all firms use the same technology and face the same market prices then for any pair of firms for any $i \neq j$, $k_i = kj$ and K_i / K_i = K_j / L_j =*K/L*, where *K/L* is capital per capita in the economy. Finally, we assume a population size of one, *L=1*, therefore, the equilibrium α (common to every firm) is

(2) $$\alpha = \max\left\{\alpha_0, \frac{K\ln k}{1+K\ln k}\right\}$$

Equation 2 tells that in the interior solution the productivity of a unit of assets devoted to capital must be equal to the productivity of a unit of assets devoted to technology, that is, $\dfrac{\partial Y}{\partial K} = (1-\alpha)\dfrac{\partial Y}{\partial \alpha}$.

Combining *K=B+ln(1-α)* with equation 2,

(3) $$\alpha = \max\{\alpha_0, \overline{\alpha}\}$$

Where $$\overline{\alpha} = \frac{\left(B+\ln(1-\overline{\alpha})\right)\ln\left(\dfrac{B+\ln(1-\overline{\alpha})}{l}\right)}{1+\left(B+\ln(1-\overline{\alpha})\right)\ln\left(\dfrac{B+\ln(1-\overline{\alpha})}{l}\right)}.$$

Note that given the amount of assets per worker there is only one α that satisfies equation 3 and, given that *K=B+ln(1-α)*, there is only one *K* that satisfies equation 3. Note also that, in the interior solution, α is an increasing function of *B* and that α converges to one as *B* goes to infinity,

(4) $$\frac{\partial \overline{\alpha}}{\partial B} = (1-\overline{\alpha})^2 \frac{1+\ln k}{1+(1-\overline{\alpha})(1+\ln k)}$$

(5) $$\lim_{B\to\infty} \overline{\alpha} = 1$$

Therefore, the capital income share depends on the amount of labor and the value of the assets in the economy. In other words, the technological parameter *α* is bigger when there ire more assets in the economy and lower when the economy is labor abundant. Moreover, if *B<l*, there are no incentives to increase the capital intensity.

Note also that there is a negative relation between *α* and *l,* so that a reduction in the labor supply constitutes an incentive for labor saving innovations, formally,

(6) $$\frac{\partial \overline{\alpha}}{\partial l} = -\frac{K}{l} \frac{(1-\overline{\alpha})^2}{1+(1-\overline{\alpha})(1+\ln k)}$$

Finally, note that the ratio of assets to capital is given by $\frac{K}{B} = 1 + \frac{\ln(1-\alpha)}{B}$. Therefore, $\lim_{a\to\infty} \frac{K}{B} = 1$ (the proof is presented in the Appendix 1).

4.2 Labor, Technology and Factor Prices

We already saw that firms choose labor in such a way that $w = (1-\alpha)Ak^{\alpha}$. Therefore, if markets are competitive and firms make zero profits, the interest rate is given by $r = \alpha A k^{\alpha-1} \frac{K}{B}$.

Recall that in equilibrium part of the assets are devoted to capital and part of the assets are devoted to the technology α. Recall also that *K=B+ln(1-α)* and that the chosen technology is the one that maximizes output, so no single firm has incentives to choose another technology. In this setting, if a single firm increases the interest rate then it may attract more assets. However, the technology and the capital labor ratio remain the same so the firm makes negative profits. If a single firm reduces the interest rate then it looses all its assets. Therefore, $r = \alpha A k^{\alpha-1} \frac{K}{B}$ is the equilibrium interest rate.

The effect of a technological change on factor prices is given by (complete derivation in the Appendix 2),

$$\frac{\partial w}{\partial \alpha} = -Ak^{\alpha}(1 - 2(1-\alpha)\ln k) \tag{7}$$

$$\frac{\partial r}{\partial \alpha} = -Ak^{\alpha}\frac{K}{B}(1 - (1-\alpha)\ln k) \tag{8}$$

Therefore, the effect on wages of an increase in the capital intensity of the technology is positive whenever $\ln k > \frac{1}{2}\frac{1}{1-\alpha}$ and negative otherwise. However, from equation 3 it follows that in the interior solution $\ln k = \frac{1}{K}\frac{\alpha}{1-\alpha}$, so an increase in *α* has a positive effect on wages if $\alpha > \frac{K}{2}$. However, this condition cannot hold if $K \geq 2$. But, if $K<2$ then α is constant unless $l<0.37$, which seems to be an extreme case. For this reason and in order to simplify the analysis from now on we assume that $l>0.37$ for any t. Therefore, increases in *α* generate a decrease in wages.

Similarly, the effect of an increase in α on interest rate is positive whenever $\ln k < \frac{1}{1-\alpha}$ and negative otherwise. This condition holds whenever $K \geq 1$. Therefore, increases in α generate an increase in the interest rate.

Now, a decrease in the labor supply has a direct effect on wages (positive) and interest rates (negative), but it also has an indirect effect. This indirect effect appears because the decrease in labor supply generates incentives to increase the capital intensity of the technology, reducing labor income and increasing capital income.[4]

The net result depends on the relative importance of each effect:

(9) $$\frac{\partial w}{\partial l} = -(1-\alpha)\frac{\alpha A k^{\alpha}}{l} + \frac{\partial w}{\partial \alpha}\frac{\partial \alpha}{\partial l}$$

(10) $$\frac{\partial r}{\partial l} = (1-\alpha)\frac{\alpha A k^{\alpha}}{B} + \frac{\partial r}{\partial \alpha}\frac{\partial \alpha}{\partial l}$$

Therefore,

- if $(1-\alpha)\frac{\alpha A k^{\alpha}}{l} < \frac{\partial w}{\partial \alpha}\frac{\partial \alpha}{\partial l}$ then $\frac{\partial w}{\partial l} > 0$
- if $(1-\alpha)\frac{\alpha A k^{\alpha}}{B} > -\frac{\partial r}{\partial \alpha}\frac{\partial \alpha}{\partial l}$ then $\frac{\partial r}{\partial l} > 0$.

Using equations 2, 6, 7, 10 and 8 we find the conditions under which $\frac{\partial r}{\partial l} > 0$ and $\frac{\partial w}{\partial l} > 0$ (complete derivation on Appendix 3),

(11) If $\ln k < \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ then $\frac{\partial w}{\partial l} > 0$ and $\frac{\partial r}{\partial l} < 0$

(12) If $\ln k > \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ then $\frac{\partial w}{\partial l} < 0$ and $\frac{\partial r}{\partial l} > 0$

From equations 11 and 12 it follows that given an initial labor supply *l=1*, there exists a capital stock k^r such that:

(i) ln $\ln k = \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ if and only if $k=k^r$,

(ii) if $k<k^r$ then $\ln k < \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ and

[4] Recall that if *k<1* then technology is constant, so changes in labor supply do no affect technology.

(iii) if $k>k^r$ then $\ln k > \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$.

Similarly, there exists a capital stock k^w such that:

(i) $\ln k = \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ if and only if k=k^{w},

(ii) if $k< k^w$ then $\ln k < \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ and

(iii) if $k> k^w$ then $\ln k > \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$.

We use the previous result to define four groups of economies depending on their capital abundance:

1. An economy is very capital abundant if $k>k^r$.
2. An economy is capital abundant if $k^w<k<k^r$.
3. An economy is labor abundant if $1<k\leq k^w$.
4. An economy is very labor abundant if $k\leq 1$.

Therefore,

1. In very capital abundant economies, a decrease in the labor supply decreases interest rates and increases wages.

2. In capital abundant economies, a decrease in the labor supply increases interest rates and wages.

3. For labor abundant economies, a decrease in the labor supply increases the interest rate and decreases wages.

4. Finally, recall that in very labor abundant economies technology α is constant. Therefore, in these economies a decrease in the labor supply increases wages and decreases interest rates.

4.3 Labor, Income and Functional Distribution.

Labor affects output in two ways. First, directly as an input and second, it affects technology and therefore output. Now, the change in technology affects output in two different ways: It affects the elasticity of output with respect to capital and given the amount of assets it reduces the stock of capital.

To see the net effect we take derivatives,

$$\frac{\partial \mathrm{Y}}{\partial l} = (1-\alpha)\frac{\mathrm{Y}}{l} + \mathrm{Y}\ln \mathrm{k}\frac{\partial \alpha}{\partial l} + \frac{\partial \mathrm{Y}}{\partial K}\frac{\partial \mathrm{K}}{\partial \alpha}\frac{\partial \alpha}{\partial l}$$

combining with equations 6 and 3 and rearranging,

$$\frac{\partial \mathrm{Y}}{\partial l} = (1-\alpha)\frac{\mathrm{Y}}{l}$$

Therefore, a reduction in labor supply always generates a decrease in total output. We proceed in a similar way to see the effects on labor income and capital income,

(13) $$\frac{\partial \mathrm{w}l}{\partial l} = \frac{\partial \mathrm{w}}{\partial l}l + w$$

(14) $$\frac{\partial ra}{\partial l} = \frac{\partial \mathrm{r}}{\partial l}B$$

Combining equation 13 with results 1, 2, 3 and 4 from section 4.2 it follows that in very capital abundant economies a decrease in the labor supply reduces capital income, in capital abundant and labor abundant economies a decrease in labor increases capital income; finally, in very labor abundant economies a decrease in the labor supply reduces capital income.

Combining equations 8 and 12 we get,

$$\frac{\partial \mathrm{w}l}{\partial l} = (1-\alpha)^2 Ak^{\alpha} + \frac{\partial w}{\partial \alpha}\frac{\partial \alpha}{\partial l}l$$

Therefore, a reduction in labor supply always generates a decrease in labor income. However, as the technology becomes more capital abundant the magnitude of this negative effect becomes smaller.

For the moment we have described the static effects of a change in labor supply. In order to analyze the possible effects on economic growth we need to add some structure to the model. In particular, we need to model the behavior of consumers in a dynamic setting. In the following section we provide two different dynamic models: the first one is a model of homogenous agents and the second one is an overlapping generations model. None of these set-ups reflect the real structure of the economy but are two extreme cases that can help understand the mechanisms through which an increase in labor supply affects economic growth.

5. Labor, Technology and Growth

5.1 Homogenous Agents and Infinite Horizon

Consumers maximize the present value of their utility which depends on the consumption path.

$$\max \int_0^{\infty} \ln c_t e^{-\rho t} dt \qquad \text{s.t.} \quad \dot{B}_t = B_t r_t + w_t l_t - c_t$$

From the First Order Conditions it follows that consumption growth rate depends on the interest rate and the discount rate,

(15) $$\frac{\dot{c}_t}{c_t} = r_t - \rho$$

Firms receive assets and choose capital and technology in order to maximize income given wages, so $r = \alpha A k^{\alpha - 1} \frac{K}{B}$ and $\alpha = max\{\alpha_0, \alpha\}$. In order to see the effect that an increase in labor supply has on the growth rate it suffices to see its effect on the interest rate.

From equation 15 and the results of section 4.2, a decrease in the labor supply:

1. reduces economic growth in very capital abundant economies.
2. increases economic growth in capital abundant economies.
3. increases economic growth in labor abundant economies.
4. reduces economic growth in very labor abundant economies

Under this setting the redistribution of income generated by the change in technology is irrelevant because agents are homogenous.

Note that this model can support two long run equilibria. The first one is a steady state where $\frac{\dot{B}}{B}=\frac{\dot{K}}{K}=\frac{\dot{\alpha}}{\alpha}=0$. The second one is a Balanced Growth Path where the production function is AK.

Under this setting, whenever the technology is changing, the interest rate grows as the economy accumulates assets and converges to A in the long run (we provide the proof in the Appendix 4). Therefore, growth effects are permanent for economies converging to the Balanced Growth Path (BGP).

For very labor abundant economies, where technology is constant, growth effects can be permanent or transitory depending on the initial conditions and on the size of the shock. Consider for example and economy with initial technology α_0 and initial labor force $l=1$, where total factor productivity and discount rate are such that $\alpha_0 A \leq \rho$. If technology is constant this economy converges to a steady sate characterized by the following capital labor ratio $k^* = \left(\frac{\alpha_0 A}{\rho}\right)^{\frac{1}{1-\alpha_0}}$. Finally, suppose that $\alpha_0 > \frac{K^* \ln k^*}{1+K^* \ln k^*}$, so there are no incentives to increase the capital intensity of the technology and there is no economic growth in the long run.

Now, suppose there is a positive shock in the supply of labor (population remains constant). The steady state capital labor ratio k^* remains the same, but the amount of capital K^* grows (from K^* to K^{**}) Therefore, if the shock is strong enough it can be the case that after the shock $\alpha_0 < \frac{K^{**} \ln k^*}{1+K^{**} \ln k^*}$ and the economy does not converges to the initial steady state.

In this case, in the short run the positive shift in labor supply generates an increase in output and an increase in the interest rate. This two effects work together accelerating the accumulation of assets. Finally, the amount of assets grows until the point where capital using innovations become profitable. Then, the economy starts a process of accumulation and technological change and converges in the long run to a BGP.

The OLG model

The economy consists of overlapping generations of agents who live two periods. In the first period individuals work, consume and save. In the second one, they use their savings to build up capital, produce and consume.

The representative consumer lives two periods and her utility depends on the consumption when young (c_t) and the consumption when old (d_{t+1}). We assume a logarithmic utility function which combines the two arguments (c_t, d_{t+1}). The income of a young consumer is given by the wage w_t. We assume full depreciation so the return to savings is the interest rate r_t and the income of the elderly is given by $\alpha A k^{\alpha}$.
The problem for the representative agent is

$$\max_{c_t, d_{t+1}} \{\ln c_t + \beta \ln d_{t+1}\}$$

$$\text{s.t.} \quad w_t = c_t + \frac{d_{t+1}}{r_{t+1}}, \; c_t \geq 0 \text{ and } d_{t+1} \geq 0$$

where β is the discount factor and A is total factor productivity. From this maximization problem, we derive consumption and savings:

(16) $$c_t = \frac{w_t}{1+\beta}$$

and

(16) $$s_t = \frac{\beta w_t}{1+\beta}$$

Therefore, total savings and future assets are given by $B_{t+1} = \frac{\beta}{1+\beta} w_t l_t$. Note that we are assuming an exogenous shock in the labor supply and constant population. For this reason, the amount of assets per capita is determined by the savings rate multiplied by total labor income.

In section 4 we showed that a decrease in labor generates a decrease in labor income. Therefore, in an OLG context, a decrease in labor supply reduces economic growth. In this case the capital income is completely consumed and savings come exclusively from labor income. Therefore, any decrease in the labor income reduces savings and, consequently, economic growth. Under this setting, if the shift in labor supply affects only one generation then growth effects are transitory.

6. Conclusions and Discussion

We show that, in the context of biased innovations the effect of an exogenous change in labor supply depends on the structure of the economy (homogeneous agents or OLG) and on its capital abundance.

A decrease in labor supply has a negative effect on current output. The magnitude of the decrease depends on the capital abundance of the economy. Indeed, in capital abundant economies the technologies are capital intensive, so the negative effect of a decrease in the labor supply is relatively small.

The dynamic effects of a change in labor supply depend on its effect on factor prices and on the structure of the economy. A decrease in labor supply has a direct effect on factor prices: positive for wages and negative for interest rates. However, a decrease in labor supply has also indirect effects since it generates incentives to use more capital intensive technologies. Moreover, if we consider an OLG structure the net effect on economic growth depends on the behavior of labor income, while if we consider a homogeneous agents structure the effect on growth depends on the behavior of interest rates.

We presented stylized economies with important simplifying assumptions. In order to have a useful tool to analyze real problems some of these assumptions have to be relaxed. In particular, we consider a one sector model, so the problem of advanced and retarded sectors, common in developing countries, is ignored. Similarly, we assume that agents are homogeneous within generations so many distributive issues are leaved aside. Indeed, the only source of redistribution appears in the OLG model and comes from the fact that only the elderly own capital.

Finally, the models we present are extreme examples that help understand the mechanisms through which a change in labor supply may affect economic growth. The structure of a real economy is by far more complex. However, extensions of this model can be used to analyze the possible effects of migration in both native and host countries.

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Appendix 1

$\frac{K}{B}$ in the long run

$\frac{K}{B} = 1 + \frac{\ln(1-\alpha)}{B}$. Therefore we have to prove that $\lim_{B\to\infty} \frac{\ln(1-\alpha)}{B} = 0$.

Applying l'Hopital, if $\frac{\partial(1-\alpha)}{\partial B} > -1$ then $\lim_{B\to\infty} \frac{\ln(1-\alpha)}{B} = 0$.

Now, $\frac{\partial(1-\alpha)}{\partial B} = -\frac{(1-\alpha)(1+\ln k)}{1+(1-\alpha)(1+\ln k)}$ so $\frac{\partial(1-\alpha)}{\partial B} > -1$.

Appendix 2

The effect of a technological change on factor prices

$$\frac{\partial w}{\partial \alpha} = -Ak^{\alpha} + (1-\alpha)Ak^{\alpha}\ln k + \alpha(1-\alpha)Ak^{\alpha-1}\frac{1}{l}\frac{\partial K}{\partial \alpha}$$

$$\frac{\partial r}{\partial \alpha} = Ak^{\alpha-1}\frac{K}{B} + \alpha Ak^{\alpha-1}\frac{K}{B}\ln k + \alpha Ak^{\alpha-1}\frac{1}{B}\frac{\partial K}{\partial \alpha}$$

Rearranging

$$\frac{\partial w}{\partial \alpha} = -Ak^{\alpha}\left(1-(1-\alpha)\ln k+\alpha(1-\alpha)\frac{1}{K}\frac{\partial K}{\partial \alpha}\right)$$

$$\frac{\partial r}{\partial \alpha} = Ak^{\alpha-1}\frac{K}{B}\left(1+\alpha\ln k+\alpha\frac{1}{K}\frac{\partial K}{\partial \alpha}\right)$$

Given that $\frac{K}{B} = 1+\frac{\ln(1-\alpha)}{B}$, $\frac{\partial K}{\partial \alpha} = -\frac{1}{1-\alpha}$ so

$$\frac{\partial w}{\partial \alpha} = -Ak^{\alpha}\left(1-(1-\alpha)\ln k-\alpha\frac{1}{K}\right)$$

$$\frac{\partial r}{\partial \alpha} = Ak^{\alpha-1}\frac{K}{B}\left(1+\alpha\ln k-\frac{1}{K}\frac{\alpha}{1-\alpha}\right)$$

From equation 2 it follows that $\alpha\frac{1}{K} = (1-\alpha)\ln k$. Therefore,

$$\frac{\partial w}{\partial \alpha} = -Ak^{\alpha}\left(1-2(1-\alpha)\ln k\right)$$

$$\frac{\partial r}{\partial \alpha} = Ak^{\alpha-1}\frac{K}{B}\left(1-(1-\alpha)\ln k\right)$$

Appendix 3

The effect of a decrease in labor supply on factor prices

Combining equations 6, 7, 8, 9 and 10,

$$\frac{\partial w}{\partial l} = -(1-\alpha)\frac{\alpha Ak^{\alpha}}{l} - Ak^{\alpha}\left(1-2(1-\alpha)\ln k\right)\frac{K}{l}\frac{(1-\alpha)^2}{1+(1-\alpha)(1+\ln k)}$$

$$\frac{\partial r}{\partial l} = (1-\alpha)\frac{\alpha Ak^{\alpha}}{B} - Ak^{\alpha-1}\frac{K}{B}\left(1-(1-\alpha)\ln k\right)\frac{K}{l}\frac{(1-\alpha)^2}{1+(1-\alpha)(1+\ln k)}$$

Rearranging

$$\frac{\partial w}{\partial l} = (1-\alpha)^2\alpha Ak^{\alpha+1}\left(\left(1-2(1-\alpha)\ln k\right)\frac{1}{1+(1-\alpha)(1+\ln k)} - \frac{\alpha}{1-\alpha}\frac{1}{K}\right)$$

$$\frac{\partial r}{\partial l} = (1-\alpha)Ak^{\alpha}\frac{1}{B}\left(\alpha-\left(1-(1-\alpha)\ln k\right)\frac{(1-\alpha)K}{1+(1-\alpha)(1+\ln k)}\right)$$

Using equation 2,

$$\frac{\partial w}{\partial l} = (1-\alpha)^2\alpha Ak^{\alpha+1}\left(\frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)} - \ln k\right)$$

$$\frac{\partial r}{\partial l} = (1-\alpha)^2 Ak^{\alpha}\frac{K}{B}\left(\ln k - \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}\right)$$

Therefore,

If $\ln k < \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ then $\frac{\partial w}{\partial l} > 0$ and $\frac{\partial r}{\partial l} < 0$

If $\ln k > \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ then $\frac{\partial w}{\partial l} < 0$ and $\frac{\partial r}{\partial l} > 0$

Note that $\frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)} < \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)} < 1$.

Therefore, if *lnk>1* then $\frac{\partial r}{\partial l} > 0$ and $\frac{\partial w}{\partial l} < 0$.

Note also that if *lnk=0* then $\frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)} > 0$ so $\frac{\partial w}{\partial l} > 0$ and $\frac{\partial r}{\partial l} < 0$.

Finally, note that if *ln k>0* then the functions $\frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ and $\frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ are strictly decreasing in k.

Therefore, given an initial labor supply *l=1*, there exists a capital stock k^r such that:

(iv) ln $\ln k = \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ if and only if $k=k^r$,

(v) if $k<k^r$ then $\ln k < \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ and

(vi) if $k>k^r$ then $\ln k > \frac{1-(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$.

Similarly, there exists a capital stock k^w such that:

(iv) $\ln k = \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ if and only if k=k^{w},

(v) if $k< k^w$ then $\ln k < \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$ and

(vi) if $k> k^w$ then $\ln k > \frac{1-2(1-\alpha)\ln k}{1+(1-\alpha)(1+\ln k)}$.

Appendix 4

Long run

4.1 Homogeneous agents

We already showed that $\lim_{B\to\infty}\frac{K}{B}=1$. Therefore, $\lim_{a\to\infty}\alpha=1$, $\lim_{a\to\infty} r = A$ and $\lim_{a\to\infty}\frac{\dot{c}}{c} = A-\rho$.

If $A<\rho$ then long run growth is not possible. Indeed, the marginal productivity of capital is always lower than the discount rate, so there are no incentives to save.

If $A>\rho$ then long run growth is possible. Moreover, if the initial conditions are such that $\left(\frac{\alpha A}{\rho}\right)^{\frac{1}{1-\alpha}} > k > 1$ then there is only one candidate for optimal path and the economy converges to a balanced growth path characterized by a production function of the AK type.

Note that for high levels of assets the interest rate is an increasing function of *B* (see next subsection). Therefore, economies where the initial conditions are such that the interest rate is positive and increasing converge to a BGP.

4.1.1 Dynamics of the interest rate.

To find the growth rate of the interest rate we take logs and derivatives

$$\frac{\dot{r}}{r}=\frac{\dot{\alpha}}{\alpha}-(1-\alpha)\frac{\dot{k}}{k}+\dot{\alpha}\ln k+\frac{\dot{k}}{k}-\frac{\dot{B}}{B}$$

To save notation we assume *l=L=1.*

Using equation 3,

$$\frac{\dot{r}}{r}=\frac{(1-\alpha)^2}{\alpha}(1+\ln k)\dot{K}+\alpha\frac{\dot{K}}{K}+(1-\alpha)^2(1+\ln k)\dot{K}\ln k-\frac{\dot{B}}{B}$$

Using equation 2 and rearranging

$$\frac{\dot{r}}{r}=\frac{\dot{K}}{K}\left((1-\alpha)\left(1+\frac{1}{\ln k}+\alpha+\alpha\ln k)\right)+\alpha\right)-\frac{\dot{B}}{B}$$

Rearranging,

$$\frac{\dot{r}}{r}=\frac{\dot{K}}{K}\left((1-\alpha)\left(\frac{1}{\ln k}(1+\ln k)+\alpha(1+\ln k))\right)+\alpha\right)-\frac{\dot{B}}{B}$$

and

$$\frac{\dot r}{r}=\frac{\dot K}{K}\left((1-\alpha)\left(\left(\frac{1}{\ln k}+\alpha\right)(1+\ln k)\right)+\alpha\right)-\frac{\dot B}{B}$$

Now, $\frac{K}{B}=1+\frac{\ln(1-\alpha)}{B}$ implies that $\frac{\dot B}{B}=\frac{\dot K}{K}\frac{K}{B}\left(1+(1-\alpha)(1+\ln k)\right)$.

Therefore,

$$\frac{\dot r}{r}=\frac{\dot K}{K}\left((1-\alpha)\left(\left(\frac{1}{\ln k}+\alpha\right)(1+\ln k)\right)+\alpha-\frac{K}{B}\left(1+(1-\alpha)(1+\ln k)\right)\right)$$

Rearranging,

$$\frac{\dot r}{r}=\frac{\dot K}{K}\left((1-\alpha)\left(\left(\frac{1}{\ln k}+1\right)(1+\ln k)\right)+\left(\alpha-\frac{K}{B}\right)\left(1+(1-\alpha)(1+\ln k)\right)\right)$$

Therefore,

1. If $\alpha\geq\frac{K}{a}$ then $\frac{\dot r}{r}>0$.

Using equation 2 it can be show that for any $a\geq 3.5478$ the inequality $\alpha\geq\frac{K}{B}$ holds.

2. If $\alpha<\frac{K}{B}$ and $(1-\alpha)\frac{1}{\ln k}\frac{1+\ln k}{\left(1+(1-\alpha)(1+\ln k)\right)}+\alpha>\frac{K}{B}$ then $\frac{\dot r}{r}>0$.

Suppose $B<3.5478$ then $K<2.42$, $\alpha<0.683$ and $\ln k<0.885$

Under this conditions $(1-\alpha)\frac{1}{\ln k}\frac{1}{\left(\frac{1}{1+\ln k}+(1-\alpha)\right)}+\alpha>1>\frac{K}{B}$

Therefore, in the interior solution $\frac{\dot r}{r}>0$.

4.2 Overlapping generations

|In this setting long run growth is not possible. The growth rate of assets is given by

$$\frac{B_{t+1}}{B_t} = (1-\alpha_t)\frac{(B_t + \ln(1-\alpha_t))^{\alpha_t}}{B_t} A \frac{\beta}{1+\beta} \text{ so } \lim_{\alpha \to 1} \frac{a_{t+1}}{a_t} = 0 .$$

Therefore, a steady state characterized by $\frac{B}{(1-\alpha)(B+\ln(1-\alpha))^{\alpha}} = \frac{A\beta}{1+\beta}$ is the only possible long run equilibrium.