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Investment and Market Structure in Common Agency Games

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Abstract

I study the incentives of a common buyer to undertake cooperative investment with a group of suppliers providing a homogeneous input. In my model, investment is not directed to increase the gains from trade but to enhance the competitive pressure among suppliers. At the same time, however, investment may strengthen the bargaining position of suppliers. Which effect dominates depends on the intensity of competition in the trading game, which also determines the equilibrium distribution of investment. Then, the model reproduces different market structures, and a firm may have higher incentives to become active in markets where competition is expected to be vigorous.

Keywords: Cooperative investment, Market structure, Competition, Bargaining position.

JEL classification: D44; L11.

1 Introduction

In the market economy, most of the manufacturing industry is outsourced. Many of the customer-supplier trading relationships nowadays are characterised by the collaboration on cost-reducing design innovations and competition over prices. Whitford (2005) calls this phenomenon "contested collaboration". We find examples of such form of trading relationships with the Original Equipment Manufacturers (OEM). In OEM transactions, a brand name company (OEM customer) who transacts with a contractor (OEM supplier) provides detailed technical blueprints, key technology, and

*This article is based on the second chapter of my thesis. I am in debt to my advisor Jacques Crémer for his insightful comments that were essential for the development of this article.

timely information to allow the supplier to produce according to specification, Ernst (2000). For instance, Dell and Hewlett Packard purchase components from other companies and sell complete systems under their own labels. Such companies source microprocessors, hard drives, and other equipment from OEM parts suppliers, who view them as OEM customers.

An interesting element of OEM transactional agreements is that transactions are often not exclusive, and a buyer trades with a different group of suppliers. Suppliers differentiate between first-tier with whom the buyer undertakes relation-specific investment and second-tier who have a more passive role and complement the production of the first-tier. Evidence suggests that the market structure in OEM transaction agreements differs considerably between industries. For instance, in the cycling industry, most of the suppliers providing intermediate and final components have a close trading relationship with the buyer who makes key investments with each one of them. This results in a homogeneous market structure in which none of the suppliers dominates over the others. The opposite happens in the IT industry. Kang et al. (2007) give evidence of the differences between suppliers trading with Dell stating that HIPRO has a dominant position with respect to its rival suppliers. What are then the incentives for a buyer to only invest with one group of suppliers but not with others? What is the origin of various market structures observed in different industries? How the investment and competition determine the number of active suppliers in an industry? I address such questions here.

In this article, I study a trading game where one side of the market decides to establish investment with the other side. Because investment directly benefits the non-investing party, investment in my model is not directed to increase the potential gains from trade but to generate more competition. A key element to understand how investment increases competition is the modelling of my trading game. Following the common agency literature, e.g., Bernheim & Whinston (1986), Segal (1999) and Chiesa & Denicolò (2009, 2012), I study a general non-linear pricing model where trade has the structure of a first-price auction. Suppliers submit a menu of trading contracts and the buyer chooses only one contract per supplier. Because the buyer can decide the group of suppliers to trade, in addition to its equilibrium contract, suppliers can offer contracts that are designed to compete for the equilibrium trade of any excluded supplier. Then, the equilibrium transfer from each supplier depends on the outside option available to the buyer after exclusion. As a result, the more the buyer invests with rival suppliers, the larger such outside option becomes, and the equilibrium transfers shrinks.

However, investment can also increase the bargaining position of suppliers. The reallocation of trade among suppliers as a result of investment difficult the generation of trading surpluses of rival suppliers with the buyer, and a supplier can appropriate more than the direct gains from investment. Then, investment to a given supplier generates a trade-off between reducing the equilibrium payoff of the rivals suppliers, *the competition effect*, and increasing the bargaining position of this supplier, *the bargaining effect*. Depending on how the competition between the suppliers in the market materialises, one effect may dominate over the other. I show that with the most intense competition, i.e., when all supplies offer contracts to compete for the equilibrium trade of an excluded supplier, only the competition effect emerges. To maximise this competition effect induces the buyer to set the same investment to each supplier. This generates a homogenous structure of suppliers with similar trading patterns. With milder competition, the bargaining effect appears, and the buyer either decides not to invest or to set the same investment only to those suppliers who compete for the trade of an excluded supplier. In this case, the resulting market structure is heterogenous with first and second-tier supplies coexsitring in the market. The competition in the trading game, therefore, determines the emerging market structure.

With regards to entry decisions, I show that a firm has more incentives to become active when anticipating fiercer competition. More competition decreases suppliers bargaining position but incentivises buyer's investment. This results in a more significant amount of the gains from trade that a supplier can appropriate. However, the number of active suppliers is always lower than efficiency. This comes from the result that in equilibrium there are also smaller levels of aggregate investment: the increase in competition that investment brings about is always of second order compared to the total gains from trade. Finally, the linkages between competition and investment that the model explores imply that not only the introduction of competition in the market is necessary, but the intensity of such competition also matters.

Existing works on investment decisions study the case where specific investment generates a direct benefit to non-investing parties, i.e., Che & Chung (1999), Che & Hausch (1999) and Hori (2006). These models show that when trade is exclusive, the implementation of simple ex-ante contracts is not sufficient to restore the efficient investment level. There is the need of complex option contracts.[1] For instance, Chih-Chi (2005) obtain over-investment with asymmetric information on the bargaining game, and only an option contract can restore efficiency. Departing from the literature, I do not restrict to the case of a bilateral monopoly, and to accommodate my theory to the case of OEM trading arrangements, I consider a case where a single buyer trades with many suppliers at the same time.

At this regard, closer to my paper is the literature that studies investment decisions without creating de-facto foreclosure.[2] In McLeod & Malcomson (1993), Bergemann & Välimäki (1996) and Burguet (1996), investment is modelled as learning by doing, and the papers explore strategic learning in a multi-period setting. The setting considered is one where supplier holds private information that reveals through experience with the buyer, and a more efficient relationship arises from experimentation. For example, Lewis & Yildirim (2002), consider a dynamic procurement game where the cost of a supplier decreases when contracting with the buyer. The authors assume that the production cost is private information, and the buyer designs a procurement mechanism to induce truth-telling. Trade allows to exploit the learning economies and reduce information rents. The buyer rotates her purchases with both suppliers to increase competition. Cabral & Riordan (1994) find similar results.[3]

[1] Arbitrarily default ex-ante contracts implements efficiency with selfish investment, Edlin & Reichelstein (1996), Konakayama et al. (1986), Nöldeke & Schmidt (1995), Rogerson (1992) and Schmitz (2002b).

[2] Under some circumstances, a relation-specific investment with a specific agent is strategically made to foreclose possible competitors, Farrell & Shapiro (1989), Valletti (2000), and Rey & Tirole (2007). Those papers solve the opportunism appearing when one of the parties is locked into the relationship. Farrell & Shapiro (1989) introduce long-term contracts to solve ex-post opportunism.

[3] In another article, Lewis & Yildirim (2005), study the situation that if the buyer deals with another supplier, the

Finally, this paper relates to the literature on the "hold-up" problem as in Klein, Crawford, & Alchian (1978) and Williamson (1979, 1983). The "hold-up" problem arises from parties being unable to bargain over non-contractible specific investment. Because my model considers purely cooperative investment, non-investing parties appropriate the direct gains from investment, the problem of being "held-up" is more severe. I show that the "hold-up" problem depends on the competition over trading contracts and the number of active suppliers in the market: the hold-up problem is severe when competition in the trading game is mild, or when the number of participating suppliers increases.

The article is organised as follows. Section 2 presents the model. A preliminary analysis where I determine the efficient trade, investment and the number of active suppliers is presented in Section 3. Section 4 presents the equilibrium analysis. After obtaining the set of equilibria in the trading game, I solve for the buyer investment and study the suppliers' entry decision. Section 5 summarises and concludes. All proofs are in the Appendix.

2 The model

I consider a non-exclusive trading game where a single buyer undertakes cooperative investment with suppliers producing a homogeneous input.[4] The game consists of three stages played sequentially. In stage 1, ex-ante equal suppliers decide to become active in the market, incurring a set-up cost $F > 0$. In stage 2, given the number of active suppliers, the buyer undertakes cooperative investment. Then, with N suppliers, the buyer decides how much to invest in each one, $k_i \geq 0$ for all $i \in N$, and the vector $\mathbf{k} = (k_1, k_2, ..., k_N)$ represents the investment allocation. I denote by $\mathbf{k}_{-i}$ the vector of investment containing all investments except supplier's i, and $K = \sum_{i=1}^{N} k_i$ stands for total investment. Finally, in the last stage trade happens.

Following Chiesa and Denicolò (2009), I model trade as a first-price auction in which suppliers

learning a supplier acquires by trading with the buyer disappears with certain probability, and the model relates to the literature on switching costs.

[4]Allowing for heterogeneity on the inputs provided by suppliers complicate the analysis without any effect on the main results of the paper.

simultaneously submit a menu of trading contracts and the buyer chooses the quantity it purchases from each supplier. I denote the menu of trading contracts for each supplier i by $M_i \subset \Re^2_+$. A trading contract consist of a pair $m_i = (x_i, T_i)$, where $x_i \geq 0$ represents the quantity of input supplied and $T_i \geq 0$ the transfer requested by each supplier i. The model belongs to private and delegated common agency. Private common agency means that a supplier cannot condition payments on the quantities others trade, and delegated implies that trade is voluntary. In what follows, I state the model more formally.

Strategies and payoffs

A strategy for each supplier i is the set of menu of contracts $M_i \subset \Re^2_+$. With a menu profile of trading contract $\mathbf{M} = (M_1, M_2, ..., M_N) \in \Gamma^N$, a strategy for the buyer is a function $\mathcal{M}(\mathbf{M}) : \Gamma^N \to (\Re^+)^N$ such that $\mathcal{M}(\mathbf{M}) \in \times_{i=1}^N M_i$ for all $\mathbf{M} \in \Gamma^N$, and $\mathbf{m} = (m_1, m_2, ..., m_n)$ is the vector of contracts accepted by the buyer. I do not impose any restriction on the number and the form of trading contracts belonging to M_i, expect that each supplier i must offer the null contract $m_i^0 = (0, 0)$ - due to the voluntary of trade - and that the menu of trading contracts is a compact set.[5]

For a given vector of contacts $\mathbf{m}$ accepted by the buyer and investment $\mathbf{k}$ the buyer's payoff is

$$\Pi(\mathbf{m} \mid \mathbf{k}) = U(X) - \sum_{i=1}^{N} T_i - \phi(K), \tag{2.1}$$

where $X = \sum_{i=1}^N x_i$ represents the total input traded and the function $U(X) : \Re^+ \to \Re^+$ denotes, in monetary terms, the value to the buyer. The payoff for the suppliers is net of the fixed cost F is

$$\pi_i(m_i \mid k_i) = T_i - C_i(x_i \mid k_i); \quad \forall i \in N, \tag{2.2}$$

where $C_i(x_i \mid \cdot) : \Re^+ \to \Re^+$ is supplier i's cost function. Because the cost function is only dependent on own output, direct externalities are absent. But because the willingness to pay for the goods

[5] This last assumption is necessary to guarantee existence of an optimal choice for the buyer. The same assumption is considered in Chiesa and Denicolò (2009).

depends on the quantities traded with all principals, contractual externalities arise.

For a given investment vector $\mathbf{k}$ and a number of active suppliers N, the maximum trading surplus gross of investment costs is

$$TS^*(\mathbf{k}, N) = \max_{x_1,\ldots,x_n} \left[U\left(x_1 + \cdots + x_N\right) - \sum_{i=1}^{N} C_i(x_i \mid k_i) \right], \tag{2.3}$$

where $\mathbf{x}^* = (x_1^*(\mathbf{k}, N), \ldots, x_N^*(\mathbf{k}, N))$ stands for the vector of trading quantities solving the problem. For later use, I denote $X^* = \sum_{i \in N} x_i(\mathbf{k}, N)$ the sum of the efficient quantities, and by $X^*_{-H} = \sum_{i \notin H} x_i^*(\mathbf{k}, N)$, for $H \subset N$, the sum of the efficient quantities without taking the quantities of the set of suppliers in H.

To ensure an interior solution and that every supplier trades a strictly positive and finite quantity with the buyer, I introduce the following regularity assumptions, subscripts denote partial derivatives.

Assumption 1. *(Regularity conditions)*

1. $U_x(\cdot) > 0,\ U_{xx}(\cdot) < 0,\ U_{xxx}(\cdot) > 0,\ \phi_k(\cdot) > 0$ *and* $\phi_{kk}(\cdot) > 0$.

2. $C_x(\cdot) > 0,\ C_{xx}(\cdot) > 0,\ C_k(\cdot) < 0,\ C_{xxk}(\cdot) < 0,\ C_{xk}(\cdot) < 0$ *and* $C_{xkk}(\cdot) > 0$.

3. $\lim_{X \to 0} U_x(\cdot) = +\infty,\ \lim_{X \to \infty} U_x(\cdot) = 0,\ \lim_{x_i \to 0} C_x(\cdot) = 0$ *and* $\lim_{x_i \to \infty} C_x(\cdot) = +\infty$.

Because the game is of complete information and played in different stages, I employ the solution concept of Subgame Perfect Nash.

3 Preliminaries: efficiency

Before the equilibrium analysis, it is interesting to consider the investment profile and the number of active suppliers that maximise the gains from trade net of the investment and entry costs. This will be a useful benchmark to compare against equilibrium outcomes and will allow identifying the inefficiencies arising in equilibrium.

First observe that for a given number of suppliers N and an investment profile $\mathbf{k}$, the amount of trade maximising trading surplus solves expression (2.3). Then, the efficient trading vector $\mathbf{x} = (x_1^*(\mathbf{k}, N), x_2^*(\mathbf{k}, \mathbf{N}), ..., x_N^*(\mathbf{k}, \mathbf{N}))$ is obtained by the system of equations

$$U_x\left(\sum_N x_i^*(\mathbf{k}, N)\right) = C_x(x_i^*(\mathbf{k}, N) \mid k_i), \quad \text{for all } \ i \in N, \tag{3.1}$$

in which marginal cost of production of any suppliers equal the marginal valuation of the buyer. With this expression, I can identify changes in the efficient trade with respect to the intensity of investment and the number of suppliers.

Lemma 1. *The efficient trade is such that:*

a) With an investment vector $\mathbf{k}$ *and a number of active suppliers* N*, with more investment to supplier* i*, the buyer trades more with supplier* i *and less with the others. The total amount of trade increases.*

$$i)\ \frac{dx_i^*(\mathbf{k}, N)}{dk_i} > 0, \quad ii)\ \frac{dx_j^*(\mathbf{k}, N)}{dk_i} < 0 \ \text{ for all } \ j \neq i, \quad \text{and} \quad iii)\ \frac{\partial}{\partial k_i} X^*(\mathbf{k}, N) > 0.$$

b) For a given investment vector $\mathbf{k}$*, the amount of trade of each suppliers and the aggregate trade are non-increasing and non-decreasing respectively with an increase in the number of active suppliers.*

Proof. All the proofs are in the Appendix. □

The Lemma indicates that when the buyer investments more to supplier i, this supplier becomes more efficient than the rest, and the increase in the relative efficiency crowds-out trade from the rest of suppliers to supplier i. The Lemma also verifies that this crowding-out effect is of second order: the increase in investment always generates a larger amount of trade. The second part of the Lemma identifies the negative externality that an extra supplier generates to the rest. A new supplier substitutes production from the existing supplier until the marginal cost of production equalises across them. As before, this effect is of second order, and there is always a larger amount of trade with an extra unit of production.

Definition 1. ***(Allocative sensitivity)*** *I call dx_j^*/dk_i for $j \neq i$ the allocative sensitivity, corresponding to the crowding-out of the equilibrium trading allocation of suppliers $j \neq i$ from an increase of investment to supplier i.*

Given the behaviour of the efficient allocation, I proceed to characterise the efficient investment profile. The changes in the trading surplus with respect to investment of supplier i gives

$$\begin{aligned}\frac{\partial TS(\mathbf{k}, N)}{\partial k_i} = U_x\left(\sum_{i\in N} x_i^*(\mathbf{k}, N)\right) \times \sum_{j=1}^{N} \frac{\partial x_j^*(\mathbf{k}, N)}{\partial k_i} - C_x\left(x_i^*(\mathbf{k}, N) \mid k_i\right) \times \sum_{j=1}^{N} \frac{\partial x_j^*(\mathbf{k}, N)}{\partial k_i} \\ - C_k\left(x_i^*(\mathbf{k}, N) \mid k_i\right) - \phi_K(K) \times \frac{\partial K}{\partial k_i}.\end{aligned}$$

The first part represents how the valuation of the buyer is affected by the change in total trade. The second part illustrates the changes originated in the production cost function, first as a consequence of the rearrangements of the trading allocation, and second, the direct effect that the investment has on the production cost for supplier i. Changes in the buyer's investment cost appears in the last part. By arranging terms and equating to zero the previous expression becomes

$$\left[U_x\left(\sum_{N} x_i^*(\mathbf{k}, N)\right) - C_x\left(x_i^*(\mathbf{k}, N) \mid k_i\right)\right] \times \sum_{j=1}^{N} \frac{\partial x_j^*(\mathbf{k}, N)}{\partial k_i} - C_k\left(x_i^*(\mathbf{k}, N) \mid k_i\right) - \phi_K(K) = 0$$

and with the envelope condition, I obtain the expression defining the efficient investment

$$\phi_K(K) = -C_k\left(x_i^*(\mathbf{k}, N) \mid k_i\right), \quad \forall i \in N. \tag{3.2}$$

The efficient investment profile indicates that the supplier's marginal reduction costs of production, as a result of investment, equals the marginal investment cost of the buyer. The expression indicates the direct link between the level of investment and the amount of input supplied. The other side of the coin presented in Lemma 1 is that a larger amount of trade from a supplier it must be optimality respond by a higher level of investment. The question that naturally arises if we solve for the optimal allocation is to study whether it is efficient for suppliers to have different levels of production. The next Proposition ascertains that due to the convex cost of production and the ex-ante symmetry of suppliers, it is never efficient to set an asymmetric allocation production among suppliers. As a result, the buyer will set the same level of investment to all suppliers. The next Proposition also characterises the efficient number of active suppliers in the market.

Proposition 1. *For a given number of active suppliers N, the buyer takes the same investment for each supplier, i.e., $k_i = k_{i'} = k^*(N)$ for all $i, i' \in N$. The efficient number of suppliers N^* is*

$$N^* = \arg\max_{N} \left[TS^* \left(N \times k^*(N), N \right) - TS^* \left((N-1) \times k^*(N-1), N-1 \right) \geq F \right]. \tag{3.3}$$

If the buyer was to set a vector of investment with asymmetric components, the optimal amount of trade will also be asymmetric, and more trade will be demanded to suppliers with a larger investment. However, due to the convex cost of production and the decreasing returns to scale of investment, there will be an increase in the gains from tarde by redistributing investment from those who receive more to those who receive less. Indeed this redistribution generates larger gains from trade until the point where investment equalises among all suppliers. Given that the buyer invests the same to all suppliers, adding an extra producing unit or supplier in the market is efficient if the resulting gains from trade are above the fixed cost of F. The Appendix shows that the marginal gains from trade of an additional suppliers decreases within the existing number of suppliers, and this guarantees that, for a given fixed entry costs, there is a unique number of active suppliers that maximises the gains from trade. At the optimum, the marginal gains from trade are equal to entry costs.

The next section, introduces equilibrium analysis where I show that the main reason for the buyer to invest is not to increase the gains from trade but rather to foster competition among suppliers.

4 Equilibrium analysis

I look for the subgame perfect equilibria of the game and start by solving the equilibria in the trading game. Then, I turn to the investment decision of the buyer, and finally analyse the entry decision of suppliers.

Trading game

In common agency games, when a set of suppliers trade with a single buyer, potential inefficiencies may arise due to externalities among suppliers. In my model, because the production cost of each supplier does not depend on the production by other suppliers, the trading contracts submitted by the rivals do not directly affect its payoff.[6] Then, take any supplier i and given a menu of trading contracts for the rest of suppliers, generating an amount of trade X_{-i}, each supplier i effectively plays a bilateral trading game with the buyer in which the former has the whole bargaining power. Then, when submitting a trading contract each supplier i maximises the potential gains from trade generated between the buyer and itself. This result is what the literature of markets and contracts has called "*bilateral efficiency*", and guarantees that in equilibrium there must exist a trading contract containing the amount of trade as characterised in expression (3.1).

The analysis with the efficient amount of trade is directly applicable when studying the equilibrium allocation. But, in addition to the efficient trade, suppliers can offer other trading quantities. As I later discuss, when eliciting the suppliers' equilibrium transfers, it will be useful to characterise the amount of trade in situations when the buyer decides not to deal with all suppliers. This is because the equilibrium transfer of a supplier depends on the trade the buyer undertakes with the rest of the suppliers. Then, conditional on the buyer not trading with a supplier, another supplier can offer a trading contract with an amount of trade that optimally replaces the efficient allocation

[6] The interested reader may look at Bernheim and Whinston (1986) and Segal (1999b) for a better understanding of trading externalities.

of the excluded supplier. Such individual quantity will, in turn, depend on the number of suppliers who also offer a trading contract aimed at competing for the excluded supplier.

Then, consider suppliers $j \in J_i \subset N$ offer an amount of trade to compete for the excluded supplier i. By bilateral efficiency, given that the buyer does not trade with supplier i and the amount of trade with the rest of the suppliers $X_{-\{j,i\}}(\mathbf{k}, N)$, each supplier $j \in J_i$ offers an amount of trade equal to

$$\tilde{x}_j(\mathbf{k}, N \setminus \{i\}) = \arg\max_{x_j \geq 0} \left[U\left(X_{-\{j,i\}}(\mathbf{k}, N) + x_j(\mathbf{k}, N) \;\middle|\; x_i(\mathbf{k}, N) = 0 \right) - C_j(x_j(\mathbf{k}, N) \mid \cdot) \right], \; \forall j \in J_i, \tag{4.1}$$

where the amount $\tilde{x}_j(\mathbf{k}, N \setminus \{i\})$ for each $j \in J_i$ constitutes the trading allocation that maximises the gains from trade when supplier i is excluded.

For later use, it is useful to compare the amount of trade in (4.1) against the efficient allocation. The convexity of the cost function implies that the aggregate trade is the largest when the buyer deals with all suppliers. However, because the amount of trade in expression (4.1) is designed to optimally replace the allocation of the excluded supplier, it is always larger that the individual efficient allocation. The next Lemma states this result.

Lemma 2. *For a given investment profile* $\mathbf{k}$ *and active suppliers* N*:*

i) The aggregate amount of trade is the largest when the buyer trades with all suppliers

$$X^*(\mathbf{k}, N) > X^*_{-\{J_i,i\}}(\mathbf{k}, N) + \sum_{j \in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i).$$

ii) The amount of trade that replaces supplier i *is larger than the efficient allocation*

$$\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i) > x^*_j(\mathbf{k}, N), \quad \forall j \in J_i.$$

Moreover, the larger the number of suppliers competing for the efficient allocation of any supplier i, the lower is the individual amount of trade solving expression (4.1), i.e., for $J_i' \subseteq J_i$, then

$\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i') \geq \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)$ for all $j \in J_i'$. Once again, this result comes from the convexity of the production function.

With the results of the trading allocation, I proceed to characterise the equilibrium transfers of the game. Given that the efficient amount of trade, presented in equation (3.1), is traded in equilibrium, the question that emerges refers to the equilibrium transfer that each supplier asks for the provision of the efficient allocation. In this regard, the common agent literature establishes that the buyer obtains the same payoffs when accepting the equilibrium contracts and when it excludes a single supplier from trade. This condition is called "individual excludability" and dictates that the equilibrium transfer of any supplier i is related to the buyer's outside option when excluded from trade. Then, crucial to determining the outside option of the buyer is the amount of trade offered for those buyers who compete for the equilibrium allocation of the excluded supplier as expressed in (4.1).

If we consider a set of suppliers $j \in J_i$ competing for the equilibrium allocation of any supplier i, individual excludability gives expression

$$U(X^*(\mathbf{k}, N)) - \sum_i T_i^* = U\left(X^*_{-\{J_i, i\}}(\mathbf{k}, N) + \sum_{j \in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)\right) - \sum_{j \in N \setminus \{J_i, i\}} T_j^* - \sum_{j \in J_i} \tilde{T}_j. \quad (4.2)$$

The left-hand side is the equilibrium payoff of the buyer. The right-hand side stands for the payoff that the buyer obtains by excluding from trade supplier i. Without supplier i, the buyer trades the efficient amount $X^*_{-\{J_i, i\}}(\mathbf{k}, N)$ with the suppliers who do not compete for the effect allocation of the excluded supplier, and the amount $\sum_{j \in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)$ with those suppliers who offer trading contacts that optimally replace supplier i. There is no other combination of a trading allocation that generates larger payoff to the buyer, and the intuition comes from the fact that the latent contracts submitted by suppliers $j \in J_i$ are designed to replace supplier i optimally. A similar equilibrium condition is shown in Laussel and LeBreton (2001) and Chiesa and Denicolò (2009).

To obtain the equilibrium transfer of supplier i, I use the condition that in equilibrium the set of suppliers in $j \in J_i$ are indifferent between supplying their equilibrium offers and the trading allocation that optimally replaces supplier i. Then,

$$T_j^* - C_j(x_j^*(\mathbf{k}, N)) = \tilde{T}_j - C_j(\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)) \quad \text{for } j \in J_i.$$

Summing over the suppliers in J_i

$$\sum_{j \in J_i} \left[T_j^* - C_j(x_j^*(\mathbf{k}, N))\right] = \sum_{j \in J_i} \left[\tilde{T}_j - C_j(\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i))\right], \tag{4.3}$$

and introducing the result into expressions (4.2) gives the equilibrium transfer

$$\begin{aligned} T_i^*(\mathbf{k}, N \mid J_i) &= U(X^*(\mathbf{k}, N)) - U\left(X^*_{-\{J_i,i\}}(\mathbf{k}, N) + \sum_{j \in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)\right) \\ &\quad + \sum_{i \in J_i} \left[C_j(\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)) - C_j(x_j^*(\mathbf{k}, N))\right] \\ &= U\left(X^*(\mathbf{k}, N)\right) - \left(V_{J_i}\left(X^*_{-\{J_i,i\}}(\mathbf{k}, N) \mid \mathbf{k}, N\right) + \sum_{j \in J_i} C_j(x_j^*(\mathbf{k}, N) \mid k_j)\right), \end{aligned}$$

where

$$V_{J_i}\left(X^*_{-\{J_i,i\}}(\cdot) \mid \mathbf{k}, N\right) = \max_{\{x_j\}_{j \in J_i}} \left[U\left(X^*_{-\{J_i,i\}}(\cdot) + \sum_{j \in J_i} x_j(\cdot) \,\middle|\, x_i = 0\right) - \sum_{j \in J_i} C_j(x_j(\cdot) \mid k_j)\right],$$

is the maximal trading surplus that buyer and the suppliers in J_i can generate conditional on $X^*_{-\{J_i,i\}}(\mathbf{k}, N)$ and the exclusion of supplier i. Observe that the convexity of the cost function makes the equilibrium transfer weakly decreasing in the set $J_i : J_i' \subseteq J_i \Longrightarrow T_i^*(\mathbf{k}, N \mid J_i) \leq T_i^*(\mathbf{k}, N \mid J_i')$.[7] Therefore, the more the number of suppliers competing for the efficient allocation of supplier i, the outside option available to the buyer increases and the equilibrium transfer of supplier i decrease.

Finally, by introducing the equilibrium transfers into the payoff functions, I obtain the equilib-

[7] In general the inequality is strict if J_i' is not equal to J_i.

rium payoffs

$$\pi_i(\mathbf{k}, N \mid J_i) = TS^*(\mathbf{k}, N) - \tilde{TS}_{-i}(\mathbf{k}, N \setminus \{i\} \mid J_i); \quad \forall i \in N, \tag{4.4}$$

$$\Pi(\mathbf{k}, N \mid J) = TS^*(\mathbf{k}, N) - \sum_i \Big(TS^*(\mathbf{k}, N) - \tilde{TS}_{-i}(\mathbf{k}, N \setminus \{i\} \mid J_i)\Big) - \phi(K), \tag{4.5}$$

for each supplier and the buyer respectively. Remember that $TS^*(\mathbf{k}, N)$ stands for the maximal trading surplus that can be generated given an investment profile $\mathbf{k}$ and a number of active suppliers N. The object $\tilde{TS}_{-i}(\mathbf{k}, N \setminus \{i\} \mid J_i)$ is the maximal trading surplus that can be generated when the buyer does not trade with supplier i. Then, what a supplier obtains in equilibrium is exactly its contribution to the surplus. Observe that this depends on the set of suppliers J_i competing for the efficient allocation of supplier i. Again due to the convex cost of production, it becomes cheaper to substitute the efficient allocation of supplier i the larger the number of suppliers in J_i and more the gains from trade become. As a result, the payoff of any supplier i decreases with the number of suppliers in J_i.

Investment

With the equilibrium in the trading allocation, I study buyer's investment decisions. Different from existing models that analyse specific investment, in my model, the buyer does not invest to increase the potential gains from trade: suppliers appropriate all the direct benefits from investment. But investment generates two opposing effects. Investment causes a competition effect that increases the competition among suppliers and a bargaining effect which makes suppliers enjoy larger market power. With the competition effect, suppliers compete fiercely for the trading allocation of any excluded supplier, resulting in better terms from trade to the buyer. The bargaining effect emerges as a result of a reallocation of the equilibrium amount of trade due to investment. The trade reallocation generates a constraint on the surplus that the buyer may produce with a group of suppliers, resulting in suppliers appropriating more than the direct benefits from investment. The incentives for the buyer to invest depends on which one of the two effects dominates, and I show that the level of competition in the trading game is crucial for the analysis.

To study the effects of investment, consider first a situation with only one active supplier. In this case, because the supplier has the whole bargaining power, it appropriates all the gains from trade, and the buyer earns no benefits. Anticipating this behaviour, the buyer does not invest as it will be entirely "hold-up". Now consider the situation with two active suppliers. In this case, I obtain the same results that with a single supplier. When the buyer invests, the increase in the equilibrium payoff for the supplier is precisely the gains from trade that investment generates

$$d\pi_i(\mathbf{k}, 2)/dk_i = dTS^*(\mathbf{k}, 2)/dk_i = -C_k(x_i^*(\mathbf{k}, 2).$$

This is because, the gains from trade without supplier i, $TS^*(\mathbf{k}_{-i}, 1)$, are not affected by the investment directed to him. Moreover, due to the envelope condition, $dTS^*(\mathbf{k}, 2)/dk_i = -C_k(x_i^*(\mathbf{k}, 2)) = dTS^*_{-j}(\mathbf{k}_{-j}, 1)/dk_i$, the equilibrium payoff of the other supplier $j \neq i$ does not change with i's investment

$$d\pi_j(\mathbf{k}, 2)/dk_i = dTS^*(\mathbf{k}, 2)/dk_i - dTS^*_{-j}(\mathbf{k}_{-j}, 1)/dk_i = 0.$$

Investment does not generate any increase in competition and the buyer decides not to invest.

To obtain a positive competition effect, we need at least three active suppliers in the trading game. The literature on common agency with non-linear pricing has established that for the characterisation of an equilibrium, suppliers need to offer more trading contracts than their efficient and null contracts.[8] In the previous section when I obtained the equilibrium of the trading game, the payoff of any supplier i was affected by the set of suppliers who were offering trading contras to compete for its equilibrium allocation if the buyer decided to exclude him from trade. Then, I will consider first an equilibrium in which all of the three suppliers offer such trading contracts, and analyse the effects of investment on any supplier i. Studying the change that investment generates

[8] Chiesa and Denicolò (2009) obtain the minimum number of trading contracts that support any equilibrium payoffs of the trading game. See their Proposition 3. Also, Roig (2017) provides the set of contracts and characterises the equilibrium of the trading game.

to supplier i's payoff, as before, I obtain that supplier i appropriates all the direct gains from the investment,

$$d\pi_i(\mathbf{k},3)/dk_i = dTS^*(\mathbf{k},3)/dk_i = -C_k(x_i^*(\mathbf{k},3).$$

This is because the investment does not affect the gains from trade that can be generated without him, $dTS^*_{-i}(\mathbf{k}_{-i},2)/dk_i = 0$. However, different from before, now the investment reduces the payoff for the rest of suppliers. The change in the equilibrium payoff for any supplier $j \neq i$ is

$$d\pi_j(\mathbf{k},3)/dk_i = dTS^*(\mathbf{k},3)/dk_i - dTS^*_{-j}(\mathbf{k}_{-j},2)/dk_i < 0, \quad \text{for } j \neq i.$$

The envelope condition does not apply, and an increase in the investment on supplier i makes him more effective in generating trading surpluses when any other supplier j is excluded from trade, i.e., $dTS^*(\mathbf{k},3)/dk_i = -C_k(x_i^*(\mathbf{k},3)) < -C_k(\tilde{x}_i(\mathbf{k}_{-j},2)) = dTS^*_{-j}(\mathbf{k},3)/dk_i$. This inequality comes from Lemma 2, which states that $\tilde{x}_i(\mathbf{k}_{-j},2) > x_i^*(\mathbf{k},3)$, and the assumption $C_{xk}(\cdot) < 0$. The competition for the equilibrium allocation of the excluded supplier intensifies with investment, and the buyer enjoys a better outside option. It is precisely the fact that the investment to supplier i decreases the equilibrium payoff to the rest of competing suppliers what makes the buyer obtain rents from the investment. This competition effect is always present when there are more than three active suppliers. As I later show, when the competition in the trading game is structured so that all suppliers submit trading contracts to compete for the equilibrium allocation of any excluded supplier, only the competition effect is present, and the buyer always sets a positive level of investment, see Proposition 2. Moreover, with this competitive environment, the next Lemma asserts that the buyer invests the same amount to all active suppliers.

Lemma 3. *When the equilibrium in the trading game is such that suppliers $j \in J_i = N \setminus \{i\}$ offer contracts to compete for the equilibrium allocation of an supplier $i \in N$, the buyer sets the same investment to all suppliers.*

Setting the same level of investment to each supplier follows the same logic as in the efficient investment profile. Now the objective though is not to maximise the total gains from trade, but the outside option available to the buyer emerging after a supplier's exclusion, $TS^*_{-i}(\mathbf{k}_{-i}, N-1)$ for all $i \in N$. Moreover, the result of the Lemma asserts that when the trading game is the most competitive, the efficient distribution of investment is achieved. Nevertheless, as I will later show, the aggregate level of investment is always lower than efficiency: the investment channel to enhance competition is not sufficient to restore investment efficiency. This is because the suppliers, who do not bear the investment costs, appropriate all the direct benefits, and the competition effect which constraint the equilibrium payoffs of the rival suppliers is always of second order with respect to the investment costs.

I proceed to show that competition with more than two active suppliers is not a sufficient condition for the buyer to set a positive level of investment. When competition in the trading game is milder, i.e., when not all suppliers will be offering trading contracts to compete for the efficient allocation of an excluded supplier, the competition effect will not be present with the investment of all suppliers. Besides, investment will also create a bargaining effect that will improve the suppliers' terms from trade in detriment to the buyer. Then, different from before, consider that supplier 3 only provides its equilibrium and the null contract, and study the effects of an increase in supplier 1's investment. The change in the equilibrium payoff for supplier 1 is

$$\begin{aligned} d\pi_1(\mathbf{k},3)/dk_1 &= dTS^*(\mathbf{k},3)/dk_1 - dTS^*_{-1}(\mathbf{k},2)/dk_1 \\ &= \underbrace{-C_k(x_1^*(\mathbf{k},3))}_{\text{Direct gain}} - \underbrace{\left[U_x\left(x_3^*(\mathbf{k},3)+\tilde{x}_2(\mathbf{k},2)\right) - C_x(x_3^*(\mathbf{k},3))\right] \times \frac{dx_3^*(\mathbf{k},3)}{dk_1}}_{\text{Bargaining effect}}. \end{aligned}$$

In addition to the direct gains from the investment, the extra element represents the bargaining effect. This last effect comes from the reallocation of trade as a result of investment, $dx_3^*(\mathbf{k},3)/dk_1$. Due to Lemma 2 and the regularity conditions, the expression inside the bracket is positive, and because of $dx_3^*(\mathbf{k},3)/dk_1 < 0$, the bargaining effect increases supplier 1's equilibrium payoff, who appropriates more than the direct gains from investment.

For a better understanding of the bargaining effect, consider the outside option available to the buyer when supplier 1 is excluded from trade. The increase in investment to supplier 1 generates a reduction in the equilibrium allocation of supplier 3. As a result, the ability of rival suppliers to create trading surpluses with the buyer decreases. This makes supplier 1 more indispensable bis a bis to the buyer, increasing its bargaining position. Notice that this bargaining effect was absent in an equilibrium where all suppliers provide trading contacts that compete for the equilibrium allocation of an excluded supplier. In this equilibrium, the amount of trade designed to compete for the excluded supplier did not depend on the investment allocated to the latter, and the outside option of the buyer was not affected.

The change in the equilibrium payoff for the rival suppliers is

$$\begin{aligned} d\pi_2(\mathbf{k},3)/dk_1 &= dTS^*(\mathbf{k},3)/dk_1 - dTS^*_{-2}(\mathbf{k},2)/dk_1 \\ &= -C_k(x_1^*(\mathbf{k},3) + C_k(\tilde{x}_1(\mathbf{k},2)) \\ &\quad - \left[U_x\left(x_3^*(\mathbf{k},3)+\tilde{x}_1(\mathbf{k},2)\right) - C_x(x_3^*(\mathbf{k},3))\right] \times \frac{dx_3^*(\mathbf{k},3)}{dk_1}, \end{aligned}$$

and

$$\begin{aligned} d\pi_3(\mathbf{k},3)/dk_1 &= dTS^*(\mathbf{k},3)/dk_1 - dTS^*_{-3}(\mathbf{k},2)/dk_1 \\ &= -C_k(x_1^*(\mathbf{k},3) + C_k(\tilde{x}_1(\mathbf{k},2)) \end{aligned}$$

for 2 and 3 respectively. Observe that for supplier 3 there is only a competition effect as in the case where all suppliers compete for the equilibrium allocation of an excluded supplier. Indeed, when supplier 3 is excluded from trade both suppliers 1 and 2 offer trading contracts that compete for the equilibrium allocation of this supplier. But for supplier 2, the competition effect is constrained by the change in the equilibrium allocation of supplier 3 in a similar way as the increase in supplier 1's bargaining position, and the investment of supplier 1 also increases the bargaining position of supplier 2. The argument for this result is the same as previously stated.

Similar results emerge when calculating the effect of the investment to supplier 2. This makes, as I later show, the buyer to invest the same amount to those suppliers who offer trading contacts that compete for the equilibrium allocation of an excluded supplier, see Lemma 4. Then, it is only left to study the incentives to invest with supplier 3. Changes in supplier 3's equilibrium payoffs are

$$d\pi_3(\mathbf{k}, 3)/dk_3 = dTS^*(\mathbf{k}, 3)/dk_3 = -C_k(x_1^*(\mathbf{k}, 3)),$$

and it only appropriates the direct gains from investment. This is because the trading surplus that rival suppliers can generate without supplier 3 is unaffected by the investment of the latter. Looking at the effect to the other suppliers, I obtain no alteration due to the envelope condition

$$d\pi_i(\mathbf{k}, 3)/dk_3 = dTS^*(\mathbf{k}, 3)/dk_3 - dTS^*_{-i}(\mathbf{k}, 2)/dk_3 = 0 \quad \text{for} \quad i = 1, 2.$$

Therefore, as the investment in supplier 3 does not generate any competition effect, the buyer decides not to invest. This result reinforces the conclusion that the only objective for the buyer to invest is to increase competition among suppliers. Investment is wholly wasted if no increase in competition materialises.

With the result that the competition effect is crucial to provide invectives for the buyer to invest, and the negative counterbalance that the bargaining effect may generate, I proceed to study when will the buyer set a positive investment level. To this aim, I consider a situation with zero level of investment and calculate the impact of an increase in investment on the buyer's equilibrium payoff. I will study a more general case with N active suppliers and where suppliers $j \in J_i$ offer contracts to compete for the equilibrium allocation of an excluded supplier $i \in N$, and suppliers $m \in N \setminus \{J_i\}$ offer only the equilibrium and the null contract. Then, the change in the buyer's equilibrium payoff with an increase of investment in one of the suppliers $j \in J_i$ is equal to:

$$\begin{aligned}
\frac{\partial \Pi(N, J_i)}{\partial k_j} &= \sum_{N \setminus \{j\}} \left[C_k(x_j^* \mid k_j) - C_k(\tilde{x}_j \mid k_j) \right] \\
&\quad + \sum_{J_i} \left(\sum_{m \in N \setminus \{J_i\}} \left[\left(U_x \left(X^*_{-\{J_i\}} + \sum_{j' \in J_i \setminus \{j\}} \tilde{x}_{j'} \right) - C_x(x_m^*) \right) \times \frac{dx_m^*}{dk_j} \right] \right), \\
&= \underbrace{\sum_{N \setminus \{j\}} \left(\int_{\tilde{x}_j}^{x_j^*} C_{xk}(\tau) d\tau \right)}_{\text{Competition effect } (+)} + \underbrace{\sum_{J_i} \left(- \sum_{m \in N \setminus \{J_i\}} \left[\left(\int_{X^*_{-\{J_i\}} + \sum_{j' \in J_i \setminus \{j\}} \tilde{x}_{j'}}^{X^*} U_{xx}(\tau) d\tau \right) \frac{dx_m^*}{dk_j} \right] \right)}_{\text{Bargaining effect } (-)}, \quad \forall j \in J_i.
\end{aligned}$$

The last line comes from the fundamental theorem of calculus together with the equilibrium condition that $U_x(X^*) = C_x(x_m^*)$ for all $m \in N \setminus \{J_i\}$. Observe that due to the assumption $U_{xx}(\cdot) < 0$ and Lemma 2 the integral is negative and because $dx_m^*/dk_j < 0$ the elements inside the bracket are positive. The negative sign at the beginning of the expression makes the bargaining effect in the buyer's equilibrium payoff to be negative. The buyer decides to set a positive level of investment to those suppliers who compete for the equilibrium allocation an en excluded supplier if it benefits from it, in other words, if the competition effect dominates the bargaining effect. Notice that a crucial element to determine if one effect dominates over the other is the allocative sensitivity dx_m^*/dk_j. Because given a level of competition in the trading game, the bargaining effect increases with the allocative sensitivity, when this allocative sensitivity is sufficiently large, the buyer decides not to invest. The contrary happens with a small allocative sensitivity, as the competition effect always dominate.

With all the previous discussion, the next Lemma characterises the investment allocation for an equilibrium in the trading game where a subset of suppliers do not compete for the equilibrium allocation of an excluded supplier.

Lemma 4. *When the equilibrium in the trading game is such that only suppliers $j \in J_i \subset N \setminus \{i\}$ offer contracts to compete for the equilibrium allocation of supplier i, for $i \in N$, and with more than two active suppliers:*

- If the allocative sensitivity is such that the competition effect dominates the bargaining effect, the buyer sets the same level of investment to all supplier $j \in J_i$*, and no investment to suppliers* $m \in N \setminus \{J_i\}$.

- Otherwise, the buyer does not invest with any supplier.

The next Proposition characterises the equilibrium investment depending on both the level of competition in the trading game and the allocative sensitivity.

Proposition 2. *The equilibrium investment fo the buyer depends on the number of active suppliers and the competition in the trading game:*

A) With $N \leq 2$ *active suppliers, the buyer does not invest.*

B) With $N > 2$ *active suppliers:*

i) For an equilibrium in the trading game where all suppliers compete to replace any other supplier, the equilibrium investment of the buyer is

$$\phi_K(K) = \underbrace{- \sum_{j \in N \setminus \{i\}} \left[\int_{x_i^*(\mathbf{k},N)}^{\tilde{x}_i(\mathbf{k}_{-j}, N \setminus \{j\})} C_{xk}(\tau) d\tau \right]}_{\textit{Competition Effect}}, \quad \forall i \in N. \tag{4.6}$$

ii) In any other equilibrium, the investment of the buyer depends on the allocative sensitivity.

- When the allocative sensitivity is small enough such that the competition effect dominates the bargaining effect, the equilibrium investment of the buyer is:

$$\phi_K(K) = \underbrace{- \sum_{i \in N \setminus \{j\}} \left(\int_{x_j^*(\mathbf{k},N)}^{\tilde{x}_j(\mathbf{k}, N \setminus \{i\} | J_i)} C_{xk}(\tau) d\tau \right)}_{\textit{Competition Effect}}$$
$$\underbrace{+ \sum_{J_i} \left(- \sum_{m \in N \setminus \{J_i\}} \left[\left(\int_{X^*_{-\{J_i\}}(\mathbf{k},N) + \sum_{j' \in J_i \setminus \{j\}} \tilde{x}_{j'}(\mathbf{k}, N \setminus \{i\} | J_i)}^{X^*(\mathbf{k},N)} U_{xx}(\tau) d\tau \right) \frac{dx_m^*(\mathbf{k},N)}{dk_j} \right] \right)}_{\textit{Bargaining Effect}}, \quad \forall j \in J_i. \tag{4.7}$$

- Otherwise the buyer does not invest.

The Proposition states that the buyer's decision to invest depends on three factors: the number of active suppliers, the competition in the trading game and the allocative sensitivity. With less than three active suppliers, investment does not materialise. In this case, investment does not increase competition, and the suppliers appropriate all the benefits from investment. With three or more active suppliers, investment depends on the competition in the trading game. When the competition is structured so that all suppliers offer contracts to compete for the exclusion of any supplier, investment always materialise. Only the competition effect is present, and investment improves the buyer's terms of trade. With milder competition, investment also generates a bargaining effect that grows with the allocation sensitivity. In this case, investment only happens when the competition effect dominates the bargaining effect. Also from the Proposition, two direct Corollaries emerge. The first, that will be important when eliciting the entry decision of suppliers, takes into consideration the evolution of the per-supplier investment with respect to changes in the level of competition. The second studies the hold-up problem and compares the aggregate level of investment with efficiency.

Corollary 1. *For $N > 2$, with a large enough allocative sensitivity, the individual investment increases with the level of competition in the trading game. Otherwise, a supplier who competes for the exclusion of any other supplier obtains larger investment with less competition in the trading game.*

Whether the level of competition generates more or less investment per-supplier also depends on the comparison between the competition and the bargaining effect. I show that the competition effect produced as a result of the investment is always increasing with less competition. With a less number of suppliers competing for the exclusion of any other supplier, those who compete have to offer a larger amount of trade to effectively replace a supplier. This result in an increase in how investment constraints the equilibrium payoff for the rest of suppliers and an increase in the competition effect. However, less competition also increases the bargaining effect. When all suppliers compete for the exclusion of a supplier, no bargaining effect emerges, see Proposition 2. But with less number of suppliers competing for an excluded supplier, the investment hurts more the outside option available to the buyer. The increase in the number of suppliers who only offer their equilibrium and null contract and the reduction of their equilibrium allocation as a result of investment explains the result.

With regards the aggregate level of investment, the next Corollary states the hold-up problem that emerges in equilibrium.

Corollary 2. *With a given number of active suppliers, the aggregate level of investment is always lower than efficiency.*

This result complements the findings of the existing literature on cooperative investment. When investment is cooperative, the direct gains from investment are appropriated by the part who does not bear the cost. It is precisely the increase in competition that investment brings about what gives incentives for the buyer to invest. However, the Corollary shows that the increase in competition as a result of investment is always of second order compared to the total gains from trade. This results in investment to be undersupplied. Moreover, in situations when competition is mild, due to the emergence of the bargaining effect, the buyer may decide not to invest. In this case, the hold-up problem is maximised. This last result implies that not only the introduction of competition in the market is necessary, but the intensity of such competition matters.

Entry decisions

With the buyer's investment decisions and the equilibrium in the trading game, I analyse the entry decision of suppliers. To this aim, I first consider the evolution of the per-supplier investment with an increase in the number of active suppliers. Then, fixing the number of suppliers who do not compete for the exclusion of any other supplier, the evolution of investment with an increase in the number of suppliers is presented in the next Lemma.

Lemma 5. *With $N > 2$ suppliers, the investment per supplier decreases with the number of suppliers in the market.*

$$k_j(N \mid J_i) > k_j(N+1 \mid J_i); \quad \forall j \in J_i \ \text{ and any } \ J_i \subset N.$$

The result comes from the reduction in the individual amount of trade due to the increase in the number of suppliers. When each supplier trades less, the competition effect decreases. The growth in competition resulting from an extra supplier substitutes the need for investment to foster competition. Moreover, the fact that the bargaining effect also increases with the number of suppliers, see the Appendix, reinforces the reduction of the per-capita level of investment with more active suppliers.

Equipped with this result, I determine the number of active suppliers in equilibrium. As it will later be clear, a supplier decides to enter the market depending on the expected competition in the trading game. Then, given a number N of suppliers and a set J off suppliers who compete for any excluded supplier, a supplier i decides to become active if its expected payoffs are above the fixed cots of entry, i.e., $\mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}, N, J\right)\right] \geq F$. The element J illustrates the level of competition in the trading game, and as previously studied, it affects not only the supplier's bargaining position but also the magnitude and distribution of investment in equilibrium. Then, the supplier's expected payoff is

$$
\begin{aligned}
&\mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}, N, J\right)\right] = \Pr\left(i \in J\right) \times \pi_i\left(k_i \mid \mathbf{k}, N, J\right) + \Pr\left(i = m \in N \setminus \{J\}\right) \times \pi_m\left(0 \mid \mathbf{k}, N, J\right) \\
&= \frac{|J|}{N} \times \Big(TS^*(\mathbf{k}, N) - \tilde{TS}_{-i}(\mathbf{k}, N \setminus \{i\} \mid J)\Big) + \frac{N - |J|}{N} \times \Big(TS^*(\mathbf{k}, N) - \tilde{TS}_{-m}(\mathbf{k}, N \setminus \{m\} \mid J)\Big).
\end{aligned}
$$

The first part represents the expected payoff of a supplier who competes for the equilibrium allocation of an excluded supplier. The second is the payoffs from a supplier who does not compete. Due to the different levels of investment applied to each supplier, it is immediate to show that for any J, we have that $\tilde{TS}_{-m}(\mathbf{k}(N), N \setminus \{m\} \mid J) > \tilde{TS}_{-i}(\mathbf{k}(N), N \setminus \{i\} \mid J)$, i.e., the gains from trade generated without a supplier to whom the buyer does not invest are always larger than without a supplier who receives investment. The previous expression is also equivalent to

$$\mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}(N), N, J\right)\right] =$$
$$TS^{*}(\mathbf{k}(N), N) - \frac{\mid J \mid}{N} \times \left[\tilde{TS}_{-i}(\mathbf{k}(N), N \setminus \{i\} \mid J)\right] - \frac{N- \mid J \mid}{N} \times \left[\tilde{TS}_{-m}(\mathbf{k}(N), N \setminus \{m\} \mid J)\right]. \tag{4.8}$$

The next Proposition states how a supplier's expected payoff evolves with the level of competition in the trading game with the number of active suppliers.

Proposition 3. *A supplier's expected profit is non-increasing with the number of active suppliers, and non-decreasing with the level of competition in the trading game.*

The first result of the Proposition, obtained from Lemma 5, is necessary to get a well-defined number of active suppliers in equilibrium. The evolution of the expected profits concerning competition is a bit more involved. This is because the expected payoff depends on the probability of being selected as a supplier who receives investment, and the per-capita level of investment. When competition becomes less intense, fewer suppliers are receiving investment, and the changes in the per-capita investment depend on the evolution of the competition and the bargaining effect as presented in the proof of Corollary 1. Remember that with less competition, the competition effect increase but so does the bargaining effect. I show that even eliminating the bargaining effect, the increase in the pare-capita investment as a result of the competition effect is always of second order compared to the reduction in the probability of being a supplier who receives investment. This proves that the expected supplier's payoff increases with competition. With the results presented in the Proposition, the next Corollary directly obtains.

Corollary 3. *The number of active suppliers in equilibrium is lower than efficiency, but entry is encouraged with the prospects of more intense competition.*

The efficient number of suppliers is obtained when the fixed entry cost is equal to the gains for trade generated from the last supplier to enter. The same decision rule is applied when the equilibrium in the trading game is the most competitive. However, because in equilibrium the per-capita level of investment is always lower makes those gains from trade to be reduced. Because the equilibrium expected profits increase with the level of competition, any supplier will have more incentives to become active if it expected competition in the trading game to be vigourous. The more homogenous distribution of investment more than compensates the reduction in the bargaining position as a result of more competition. This last result is important. Not only the introduction of competition to one side of the market matters but also the intensity of such competition. Indeed, my model shows that the more competitive the trading game becomes, the more it encourages the entry of new suppliers which also generates more competition.

5 Conclusion

The economics of specialisation make relation-specific investment a current growing phenomenon, whose analysis is essential to understand the well functioning of market transactions. In this paper, I have considered the case where a single buyer undertakes relation specific investment with a group of suppliers. I have studied cooperative rather than selfish investment and demonstrated that both the distribution and aggregate level of investment depends on the competition in the trading game.

The paper demonstrates that the buyer's investment incentives shape the market structure of the supply side of the market. When the equilibrium in the trading game is structured so that all suppliers offer trading contracts to compete for the equilibrium allocation of an excluded supplier, the buyer sets the same level of investment to each one of them. This results in an ex-post homogenous market structure, where the buyer trades only with first-tier suppliers. Evidence suggests this market structure for big automobile companies and large cycling brands. When competition in the trading game is milder, i.e., a group of suppliers does not compete for the trading allocation of an excluded supplier, the investment of the buyer is asymmetric. The buyer only invests in those suppliers whose investment reduces the equilibrium payoff of the rest of the suppliers. An asymmetric market structure emerges in which the buyer trades significant amounts with first-tier suppliers and smaller amounts with second-tier. The information technology industry gives testimony of this market structure. In computing manufacturing, Dell buys components from many different suppliers where HIPRO has a dominant position. In this latter case, my model also finds situations when the buyer may decide not to invest. If the increase in the bargaining position of the invested suppliers is above the rise in the competitive pressure that investment brings about, the buyer does not appropriate any benefits from investment.

In terms of efficiency, my model demonstrates that for a given number of active suppliers, the equilibrium level of investment is always lower than efficiency. This result complements the findings of the existing literature on cooperative investment. When investment is cooperative, the direct gains from investment is appropriated by the part who does not bear the cost. The introduction of competition to one side of the market reduces the problem of being "held-up" without the introduction of ex-ante contracts. However, because the increase in competition due to investment is always of second order compared to the total gains from trade, investment is always undersupplied. In my model the lower level of investment will also generate less participation of suppliers in the market. Surprisingly, because investment depends on the level of competition. The prospects of higher competition in the market attracts a larger number of suppliers.

Some interesting issues lie beyond the scope of the article. In particular, the current model has studied the incentives of a single buyer to invest. The analysis may be different if there are many buyers in the market who compete with suppliers. Then, the investment that a buyer undertakes with a supplier may generate positive spillovers to other buyers. Making investment conditional on exclusive dealing may be a possibility in this new environment. However, the modelling of a trading game where suppliers can offer a menu of trading contracts to different potential buyers is daunting to analyse, and I leave this analysis for further research.

A Proofs of Lemmas and Propositions.

Proof of Lemma 1: I prove point (a) of the Lemma by contradiction. For a given investment profile $\mathbf{k}$ and number os suppliers N, differentiating the first-order conditions for x_j^* given in (3.1) with respect to k_i I obtain

$$U_{xx}(X^*(\mathbf{k}, N)) \times \sum_{h=1}^{N} \frac{dx_h^*(\mathbf{k}, N)}{dk_i} = C_{xx}(x_j^*(\mathbf{k}, N) \mid k_j) \times \frac{dx_j^*(\mathbf{k}, N)}{dk_i}. \tag{A.1}$$

Because the left hand side is independent of j, I find that all dx_j^*/dk_i have the same sign. Now suppose also that dx_i^*/dk_i has that same sign. Then also the sum has that same sign and since $U_{xx}(\cdot) < 0$ and $C_{xx}(\cdot) > 0$ the right hand side and the left hand side have different signs. This leads into a contradiction.

Now suppose $dx_i^*/dk_i < 0$. The other signs therefore have to be positive. By (A.1) I find $\sum_{h=1}^{N} (dx_h^*/dk_i) < 0$, but the first-order condition for x_i^* differentiated with respect to k_i is

$$U_{xx}(X^*(\mathbf{k}, N)) \times \sum_{h=1}^{N} \frac{dx_h^*(\mathbf{k}, N)}{dk_i} = C_{xx}(x_i^*(\mathbf{k}, N) \mid k_i) \times \frac{dx_i^*(\mathbf{k}, N)}{dk_i} + C_{xk}(x_i^*(\mathbf{k}, N) \mid k_i),$$

which would then have a positive left hand side and a negative right hand side due to $C_{xk}(\cdot) < 0$. This also leads to a contradiction. I thus have shown point (i) and (ii) of (a) in the Lemma. Again by (A.1) point (iii) follows from $\partial X^*/\partial k_i = \sum_{h=1}^{N} (dx_h^*/dk_i)$.

The proof of point (b) comes directly from the concavity of the utility function and the convexity of the cost function. Without loss of generality, I take an symmetric efficient trade and prove point (b) by contradiction. Then, take $x^*(\mathbf{k}, N+1) \geq x^*(\mathbf{k}, N)$. This implies that $(N+1) \times x^*(\mathbf{k}, N+1) > N \times x^*(\mathbf{k}\cdot, N)$. However, by the concavity of $U_x(\cdot)$ and the efficiency condition, it has to be that

$$\begin{aligned} C_x\left(x^*(\mathbf{k}, N+1) \mid k\right) &= U_x\left((N+1) \times x^*(\mathbf{k}, N+1)\right) \\ &< U_x\left(N \times x^*(\mathbf{k}, N)\right) = C_x\left(x^*(\mathbf{k}, N) \mid k\right), \end{aligned}$$

but the convexity of $C_x(\cdot)$ implies that $x^*(\mathbf{k}, N+1) < x^*(\mathbf{k}, N)$, which leads to a contradiction. From the previous, I see that that $X^*(\mathbf{k}, N+1) > X^*(\mathbf{k}, N)$ comes directly.

Proof of Proposition 1: To show that a symmetric allocation of investment is optimal, i.e., $k_i = k_{i'} = k$ for all $i, i' \in N$. Consider an asymmetric allocation of the same amount of total investment such that $k'_1 = k + \Delta k$, $k'_2 = k - \Delta k$, and $k'_j = k$ for $j \in N \setminus \{1, 2\}$. Assume a reallocation of investment such the total amount of trade stays constant and that the loss of trade form supplier 2, due to a lower level of investment, is compensated by the increase of trade form supplier 1, who now enjoy a larger investment. Then, this asymmetric investment allocation will only reduce the gain from trade compared to the symmetric allocation if the same amount of trade is more costly to produce. Form Lemma 1, we know that $x_1^*(\mathbf{k}', N) \mid k'_1) > x_1^*(\mathbf{k}, N) \mid k_1)$ and $x_2^*(\mathbf{k}', N) \mid k_2) < x_2^*(\mathbf{k}, N) \mid k_2)$, and applying the fundamental theorem of calculus, the difference in production cost becomes

$$\int_{x_1^*(\mathbf{k},N)|k_1)}^{x_1^*(\mathbf{k}',N)|k'_1)} C_{xk}(\cdot) + \int_{x_2^*(\mathbf{k}',N)|k'_2)}^{x_2^*(\mathbf{k},N)|k_2)} C_{xk}(\cdot) < 0,$$

and is always negative due to assumption $C_{xk}(\cdot) < 0$. Then, the total cost of production is larger with an asymmetric allocation of investment and the buyer decides to reallocate investment until symmetry.

To show that there exist a unique number of suppliers N^* that satisfies expression (3.3) in the Proposition, I first assume that the fixed entry cost is sufficiently small to ensure entry from a single supplier, i.e., $TS^*(k^*, 1) > F$. Otherwise, there will be no beneficial exchange. I proceed to show that the marginal trading surplus is decreasing with the number of suppliers.

Consider first a situation without investment from the buyer, then, with on active supplier the marginal trading surplus is

$$TS^*(2) - TS^*(1) = U(2x^*(2)) - 2C(x^*(2)) - (U(x^*(1)) - C(x^*(1)))$$

and with two active suppliers

$$TS^*(3) - TS^*(2) = U(3x^*(3)) - 3C(x^*(3)) - (U(2x^*(2)) - 2C(x^*(2)))$$

Using both expressions, I obtain

$$\begin{aligned} &TS^*(2) - TS^*(1) \geq TS^*(3) - TS^*(2) \\ &\qquad \Longrightarrow 2U(2x^*(2)) - U(3x^*(3)) - U(x^*(1)) \geq 4C(x^*(2)) - 3C(x^*(3)) - C(x^*(1)) \end{aligned}$$

which is always true from then concavity of the utility function and the convexity of the cost function and that $x^*(1) < 2x^*(2) < 3x^*(3)$ and $x^*(1) > x^*(2) > x^*(3)$ from Lemma 1. Then, using an inductive argument, it can be shown that for any N

$$TS^*(N) - TS^*(N-1) > TS^*(N+1) - TS^*(N).$$

Finally, as the next Claim show that individual investments are not incasing with the number of active suppliers, proves that the marginal trading surplus is a decreasing function of the number of suppliers and that N^* as stated in the Proposition exists.

Claim 1. *The per-supplier investment is non increasing with the number of active suppliers.*

Proof. From Lemma 1, the efficient allocation is not increasing with the number of suppliers, i.e., for any supplier i, then $x_i^*(N) \geq x_i^*(N+1)$. This, together with the condition of efficient investment and the assumption $C_{xxk}(\cdot) < 0$ prove the Claim.

Proof of Lemma 2: I have to show that for any $J_i \subset N$ I obtain

$$X^*(\mathbf{k}, N) > X^*_{-\{J_i,i\}}(\mathbf{k}, N) + \sum_{j \in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i). \tag{A.2}$$

For any investment profile $\mathbf{k}$ and a set J_i, I know that $\sum_{h \in N \setminus \{J_i,i\}} x_h^*(\mathbf{k}, N) = X^*_{-\{J_i,i\}}(\mathbf{k}, N)$.

Hence, the expression above is equivalent to $\sum_{j\in J_i} x_j^*(\mathbf{k}, N) + x_i^*(\mathbf{k}, N) > \sum_{j\in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)$. Therefore since $x_i^*(\mathbf{k}, N) > 0$ if $\sum_{j\in J_i} \Big(x_j^*(\mathbf{k}, N) - \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)\Big) > 0$ expression (A.2) is satisfied. Observe that for a given investment profile, if the above is true it also has to be true for any $j \in J_i$, hence $x_j^*(\mathbf{k}, N) > \tilde{x}_j(\mathbf{k}, N\setminus\{i\} \mid J_i)$. If the contrary occurs, $x_j^*(\mathbf{k}, N) < \tilde{x}_j(\mathbf{k}, N\setminus\{i\} \mid J_i)$, then from the equilibrium allocation I have

$$U_x\left(X^*_{-\{J_i,i\}}(\mathbf{k}, N) + \sum_{j\in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)\right) = C_x(\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i))$$

$$> C_x(x_j^*(\mathbf{k}, N)) = U_x(X^*(\mathbf{k}, N)),$$

and by the concavity of $U(\cdot)$ I prove the claim. The previous also implies that for any $j \in J_i$ I have $\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i) > x_j^*(\mathbf{k}, N)$. Using the same procedure I can easily prove that for any $J_i' \subseteq J_i$ I have

$$X^*_{-\{J_i,i\}}(\mathbf{k}, N) + \sum_{j\in J_i} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i) \geq X^*_{-\{J_i',i\}}(\mathbf{k}, N) + \sum_{j\in J_i'} \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i'),$$

and I also obtain that $\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i') \geq \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)$. □

Proof of Proposition 2: With only two active suppliers, the main text has shown that all the gains from investment are appropriated by the suppliers, and the buyer does not invest. With a larger number of active suppliers, the equilibrium investment depend on the level of competition in the trading game. With an equilibrium in the trading game where all supplies offer trading contract to compete for the equilibrium allocation of an excluded supplier, Lemma 3 shows that investment is always positive and equal to each supplier. Then magnitude of investment is obtained by calculating the first-order condition of the equilibrium payoff of buyer with respect to investment

$$\begin{aligned}\frac{\partial \Pi(\mathbf{k},N)}{\partial k_i} &= -C_k\left(x_i^*(\mathbf{k},N)\mid k_i\right) - \left[-C_k\left(x_i^*(\mathbf{k},N)\mid k_i\right)\right] \\ &\quad - \sum_{j\in N\setminus\{i\}} \left[-C_k\left(x_i^*(\mathbf{k},N)\mid k_i\right) + C_k\left(\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})\mid k_i\right)\right] - \phi_K(K)\times\frac{\partial K}{\partial k_i} = 0 \\ &\Longrightarrow \phi_K(K) = -\sum_{j\in N\setminus\{i\}}\left[\int_{x_i^*(\mathbf{k},N)}^{\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})} C_{xk}(\tau)d\tau\right], \ \forall i\in N,\end{aligned}$$

where the last line comes from the fundamental theorem of calculus.

Point (ii) in the Proposition is obtained from the analysis in the main text and the result of Lemma 4.

Proof of Corollary 1: To see how investment evolves with respect to competition, I set a number of active suppliers N, and compare the right-hand side of expression (4.7) with respect to changes in the number of suppliers who offer trading contracts that compete for the exclusion of a given supplier. Then, consider $J_i' \subset J_i$ where suppliers $j \in \hat{J}_i$ are in J_i but not in J_i'. Whether individual investment increases with the leve of competition will depend on the allocative sensitivity.

Then, denoting by $\aleph(J_i')$ and $\aleph(J_i)$ the right-hand side for J_i' and J_i respectively, the difference is equal to

$$\begin{aligned}\aleph(J_i') - \aleph(J_i) = &\underbrace{-\sum_{i\in N\setminus\{j\}}\left(\int_{x_j^*(\mathbf{k},N)}^{\tilde{x}_j(\mathbf{k},N\setminus\{i\}|J_i')} C_{xk}(\tau)d\tau\right) + \sum_{N\setminus\{j\}}\left(\int_{x_j^*(\mathbf{k},N)}^{\tilde{x}_j(\mathbf{k},N\setminus\{i\}|J_i)} C_{xk}(\tau)d\tau\right)}_{\text{Differences in the Competition Effect}} \\ &\underbrace{\begin{aligned}&+\sum_{J_i'}\left(-\sum_{m\in N\setminus\{J_i'\}}\left[\left(\int_{X^*_{-\{J_i'\}}(\mathbf{k},N)+\sum_{j'\in J_i'\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J_i')}^{X^*(\mathbf{k},N)} U_{xx}(\tau)d\tau\right)\frac{dx_m^*(\mathbf{k},N)}{dk_j}\right]\right) \\ &+\sum_{J_i}\left(\sum_{m\in N\setminus\{J_i\}}\left[\left(\int_{X^*_{-\{J_i\}}(\mathbf{k},N)+\sum_{j'\in J_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J_i)}^{X^*(\mathbf{k},N)} U_{xx}(\tau)d\tau\right)\frac{dx_m^*(\mathbf{k},N)}{dk_j}\right]\right)\end{aligned}}_{\text{Differences in the Bargaining Effect}}\end{aligned} \tag{A.3}$$

The term representing the competition effect

$$-\sum_{i\in N\setminus\{j\}}\int_{\tilde{x}_j(\mathbf{k},N\setminus\{i\}|J_i)}^{\tilde{x}_j(\mathbf{k},N\setminus\{i\}|J'_i)} C_{xk}(\tau)d\tau > 0,$$

is always positive due to $C_{xk}(\cdot) < 0$ and $\tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J'_i)) > \tilde{x}_j(\mathbf{k}, N \setminus \{i\} \mid J_i)$ from Lemma 2. The difference in the bargaining effect is equal to

$$\begin{aligned}
&\sum_{J'_i}\left(-\sum_{m\in N\setminus\{J_i\}}\left[\left(\int_{X^*_{-\{J'_i\}}(\mathbf{k},N)+\sum_{j'\in J'_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J'_i)}^{X^*_{-\{J_i\}}(\mathbf{k},N)+\sum_{j'\in J_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J_i)} U_{xx}(\tau)d\tau\right)\frac{dx^*_m(\mathbf{k},N)}{dk_j}\right]\right)\\
&+\sum_{J'_i}\left(-\sum_{m\in \hat{J}_i}\left[\left(\int_{X^*_{-\{J'_i\}}(\mathbf{k},N)+\sum_{j'\in J'_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J'_i)}^{X^*(\mathbf{k},N)} U_{xx}(\tau)d\tau\right)\frac{dx^*_m(\mathbf{k},N)}{dk_j}\right]\right)\\
&+\sum_{\hat{J}_i}\left(\sum_{m\in N\setminus\{J_i\}}\left[\left(\int_{X^*_{-\{J_i\}}(\mathbf{k},N)+\sum_{j'\in J_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J_i)}^{X^*(\mathbf{k},N)} U_{xx}(\tau)d\tau\right)\frac{dx^*_m(\mathbf{k},N)}{dk_j}\right]\right).
\end{aligned}$$

The first element of the expression is negative because $U_{xx}(\cdot) < 0$, $dx^*_m(\mathbf{k}, N)/dk_j < 0$ and $X^*_{-\{J_i\}}(\mathbf{k}, N) + \sum_{j'\in J_i\setminus\{j\}} \tilde{x}_{j'}(\mathbf{k}, N \setminus \{i\} \mid J_i) < X^*_{-\{J'_i\}}(\mathbf{k}, N) + \sum_{j'\in J'_i\setminus\{j\}} \tilde{x}_{j'}(\mathbf{k}, N \setminus \{i\} \mid J'_i)$ due to Lemma 2. The second and the third elements have different signs. The second element represents the increase in the bargaining effect emerging from the distortion of a larger number of suppliers not competing for an excluded supplier. The third element considers the reduction in the bargaining effect as a result of a lower number of suppliers competing for an excluded supplier. This last effect can be shown to be of second order, and when competition becomes milder, the bargaining effect increases, and it negatively affect the difference in (A.3). Because the magnitude of the bargaining effect directly depends on the allocative sensitivity. This proves that with a larger enough allocative sensitivity, the differences in the bargaining effect dominates over the differences in the competition effect. Then, the investment per-supplier may decrease with less competition in the trading game.

Proof of Corollary 2: For a given number of active suppliers N, I compare the equilibrium investment, characterised in Proposition 2, against efficiency. In the most competitive equilibrium, the efficient investment will be larger than in the equilibrium if the right-hand side for the expression that characterise investment are

$$
\begin{aligned}
&-C_k\left(x_i^*(\mathbf{k},N)\mid k_i\right) > -\sum_{j\in N\setminus\{i\}}\left[\int_{x_i^*(\mathbf{k},N)}^{\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})} C_{xk}(\tau)d\tau\right]\\
&\quad\Longrightarrow \sum_{j\in N\setminus\{i\}}\left[C_k\left(\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})\mid k_i\right)-C_k\left(x_i^*(\mathbf{k},N)\mid k_i\right)\right]-C_k\left(x_i^*(\mathbf{k},N)\mid k_i\right)>0\\
&\quad\Longrightarrow \int_{X^*(\mathbf{k},N)}^{(N-1)\times\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})} C_{xk}(\tau)d\tau > 0.
\end{aligned}
$$

Both lines come from the fundamental theorem of calculus, and the last line is positive due to assumption $C_{xk}(\cdot)<0$ and Lemma 2 showing that $X^*(\mathbf{k},N)>(N-1)\times\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})$.

The same analysis also shows that when competition in milder, the competition effect represented in the first part of the right-hand side is lower than the right-hand side representing the efficient investment. Also, there is the extra bargaining effect that reinforces the result that the efficient level of investment is larger.

Proof of Lemma 5: I first consider the situation in the most competitive equilibrium. In this equilibrium, the buyer sets the same level of investment to each supplier, see Lemma 3. Here I show that the buyer invests less with each supplier when the number of active suppliers increase. To see this, I compare the right-hand side of the investment decision of the buyer with N and $N+1$ active suppliers. Since the left-hand side of expression (4.6) in Proposition 2 increases with the aggregate level of investment, the buyer invests less with $N+1$ than with N suppliers if

$$
\begin{aligned}
&-\sum_{j\in N\setminus\{i\}}\left[\int_{x_i^*(\mathbf{k},N)}^{\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})} C_{xk}(\tau)d\tau\right]+\sum_{j\in(N+1)\setminus\{i\}}\left[\int_{x_i^*(\mathbf{k},N+1)}^{\tilde{x}_i(\mathbf{k}_{-j},(N+1)\setminus\{j\})} C_{xk}(\tau)d\tau\right]>0\\
&\quad\Longrightarrow \int_{(N+1)\times x_i^*(\mathbf{k},N+1)-N\times x_i^*(\mathbf{k},N)}^{(N+1)\times\tilde{x}_i(\mathbf{k}_{-j},(N+1)\setminus\{j\})-N\times\tilde{x}_i(\mathbf{k}_{-j},N\setminus\{j\})} C_{xk}(\tau)d\tau>0.
\end{aligned}
$$

Due to assumption $C_{xk}(\cdot) < 0$, the above expression is positive if the lower bound of the integral is larger than the upper bound, i.e.,

$$N\left[\tilde{x}_i(\mathbf{k}_{-j}, N \setminus \{j\}) - x_i^*(\mathbf{k}, N)\right] > (N+1)\left[\tilde{x}_i(\mathbf{k}_{-j}, (N+1) \setminus \{j\}) - x_i^*(\mathbf{k}, N+1)\right].$$

Both, the left and the right-hand side are positive, and the inequality is due to the concavity of the problem.

When the competition in the trading game is milder, remember that to determine the investment for any supplier $j \in J_I$, two effects go into opposite directions: the competition and the bargaining effect. Similar to the previous case, the competition effect decreases with an increase in the number of active suppliers. This will generate a lower investment to the supplier. Then, a sufficient condition for the result of the Lemma to be true is to prove that the bargaining effect increases with the number of active suppliers. Then, denoting $\tilde{J}_i = J_i \cup \{k\}$, the set of suppliers who compete for the equilibrium allocation of supplier i, where supplier k represents the new supplier who has entered the market, the differences in the bargaining effect as a result of an extra suppliers is

$$\begin{aligned}
&\sum_{J_i}\left(-\sum_{m \in N \setminus \{\tilde{J}_i\}}\left[\left(\int_{X^*_{-\{\tilde{J}_i\}}(\mathbf{k},N+1)+\sum_{j' \in \tilde{J}_i \setminus \{j\}} \tilde{x}_{j'}(\mathbf{k},(N+1)\setminus\{i\}|\tilde{J}_i)}^{X^*(\mathbf{k},N+1)} U_{xx}(\tau)d\tau\right)\frac{dx_m^*(\mathbf{k},N+1)}{dk_j}\right]\right) \\
&-\sum_{m \in N \setminus \{\tilde{J}_i\}}\left(\int_{X^*_{-\{\tilde{J}_i\}}(\mathbf{k},N+1)+\sum_{j' \in \tilde{J}_i \setminus \{j\}} \tilde{x}_{j'}(\mathbf{k},(N+1)\setminus\{i\}|\tilde{J}_i)}^{X^*(\mathbf{k},N+1)} U_{xx}(\tau)d\tau\right)\frac{dx_m^*(\mathbf{k},N+1)}{dk_j} \\
&+\sum_{J_i}\left(\sum_{m \in N \setminus \{J_i\}}\left[\left(\int_{X^*_{-\{J_i\}}(\mathbf{k},N)+\sum_{j' \in J_i \setminus \{j\}} \tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J_i)}^{X^*(\mathbf{k},N)} U_{xx}(\tau)d\tau\right)\frac{dx_m^*(\mathbf{k},N)}{dk_j}\right]\right).
\end{aligned}$$

The first and the second line represent the bargaining effect of each suppler when there are $N+1$ active suppliers, the last line is the bargaining effect with N suppliers. The second line is always negative and the sign for the first plus the third line is determined by expression

$$\eta(N) \equiv -\left(\int_{X^*_{-\{\tilde{J}_i\}}(\mathbf{k},N+1)+\sum_{j'\in\tilde{J}_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},(N+1)\setminus\{i\}|\tilde{J}_i)}^{X^*(\mathbf{k},N+1)} U_{xx}(\tau)d\tau\right)\frac{dx^*_m(\mathbf{k},N+1)}{dk_j}$$
$$+\left(\int_{X^*_{-\{J_i\}}(\mathbf{k},N)+\sum_{j'\in J_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}|J_i)}^{X^*(\mathbf{k},N)} U_{xx}(\tau)d\tau\right)\frac{dx^*_m(\mathbf{k},N)}{dk_j}.$$

Because of efficiency, we have that

$$X^*_{\{\tilde{J}_i\}}(\mathbf{k},N+1) - X^*_{\{J_i\}}(\mathbf{k},N) \geq \sum_{j'\in\tilde{J}_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},(N+1)\setminus\{i\}\mid\tilde{J}_i) - \sum_{j'\in J_i\setminus\{j\}}\tilde{x}_{j'}(\mathbf{k},N\setminus\{i\}\mid J_i),$$

which again goes in favour of an increase in the bargaining position with more active suppliers. However, the next Claim states that the allocative sensitivity decreases with more suppliers

Claim 2. *For a set of supplies $m \in N \setminus \{J_i\}$, the allocative sensitivity decreases with the number of active suppliers, i.e., $\mid dx^*_m(\mathbf{k},N+1)/dk_j \mid < \mid dx^*_m(\mathbf{k},N)/dk_j \mid$.*

Proof. For a given vector of investment $\mathbf{k}$, from Lemma 1, with more active suppliers, the buyer trades less with any supplier. A direct result is that the change in the equilibrium allocation derived from an increase in investment is also reduced. □

As a result, the sing of $\eta(N)$ depends on which of the two previous elements dominate. Even if the allocative sensitivity has a larger effect, the bargaining effect may still be increasing with the number of suppliers. This is because of the extra condition from supplier k. Indeed, I assume that the fundamentals of the model are such that this is always the case.

Proof of Proposition 3: To show that the expected payoff is non-deceasing with the level of competition, consider $J' \subset J$ and I calculate the difference in expected payoffs, i.e., $\mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}(N,J),J\right)\right] - \mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}(N,J'),J'\right)\right]$. By further considering the case without a bargaining effect, the previous difference represents a lower bound. Then, if I prove a positive difference this will always be for any positive bargaining effect. Then, the lower-bound is

$$\begin{aligned}
&\mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}(N,J),J\right)\right] - \mathbb{E}\left[\pi_i\left(k_i \mid \mathbf{k}(N,J'),J'\right)\right] = \\
&TS^*(\mathbf{k}(N,J)) - \frac{\mid J \mid}{N} \times \left[\tilde{TS}_{-i}(\mathbf{k}(N,J), N\setminus\{i\} \mid J)\right] - \frac{N - \mid J \mid}{N} \times \left[\tilde{TS}_{-m}(\mathbf{k}(N,J), N\setminus\{m\} \mid J)\right] \\
&-TS^*(\mathbf{k}(N,J')) + \frac{\mid J' \mid}{N} \times \left[\tilde{TS}_{-i}(\mathbf{k}(N,J'), N\setminus\{i\} \mid J')\right] + \frac{N - \mid J' \mid}{N} \times \left[\tilde{TS}_{-m}(\mathbf{k}(N,J'), N\setminus\{m\} \mid J')\right].
\end{aligned}$$

The more asymmetric distribution of investment implies that $TS^*(\mathbf{k}(N,J)) > TS^*(\mathbf{k}(N,J'))$. The main texts sates that for any for any $J \in N$, $\tilde{TS}_{-m}(\mathbf{k}(N,J'), N\backslash\{m\} \mid J') > \tilde{TS}_{-i}(\mathbf{k}(N,J'), N\backslash\{i\} \mid J')$ and because $(N- \mid J' \mid)/N > (N- \mid J \mid)/N$, the difference is always positive.

Proof of Corollary 3: To establish that the number of active suppliers cannot be larger than efficiency, I compare the entry decision rule presented in Proposition 1, establishing the optimal number of active suppliers N^*, against the expected payoff that a supplier will obtain in equilibrium if there were N^* active suppliers. Because the equilibrium payoffs are always lower implies that the number of active suppliers cannot be larger than efficiency.

Remember from Proposition 1, that the efficient number of active suppliers N^*, is the maximum integer that solves

$$TS^*\left(\mathbf{k}^*(N^*), N^*\right) - TS^*\left(\mathbf{k}^*(N^*-1), N^*-1\right) = F.$$

With this number of active suppliers, in the most competitive equilibrium, a suppliers obtains equilibrium payoffs

$$\pi_i(\mathbf{k}(N^*), N^*) = TS^*\left(\mathbf{k}(N^*), N^*\right) - TS^*_{-i}\left(\mathbf{k}_{-i}(N^* \setminus \{i\}), N^* \setminus \{i\}\right).$$

With the same level of investment, both expressions are the same, but because the per-supplier investment is lower in equilibrium, as show in the proof of Corollary 2 it is always the case that

$$TS^*\left(\mathbf{k}^*(N^*), N^*\right) - TS^*\left(\mathbf{k}^*(N^*-1), N^*-1\right) \geq TS^*\left(\mathbf{k}(N^*), N^*\right) - TS^*_{-i}\left(\mathbf{k}_{-i}(N^* \setminus \{i\}), N^* \setminus \{i\}\right).$$

$$\begin{aligned} \Longrightarrow & TS^*\left(\mathbf{k}^*(N^*), N^*\right) - TS^*\left(\mathbf{k}(N^*), N^*\right) \\ & \geq TS^*\left(\mathbf{k}^*(N^*-1), N^*-1\right) - TS^*_{-i}\left(\mathbf{k}_{-i}(N^* \setminus \{i\}), N^* \setminus \{i\}\right) \\ & = TS^*_{-i}\left(\mathbf{k}^*_{-i}(N^* \setminus \{i\}), N^* \setminus \{i\}\right) - TS^*_{-i}\left(\mathbf{k}_{-i}(N^* \setminus \{i\}), N^* \setminus \{i\}\right). \end{aligned}$$

This is because the difference in the amount of trade as a result of lower investment, is larger with all active suppliers. This shows that in the most competitive equilibrium, the number of active suppliers cannot be larger than efficiency. Finally, as the profit of a supplier increase with the level of competition, see Proposition 3, the number of active suppliers in equilibrium cannot be larger than efficiency. This last remark proves that when competition is more intense, a larger number of suppliers can be sustained in equilibrium.

References

[1] Bergemann, D., and J. Välimaki (1996): "Learning and Strategic Pricing", *Econometrica*, 64, 1125-1150.

[2] Bernheim, B., and M. Whinston (1986): "Common Agency", *Econometrica*, Vol. 54, No. 4, 923-942.

[3] Burguet, R. (1996): "Optimal Repeated Purchases when Sellers are Learning about Costs", *Econometrica*, 68, 440-455.

[4] Cabral, L., and M. Riordan (1994): "The Learning Curve, Market Dominance, and Predatory Pricing", *Econometrica*, 62, 1115-1140.

[5] Chiesa, G., and V. Denicolò (2009): "Trading with a Common Agent under Complete Information: A Characterization of Nash Equilibria", *Journal of Economic Theory*, 144, 296-311.

[6] ——— (2012): "Competition in Non-linear Pricing, Market Concentration and Mergers", *Economic Letters*, 117, 414-417.

[7] Che, Y., and T. Chung (1999): "Contract Damages and Cooperative Investment", *The RAND Journal of Economics*, 84-105.

[8] Che, Y., and D. Hausch (1999): "Cooperative Investment and the Value of Contracting", *The American Economic Review*, 125-147.

[9] Chih-Chi (2005): "Incomplete Contract and Overinvestment", *Academia Economics Papers,* 33, 303-321.

[10] Edlin, A. S. and S. Reichelstein (1996): "Holdups, Standard Breach Remedies and Optimal Investment, *American Economic Review*, 86, 478-501.

[11] Farrell, J., and C. Shapiro (1989): "Optimal Contracts with Lock-in", *American Economic Association*, 79, 51-68.

[12] Hori, K. (2006): "Inefficiency in a Bilateral Trading Problem with Cooperative Investment", *Contributions to Theoretical Economics*, Vol 6.

[13] Kang, M., J. Mahoney and D. Tan (2007): "Why Firms Make Unilateral Investments Specific to Other Firms: The Case of OEM Suppliers".

[14] Klein, B., R. Crawford, and A. Alchian (1978): "Vertical Integration, Appropriable Rents and the Competitive Contracting Process", *Journal of Law and Economics*, 21, 297-326.

[15] Lewis, T., and H. Yildrim (2002): "Managing Dynamic Competition", *The American Economic Review*, 779-797.

[16] ——— (2005): "Managing Switching Costs in Multi-period Procurement with Strategic Buyers", *Quarterly Journal of Economics*, 101, 1-31.

[17] McLeod, W. B., and J. M. Malcomson (1993): "Investments, Holdup and the Form of Markets Contracts", *American Economic Association*, 83, 811-837.

[18] Nöldeke, G. and K. M. Schmidt (1995): "Option Contracts and Renegotiation: A Solution to the Hold-up Problem", *RAND Journal of Economics*, 26, 163-179.

[19] Rogerson, W. P. (1992): "Contractual Solution to the Hold-up Problem", *Review of Economic Studies*, 59, 777-793.

[20] Segal, I. (1999): "Contracting with Externalities", *The Quarterly Journal of Economics*, 114, 337-388.

[21] Schmitz, P. W. (2002b): "Simple Contracts, Renegotiation under Asymmetric Information, and the Hold-up Problem", *European Economic Review*, 46, 169-188.

[22] Valletti, T. (2000): "Switching Costs in Vertically Related Markets", *Review of Industrial Organization*, 17, 395-409.

[23] Williamson, O. (1979): "The Transaction-Cost Economics: the Governance of Contractual relations", *Journal of Law and Economics*, 22, 233-261.

[24] ——— (1983): "Credible Commitment: Using Hostages to Support Exchange", *The American Economic Review*, 73, 519-540.

[25] Whitford, J. (2005): "The New Old Economy: Networks, Institutions, and Organizational Transformation of American Manufacturing", *Oxford University Press*.