Productivity, Misallocation and Welfare in Services and Industry in Colombia, 2007-2019

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Productividad, Mala Asignación y Bienestar en Servicios e Industria en Colombia, 2007 - 2019*

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Resumen Este trabajo estudia el crecimiento de la productividad agregada (APG, por sus siglas en inglés) en los sectores de Servicios e Industria en Colombia de 2007 a 2019 aplicando la metodología de Petrin y Levinsohn (2012); en ella, el APG se descompone en dos efectos que lo explican: reasignación de recursos y eficiencia técnica. Utilizando datos de la Encuesta Anual de Servicios y la Encuesta Anual Manufacturera, se encuentra que para los dos sectores como un todo, el APG presenta un decrecimiento en promedio de -0.69 % en el periodo examinado, explicado porque el efecto de la asignación de recursos (negativo) domina sobre el de eficiencia técnica. Al estudiar el sector de Servicios de forma independiente, el APG cae en promedio un -0.96 %, explicado también por un efecto negativo de la asignación de recursos y mayor en magnitud al de eficiencia técnica. En el caso de la Industria, el APG es negativo e igual a -0.11 %, pero en este caso, el efecto que domina la explicación es el de la eficiencia técnica. También se calcula una estimación del Costo de Uso del Capital para estos sectores con un resultado de 16.2 % en promedio para el periodo de estudio. Finalmente, este documento provee una interpretación en términos de bienestar del APG.

Keywords: Productividad · Mala Asignación · Bienestar · Servicios· Industia · Eficiencia Técnica · Crecimiento · Costo de Uso del Capital

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Productivity, Misallocation and Welfare in Services and Industry in Colombia, 2007-2019*

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Abstract. This paper studies aggregate productivity growth (APG) in Services and Industry in Colombia from 2007 to 2019 applying Petrin and Levinsohn (2012) framework, where APG is decomposed in two effects: resource reallocation and technical efficiency. Using data from National Surveys of Services and Industry, the results for both sectors combined show that APG decreased on average -0.69% over the period examined, explained because the reallocation effect (negative) dominates over the technical efficiency one. When studying each sector independently, Services present a negative APG on average of -0.96%, explained also by the negative effect of reallocation of resources, bigger in magnitude compared to the technical efficiency effect. In the case of Industry, APG is negative and equal to -0.11%, but in this case, the effect that dominates the explanation is the technical efficiency. This paper also estimates the Cost of Use of Capital for these sectors with a result of 16.2% on average for the period of study. Finally, this document provides a welfare interpretation of APG.

Keywords: Productivity · Misallocation · Welfare · Services· Industry · Technical Efficiency · Growth · Cost of Use of Capital

* Paper to apply for the Master's degree in economics. All remaining errors are my own. rafaela.paez@urosario.edu.co

1 Introduction

The journey an economy sails towards development is carved through change; from agricultural societies to industrial, from people living in the countryside producing for themselves to a population primarily urban producing for big markets connected to the world economy, in this process of transformation the industry has played a major role and lately services are taking over as one if not the most important sector in many middle-income countries absorbing a huge amount of the workforce and contributing strongly in GDP growth.

Development processes differ in time and space, many developed countries follow a path of industrialization that foster productivity and employment to grow at relevant paces, the manufacturing sector played a key role in the economies of Europe and United States, this is also the case of Asian countries such as Korea or Taiwan where industrialization allowed them to catch up with western countries. Most advanced economies have gone to a new phase of development: post-industrial with the manufacturing sector decreasing in importance in terms of employment and income turning to service economies (Rodrik, 2016).

Conversely, developing countries especially in Latin America and Africa are not following the same way to progress but, "are turning into service economies without having gone through a proper experience of industrialization" (Rodrik , 2016, p. 2), phenomenon referred to as "premature deindustrialization" that can be seen in the low capacity of these countries to build strong manufacturing sectors to help economic growth and the increasing share of services in output and employment.

Productivity plays an important role in economic growth, and is useful to understand how the economic activity is moving and the relevant sectors inside. There are few productivity estimations for industry and no estimation in Colombia (to the extent of our knowledge) of a production function for services and what explains productivity growth. This paper tries to fill this void, first, it gives a measure of Aggregate Productivity Growth (APG from now on) for Services and Industry in Colombia for the period 2007-2019, second, it tests the hypothesis that productivity growth is explained primarily by misallocation of resources following the methodology proposed by Levinsohn and Petrin (2012), that disentangles aggregate productivity growth in two effects, technical efficiency and reallocation of resources, so, it quantifies how much each one accounts for productivity growth, third, it provides a welfare interpretation of APG, and finally, it calculates an estimation of the Cost of use of capital in the economy.

The amount of output a country produces is an important indicator of the performance of that economy and shows how resources are used, a relevant concept to understand growth and production is productivity, defined as "efficiency in production: how much output is obtained from a set of given inputs" (Syverson, 2011, p. 4). The most common way to measure it is the Total Factor Productivity (TFP) where output is a function of a set of observable inputs (usually capital and labor) and a factor-neutral shifter; TFP is essentially a residual: the output growth not explained by inputs (Syverson, 2011).

Studying and understanding the movements in productivity growth of services and industry is necessary and important in developing economies, in this sense Djellal and Gallouj (2008) state four reasons to do so, first because in developed countries, services are responsible for a big share of wealth and employment, also developing countries are moving towards a stronger service sector rapidly. Productivity has a strong link with economic growth (as one of its determinants) and living standards (through prices and earnings, increases in productivity lead to falls in product prices and hence an increase in consumers purchasing power also increases labor productivity and rise wages).

The second reason is that these sectors have an important influence on the rest of the economy, one mechanism is through the connections with other sectors that lead to a better division of labor and promotes the creation of jobs and therefore an increase in demand; other mechanism is through the link between productivity and firm competitiveness (ability to generate profits by selling its goods or services in a competitive market) because it decreases costs of production allowing firms to better compete in the market. Third, public services need to be properly accounted for to tax-payers, and finally, services pose problems regarding technical productivity measurement thanks to the nature of the output (not stockable) in these sectors.

There are several determinants of productivity growth like managerial practice, the quality of inputs, technology information and research, capacity of innovation, firm structure, competition, trade, government policy and regulations (Syverson, 2011). Economists have tried to study productivity and its determinants using production functions that relate inputs (mainly capital and labor) with output. Later advances in economics provide econometric techniques to identify the shares of inputs in total production and estimate productivity growth and lately, research based on plant level data is placing the idea that productivity growth can be explain, mostly, thanks to a combination of technical efficiency and how resources are allocated across firms. Determining what drives productivity growth in the economy and a specific sector will prove to be a big tool for policy making in the future.

According to World Bank Data, in Colombia the industry represented 31% of GDP in 2010 and move to 27% in 2018, while Services increased from 53.5% to 57.7% in the same period, in terms of employment industry kept a stable 20% of share of all employment during those years, while services shifted from 61.6% to 64.2%. This shows signs that as Rodrik (2016) explains the economy is specializing in services leaving industry behind and part of this phenomenon should be explain by differences in sector productivity growth. Even when these sectors are relevant and important for the economy little is known about the data available, the trends and functioning of services and industry in Colombia and how the productivity in this sectors has evolved.

An interesting implication of productivity growth is that affects the production of real goods to consume, this influence economic welfare as there are more or less goods available for the consumer, however, studies about productivity usually do not present the connection with welfare, so this paper also presents a welfare interpretation for productivity growth. In this paper, we use plant level data from 2007 to 2019 from Annual Survey of Services (EAS) and Annual Manufacturing Survey (EAM) provided by the Colombian Department of Statistics (DANE) in Colombia.

The rest of the paper is organized as follows, section two presents the literature review related to the topics of the study, section three explains the theory behind Aggregate Productivity Growth and Welfare, the next section describes the data, section five shows the estimation of technical efficiency, followed by section six that shares and discusses the results, and finally, the last section concludes.

2 Literature Review

This study relates to four big ideas in economics, first, aggregate productivity and economic growth, second (a more econometric issue), production function estimation derived from the previous one, third, misallocation of resources, and fourth, welfare.

2.1 Aggregate Productivity and Economic Growth

The first topic this paper addresses is productivity growth measurement and economic growth. A seminal paper in the matter and on of the most important works in productivity and growth is Solow (1957) where using Cobb-Douglas production functions is possible to reach an aggregate productivity level called total factor productivity that measures growth not explain by inputs of production. Domar (1961) follows this approach and emphasizes the importance of weighted measurements in the accountability of growth opening the door for inter and intra-sectorial studies.

Later, Hulten (1978) uses national income identity to connect final demand and production, he uses plant level data and shows how to aggregate technical efficiency shocks to the change in final demand in the neoclassical setting, and Basu and Fernald (2002) allow for the existence of markups meaning that growth can occur when resources are re-allocated from low-capacity firms to higher ones.

A different set of papers define productivity growth only based on technical efficiency residuals on plant level data, that is the case of Baily, Hulten and Campbell (1992) and Foster Haltiwanger and Krizan, (2001). Other models, those of creative destruction like Aghion and Howitt (1992) and Lentz and Mortensen (2008), use endogenous growth and establish the size of innovations, skilled labor force and productivity of research as determinants of productivity growth. Finally, Levinsohn and Petrin (2012, p.706) extend these previous works and define aggregate productivity growth "as the change in aggregate final demand minus the change in the aggregate expenditures on labor and capital", used to evaluate the components and find their contribution to productivity growth, this last approach is preferable because it allows the presence of jumps in APG, allows for entry and exit of firms and is able to "characterize the source of every plant's contribution to APG" Levinsohn and Petrin (2012, p.706).

2.2 Production Function Estimation

To quantify economic growth is very important the estimation of a production function, however from an econometric point of view this presents a basic problem to reach consistent parameters since a problem of endogeneity arises, given by the fact that "there are determinants of production that are unobserved to the econometrician but observed by the firm" (Ackerberg, et al, 2015, p.1). To deal with this issue, the first approaches come with the papers by Hoch (1955, 1962), Mundlak (1961, 1963), and Mundlak and Hoch (1965) using panel data with fixed effects, here the authors solve the problem assuming endogeneity is linked to a time invariant shock (unobservables are constant across time). However, according to Ackerberg, et al (2015) these approaches have proven inefficient in solving the endogeneity issues.

A second approach followed by Klein (1953), Griliches and Ringstad (1971), McElroy (1987) and Hall (1988) propose to use first order conditions of inputs parameters assuming that choosing inputs is a static process, this of course wrestles with the dynamic nature of capital or adjustment costs. An extension to this approach is the use of input prices as instrumental variables because they are exogenous, but this raises the issue that differences in quality of inputs cannot be reflected on their prices.

The last and most recent set of techniques developed by Olley and Pakes (1996) and Levinsohn and Petrin (2004) take a more structural approach using panel data allowing input endogeneity with respect to a time varying unobservable and the dynamic nature of inputs, estimating production functions in two stages using investment and intermediate inputs to control for unobservables. Later Ackerberg, Caves and Frazer (2006) identify a collinearity problem in these methods and propose a correction for the estimators. Wooldridge (2009, p.1) expands the literature showing that these methods "can be implemented by specifying different instruments for different equations and applying generalized method of moments". Finally, Wexler and De Loecker (2016) recognize the importance of capital measurement error and its impact on estimation of production functions using investment expenditures to create a "hybrid IV-Control function approach that instruments capital with (lagged) investment, while relying on standard intermediate input demand equations to offset the simultaneity bias".

2.3 Missallocation

The third important topic of literature for this paper is misallocation of resources and the relationship with productivity. Reallocation of inputs means moving resources from units of production that barely or do not use them to units that use them properly, and this sole fact of taking resources from idle hands and put them to better use generates increases in productivity growth.

Research have approach misallocation in two forms, following Restuccia and Rogerson (2013) the first, the direct approach tries to obtain direct measures to find the effect on productivity growth, the second, the indirect approach, deals with the difficulty of measuring multiple possible sources of misallocation so it focuses on the net effect of all underlying factors without determining the individual effect of each factor.

For the first approach, policy barriers are recognized as a source of misallocation, for example, Hopenhayn and Rogerson (1993) show how firing taxes disturb the allocation of resources and affects negatively productivity. Lagos (2006) study the effects on productivity of policies like employment protection and unemployment insurance. Leal (2010) studies the connection of policy to informality with heavy taxation and huge costs for business as responsible for sending workers to informality.

Banerjee and Duflo (2005) provides evidence of credit constraints and financial market imperfections leading to capital misallocation. Adamopoulos and Restuccia (2011) study productivity gaps because of misallocation in agriculture in poor economies and Epifani and Gancia (2011) present trade barriers affect competition and markups, then the difference in markups produces a poor allocation of resources.

On the other hand, the indirect approach, Restuccia and Rogerson (2008, p. 707) study policy distortions and aggregate productivity and propose a growth model with heterogeneous establishments, they find that reallocation in firms with different productivity accounts in cross-country differences in output per capita, and show that policies that develop differences in prices faced by the firm "can lead to sizeable decreases in output and measured total factor productivity (TFP) in the range of 30 to 50 percent". Hsieh and Klenow (2012) argue that policies that increase distortions on large plants moves productivity across countries, suing data from India, Mexico and the US.

2.4 Welfare

Since Smith published the Wealth of Nations in 1776, economics has tried to established the conditions to create wealth and bring welfare to the individuals in the society. A way to achieve wellness is through markets and individual self-interest because produces an efficient economic organization where people can access all type of good and services at affordable prices.

The study of welfare led to the Fundamental Theorems of Welfare Economics proposed by Kenneth Arrow (1951) and Gerard Debreu (1959). "The first theorem states that (under certain conditions) the competitive economy is always Pareto efficient; the second theorem says that every Pareto efficient allocation can be attained through the price system. All the government needs to do is engage in some initial lump sum transfers (taxes and subsidies)" (Stiglitz, 1991, p. 3).

To find the welfare of an individual economists construct utility functions, mathematical objects that quantify the well-being or the happiness that a person gets from consumption. The welfare of a community is seen as the sum of of the utilities of all constituent individuals, what implies interpersonal comparability of utilities. (Lange, 1942)

Some works like Weitzman (1976) started to relate the Welfare analysis to the macroeconomic results of a country, meaning that macro variables could speak about welfare in a society. He arguments that Net National Product is a better concept than Gross National Product to reflect welfare as "in theory it is a proxy for the present discounted value of future consumption" (1976, p.2), and the ultimate end of economic activity is consumption.

Basu and Fernald (2002) continue this path and demonstrate that "the change in utility is proportional to the change in the modified Solow residual" (2002, p. 971). This permits welfare analysis without a general equilibrium model with consumer and producer sides, but is needed only the production side and a correct calculation of productivity growth; the result holds when there are economic profits, when consumers preferences do not change, when the ratio of factor prices equals the consumer's marginal rate of substitution and consumers face a single prices for each good; even thou is not a general result, it holds in a set reasonable conditions. Then, the authors explain the interpretation of this result, welfare rise when inputs rise, because the labor-leisure choice is affected since the utility of the goods outruns the dis-utility of producing them and technology improvements rise welfare because more good can be produced with a set of given inputs.

3 Theory

3.1 Aggregate Productivity Growth

We follow the Petrin and Levinsohn (2012) framework for the definition of aggregate productivity growth (APG). To begin, consider a continuous-time setting that has N(t) potential goods indexed by i. Every product i may have a different technology, expressed by the production function for good i as:

$$Q_i = Q_i(X_i, M_i, \omega_i) \tag{1}$$

With $X_i = (X_{i1}, ...X_{iK})$ as the vector of primary inputs K used in production of plant i, Where $M_i = (M_{i1}, ...M_{ij})$ is the vector of intermediate inputs (Final good used as input in the production of itself or another product) used in production at plant i and ω_i is the level of plant $i's$ technical efficiency.

$Z_i = (X_i, M_i, \omega_i)$ gathers primary and intermediate inputs and productivity shocks for firm i. Output is determined by the production technologies $Q = (Q_1(Z_1), ..., Q_N(Z_N))$. Prices are assumed to be uniquely determined by Q, given as $P = (P_1(Q), ..., P_N(Q))$, and similarly for primary input costs $W = (W_1(Z), ..., W_K(Z))$. Fixed and sunk costs for all i are assumed to be deterministic given Z and its past values, collected in vector $F = (F_1, ..., F_N)$.

So, F_i denotes the sum of all fixed and sunk costs paid by plant i that we discount from the production:

$$Q_i = (X_i, M_{i,i}) - F_i \tag{2}$$

Now, Let's designate final demand as Y_i, so, the total amount of output from plant i that goes to final demand Y_i is

$$Y_i = Q_i - \sum_j M_{ij} \tag{3}$$

With the part after the minus being the total amount of $i's$ output that serves as intermediate input within the plant and at other plants. And the change in aggregate final demand is:

$$\sum_{i=1}^{N} P_i dY_i \tag{4}$$

So, now let us define APG at time t as the change in aggregate final demand minus the change in aggregate expenditures on labor and capital at time t:

$$APG(t) = \sum_{i=1}^{N} P_i dY_i - \sum_i \sum_k W_{ik}(t) dX_{ik}(t) \tag{5}$$

When using plant level data, is not possible to observe the output that goes to final demand, so in the previous equation we do not have dY_i, to solve this we need to make use of national accounting identities, from where we know that aggregate final demand is equal to aggregate value-added:

$$\sum_i P_i Y_i = \sum_i VA_i \tag{6}$$

With

$$VA_i = P_i Q_i - \sum_j P_j M_{ij} \tag{7}$$

And

$$dVA_i = P_i dQ_i - \sum_j P_j dM_{ij} \tag{8}$$

So, we finally replace (7) in (5) to get APG:

$$APG = \sum_i dVA_i - \sum_i \sum_k W_{ik} dX_{ik} \tag{9}$$

Now, multiplying dY_i by P_i and aggregating over all i we get an expression for the change aggregate value-added, then totally differentiating Q_i and normalizing $Q_i/ = 1$ we are able to decompose APG in a technical efficiency (TE) term, a reallocation term (RE) and a costs term (F):

$$TE = \sum_i P_i d\omega_i \tag{10}$$

$$RE = \sum_i \sum_k (P_i \frac{\delta Q_i}{\delta X_k} - W_{ik}) dX_{ik} + \sum_i \sum_j (P_i \frac{\delta Q_i}{\delta M_j} - P_j) dM_{ij} \tag{11}$$

$$F = -\sum_i P_i dF_i \tag{12}$$

Then,

$$APG = TE + RE + F \tag{13}$$

Finally, the growth rate formulation follows as:

$$\begin{aligned} APG_G = & \textstyle\sum_i D_i dln\omega_i + \sum_i D_i \sum_k (\epsilon_{ik} - s_{ik}) dlnX_{ik} \\ & + \textstyle\sum_i D_i \sum_k (\epsilon_{ij} - s_{ij}) dlnM_{ij} - \sum_i D_i dlnF_i \end{aligned} \tag{14}$$

Where ϵ_{im} denotes output elasticities with respect to input m, ϵ_{ik} and ϵ_{ij} are the elasticities of output with respect to each potential $K+N$ input, and s_{ik} and s_{ij} are the respective revenue shares for each input: $s_{ik} = \frac{W_{ik} X_{ik}}{P_i Q_i}$, $s_{ij} = \frac{W_{ij} M_{ij}}{P_i Q_i}$, $dln\omega_i$ and $dlnF_i$ are growth rates in technical efficiency and fixed costs, and the Domar weight is $D_i = \frac{P_i Q_i}{\sum_{i=1}^{N} VA_i}$.

3.2 Welfare

Following Basu and Fernald (2002) we will present the derivation of the interpretation of productivity as welfare: Suppose a large number of identical households solving the problem:

$$\begin{aligned}
&maxU = \textstyle\sum_{s=0}^{\infty} \beta^s u(C_{1,t+s}, C_{2,t+s}, ..., C_{N-1,t+s}, \overline{L} - Lt + s)\\
&s.t.\\
&A_{t+1} = A_t + P_{Lt}L_t + P_{Kt}K_t - \delta P^I_{t+1}K_t +\\
&(P^I_{t+1} - P^I_t)K_t + r_tB_t + \varPi_t - \textstyle\sum_{i=1}^{N-1} P_{i,t}Ci,t
\end{aligned} \tag{15}$$

Where K is capital, L labor, P^I price of investment goods, P_{Kt} rental price of capital, and P_{Lt} real wage. We assume for any input J and firm i, $P_{Jit} = P_{Jt}$. $\overline{L}$ is each consumer's per period endowment of labor. A are household assets, B are bonds, capital depreciates at rate δ, r is the rate of return on bonds, and $\varPi$ are pure profits that are rebated lump-sum to consumers.

In equilibrium arbitrage requires households to be indifferent between holding capital and holding a bond, so: $(1+r_t)P^I_t = (1+r_t)P_{Kt} + (1-\delta)P^I_{t+1}$. Let λ_t be the shadow value of assets at time t, the consumer's first order conditions are:

$$\begin{aligned}
&u_{Lt} = \lambda_t w_t,\\
&u_{Ci,t} = \lambda_t P_{i,t}, i = 1, ..., N-1,
\end{aligned} \tag{16}$$

And the Euler equation:

$$\lambda_t = \beta(1 + r_t)\lambda_{t+1} \tag{17}$$

Suppose a temporary shock at time t that lasts one period and affects output, consumption, and labor supply, this affects lifetime utility by:

$$\begin{aligned}
&dU = \textstyle\sum_{i=1}^{N-1} u_{Ci,t}dC_{i,t} - u_{Lt}dL_t + \beta\lambda_{t+1}dA_{t+1}\\
&= \lambda_t(\textstyle\sum_{i=1}^{N-1} P_{i,t}dC_{i,t} - P_{Lt}dL_t) + \beta\lambda_{t+1}dA_{t+1}
\end{aligned} \tag{18}$$

From national income identity, the change in the value of consumption can be expressed in terms of changes in aggregate output and investment:

$$\sum_{i=1}^{N-1} P_{i,t}dC_{i,t} = P^C_t C_t(\sum_{i=1}^{N-1} \frac{P_{i,t}C_{i,t}}{P^C_t C_t}\frac{dC_{i,t}}{C_{i,t}}) = P^V_t V_t \frac{dV_t}{V_t} - P^I_t dI_t \tag{19}$$

Using Euler's equation and the fact that the only asset in this economy is capital we can write (19) as:

$$\begin{aligned}
&dU = \lambda_t(P^V_t V_t \tfrac{dV_t}{V_t} - P^I_t dK_t - P_L t dL_t) + \beta\lambda_{t+1}dA_{t+1}\\
&= \lambda_t(P^V_t V_t \tfrac{dV_t}{V_t} - P^I_t dK_t - P_L t dL_t \tfrac{(1-\delta)P^I_{t+1}}{1+r_t} dK_t)
\end{aligned} \tag{20}$$

Rearranging the equation in logarithmic form and pulling nominal output $P^V V$ outside the brackets we have:

$$dU = (\lambda_t P_t^V V_t)(dv_t - \frac{P_{Kt} K_t}{P_t^V V_t} dk_t - \frac{P_{Lt} L_t}{P_t^V V_t} dl_t) = (\lambda_t P_t^V V_t) dp_t \tag{21}$$

Equation (21) "says that the change in utility is proportional to the change in the modified Solow Residual dp_t" (Basu and Fernald, 2002, 971). For the purpose of this paper, what the authors call modiefied Solow Residual for us is the derivation of APG (previous section), then the welfare change in any period is equivalent to the growth rate of productivity (APG); this allows APG to gain a new interpretation as a measure of welfare in the economy, and as explained before, it is possible to obtain the loss of welfare using the difference between APG and technical efficiency without reaching to the consumer side of the economy like is usual in general equilibrium models.

4 Data

We use data from the Colombian Department of Statistics DANE (Departamento Administrativo Nacional de Estadistica), specifically from the Annual Survey of Services (Encuesta Anual de Servicios-EAS) and the Annual Manufacturing Survey (Encuesta Anual Manufacturera-EAM) for the period 2007-2019. Data are unbalanced panels (for services) per year including plant level data classified at 4-digit level, we take sectors as firms, so, we aggregate companies at each 4-digit level and use it as our unit of study. To this analysis we use labor (number of workers) and the next variables deflated by a 2-digit GDP deflator: gross output, value-added (we stumble with some firms reporting negative value-added data, those firms were removed from the data set), intermediate consumption, salaries (cost of labor) and capital (constructed using the perpetual inventory method).

The Services sectors and type of companies included in the study are as follows: Firms with 40 or more employees or annual income equal or superior to 3.000 million pesos for: Storage and complementary activities of transportation, mail and messenger services, lodging, food and beverage service activities, telecommunications, travel agencies, tour operators, reservation services, human health care activities and specialized medical care.

Companies with 75 or more employees or annual income equal or superior to 3.000 million pesos for sectors: Development of computer systems (planning, analysis, design, programming, testing), computer consulting and related activities, information services activities, real estate activities, rental and leasing activities, legal and accounting activities, business management activities, management consulting, architectural and engineering activities, tests and technical analysis, scientific research and development, market studies and conducting opinion surveys and other professional, scientific and technical activities.

Firms with 40 or more employees or annual income equal or superior to 2.000 million pesos for sectors: Cinema, video and television program activities, programming, transmission and/or dissemination activities and activities of news agencies and other personal service activities. Companies with 20 or more employees or annual income equal or superior to 1.000 million pesos for sector: Higher education.

The Industry sectors and type of companies included in the study are firms with 10 or more employees for the following sectors: Production of food products, beverage production, manufacture of textile products, clothing making, textile products, leather goods tanning and manufacturing production of wood and cork products, paper, cardboard and its products, printing and copying, oil refining and fuel mixing, chemical substances and products, pharmaceuticals and medicinal chemicals, rubber and plastic products, other non-metallic mineral products, basic metallurgical products, metal products, except machinery and equipment, computer, electronic and optical products, electrical appliances and equipment, machinery and equipment, motor vehicles, trailers and semi-trailers, other types of transport equipment, furniture, mattresses and bed bases, and rest of the industry.

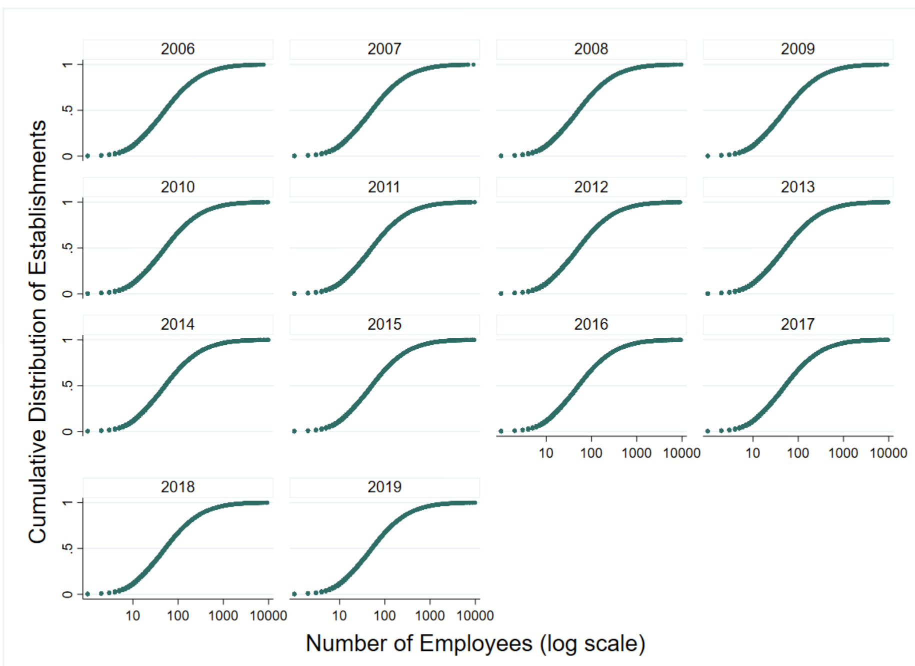
Table 1: Total Firms and Workers

Year	Firms	Workers (Millions)
2007	10.917	1.6
2008	10.328	1.7
2009	10.974	1.8
2010	11.877	2.0
2011	11.968	2.0
2012	12.063	2.1
2013	12.261	2.2
2014	12.372	2.3
2015	12.231	2.3
2016	12.267	2.3
2017	11.902	2.5
2018	12.202	2.6
2019	12.264	2.5

Table 1 presents the total amount of firms we use for each year and the total amount of workers both sectors employ. Firms move around 10.000 to 12.000 with the first 3 years with the lowest quantities and stabilizing in 2012 and beyond around 12200; in 2007, 33% of firms are from services and 69% from industry, this composition changes over time to reach 43% of services and 57% of industry in 2017 and the years after. In terms of workers, we see a rise from 1.6 millons to 2.5 for the whole term with no periods decreasing the number of hired employees. The composition of employment shows a different pattern, as in 2007 62% of workers went to the services firms and 38% to the industrial firms and the breach widened with the years to finish on 2019 with 76% of the workforce going to services and the remaining %24 to industry. This could be explained by the nature of the surveys we use that include bigger firms in the services sectors as described before, but also to the economic process of both sectors, where services tend to need more people to grow while industry moves to be more capital intensive leading to maintain or decrease the job positions available.

Figure 1 depicts the distribution of establishments by employment per year for our data, the graph shows how labor is allocated in companies since every point is a firm, the vertical axis is the proportion of firms and the horizontal the number of workers (in logarithmic scale). The first insight from figure 1 is that labor allocates virtually in the same way for every year of the period of study, as the graphs have similar shape. The second insight is that the majority of firms are small or medium in terms of number of employees, around half of the firms studied have 100 or less employees, almost the other half of firms have between 500 and 1000 people of labor force, and a really small proportion of firms employ over 1000 people.

Fig. 1: Distribution of Establishments by Employment per Year

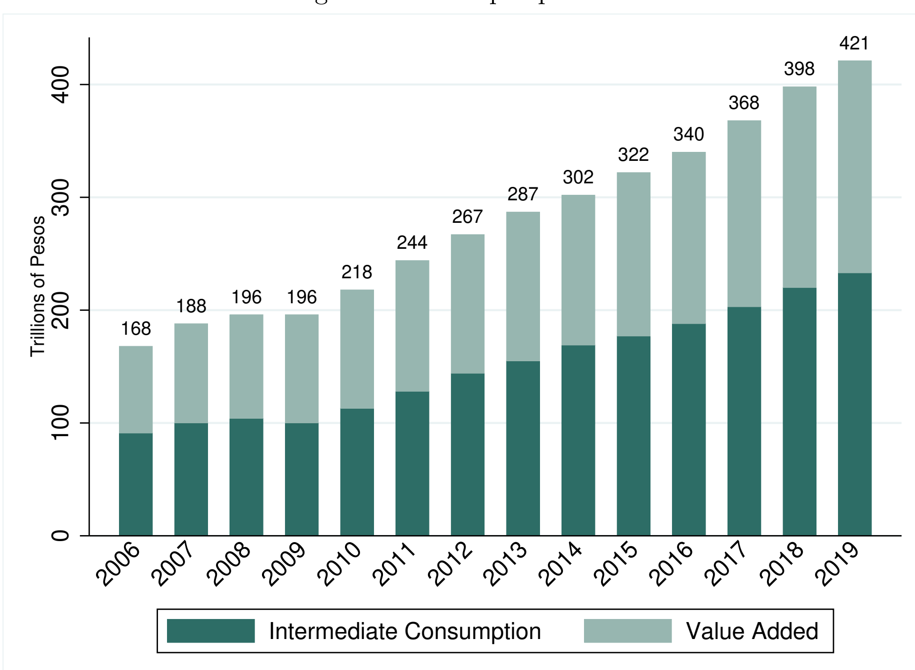


In Appendix B, we present the distribution only for services and only for industry, on one hand, services present a similar behaviour to the on exposed here with almost half of firms with 100 workers or less and the other half virtually under 1000. For industry, half of the firms are below 100 employees, closer to 50 and the other half is under 1000 employees, showing that on average firms in industry, in general, are smaller than firms in services; again, this may come as a result of the type of companies present in each survey, but also may indicate the nature of each type of activities, services are in general more labor intensive, while industry relies more on capital, so they may need less people to produce.

Figure 2 presents gross output per year for the whole economy studied here. It shows its decomposition into valued-added and intermediate consumption in Colombian currency (current prices). The numbers on top of the bars are the total gross output in trillions of pesos, the light green bars are the part that is value-added, and the dark green bars represent the part corresponding to intermediate consumption.

In the graph we have a clear tendency with a steep increase of output from 188 trillions in 2007 to 421 in 2019. The composition is very steady for the 13 years, on average 46% of gross output corresponds to value-added and 54% to intermediate consumption. In terms of sectors, services provide 28% of gross output and industry 72% in 2007, that composition changes over time to a 41% and 59% respectively in 2019. This movement, along with the composition of workers between sectors, state a continuous increase in the relative importance of services in the economy, participating more and more in the total output produced and hoarding more and more labor force.

Fig. 2: Gross Output per Year



5 Estimation

We use a Cobb-Douglas production function to estimate technical efficiency, the estimatation is done separately for each 2-digit industry code using ordinary least squares, fixed effects and the Wooldridge (2009) variant of Levinsohn and Petrin (2003) estimator. The latter estimator is the preferred one because: it "corrects for the simultaneous determination of inputs and technical efficiency, does not maintain constant returns to scale or require cost minimization without input adjustment costs to identify production function parameters, is robust to the Ackerberg, Caves, and Frazer (2006) criticism" (Petrin and Levinsohn, 2012, p.718).

Therefore, the estimate of technical efficiency is

$$ln\omega_{it}^{\nu} = ln(VA_{it}) - (\beta_j^{\nu} + \epsilon_{j\rho}^{\nu} lnL_{it}^{\rho} + \epsilon_{jN\rho}^{\nu} lnL_{it}^{N\rho} + \epsilon_{jK}^{\nu} lnK_{it}) \tag{22}$$

Where β_j^{ν} and ϵ_j^{ν} denote the estimated intercept and elasticities of value-added with respect to the inputs in industry j, L_{it}^{ρ} are permanent workers, $nL_{it}^{N\rho}$ are temporal workers, K_{it} is capital and VA_{it} is value-added.

From equation (9) we can obtain APG; from the previous estimation we have technical efficiency; so the only term missing for the decomposition is the reallocation one, to get it we just need to subtract APG minus the estimated technical efficiency; in this case reallocation comes as a residual.

6 Results

Table 2 shows the comparison for aggregated growth rates for value-added, APG and input costs (capital and labor, given the characteristics of the survey we can separate and calculate separately the cost of temporal and permanent labor) with the formula presented in equation (9) per year for the period of study (2007-2019) for our economy (Services + Industry). Value-Added grew 3.5 on average with only two years (2014 and 2016) growing at negative rates; the cost of labor grew at the same rate on average, 0.94% for both types of labor, but the cost for temporal labor was more volatile; capital costs grew on average 2.38% with 2016 and 2017 presenting growth rates over 7%; finally, APG grew at a negative rate of -0.69% on average driven especially by the negative results of the last 4 years, with 2017 as the worst year fro APG with a decrease of -8.37%.

The results in table 2 indicate that APG is not tracking particularly well the changes in value-added, this means that the expenditures in inputs are relatively big to changes in valued-added, so an important portion of what is produced goes to pay for inputs.

Table 2: Comparing the Rate of Growth in Aggregate Value, Added with APG, 2007-2019

APG = Change in Value Added - Change in Input Costs					
Year	Value Added	Temporal Labor	Permanent Labor	Capital	APG
2007	5.02	1.32	1.42	0.37	1.91
2008	0.07	1.30	-0.09	0.5	-1.64
2009	2.88	1.52	0.17	1.76	-0.57
2010	7.12	1.48	2.16	2.00	1.48
2011	4.20	0.80	0.52	3.74	-0.86
2012	5.96	0.03	1.26	2.23	2.44
2013	8.64	2.64	1.35	-2.31	6.96
2014	-1.66	-0.53	1.21	1.53	-3.87
2015	4.94	0.58	0.85	0.02	3.49
2016	-0.70	-5.44	1.02	7.59	-3.87
2017	3.67	2.63	1.80	7.61	-8.37
2018	4.96	6.55	0.15	1.99	-3.73
2019	1.27	-0.7	0.36	3.95	-2.34
Mean	3.57	0.94	0.94	2.38	-0.69
Standard Deviation	3.08	2.66	0.68	2.82	3.97

Table 3 offers the decomposition of APG across estimators.To compare, we present 3 different methods: OLS, Fixed Effects and the Wooldridge Levinshon-Petrin estimator. APG is in column 2, technical efficiency estimations are in columns 3-5, reallocation effects are in columns 6-8. Following equation (13), for each estimator, APG is the sum of technical efficiency and reallocation, then, column 2 equals column 3 + column 6; column 2 equals column 4 + column 7; and column 2 equals column 5 + column 8.

In the table we can see for each year the part of APG explained thanks to the change in technical efficiency and the part due to reallocation of resources. Positive numbers in technical efficiency indicate firms use better technologies or use the resources they already have in a more efficient way, and positive numbers in reallocation imply that resources in the economy are moving from less productive hands or from idle use to more productive firms, this is consistent with how a good economy should work.

We present three estimators to compare, but focus on the Wolldridge - LP as this is the preferable one, following the reasons presented in section five. We find that the two effects that explain APG on average, go in different directions. While on average technical efficiency display a positive result of 0.61%, meaning that firms are able to produce more using better technologies, the reallocation effect is on average negative with -1.29% meaning that resources are moving from activities with low distortions to activities with large distortions.

Throughout the years technical efficiency moves mainly between -3.00% and 3.00%, with some exceptions in 2012 and 2013 with important results over 8.00% and a relevant decrease in 2016 with -9.91%. The results jump from periods of negative numbers to positive ones, we do not observe continuous improvement nor continuous decay, indicating this sectors struggle to apply and implement better technologies even though in the end, on average this component has a positive grow, but quite small with a 0.61%.

In terms of reallocation, seven years presented negative values (with 2012 and 2017 as especially tough years) and the resting six years showed positive ones, in general the negative numbers in magnitude are larger, therefore on average the reallocation effect is negative an equal to -1.29%. These results indicate that these sectors are being constantly unable to locate resources properly.

Table 3: Comparing the Decomposition of APG across Estimators, 2007-2019

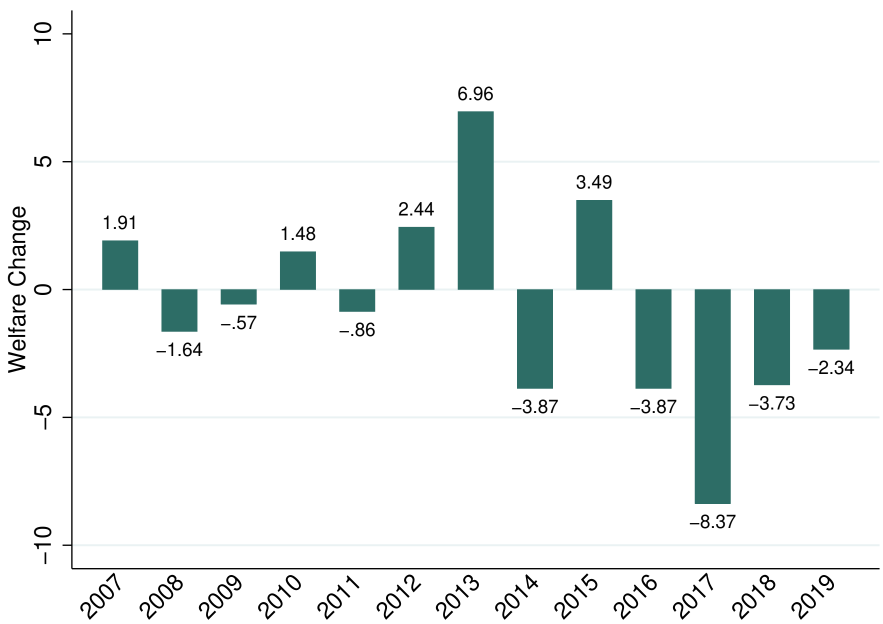
		APG Technical Efficiency			APG Reallocation		
Year	APG	OLS	Fixed Effects	Wooldridge - LP	OLS	Fixed Effects	Wooldridge - LP
2007	1.91	-1.24	-1.78	-0.11	3.15	3.69	2.03
2008	-1.64	-4.13	-3.64	1.37	2.49	2.00	-3.01
2009	-0.57	2.68	4.04	-0.98	-3.25	-4.61	0.41
2010	1.48	-1.79	-2.07	-2.00	3.27	3.55	3.48
2011	-0.86	-1.21	-0.10	-2.08	0.35	-0.76	1.22
2012	2.44	0.71	-1.02	10.81	1.73	3.46	-8.37
2013	6.96	2.78	4.66	8.65	4.18	2.30	-1.69
2014	-3.87	-4.79	-6.24	-1.03	0.92	2.37	-2.84
2015	3.49	1.42	0.67	0.61	2.07	2.82	2.88
2016	-3.87	-3.01	-2.48	-9.91	-0.86	-1.39	6.04
2017	-8.37	-0.86	0.45	2.66	-7.51	-8.82	-11.03
2018	-3.73	-1.11	0.51	0.79	-2.62	-4.24	-4.52
2019	-2.34	0.10	-0.49	-0.91	-2.44	-1.85	-1.43
Mean	-0.69	-0.80	-0.58	0.61	0.11	-0.11	-1.29
Standard Deviation	3.97	2.35	2.91	5.05	3.33	3.91	4.79

The results presented here have some interesting policy implications, in the short term, policies focused on understanding and correcting misallocation of resources will impact better APG rather than policies focused on technical advance. Nonetheless, the results for the whole economy might not be the same for each sector, or might not reflect what happens within in each specific sector. Therefore, applying this method for each sector will prove to be beneficial because it will show the source of productivity growth in a sector specific way.

We take a step forward in this direction and present the results only for services and only for industry (Appendix B). In services we find an APG on average of -0.96%, explained by 3.21% of technical efficiency and -4.17% by resource reallocation. In industry APG on average is equal to -0.11%, explained by -2.50% of technical efficiency and 2.39% by reallocation. When studying each sector, we find that the explanation for the poor APG performance stays the same in services, but switches in industry (the negative effect comes from the technical efficiency part), then, a good policy in services should focus on correcting poor resource allocation, but the proper policy in industry should target technical efficiency issues.

Lastly, the welfare derivation of Basu and Fernald (2002) states that the change in productivity (APG) is proportional to the utility change, so positive numbers in APG indicate a gain in welfare for the society in that exact proportion, while negative results represent a welfare loss. Figure 3 depicts the gains and losses in welfare for every year, the economy has experienced 5 years of improvement in welfare and 8 of losses that leads to say that on average, society is worse off during the years we studied. In appendix B are the welfare graphs for services and industry separately.

Fig. 3: Welfare Change per Year



7 Conclusion

This article provides an application of the methodology proposed by Petrin and Levinsohn (2012) of aggregate productivity growth (APG) and a decomposition between technical efficiency and reallocation of resources for Colombian sectors of Services and Industry from 2007 to 2019. This procedure is interesting because it uses aggregate data to track final demand, is robust to jumps in APG and is comparable across industries and time. We find that, while value-added grew 3.57% on average, productivity decreased on average -0.69% explained by an increase in technical efficiency of 0.61% on average and a decrease of -1.29% on average in reallocation of resources, meaning that the effect of reallocation dominates over the technical one, implying that even when firms are more efficient and innovative (not by much), a poor movement of resources among activities does not allow productivity to grow. We also present an interpretation of welfare for APG following Basu and Fernald (2002) that states that: welfare change is proportional to productivity growth, we observe 5 years increasing welfare and 8 decreasing it.

We also produce results for every sector individually, services present a similar behaviour to the one of all the economy, but in industry the explanation changes and the technical efficiency effect is negative, and dominates over the reallocation effect that in this case is positive. This change means that for manufacturing sectors resources move from poor to better uses, but the issue is the technological advance. The difference in results per sector has policy implications as services need policies that understand and correct the bad allocation of resources, while industry needs a policy that promotes innovation, the use of new and better technologies and more efficient production processes.

Finally, this paper also provides an estimation of the Cost of Use of Capital (the rent that pays a company to add a unit of capital into the production process) for the economy and period studied moving between 12% and 21% with an average of 16.2%. The years 2009, 2012 and 2017 have the biggest cost, and the years 2015, 2016 and 2019 have the lowest cos for all the economy.

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A Appendix A: Cost of Use of Capital

Cost of use of capital can be interpreted in different forms, though equivalent in equilibrium, the simpler way to understand it is as the rent that pays a company to add a unit of capital into the production process, this rent, of course, depends on various factors such as tax rates, interest rates, the price of the capital good itself and depreciation rates.In this interpretation, the estimation comes from the idea of profit maximization, where the company adds capital to the point the marginal productivity of capital equals the cost of using it, hence, marginal productivity of capital represents the cost of use of capital (Steiner and Soto, 1998).

In a different approach, the cost of use of capital may be associated with the minimum return that a capital good rents besides taxes and the minimum return required by shareholders and lenders; here the cost of use of capital depends on composition of investment in type of assets, economic activity, tax structure and way of funding. Le us show the derivation of the first approach following Rhenals (2005):

Profit maximization problem of the company can be expressed as:

$$MaxV_t = \int_t^\infty \mathrm{e}^{-rt}((1-\tau)(p_t F(K_t, L_t) - w_t L_t) - (1 - A_t + t_v) q_t I_t) d_t \quad (23)$$

Subject to

$$\dot{K}_t = I_t - \delta K_t \quad (24)$$

Where, τ: Income Tax rate, p_t: Price of the good produced, q_t: Price of the capital good, δ: Depreciation rate, r: Discount rate, F: Production Function, w_t: Salary, A_t: Tax discounts granted to the investor, t_v: Value added tax, K_t: Capital, L_t: Labor, I_t: Investment.

The resulting Hamiltoninan is:

$$H_t = \mathrm{e}^{-rt}((1-\tau)(p_t F(K_t, L_t) - w_t L_t) - (1 - A_t + t_v) q_t I_t + \beta_t (I_t - \delta K_t)) \quad (25)$$

The first order conditions with respect to I and L are:

$$\frac{\delta H_t}{\delta I_t} = \mathrm{e}^{-rt}(-(1 - A_t + t_v) q_t + \beta_t) = 0 \quad (26)$$

$$\beta_t = (1 - A_t + t_v) q_t \quad (27)$$

$$\frac{\delta H_t}{\delta L_t} = \mathrm{e}^{-rt}(1-\tau)(p_t F'_L - w_t) = 0 \quad (28)$$

$$F'_L = \frac{w_t}{p_t} \quad (29)$$

Then, by Euler's rule, differentiation with respect to K requires marginal cost and marginal productivity of capital to be equal:

$$-\frac{\delta H_t}{\delta K_t} = \frac{d(\mathrm{e}^{-rt}\beta_t)}{dt} \tag{30}$$

$$-\mathrm{e}^{-rt}((1-\tau)(p_t F'_K - \beta_t \delta) = -r\mathrm{e}^{-rt}\beta_t + \mathrm{e}^{-rt}\dot{\beta}_t \tag{31}$$

$$\dot{\beta}_t = (r+\delta)\beta_t - (1-\tau)p_t F'_K \tag{32}$$

Replacing (27) in (32):

$$\dot{\beta}_t = (r+\delta)(1-A_t+t_v)q_t - (1-\tau)p_t F'_K \tag{33}$$

With taxes unchanged, equation (20) turns to:

$$\dot{\beta}_t = (1-A_t+t_v)\dot{q}_t \tag{34}$$

Matching (33) and (34):

$$F'_K = C_u = \frac{(1-A_t+t_v)}{(1-\tau)}((r+\delta-\frac{\dot{q}_t}{q_t})\frac{q_t}{p_t}) \tag{35}$$

Finally, we assume price of produced and capital goods grow with inflation, then $\frac{\dot{q}_t}{q_t} = \pi$, and relative price of the capital good with respect to product is assumed to be 1, so $\frac{q_t}{p_t} = 1$

Then we get the final equation of the cost of use of capital:

$$C_u = \frac{(1-A+t)}{(1-\tau)}(r+\delta-\pi) \tag{36}$$

Where, A: Tax discounts granted to the investor, t: Value added tax, τ: Income tax rate, r: Nominal interest rate, δ: Depreciation rate, π: Inflation rate.

This expression is calculated for every type of asset j: Land, Structures and buildings, Machinery and equipment, Transport, Office and computer equipment, Other Fixed Assets. Then to get the total cost we weight each asset by the participation in the total investment of the company. And to get the total cost for the economy we weight the by the participation of each sector i on the total investment of the economy:

$$C_u = \sum_{i=1}^{n} \psi_i C_u^i \tag{37}$$

The data we use in this section comes from Direccion de Impuestos y Aduanas Nacionales (DIAN) where we find implicit rates per sector at 4-digit level for A and τ; for t, we use the general rate of the value-added tax for every year; from Banco de la Republica we get r with the series of fixed term deposit (DTF for the acronym in spanish) following Steiner and Soto (1998) and π with the series of inflation; and we use the following depreciation rates for each type of asset: Land 0%, Structures and buildings at 5%, Machinery and equipment 10%, Transport 20%, Office and computer equipment 10%, Other Fixed Assets 10%.

There have been few studies about the cost of capital in colombia, many of them are old but serve as a reference point, Fainboim (1990) study investment, taxes and cost of use of capital in Colombia from 1950 to 1987 using a neoclassical model to determine relative prices of capital goods and income tax as determinants of cost of use of capital finding that cost was around 12%-15%. Steiner and Soto (1998) are able to find a cost per sector in the economy in 1996 with industry at 32.7% and services at 30.3%. Rhenals (2005) estimates the cost between 1997-2003 decreasing from 39.5% to 25.1%. Botero, Ramirez and Palacio (2007) calculate the cost per quarter from 1990 to 2007, it moves from 27.69% to 10.01%, a relevant decline for that period.

Table 4 presents our the results for the cost of use of capital for the period of study of this paper, the first column shows the total result for the economy, while the second and third for Services and Industry, respectively.For the total, on average we find a cost of 16% with 2009 with the higher value with 20% and the lowest in 2015 with 10%. We observe a general decline in the results compared to the papers discussed before (with different time frames), this suggests an effort in the country the last few decades to reduce the cost of capital, especially via taxes, to foster capital accumulation and competitiveness.

For services and industry, the average remains on %16 for both sectors, the first one experienced in 2012 the most expensive year with %22 and 2015 as the cheapest with 10%, the later had a particularly rough year on 2017 when the cost went up to 28% (this peak is in part explained by change of value-added tax from 16% to 19%)just to years after getting a 9% in 2015, the lowest value.

Figure 4 graphs an index for the cost of use of capital, with base 2007. The 3 results move similarly all the time as the impacts of factors that explain the cost hits everyone in the same direction, but different magnitude. Until 2013, the cost moves up and down, then decreases the next two years to rise again in 2016 and especially in 2017 (mainly due to the high impact on industry) and then goes back again the last years.

Table 4: Cost of Use of Capital

Year	Total	Services	Industry
2007	17.48	19.11	16.60
2008	13.75	12.31	15.85
2009	20.37	20.50	20.11
2010	17.09	20.19	13.93
2011	17.01	19.38	13.68
2012	20.93	22.84	18.81
2013	18.28	19.02	16.96
2014	15.50	15.46	15.58
2015	10.32	10.66	9.57
2016	12.93	12.76	14.83
2017	19.59	15.02	28.01
2018	14.69	15.28	13.56
2019	12.78	11.61	15.84

Fig. 4: Cost of Use of Capital Index

B Appendix B: Results per Sector

B.1 Services

Fig. 5: Services: Distribution of Establishments by Employment per Year

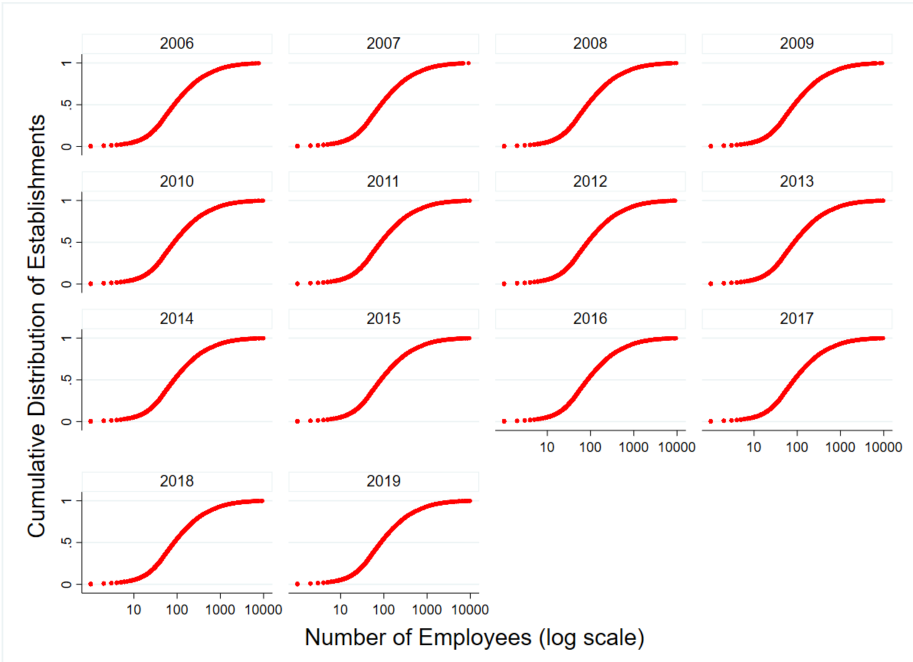


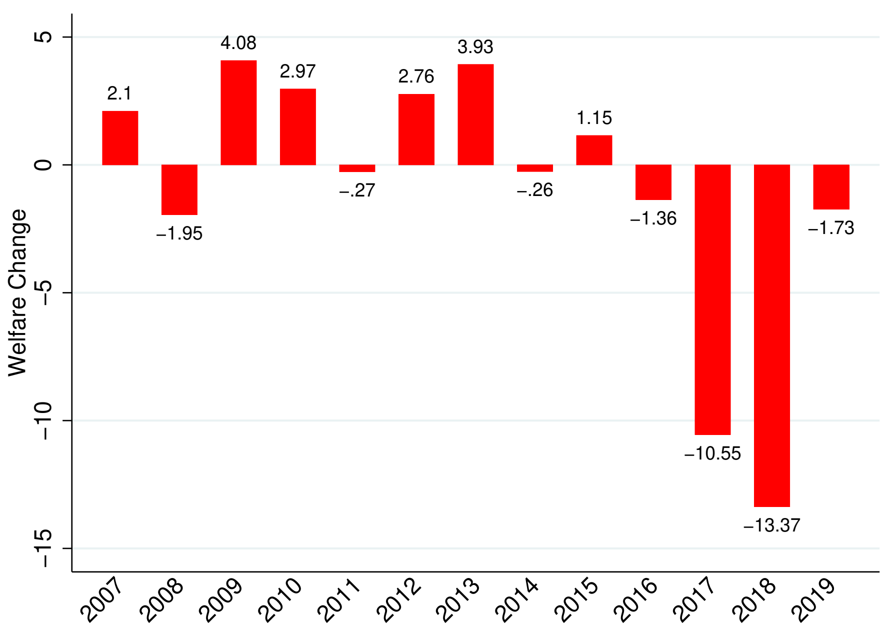
Table 5: Services: Comparing the Rate of Growth in Aggregate Value, Added with APG, 2007-2019

APG = Change in Value Added - Change in Input Costs					
Year	Value Added	Temporal Labor	Permanent Labor	Capital	APG
2007	8.99	2.98	3.01	0.90	2.10
2008	0.63	3.12	-0.57	0.03	-1.95
2009	11.16	3.56	0.40	3.12	4.08
2010	12.81	2.87	4.22	2.75	2.97
2011	5.65	1.42	0.68	3.82	-0.27
2012	5.93	-0.16	1.39	1.94	2.76
2013	8.55	5.55	2.84	-3.77	3.93
2014	2.70	-1.28	2.08	2.16	-0.26
2015	2.96	1.10	1.43	-0.72	1.15
2016	0.46	-10.72	1.69	10.85	-1.36
2017	8.90	5.11	3.27	11.07	-10.55
2018	5.89	12.12	0.48	6.66	-13.37
2019	3.93	-1.25	0.62	6.29	-1.73
Mean	6.04	1.88	1.66	3.47	-0.96
Standard Deviation	3.88	5.15	1.37	4.32	5.33

Table 6: Services: Comparing the Decomposition of APG across Estimators, 2007-2019

Year	APG	APG Technical Efficiency			APG Reallocation		
		OLS	Fixed Effects	Wooldridge - LP	OLS	Fixed Effects	Wooldridge - LP
2007	2.10	-3.74	-4.89	10.31	5.84	6.99	-8.21
2008	-1.95	0.82	2.84	-2.52	-2.77	-4.79	0.57
2009	4.08	4.53	6.99	2.85	-0.45	-2.91	1.23
2010	2.97	-0.54	-0.09	7.78	3.51	3.06	-4.81
2011	-0.27	1.19	2.24	4.23	-1.46	-2.51	-4.50
2012	2.76	7.18	5.31	6.78	-4.42	-2.55	-4.02
2013	3.93	0.60	3.56	3.53	3.33	0.37	0.40
2014	-0.26	-0.90	-1.94	2.67	0.64	1.68	-2.93
2015	1.15	0.48	-0.89	-0.13	0.67	2.04	1.28
2016	-1.36	0.68	0.19	2.40	-2.04	-1.55	-3.76
2017	-10.55	-0.90	1.24	1.62	-9.65	-11.79	-12.17
2018	-13.37	-5.73	-1.62	-4.80	-7.64	-11.75	-8.57
2019	-1.73	3.59	2.45	7.02	-5.32	-4.18	-8.75
Mean	-0.96	0.56	1.18	3.21	-1.52	-2.15	-4.17
Standard Deviation	5.33	3.33	3.21	4.20	4.49	5.37	4.33

Fig. 6: Services: Welfare Change per Year



B.2 Industry

Fig. 7: Industry: Distribution of Establishments by Employment per Year

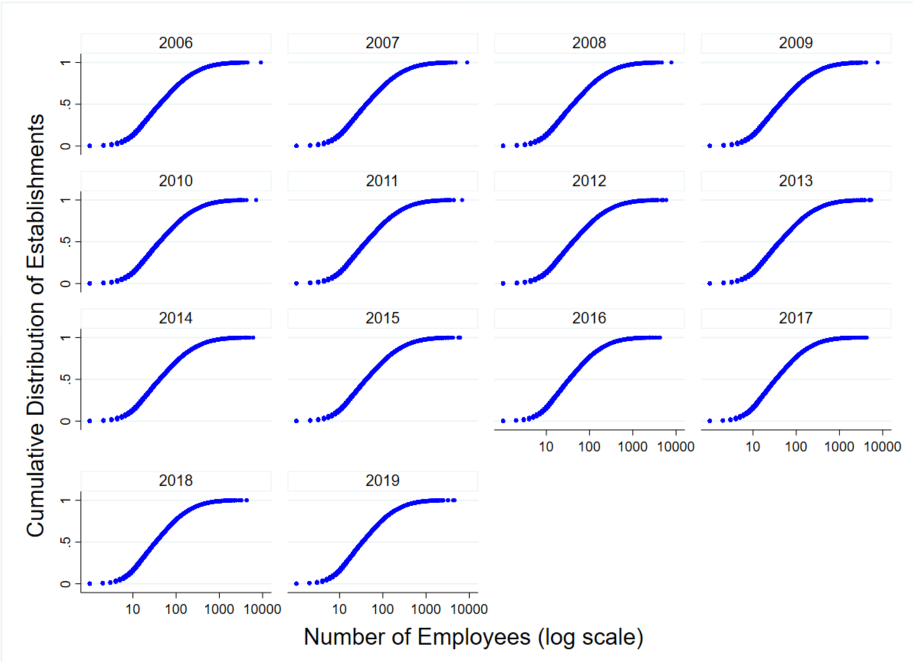


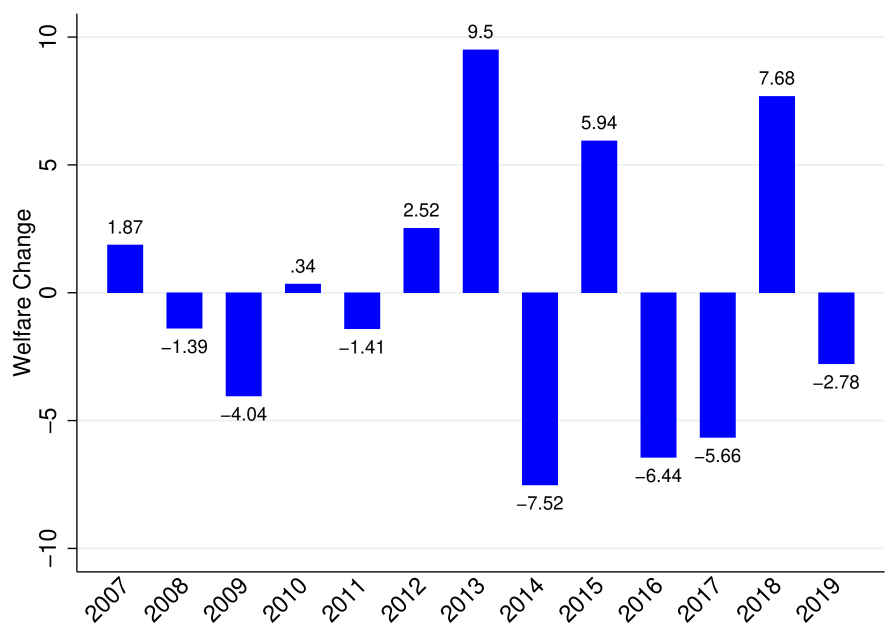
Table 7: Industry: Comparing the Rate of Growth in Aggregate Value, Added with APG, 2007-2019

APG = Change in Value Added - Change in Input Costs					
Year	Value Added	Temporal Labor	Permanent Labor	Capital	APG
2007	2.23	0.14	0.26	-0.04	1.87
2008	-0.33	-0.02	0.25	0.83	-1.39
2009	-3.68	-0.14	-0.11	0.61	-4.04
2010	2.01	0.20	0.26	1.21	0.34
2011	2.80	0.19	0.35	3.66	-1.41
2012	6.00	0.21	0.83	2.43	2.52
2013	8.72	-0.18	0.20	-0.8	9.50
2014	-6.10	0.24	0.32	0.86	-7.52
2015	7.01	0.04	0.25	0.79	5.94
2016	-1.91	0.02	0.33	4.18	-6.44
2017	-2.11	-0.11	0.16	3.50	-5.66
2018	3.86	-0.03	-0.25	-3.54	7.68
2019	-1.99	-0.04	0.01	0.82	-2.78
Mean	1.27	0.04	0.22	1.12	-0.11
Standard Deviation	4.42	0.14	0.26	2.05	5.42

Table 8: Industry: Comparing the Decomposition of APG across Estimators, 2007-2019

Year	APG	APG Technical Efficiency			APG Reallocation		
		OLS	Fixed Effects	Wooldridge - LP	OLS	Fixed Effects	Wooldridge - LP
2007	1.87	0.00	-0.22	0.17	1.87	2.09	1.70
2008	-1.39	-6.83	-7.19	-12.25	5.44	5.80	10.86
2009	-4.04	1.50	2.15	0.22	-5.54	-6.19	-4.26
2010	0.34	-2.67	-3.49	-0.31	3.01	3.83	0.65
2011	-1.41	-2.95	-1.81	-2.17	1.54	0.40	0.76
2012	2.52	-3.51	-5.22	-9.61	6.03	7.74	12.13
2013	9.50	3.89	5.01	0.53	5.61	4.49	8.97
2014	-7.52	-8.26	-10.08	-9.47	0.74	2.56	1.95
2015	5.94	2.30	2.15	6.04	3.64	3.79	-0.10
2016	-6.44	-6.41	-4.94	-2.32	-0.03	-1.50	-4.12
2017	-5.66	-0.84	-0.36	-3.61	-4.82	-5.30	-2.05
2018	7.68	3.83	2.80	3.19	3.85	4.88	4.49
2019	-2.78	-3.82	-3.79	-2.85	1.04	1.01	0.07
Mean	-0.11	-1.83	-1.92	-2.50	1.72	1.82	2.39
Standard Deviation	5.42	4.01	4.37	5.24	3.62	4.13	5.31

Fig. 8: Industry: Welfare Change per Year



C Appendix C: Aggregate Productivity Growth Based Only on Technical Efficiency Comparison

This section compares the results of APG to a different definition of aggregate productivity growth given by Bailey, Hulten, and Campbell (1992): "output- or input-share weighted changes in the distribution of plant level technical efficiencies. They are decomposed into aggregate growth from technical efficiency or reallocation" which will be referred to as BHC. So, let us define BHC as:

$$BHC = d\sum_i (s_i ln\omega_i) = \sum_i (s_i ln\omega_i) + \sum_i (ln\omega_i \ ds_i) \tag{38}$$

Where the first term in the right-hand side refers to the aggregate technical efficiency and the second to the aggregate reallocation.

Table 9 compares the rate of growth between APG and the different estimators of BHC. It is noteworthy that on average, APG produces negative numbers, while BHC OLS, Fixed effects and Wooldridge-LP estimators give a positive growth average rate. BHC estimators have higher volatility, especially the Wooldridge-LP estimator reaching a standard deviation of 57.20. The variance of the BHC estimators is large regardless of the production function estimator, as this methodology tends to produce high dispersion that might be explained by the distribution of the data across units.

Table 10 shows why APG and BHC differ, we present the results for Wooldridge-LP estimator, technical efficiency and the reallocation term, for both methodologies the technical efficiency term is the same, but the reallocation terms differ (reported in the last two columns). BHC reallocation is positive on average and have more dispersion in comparison to the reallocation of APG, the reason for the difference in the reallocation for the two methodologies, is that in plant level data there is much dispersion in the log level of productivity that translates into volatility in the BHC reallocation term.

Finally, results are divided in two, presenting the tables just for services and just for industry. In services, dispersion decreases, but the Wooldridge-LP estimator maintains the largest standard deviation with 36.79. In this case on average all the estimators have negative sign just as APG. The BHC reallocation is sharper than the APG's. For industry, only Wooldridge-LP estimator keeps the same sign as APG with a higher standard deviation, but the BHC reallocation is negative and more disperse.

Table 9: Comparing the Rate of Growth in Aggregate Value-Added, APG and BHC, 2007-2019

Year	APG	BHC Productivity Growth Across Estimators		
		OLS	Fixed Effects	Wooldridge -LP
2007	1.91	6.83	5.05	-27.68
2008	-1.64	3.5	8.41	55.3
2009	-0.57	0.18	3.71	-114.9
2010	1.48	-5.76	-9.21	-24.48
2011	-0.86	-0.4	0.75	40.98
2012	2.44	31.25	43.42	112.95
2013	6.96	14.77	20.79	56.73
2014	-3.87	19.64	21.73	34.58
2015	3.49	-12.28	-20.4	-37.87
2016	-3.87	-7.27	-10.78	-38.15
2017	-8.37	9.98	17.94	14.65
2018	-3.73	3.4	7.51	14.32
2019	-2.34	0.59	-3.87	-3.09
Mean	-0.69	4.96	6.54	6.41
Standard Deviation	3.97	11.79	16.72	57.20

Table 10: Comparing the Rate of Growth in Aggregate Value-Added with APG and BHC, including Decomposition, Wooldridge - LP Estimator, 2007-2019

Year	APG	BHC	Technical Efficiency	BHC Reallocation	APG Reallocation
2007	1.91	-27.68	0.92	-28.6	0.99
2008	-1.64	55.3	7.92	47.38	-9.56
2009	-0.57	-114.9	-4.57	-110.33	4
2010	1.48	-24.48	-2.28	-22.2	3.76
2011	-0.86	40.98	-2.87	43.85	2.01
2012	2.44	112.95	22.56	90.39	-20.12
2013	6.96	56.73	16.55	40.18	-9.59
2014	-3.87	34.58	3.42	31.16	-7.29
2015	3.49	-37.87	-1.35	-36.52	4.84
2016	-3.87	-38.15	-18.58	-19.57	14.71
2017	-8.37	14.65	5.68	8.97	-14.05
2018	-3.73	14.32	2.32	12	-6.05
2019	-2.34	-3.09	-1.84	-1.25	-0.5
Mean	-0.69	6.41	2.14	4.27	-2.83
Standard Deviation	3.97	57.20	10.09	50.09	9.34

C.1 Services

Table 11: Services: Comparing the Rate of Growth in Aggregate Value-Added, APG and BHC, 2007-2019

Year	APG	BHC Productivity Growth Across Estimators		
		OLS	Fixed Effects	Wooldridge -LP
2007	2.1	7.92	1.86	58.84
2008	-1.95	-1.95	5.74	-27.29
2009	4.08	4.08	-4.33	-89.31
2010	2.97	2.97	-57.36	-22.88
2011	-0.27	-0.26	-11.27	28.92
2012	2.76	2.76	54.88	-18.4
2013	3.93	3.94	0.94	-11.69
2014	-0.26	-0.26	-0.42	3.95
2015	1.15	1.14	-27.09	-18.11
2016	-1.36	-1.36	-5.91	-21.34
2017	-10.55	-10.56	-7.55	-54.41
2018	-13.37	-13.37	10.48	15.3
2019	-1.73	-1.37	-27.9	3.98
Mean	-0.96	-0.49	-5.23	-11.73
Standard Deviation	5.33	5.83	25.57	36.79

Table 12: Services: Comparing the Rate of Growth in Aggregate Value-Added with APG and BHC, including Decomposition, Wooldridge - LP Estimator, 2007-2019

Year	APG	BHC	Technical Efficiency	BHC Reallocation	APG Reallocation
2007	2.1	58.84	8.48	50.36	-6.38
2008	-1.95	-27.29	-1.8	-25.49	-0.15
2009	4.08	-89.31	-4.28	-85.03	8.36
2010	2.97	-22.88	6.82	-29.7	-3.85
2011	-0.27	28.92	6.05	22.87	-6.32
2012	2.76	-18.4	7.03	-25.43	-4.27
2013	3.93	-11.69	4.31	-16	-0.38
2014	-0.26	3.95	0.46	3.49	-0.72
2015	1.15	-18.11	0.09	-18.2	1.06
2016	-1.36	-21.34	-4.01	-17.33	2.65
2017	-10.55	-54.41	1.44	-55.85	-11.99
2018	-13.37	15.3	3.84	11.46	-17.21
2019	-1.73	3.98	3.98	0	-5.71
Mean	-0.96	-11.73	2.49	-14.22	-3.45
Standard Deviation	5.33	36.79	4.21	34.14	6.52

C.2 Industry

Table 13: Industry: Comparing the Rate of Growth in Aggregate Value-Added, APG and BHC, 2007-2019

		BHC Productivity Growth Across Estimators		
Year	APG	OLS	Fixed Effects	Wooldridge -LP
2007	1.87	-1.54	-2.51	-0.19
2008	-1.39	-2.25	0.06	-33.44
2009	-4.04	-16.65	-22.84	-23.75
2010	0.34	7.71	10.3	21.49
2011	-1.41	5.61	11.84	-3.53
2012	2.52	10.39	17.45	-12.69
2013	9.5	18.86	27.88	-3.5
2014	-7.52	15.9	21.75	22.47
2015	5.94	-4.18	-8.94	13.33
2016	-6.44	-11.87	-15.65	-8.41
2017	-5.66	9.91	17.99	-16.82
2018	7.68	2.35	3.57	-3.66
2019	-2.78	8.87	13.54	12.25
Mean	-0.11	3.32	5.73	-2.80
Standard Deviation	5.42	10.37	15.13	16.90

Table 14: Industry: Comparing the Rate of Growth in Aggregate Value-Added with APG and BHC, including Decomposition, Wooldridge - LP Estimator, 2007-2019

Year	APG	BHC	Technical Efficiency	BHC Reallocation	APG Reallocation
2007	1.87	-0.19	0.45	-0.64	1.42
2008	-1.39	-33.44	-12.85	-20.59	11.46
2009	-4.04	-23.75	-0.49	-23.26	-3.55
2010	0.34	21.49	-0.93	22.42	1.27
2011	-1.41	-3.53	-1.83	-1.7	0.42
2012	2.52	-12.69	-8.7	-3.99	11.22
2013	9.5	-3.5	1.32	-4.82	8.18
2014	-7.52	22.47	-9.34	31.81	1.82
2015	5.94	13.33	6.36	6.97	-0.42
2016	-6.44	-8.41	-3.1	-5.31	-3.34
2017	-5.66	-16.82	-3.82	-13	-1.84
2018	7.68	-3.66	3.88	-7.54	3.8
2019	-2.78	12.25	-2.93	15.18	0.15
Mean	-0.11	-2.80	-2.46	-0.34	2.35
Standard Deviation	5.42	16.90	5.35	15.95	5.01