PPP, UIP and Fisher parity: speculation or rational expectations?

Evidence for four Latin American countries

(Thesis Proposal)

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Abstract

This work aims to test the equilibrium relations of two international macroeconomics models for Colombia, Chile, Mexico and Brazil. The first model is the rational expectation hypothesis (REH) where three key relations will be tested: Purchasing Power Parity (PPP), Uncovered Interest Rate Parity (UIP) and the Fisher Parity condition. The second model follows the line of though of Imperfect Knowledge Economics (IKE) where two equilibrium relations will be tested. According to IKE, even under the assumption that agents are rational, the presence of speculative behavior in financial markets helps explain the long swings often observed in the behavior of exchange rates. The results support the view that the predictions of the IKE model hold for Colombia, while those of the REH hold for both Brazil and Mexico. Mixed findings are obtained for Chile.

Keywords: Long Swings, Imperfect Knowledge, CVAR, Currency markets, Speculation.

JEL Classification: E24, F31, F41.

1 Introduction

"All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive... When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious. the results are suspect"

Robert Solow (1956)

"I confess that I prefer true but imperfect knowledge, even if it leaves much indetermined and unpredictable, to a pretence of exact knowledge that is likely to be false"

Hayek. Nobel Prize lecture (1974)

The conventional approach to exchange rate dynamics assumes rational individuals interacting in a market where the driving forces of the system, or equilibrium relations, are led by the macroeconomic fundamentals (see Mundel, 1963., Fleming, 1962., Dornbusch, 1976a., Dornbusch, 1976b., Frenkel, 1976). However, the empirical failures or puzzles of these macroeconomic models have opened the door to the notion that macroeconomic fundamentals may not play an important role in driving macroeconomic outcomes, and thus ideas such as irrational behavior of market participants have begun to appear in the literature.

The majority of international macroeconomic models rest on the assumptions of perfect information and rational expectations. In these models, Purchasing Power Parity (PPP), Uncovered Interest rate Parity (UIP) and the Fisher Parity hypothesis are presented as equilibrium relations. However, as Solow noted in 1956, when economic theory explains individual decision making and market outcomes, it makes assumptions that may not be true. This thesis aims to examine what happens to international macroeconomic outcomes, when individuals are assumed to hold imperfect knowledge. Several important questions arise from this approach: Do PPP, UIP and Fisher Parity still hold as equilibrium relations?. Does the Rational

Expectation Hypothesis make sense?. Are macroeconomic fundamentals the driving forces of market outcomes?. Can policy makers influence the market?.

In order to answer these questions, this thesis follows a contemporary approach of economic analysis known as Imperfect Knowledge Economics (IKE), see Frydman and Goldberg (2007). The IKE model, in the tradition of early modern economics, links mathematically the aggregate outcomes with the behavior and forecasting strategies of individuals. However, models based on IKE reflect modesty about how complete the representations of individual behavior can be.

The idea that exchange rate dynamics are driven by 'irrational noise' traders who do not rely on macroeconomic fundamentals is not an assumption in IKE. On the contrary, the model assumes that market participants must cope with imperfect knowledge, which is not the same as the presumption that they are irrational. But when perfect knowledge does not exist, although macroeconomic fundamentals play an important role in the model, individuals speculate in their forecasting behavior and the hypothesis of rational expectation fails to hold. This approach differs from the speculative economic bubbles approach in the sense that in the IKE model, macroeconomic fundamentals play an important role by being the driving force of the system; as will be seen later, macroeconomic fundamentals are a reference point in forecasting strategies, and therefore policy makers do interact and influence the market.

The objective of this thesis is to test three equilibrium relations derived from the REH, namely PPP, UIP and Fisher Parity, against two equilibrium relations that are derived from IKE. To do this we apply the theoretical model of Frydman and Goldberg (2007, 2011) under a methodology based on the Cointegrated Vector Autoregressive model (CVAR), with a particular focus on the foreign exchange markets of Colombia, Chile, Mexico and Brazil. Both REH and IKE postulate relations that can be estimated and tested within a multivariate cointegrating framework. One of the advantages of the CVAR approach is that the three hypotheses of the REH model mentioned above can and will be tested jointly. This is in sharp contrast

to the existing literature which, to the best of our knowledge, generally tests each hypothesis separately.

This rest of the thesis is organized as follows. Section 2 briefly presents both the REH and IKE model. Section 3 presents a brief review of the literature of REH and IKE, including some empirical applications. Section 4 presents the econometric methodology and summarizes the cointegration relations that will be tested under REH and IKE. Lastly, section 5 concludes.

2 Rational Expectation Hypothesis vs. Imperfect Knowledge Economics

2.1 REH framework

The ex-post return on a pure long position, r_{t+1}, of an individual who invests abroad can be written as:

$$r_{t+1} = s_{t|t+1} - s_t + i_{f_t} - i_{d_t}, \tag{1}$$

where $s_{t|t+1} - s_t$ is the expected change in nominal exchange rates, and i_{f_t} and i_{d_t} are the foreign and domestic interest rates, respectively.

For each unit of investment, an individual would get $\frac{1}{S_t}$ units at i_{f_t}, that is the money they receive in foreign currency. Then, the next period the individual sells $\frac{1}{S_t}(1 + i_{f_t})$ at the exchange rate S_{t+1} in order to get the earnings in the national currency. Therefore the total return of the long position would be $\frac{S_{t+1}}{S_t}(1 + i_{f_t}) - (1 + i_{d_t})$, which is the amount that an individual receives from the investment abroad less what the individual would have received by investing in his/her own country. Taking log approximation yields equation (1) . Note that the individual takes a long position when a rise in s_{t+1} is expected.

To model this return, economists assume that the exchange rate adjusts to equate demand and supply. Let $\hat{r}_{t|t+1}$ and $\hat{up}_{t|t+1}$ denote all market participants' point forecast of r_{t+1} and of the premium that they require to hold an open position in foreign exchange rate market, respectively. Equilibrium in the foreign exchange market under perfect capital mobility can be written as:

$$\hat{r}_{t|t+1} = \hat{up}_{t|t+1}.$$

This equation holds whether individual preferences are represented as risk neutral or risk averse, or are based on prospect theory. However, the assumption of REH is that the conditional expectation $\hat{r}_{t|t+1}$ is equal to the conditional expectation of r_{t+1}. Further, if individual preferences are risk neutral, as generally assumed in the REH models, then $\hat{up}_{t|t+1} = 0$. This leads to the conclusion that the ex-post return has a conditional mean of zero and is uncorrelated with the causal variables that an economist includes in his/her representation. The equilibrium of the REH is a no-arbitrage condition where individuals are indifferent to the interest rates available in the two countries.

With these assumptions, equation (2) is the well known Uncovered Interest Parity condition:

$$\Delta s_{t+1} = i_{d_t} - i_{f_t}. \tag{2}$$

Another equilirbuim condition generally invoked by REH is Purchasing Power Parity (PPP). PPP asks how much money would be needed to purchase the same basket of goods and services in two countries. Then, if prices rise abroad, the exchange rate S_t adjusts in order for PPP to hold. Thus $P_{d_t} = S_t \cdot P_{f_t}$ or $S_t = \frac{P_{d_t}}{P_{f_t}}$. Where P_{d_t} is the domestic price, P_{f_t} is the foreign price. Taking logs yields:

$$s_t = p_{d_t} - p_{f_t}. \tag{3}$$

If PPP holds for Colombia, Chile, Brazil and Mexico, then one would see in Figure 1 that the black line should be equal to the gray line, or at least the difference is stationary. With the series in logs. Table 2 shows the results of applying Dickey-Fuller test to the real exchange rates of the countries under consideration. The results suggest that for four countries it is not possible reject the null hypothesis of a unit root[1]. Although for Mexico rejection occurs at the 5% (but not 1%) significance level.

Taking the first difference of equation (3) and combining it with equation (2) yields the Fisher Parity condition of the REH model:

$$i_{d_t} - \Delta p_{d_{t+1}} = i_{d_t} - \Delta p_{d_{t|t+1}}, \tag{4}$$

where $r_{d_t} = i_{d_t} - \Delta p_{d_{t|t+1}}$ is the real interst rate or the Fisher parity condition for country d. Under REH, equation (4) becomes $r_{d_t} = i_{d_t} - \Delta p_{d_{t+1}}$.

Therefore, PPP, UIP and Fisher Parity can be viewed as equilibrium relationships with risk averse individuals, with no risk premium[2] and where the efficient-market hypothesis holds. The individuals interact in a market where prices reflect all information and change instantly to reflect new information.

IKE framework[3]

The IKE model assumes that individuals agents are rational, in the sense that they exploit profit opportunities. However, agents are assumed to have only imperfect knowledge concerning the relationships driving the future payoffs of their decisions. With theses assumptions equilibrium under REH does not hold and other equilibrium relation arise.

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[1] The univariate unit root test is not the most appropriate test in a multivariate context. In the next section stationarity will be tested in a multivariate context. In this section these results are presented for illustration purpose only.

[2] Under REH and Imperfect capital mobility, the risk premium, $\hat{u}p_{t|t+1}$, is different from zero, but PPP and fisher Parity holds. However Under REH the risk premium is stationary. Under IKE the risk premium is not stationary. An important property in the dynamic of the model that will be shown later.

[3] The presentation of the model in this section draws heavily on the IKE model presented by Frydman and Goldberg (2004, 2007). Interpretation and simplifications in this sections are our.

To build the IKE model, Frydman and Goldberg assumed that there are two type of individuals: bulls who gamble on appreciation, and bears on depreciation; also, the authors follow Kahneman and Tversky (1979) and assume that speculators are loss averse, but they also augment prospect theory by assuming that an agent's degree of loss aversion increase with the size of her speculative position (endogenous prospect theory). The above framework of the model leads to a key result: All agents require a minimum premium before they are willing to commit any capital to speculate in the foreign exchange market. This premium is called individual uncertainty premium and denoted by $\tilde{up}_t$.

In this model, potential losses are not related to volatility, but to the divergence of an agent's forecast of an asset price, in this case the exchange rate $\hat{s}_{t+1}$. The causal variables that an individual uses to form her forecast is made through her evaluation of the gap defined as:

$$g\tilde{a}p_t^i(z_t) = \hat{s}_{t+1}^i(z_t) - \hat{s}_t^{HB_i}(z_{t+1}).$$

where $\hat{s}_t^{HB_i}(z_t)$ denotes an individual's assessment at time t of the historical benchmark exchange rate.[4]If an agent is a bull (i.e., holds a long position), then a rising gap creates more fear of an eventually countermovement. This greater fear leads him/her to simultaneously revise up his/her assessment of the likelihoods and/or magnitudes of the potential losses, which would result if the price were to revert to its benchmark level. If the agent is a bear (i.e., holds a short position) then a rising gap enhances his/her confidence that a countermovement is likely to occur. Then if agents forecasts depend on their gaps, then as they raise these forecasts, they simultaneously raise (lower) their uncertainty premium if they are bulls (bears).

In the aggregate, the uncertainty premium, up_t, is equal to uncertainty premium of the groups of bulls *minus* the uncertainty premium of bears. Note that market uncertainty premium,up_t, depends negatively on the bear's uncertainty premium because profits of bear occur when the return on holding foreign exchange is negative.

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[4]This gap can also be defined in terms of the exchange rates, rather than the forecast of the future exchange rate, or some weighted average of the two. The conclusion of the analysis are not affected by either of these specification of the gap variable. (Frydman and Goldberg, 2007,p-199)

The proceeding analysis replaces the UIP condition of REH with the Uncertainty Adjusted UIP (UAUIP) of IKE condition:

$$(i_{d,t} - i_{f,t}) = \Delta s_{t+1}^e + up_t. \tag{5}$$

UAUIP seems similar to the UIP with imperfect capital mobility. However, the IKE model assumes that uncertainty premium is a function of the gap effect, the difference of s_t from his benchmark value, and so $up_t = f(s_t - p_{d_t} + p_{f_t})$. Replacing up_t in equation (5) gives the expression:

$$(i_{d_t} - i_{f_t}) - f(s_t - p_{d_t} + p_{f_t}) = \Delta s_{t+1}^e.$$

Lastly, IKE derive $f(s_t - p_{d_t} + p_{f_t}) = \sigma(s_t - p_{d_t} + p_{f_t})$, where σ is an unknown parameter related to loss aversion, $0 < \sigma < 1$. Thus equation (5) becomes:

$$(i_{d_t} - i_{f_t}) = \Delta s_{t+1}^e + \sigma(s_t - p_{d_t} + p_{f_t}). \tag{6}$$

An important implication of UAUIP is that the expected change of the nominal exchange rates is not only related with the observed interest rate, but also with the interest rate differential corrected by the uncertainty premium. Figure (2) depicts, ppp_{d_t} condition, along with $up_t = \Delta s_{t+1} - (i_{d_t} - i_{f_t})$ for Colombia, Chile, Brazil and Mexico. According to IKE, the graphs should reflect a similar movement between these series.

UAUIP enables us to explain the frequency of sign reversals in foreign exchange rates returns that models based on risk-averse preference have found anomalous. There are two types of forecasting strategies in the model: bulls expecting appreciation while bears bet on depreciation. As the uncertainty premium is the main determination of the return on foreign exchange rate. these different strategies are the crucial keys to explain the speculative behavior of foreign exchange rate returns. When there is a change in the causal variables from one time period to the next,

such changes lead market participant to revise their forecast of the return and unit loss form holding open position.

Fluctuations in the equilibrium depend on new realizations of the causal variables and the exchange rate, and on how market participants revise their forecasting strategies.The equilibrium premium on foreign exchange and the market´s forecast of the future spot exchange rate can change in opposite directions between two consecutive time periods. The intuition behind this result is that a higher s_{t+1}^e, for example, creates an excess demand for foreign exchange, which bids up the exchange rate and creates a gain for bulls and a loss for bears; bulls lower their degree of loss aversion while it rises for bears. The resulting fall in the uncertainty premium for the bulls and rise for the bears leads to a lower equilibrium premium, UAUIP implies a large equilibrium premium when one side of the market, either the bulls or the bears, forecast a large potential unit loss from speculation while the other side does not.

In the model, forecast strategies of future price must change almost all the time; strategies that initially generate profits, sooner o later lose the ability to continue producing profits. That is the point that the REH approach ignores and at the same time constitutes the foundation of IKE. The swings away from PPP are related with these strategies, the swings occur because the market forecasting strategies of the exchange rate s_{t+1} moves persistently away from PPP, causing the current exchange rate to follow it.

A further difference between REH and IKE is that under REH a stationary real exchange rate is consistent with a stationary real interest rate differential, whereas under IKE equilibrium is defined as a stationary cointegration relation between the two. Indeed, when the real exchange rate is moving away from its benchmark value, the real interest rate differential has to move in a way that the equilibrium in the product market is restored, that is:

$$(s_t - p_{d,t} - p_{f,t}) = \omega \left\{ (i_{d,t} - i_{f,t}) - (\Delta p_{d,t} - \Delta p_{f,t}) \right\} + \epsilon_t. \tag{7}$$

3 Brief literature review

Macroeconomic models of the REH are compatible with equilibrium in the goods market, a stationary real exchange rate and a stationary real interest rate differential. However, some authors have suggested that PPP, UIP and the Fisher Parity condition may in fact be non-stationary; see for instance Froot *et al.* (1995), Rogoff (1996), Rogoff *et al.* (2008), Lopez *et al.* (2005), and Papell *et al.* (2006). Persistent deviations from the steady state are commonly known as puzzles or anomalies in the international macroeconomic literature and as such have been the topic of extensive research by several authors.

As an illustration, let us consider once again Figure 1 which presents the nominal exchange and the price relations of Colombia, Chile, Mexico and Brazil with respect to the price level in the United States. The systematic and persistent deviations between relative prices and the exchange rate are known in the literature as exchange rate swings. In order to explain such swings away from PPP, economists have constructed two types of monetary models, namely, those based on flexible prices and those based on sticky prices.

Flexible price monetary models assume that all prices including wages, adjust instantaneously to their equilibrium. In these models, deviations from PPP are caused by 'real disturbances' to supplies. However long swings (as in Figure 1) require that such shifts be large enough, and in the same direction over several periods. But this behavior is not plausible in these models. Usually, in flexible-price models shifts occur in one direction in one period but in the opposite direction in the next one.

In sticky price monetary models, by contrast, goods and wages adjust sluggishly in the short run, resulting in temporary deviations of exchange rate from PPP. Dornbusch (1976) showed that in these models exchange rates are characterized by an overshooting effect, so that temporary deviations from PPP are plausible and also help to explain exchange rate volatility. However, such models can not generate long swings such as the ones observed empirically.

Literature about these anomalies or puzzles have been extensive. For example, Obstfeld and Rogoff (2000) indicate that it is likely that the pattern of exchange rate expectations lies behind many empirical puzzles found in international macroeconomics. An idea that has been taken up under IKE by Frydman and Goldberg (2004, 2007, 2011), Frydman et al (2007) and Juselius (2009).

As indicated by Frydman and Goldberg (2004) There are two distinctive features of IKE. First, the authors develop an alternative framework for modeling forecasting behavior that recognizes that market participants posses only imperfect knowledge about the relationships driving asset prices. Second, the authors replace expected-utility theory with the prospect theory of Kahneman and Tversky (1979). Thus the authors build on prospect theory to model an idea due to Keynes (1936) according to which the risk in financial markets is connected to departures of asset prices from historical benchmark levels. As will be seen later, the uncertainly premium of the model depends on discrepancies between exchange rate and the relative prices.

Within this framework two major conclusions arise. First, swings away from parity can be based solely on macroeconomic fundamentals in contrast to the bubble view. Therefore policy makers can alter the course of macroeconomic fundamentals. Second, target zones and other intermediate exchange rate regimes may be effective even in situations in which policy institutions are not credible.

Among the authors that have worked on the rational expectation hypothesis for Latinamerican countries, Jongen *et al* (2008) present evidence in favor of agents with irrational behavior for several countries. Using a different approach, Pesaran (1987) examines the case of Brazil where rational expectation fails; see also Kaltenbrunner and Nissanke (2009) on exchange rate dynamics in Brazil, not as a result of market equilibrating forces, but as one of changing expectations of heterogeneous traders. For Colombia, Otero and Ramirez (2006) use cointegration analysis and find support for a weak version of PPP between Colombia and the US; however the authors focus the analysis on PPP separately and do not take into account the two other equilibrium relationships derived from the REH, Echavarría, *et al* (2008) and Echavarría

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and Villamizar (2012) open the discussion about expectation of exchange rates by arguing that "the forward rate is generally different from the future spot rate, mainly because forecast errors are on average different from zero. This suggests that exchange rate expectations are not rational" (pp.1), Ojeda (2009) provides evidence in favor of REH with unit root tests of PPP, but only after allowing for the presence of structural breaks. Also Ojeda, *et al (2013)* show evidence that fundamentals also play an important rate in the exchange rate dynamics at least for Colombia.

4   Empirical approach

As indicated in the Introduction, the aim of the thesis is to apply the cointegration analysis through the formulation and estimation of a cointegrated vector autoregressive (CVAR) model to a data set of exchange rates and prices in four Latinamerican countries. In doing so, hypotheses derived from two fundamentally different approaches will be tested. On the one hand, the REH implies that an agent can fully prespecify an economic model, leaving essentially no role for expectations. In this modeling approach an equilibrium in the good market is compatible with a stationary real exchange and a stationary real interest rate differential. On the other hand, IKE implies that individual's forecasting strategies play an important role in driving market outcomes. Because the individuals do not know the right model they have different forecasting strategies. It means that a speculative behavior is present in the model and when the real exchange moves away from its benchmark value (PPP), the real interest rate differential has to move in order to compensate and restore the product market.

The CVAR methodology, as laid out in Johansen (1988, 1996) and Juselius (2006), provides economists, with a powerful tool to empirically asses these models. The scenario approach presented in Juselius (2006) and Juselius (2010) is a useful way of directly testing hypotheses derived from theory, thus not only guaranteeing the theoretical but also the empirical validity of economic models. The following variables will be used in the CVAR:

$$\left[\begin{array}{ccccccc} (s_t - p_{d_t} + p_{f_t}) & \Delta p_{d_t} & \Delta p_{f_t} & i_{d_t} & i_{f_t} & \Delta s_{t+1} \end{array} \right]'.$$

The following table presents a summary of the cointegration relations that arise in both REH and IKE theoretical models:

Table 1. Cointegration relations

REH	IKE
1. $(s_t - p_{d_t} - p_{f_t}) \sim I(0)$	1. $b_1(\Delta p_{d_t} - \Delta p_{f_t}) - (i_{d_t} - i_{f_t}) + \sigma(s_t - p_{d_t} + p_{f_t}) \sim I(0)$
2. $(i_{d_t} - i_{f_t} - \Delta s_{t+1}) \sim I(0)$	2. $b_2 \Delta s_{t+1} - (i_{d_t} - i_{f_t}) + \sigma(s_t - p_{d_t} + p_{f_t}) \sim I(0)$
3. $(i_{d_t} - \Delta p_{d_t}) \sim I(0)$	
4. $(i_{f_t} - \Delta p_{f_t}) \sim I(0)$	

One of the postulates of IKE is that when there is instability, that is when the market outcomes are away from their fundamentals, the individuals behave as postulated by the cointegration relations 1 and 2 of the IKE model (see Table 1). But also note that, if $s_t = p_{d_t} - p_{f_t}$, that is when the exchange rate is driving by its fundamentals, two things happen: First, the first cointegration relation of IKE is simply a lineal combination of relations 1, 3 and 4 of the REH model. Second, the second cointegration relation of REH is the same as the second cointegration relation of IKE. Thus, one can conclude that in those cases where individuals observe that outcomes are driven by fundamentals, there is no speculation and the REH holds.

4.1 Data

We are interested in testing the validity of the cointegration relations that arise from both the REH and IKE models over a similar period of time for four Latin American countries, namely: Colombia, Chile, Mexico and Brazil. However, in the database of the International Monetary Fund, there are data available for Colombia since 1986

for all the variables needed to apply the CVAR-methodology. Therefore, the sample period to be considered for the four countries runs from 1986 to 2012.

Let d denotes *Colombia*, *Chile*, *Mexico* and *Brazil* domestic countries, while the United States is the foreign one. The data consist of the following variables:

ΔP_d = variation of consumer prices index for country d, Δp_d in logs.

ΔP_{us} = variation of United States consumer prices index, Δp_{us} in logs.

i_d =the country d loan rate.

i_{us} =the Unites States loan rate.

S =the exchange rate of country d/ US, s in logs.

$ppp = s - p_d + p_{us}$ the real exchange rate between country d and United State in logs.

Lower case letters denote variables in logarithms, with the exception of interest rates. All annual effective interest rate have been divided by 4 to achieve comparability between the quarterly inflation rates in log differences. Figure 3 presents the series used to build the models.

4.2   The estimated model

The following CVAR model is estimated for each country:

$$\triangle x_t = \Pi x_{t-1} + \Gamma_1 \triangle x_{t-1} + \cdots + \Gamma_{k-1} \triangle x_{t-k+1} + \mu + \varepsilon_t \tag{8}$$

Within the CVAR model, the cointegration hypothesis can be formulated as a reduced rank restriction on the Π matrix, also called the long-run matrix. If $x_t \sim I(1)$, then $\triangle x_t \sim I(0)$ implying that Π cannot have full rank. Therefore Π must have reduced rank, and so it can be decomposed as:

$$\Pi = \alpha\beta',$$

where α is a $p \times r$ matrix that contains the adjustment coefficients, β is a $p \times r$ matrix with the cointegration relations, r is the rank of Π, p is the number of endoge-

nous variables in the VAR model, $r \leq p$. μ denotes the deterministic components, and ε_t is the error term. The term $\beta' x_{t-1}$ is an $r \times 1$ vector of stationary cointegration relations. Then, all stochastic components are stationary in the model and equation (8) is consistent.

For each country, the CVAR model contains 6 variables: Δp_t, r_t, ppp_t, Δs_{t+1}, r_t^{us} and Δp_t^{us}. Results not reported here suggest that r_t^{us} and Δp_t^{us} can be regarded as exogenous variables, that is, they are variable that affect the other country-specific variables in the system, but are not affected by them. These results can be also justified based on economic intuition. Thus, in all models under consideration, both the United States inflation and interest rate are regarded as exogenous variables. Therefore the CVAR model contains, $p = 4$ endogenous variables, Δp_t, r_t, ppp_t and Δs_{t+1}; and 2 exogenous ones, r_t^{us} and Δp_t^{us}.

Due to the regime changes in the Latinamerican's exchange rate systems, structural level breaks were also included in the vectors. In September 1999 the Colombia's exchange rate band system was eliminated to allow a freely floating regime. Also Chile adopted inflation targeting scheme as well as flexible exchange rate system in September 1999. In Mexico at the end of 1994 fixed exchange regime was eliminated. Finally, in January 1999 Brazil adopted a free exchange rate system. However, these level breaks were statistically significant in the cointegrated vector only for Colombia and Mexico. Additionally, due to the several changes in the fiscal and monetary policy in Brazil 1994, it was necessary introduce a break level variable in that period. For the four countries in the sample, there are 4 lags within the CVAR models and seasonal centered dummy variables. Lately, to improve the diagnostic of the residuals, impulse dummy variables were incorporated in the models for Colombia and Mexico [5].

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[5]More specifically, for Colombia the dummy variable (denoted 1999q2) takes the value of one in the second quarter of 1999, and zero elsewhere. For Mexico the dummy variable (denoted 1995q1) takes the value of one in the first quarter of 1995, and zero elsewhere.

4.3 Rank determination

Juselius (2006) argues that the determination of the cointegration rank may be a difficult choice. Indeed, the rank influences all subsequent inference and itself is a decisive step in the empirical analysis. However, macroeconomic theory gives insights about the rank of the Π matrix. The REH model postulates at least r=3 (PPP, UIP, FP and/or Interest rate differential), while IKE postulates r=2.

Table 3 presents the rank test for all countries in the sample, $p-r$ are the common trends or pushing forces and r are the cointegration relations. Crucial results emerge from these results, essentially the rank of Π let us classify the four countries in two groups. Colombia and Chile form the first group with r=2, Mexico and Brazil, the second group with r=3. For the former group we can postulate that IKE describes better the macroeconomic outcomes, for the latter is the REH who better explains these outcomes. However, at this point it is premature to make these conclusions since we have not yet identified the cointegrated vectors.

4.4 Testing hypotheses on cointegration: REH or IKE?

This section identify the long-run structure by introducing restrictions on $\beta' x_{t-1}$, The test tests whether the cointegrated vector is stationary, that is the null hypothesis $\beta' x_{t-1} \sim I(0)$.

Under r=2

- Model IKE_1 :

$$
\begin{bmatrix} \Delta p_t & r_t & ppp_t & \Delta s_{t+1} & r_t^{us} & \Delta p_t^{us} \end{bmatrix} \cdot \begin{bmatrix} 0 & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} & 0 \\ \beta_{21} & \beta_{22} & \beta_{23} & 0 & \beta_{24} & \beta_{25} \end{bmatrix}'
$$

- Model IKE_1^*:

$$
\begin{bmatrix} \Delta p_t & r_t & ppp_t & \Delta s_{t+1} & r_t^{us} & \Delta p_t^{us} \end{bmatrix} \cdot \begin{bmatrix} 0 & -\beta_{12} & \beta_{13} & \beta_{14} & \beta_{12} & 0 \\ \beta_{21} & -\beta_{22} & \beta_{23} & 0 & -\beta_{22} & -\beta_{21} \end{bmatrix}'
$$

- Model REH_1 :

$$\begin{bmatrix} \Delta p_t & r_t & ppp_t & \Delta s_{t+1} & r_t^{us} & \Delta p_t^{us} \end{bmatrix} \cdot \begin{bmatrix} 0 & \beta_{12} & 0 & \beta_{14} & \beta_{15} & 0 \\ \beta_{21} & \beta_{22} & 0 & 0 & \beta_{24} & \beta_{25} \end{bmatrix}'$$

- Model REH_1^*:

$$\begin{bmatrix} \Delta p_t & r_t & ppp_t & \Delta s_{t+1} & r_t^{us} & \Delta p_t^{us} \end{bmatrix} \cdot \begin{bmatrix} 0 & -\beta_{12} & 0 & \beta_{14} & \beta_{12} & 0 \\ \beta_{21} & -\beta_{22} & 0 & 0 & -\beta_{22} & -\beta_{21} \end{bmatrix}'$$

Note that the models with stars IKE_1^* and REH_1^*, have the same zero restrictions on beta that IKE_1 and REH_1, but also the first ones have a magnitude and sign restrictions.

For both the IKE_1 and IKE_1^* we are testing the equilibrium conditions of the IKE model (see Table 1). However, it is important to clarify that if ppp_t is stationary, then it is possible not reject the nule hypothesis of stationarity in the IKE model. However, this does not mean that IKE is the right model, this is because the sum of stationary variables is also stationary. So in that particular case the model is also compatible with the sum of stationary relations of the REH. Then REH could be the right model as well.

For that reason, although it makes not sense to test REH under r=2, because REH implies at least r=3. We present the cases $\beta_{13} = \beta_{23} = 0$, which are the cases where IKE_1 and IKE_1^* change to REH_1 and REH_1^*. In these cases we are testing UIP and Interest rate differential, both equilibrium condition of REH model. Thus, if REH_1 and REH_1^* are not equilibrium relation but IKE_1 and IKE_1^* are, in these cases we can be sure that IKE is the right model because it's clear that ppp_t is needed in the cointegrated vector.

Under r=3

- Model REH_1:

$$\begin{bmatrix} \Delta p_t & r_t & ppp_t & \Delta s_{t+1} & r_t^{us} & \Delta p_t^{us} \end{bmatrix} \cdot \begin{bmatrix} 0 & \beta_{12} & 0 & \beta_{14} & \beta_{15} & 0 \\ \beta_{21} & \beta_{22} & 0 & 0 & \beta_{24} & \beta_{25} \\ 0 & 0 & \beta_{33} & 0 & 0 & 0 \end{bmatrix}'$$

- Model REH_2^*:

$$\begin{bmatrix} \Delta p_t & r_t & ppp_t & \Delta s_{t+1} & r_t^{us} & \Delta p_t^{us} \end{bmatrix} \cdot \begin{bmatrix} 0 & -\beta_{12} & 0 & \beta_{14} & \beta_{12} & 0 \\ \beta_{21} & -\beta_{22} & 0 & 0 & -\beta_{22} & -\beta_{21} \\ 0 & 0 & \beta_{33} & 0 & 0 & 0 \end{bmatrix}'$$

Table 4 presents the results of the $\chi^2_{(v)}$ tests of the cointegrated vectors. As we can see, Colombia is the strongest case where the IKE equilibrium relation cannot be rejected. Imposing the restrictions implied by model IKE_1. Colombia has a $p-value$ of 0.954 which is clearly not rejected. Also, the $p-value$ of IKE_1^* is 0.803 which is high. Note, that in the models REH_1 and REH_1^*, the nule hypotheses of stationarity are rejected. Therefore, IKE appears to be the model which better describes the macroeconomic outcomes of Colombia over the sample period under consideration.

For Chile the results reported in Table 4 suggest that the equilibrium relation implied by IKE cannot be rejected. However, these results are not as stronger as those observed for the case of Colombia. Additionally, the $p-value$ of the model REH_1 has a $p-value$ of 0.10, a value near to the one of the IKE_1^*. Therefore, we opt for adopting the view of chile being a borderline case.

Regarding, Mexico and Brazil, these are countries in favor of the REH. Indeed, in both cases the cointegration analysis suggests $r = 3$ and the equilibrium relations implied by REH are not rejected by the data either. However in Brazil ppp_t does not hold as an equilibrium, while the Fisher Parity condition, the interest rate differential and the UIP conditions were found to be stationary relations. For Mexico the the equilibrium relations of the REH model behave as stationary cointegrated vectors.

4.5   The cointegrated vectors

As the IKE_1^* and REH_1^* vectors appears to be the best specification, in the sense they include sign and magnitude restrictions, these cointegrated vectors are reported in this subsection. To unify the analysis, β vector was normalized by the coefficient of the interest rate in the cases of Colombia, Chile and Mexico. Table 5 presents

the β and α vectors for all countries. As can be seen, in all models the $t-statitic$ of the β vectors, presented in parentheses, are always higher than 1.96, Therefore, all variables are needed in the cointegrated vectors. Also, all the vectors have the correct sign postulated by the IKE and REH models.

In contrast to the REH model, one of the main implications of the IKE is that ppp_t is not stationary by itself. Instead, other variables are needed to build a stationary relation. Therefore, the inclusion of ppp_t in the IKE cointegrated vectors has a particular interest, since this variable should be always statistically significant, ppp_t represents the uncertainty premium function, which is a function of the gap effect $up_t = \sigma(s - p_{d_t} + p_{f_t})$. In equation (6) we can see up_t implies that an increase in Δs_{t+1}^e will lead an excess of demand for the foreign exchange rate, where the σ coefficient is related to the loss averse parameter. For Colombia and Chile this parameter is significant. Also, within each country the magnitude of the coefficient in the two cointegrated vectors is quite similar, between zero and one, as it was expected.

Although the cointegrated vectors of Mexico and Brazil are in favor of the REH, Mexico is the only country in which it is possible to find a lineal combination that makes ppp_t stationary by itself. These results are in line with the stationary test presented in table 6[6]. The test shows that Mexico is the only country where ppp_t is a stationary relation.

A final test that shows the importance of the ppp_t variable in the IKE models, is the test of unit vector in *alpha* presented in Table 7. This test shows which variable are the pushing forces of the system and which are purely of adjusting (they only have transitory effects on the other variables). As we can see in the countries where IKE holds, ppp_t is presented as one of the pushing force of the system. Countries where REH holds the hypotheses of ppp_t as a pushing force is rejected.

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[6]Juselius and Jonhansen postulate the multivariate stationary test more appropriated than an univariate test

5   Final considerations

Using a CVAR approach, we find that macroeconomic outcomes for Colombia and Chile appear to be better described by the IKE model, while REH seems to hold best for Mexico and Brazil. In the particular case of Colombia, in 2012 the Colombian peso was the most appreciated currency against the US dollar within a sample of 170 currencies. This thesis argues that such appreciation could have taken place because of speculative behavior of agents with imperfect knowledge regarding future payoffs. Therefore, it may well be the case that individuals are either not receiving the right signals from the market or that they do not have the right model to form their expectations.

According to the 2012 Financial Development Report of the World Economic Forum, Brazil is the country with the best financial stability indicator, followed by Chile, Mexico and Colombia. Therefore, one can conjecture that the more advanced the financial market, the more knowledge individuals will have about their markets. Thus, coutries with developed financial systems are more likely to be economies where the REH hold, while IKE appears more consistent with economies with less developed financial markets.

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Figure 1: Series used to test purchasing power parity (PPP)

COL Nominal exchange rate — COL/ US Prices

CHI-US Exchange rate — CHI/US Prices

MEX Nominal exchange rate — MEX/ US Prices

BRA Nominal exchange rate — BRA/ US Prices
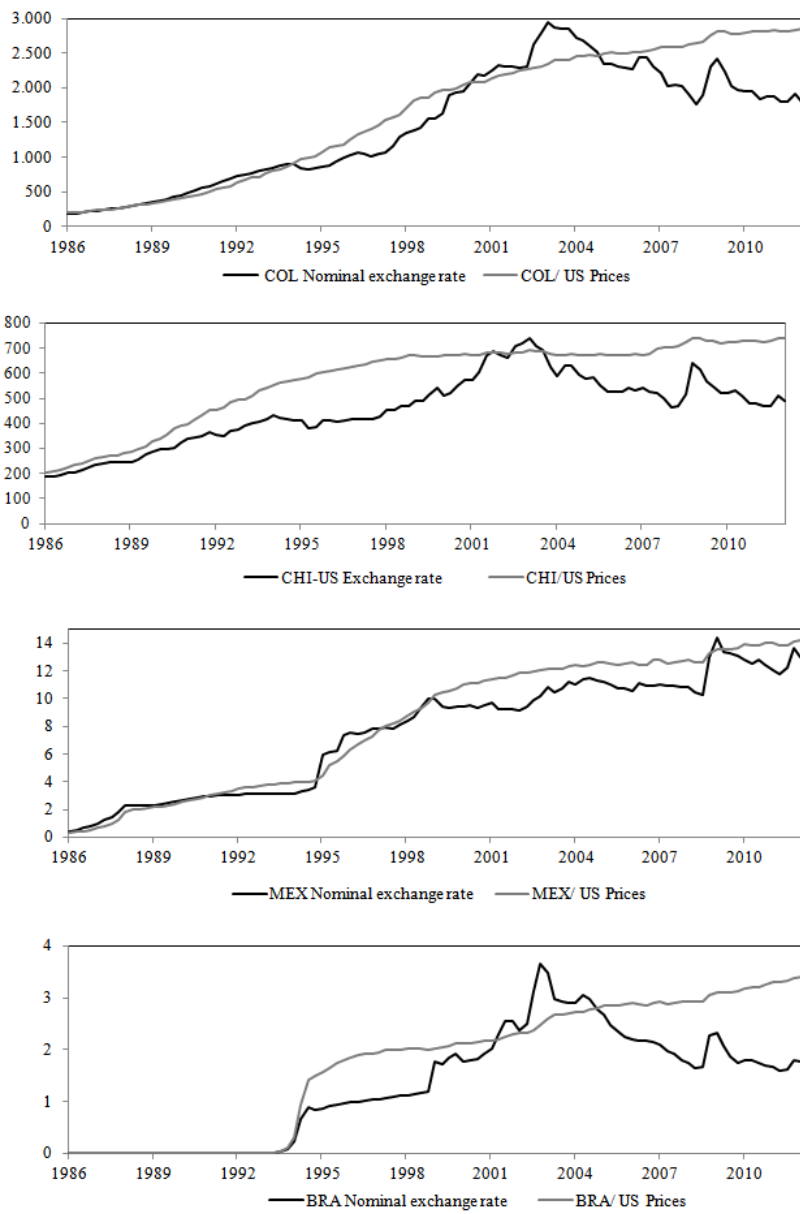
Figure 2: Series used to test uncertainty adjusted uncovered interest parity (UAUIP)
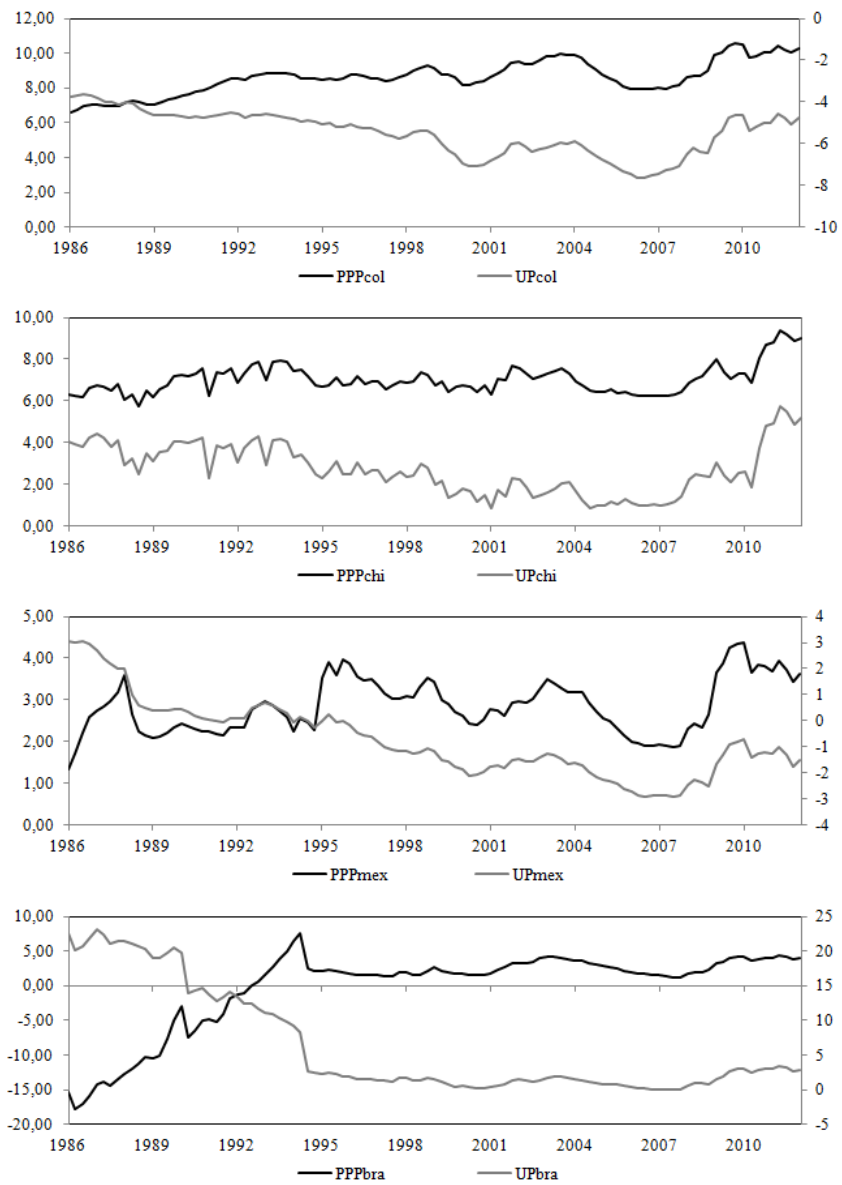
Figure 3: Time series used to the CVAR model

Colombia

Exchange rates in logs

Inflation

Interest rate

Chile

Exchange rates in logs

Inflation

Interest rate

Mexico

Exchange rates in logs

Inflation

Interest rate

Brazil

Exchange rates in logs

Inflation

Interest rate
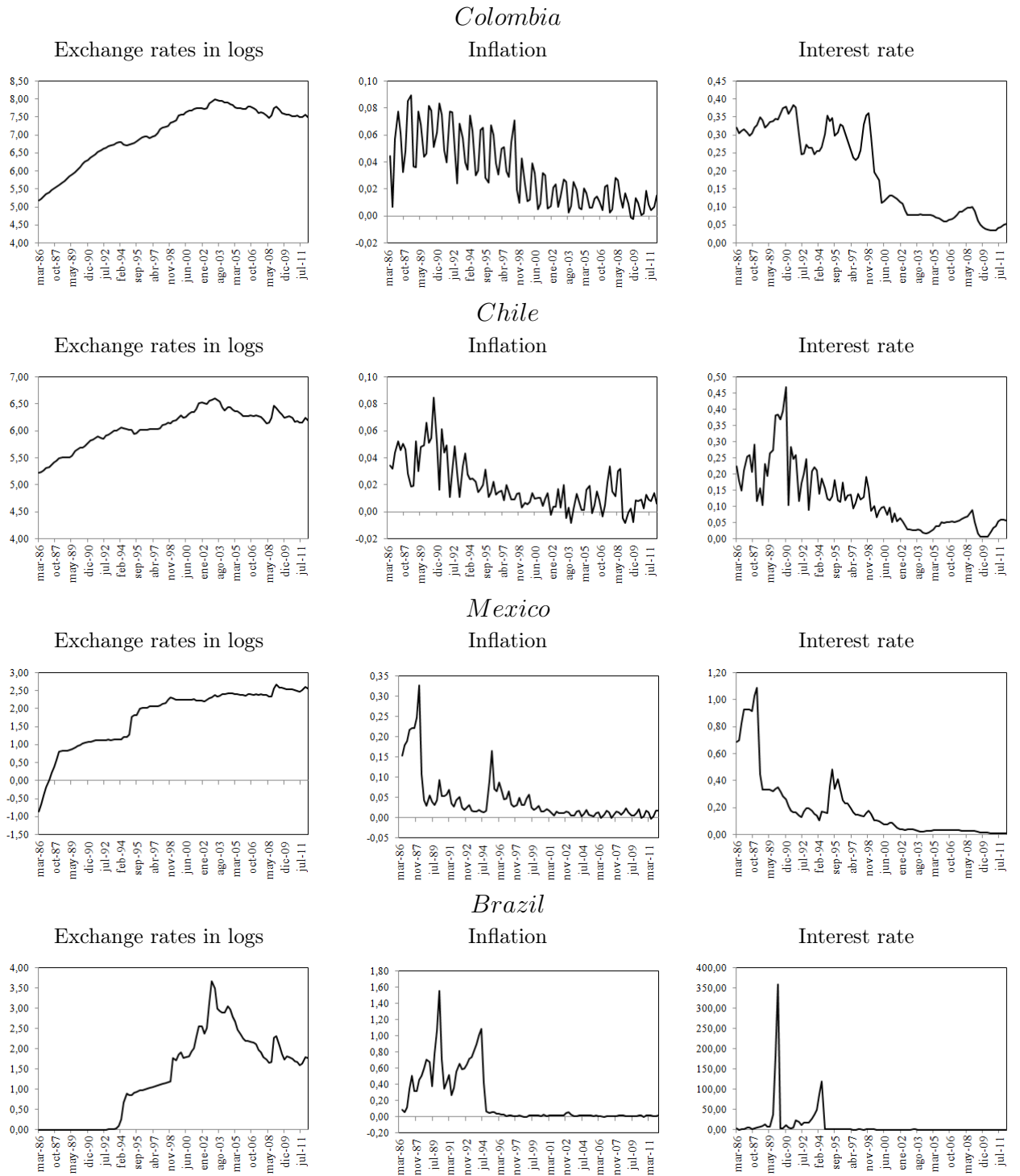
Table 2. Augmented Dickey-Fuller Test Equation for ppp_t

	Colombia	Chile	Mexico	Brazil
$t-satistics$	-0.421	-2.035	-3.131	-2.036
$Prob$	0.9004	0.2716	0.027	0.2710
Observations	102	102	102	102

Note: The tests are performed using 12 lags of the dependent variable.

Table 3: Rank trace tests

$p-r$	r	Colombia	Chile	Mexico	Brazil
4	0	97.335	121.919	129.723	118.740
		(0.000)	(0.000)	(0.000)	(0.000)
3	1	57.924	46.507	77.608	79.172
		(0.03)	(0.081)	(0.000)	(0.000)
2	2	27.596	20.650	42.162	43.151
		(0.26)	(0.402)	(0.010)	(0.006)
1	3	5.676	2.630	6.929	11.040
		(0.87)	(0.964)	(0.759)	(0.359)

Note: Structural change and dummy variables are included in the test. Therefore, test and $p-value$ in parentheses are generated using the simulation techniques available in the CATS version 2.0 for RATS software.

Table 4: Testing the stationarity

	Colombia	Chile	Mexico	Brazil
			r=2	
IKE_1	0.331 (0.954)	4.879 (0.186)	n.d.	n.d.
IKE_1^*	3.048 (0.803)	4.997 (0.150)	n.d.	n.d.
REH_1	13.830 (0.017)	10.411 (0.108)	n.d.	n.d.
REH_1^*	21.484 (0.006)	12.383 (0.091)	n.d.	n.d.
			r=3	
REH_2	n.d.	n.d.	4.502 (0.480)	5.864[a] (0.556)
REH_2^*	n.d.	n.d.	10.530 (0.230)	6.963[a] (0.541)

Note: $p-values$ of the $\chi^2_{(v)}$ test in parentheses. Structural change and dummy variable are included in the test. (a) This replaces the third cointegration relation by the restrictions implied by the Fisher parity condition.

Table 5: α and β vectors

	Colombia		Chile		Mexico			Brazil		
	$\hat{\beta}_1^{IKE}$	$\hat{\beta}_2^{IKE}$	$\hat{\beta}_1^{IKE}$	$\hat{\beta}_2^{IKE}$	$\hat{\beta}_1^{REH}$	$\hat{\beta}_2^{REH}$	$\hat{\beta}_3^{REH}$	$\hat{\beta}_1^{REH}$	$\hat{\beta}_2^{REH}$	$\hat{\beta}_3^{REH}$
Δp_t	0.357	0	1.316	0	0	1.151	0	1.000	−1.000	0
	(3.549)	(n.a)	(16.687)	(n.a)	(n.a)	(32.157)	(n.a)	(n.a.)	(n.a)	(n.a)
r_t	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	0	−0.064	0.069	−0.048
	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(−9.876)	(9.708)	(−8.130)
ppp_t	0.600	0.565	0.053	0.051	0	0	1.000	0	0	0
	(9.970)	(16.124)	(n.a)	(2.053)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a.)
Δs_{t+1}	0	0.417	0	1.342	1.068	0	0	0	0	1
	(n.a)	(7.521)	(n.a)	(17.847)	(29.156)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a.)
r_t^{us}	1.000	1.000	1.000	1.000	1.000	1.000	0	0	−0.069	0.048
	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(n.a)	(−9.708)	(8.130)
Δp_t^{us}	−0.357	0	−1.316	0	0	−1.151	0	0	1.000	0
	(−3.549)	(n.a)	(−16.687)	(n.a)	(n.a)	(−32.157)	(n.a)	(n.a)	(n.a)	(n.a)
$constant$	n.a.	n.a.	n.a.	n.a.	0.008	0,008	−2.419	0	−6.056	43.512
					(2.227)	(2,250)	(122,107)	(n.a)	(−21.942)	(17.014)
$t_{99.q4}$	−3.586	−3.434	n.a.	n.a.	n.a	n.a.	n.a.	n.a.	0	0
	(−8.738)	(−13.450)							(n.a)	(n.a)
$t_{96.q3}$	n.a.	n.a.	n.a.	n.a.	−0.013	0.008	0	n.a.	n.a	n.a
					(−3.664)	(2.250)	(n.a)			
$t_{94.q3}$	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	0	5.376	−44.019
								(n.a.)	(17.367)	(−15.337)

	Colombia		Chile		Mexico			Brazil		
	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
Δp_t	0.161	0.052	0.48	−0.337	−0.155	−0.602	0.061	−2.337	−2.188	0.175
	(0.706)	(0.260)	(1.029)	(−1.626)	(−0.656)	(−2.533)	(4.437)	(−1.273)	(−1.232)	(0.963)
r_t	0.102	0.314	0.740	−0.317	−0.014	−0.491	0.018	258.903	233.728	15.872
	(0.923)	(3.191)	(2.950)	(−1.470)	(−0.077)	(−2.656)	(1.704)	(1.892)	(1.766)	(1.169)
ppp_t	−0.050	0.036	−0.019	0.015	0.854	−1.147	−0.060	−0.023	−0.021	−0.001
	(−4.451)	(3.603)	(−2.377)	(2.263)	(1.967)	(−2.624)	(−2.387)	(−1.825)	(−1.750)	(−0.581)
Δs_{t+1}	−1.524	0.485	0.419	−1.182	−3.191	0.392	−0.092	−6.121	−6.361	−0.936
	(−0.931)	(0.334)	(0.371)	(−1.216)	(−3.547)	(0.434)	(−1.775)	(−2.418)	(−2.599)	(−3.727)

Note: $t-ratios$ in parentheses.

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Table 6: Test of stationarity

	Δp_t	r_t	ppp_t	Δs_{t+1}
Colombia	9.970	2.416	18.679	4.991
	(0.007)	(0.299)	(0.000)	(0.082)
Chile	1.135	4.687	21.103	1.308
	(0.567)	(0.096)	(0.000)	(0.520)
Mexico	1.573	2.298	2.990	1.030
	(0.210)	(0.130)	(0.084)	(0.164)
Brazil	6.184	6.494	25.586	6.512
	(0.045)	(0.039)	(0.000)	(0.039)

Table 7: Test of Unit vector in alpha

	Δp_t	r_t	ppp_t	Δs_{t+1}
Colombia	18.027	2.893	1.403	14.001
	(0.000)	(0.235)	(0.496)	(0.001)
Chile	10.642	5.793	0.503	8.440
	(0.005)	(0.055)	(0.778)	(0.015)
Mexico	3.639	17.572	4.626	1.228
	(0.056)	(0.000)	(0.031)	(0.268)
Brazil	1.049	1.970	10.176	0.029
	(0.306)	(0.160)	(0.001)	(0.865)

Table 8: Residual Analysis

		Colombia[(a)]	Chile	Mexico	Brazil[(b)]
Test for autocorrelation					
LM(1)	ChiSqr(16)	4.537 [0.998]	24.698 [0.075]	27.882 [0.173]	16.454 [0.422]
LM(2):	ChiSqr(16)	3.604 [0.998]	4.908 [0.996]	16.519 [0.672]	14.772 [0.541]
Test for Joint Normality	ChiSqr(8)	18.597 [0.017]	8.476 [0.388]	122.088 [0.000]	143.242 [0.000]
Test for Normality					
Δp_t		3.253 [0.197]	2.756 [0.252]	14.480 [0.001]	55.145 [0.057]
r_t		7.133 [0.028]	5.267 [0.072]	61.049 [0.000]	47.406 [0.000]
ppp_t		0.742 [0.690]	0.528 [0.768]	0.84 [0.657]	37.943 [0.000]
Δs_{t+1}		5.769 [0.055]	2.045 [0.360]	33.650 [0.667]	18.673 [0.000]

(a) with a dummy variable (dum99q2) is not possible reject the nule hypotheses of individual normality.

(b) with three dummy variables (dum 90q1. dum 89q3. dum99q) is not possible reject the nule hypotheses of individual normality.