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Abstract

In spite of increasing globalization around the world, the effects of international trade on economic growth are not very clear. I consider an endogenous economic growth model in an open economy with the Home Market Effect (HME) and non-homothetic preferences in order to identify some determinants of the different results in this relationship. The model shows how trade between similar countries leads to convergence in economic growth when knowledge spillovers are present, while trade between very asymmetric countries produces divergence and may become trade in a poverty or growth trap. The results for welfare move in the same direction as economic growth since convergence implies increases in welfare for both countries, while divergence leads to increases in welfare for the largest country and the opposite for its commercial partner in the absence of knowledge spillovers. International trade does not implicate greater welfare as is usual in a static context under CES preferences.

Keywords: International Trade, Economic Growth, Home Market Effect, Non-homothetic Preferences.

Jel Classification: F12, F43, O41,O33.

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1 Introduction

The existence of a positive relationship between international trade and economic growth is one of the central proposals upon which the so-called Washington Consensus is based, as well as, and in particular, the policies of multilateral organisms of credit and economic development, such as the World Bank and the International Monetary Fund (Williamson 1990, Williamson 2000, Dollar 2005, etc.). A wide spectrum of trade agreements made the world over are explained by the assumption that international trade generates positive effects for economic growth and welfare. However, the causality between international trade and economic growth is still ambiguous. From one side the classical international trade literature presents trade as the mechanism to generate economic growth. On the other side, the new literature like Young (1991) and Melitz (2005) suggest that the infant industry should be protected to generate economic growth previous to trade. International trade effects can be positive or negative depending on the countries engaged in trading. Singh (2010).

The aim of this article is to determine under which circumstances international trade increases economic growth and under which conditions it does not. This objective seeks to avoid general and ambiguous questions about the effects of international trade on economic growth and move instead towards a particular field in which the characteristics that generate a positive relationship are defined. In short, what are the elements that determine the effects of international trade on economic growth? What are the dynamic results for welfare?

This problem is analyzed through an endogenous growth model and an international trade model based on the existence of the Home Market Effect with non-homothetic preferences.[1] HME is generated through economic market size, as shown in Giraldo and Jaramillo (2016), and thus allows for interactions between elements of demand, such as population size and the purchasing power of agents, and elements of offer, such as productivity between sectors. This structure is brought to a dynamic field in a model of endogenous growth with knowledge spillovers and learning by doing in production of heterogeneous goods. The results present new findings for the effects of international trade, as defined by the characteristics of the associated countries and the commercial legislation of the trading countries.

In contrast to standard models of international trade effects on economic growth and some models that use HME, the model herein gives particular relevance to demand-side variables in the determination of the dynamics of the model. Most of the models that use HME in dynamic environments only utilise the static effects of demand, but the dynamic is addressed by recourse to supply variables. In

[1] The Home Market Effect (hereafter HME) is generally difined as "a more than-proportional realtionship between a country's share of world production of a good and its share of world demand for the same good" Crozet and Trionfetti (2008).

contrast, the present model allows for the interplay of supply and demand side elements in the dynamic determination of variables because of the assumption of non-homothetic preferences. The dynamic of the model is determined by supply and demand variables but it is addressed through demand variables.

Knowledge spillovers, transportation costs and differences in the per capita income of countries are key variables in the results of trade relations. The model shows how a commercial partnership between very different countries leads to a divergence between them, trade may become in a poverty or growth trap in the divergent cases. Trade between similar countries may lead to a converging growth path and a stationary equilibrium, that is, conditional convergence.

Contrary to the results presented in static models of international trade, and in some models that relate this to economic growth, the results of this relationship are not always positive for welfare. In particular, although levels of welfare initially increase after trade, the scenarios that present a divergence in growth also present a divergence in welfare in the absence of knowledge spillover. In these particular scenarios, autarky is strictly preferable to trade.

The results obtained with the model are consistent with widely known stylized facts about economic growth around the world. During the last two centuries, the global economy has been characterized by a meaningful economic growth rate - which began after the Industrial Revolution - the expansion of international trade, and a convergence in income (and productivity) within developed countries and divergences in developing countries, Maddison (1983), Williamson (2002), Baldwin, Martin, and Ottaviano (2001), Acemoglu, Johnson and Robinson (2005).

Empirical research has found conflicting results that prevent the presentation of definitive answers about the effects of international trade on economic growth.[2] Endogeneity problems in estimations, errors in the measurement of economic policy variables, and sample selection bias are some of the arguments that have been presented in the empirical field as the causes of such inconsistencies in the results. Singh (2010).

The current theoretical literature is built from the supply side, showing how levels of productivity in each country and the possibilities of a transfer of technology after trade agreements play central roles in the results of these models. For example, Rivera-Batiz and Romer (1991), Young (1993) and, more recently, Gancia and Zilibotti (2005) present variations of the model by Romer (1990) and show how these are applied to an open economy, where dynamic gains are not presented via integration among symmetric economies; static gains are shown, raising the welfare of both countries. However, these conclusions are not preserved when the intertemporal dynamic among asymmetric countries is considered, since the models show a pattern of specialization. The authors do mention that the model does not fit the reality of asymmetric countries due to the absence of determinant variables, such as

[2] The most important research in this area includes: Dollar (1992), Ben-David (1993), Sachs and Warner (1995), Edwards (1998), Frankel and Romer (1999), Rodriguez and Rodrik (2001), Chang , Kaltani, and Loayza (2009).

product cycle and knowledge spillover.

Aghion and Howitt (2005) have built a model of endogenous growth based on innovation quality and not quantity, as has also been done by the above-mentioned authors. The results show a trade-off in the dynamic gains of trade between innovation and amount of skilled labor with size of innovation, degree of competition in the market and possibilities of imitation. In addition, the authors avoid presenting absolute results and state that the introduction of additional variables could modify the results, as is the case with the development of financial markets and property rights legislation.

This general framework of analysis, in which modern theories of economic growth are used to study the relationship between trade and economic growth from the supply side, has served as a basis of analysis for some authors who study particular aspects of the aforementioned relationship. For example, Ventura (1997) uses the trade-growth relationship to explain conditional convergence among Asian countries during the postwar period. The results show the convergence hypothesis, but the effects of trade on growth are conditioned in relation to parallel policy decisions. Thoenig and Verdier (2003) present a model whereby the incentives for innovation are carried out to prevent exporter firms from being easily plagiarized by the firms of other countries, with monopoly time protection provided for each innovation. The results of this model are determined by legislation regarding property rights, again showing the importance of this variable in the frameworks of these models. Galor and Mountford (2008) model the trade-growth relationship and its implications for the demographic transition of countries, as well as the direct effects on economic growth and income distribution. The result predicts the negative effect of trade on growth, but this excludes other variables, such as education, strength of institutions or the level of knowledge spillover that international trade might generate, all of which could change the findings.

The generality of models linking international trade with economic growth is based on the theory of comparative advantage or specific factors. The implementation of dynamic models in an open economy based on the theory of the Home Market Effect is quite scarce. However, this strategy for modeling international trade allows for an analysis of the effects of supply-side and demand-side variables on the dynamic effects of international trade.

From the literature related to the concerns of the present article, the work of Martin and Ottaviano (1999) stands out. They make use of HME to build a model of industrial localization within an endogenous growth environment, with knowledge spillover and transaction costs being the determinants of the location of a firm and, therefore, the rate of economic growth in global regions. Baldwin, Martin and Ottaviano (2001) use this same strategy to show that after the specialization generated by trade liberalization, large countries tend to grow rapidly, while small countries are left behind with a slower rate of growth. Similarly, Kind (2002) does not find any concrete results regarding the effects of trade on growth, leaving an ambiguity to be solved by additional parameters such as transportation costs, knowledge spillover or some form of trade friction. Even if these models use HME to introduce demand elements into the determination of trade effects, the dynamics of the models are dominated by supply side variables, just as they are in standard economic growth models. Contrary to this, the present model allows for interactions between the supply-side and demand-side variables and gives high relevance to the demand-side variables in the determination of the dynamic of the model. The dynamic effects of international trade depend on the economic sizes of the markets that are trading and the income levels of the agents from each country.

This article consists of this introduction and three additional sections. The second section exposes the characteristics of the model, the third develops the model in an open economy with its static and dynamic implications, and the fourth section concludes.

2 The Model

The model is based on the basic structure of HME with non-homothetic preferences, Giraldo and Jaramillo (2016), and the dynamic is modeled from an endogenous growth model. This model allows for the study of the intertemporal implications of international trade on economic growth. The fundamentals of the model involve a dynamic Stone-Geary utility function, a two-sector economy, productivity differences among countries and the equality of these among sectors.

Two regions are assumed, domestic and foreign ($*$).[3] There are two sectors. First, a sector that produces a homogeneous good (X), which represents agricultural goods, and presents constant returns to scale in production. Second, a manufacturing sector that produces a set of heterogeneous goods (Y), with increasing returns to scale in production. The varieties of heterogeneous good are horizontally differentiated *à la* Dixit-Stiglitz and the firms in this sector maximize their benefits under monopolistic competition. Labor (L), is the only existing factor of production and is mobile among sectors but immobile among countries.

Following Chung (2006), countries differ in terms of amount of labor (L and L^*) and population size (N and N^*), but these variables are constant over time. The number of people who consume is different from the number of people who produce. It is thus supposed that domestic households offer $\frac{1}{\gamma}$ of labor for each resident ($N = \gamma L$) while foreign households offer $\frac{1}{\gamma^*}$, meaning ($N^* = \gamma^* L^*$). This allows for the simple entering of differences in per capita income between countries in a scenario in which wages are equal.

Intuitively, γ captures the demographic and redistribution factors that affect the relative demand for heterogeneous goods in comparison to that for homogeneous goods. Through this modification it is possible to interpret γ as the number of dependents under economic responsability of each worker.

[3]Hereafter, the variables corresponding to foreign have the superscript $*$.

I assume that all households demand both types of good and symmetrically demand each variety of heterogeneous good (Y). Households in both countries have the same utility function with non-homothetic preferences

$$U_t = \int_0^\infty e^{-\rho t} u_t dt \tag{1}$$

$$\text{With } u_t = \alpha \ln\left(X_t - \overline{X}\right) + (1-\alpha)\ln Y_t \tag{2}$$

$$\text{and } Y_t = \left(\int_1^{n_t} y_{it}^\sigma di\right)^{\frac{1}{\sigma}}, \ 0 < \sigma < 1, \quad n = \text{ the number of varieties consumed} \tag{3}$$

Where $\overline{X}$ is the minimum consumption (of survival) of the homogeneous good and X_t is the consumption of this good at time t, beyond the threshold of survival. Y_t is the aggregate consumption of all n varieties of heterogeneous good at time t and y_{it} is the consumption of the i-th variety at every moment.

Both goods use the same factor of production, namely labor. The production of each good is determined by the amount of labor used and its productivity. The production of homogeneous goods and all varieties of the heterogeneous goods sector is conducted with the same production function in both countries. The homogeneous goods sector has the following production function:

$$NX_t = D_{xt} = L_{xt}A_t \tag{4}$$

Where D_x is the aggregate demand of the homogeneous good, L_x is the amount of labor used in the production of such goods, and A_t is the productivity. The cost function in the heterogeneous goods sector is given by:

$$l_{it} = \frac{\mu}{A_t} + \frac{\beta D_{it}}{A_t} \qquad i = 1, 2...n \quad \text{where } D_{it} = Ny_{it} \tag{5}$$

D_{it} is the aggregate demand of the i-th variety, l_{it} is the amount of labor used in the production of each variety and A_t is the productivity at time t. Moreover, μ and β are the parameters of fixed and variable costs respectively.

Technological progress only occurs in the production of heterogeneous goods, which operates by way of a learning by doing process specific to each country, as Krugman (1987) and Lucas (1988) explain. The evolution of productivity depends on both the domestic manufacturing sector of production, and a proportion of the productivity of this sector abroad, representing knowledge spillover from the outside towards the domestic economy. In the case of autarky, this final factor is equal to zero ($\delta = 0$)

$$A_t = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right) \, ds \tag{6}$$

Where K_s and K_s^* are the levels of knowledge of each economy, which increase with the production of heterogeneous goods, thus:

$$K_s = \int_1^{n_t} y_{it} di \text{ and } K_s^* = \int_1^{n_t^*} y_{jt} dj \tag{7}$$

Finally, the full-employment condition is assumed:

$$L = L_{Xt} + L_{Yt} = \frac{D_{Xt}}{A_t} + \sum_{i=1}^{n} \left(\frac{\mu}{A_t} + \frac{\beta D_{it}}{A_t} \right) \tag{8}$$

2.1 Equilibrium in a Closed Economy

2.1.1 Consumer

The intratemporal optimization problem of agents is conventional, maximizing (2) subject to its budget restriction. As previously mentioned, the methodology presented by Chung (2006) is used to enter income differences. This model differentiates between the members of the household who work and those who only consume. In more formal terms, each worker in the home has (γ) additional agents under its responsibility that only consume. The dependency ratio being $\frac{N}{L} = \gamma$.

$$Max\ u_t = \alpha \ln \left(X_t - \overline{X} \right) + (1-\alpha) \ln Y_t \text{ s.t. } P_{xt} X_t + P_{yt} Y_t = \frac{w_t}{\gamma} \tag{9}$$

The optimal demands of the agricultural good X_t and the aggregate manufacturing goods Y_t result from the optimization program of every agent at every moment of time.

$$Y_t = \frac{1-\alpha}{P_{yt}} \left(\frac{w_t}{\gamma} - P_{xt} \overline{X} \right) \tag{10}$$

$$X_t = \frac{\alpha}{P_{xt}} \left(\frac{w_t}{\gamma} - P_{xt} \overline{X} \right) + \overline{X} \tag{11}$$

The optimal demands at every moment in time depend on supernumerary income, which is weighted by price and which in the case of manufactured goods is an index price established by the price of each of the existing varieties. The continuation of the optimization process allows for a determination of the demand of each of the varieties of heterogeneous good at every moment, which depends on the supernumerary income and the relative price of each variety.

$$y_{it} = \frac{p_{it}^{\frac{1}{\sigma-1}} (1-\alpha) \left(\frac{w_t}{\gamma} - P_{xt} \overline{X} \right)}{P_{yt}^{\frac{\sigma}{\sigma-1}}} \tag{12}$$

$$\text{Where } P_{yt} = \left(\sum_{i=1}^{n_t} p_{it}^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} \tag{13}$$

P_{yt} is the index price of heterogeneous goods, which is established as an aggregate of the prices of all varieties that exist in every moment of time, weighted by the degree of substitution between them.

2.1.2 Producer

The production of homogenous goods is supposed under perfect competition. This implies that the equilibrium price is equal to the labor cost. The price of this good is established as a numeraire, so productivity determines salary levels in this economy.

$$P_{xt} = 1 = \frac{w_t}{A_t} \tag{14}$$

I assume monopolistic competition in the production of heterogeneous goods. Since every variety of heterogeneous good uses the same technology of production, the price of each of the varieties is the same, and it is determined by wages, productivity, and fixed and variable cost parameters.

$$p_t = p_{it} = \frac{\beta w_t}{\sigma A_t} \tag{15}$$

Replacing the last equation in the zero-benefits condition, determined by the free entry and exit of firms in the manufacturing sector, I find the aggregate production of each variety, which is equal to its total demand at each instant of time.

$$D_t = D_{it} = \frac{\mu\sigma}{(1-\sigma)\,\beta} \tag{16}$$

Finally, from the full-employment condition (8) it is possible to obtain the number of varieties of heterogeneous good produced under autarky at each time t

$$n_t = \frac{L_{yt}\,(1-\sigma)\,A_t}{\mu} \tag{17}$$

The full-employment condition determines the amount of labor used in the heterogeneous goods sector L_{yt}, which is equal to the total available labor force (L), minus the quantity used in the production of homogeneous goods $\left(L_{xt} = N\left(\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{A_t}\overline{X}\right)\right)$. Accordingly, the number of varieties produced at every moment can be rewritten as:

$$n_t = (A_t - \gamma\overline{X})(1-\sigma)(1-\alpha)\frac{N}{\mu}\frac{1}{\gamma} \tag{18}$$

This expression shows how the number of varieties of heterogeneous good produced in the domestic market under autarky - which we assume to be the level of industrialization of a country - is determined by demand and supply factors. From one side the dependency ratio and the productivity levels determine the supernumerary income, which represents the purchasing power of the agents in each country. On the other side the population size determines the market size in the standard way.

At the same time, the dynamic is determined by a learning process generated in the production of manufactured goods. Thus the productivity variation rate is proportional to the number of varieties produced in this sector

$$\dot{A}_t = K_t = n_t y_i = n_t\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) = (A_t - \gamma\overline{X})(1-\alpha)\frac{N}{\gamma}\frac{\sigma}{\beta} = (A_t - \gamma\overline{X})(1-\alpha)L\frac{\sigma}{\beta} \tag{19}$$

The productivity growth rate is obtained from this last equation:

$$\frac{\dot{A}_t}{A_t} = \frac{n_t}{A_t}\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{20}$$

The dynamics of the economy under autarky are determined by equations (20) and (18). Welfare levels in a closed economy, which depend mainly on the number of varieties available for agent consumption and productivity levels, can be determined via the previous result. The level of intertemporal utility in this scenario is:

$$U_t = \int_0^\infty e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t}\left[\alpha \ln\left(\alpha\left(\frac{A_t}{\gamma} - \overline{X}\right)\right) + (1-\alpha)\ln\left(\frac{(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)}{n_t^{\frac{\sigma-1}{\sigma}}\frac{\beta}{\sigma}}\right)\right] dt \tag{21}$$

3 Open Economy

After establishing the model implications under autarky, it is presented in an open economy setting. The international trade result is determined by the HME with non-homothetic preferences presented in Giraldo and Jaramillo (2016), plus the dynamic effects generated according to the model presented. In general, the number of varieties produced in each country is determined by HME, and the dynamic effects are determined by the levels of learning in each country and knowledge spillovers from foreign technologies.

Assuming costless international trade for homogenous good (X), its price is equalized in the two countries. This price is taken as a numeraire ($P_x = P_x^* = 1$), so that productivity determines the salary levels for each economy in the same way as in a closed economy.

The international trade of heterogeneous goods generates positive transportation costs, which are modeled as iceberg costs.[4] Following Giraldo and Jaramillo (2016), in the presence of positive transportation costs for the heterogeneous goods trade, market size is determined by three basic elements: population size, relative income and productivity levels. In turn, after trade liberalization the economy with the greater market size gathers the majority of production of the varieties of heterogeneous good. In accordance with the international trade model, the aggregate demand for heterogeneous goods in each country is the sum of the domestic and foreign demand for this type of good:

$$n_t p_t D_t = \frac{n_t}{n_t + \theta\frac{p_t^*}{p_t}n_t^*}(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)N + \frac{\theta^* n_t}{\theta^* n_t + \frac{p_t^*}{p_t}n_t^*}(1-\alpha)\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^* \tag{22}$$

$$n_t^* p_t^* D_t = \frac{\theta n_t^*}{n_t\frac{p_t}{p_t^*} + \theta n_t^*}(1-\alpha)\left(\frac{A_t}{\gamma} - \overline{X}\right)N + \frac{n_t^*}{\theta^*\frac{p_t}{p_t^*}n_t + n_t^*}(1-\alpha)\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^* \tag{23}$$

Where $\theta = \left(\frac{p}{p^*}\right)^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}$ is the demand rate for foreign heterogeneous goods in terms of the domestic ones, and $\theta^* = \left(\frac{p}{p^*}\right)^{\frac{-1}{1-\sigma}}\tau^{\frac{\sigma}{1-\sigma}}$ is the corresponding foreign rate.

[4] The "iceberg cost" supposes that a τ portion of transported good arrives, and that $(1-\tau)$ is lost in transit.

The previous equations are given a HME dynamic equation (24), which is determined by the relationship between the demand elements mentioned above and the evolution of productivity levels in each country

$$\frac{n_t}{n_t^*} = \frac{\frac{\left(\frac{A_t}{\gamma} - \overline{X}\right)N}{\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^*} - \tau^{\frac{\sigma}{1-\sigma}}}{1 - \tau^{\frac{\sigma}{1-\sigma}} \frac{\left(\frac{A_t}{\gamma} - \overline{X}\right)N}{\left(\frac{A_t^*}{\gamma^*} - \overline{X}\right)N^*}} \tag{24}$$

The last equation shows HME in terms of the number of varieties produced in each country. HME is determined for the interplay of demand and supply elements. After trade, heterogeneous goods production in each country depends on supernumerary income, population size and level of productivity. The interactions between these aforementioned variables yield different trade scenarios with or without complete specialization in each countries' production.[5]

Given that both countries have the same production technologies and the same learning functions, one can determine the contemporary effects of international trade and its implications in the long term. With the learning function of the economy, which presents knowledge spillover, $\delta > 0$, in the case of an open economy, it is possible to identify the evolution of productivity and thus the number of varieties produced in the manufacturing sector at each time

$$A_t = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right)\, ds \tag{25}$$

In order to simplify the model, productivity is redefined in relation to agricultural survival consumption, weighted by the relationship between people who integrate the home and who are part of the labor force (supernumerary income) $\widehat{A_t} = A_t - \gamma\overline{X}$. This redefinition of variables reduces the mathematical processes and allows one to introduce a new state variable with all the determinants of intertemporal market size (population, workforce, purchasing power and, of course, productivity), allowing it to facilitate a dynamic analysis. So productivity is determined as:

$$\widehat{A_t} = A_t - \gamma\overline{X} = \int_{-\infty}^{t} \left(K_s + \delta K_s^*\right) ds - \gamma\overline{X} \tag{26}$$

Differentiating over time, it is possible to obtain the rates of adjustment of productivity in domestic and foreign markets respectively:

$$\dot{\widehat{A_t}} = K_t + \delta K_t^* = n_t \left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) + \delta n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{27}$$

$$\dot{\widehat{A_t^*}} = K_t^* + \delta K_t = n_t^* \left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) + \delta n_t \left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{28}$$

[5] For more details, see Giraldo and Jaramillo (2016).

The productivity growth rate in each country can then be obtained by dividing (27) and (28) by their respective productivities, which in turn determines the growth rates for the rest of the variables in the economy:

$$\frac{\dot{\widehat{A_t}}}{\widehat{A_t}} = \frac{n_t\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t^*\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t}} \tag{29}$$

$$\frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = \frac{n_t^*\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t^*}} \tag{30}$$

I define the relative productivity of the two countries $\left(H_t = \frac{\widehat{A_t}}{\widehat{A_t^*}}\right)$ as the new state variable. The rate of growth of this variable is then the subtraction among domestic and foreign productivity growth rates. The growth rate of H_t is determined by productivity levels, the number of varieties produced in each country and the spillovers between countries.

$$\frac{\dot{H_t}}{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = \frac{n_t\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t^*\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t}} - \frac{n_t^*\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) + \delta n_t\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right)}{\widehat{A_t^*}} \tag{31}$$

These variable changes must be brought to the other equations of the model. After the respective replacements in the HME equation (24):

$$\frac{n_t}{n_t^*} = \frac{\frac{\widehat{A_t} N\gamma^*}{\widehat{A_t^*} N^*\gamma} - \tau^{\frac{\sigma}{1-\sigma}}}{1 - \tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{A_t} N\gamma^*}{\widehat{A_t^*} N^*\gamma}} \tag{32}$$

After computations, this equation may be defined as a function of the ratio of varieties produced and the gap in productivity levels. Defining $\widehat{n_t} = \frac{n_t}{\widehat{A_t}}$, and $\widehat{n_t^*} = \frac{n_t^*}{\widehat{A_t^*}}$:

$$\frac{\frac{n_t}{\widehat{A_t}}}{\frac{n_t^*}{\widehat{A_t^*}}} = \frac{\widehat{n_t}}{\widehat{n_t^*}} = \frac{\left(\frac{N\gamma^*}{N^*\gamma} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t}\right)}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}} H_t \frac{N\gamma^*}{N^*\gamma}\right)} \tag{33}$$

This last equation is the HME equation in terms of the productivity gap. The HME equation is now dynamic and it is determined for the same variables as in its static version (24) at each moment in time, Giraldo and Jaramillo (2016). Similar to the HME equation, one could also redefine (22 and 23) in these terms:

$$\widehat{n_t} = \left(\frac{1}{1 + \frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t}} \frac{N}{\gamma} + \frac{\tau^{\frac{\sigma}{1-\sigma}}}{\tau^{\frac{\sigma}{1-\sigma}} + \frac{1}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t}} \frac{1}{H_t} \frac{N^*}{\gamma^*}\right) \frac{(1-\alpha)(1-\sigma)}{\mu} \tag{34}$$

$$\widehat{n_t^*} = \left(\frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + \tau^{\frac{\sigma}{1-\sigma}}} H_t \frac{N}{\gamma} + \frac{1}{\tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + 1} \frac{N^*}{\gamma^*}\right) \frac{(1-\alpha)(1-\sigma)}{\mu} \tag{35}$$

Consequently, there is a system with three equations and three unknowns that determines the equilibrium and the evolution of these open economies. Synthesizing, the equations of the system are:

$$\frac{\widehat{n_t}}{\widehat{n_t^*}} = \frac{\left(\frac{N\gamma^*}{N^*\gamma} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t}\right)}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}} H_t \frac{N\gamma^*}{N^*\gamma}\right)} \tag{36}$$

$$\widehat{n_t^*} = \left(\frac{\tau^{\frac{\sigma}{1-\sigma}}}{\frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + \tau^{\frac{\sigma}{1-\sigma}}} H_t \frac{N}{\gamma} + \frac{1}{\tau^{\frac{\sigma}{1-\sigma}} \frac{\widehat{n_t}}{\widehat{n_t^*}} H_t + 1} \frac{N^*}{\gamma^*}\right) \frac{(1-\alpha)(1-\sigma)}{\mu} \tag{37}$$

$$\frac{\dot{H_t}}{H_t} = \left(\widehat{n_t}\left(1 - \delta H_t\right) - \widehat{n_t^*}\left(1 - \frac{\delta}{H_t}\right)\right)\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{38}$$

Solving the system, the dynamics of the model can be found. The first two equations of the system are entered into the dynamic equation of productivity differences between countries, and the dynamic equation of the model is found:

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\widehat{n_t}}{\widehat{n_t^*}}\left(1 - \delta H_t\right) - \left(1 - \frac{\delta}{H_t}\right)\right]\widehat{n_t^*}\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{39}$$

The parameter of the demand relation between foreign and domestic heterogeneous goods $\theta = \tau^{\frac{\sigma}{1-\sigma}}$, is determined by the assumptions of transportation costs and productivities. $\phi = \frac{N\gamma^*}{N^*\gamma} = \frac{L}{L^*}$ is defined as a new variable. Thus the equation can be written in the following way:

$$\frac{\dot{H_t}}{H_t} = \left[\frac{\left(\phi - \frac{\theta}{H_t}\right)}{\left(1 - \theta H_t \phi\right)}\left(1 - \delta H_t\right) - \left(1 - \frac{\delta}{H_t}\right)\right]\widehat{n_t^*}\left(\frac{\mu\sigma}{(1-\sigma)\beta}\right) \tag{40}$$

The growth rate of H_t (the relative supernumerary income between countries) determines the short-term and long-term effects of international trade on economic growth. This growth is determined by the level of the productivity gap H_t, knowledge spillover δ, and the relative labor force between countries ϕ. In short, interactions between supply and demand variables establish the evolutions that will occur for the two economies after trade.

From the growth rate of H_t (equation 40) the dynamics of the other variables of the model can be deduced. The steady equilibrium properties (existence, unicity and stability) are shown in the appendices and are described in the following propositions.

Proposition 1 *: There is one unique stationary equilibrium possible in this economy, the stability of which depends on the expansion path that crosses this and the value of which is determined by:*

$$H_t^{Equilibrium} = \frac{-\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right) + \sqrt{\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)^2 + \frac{4}{\phi}}}{2} \tag{41}$$

The proposition proves the existence of the long-term equilibrium and presents the steady state value of the relative supernumerary income of the economies that are trading. There is one unique steady state in which the productivity gap remains constant and its growth rate is equal to zero. Knowledge spillover, transportation costs and relative labor force between countries determine the value of the long run $H^{Equilibrium}$.[6]

Proposition 2 *: There is only one convergent expansion path of the state variable H, the productivity gap between countries, which guarantees the stability of the stationary equilibrium. This expansion path satisfies the following characteristics:*

$$\frac{\dot{H}_t}{H_t} = Convergent \Leftrightarrow \delta > \theta \ \wedge \frac{\delta}{\theta} > \phi \tag{42}$$

There is only one convergent expansion path that guarantees the stability of the stationary equilibrium. This path only exists in the cases in which the next two conditions are achieved: first, knowledge spillovers between the countries (δ) are greater than the relationship of their demands (θ), which is a price ratio determined by transportation costs and the substitutability between varieties. Second, the relative labor force between the countries is lower than the ratio of the above two variables ($\frac{\delta}{\theta}$). Synthesizing then, the equilibrium is stable only if the knowledge spillovers are large enough in relation to transportation costs and the relative labor force between countries.

Figure 1 represents the stable equilibrium for the convergent expansion path and the unstable equilibrium for one of the divergent expansion paths with progressive specialization by low levels of knowledge spillover.

Figure1:Convergent and divergent path of H

Convergence in productivity between countries after trade can be achieved for similar countries in market size, per capita income and productivity, whenever there is a high level of knowledge spillover between the participating economies. The more similar (different) the countries that trade are, the more (less) likely a convergence in the productivity gap. This result is known in the economic literature as conditional convergence and it is a stylized fact that has been amply demonstrated by different authors.[7]

When the parameters do not meet the conditions described by *proposition 2*, the equilibrium is unsteady (see figure 1) and the paths are divergent. This divergence is generated by the trade association among very different countries or the absence of knowledge spillovers that increase production learning after trade liberalization.

[6] In other words, the last proposition summarizes the main information that provides the functional form of the state variable H, which is a quadratic function, presenting two roots that could become points of the stationary equilibrium. However, only the positive root makes economic sense and it is established as the only equilibrium present in the function. The stability of this equilibrium is determined by each of the possible expansion paths that cross it.

[7] Barro and Sala-i-Martin (1992) and Sala-i-Martin (1996) provide some of the most cited articles for this empirical estimation.

On the one hand, if the countries that trade are very different, the productivity gap will be large and it will generate a complete specialization after trade due to HME. The country with a greater supernumerary income will specialize in the production of heterogeneous goods and increase productivity through the learning by doing process, while the partner country will specialize in homogenous goods and only improve productivity via knowledge spillovers. The more different the countries, the more likely the divergence. On the other hand, the absence of knowledge spillovers limits the learning process among economies after trade, so the country with higher levels of productivity will begin to specialize in heterogeneous goods production until it achieves complete specialization for such goods. The greater the knowledge spillovers, the greater the likelihood of convergence between countries.

In figure 1, the solid line represents the case in which $\delta > \theta$ and $\frac{\delta}{\theta} > \phi$, where the expansion path converges to a stationary equilibrium. The growth rates of productivity among the associated countries are equal and converge to a dynamically stable equilibrium. The dotted line represents the divergent path ($\theta > \delta$), where there also exists an equilibrium but it is unstable. Here the productivity of partner countries tends to diverge over time, generating a complete specialization in the production of manufactured goods in one of the countries involved in the trade. The production in which a country specializes depends on the conditions of the countries at the moment that trade starts. The country with a greater supernumerary income (productivity and per capita income) specializes in manufactured goods, while its partner specializes in the production of homogeneous goods.

The results on the state variable show the determinants of international trade effects on economic growth under this analysis framework. First, the economic market size of the countries that are trading, which is defined through the relative supernumerary income, determines the international trade effects on economic growth in the same way as that of the conditional convergence theory - similar countries have similarly steady states. Second, the knowledge spillovers.The more knowledge spillovers there are the easier the convergence among countries that trade.[8]

3.1 Complete Specialization

Complete specialization scenarios come from trading between very asymmetrical countries in their supernumerary incomes or from a low level of knowledge spillovers. The mechanism operates through HME and HME produces a concentration of heterogeneous goods production in the largest market.[9] The productivity functions then change to a new regime of complete specialization. When the relationship of productivities H_t is greater than $\frac{1}{\theta\phi}$, the value of the asymptote of the dynamic function (40), or the parameters value generates an equilibrium point greater than the this asymptote, scenarios

[8] Some authors relate the knowledge spillovers with the property rights institutions or some institutional features, Acemoglu (2009)

[9] This means the country with the greater supernumerary income.

of complete specialization occur. In such cases, when countries are very different the country with a greater economic market size specializes in the production of manufactured goods, raises productivity levels and therefore expands the productivity gap with the trading partner. At the same time, the commercial counterpart specializes in the production of homogeneous goods, thereby reducing its growth. The convergence or divergence in the growth rate depends on the existence of knowledge spillovers, yet divergence is always present in the levels of income and productivity.

The other scenarios of complete specialization are presented when the countries are not very different in their supernumerary incomes but the transportation costs are greater than the levels of knowledge spillover ($\delta < \theta$), and/or the quotient of spillovers on transportation costs is less than the relative labor force between countries $\left(\frac{\delta}{\theta} < \phi\right)$. In these cases, trade generates incomplete specialization between countries in the short term but the net productivity gap is expanded through time. The income and productivity levels are more distant each time, until they find a long-term steady state in the new regime of complete specialization.

The dynamic of complete specialization scenarios is determined by equations (22) and (23). Complete specialization implies that the production of varieties of heterogeneous good is zero in the country specialized in the production of homogeneous goods. By inserting these assumptions into the equations and applying them to the same notation of supernumerary income ($\widehat{A_t}$ y $\widehat{n_t}$) the results are:

Under complete specialization in heterogeneous goods at the domestic level:

$$\widehat{n_t} = (1-\alpha)\left(\frac{N_t}{\gamma} + \frac{N^*}{\gamma^* H_t}\right)\left(\frac{1-\sigma}{\mu}\right) \text{ and } \widehat{n_t^*} = 0 \tag{43}$$

Under complete specialization in heterogeneous goods at the foreign level:

$$\widehat{n_t} = 0 \text{ and } \widehat{n_t^*} = (1-\alpha)\left(\frac{N_t H_t}{\gamma} + \frac{N^*}{\gamma^*}\right)\left(\frac{1-\sigma}{\mu}\right) \tag{44}$$

The equation (43) represents the number of varieties produced in terms of productivity levels for the case of complete specialization in heterogeneous goods in the domestic market, while (44) corresponds to the case of complete specialization in heterogeneous goods abroad. The growth rate of the state variable H_t, in scenarios of complete specialization, can be defined in order to identify the results in the best way. However, there exist two different scenarios for this variable depending on the presence, or not, of knowledge spillovers.

3.1.1 Complete specialization with knowledge spillover ($\delta > 0$)

In the presence of knowledge spillover ($\delta > 0$), the two possible scenarios under complete specialization are:

$$\frac{\dot{H_t}}{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = (1-\delta H_t)\left(L + \frac{L^*}{H_t}\right)\frac{(1-\alpha)\,\sigma}{\beta} \text{ With } \widehat{n_t} > 0 \text{ and } \widehat{n_t^*} = 0 \tag{45}$$

$$\frac{\dot{H_t}}{H_t} = \frac{\dot{\widehat{A_t}}}{\widehat{A_t}} - \frac{\dot{\widehat{A_t^*}}}{\widehat{A_t^*}} = -\left(1 - \frac{\delta}{H_t}\right)(LH_t + L^*)\frac{(1-\alpha)\sigma}{\beta} \text{ With } \widehat{n_t} = 0 \text{ and } \widehat{n_t^*} > 0 \tag{46}$$

The equation (45) corresponds to the case of complete specialization in heterogeneous goods at the domestic level and the equation (46) is the analogous version for the foreign level.

The country that specializes in heterogeneous goods grows according to the number of varieties produced, while the country that specializes in homogeneous goods grows according to knowledge spillovers. After trade, the net productivity gap increases because one country stops producing heterogeneous goods and the other starts to produce all sets of varieties. Consequently, the growth rate of the productivity gap is positive. Nevertheless, in the long term the economies arrive at a steady state under the complete specialization regime. In this steady state, both countries grow at the same rate but their levels of income and productivity differ. Trade becomes in a poverty trap.

Proposition 3 *: If knowledge spillovers are present and the countries that trade are very different in their supernumerary incomes ($H_t > \frac{1}{\theta\phi}$ or $H_t^{Equilibrium} > \frac{1}{\theta\phi}$), or the parameters value does not meet the conditions of proposition 2, there is complete specialization in the steady state. In this steady state there exists convergence in the growth rate but not at the level of the variables. The countries grow at the same rate, but with different levels of productivity and income.*[10]

The country that specializes in homogeneous goods grows at the same rate as the country that specializes in heterogeneous goods because of the presence of knowledge spillovers. In the long run, the growth rate is the same but the levels of productivity and income are different; this is conditional convergence.

3.1.2 Complete specialization without knowledge spillover ($\delta = 0$)

In the absence of knowledge spillover ($\delta = 0$), the two possible scenarios under complete specialization are:

$$\frac{\dot{H_t}}{H_t} = \left(L + \frac{L^*}{H_t}\right)\frac{(1-\alpha)\sigma}{\beta} \text{ With } \widehat{n_t} > 0 \text{ and } \widehat{n_t^*} = 0 \tag{47}$$

$$\frac{\dot{H_t}}{H_t} = -(LH_t + L^*)\frac{(1-\alpha)\sigma}{\beta} \text{ With } \widehat{n_t} = 0 \text{ and } \widehat{n_t^*} > 0 \tag{48}$$

Equation (47) corresponds to the case of complete specialization in heterogeneous goods at the domestic level and the equation (48) is the analogous equivalent for the foreign level.

Without knowledge spillovers, the productivity gap increases after trade, and the countries diverge in growth rates and levels of income and productivity. The country that specializes in heterogeneous

[10] Under complete specialization at the domestic level, the long-term productivity gap is $H^{ss} = \frac{1}{\delta}$. Under complete specialization at the foreign level, the long-term productivity gap is $H^{ss} = \delta$.

goods grows according to the number of varieties produced, while the country that specializes in homogeneous goods stops growing because it does not produce heterogeneous goods and technology transfer is absent. Trade becomes in a growth trap.

The last two scenarios show how knowledge spillovers are a fundamental source of technology transfer between countries and how they contribute to reducing the gap between asymmetric countries. It has been explained in both development theory and economic history how developed (United States, England, Japon, etc.) and emerging economies (China, Taiwan, Singapore, etc.) have taken advantage of knowledge spillovers to increase technology transfers and reduce the productivity gap for the greatest economies, Chang (2001).

These scenarios of complete specialization, as well as the other cases mentioned above, have direct implications for the dynamics of productivity of the countries, as well as for the different economic variables that compose the model, determining their path and their levels at every moment in time. The expansion path of the state variable of the productivity gap in the last cases of complete specialization is divergent, and so supernumerary income, productivity and production are divergent too.

3.2 Welfare

In this section, I analyze the dynamic effects of international trade on welfare. Welfare, in the case of an open economy, is determined by the following utility expression, which relates to different variables present in the model. The state variable $\widehat{A_t}$, which drives the dynamics of the model and determines the number of varieties produced in each country, as well as the supernumerary income in each region; furthermore, the earnings for diversity in the products, represented by the number of varieties of manufactured goods available after trade

$$U_t = \Omega + \int_0^\infty e^{-\rho t}\left[\ln \widehat{A_t} - (1-\alpha)\frac{\sigma-1}{\sigma}\ln\left(n_t + n_t^*\left(\frac{1}{\tau}\right)^{\frac{\sigma}{\sigma-1}}\right)\right]dt \tag{49}$$

$$\text{Where } \Omega = \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha) - \ln\gamma - (1-\alpha)\ln\left(\frac{\beta}{\sigma}\right) \tag{50}$$

Looking for a comparative framework for the effects of international trade, the results of trade in terms of welfare can be contrasted with welfare levels under autarky. Rewriting the intertemporal utility equation at autarky:[11]

$$U_t^A = \Omega + \int_0^\infty e^{-\rho t}\left[\ln \widehat{A_t^A} - (1-\alpha)\frac{\sigma-1}{\sigma}\ln\left(n_t^A\right)\right]dt \tag{51}$$

The subtraction between the two intertemporal utilities leads to the following expression, which presents the welfare differential in terms of the evolution of the productivity level and the number of

[11] Hereafter, the variables corresponding to autarky have the superscript A.

available varieties of heterogeneous good

$$U_t - U_t^A = \int_0^\infty e^{-\rho t} \left[\ln \left(\frac{\widehat{A_t}}{\widehat{A_t^A}} \right) + (1-\alpha) \frac{(1-\sigma)}{\sigma} \ln \left(\frac{n_t}{n_t^A} + \frac{n_t^*}{n_t^A} \left(\frac{1}{\tau} \right)^{\frac{\sigma}{\sigma-1}} \right) \right] \tag{52}$$

The variation in the intertemporal utility - which is a discounted sum of intratemporal variations - fundamentally depends on two differentiated effects. First, the effects of the available varieties of heterogeneous goods, which increases utility levels after trade, as shown in Giraldo and Jaramillo (2016). This effect is always positive.

$$Varieties\ Effect = (1-\alpha) \frac{(1-\sigma)}{\sigma} \ln \left(\frac{n_t}{n_t^A} + \frac{n_t^*}{n_t^A} \left(\frac{1}{\tau} \right)^{\frac{\sigma}{\sigma-1}} \right) \tag{53}$$

Second, the income effect (or supernumerary income effect), which is a direct result of trade effects on the variable of the productivity gap between countries.

$$Income\ Effect = \ln \left(\frac{\widehat{A_t}}{\widehat{A_t^A}} \right) \tag{54}$$

This income effect can go in any direction, depending on the effects of international trade on H_t. Thus the income effect depends directly on the convergence or divergence scenario, which entails a trading relationship. Convergent scenarios after trade will generate a positive dynamic effect that raises the level of welfare. Divergent scenarios after trade will produce a dynamic effect that is positive in countries that specialize in the production of heterogeneous goods. However, the dynamic effect on countries that specialize in the production of homogenous goods will be positive or negative, depending on the presence, or not, of knowledge spillovers.

Figure 2 shows the simulation of trade effects on welfare for the domestic country by comparing utility levels under trade and autarky in three possible scenarios with positive knowledge spillovers. Trade instantaneously increases utility in relation to autarky levels in the three simulated scenarios. However, in time, the effects of international trade on economic growth are only completely positive for a convergence scenario, or a divergence scenario with complete specialization in heterogeneous goods for the domestic country.

For the scenario in which trade generates divergence with complete specialization in heterogeneous goods for the foreign country, the levels of welfare are worse in the domestic country after the first period but they start to improve after about twenty periods.[12] The effect of the first period represents the standard static effect of greater available varieties, which improves welfare instantaneously. The negative effect of the following periods demonstrates the income effect, which is negative and greater than the varieties effect for these periods due to the loss of a manufactured sector for the domestic

[12] The number of periods before than the welfare levels start to improve depends on the levels of knowledge spillovers (δ).

country. However, productivity growth through knowledge spillovers allows for an increase in income levels for the domestic country and counteracts the negative productivity effect until welfare after trade starts to become better than under autarky. In the long run, welfare is better under trade than under autarky in any scenario with positive knowledge spillovers.

Figure 2: Welfare levels in relation to autarky with positive knowledge spillover

Figure 3 shows the simulation under complete specialization without knowledge spillover between countries after trade. There are two possible divergent scenarios, with complete specialization in heterogeneous goods production at the domestic or foreign level. Trade produces an instant positive effect in both scenarios (the static varieties effect). However, in time, the effects of international trade on welfare are only positive for a divergent scenario with complete specialization in heterogeneous goods at the domestic country level. For the case in which trade generates divergence with complete specialization in heterogeneous goods at the foreign country level, the levels of welfare will deteriorate over time in relation to autarky levels in the domestic market.

The effects are clear and show the importance of knowledge spillovers in the dynamic effects of trade on economic growth. Without technology transfer, trade only carries implications for the number of available varieties in the global market, while income levels are self-determined in each market. As a result, productivity, per capita income and utility levels depend on the types of goods in which each country specializes.

Figure 3: Welfare levels in relation to autarky without positive knowledge spillovers

In short, the effects of trade on economic growth and welfare are only positive for domestic markets in the following cases. Trade with a similar country in the presence of high levels of knowledge spillover and low transportatio costs, or trade with a country with a smaller market size, which allows it to take a divergence path with complete specialization in heterogeneous goods in the domestic market through HME.

Trade with a large country generates negative implications for domestic growth and welfare. Under positive knowledge spillovers there exists income divergence and less welfare than under autarky in the short term. Without knowledge spillovers, divergence in growth rate and income occurs and, furthermore, welfare levels are at their worst, even when compared with the conditions under autarky. According to this section, autarky is strictly preferred to free trade in this last case.

In contrast to the mainstream theory of international trade, the model presented in this paper shows that the trade effects on economic growth and welfare may be positive or negative. The long term result of international trade depends on the types of countries that are trading and the economic policy that regulates any trade. The best results of the trade are presented when similar countries trade, or different countries trade with high rates of knowledge spillover.

4 Conclusions

In this paper, I analyze the conditions under which trade has positive and negative effects on growth. The results show some elements that contribute to this paradigmatic discussion.

The existence of different productivity expansion paths shows the diversity of scenarios that might be generated according to the countries that trade. The divergence or convergence of these paths reveals the impact of international trade on economic growth. In particular, the results show that productivity levels, per capita income, knowledge spillovers and population size are determinants of the effects of international trade on economic growth.

Trade between similar countries in the presence of knowledge spillover generates positive effects for the countries. The countries converge to a steady state with equal long-term growth rates and better welfare than in autarky. When there are low levels of knowledge spillover or the countries' sizes are very asymmetrical, international trade causes different effects on growth. The divergent scenarios show that the country with a greater market size improves its productivity and income levels after trade, while the smaller country would have positive or negative results depending on the existence of knowledge spillovers. The productivity gap between countries expands over time, so that income and productivity levels are more distant every time, until a new steady state in a new regime of complete specialization is established. Under the complete specialization scenario with knowledge spillover the long-term growth rate is equal between the countries, but the levels of productivity and income are divergent. However, the complete specialization scenario without knowledge spillover produces divergence in both the long-term growth rate and income and productivity levels.

The effects on welfare go in the same direction as the effects on growth. In cases in which countries converge, welfare is greater in relation to the welfare levels under autarky for both countries. For divergent cases, welfare after trade is better for both countries in the long run only when technology transfer is present (positive knowledge spillover). Without technology transfer, only the country that keeps growing after trade sees an increase in its welfare levels with respect to autarky, while its counterpart reduces these utility levels permanently. The last dynamic effect occurs despite the fact that initially both countries increase their welfare, which is reflected in the positive static effect widely presented in static international trade models.

The discussion surrounding this relationship continues and these results are nothing more than a contribution that aims to direct the discussion towards the search for more scenarios and determinants that clarify the different implications of economic globalization on country development.

References

[1] Acemoglu, D., Johnson, S., & Robinson, J. (2005). The Rise of Europe: Atlantic trade, institutional change and economic growth. American Economic Review, 546-579.

[2] Aghion, P., & Howitt, P. (2005). Growth with Quality-improving Innovations: An integrated framework. Handbook of Economic Growth.

[3] Baldwin, R., Martin, P., & Ottaviano, G. (2001). Global Income Divergence, Trade and Industrialization: The geography of growth take-offs. Journal of Economic Growth, 5-37.

[4] Barro, R., & Sala-i-Martin, X. (1992). Convergence. Journal of Political Economy, 223-251.

[5] Ben-David, D. (1993). Equalizing Exchange: Trade liberalization and income convergence. Quarterly Journal of Economics, 653-679.

[6] Chang, H.-J. (2001). Intellectual Property Rights and Economic Development: Historical lessons and emerging issues. Journal of Human Development: A Multi-disciplinary journal for people-centred development, 287-309.

[7] Chang, R., Kaltani, L., & Loayza, N. (2009). Openness can be Good for Growth: The role of policy complementarities. Journal of Development Economics, 33-49.

[8] Chung, C. (2006). Nonhomothetic Preferences and the Home Market Effect: Does Relative Market Size Matter? Georgia Institute of Technology.

[9] Crozet, M., & Trionfetti, F. (2008). Trade costs and the Home Market Effect. Journal of International Economics, 309-321.

[10] Dollar, D. (1992). Outward-oriented developed economies really do grow more rapidly: Evidence from 95 LDC's. Economic Development and Cultural Change, 523-544.

[11] Dollar, D. (2005). Globalization, poverty, and inequality since 1980. Oxford University Press, 145-175.

[12] Edwards, S. (1998). Openness, prosuctivity and growth: What do we really know? Economic Journal, 383-398.

[13] Frankel, J. A., & Romer, D. (1999). Does Trade Cause Growth? The American Economic Review, 379-399.

[14] Galor, O., & Mountford, A. (2008). Trading Population for Productivity: Theory and Evidence. The Review fo Economic Studies, 1149-1179.

[15] Gancia, G., & Zilibotti, F. (2005). Horizontal Innovation in the Theory of Growth and Development. Handbook of Economic Growth.

[16] Giraldo Salazar, I., & Jaramillo, F. (2016). Productivity, Demand and the Home Market Effect. Serie documentos de trabajo Universidad del Rosario(186).

[17] Kind, H. (2002). Endogenous Growth and Trade Liberalization Between Small and Large Countries. Review of International Economics, 151-165.

[18] Krugman, P. (1987). The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Tatcher. Journal of Development Economics, 41-55.

[19] Lucas, R. (1988). On the Mechanics of Economics Develpment. Journal of Monetary Economics, 3-42.

[20] Maddison, A. (1983). A Comparison of Levels of GDP Per Capita in Developed and Developing Countries, 1700-1980. The Journal of Economic History, 27-41.

[21] Martin, P., & Ottaviano, G. (1999). Growing locations: Industry location in a model of endogenous growth. European Economic Review, 281-302.

[22] Melitz, M. J. (2005). When and How should Infant Industries be Protected? Journal of International Economics, 177-196.

[23] Rivera-Batiz, L., & Romer, P. (1991). Internationaltrade with endogenous technological change. European Economic Review, 971-1001.

[24] Rodriguez, F., & Rodrik, D. (2001). Trade Policy and Economic Growth: A Skeptic's Guide to the Cross-National Evidence. NBER Macroeconomics Annual 2000, 261-338.

[25] Romer, P. (1990). Endogenous technological change. Journal of Political Economy, 71-102.

[26] Sachs, J., & Warner, A. (1995). Economic reform and the process of global integration. Brookings Papers on Economic Activity, 647-668.

[27] Sala-i-Martin, X. (1996). The Classical Approach to Convergence Analysis. The Economic Journal, 1019-1036.

[28] Singh, T. (2010). Does International Trade Cause Economic Growth? The World Economy, 1517-1563.

[29] Thoenig, M., & Verdier, T. (2003). A Theory of Defensive Skill-Biased Innovation and Globalization. The American Economic Review, 709-728.

[30] Ventura, J. (1997). Growth and Interdependence. The Quaterly Journal fo Economics, 57-84.

[31] Williamson, J. (1990). Latin American Adjustment: How Much Has It Happened? Washington: Institute for International Economics.

[32] Williamson, J. (2000). What Should the World Bank Think about the Washington Consensus? The World Bank Research Observer, 251-264.

[33] Williamson, J. (2002). Winners and Losers Over Two Centuries of Globalization. NBER working papers.

[34] Young, A. (1993). Substitution and Complementarity in Endogenous Innovation. The Quaterly Journal of Economics, 775-807.

5 Appendices

5.1 Appendix 1: Properties of the dynamic function of the net productivity gap (propositions 1 and 2)

This appendix shows a particular analysis of the characteristics of the dynamic function of the productivity gap, which supports the results presented in the article. In the first instance, it can be said that the part outside the square bracket of the next equation is a positive constant that only modifies the speed rate in the variable H. Therefore, the functional form is determined by the equation within the square brackets.

$$\frac{\dot{H}_t}{H_t} = \left[\frac{\left(\frac{N\gamma^*}{N^*\gamma} - \frac{\tau^{\frac{\sigma}{1-\sigma}}}{H_t} \right)}{\left(1 - \tau^{\frac{\sigma}{1-\sigma}} H_t \frac{N\gamma^*}{N^*\gamma} \right)} (1 - \delta H_t) - \left(1 - \frac{\delta}{H_t} \right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{55}$$

The assumptions about transportation costs and productivity in the model establish $\theta = \tau^{\frac{\sigma}{1-\sigma}}$, and a new variable can be defined $\phi = \frac{N\gamma^*}{N^*\gamma} = \frac{L}{L^*}$.

$$\frac{\dot{H}_t}{H_t} = \left[\frac{\left(\phi - \frac{\theta}{H_t} \right)}{(1 - \theta H_t \phi)} (1 - \delta H_t) - \left(1 - \frac{\delta}{H_t} \right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{56}$$

$$\frac{\dot{H}_t}{H_t} = \left[\frac{\left(\phi - \delta\phi H_t - \frac{\theta}{H_t} + \delta\theta \right)}{(1 - \theta H_t \phi)} - \left(1 - \frac{\delta}{H_t} \right) \right] \widehat{n_t^*} \left(\frac{\mu\sigma}{(1-\sigma)\beta} \right) \tag{57}$$

The functional form is determined by the function inside the square brackets, thus redefining the part of function to be analyzed as $Z(H_t)$:

$$Z_t = \left[\frac{\left(\phi - \delta\phi H_t - \frac{\theta}{H_t} + \delta\theta \right)}{(1 - \theta H_t \phi)} - \left(1 - \frac{\delta}{H_t} \right) \right] \tag{58}$$

First, it is possible to determine the zeros of the function to identify its functional form and its possible equilibriums. Reorganizing the function Z_t.

$$Z_t = \frac{\phi - \delta\phi H_t - \frac{\theta}{H_t} + \delta\theta - 1 + \frac{\delta}{H_t} + \theta\phi H_t - \delta\theta\phi}{(1 - \theta H_t \phi)}$$

The numerator determines the zeros of the function. Equalizing the numerator to zero and multiplying by H finds the following quadratic expression with its respective solution:

$$H_t^2 + \frac{(\phi + \delta\theta - 1 - \delta\theta\phi)}{(\theta - \delta)\,\phi} H_t - \frac{1}{\phi} = 0$$

$$H_{1,2} = \frac{-\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right) \pm \sqrt{\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)^2 + \frac{4}{\phi}}}{2} \tag{59}$$

After solving the quadratic expression one could conclude that the function has two roots, one positive and one negative, the values of which depend, in particular, on the parameter values. The negative segment of this function is of no interest within the context of this model, since it does not have any valid economic interpretation (negative productivity). Similarly, a vertical asymptote is verified when H_t takes the value $\frac{1}{\theta\phi}$ which adds to the information used to define the form that takes different expansion paths. The information obtained about the function's characteristics proves the existence of a stationary equilibrium (*proposition 1*) in the positive root of the function, and the existence of a vertical asymptote which denotes a regime change towards a complete specialization scenario.

After obtaining the above information, one should determine the shape of the expansion paths that cross the equilibrium. Z_t is differentiated in order to determine these paths:

$$\frac{\partial Z_t}{\partial H_t} = \frac{\theta - \delta\phi H_t^2 - 2\theta^2\phi H_t + \theta\phi^2 H_t^2 + \delta\theta^2\phi H_t^2 - \delta + 2\delta\theta\phi H_t - \delta\theta^2\phi^2 H_t^2}{(1 - \theta H_t \phi)^2 H_t^2}$$

Organizing the expression, the following is found:

$$\frac{\partial Z_t}{\partial H_t} = \frac{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) H_t^2 + \left(2\delta\theta\phi - 2\theta^2\phi\right) H_t + (\theta - \delta)}{(1 - \theta H_t \phi)^2 H_t^2}$$

The denominator is always positive. The differentiation sign will depend on the values that take the numerator. In particular, for the following calculations we will call $b(H_t)$ the numerator, which fundamentally defines the values of the differentiation.

$$b(H_t) = \left[\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) H_t^2 - 2\theta\phi\,(\theta - \delta)\, H_t + (\theta - \delta)\right]$$

A particular analysis shows:

$$\begin{aligned} b(0) &= (\theta - \delta) \\ \frac{\partial b}{\partial H_t} &= 2\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) H_t - 2\theta\phi\,(\theta - \delta) \\ b'(0) &= -2\theta\phi\,(\theta - \delta) \\ \frac{\partial^2 b}{\partial H_t^2} &= 2\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right) \end{aligned} \tag{60}$$

These criteria determine intervals in which the function is concave or convex and, therefore, if it has a minimum or a maximum in the critical value where the first differentiation is zero:

$$\frac{\partial b}{\partial H_t} = 0 \Leftrightarrow H_t^{critical} = \frac{\theta\phi\,(\theta-\delta)}{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right)}$$

Entering this critical value $H_t^{critical}$ in the function $b(H_t)$:

$$b(H_t^{critical}) = \frac{-\theta^2\phi^2\,(\theta-\delta)}{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right)} + 1$$

If the parameter values generate a $b(H_t)$ function with positive concavity and the function evaluated at the critical value is positive, it means that the slope of the function Z_t is always positive. If the parameter values generate a $b(H_t)$ function with negative concavity and the function evaluated at the critical value is negative, it means that the slope of the function Z_t is always negative. In other cases, a change of slope that modifies the expansion paths is presented. After evaluating $b(H_t^{critical})$ it has the following inequality:

$$b(H_t^{critical}) > 0 \Leftrightarrow \frac{\theta^2\phi^2\,(\theta-\delta)}{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right)} < 1 \tag{61}$$

The inequality results depend in the first instance on the value that takes the denominator. This value is determined by the values of the parameter ϕ, which is related to the labor force size of the countries trading.

$$\begin{aligned} \text{Denominator} &> 0 \Leftrightarrow \theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2 > 0 \\ &\Leftrightarrow \theta\,(1-\delta\theta)\,\phi^2 + \delta\left(\theta^2 - 1\right)\phi > 0 \\ \phi &> \frac{\delta\left(1-\theta^2\right)}{\theta\,(1-\delta\theta)} = \frac{\delta-\delta\theta^2}{\theta-\delta\theta^2} \end{aligned} \tag{62}$$

Solving for ϕ the polynomial in the denominator, the root of the same is obtained $\phi = \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ which establishes the threshold where the inequality changes direction. This value is the same critical point that determines the sign of the second differentiation, that is, the concavity or convexity of the function. These results define the function's form, given the values that take different parameters. Two cases may occur in the resolution of the inequality:

When $\phi > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ the denominator is positive and the function is convex. If in addition the next condition on the parameter ϕ is accomplished then together these two criteria guarantee the existence of a positive minimum.

$$\begin{aligned} \frac{\theta^2\phi^2\,(\theta-\delta)}{\left(\theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2\right)} &< 1 \Leftrightarrow \theta^2\phi^2\,(\theta-\delta) < \theta\phi^2 + \delta\theta^2\phi - \delta\phi - \delta\theta^2\phi^2 \\ \theta^3\phi^2 &< \theta\phi^2 + \delta\theta^2\phi - \delta\phi \\ 0 &< \theta\left(1-\theta^2\right)\phi^2 + \delta\left(\theta^2 - 1\right)\phi \\ \frac{\delta}{\theta} &< \phi \end{aligned} \tag{63}$$

When $\phi < \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ the denominator is negative and the function is concave. If in addition the next condition on the parameter ϕ is accomplished then together these two criteria guarantee the existence of a maximum negative.

$$\begin{aligned}
\frac{\theta^2\phi^2\left(\theta-\delta\right)}{\left(\theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2\right)} &< 1 \Leftrightarrow \theta^2\phi^2\left(\theta-\delta\right) > \theta\phi^2+\delta\theta^2\phi-\delta\phi-\delta\theta^2\phi^2 \\
\theta^3\phi^2 &> \theta\phi^2+\delta\theta^2\phi-\delta\phi \\
0 &> \theta\left(1-\theta^2\right)\phi^2+\delta\left(\theta^2-1\right)\phi \\
\frac{\delta}{\theta} &> \phi
\end{aligned} \tag{64}$$

These findings for the characteristics of the differentiation of function Z_t generate six different scenarios according to the parameter values and the interval in which the parameter ϕ occurs.

Condition 1: If $\theta > \delta$ there are three different criteria according to the interval in which the parameter ϕ is, and these determine the function's form.

Case 1: If $\phi > \frac{\delta}{\theta} > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ the two conditions of ϕ (see (63)) are reached, the minimum value is positive, therefore the function $b(H_t)$ is always positive. The function $Z(H_t)$ always increases and describes a divergent expansion path of the productivity gap between countries after trade.

Case 2: If $\frac{\delta}{\theta} > \phi > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)}$ only one of the two conditions of the parameter ϕ (see (63)) is reached. This implies that the minimum in the function is not always positive and that there is a slope change in the $Z\left(H_t\right)$ function. However, appendix 2 indicates that in this case the equilibrium $H^{Equilibrium}$ is always subsequent to the asymptote, which means that the path is still divergent since it occurs in a complete specialization regime.

Case 3: If $\frac{\delta}{\theta} > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \phi$ the two conditions on the parameter ϕ (see (64)) are reached, but given the condition $\theta > \delta$ the maximum is positive (see abscissa intercept (60)) and the function decreases in the interval (see second derivative criterion). This result presents the existence of a slope change in the $Z\left(H_t\right)$ function, so this variation also involves a change in the direction of the expansion path, meaning that it decreases. However, as in the previous criterion the equilibrium point is always more to the right than the asymptote of the function, which implies a divergence in the expansion path despite its change of direction because it occurs in a complete specialization regime (appendix 2).

Condition 2: If $\theta < \delta$, as in the previous condition, there are three different criteria determined by ϕ values that describe the function's shape.

Case 1: If $\frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \frac{\delta}{\theta} > \phi$ the two conditions of ϕ (see (64)) are reached, the maximum value is always negative, so the function $b(H_t)$ is always negative. This demonstrates that the $Z(H_t)$

function always decreases and describes a convergent expansion path for the productivity gap between countries after trade (*proposition 2*).

Case 2: If $\frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \phi > \frac{\delta}{\theta}$ only one of two conditions of the parameter ϕ (see (64)) is reached. This implies that the maximum in the function is not always negative and that there is a slope change in the $Z(H_t)$ function. With appendix 2 in mind, this may show that this case involves a divergent complete specialization scenario.

Case 3: If $\phi > \frac{\delta\left(1-\theta^2\right)}{\theta(1-\delta\theta)} > \frac{\delta}{\theta}$ the two conditions of the parameter ϕ (see (63)) are reached, but given the condition $\delta > \theta$ the minimum is negative (see intercept abscissa (60)) and the function increases. This implies a slope change in the $Z(H_t)$ function and therefore in the path direction. As in previous atypical cases, appendix 2 shows that this case represents a divergent complete specialization scenario.

The analysis of the different possible cases in the trade between different countries shows that there is only one equilibrium $H^{Equilibrium}$ (*proposition 1*) and one convergent path achieved when $\delta > \theta$ and $\frac{\delta}{\theta} > \phi$ (*proposition 2*). The other cases generate complete specialization for the production of countries after trade and a divergent expansion path in H_t.

After knowing the behaviour of function $Z(H_t)$ via its properties and the properties of its differentiations $b(H_t)$, we can return to the function (56) to determine the trajectories of the state variable H_t, as defined above. The dynamic productivity function will be:

$$\frac{\dot{H_t}}{H_t} = Z(H_t)\widehat{n_t^*}\left(\frac{\mu\sigma}{(1-\sigma)\,\beta}\right) \tag{65}$$

Accordingly, the different expansion paths are determined by the Z function multiplied by a positive constant (in intervals with economic interpretation), which does not alter its functional form.

5.2 Appendix 2: The relation between equilibrium and asymptote

Cases 2 and 3 from the two conditions in the previous appendix can be solved by determining whether the equilibrium comes before the asymptotic behavior of the function where economic interpretation is present under the incomplete specialization of the countries, or, outside of this, where a complete specialization scenario exists and the function that determines the expansion path is different. In particular, it is necessary to determine whether:

$$\frac{-\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)+\sqrt{\left(\frac{\phi+\delta\theta-1-\delta\theta\phi}{(\theta-\delta)\phi}\right)^2+\frac{4}{\phi}}}{2} > \frac{1}{\theta\phi}$$

It can be shown that this inequality is always reached for the criteria in question. Therefore, regardless of the slope change in the expansion path for criteria 2 and 3 of both conditions, these are always divergent, since they occur in complete specialization scenarios.

6 Figures

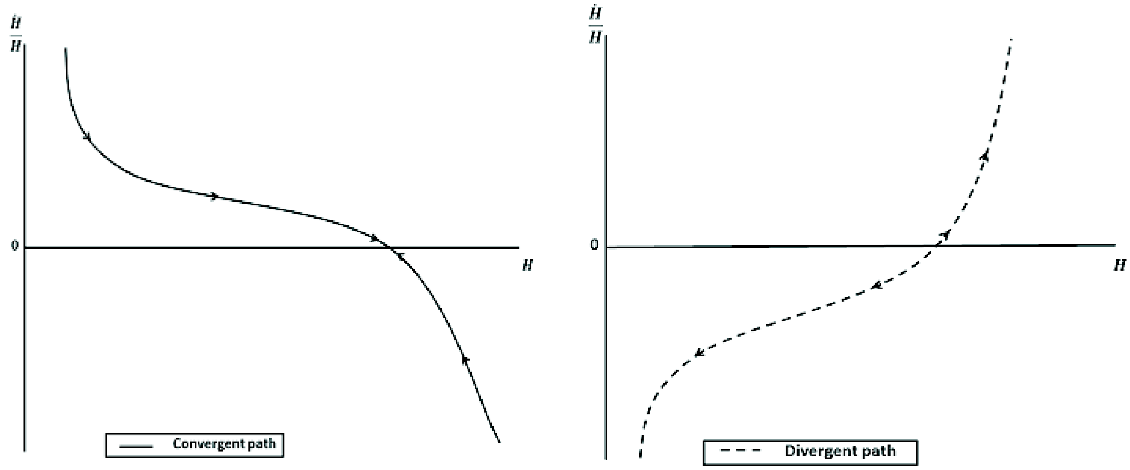

Figure1:Convergent and divergent path of H

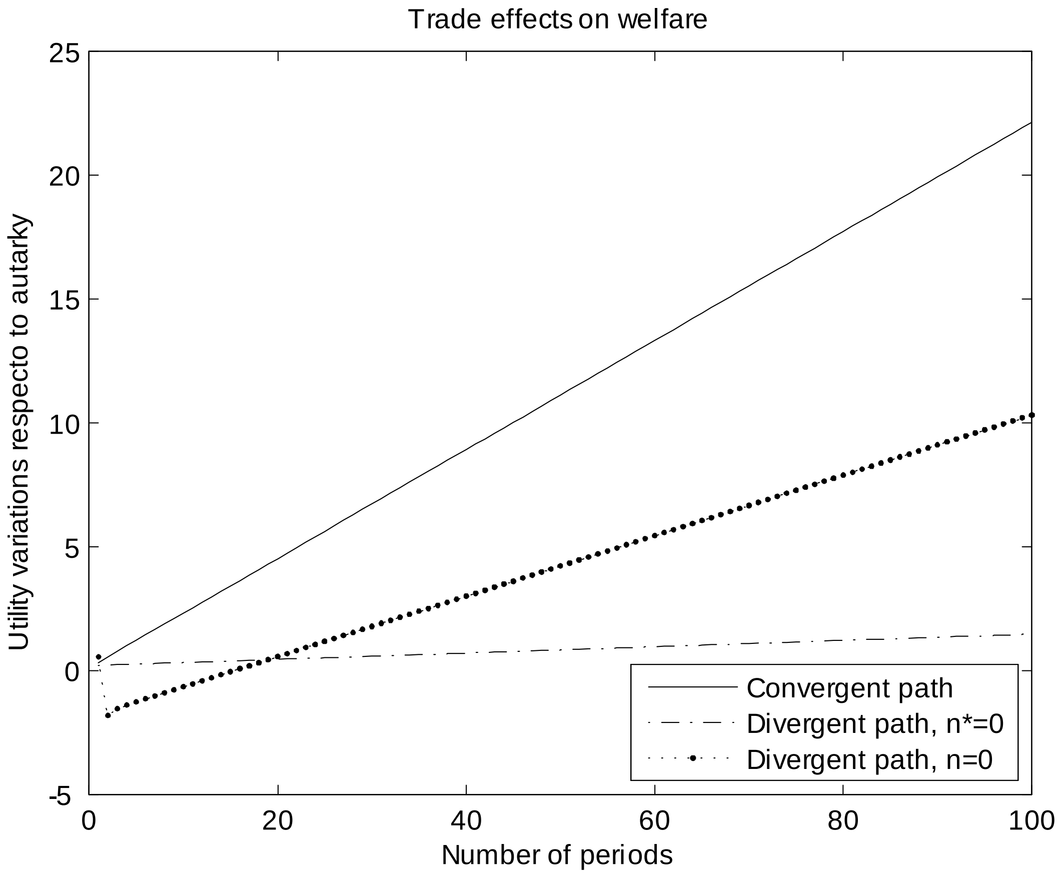

Figure 2: Welfare levels in relation to autarky with positive knowledge spillover

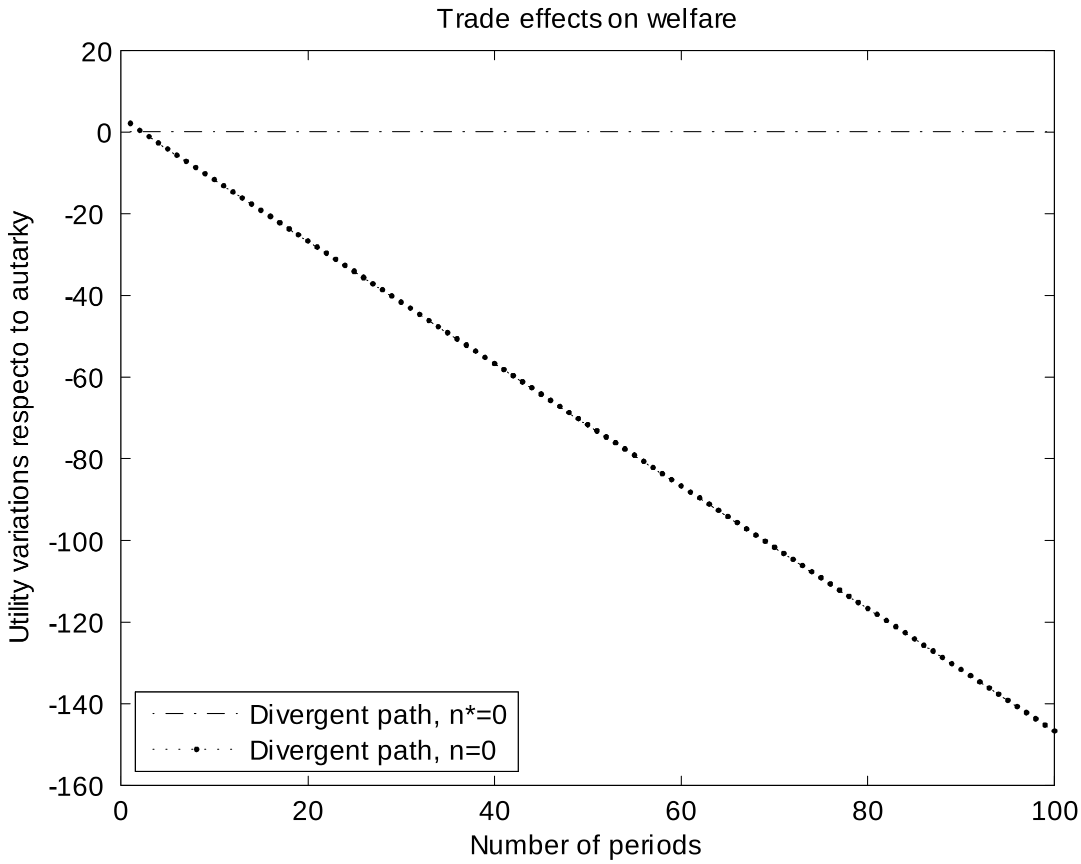

Figure 3: Welfare levels in relation to autarky without positive knowledge spillovers