# Optimal Redistribution with a Shadow Economy

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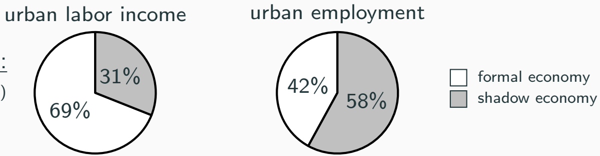
A shadow economy/informal sector: economic activity which evades taxation.

The shadow economy is **large** in many low and middle income countries.

- Share in urban emp. in 90s: Latin America 54%, SE Asia 70% (source: OECD).

Colombia in 2013:

(source: own calculations)




Why is the shadow economy important for income redistribution?

- Underreporting income makes income taxation more difficult.
- Poor workers engage more often in informal activity.

**What is the optimal income tax with a shadow economy?**

What is the optimal size of the shadow economy? How does it affect welfare?

# This paper

## Environment

1. Distribution of workers with heterogenous productivities.
2. Private information about individual productivity.
3. Formal labor market (observed) and shadow labor market (hidden).

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## Results

The theory of optimal income taxation with a shadow economy.

- The shadow economy can play two welfare enhancing roles:
  - a *shelter against tax distortions* and a *screening device*.
- Novel optimal tax formula.
- Quantitative results for Colombia.

Technical contribution: the optimal taxation model with two sectors, sector-specific taxes and the possibility of working in the two sectors simultaneously.

# Related literature & contribution

## Models of the shadow economy

Rauch (1991); Amaral and Quintin (2006); Albrecht, Navarro, and Vroman (2009); Meghir, Narita, and Robin (2015).

This paper: focus on the workers' sectoral choice and workers' heterogeneity.

## Taxation and tax evasion

Allingham and Sandmo (1972); Kopczuk (2001); Alvarez-Parra and Sánchez (2009); Waseem (2013); Frías, Kumler, and Verhoogen (2013); Pappadá and Zylberberg (2015).

This paper: the optimal non-linear income tax.

## Optimal non-linear income taxation

Mirrlees (1971); Diamond (1998); Saez (2001); Gomes, Lozachmeur, and Pavan (2014); Rothschild and Scheuer (2014).

This paper: optimal tax formula in environment where a part of income is unobserved.

## Plan of the presentation

Intro

Simple model

Optimal tax formula

Empirical analysis

# Workers

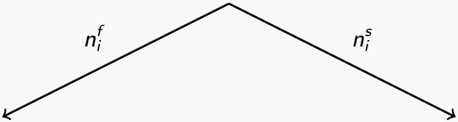
Two types of workers $i \in \{H, L\}$ with population shares $\mu_H$ and $\mu_L$.

Workers have a quasilinear utility function over consumption and labor

$$U(c, n) = c - v(n),$$

where $v'' > 0$ and $v'(0) = 0$.

Worker $i$ can supply labor to




formal labor market
productivity $w_i^f$
income $y_i^f \equiv w_i^f n_i^f$

shadow labor market
productivity $w_i^s$
income $y_i^s \equiv w_i^s n_i^s$

Total labor supply is $n_i \equiv n_i^f + n_i^s$.

## Assumptions on productivities

**Ordering of types:** Type $H$ is more productive formally: $w_H^f > w_L^f$.

**Simplifying assumption:** Each type is more productive formally $\forall_i \; w_i^f > w_i^s$.

$\implies$ in the first best nobody works in the shadow economy.

- we drop this assumption in the full model.

# The planner's problem: direct revelation mechanism

The planner maximizes the social welfare function

$$\max_{\left(y_i^f, T_i\right)_{i \in \{H,L\}}} U(c_L, n_L)$$

subject to the resource constraint

$$\mu_L T_L + \mu_H T_H \geq 0,$$

workers' choice of shadow labor

$$\forall_{i \in \{H,L\}} \; n_i^s = \arg\max_{n^s} U\left(y_i^f - T_i + w_i^s n^s, \frac{y_i^f}{w_i^f} + n^s\right),$$

and incentive compatibility constraints, preventing type misreporting

$$\forall_{i,j \in \{H,L\}} \; U\left(c_i, n_i\right) \geq \max_{n^s} U\left(y_j^f - T_j + w_i^s n^s, \frac{y_j^f}{w_i^f} + n^s\right).$$

# Shadow economy in the optimum.

**Assumption 1.** $\lambda_H = 0$ and $v''$ is non-decreasing.

**Assumption 2.** $w_H^f \left[v'\right]^{-1}(w_H^s) \geq w_L^f \left[v'\right]^{-1}(w_L^s)$.

**Proposition**

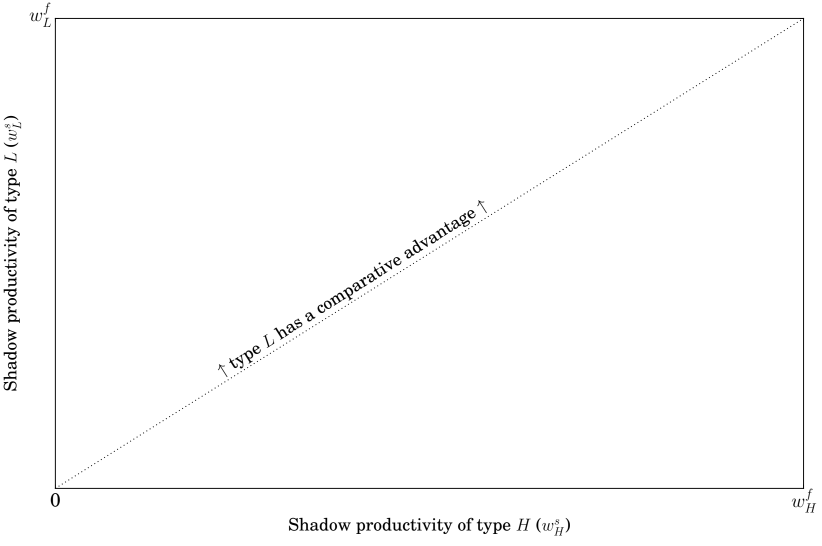
*Under Assumption 1, type L optimally supplies shadow labor only if*

$$\left(\frac{w_L^s}{w_L^f} - \frac{w_H^s}{w_H^f}\right)\mu_H \geq \frac{w_L^f - w_L^s}{w_L^f}\mu_L.$$

*Under Assumptions 1 and 2, this condition is both necessary and sufficient.*

| | |
|---|---|
| Benefit: | more redistribution due to the relaxed IC constraint. |
| Cost: | output loss due to lower productivity. |

# Shadow economy in the optimum.



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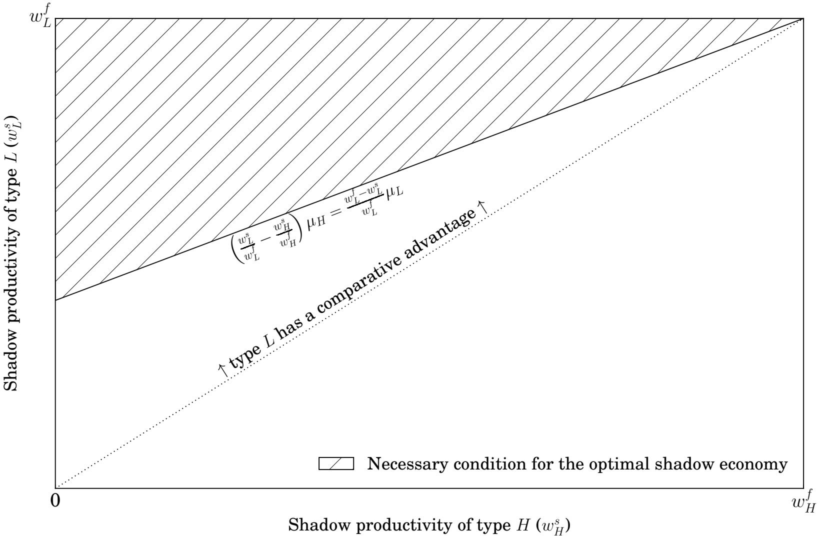
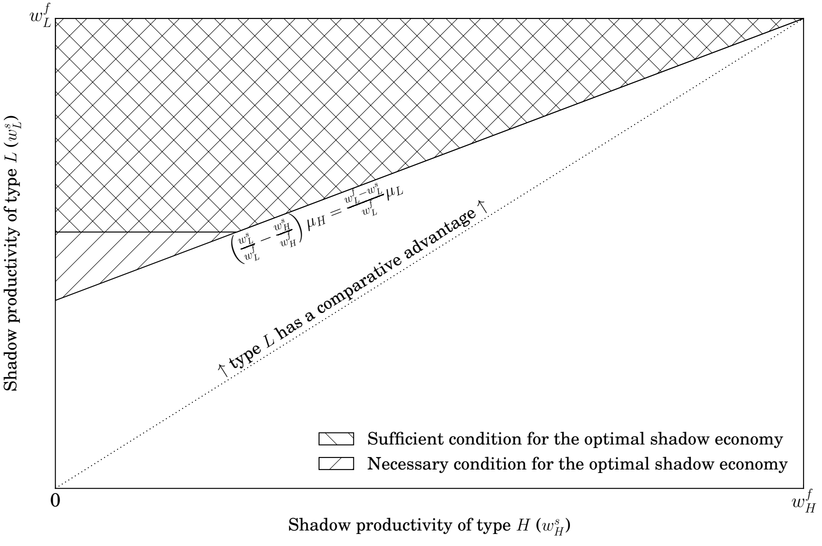


# Shadow economy in the optimum.



## Shadow economy and welfare.

Let's compare two allocations:

$$SE: \text{ allocation in which type } L \text{ works only in the shadows,}$$
$$M: \text{ optimum of the standard Mirrlees model } (\forall_i w_i^s = 0).$$

We can decompose the welfare difference between the two allocations

$$U\left(w_L^s n_L^{SE} - T_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M - T_L^M, n_L^M\right)$$

into

$$\underbrace{U\left(w_L^s n_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M, n_L^M\right)}_{\text{efficiency gain}} \quad + \quad \underbrace{T_L^M - T_L^{SE}}_{\text{redistribution gain}}.$$

# Efficiency gain.

$$U\left(w_L^s n_L^{SE}, n_L^{SE}\right) - U\left(w_L^f n_L^M, n_L^M\right)$$

Efficiency gain measures the difference in distortions between $M$ and $SE$.

- Distortions in $M$: positive tax rate on formal income.
- Distortions in $SE$: lower shadow productivity.

Efficiency gain is strictly increasing in $w_L^s$, positive for $w_L^s > \bar{w}_L^s \in (0, w_L^f)$.

Positive efficiency gain $\rightarrow$ shadow economy as a *shelter against tax distortions*.

# Redistribution gain.

$$T_L^M - T_L^{SE}$$

Redistribution gain is a difference in transfers of type $L$.

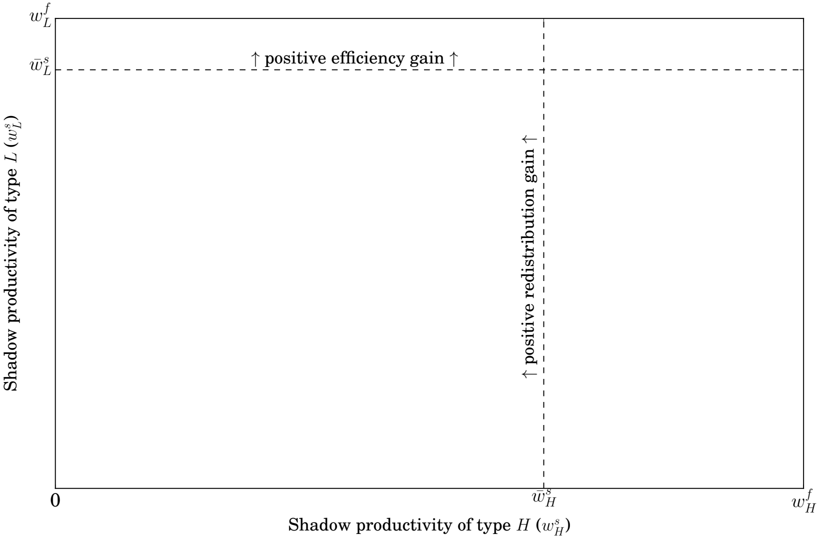
It depends on the utility of $H$ from mimicking $L$:

- Utility of mimicker in $M$: $U\left(y_L^M - T_L^M, y_L^M / w_H^f\right)$
- Utility of mimicker in $SE$: $\max_{n^s} U\left(w_H^s n^s - T_L^{SE}, n^s\right)$

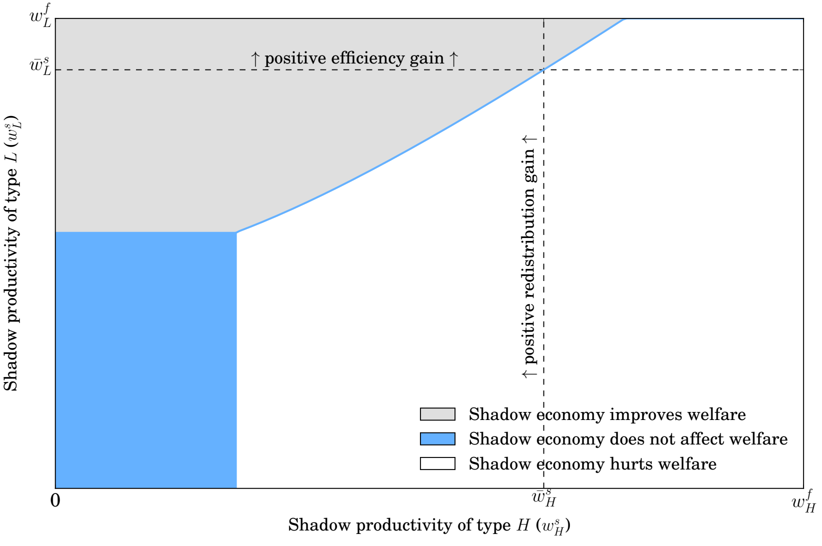
Redistribution gain is strictly decreasing in $w_H^s$, positive for $w_H^s < \bar{w}_H^s \in (0, w_H^f)$.

Positive redistribution gain $\rightarrow$ shadow economy as a *screening device*.

# Shadow economy and welfare.



# Shadow economy and welfare.



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# Continuum of types

Workers differ in productivity and in a fixed cost of shadow employment.

A continuum of productivity types $\theta \in [0,1]$ distributed with $F(\theta)$, $f(\theta)$.

$\theta$ determines formal productivity $w^f(\theta)$ and shadow productivity $w^s(\theta)$.

- Ordering of types: $w^f(\theta)$ is increasing in $\theta$.
- Assumption for single-crossing: $w^s(\theta)/w^f(\theta)$ is non-increasing in $\theta$.

An idiosyncratic fixed cost of shadow employment $\kappa \in [0, \infty)$.

- Conditional on $\theta$, $\kappa$ is distributed with $G_\theta(\kappa)$, $g_\theta(\kappa)$.

# The optimal tax formula

The optimal tax formula can be expressed as

$$\frac{\tau(\theta)}{1-\tau(\theta)} h^f\left(y^f(\theta,\infty)\right) y^f(\theta,\infty)\tilde{\varepsilon}_{y^f,1-\tau}(\theta,\infty) + \left(\frac{w^f(s(\theta))}{w^s(s(\theta))}-1\right) h^s\left(y^f(s(\theta),0)\right) y^f(s(\theta),0)\tilde{\varepsilon}_{y^f,1-\tau}(s(\theta),0) = \int_{y^f(\theta,\infty)}^{\infty}\left[1-\bar{\lambda}(y)\right]h(y)dy - \int_{y^f(\theta,\infty)}^{y^f(s(\theta),\infty)}\pi(y)h^f(y)dy,$$

where

| | |
|---|---|
| $h^f$, $h^s$, $h$ | density of formal income of formal workers, shadow workers and all workers, respectively |
| $\tilde{\varepsilon}_{y^f,1-\tau}(\theta,\kappa)$ | elasticity of $y^f(\theta,\kappa)$ with respect to the marginal tax rate along the non-linear tax schedule |
| $\pi(y^f(\theta,\infty))$ | the elasticity of formality of workers with productivity type $\theta$ with respect to $\Delta T(\theta)$ |
| $\bar{\lambda}(y)$ | the mean Pareto weight of all workers at formal income $y$ by $\bar{\lambda}(y)$ |
| $s(\theta)$ | a productivity type of low-cost workers distorted by $\tau(\theta)$ |

# The optimal tax formula

$$
\underbrace{\frac{\tau(\theta)}{1-\tau(\theta)} h^f\left(y^f(\theta,\infty)\right) y^f(\theta,\infty)\tilde{\varepsilon}_{y^f,1-\tau}(\theta,\infty)}_{\text{deadweight loss from formal workers}}
$$

$$
+ \underbrace{\left(\frac{w^f(s(\theta))}{w^s(s(\theta))}-1\right) h^s\left(y^f(s(\theta),0)\right) y^f(s(\theta),0)\tilde{\varepsilon}_{y^f,1-\tau}(s(\theta),0)}_{\text{deadweight loss from shadow workers}}
$$

$$
= \underbrace{\int_{y^f(\theta,\infty)}^{\infty}\left[1-\bar{\lambda}(y)\right] h(y)dy}_{\substack{\text{welfare adjusted} \\ \text{tax revenue gain}}} - \underbrace{\int_{y^f(\theta,\infty)}^{y^f(s(\theta),\infty)} \pi(y)h^f(y)dy}_{\substack{\text{increased participation} \\ \text{in the shadow economy}}}
$$

# The optimal tax formula

$$\underbrace{\frac{\tau(\theta)}{1-\tau(\theta)} h^f\left(y^f(\theta,\infty)\right) y^f(\theta,\infty)\tilde{\varepsilon}_{y^f,1-\tau}(\theta,\infty)}_{\text{deadweight loss from formal workers}}$$

$$+\underbrace{\left(\frac{w^f(s(\theta))}{w^s(s(\theta))}-1\right) h^s\left(y^f(s(\theta),0)\right) y^f(s(\theta),0)\tilde{\varepsilon}_{y^f,1-\tau}(s(\theta),0)}_{\text{deadweight loss from shadow workers}}$$

$$=\underbrace{\int_{y^f(\theta,\infty)}^{\infty}\left[1-\bar{\lambda}(y)\right] h(y)dy}_{\substack{\text{welfare adjusted}\\ \text{tax revenue gain}}}-\underbrace{\int_{y^f(\theta,\infty)}^{y^f(s(\theta),\infty)}\pi(y)h^f(y)dy}_{\substack{\text{increased participation}\\ \text{in the shadow economy}}}$$

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# Colombia as a case study

58% of the workers are part of the shadow economy according to our estimates.

We estimate the three key objects of the model:

- The formal productivity ($w^f(\theta)$).
- The informal productivity ($w^s(\theta)$).
- The distribution of types ($F(\theta)$) and ($G_\theta(\kappa)$).

**What is the optimal income tax?**

What is the optimal size of the shadow economy? How does it affect welfare?

# Identifying informality and productivity

Our source of information is the household survey collected on a monthly basis by the official statistical agency. Our sample is for the year 2013 and comprises 170.000 observations of workers.

A shadow worker is a worker that is not a contributor to the social security system (health, pension and accidents insurance). Payroll taxes and social security contributions are collected jointly.

The gross hourly wage is our proxy for the labor productivity.

# Empirical specification

Our sample consist of $\{w_i, j_i, X_i\}_{i=1}^N$ where $w_i$ is the hourly wage, $j_i$ the sector where the individual works (shadow or formal) and $X_i$ a vector of worker and job characteristics of $N$ individuals in their first and second job.

The probability distribution of the type conditional on a factor $F_i$ that comprises the workers-job characteristics, is given by

$$\theta_i \sim N(F_i, \sigma_\epsilon)$$

where

$$F_i = X_i \beta$$

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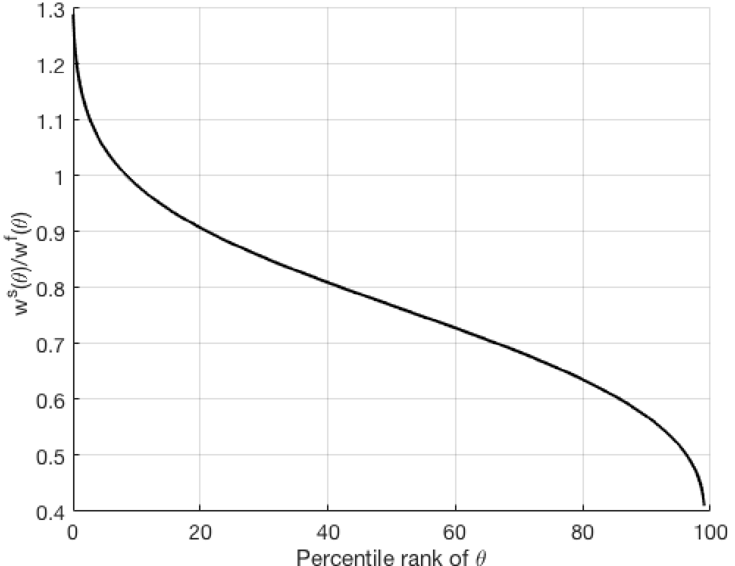
where

$$F_i = X_i \beta$$

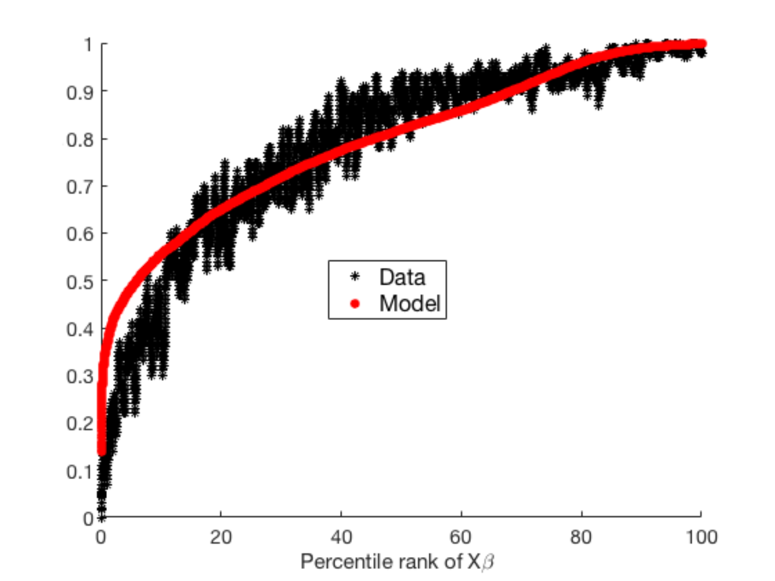
The mapping of the productivity type to the productivity levels in each market are given by:

$$\begin{aligned} \log(w_i^s) &= \gamma_0^s + \gamma_1^s \theta_i \\ \log(w_i^f) &= \gamma_0^f + \theta_i \end{aligned}$$

# Producitivity profiles: Relative advantage



# Formal Participation.



# Social welfare function

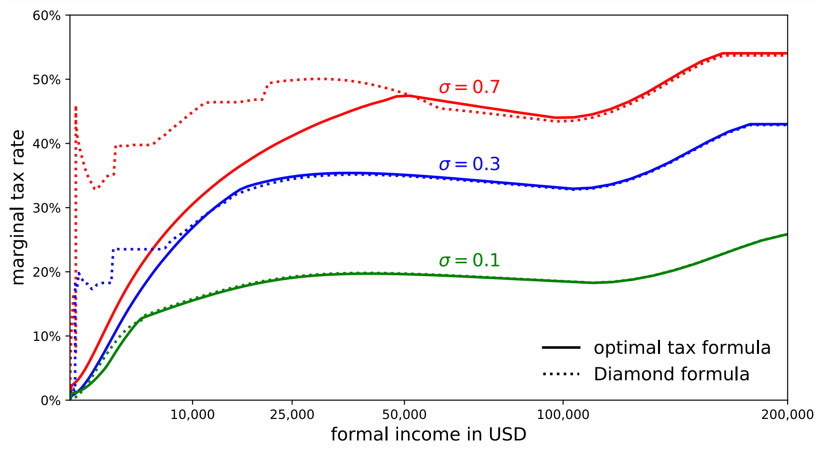
In the quantitative exercise we consider the following social welfare function

$$\int_0^1 \int_0^\infty \frac{V(\theta,\kappa)^{1-\sigma}}{1-\sigma} dG_\theta(\kappa) dF(\theta),$$

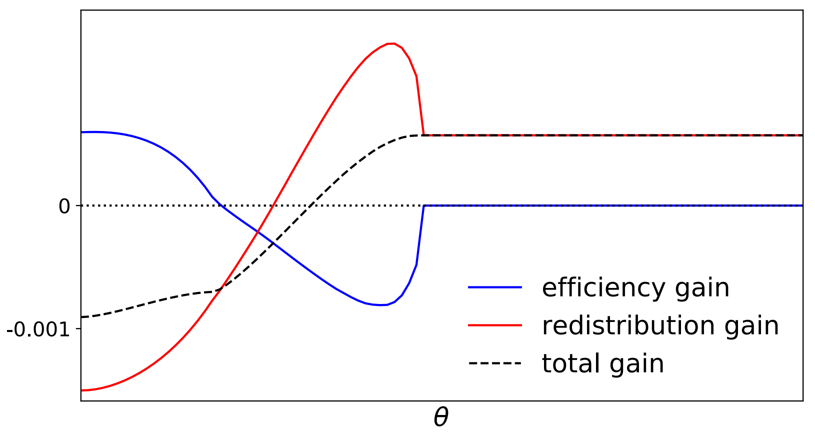
where $V(\theta,\kappa)$ is the utility of agent with productivity type $\theta$ and fixed cost $\kappa$.

Parameter $\sigma$ controls the redistributive taste of the planner.

# Optimal tax schedule



# Welfare decomposition: Low participation cost workers



# Conclusions

Although the shadow economy allows for income underreporting, it can be useful as a *shelter against tax distortions* or as a *screening device*.

We derive a novel optimal tax formula.

In comparison to the actual Colombian tax, the optimal tax has:

- lower rates at low income levels,
- higher rates elsewhere.

# The optimal shadow economy: full results ↶

**Proposition**

*Suppose that $v''$ is nondecreasing.*

*(i) When $w_H^f [v']^{-1} (w_H^s) \geq w_L^f [v']^{-1} (w_L^s)$, type L works in the shadow economy if and only if*

$$\left( \frac{w_L^s}{w_L^f} - \frac{w_H^s}{w_H^f} \right) \mu_H \geq \frac{w_L^f - w_L^s}{w_L^f} \mu_L. \tag{1}$$

*(ii) When $w_H^f [v']^{-1} (w_H^s) < w_L^f [v']^{-1} (w_L^s)$, (1) is a necessary condition for type L to work in the shadow economy. The sufficient condition is given by*

$$\left( \frac{w_L^s}{w_L^f} - v' \left( \frac{w_L^f}{w_H^f} [v']^{-1} (w_L^s) \right) \Big/ w_H^f \right) \mu_H \geq \frac{w_L^f - w_L^s}{w_L^f} \mu_L. \tag{2}$$

# Likelihood

The likelihood to observe the realisation $\{w_i, j_i, X_i\}$ is given by:

$$P\left(w = w_i, j = j_i, X = X_i; \sigma_\epsilon, \beta, \Gamma, T, \alpha, \gamma\right) = P\left(w^j = w_i \mid X_i; \sigma_\epsilon, \beta, \gamma\right) P\left(X_i\right) \cdots \cdots P\left(j = j_i \mid w_i^j; \gamma, \beta, \Gamma, T, \alpha\right) \quad (3)$$

- $P\left(w^j = w_i \mid X_i; \sigma_\epsilon, \beta\right)$: It is the probability that the agent $i$ has a productivity level $w_i$ at sector $j$.
- $P\left(X_i\right)$: It is the sampling weight assigned in the survey.
- $P\left(j = j_i \mid w_i^j; \gamma, \beta, \Gamma, T\right)$: This is the probability that worker $i$ selects to work on sector $j$ given that the wage in sector $j$ is $w_i^j$.