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SEASONS, SAVINGS AND GDP

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Abstract

The industrial revolution and the subsequent industrialization of the economies occurred first in temperate regions. We argue that this and the associated positive correlation between absolute latitude and GDP per capita is due to the fact that countries located far from the equator suffered more profound seasonal fluctuations in climate, namely stronger and longer winters. We propose a growth model of biased innovations that accounts for these facts and show that countries located in temperate regions were more likely to create or adopt capital intensive modes of production.

The intuition behind this result is that savings are used to smooth consumption; therefore, in places where output fluctuations are more profound, savings are bigger. Because the incentives to innovate depend on the relative supply factors, economies where savings are bigger are more likely to create or adopt capital intensive technologies.

Journal of Economic Literature Classification: N00, O00, 011, 031, 033.

Keywords: absolute latitude, seasons, endogenous growth, capital using innovations.

1 Introduction

The industrial revolution occurred first in temperate regions and since then the world have experienced a massive absolute divergence in the distribution of incomes across countries. Indeed, today there exists a positive correlation between absolute latitude and GDP per capita (see figure 1). Moreover, only three tropical economies (Hong Kong, Singapore, and Taiwan) are classified as high-income by the World Bank, while all countries within regions zoned as temperate had either middle or high-income economies.

[Insert Figure 1 about here]

We argue that climatic conditions affect incentives to save in primitive economies in such a way that economies located in temperate regions are more likely to become capital abundant. In primitive economies the main source of output fluctuations is climatic and these are bigger in places located far away from the equator. Indeed, both the harvest per year and the diversification of crops are higher in tropical countries (see Chang, 1997). During the frost days in winter there is no harvest, transportation is difficult, people need more energy and, in general, surviving demands much more work. The response of people to these natural forces is saving during the good days to make bad days better. The stronger the winter the higher the level of savings needed to survive during the frost days. Finally, economies where the savings are systematically bigger are more likely to both, create and adopt capital intensive technologies. Once an economy begins to use capital intensive technologies a process of capital accumulation and capital improvement starts. Capital abundance generates the incentives for capital-using innovations and capital-using technologies generate the incentives for capital accumulation.

Seasonal fluctuations have been economically important in the past, when people

were more dependant on nature. Communities with higher savings rates were more likely to adopt capital-using innovations and such innovations had permanent effects in the economic growth path because of the relation between capital abundance and innovations described above. Nowadays, living infrastructure and new technologies of production have reduced the need of savings to smooth consumption but the effect of seasons on savings and GDP can help to explain the relation between geography and income in the present.

Summarizing, since in primitive economies savings depended on seasonal fluctuations, communities located in temperate regions were more likely to adopt capital-using technologies and, as stated before, after the adoption of a capital intensive technology a virtuous circle derived the economy to long-run growth.

Agents can respond to seasonal changes in output in different ways: (i) using financial institutions that help people smooth consumption and allocate resources to productive projects, (ii) increasing trade with economies where the timing of seasons is different or, (iii) inventing or adopting machines that allow people to produce goods using less quantities of other factors, namely, land and raw labor. These three activities have positive effects on economic growth, and can be undertaken simultaneously and, in many times, are complementary. In addition, any of these activities can be interpreted as capital-using innovations. However, we want to stress the role of machines for two reasons: first, the industrial revolution was characterized by new ways of production that made use of machines and reduced the need of land; second, high machinery investment have generated rapid economic growth over the 19th century in Canada, Germany, Italy, Japan, the United Kingdom and the United States, and a similar association holds since World War II for a broader sample of nations (De Long, 1992). Thus, the role of machines seems to be important to explain both, the beginning of the industrial era and the subsequent economic growth of

industrialized countries.

We refer to capital goods and capital-using innovations as goods and technologies used in the production process. Durable goods and technological advances that are not used to produce marketable goods are considered as consumption goods [1]. This distinction is important because long before the industrial revolution communities located in tropical regions developed techniques and built facilities which increased their welfare but were not used to produce new goods.

Similarly, we assume that any capital intensive technology can be adopted paying a cost and agents decide whether or not to pay the cost.[2] Therefore, capital-using modes of production are only adopted where agents have economic incentives to pay for them. Innovations like the ones that led to the industrial revolution had been made much earlier and, in many times, outside Europe. We argue that these innovations were not used for production before because a minimum level of capital was needed in order to make profitable the introduction of the new techniques. Similarly, the reason why some economies used the innovations for productive purposes and others did not was the difference in factor abundance. If this hypothesis is correct, the economies where industrialization occurred first must had higher capital labor ratios. We do not have a direct measure of capital stocks before the industrial revolution. However, according to Maddison (2003) GDP per capita was higher in the countries where industrialization occurred first: UK, Belgium and Netherlands (see table 1). Therefore, under reasonable assumptions, we can infer that the capital-labor

[1] Cathedrals, swimming pools and armies are not capital goods.

[2] According to the literature of biased technological change the incentives to innovate depend on the relative supply of factors: in capital abundant economies individuals have incentives to make capital-using innovations while in economies where capital is scarce there are no incentives for this type of innovation (Zeira, 1998 and 2006; Acemoglu, 2002; Boldrin and Levine, 2002; Zuleta, 2006; Peretto and Seater, 2006 among others). These innovations increase the elasticity of output with respect to capital as well as the incentives to save, so a virtuous circle derives capital abundant economies to long-run growth. Models of factor saving innovations generally predict that both the elasticity of output with respect to capital and the capital income share must be higher in richer economies.

ratio was higher in UK, Belgium and Netherlands than in the rest of Europe.

[Insert Table 1 about here]

Additionally, four pieces of evidence motivate this work:

(i) During the process of industrialization, the shift of labor away from agriculture and the increase in the size of the firms led to a decline in the proportion of self employed (see Prados de la Escosura and Roses, 2003). Therefore, we should observe an increase in the labor income share during this process. However, the share of labor decreased between 1856 and 1913 for the UK, the Netherlands and the US: in the UK the wages and salaries as a percentage of national income fell from 50 to 48 (see Mathews, Feinstein and Odlig-Smee, 1982). In the Netherlands wages and salaries fell from 45 to 38 percent of national income. In the US labor share as a percentage of national income fell from 66 to 62. These facts imply that, as the proportion of self employed declined, another process was driving up the capital income share. We claim that the force behind the decline in labor shares was a process of capital-using innovations. In a related work, regarding the British industrial revolution, Allen (2005) states that between 1800 and 1840, GDP per worker rose 37%, real wages stagnated, and the profit rate doubled. In summary, the share of profits in national income expanded at the expense of labor and land.

(ii) High rates of economic growth generally coincide with high levels of savings (see Aizenman, Pinto and Radziwill, 2004 and Hausmann, Pritchett, and Rodrick, 2004).

(iii) Previous empirical works in economic growth find that both the GDP per capita (Theil and Galvez, 1995; Irwin and Tervio, 2002) and the growth rate of GDP per capita (Sala-i-Martin 1997) depend positively on the absolute latitude.

(iv) Masters and McMillan (2001) find that the frequency of frost in winter, after frost-free summer, is a key variable to explain differences in growth paths among

countries. We claim that the variables used by Masters and McMillan (2001) are a good proxy for seasons strength.

The model we present cannot account for the timing of the industrial revolution and cannot explain why it occurred first in Great Britain. However, our story is consistent with some sensible explanations.

Voth and Voigtländer (2006) explain why the industrial revolution occurred in Great Britain using a probabilistic two-sector model. They argue that weather-induced shocks to agricultural productivity were the ultimate causes of the industrialization. As it will become apparent, in our model an exogenous shock to agricultural productivity generates an increase in savings and may trigger the industrial revolution.

Some scholars identify the ultimate causes of the industrial revolution in the Crusades which re-established traffic between the East and West after having been suspended for several centuries[3]. Along with trade, new scientific discoveries and inventions made their way from east to west and the Arabic advances including the development of algebra, optics, and refinement of engineering arrived to Europe. The new knowledge coming from the east contributed to the development of usable techniques of production that impulsed the Industrial Revolution (Sabato, 1991). Additionally, the Crusades exposed European people to new germs and viruses coming from eastern countries. As a result, new plagues with destructive social effects devastated European cities. Now, the decline in population increased the amount of food available per capita, raised nutritional status and population growth. Population growth, on its turn, fostered urbanization, knowledge creation and the expansion of the market. "The cycle of population growth, capital accumulation, market

[3] Alternative explanations focused mostly on the role played by demography in general and life expectancy in particular (see Galor and Moav, 2002).

expansion, crisis, followed by population growth again could proceed until by the eighteenth century the European societies were sufficiently advanced to break out of the Malthusian trap." (J. Komlos, 1989)

In terms of our model, the new scientific discoveries and inventions are the introduction of a new set of technologies and the decline in population is an increase in the capital labor ratio. As it will become apparent, both events can trigger the adoption of capital intensive technologies.

We formalize our argument with a model of endogenous growth with biased technological change. The model uses the notion of capital-using innovations to explain how economies switch from a storage technology to a technology of capital accumulation. We start by assuming a primitive economy where output is produced with not-reproducible factors (land and labor) and a storage technology. We also assume that output does not have a trend but fluctuates seasonally. In particular, we assume that output per worker can take two values, A_l (low) and A_h (high). Since output behaves cyclically, a storage technology is used to smooth consumption. Under such circumstances, savings (storage) are bigger in places where changes in output are bigger. Besides the primitive technology, there exists a set of capital intensive technologies differentiated by their capital intensity, that is, by the elasticity of output with respect to capital. These technologies are costly and the cost is increasing in the capital intensity. Under this setting, a minimum amount of savings is needed for the adoption of capital intensive technologies to be profitable. Therefore, economies where seasonal fluctuations are stronger are more likely to adopt capital intensives modes of production. Finally, once an economy is using capital intensive technologies a process of capital accumulation and technological improvement drives the economy to sustained growth.

The institutional view tells a story of why the industrial revolution spread to

certain regions faster than others, but not why it happened in Europe in the first place. The traditional geographical view can explain why countries where commerce and production are obstructed by natural conditions have, in general, low relative levels of development, but this approach cannot account for the correlation between latitude and GDP. Finally, Diamond (1997) explains why industrialization happened on the Eurasian continent, but is not equally convincing on the issue of why South Asia did not industrialize before Europe. In this sense, our work complements the previous approaches and adds a missing link among their findings.

The rest of the paper is organized as follows. In the second section, we explain how our theory fits into the debate "geography versus institutions". In the third section, we present the general model and its results. In the fourth section we provide an example assuming an explicit production function. In the fifth section we provide some empirical evidence. Finally, some concluding remarks are provided.

2 Geagraphy and Institutions

Several authors, following the classic work of Max Weber (1905), argue that differences in economic performance can be explained by religion (Barro and MacCleary, 2003). They explain the different performance of former English colonies compared to former Spanish or French colonies by the institutional bequest. However, the relation between geography and GDP holds even controlling for religion and language. Additionally, the superiority of British institutions is usually based on the records of the USA and Canada, but Barbados; Belize, Guyana and Jamaica did not perform well in economic terms. They were British colonies located closer to the equator.

In the same vein, Hall and Jones (1999) argue that institutions (instead of climate or location) play a fundamental role in the economic performance of different countries. Along this line, Acemoglu, Johnson and Robinson (2001 and 2002) state

that the disease environment in former European colonies determined the number of European settlements and their subsequent institutional development. Therefore, countries where the environment was friendlier to European settlers benefited from better institutions and, for this reason, have had better economic performance. In the same line, Easterly and Levine (2003) claim that geographical variables affect country income through institutions. Rodrik, Subramanian and Trevi (2004) confirm that, controlling for institutions, geography does not affect GDP.

In principle, the story of seasons and savings can be consistent with the institutional approach: the seasonality of output generates incentives to save and accumulate capital, strengthening the incentives for capital owners to vote, impose or ask for an institutional arrangement that protects their property rights. In countries where property rights are effectively protected the incentives to accumulate capital are bigger. Therefore, economic fluctuations and savings are determining capital abundance and capital abundance is the main cause of both capital intensive technologies and property rights protection. In this case, the result that geography affects GDP only through institutions can be hiding the fact that capital abundance and capital intensive technologies are determining both economic growth and institutions.

Along the institutional line, Engerman and Sokoloff (2002) study countries in the Americas and claim that the differences in economic performance can be explained by the differences in factor abundance. Their idea consists of 3 arguments: (i) differences in factor endowments determine differences in income distribution; (ii) income distribution affects the choice of institutions; and (iii) democratic institutions stimulate economic growth. The idea of Engerman and Sokoloff (2002) is the observational equivalent to the predictions of a Biased Technological Change Model in the sense that factor abundance determines economic growth. According to this view, factor abundance affects the choice of technology, i.e., in labor abundant economies the

optimal technologies are more labor intensive than in labor scarce economies. On their turn, technologies affect the marginal productivity of capital and the incentives to save. Therefore, independently of the institutional framework, in labor abundant economies the incentives to save are weaker because the predominant production factor is labor, not capital.

As stated before, the story of seasons, savings and biased technological change complements the institutional approach. In essence, we find three reasons to believe that the effect of seasonality and biased innovations is important in its own right to explain economic development:

(i) There exists a positive correlation between GDP and both absolute latitude and frost frequency.

(ii) Huber, Rueschemeyer and Stephens (1993) point out that in Europe democracy was a result of economic development. To illustrate this point they indicated that in 1870, only one European country, Switzerland, was a democracy. In contrast, by 1920, almost all Western European countries were fully democratic.

(iii)If institutions are determined by GDP, as suggested by Huber *et. al.* (1993), and Glaeser, La Porta, Lopez-de-Silanes and Shleifer (2005), the result that geography affects GDP only through institutions can be reflecting a positive correlation between current and past income level. Other limitations of the "only institutions matter" approach have been pointed out by a number of scholars. Indeed, this model implies that good institutions should generate higher growth rates during any period. However, Przeworski (2004) finds that during the period 1950-1999 for countries that were colonies as of 1945 "the rate of growth of total output does not depend on institutions". Similarly, Hausmann, Pritchett and Rodrick (2004) show that growth accelerations tend to be correlated with increases in savings and most growth accelerations are not preceded or accompanied by major changes in institutional arrangements.

3 The Model

We first present the case where only a primitive technology, without capital accumulation, is available and explain why a relation between absolute latitude and savings is likely to appear. After that, we consider the possibility of creating or adopting new technologies that use capital and show that economies where savings are bigger, are more likely to adopt new technologies.

3.1 Storage and Technology

We assume identical agents; each agent devotes a constant amount of time to work, $L = 1$ and there is no population growth. We also assume that the production function is linear in not reproducible factors (AL) and that agents can make use of a storage technology. Therefore, the budget constraint for each agent is given by, $c_t \leq A_t + \eta\varsigma_{t-1} - \varsigma_t$, where ς is the amount of stored goods, η is the proportion of stored products that can be consumed after one period ($\eta \leq 1$) and A is the output per worker.

Note that in this model savings (s) are completely allocated to storage, namely, $s_t = \varsigma_t$.

We also assume that output behaves seasonally in such a way that if current output is low ($A_t = A_l$) then future output is high ($A_{t+1} = A_h$).

The problem of the representative consumer is the standard one,

$$\begin{aligned} & Max_{\{c_t\}} \sum_{t=0}^{\infty} \log(c_t)\beta^t \\ s.t. \quad & c_t = A_t + \eta\varsigma_{t-1} - \varsigma_t \\ & \varsigma_t \geq 0 \end{aligned} \tag{1}$$

Form where,

$$c_{t+1} = \begin{cases} c_t \eta\beta & \text{if } A_{t+1} + \eta\varsigma_t \geq \beta\eta c_t \\ A_{t+1} + \eta s_t & \text{if } A_{t+1} + \eta\varsigma_t < \beta\eta c_t \end{cases} \tag{2}$$

Therefore, the most desirable consumption path is given by $\frac{c_{t+1}}{c_t} = \eta\beta$. However, the fluctuations of output and the fact that storage cannot be negative constitute an impediment to achieve this path. For this reason, in some periods the consumer chooses to have zero or negative savings[4].

Formally, savings in the loung-run equilibrium can be characterized as follows (the proof is presented in the Appendix 7.1):

$$\varsigma_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } A_{t+1} \geq \eta\beta A_t \end{cases} \tag{3}$$

Thus, the average size of assets per capita depends on the difference between A_l and A_h. Moreover:

(i) If $\frac{1}{\eta\beta} \geq \frac{A_h}{A_l}$, then $\varsigma_t = 0\ \forall t$, that is, in economies where fluctuations in output are small the optimal amount of savings is zero.

(ii) If $\frac{A_h}{A_l} > \frac{1}{\eta\beta}$, then savings are an increasing function of $\eta\beta A_h - A_l$. In other words, savings are higher in places where changes in output are higher.

(iii) If $\frac{A_h}{A_l}$ is constant and $\frac{A_h}{A_l} > \frac{1}{\eta\beta}$, then savings are an increasing function of average output.

(iv) Holding the rest constant, savings are an increasing function of the productivity of the storage technology.

In primitive economies the main source of fluctuations is climatic and such fluctuations are bigger in places located far away from the equator. Therefore, results (i)

[4]Note that individuals store goods only for the one period because it is optimal. They could store for more than one period but they choose no to do it.

and (ii) imply that in places located far away from the equator savings are likely to be higher. Result (iii) implies that savings are higher in places where land productivity is higher. Finally, result (iv) implies that savings are higher in places where the storage technology is better.

Another feature of the model is that, holding the rest constant, the consumers utility is higher in places where fluctuations are smaller (because $\eta \leq 1$). This means that in primitive economies, welfare was likely to be higher in places close to the equator[5].

Recall that consumption goods include food, leisure, housing and public goods. Therefore, the fact that some countries located in tropical zones enjoyed this type of goods before any European country is consistent with our story.

3.2 Innovations

Now consider that savings (s) may be devoted to storage (ς), to create or adopt new technologies (α) and to accumulate capital (K) that can be used in the new technologies ($F(K, \alpha)$).

Here, we assume that better technologies make capital goodsmore productive, that is, if $\alpha_1 > \alpha_0$ then $F(K, \alpha_1) > F(K, \alpha_0)$, but they are also more costly. The cost of a technology α is given by $g(\alpha)$ where $g'(\alpha) > 0$.

Therefore, the cost of the technology increases as the technology becomes more capital intensive. This assumption may be justified in two ways. On the one hand, since Jones (1995) diminishing returns have been a standard assumption in growth models. On the other hand, relaxing this assumption does not affect qualitatively the main predictions of the model.

[5] A possible extension of this model, including migrations, may help to explain why the first civilizations appeared in regions that were not temperate (Southern China, India, Mesopotamia, Peru, Guatemala and Mexico).

We also refer to α as the capital intensity of the technology because an increase in α augments the return on capital and, for this reason, stimulates capital accumulation. Similarly, we refer to increases in α as capital-using innovations.

In this setting, the difference between production and consumption can be stored or exchanged for capital goods. In the latter case, savings can be used to increase the number of capital goods of a given quality or to improve the quality of a given number of capital goods. Therefore, $s_t = \varsigma_t + \Delta K_t + g(\alpha_t)$, where $\Delta K_t = K_{t+1} - K_t$.

The new production function, $F(K, \alpha)$, has the following properties:

- If $K \geq 0$, the production function is increasing and concave in the stock of capital $\left(\frac{\partial F(\cdot)}{\partial K} > 0\right)$ for any technology $\alpha \in [0, 1)$ and linear in K for $\alpha = 1$.

- It is increasing in the technology $\left(\frac{\partial F(\cdot)}{\partial \alpha} > 0\right)$ for any $K \geq 0$.

- Technology and capital are essential ($F(0, \alpha) = 0$ and $F(K, 0) = 0$).

- Technology and capital are complementary $\left(\frac{\partial^2 F(\cdot)}{\partial \alpha \partial K} > 0\right)$ for any $K \geq 0$.

- The stock of capital depreciates at a rate δ.

As in the previous section, the consumer has to choose savings and consumption. In a period of high output the consumer saves part of it. Savings can be stored and consumed during the next period or invested in a technology of capital accumulation. In the second case, savings have to be divided between capital accumulation and technology improvement.

The problem of the representative consumer is the following:

$$\begin{aligned} & Max_{\{c,s,K,\alpha\}} \sum_{t=0}^{\infty} \log(c_t)\beta_t \\ s.t. \quad & c_t = A_t + F(K_t, \alpha_t) - (\varsigma_{t+1} - \eta\varsigma_t) - (K_{t+1} - (1-\delta)K_t) - g(\alpha_{t+1}) \\ & \varsigma_t \geq 0, K_t \geq 0, \alpha_t \geq 0, \alpha_0 = 0 \end{aligned} \tag{4}$$

From the first order conditions it follows that the growth rate of consumption is given

by,

$$\frac{c_{t+1}}{c_t} = \max\left\{\eta\beta, \beta\left[(1-\delta) + \frac{\delta F(\cdot)}{\delta K_{t+1}}\right], \beta\left[\frac{\partial F(\cdot)}{\partial \alpha_{t+1}} - g'(\alpha_{t+1})\right]\right\} \tag{5}$$

Since there are three different ways to accumulate wealth, the consumer chooses the most productive one, namely,

(i) If $\frac{\partial F(K,\alpha)}{\partial K} < (\delta + \eta - 1)$ and $\frac{\partial F(K,\alpha)}{\partial \alpha} - g'(\alpha) < \eta$, only the storage technology is used ($K = 0$ and $\alpha = 0$).

(ii) If $\frac{\partial F(K,\alpha)}{\partial K} > (\delta + \eta - 1)$ or $\frac{\partial F(K,\alpha)}{\partial \alpha} - g'(\alpha) > \eta$, the storage technology is not used anymore and the economy accumulates capital ($\varsigma = 0, K > 0, \alpha > 0$).

(iii) If $\frac{1}{\eta\beta} \geq \frac{A_h}{A_l}$, $\frac{1}{\left((1-\delta)+\frac{\delta F(\cdot)}{\delta K}\right)\beta} \geq \frac{A_h}{A_l}$, there are no incentives to save, $s = 0$.

(iv) If the economy accumulates capital, savings are devoted to new technologies and capital accumulation in such a way that the marginal productivity of capital must be equal to the marginal productivity of innovation, namely,

$$(1-\delta) + \frac{\partial F(\cdot)}{\partial K_{t+1}} = \frac{\partial F(\cdot)}{\partial \alpha_{t+1}} - g'(\alpha_{t+1}) \tag{6}$$

Notice that $\alpha_0 = 0$, so initially only the primitive technology is used and there is no capital accumulation. Note also that when the new technology is used, savings are partially devoted to capital accumulation and partially to technological change. Any increase in α generates an increase in the marginal productivity of capital $\frac{\partial F(\cdot)}{\partial K}$, and an increase in K generates an increase in productivity of technology $\frac{\partial F(\cdot)}{\partial \alpha}$. Therefore, when both K and α are growing, the production function can be convex in the amount of savings. If this is the case, there exists a minimum level of savings such that agents have incentives to use capital intensive technologies (in the next section we provide an example).

From results (i) and (ii) if follows that $s > 0$ is a necessary condition for new technologies to be adopted ($\alpha > 0$). The indivisibility of capital goods imposes a

minimum level of savings needed to adopt the technology of capital accumulation. Moreover, when the function $F(\cdot)$ is convex in s, the minimum level-of-savings condition holds. Similarly, from (i), (ii), (iii) and (iv) it follows that when the accumulation technology is used there exists a positive relation between capital intensity (α) and stock of capital (K) (see proof in the Appendix 7.2).

Finally, when the capital intensive technology is used, both the number and the quality of capital goods grow, so the marginal productivity of capital does not necessarily decrease as the stock of capital grows.

From results (i) and (ii) it also follows that, the lower the cost of storage (higher η), the higher the savings needed to adopt a technology of capital accumulation. Therefore, the net effect of the productivity of the storage technology is ambiguous.

These results can be summarized by saying that when savings are low it is optimal for the individuals to use the primitive technology. If savings are high enough it is optimal to create or adopt new technologies and start accumulating capital. Now, since technology is embodied in capital goods which are costly and indivisible, communities with low levels of savings are not able to adopt capital intensive technologies. Finally, once a capital intensive technology ($\alpha > 0$) is adopted, a process of capital accumulation and technological progress starts. Depending on the functions $F(\cdot)$ and $g(\cdot)$, the process of capital accumulation can last forever or end with a finite stock of capital (see Boldrin and Levine, 2002, Zuleta, 2006 and Peretto and Seater, 2006). If there is long run growth then the differences in GDP per worker are likely to increase over time[6].

The main implications of the model are consistent with the empirical evidence. First, GDP per capita correlates with absolute latitude and with frost frequency.

[6] If TFP behaves in such a way that there is long run growth indepently of the shape of $g(\cdot)$ the result would still be the same.

Second, the industrial revolution occurred first in countries located far away from the equator. Third, there was an important increase in agriculture productivity preceding industrial revolution. Fourth, the share of capital increased during the industrialization process in the UK. Fifth, both the share of reproducible factors and the elasticity of output with regard to reproducible factors are positively correlated with GDP per capita. Sixth, growth accelerations are generally associated with high saving rates.

The model also predict that any increase in the capital-labor ratio generates incentives to increase the capital intensity of the technology. Therefore, holding the rest constant, a reduction in the size of the populations generates an increase in the capital intensity of the technology.

4 Example

In this section we present an example where the production function of the capital intensive sector is a Cobb-Douglas, $Y = Bk^{\alpha}$. Thus, increasing α is the only way to have *capital-using* technological change. Savings can be restored and consumed during the next period or invested in technology of capital accumulation. In the latter case, savings have to be divided between capital accumulation and technological change. b_t is the fraction of savings devoted to storage, u_t the fraction of savings devoted to increase the number of capital and $(1 - b_t - u_t)$ is the fraction of savings devoted to increase the quality of capital goods.

Even if technologies were free, in capital scarce economies there are no incentives to adopt capital-using technologies. Figure 2 illustrates this fact: when the capital labor ratio is smaller than one ($k < 1$) a capital-using innovation (increase in α) reduces output, so it is better to use the primitive technology. Now, for capital abundant economies ($k > 1$) it is better to adopt capital-using innovations because

such innovations increase output.

[Insert figure 2 about here]

The cost of increasing α is captured by the following function: $\alpha_{t+1} = \alpha_t + (1 - \alpha_t)\left[1 - \exp\{-\left[(1 - b_t - u_t)\, s_t\right]\right]\}$. Therefore, the capital intensity of the technology is a function of the technology used in the past and of the amount of savings devoted to increase the capital intensity of the technology.

For simplicity the depreciation rate is assumed to be zero and the storage technology is assumed to be efficient $\eta = 1$. The results of the model do not depend on this assumption.

Under this setting, the problem of the representative agent is the following:

$$\begin{aligned} & Max \sum \log(c_t)\beta^t \\ s.t. \quad & \varsigma_{t+1} = \varsigma_t + b_t\left(s_t\right) \\ & k_{t+1} = k_t + u_t\left(s_t\right) \\ & \alpha_{t+1} = \alpha_t + (1 - \alpha_t)\left[1 - \exp\{-\left[(1 - b_t - u_t)\, s_t\right]\right]\} \\ & k_t, \alpha_t, \varsigma_t \geq 0 \end{aligned} \tag{7}$$

To find the solution we combine the first order conditions (complete derivation in the Appendix 7.3), The optimal growth rate of consumption is the following:

$$\frac{c_{t+1}}{c_t} = \beta \max\left[1, \{1 + \alpha_{t+1} B\left(k_{t+1}\right)^{\alpha_{t+1}-1}\}, \{1 + (1 - \alpha_{t+1})\, B\left(k_{t+1}\right)^{\alpha_{t+1}} \ln\left(k_{t+1}\right)\}\right] \tag{8}$$

At any t, the agent chooses the share of savings devoted to increase the number of capital goods u, to store b and to increase the capital intensity of the technology $1 - u - b$. If the marginal productivity of savings is higher in the capital intensive technology than in the storage technology there is no storage, $b = 0$. In this case,

savings are allocated in such a way that the marginal productivity of capital is equal to the marginal productivity of technology, that is,

$$\alpha_t B\left(k_t\right)^{\alpha_t-1}=\left(1-\alpha_t\right) B\left(k_t\right)^{\alpha_t} \ln\left(k_t\right) \tag{9}$$

Note that $k>1$ is a necessary condition for innovations to be profitable. Therefore, $s>1$ is a necessary condition for the representative agent to adopt capital-using innovations and equation 9 can be rewritten in the following way:

$$\alpha_t=\max\left(0, \frac{k_t \ln\left(k_t\right)}{k_t \ln\left(k_t\right)+1}\right) \tag{10}$$

So, in equilibrium α_t is a function of k_t, namely, $\alpha_t=\alpha\left(k_t\right)$, where $\alpha\left(k_t\right)=0$ for any $k_t<1$ and $\lim_{k\to\infty}\alpha\left(k_t\right)=1$. Additionally, for any $k>1$, the marginal productivity of capital increases as the capital stock grows (proof in the Appendix 7.3). Therefore, for $k>1$, the growth rate of the economy increases as the capital stock augments and converges to a finite number when the capital stock goes to infinity $\left(\lim_{k\to\infty}\frac{c_{t+1}}{c_t}=\beta\left(1+B\right)\right)$. Finally, since $\eta=1$ then for any $k>1$ the technology of capital accumulation is preferred.

Note also that depending on the values of α_t, B and β the growth rate of consumption can be positive or negative. Indeed, from equation 8 it follows that $\frac{c_{t+1}}{c_t}\geq 1$ implies $\alpha_{t+1}B\left(k_{t+1}\right)^{\alpha_{t+1}-1}\geq\frac{1}{\beta}-1$.

Now, the marginal productivity of capital increases as the capital stock grows, so it is possible to define k_m as the capital stock such that the growth rate of consumption is equal to zero, namely,

$$\alpha\left(k_m\right)k_m^{\alpha(k_m)-1}=\frac{1}{B}\left(\frac{1}{\beta}-1\right) \tag{11}$$

where $\frac{c_{t+1}}{c_t} < 1$ for any $k_{t+1} < k_m$ and $\frac{c_{t+1}}{c_t} > 1$ for any $k_{t+1} > k_m$.

Equation 11 indicates the levels of the state variables for which the discount rate is equal to the marginal productivity of savings. If under the initial conditions the state variables are high ($\alpha > \alpha(k_m)$ and $k > k_m$) then the optimal consumption growth rate is positive and, consequently, it is optimal to save and increase both the number and the quality of capital goods. If under the initial conditions the state variables are low ($\alpha < \alpha(k_m)$ and $k < k_m$) then the optimal consumption growth rate is negative and, consequently, it is optimal to consume part of the capital stock. Therefore, for $1 < k < k_m$, the new technology is preferred over the storage technology but there are no incentives to accumulate capital. In general, this implies that, under such circumstances, capital goods instead of consumption goods are used to smooth consumption. However, if capital is irreversible[7], that is, if capital goods cannot be consumed then the new technology is preferred over the storage technology only if $\alpha_t B(k_t)^{\alpha_t - 1} > 1$. Now, given that $k < k_m$, for this condition to hold the discount factor must be unrealistically low[8], $\beta < 0.5$.

Summarizing, if savings are low the storage technology is preferred over the capital accumulation technology and if savings are high agents prefer the capital intensive technology. In general, once the capital intensive technology is used, a process of capital accumulation and technological improvement drives the economy to sustained growth.

Finally, note that once the capital intensive technology is adopted, the production function becomes

[7] Under the following conditions agents save and allocate their saving to capital goods when $A_t = A_H$ and consume both the capital stock and its returns when $A_t = A_H$: 1) Capital is irreversible; 2) $K\min < s$; 3) $A_H - A_L > 2 + \alpha(k)Bk^{\alpha(k)-1}$. Conditions 1) and 2) are straightforeward, so we prove in the appendix that condition 3) is necessary.

[8] Note that $k < k_m$ implies $\alpha_t B(k_t)^{\alpha_t - 1} < \left(\frac{1-\beta}{\beta}\right)$, so if $\alpha_t B(k_t)^{\alpha_t - 1} > 1$ then $\beta < 0.5$.

$$Y_t = A_t + Bk_t^{\alpha(k_t)} \tag{12}$$

So there are two sources os seasonality, the natural sources, that is seasonal fluctuations in A, and fluctuations in the stock of capital k. However, if savings are high enough ($s > k_m$) the capital intensive technology is used and the optimal growth rate of consumption is positive for every t, so savings must be positive for every t and the stock of capital k grows every period. Now, capital growth is also seasonal because output is seasonal. However, as the economy accumulates capital the seasonal component of output (and savings) becomes less important.

In summary, the output growth rate predicted by the model is zero for primitive economies but economies where savings are high have incentives to adopt capital intensive technologies. Once an economy begins to use capital intensive technologies, growth rates may become positive. Moreover, the rate of economic growth increases as the economy accumulates capital and converges to a finite number in the long run. This result is consistent with the evolution of the growth rate of GDP per capita for Western European Countries and for Western Offshoots (see table 2): Until 1800 GDP per capita grows at a very small rate and, since then, the growth rate augments period by period with the only exception of 1900-1950 characterized by the two world wars.

[Insert table 4 about here]

5 Empirical Evidence

5.1 Latitude, Institutions and GDP

As stated above, previous empirical work has found that both GDP per capita and its growth depend positively on absolute latitude. This result, however, was recently

questioned by scholars who claimed that after controlling for institutions, the effect of absolute latitude on GDP per capita disappears.

Acemoglu, Johnson and Robinson (2001 and 2002) argue that the disease environment in former European colonies determined the number of European settlements and their subsequent institutional development. Therefore, countries where the environment was more friendly to European settlers benefited from better institutions and, for this reason, had better economic performance. According to these authors geography affects GDP only through institutions.

However, if institutions are determined by GDP, the results that geography affects GDP only through institutions can be reflecting a positive correlation between current and past income level. The fact that the average protection of property rights between 1985 and 1995 is positively correlated with the GDP per capita in 2000 can reflect that GDP affects property rights protection and that there exists a positive correlation between GDP per capita before 1985 and GDP per capita in 2000. In table 3 we illustrate this point. In column 1 we regress the logarithm of GDP per capita in 1960 against absolute latitude and other geographical variables. In column 2 we repeat the exercise including religion and colonial origin (we exclude non significant variables). In column 3 we include European settlers mortality. In column 4 we regress the index of property rights (1985-1995) against the logarithm of GDP per capita in 1960 and the geographical variables. Three results call the attention: (i) absolute latitude, among other geographic variables, positively affects the level of GDP per capita in 1960; (ii) when the logarithm of GDP per capita in 1960 is the dependent variable, the coefficient of European Settlers Mortality is not significantly different from zero at the 30 percent significance level; (iii) when property rights protection is the dependent variable, the coefficient of the logarithm of GDP per capita in 1960 is positive and significantly different from zero at the 1 percent significance level and,

when controlling for GDP per capita in 1960, the coefficient of European Settlers Mortality is not significantly different from zero at the 20 percent significance level.

In summary, absolute latitude positively affects GDP per capita even controlling for European Settlers Mortality.

[Insert Table 3 about here]

5.2 Latitude, Seasons Strength and GDP

We claim that in primitive economies the main source of fluctuations is the climate, and such fluctuations are bigger in places located far from the equator. Now, the strength of the winter can be captured by variables like *Proportion of land with more than N frost-day per month in winter* (*PLNW*) or *Average number of frost days per unit of land area (ANFD)* (see Masters and McMillan (2001)).

As we stated before, during frost days there are no harvests, transportation is difficult, people need more energy and, in general, surviving demands much more work, and, for this reason, people save part of the output produced during the rest of the year. Therefore, holding the rest constant, savings should be higher in places where the winter is stronger and longer.

Masters and McMillan (2001) find that a key variable explaining the different economic performance between tropical and temperate countries is Frost Frequency. According to them, this relation is explained by the effect of ground frost on productivity. As we stated before, the arguments of Masters and McMillan (2001) are captured in our model: if ground frost implies higher productivity, holding the rest constant, it also implies higher savings.

Here we consider the variables *PLNW* and *ANFD* but instead of using growth as the dependent variable we use GDP per capita and savings rate. Our claim is that in regions where seasons are stronger agents are likely to have higher savings and,

for this reason, are more likely to adopt capital intensive technologies. The adoption of such technologies, on their turn, increase the return on savings (investment).

A complete test of the model would require a historical data set containing savings rates for a broad sample of countries. Unfortunately this information is not available. However, we can test the hypothesis that the savings rate in 1960 depend on seasonal fluctuations. Indeed, today part of the output depends on seasonal fluctuations and some expenditures fluctuate with the seasons (heating, cleaning streets and roads, etc.).

5.2.1 Seasons and GDP

PLNW and *ANFD* can be good proxies for climatic seasonal variations in countries with frost days but these variables do not capture the strength of seasons in countries with no frost days. Even without frost days, seasonal climatic fluctuations are correlated with seasonal variations in agricultural output. Similarly, the difficulties of the winter may be important, even though smaller than in places with frost days. Therefore, to test our model we must verify if the effect of seasons is relevant for the whole sample of countries. Unfortunately we do not have a direct measure of seasons strength for countries with no frost days so we keep using absolute latitude as a proxy.

To see if the effect of latitude on GDP is positive for countries with no frost days we include the following variables in our estimations:

1. *Latitude No Frost (LNF)*: equal to zero for countries with at least one frost day and equal to absolute latitude for countries with no frost days.

2. *Latitude Frost (LF)*: equal to zero for countries with no frost days and equal to absolute latitude for countries with at least one frost day.

3. *Proportion of land with more than 3 frost-day per month in winter (PL3W).*

4. *Average number of frost days per unit of land area (ANFD).*

The last two variables are strongly correlated so we include them in separate regressions.

Table 4 presents the results when GDP per capita is the dependent variable. In column 1 we only include geographical variables with coefficients significantly different from zero and instead of absolute latitude we include *LNF* and *LF.* In column 2 we repeat the exercise controlling for institutional variables and excluding variables with coefficients not significantly different from zero. The results confirm that the effect of latitude on GDP per capita is positive for the complete sample of countries. In columns 3 and 4 we include *ANFD* and *PL3W* respectively controlling for institutional variables. The results confirm our claim that the effect of latitude, *ANFD* and *PL3W* is positive and significantly different from zero.

These results suggest that seasonal fluctuations had a positive effect on GDP for countries with frost days and also for countries with no frost days.

[Insert Table 4 about here]

5.2.2 Seasons and Savings

As we stated above the lack of information does not allow us to test the effect that seasonal fluctuations had on savings before the industrial revolution or during the industrialization process. However, we can use more recent data to see if there exists a positive relation. If such a relation exists in the 20th century, when the share of the output depending of climatic conditions is smaller, it should also have been present in the previous centuries.

The savings rate can be associated with variables like productivity and discount factors, among others. For this reason we use GDP per capita as a control variable. As we stated before, the income per capita is also determined by institutions and

geography. Therefore, if controlling for GDP the proxy variables for seasonal fluctuations positively affect savings in 1960 then the effect of seasons on savings was important in those days.

Table 3 presents the results when the savings rate is the dependent variable. In column 1 the independent variables are absolute latitude, geographical variables and GDP per capita (we exclude non-significant variables). In column 2 we include *Average number of frost days per unit of land area (ANFD)* instead of absolute latitude. In column 3 we repeat the exercise of column 1 controlling for institutional variables and excluding variables with coefficient not significantly different from zero. Finally, in column 4 we repeat the exercise of column 2 controlling for institutional variables and excluding variables with coefficients not significantly different from zero.

The effect of the two proxies for seasonal fluctuations, absolute latitude and *ANFD,* on savings is positive and significantly different from zero.

[Insert Table 5 about here]

6 Conclusions

We have studied a simple general equilibrium model of capital accumulation. Assuming that primitive economies start with a production function technology that uses only not-reproducible factors and a storage technology, we find that savings are bigger in economies where changes in output are greater. In economies with high savings, individuals have incentives to make capital-using innovations. In other words, these economies are more likely to create or adopt technologies that use capital more intensively. Finally, in primitive economies the main source of fluctuations is the climate and such fluctuations are bigger in places located far away from the equator. Therefore, economies where seasonal changes in output are higher are more likely to

make capital-using and labor saving innovations.

Once the process of capital accumulation begins, there is a positive relation between technology and capital. Thus, as long as economies accumulate capital the differences in GDP are not likely to be reduced.

The main implications of the model are consistent with the empirical evidence related with the industrial revolution, the subsequent industrialization of some economies and the economic stagnation of others.

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7 Appendix

7.1 Savings

7.1.1 Existence

In this section we proof that $s_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } \eta\beta A_t \leq A_{t+1} \end{cases}$ is a long run equilibrium.

1. Suppose a situation where $s_{t-1} = 0$. In this case, a rational consumer decides the consumption level according to the following rule:

(i) If $A_t = A_l$ then $s_t = 0$ and $c_t = A_l$.

(ii) If $A_t = A_h$ and $\frac{A_h}{A_l} \leq \frac{1}{\eta\beta}$ then $c_t = A_h$.

In cases (i) and (ii), the current consumption level is lower than the most desirable level $\left(c_t = \frac{c_{t+1}}{\eta\beta}\right)$. However, savings cannot be negative, so the best choice is to consume the entire output.

(iii) If $A_t = A_h$ and $\frac{A_h}{A_l} > \frac{1}{\eta\beta}$ then $s_t > 0$ and $c_t < A_h$. Moreover, since the optimal ratio between future consumption and present consumption is given by $c_{t+1} = c_t\eta\beta$ then the optimal savings are $s_t = \frac{\eta\beta A_h - A_l}{(1+\beta)\eta}$ and consumption is given by $c_t = \frac{A_h\eta + A_l}{(1+\beta)\eta}$.

2. Now suppose that $s_{t-1} = \frac{\eta\beta A_h - A_l}{(1+\beta)\eta}$. In this case, a rational consumer decides the consumption level according to the following rule:

If $A_t = A_l$ then $s_t = 0$ and $c_t = \frac{\beta}{(1+\beta)}(A_l + \eta A_h)$. From (iii) we know that if $s_t = 0$ then $c_{t+1} = \frac{A_h\eta + A_l}{(1+\beta)\eta}$. But note that $\frac{A_h\eta + A_l}{(1+\beta)\eta} > \frac{\beta}{(1+\beta)}(A_l + \eta A_h)$ because $\frac{1}{\eta\beta} > \frac{1}{(1+\beta)}$. Therefore, $c_{t+1} > c_t$ and $c_{t+1} > c_t\eta\beta$. Again, the current consumption level is lower than the most desirable $\left(c_t = \frac{c_{t+1}}{\eta\beta}\right)$. However, savings cannot be negative, so the best choice is to consume the entire output.

From 1 and 2 it follows that:

If $\eta\beta A_t > A_{t+1}$ and $s_{t-1} = 0$ then $s_t = \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta}$ and if $\eta\beta A_t < A_{t+1}$ then $s_t = 0$, namely, $s_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } \eta\beta A_t \leq A_{t+1} \end{cases}$ is a long run equilibrium.

7.1.2 Uniqueness

The long run equilibrium $s_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } \eta\beta A_t \leq A_{t+1} \end{cases}$ is unique.

Suppose an equilibrium where $s_t > 0$ for every t.

If $s_t > 0$ for every t then the solution of the optimization problem is interior for every t, namely,

$\frac{c_t}{c_{t-1}} = \frac{c_{t+1}}{c_t} = \frac{c_{t+2}}{c_{t+1}} = \frac{c_{t+3}}{c_{t+2}} = ... = \eta\beta$

So, for any finite t, $\lim_{T\to\infty} \frac{c_T}{c_t} = 0$.

Therefore, an equilibrium where $s_t > 0$ for every t does not exist. So the long run equilibrium must be characterized by $s_t = 0$ for some t. Finally, from the proof of existence it follows that, starting from $s_{t-1} = 0$ the only possible equilibrium is $s_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } \eta\beta A_t \leq A_{t+1} \end{cases}$.

7.2 Technology and Capital

From equation 3,

$$(1-\delta) + \frac{\partial F(\cdot)}{\partial K} + g'(\alpha) - \frac{\partial F(\cdot)}{\partial \alpha} = 0$$

Define,

$$G = (1-\delta) + \frac{\partial F(\cdot)}{\partial K} + g'(\alpha) - \frac{\partial F(\cdot)}{\partial \alpha}$$

Using the implicit function theorem we can conclude $\frac{\partial \alpha}{\partial K} = -\frac{\frac{\partial G}{\partial K}}{\frac{\partial G}{\partial \alpha}}$.

$$\frac{\partial G}{\partial \alpha} = \frac{\partial^2 F(\cdot)}{\partial K \partial \alpha} + g''(\alpha) - \frac{\partial^2 F(\cdot)}{\partial \alpha^2}$$

Notice that $g''(\alpha) > \frac{\partial^2 F(\cdot)}{\partial \alpha^2}$ and $\frac{\partial^2 F(\cdot)}{\partial K \partial \alpha} > 0$ so $\frac{\partial G}{\partial \alpha} > 0$.

Similarly,

$$\frac{\partial G}{\partial K} = \frac{\partial^2 F(K, \alpha)}{\partial K^2} - \frac{\partial^2 F(K, \alpha)}{\partial \alpha \partial K} < 0$$

Since $\frac{\partial G}{\partial \alpha} > 0$ and $\frac{\partial G}{\partial K} < 0$ then,

$$\frac{\partial \alpha}{\partial K} > 0.$$

7.3 The Cobb-Douglas Example

7.3.1 The problem

The problem can be written as,

$$Max \sum \log(y_t - s_t)\beta^t$$

$$\text{s.t. } s_t = (\varsigma_{t+1} - \varsigma_t) + (k_{t+1} - k_t) + (\log(1 - \alpha_t) - \log(1 - \alpha_{t+1}))$$

$$\varsigma_t, k_t, \alpha_t \geq 0 \text{ for every } t$$

which is equivalent to

$$Max \sum \log\left(y_t - (\varsigma_{t+1} - \varsigma_t) - (k_{t+1} - k_t) + (\log(1 - \alpha_{t+1}) - \log(1 - \alpha_t))\right)\beta^t$$

$$\varsigma_t, k_t, \alpha_t \geq 0 \text{ for every } t$$

The first order conditions for ς is,

$$\frac{1}{c_t}\beta^t - \frac{1}{c_{t+1}}\beta^{t+1} + \chi_1 = 0$$

where χ_1 is the multiplier of the restriction $\varsigma_t \geq 0$.

The first order conditions for k and α are,

$$\frac{1}{c_t}\beta^t - \frac{1}{c_{t+1}}\beta^{t+1}\left(1 + \alpha_{t+1}B\left(k_{t+1}\right)^{\alpha_{t+1}-1}\right) + \chi_2 = 0$$
$$-\frac{1}{c_t}\frac{1}{1-\alpha_{t+1}}\beta^t + \frac{1}{c_{t+1}}\beta^{t+1}\left(\frac{1}{1-\alpha_{t+1}} + B\left(k_{t+1}\right)^{\alpha_{t+1}}\ln\left(k_{t+1}\right)\right) + \chi_3 = 0$$

where χ_2 and χ_3 are the multipliers of the restriction $k_t \geq 0$ and $\alpha_t \geq 0$, respectively.

It is straightforward that:

(i) $\chi_1 = 0$ implies $\chi_2 \neq 0$.

(ii) $\chi_2 \neq 0$ implies $\chi_3 \neq 0$ and $\chi_3 \neq 0$ implies $\chi_2 \neq 0$.

(iii) $\chi_2 = \chi_3 = 0$ implies $\chi_1 \neq 0$ and

$1 + \alpha_{t+1}B\left(k_{t+1}\right)^{\alpha_{t+1}-1} = 1 - \alpha_{t+1} + (1-\alpha_{t+1})B\left(k_{t+1}\right)^{\alpha_{t+1}-1}\ln\left(k_{t+1}\right)$

$$\alpha_t = \frac{k_t \ln\left(k_t\right)}{k_t \ln\left(k_t\right) + 1} \tag{13}$$

Therefore, if the capital using technology is used the storage technology is not used and the other way around.

Differentiating equation 7 yields, $\dot{\alpha} = \frac{(\ln(k_t)+1)}{(k_t \ln(k_t)+1)^2}\dot{k}$.

Using equation 7 again,

$$\dot{\alpha} = (1-\alpha)^2\left(\ln\left(k_t\right) + 1\right)\dot{k} \tag{14}$$

7.3.2 The Marginal Productivity of Capital Increases with k

Consider the marginal productivity of capital and take logarithms:

$$\ln(\alpha_t A_t \left(k_t\right)^{\alpha_t - 1}) = \ln \alpha + \ln A - (1-\alpha) \ln k$$

Differentiating,

$$\frac{\dot{\alpha}}{\alpha} - (1-\alpha) \frac{\dot{k}}{k} + \dot{\alpha} k$$

Replacing $\dot{\alpha}$ from equation 8 and rearranging,

$$(1-\alpha) \dot{k} \left(\frac{(1-\alpha)}{\alpha} \left(\ln \left(k_t\right) + 1\right) - \frac{1}{k} + (1-\alpha) \left(\ln \left(k_t\right) + 1\right) k \right)$$

From equation 7, $\frac{1}{k} = \frac{(1-\alpha)}{\alpha} \ln(k_t)$, so

$$(1-\alpha) \dot{k} \left(\frac{(1-\alpha)}{\alpha} + (1-\alpha) \left(\ln \left(k_t\right) + 1\right) k \right) > 0$$

Using equation 8 it is also possible to find the value of u:

$$\begin{aligned} (1-u) &= (1-\alpha) \left(\ln k + 1\right) u \\ \frac{(1-u)}{u} &= (1-\alpha) \left(\ln k + 1\right) \\ u &= \frac{1}{2 - \alpha + \frac{\alpha}{K}} \end{aligned}$$

Therefore, the share of savings devoted to increase the number of capital goods positively depends on the amount of capital goods in the long run all savings are devoted to increase the amount of capital goods.

7.3.3 Using Capital Goods to Smooth Consumption

If agents save and allocate their saving to capital goods when $A_t = A_H$ and consume both the capital stock and its returns then $A_H - A_L > 2 + \alpha(k) Bk^{\alpha(k)-1}$.

Proof:

(i) If $k < k_m$ then $1 + \alpha(k) Bk^{\alpha(k)-1} < \frac{1}{\beta}$.

(ii) $s > 1$ implies $\left(1 + \alpha(k) Bk^{\alpha(k)-1}\right)(\beta(A_H - 1)) > A_L + \left(1 + \alpha(k) Bk^{\alpha(k)-1}\right)$

(iii) From (i) it follows that $\left(1 + \alpha(k) Bk^{\alpha(k)-1}\right)(\beta(A_H - 1)) < (A_H - 1)$

From (ii) and (iii) $A_H - A_L > 2 + \alpha(k) Bk^{\alpha(k)-1}$.

Table 1
Per Capita GDP
Source: Maddisson (2003)

	1820	1850	1900
United Kingdom	1706	2330	4492
Netherlands	1838	2371	3424
Belgium	1319	1847	3731
Denmark	1274	1767	3017
Austria	1218	1650	2882
Sweden	1198	1289	2561
France	1135	1597	2876
Italy	1117	1350	1785
Switzerland	1090	1488	3833
Germany	1077	1428	2985
Norway	801	956	1877
Finland	781	911	1668

Table 2
Growth Rates GDP per capita
Source Maddison (2003)

	1700-1820	1820-1850	1850-1900	1900-1950	1950-2000
Austria	0,17%	1,02%	1,12%	0,50%	3,50%
Belgium	0,12%	1,13%	1,42%	0,77%	2,69%
Denmark	0,17%	1,10%	1,08%	1,68%	2,42%
Finland	0,17%	0,51%	1,22%	1,89%	3,10%
France	0,18%	**1,14%**	**1,18%**	**1,22%**	**2,83%**
Germany	0,14%	**0,94%**	**1,49%**	**0,53%**	**3,23%**
Italy	0,01%	**0,63%**	**0,56%**	**1,36%**	**3,42%**
Netherlands	-0,12%	0,85%	0,74%	1,13%	2,60%
Norway	0,09%	0,59%	1,36%	2,15%	3,11%
Sweden	0,17%	0,24%	1,38%	1,95%	2,27%
Switzerland	0,17%	1,04%	1,91%	1,74%	1,83%
United Kingdom	**0,26%**	**1,05%**	**1,32%**	**0,87%**	**2,16%**
Total 12 Western Europe	**0,16%**	**0,97%**	**1,24%**	**0,98%**	**2,81%**
Greece	0,16%	0,81%	1,01%	0,70%	3,76%
Portugal	0,10%	0,00%	0,69%	0,95%	3,85%
Spain	0,14%	0,23%	1,01%	0,41%	4,01%
Total 13 small WEC	0,16%	0,96%	1,24%	0,98%	3,29%
Total 29 Western Europe	**0,16%**	**0,90%**	**1,23%**	**0,92%**	**2,92%**
Australia	0,22%	4,56%	1,43%	1,23%	2,16%
New Zealand	0,00%	3,57%	2,68%	1,36%	1,31%
Canada	0,62%	1,29%	1,58%	1,85%	2,27%
United States	**0,73%**	**1,21%**	**1,65%**	**1,71%**	**2,20%**
Total Western Offshoots	**0,77%**	**1,29%**	**1,66%**	**1,69%**	**2,19%**

Table 3: Absolute Latitude and GDP. The relation is robust controlling for settlers mortality.				
Dependent Variable	(1) log GDP pc 1960	(2) log GDP pc 1960	(3) log GDP pc 1960	(4) Protection of Property Rights
C	6.4951 (0.000)	6.379 (0.000)	6.979 (0.000)	-3.9919 (0.141)
Absolute Latitude	**0.0283 (0.000)**	**0.0274 (0.000)**	**0.0193 (0.027)**	
Climate	0.1700 (0.006)	0.2667 (0.000)		
Elevation	-0.0002 (0.159)	-0.0004 (0.000)		
Area (Km2)	1.33E-07 (0.000)	7.87E-08 (0.004)	1.91E-07 (0.000)	3.17E-07 (0.000)
Mean distance to nearest navigable river	-0.0005 (0.023)	3.98E-05 (0.013)	-0.0010 (0.002)	
Settlers Mortality			**-0.0002 (0.309)**	**-0.0003 (0.229)**
Spanish Colony		0.4881 (0.001)		
Percentage of Muslims		-0.8148 (0.000)		
Percentage of Catholics				-1.3337 (0.000)
Log GDP pc 1960				**5.629 (0.000)**
Adjusted R-squared	0.64	0.77	0.44	0.62
Observations	91	91	54	54

p-values in parenthesis
Sources:
GDP60, Absolute Latitude, Religion, Democracy Index, Colonial Origin: Barro-Lee Data set.
Social Infrastructure: Hall and Jones (1999).
Settlers Mortality, Property Rights Protection: Acemoglu et. al. (2001)
Climate: Olsson and Hibbs (2004).
Elevation, Ratio of population within 100 Km of ice-free coast, Area, Mean distance to nearest navigable river: Gallup et. al. (1999)

Table 4 **Dependent Variable log GDP pc 2000**				
	(1)	(2)	(3)	(4)
C	5.157 (0.000)	3.853 (0.000)	3.999 (0.000)	4.025 (0.000)
Absolute Latitude			**0.0330** **(0.000)**	**0.024** **(0.005)**
Latitude Frost	**0.0487** **(0.000)**	**0.0437** **(0.000)**		
Latitude No Frost	**0.0433** **(0.060)**	**0.0288** **(0.057)**		
Proportion of land with more than 3 frost-day per month in winter				**0.9876** **(0.002)**
Average number of frost days per unit of land area	**0.0444** **(0.000)**		**0.0330** **(0.057)**	
Ratio of population within 100 Km of ice-free coast	1.902 (0.000)	1.004 (0.000)	0.9896 (0.000)	0.9909 (0.000)
Area (Km2)	1.04E-07 (0.066)	7.95E-08 (0.095)		
Catholics		0.664 (0.003)	0.7976 (0.000)	0.7162 (0.001)
Former Spanish colonies		0.605 (0.010)	0.4886 (0.034)	0.5775 (0.010)
Democracy Index		0.725 (0.008)	0.7521 (0.006)	0.8122 (0.002)
Social Infrastructure		2.771 (0.000)	2.5254 (0.000)	2.446 (0.000)
Adjusted R-squared	0.66	0.90	0.89	0.90
Number of Observations	102	77	78	77

p-values in parenthesis
Sources:
Proportion of land with more than 3 frost-day per month in winter and Average number of frost days per unit of land area: Masters and McMillan (2001).
GDP60, Absolute Latitude, Religion, Democracy Index, Colonial Origin: Barro-Lee Data set.
Social Infrastructure: Hall and Jones (1999).
Ratio of population within 100 Km of ice-free coast: Gallup et. al. (1999)

Table 5 **Dependent Variable Savings Rate 1960**				
	(1)	(2)	(3)	(4)
C	-0.814 (0.000)	-0.566 (0.000)	-0.749 (0.000)	-0.691 (0.000)
Average number of frost days per unit of land area		**0.0012** **(0.000)**		**0.0066** **(0.005)**
Absolute Latitude	**0.0042** **(0.000)**		**0.0024** **(0.023)**	
Ratio of population within 100 Km of ice-free coast	0.122 (0.000)	0.154 (0.000)	0.116 (0.000)	0.144 (0.000)
Elevation		-5.48E-05 (0.023)		
Log GDP 1960	**0.118** **(0.000)**	**0.089** **(0.000)**	**0.106** **(0.000)**	**0.101** **(0.000)**
Catholics			-0.091 (0.049)	-0.082 (0.012)
Social Infrastructure			0.192 (0.036)	0.159 (0.019)
Adjusted R-squared	0.712	0.76	0.78	0.79
Number of Observations	101	99	80	79

p-values in parenthesis
Sources:
Average number of frost days per unit of land area: Masters and McMillan (2001).
GDP60, Absolute Latitude, Religion: Barro-Lee Data set.
Social Infrastructure: Hall and Jones (1999).
Ratio of population within 100 Km of ice-free coast: Gallup et. al. (1999)

Figure 1: Latitude and GDP

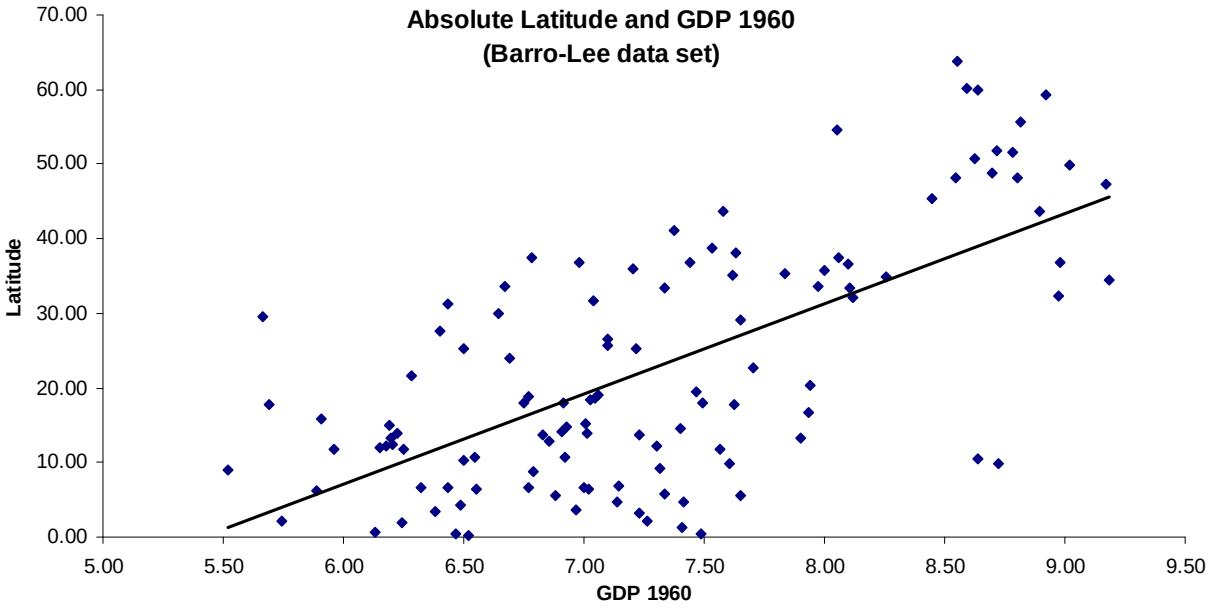

Figure 2: Effect of an increase in α