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# Characterization of the Instrumental Value of Money as a Medium of Exchange, as a Property Within the Sidrauski: Money in Utility Model

by

Natalia Salazar Vesga

Student registration number: 2004802

Supervised by

Mr. M.L. Kobielarz

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# *Abstract*

The Sidrauski: Money in Utility model, represents the decision-making process of an agent facing a choice on how to allocate his resources between consumption and money holdings. To do so, the individual needs to take into account not only the present value of the assets owned but also the opportunity cost of transferring them into the future by moving his endowments across the inter-temporal setting employing savings or investments. Despite being a useful model to analyze the consequences of the individual preferences in the general equilibrium, it does an oversimplified inclusion of the monetary assets which fails to understand their complexity. In this document, a possibly more comprehensive way of including money is explored, in such a way, that it accomplishes to capture the instrumental value of this asset instead of assuming it as intrinsic. The conclusions under this new setting are further analyzed, to see how the general equilibrium outcomes are affected and the consequences that this has on the macroeconomic outcome when using the original Sidrauski model as a benchmark.

# Contents

# Symbols

| | |
|---|---|
| $\beta^t$ | Intertemporal discount factor |
| $\beta = \frac{\beta^{t+1}}{\beta^t}$ | Simplification term |
| $c_t$ | Per capita consumption |
| $M_t$ | Nominal money holdings |
| $k_t$ | Per capita capital accumulation |
| $e_t$ | Conversion rate of money into consumption via smooth transactions |
| $P_t$ | Prices in period t |
| $f(k_t)$ | Production function in terms of capital per capita |
| $b_t$ | Real bond holdings |
| $r_t$ | Real interest rate |
| $i_t$ | Nominal interest rate |
| $\pi_t$ | Inflation on period t |
| $U(x_1, x_2)$ | Utility function in terms of the arguments $\mathrm{x}_1, x_2$ |
| $\delta_t$ | Capital depreciation rate |
| $t_t$ | Lump sum government transfers to individuals |
| $x_{ss}$ | Variables in the long run Steady State |

# Chapter 1

# Introduction

## 1.1 Why Worry About Money?

Money is an element present in all daily economic interactions; its primary use relies on facilitating payments and the process of exchange between individuals. Although monetary flows are usually associated with cash, modern payment instruments have made transactions possible even without the use of physical notes or bills. In this order of ideas, the quantity of money held by an individual at a particular moment in time could be better defined as the nominal value of assets held which have a high degree of liquidity. This level of liquidity occurs when assets are easy and fast to exchange, experiencing little or no losses in their nominal value.

The criteria defining the degree of liquidity that can be said to be high, determines the way in which the amount of money in the economy is counted. There are various definitions of money under this approach; the narrowest one, corresponds to the monetary base, which encompasses physical cash that is being exchanged in the market (could be said the one that is "in the pockets" of the people) along with bank reserves and bank's deposits in the central bank, which can be claimed by the latter entity without significant difficulty. A more extensive definition, referred to as $M1$, takes into account not only cash but also balances in current bank accounts, while $M2$, includes in addition to the aforementioned values, the balances on the individual's savings accounts. These monetary aggregates can be extended even further by including the liquid liabilities of monetary institutions and even some interest-bearing assets.

Since money, in any of the previous forms, is necessary for most economic transactions to take place, its quantity limits the number of exchanges that can be undertaken within the economy. Therefore, it is essential to understand the mechanisms underlying its use, as it is a primary determinant of the short run economic activity. Money defined in its narrowest form is mainly influenced by the central bank authorities; whereas depositors, investors and banks, take a more significant role when determining the broader aggregates. Such dynamic implies that to understand money on an extended sense, is necessary to analyze both the individuals and the government, as the correct determination of their incentives would have a crucial role in determining the money quantities that are in line with the best development of the economy.

## 1.2 Functions of Money

The value attributed to the usage of money is merely social since its financial worth relies on the fact that all the individuals that employ it acknowledge it as a valid transaction mean. After the central banks abandoned the gold standard and switched to a fiat system, the value of money stopped being intrinsic and became instrumental instead. Under this framework, money acquires its value because it is backed up by the banks that produce it, and also because there is an agreement on its worth by part of the parties that use it. Because of the existence of credibility mechanisms around the institutions that emit the money and due to its widespread adoption, it is possible for the economy to "be structured in a reasonable way so that there are equilibria in which an apparently worthless commodity has a positive price" (Ritter [1995]). Despite its lack of commodity value, money serves several functions in the economy, mainly as: a unit of count, a store of value and a medium of exchange.

### 1.2.1 Money as a Unit of Count

Being a unit of count awards money the property of being a common standard used to measure the relative worth of goods and services, thus facilitating the evaluation of the investment and consumption decisions, as prices and returns can be evaluated under a homogeneous indicator. In the model to be presented, this function of money is captured by the use of prices, as their inclusion carries the underlying assumption that all individuals in the economy can observe the value of the goods and services that they want to acquire and consider them as costs within their budget constraint. It also means that there is a price in the market for the outcome the individuals produce as well as for the revenues of their investments, which will later constitute their source of income. Since all parties in the economy observe these prices, all the instruments and goods are attributed a similar market value, which later facilitates the completion of the transactions.

### 1.2.2 Money as Value Storage

The function of value storage comes from the fact that money is a particularly liquid asset. Liquidity ensures that the money-holder will be able to trade the currency easily in the future, which makes it a convenient way to store and pass wealth from one period to another. On an inter-temporal setting, the major risk associated with cash holdings is the risk of its depreciation due to the inflation of overall prices on the economy, which causes each monetary unit to afford less consumption in real terms. In other words, it is possible to interpret inflation as the price paid for holding money. In the setting of the model, this value storage property is attained in two ways, as there is the possibility of inter-temporal wealth transactions by means of investments on capital that will be available for production in the next period (but depreciates at a rate $\delta$), or through the investment of risk-less government bonds that have a nominal rate of return $i_t$.

### 1.2.3 Money as a Medium of Exchange

Finally, the motivation of this paper relies on the function of money as a medium of exchange. Serving such purpose implies that money can be used directly to purchase goods and services without the need for a bilateral incentive alignment. In the absence of money, goods have to be exchanged through a process of barter where the double coincidence of wants is difficult to attain. Such a negotiation hinders the economic processes within large economies: "To provide a function for money, there must be sufficient variety in agents and goods to make trade desirable, and to generate impediments to trade so that money can ease the difficulty" (Jevons [1989]). The model presented, argues that for this role of money to be captured, it is necessary to define a mechanism that captures its exchange value that does not attribute the existence of an intrinsic, non-existent, value. Extensions on the implication of this function and the implications this has on the model's assumptions will be addressed in the coming sections.

## 1.3 The Inclusion of Money in Macroeconomic Models

Acknowledging the fundamental role of money in the economy, inspired various attempts to model its effects within the macroeconomic outcomes. Several approaches have been made to analyze the optimal levels of the money supply. The IS-LM-FE model shows how the liquidity preference money supply reacts to changes in the market of real goods in the context of an open economy (Hicks [1980]). The Baumol-Tobin model provides an understanding the dynamics of the money demand by exploring the trade-off between the liquidity gained with money holdings and the opportunity cost of doing so in terms of the interest rate (Tobin [1989]). The shopping-time model characterizes the utility gains arising from the usage of money for transactions and explores how labor supply reacts to growth to changes in the monetary supply. The Cash-in-Advance model sets a constraint that transforms money holdings into a necessary requisite for purchasing a good (Clower [1967]). Moreover, several applications of game theory have been used to formalize the relationship between the characteristics of the policymaker and the decision making process of the consumer (Barro 1983, Barro 1984, Barro 1986), whereas empirical research investigates how various monetary transmission mechanisms allow the macroeconomic conditions to respond to monetary policy (Taylor [1995]).

### 1.3.1 The Sidrauski Model: Advantages and Issues

The Argentinean economist, Miguel Sidrauski, proposed in his 1967 article *"Rational Choice and Patterns of Growth in a Monetary Economy"*, an approach to model money in the economy. Under this setting, he analyzed the choices of a representative consumer who maximizes his utility, which is a function of the consumption of goods and real money holdings, across an inter-temporal horizon. The model aims to observe the effects of money in the long run, or as said by the author himself: "to break monetary theory loose from the mold of short-run equilibrium analyses, conducted in abstraction from the process of economic growth and accumulation" (Sidrauski [1967]). The introduction of money as an alternative asset to real capital in the form of the government's non-interest bearing debt, results in a steady state with super-neutrality of money; where the intensity of capital, as well as other real variables derived from it, are invariant to the rate of change of the money supply.[1]

The configuration of the model allows to make a micro-economically founded analysis of a macroeconomic outcome through a general equilibrium process. This setting is advantageous, as it shows the way in which the aggregated decisions of the individuals have a direct effect on the large-scale economy, meaning that it is possible to see the how individual incentives play a role on the overall economic outcome. This aspect is especially important from the policy formulation process perspective, as it inspires action mechanisms that can be easily targeted to the individuals, instead of the market as a whole, increasing their power of action. On the other hand, the inter-temporal horizon, is more realistic, as it tries to capture the final convergence, but also the evolution towards it, allowing the possibility to model changes over time, which results on a better understanding of the examined process. Moreover, individual incentives are also relevant in the form of a more realistic setting for modeling expectations which follows Cagan's Model on individual's reactions to monetary incentives (Cagan [1956])). This construction adds a layer of complexity to the analysis, which results in a more real and accurate picture than the one made under the standard rationality assumptions. Finally, the possibility to derive a demand for money that is responsive to both the production capacity of the economy and the opportunity cost of holding the cash balances is consistent with other theoretical approximations such as that of the Baumol-Tobin optimal cash balance proposition (Tobin [1989]). All in all, these features are the reason why Sidrauski's model is the "starting point for most monetary models that feature capital accumulation" (Reis [2007]).

Despite all the theoretical advantages highlighted in the model, its setup has also been a source of significant criticism. Some authors point at the fact that the long run super-neutrality can be easily disturbed, especially in the presence of changes to the aggregate savings rate, the process of capital accumulation and the existence of differences among agents. These claims are used to support the argument that the model is yet too simplistic, which is what allows its long-run conclusions to be close to reality: "The main lessons were thus already implicit in the work of Tobin and Sidrauski. For those

[1]To see the original model setup refer to appendix A

who can bring themselves to accept the single-consumer, infinite-horizon, maximization model as a reasonable approximation to economic life, super-neutrality is a defensible presumption. All others have to be ready for a different outcome" (Orphanides [2007]). However, one of the main issues is related to the model's main characteristic: the consumer's utility function. When defining the preferences of the individual, it is assumed that real money balances enter directly in the welfare function through the assumption of a one-to-one proportion between the flow of goods and services and the real cash balances.

> *"The basic economic unit in our model is the representative family. Its welfare at any point in time is measured by a time-invariant utility function of the form $U_t = U(c_t, z_t)$, where $c_t$ stands for the flow of real consumption per unit of time, and $z_t$ for the flow of services per unit of time derived from holdings of real cash balances, both variables being expressed in per capita terms. To simplify, we will assume that the flow of services derived from the holdings of real cash balances is proportional to the stock and, by an appropriate choice of units, we make the factor of proportionality equal to one. $z_t = m_t = \frac{M_t}{P_t N_t}$. $M_t$ represents the holdings of nominal cash balances by the economic unit, $N$, the number of individuals in the economic unit and $P_t$ the money price of the only commodity produced in our model. The instantaneous utility function can then be written $U_t = U(c_t, m_t)$"* (Sidrauski [1967]).

This simplifying assumption represents an intuitive issue, which is the focus of the former paper. Such supposition is problematic since by including money directly in the utility function, it ends up giving it intrinsic value, even though in the context of fiat currency this representation is not accurate. It is more realistic to assume a conversion factor instead of a one-to-one relation between cash holdings and consumption, as it allows to capture the fact that there is not an increase in utility from cash itself, but that holding this asset only increases welfare up to the extent that the individual is interested in using it to acquire goods. The above logic is what drives the derivation of the conversion factor of money holdings to be in terms of the size of the consumption basket. The quantity of consumption that the individual wants to afford defines the amount of cash that would be useful for him. Therefore, holdings beyond that given amount provide no additional marginal utility, as the individual surpasses the optimal level of cash holdings. Moreover, such a situation represents a loss in the amount of current wealth, as those additional resources could be better invested in another productive asset (either capital or bonds) that would generate a higher return since there is no longer room to gain utility form the alleviation of the transaction costs when holding notes. Hence, it is clear that the individual does not value the money holdings, not even if they are expressed in real terms, but what is valued instead, is the consumption attained from the real basket of goods that can be acquired more conveniently with such money balances.

The subsequent sections will analyze the way in which the original one-to-one conversion factor assumption makes the model faulty when capturing the function of money as a medium of exchange. The aim of the former document is to respond to the following research question: ***Up to which extent a more comprehensive understanding of the role and value of money would affect the general equilibrium results attained in the Sidrauski: Money In Utility model?*** This would be done, by exploring a possibly more comprehensive way of including the cash balances within the model's context. To do so, the second section presents the derivation of the modified model, along with the theoretical motivation for the adjustments proposed and an illustration of the results using specific functional forms. Subsequently, the third section presents an evaluation of the modified model, using the results of the original Sidrauski model as a benchmark. Finally, the last section presents a wrap up of the most relevant insights of the proposed analysis, as well as some suggestions for further studies.

# Chapter 2

# Modified Model: An instrumental Conception of Money

## 2.1 Setup and Derivation

Just as in the original setup, this version of the model considers a rational representative consumer that maximizes his utility across all the inter-temporal horizon. To be able to have an accurate comparison of the evolution of his utility through time, the individual optimizes the present value of his utility. Periods can be easily distinguished from each other and are evaluated independently (i.e., Days or months), therefore a discrete discount factor $\beta$ is used.

The optimization problem of the individual is characterized as follows:

$$Max_{(c_t, M_{t+1}, k_{t+1}, b_{t+1})_{t=0}^{\infty}} : \sum_{t=0}^{\infty} \beta^t U\left(c_t, e_t\right) \tag{2.1}$$

$$s.t. : c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} \leqslant f(k_t) + (1-\delta)k_t + \frac{M_t}{P_t} + (1+r_t)b_t + t_t \tag{2.2}$$

$$s.t. : e_t = \frac{M_t}{c_t P_t} \tag{2.3}$$

Here, the individual's utility function is concave and positive in two arguments: consumption and the exchange conversion rate of money into consumption goods. This second argument is a variable that is further defined in equation 2.3 and whose outline also constitutes a restriction for the maximization problem. This exchange factor represents the real money holdings that are normalized not only in terms of prices but also in terms of the nominal basket of goods that they can afford. In this order of ideas, the exchange factor accounts for the way in which cash is translated into nominal consumption. The introduction of this term into the utility function aims to capture the value of money as a medium of exchange since unlike the original Sidrauski model, money is not valuable per se to the individual, but it is valuable regarding how it can be transformed into goods for consumption. Under this setting, the utility acquired from owning cash is measured in terms of the basket of goods that the individual wishes to acquire. Money alleviates the transaction costs associated with the bartering process needed to achieve a certain level of consumption and simplifies those economic transactions by reducing the expenses that arise when two parties aim to engage in an exchange. In this order of ideas, money only represents a gain regarding cost reduction and business dealing efficiency.

On the other hand, the individual has a budget constraint that depicts the various possibilities that he has when allocating his assets. The right-hand side of equation 2.2 refers to the sources of income the individual has on each period. Taking into account that this representative subject is at the same time a consumer and the owner of the factors of production, as well as the production facilities, he gets an income for the current production that the capital invested at the beginning of the period yields. This income is reduced since he needs to take into account the value lost from the physical depreciation of the capital after being used for production. Another source of income, are the real money balances held at the beginning of the period, which are further incremented by the real return of the risk-less government bonds held at that same moment in time. Finally, the government has a direct effect on the budget constraint of the individual as it can increase his endowments through a positive lump-sum transfer (i.e., subsidy) when $t_t < 0$ or a lump sum tax when $t_t > 0$. Furthermore, the left-hand side of the budget constraint refers to the possibilities that the individual has to spend his income. He has a choice between consumption in the current period, investment in the form of physical capital that will get transferred into the next period, real money balances also for the next period (accounted for at current prices), and investment in government bonds that will be carried into the future.

The setup of the optimization leads to the following nonlinear programming:

$$\Gamma = \sum_{t=0}^{\infty} \beta^t \left\{ U\left(c_t, \frac{M_t}{c_t P_t}\right) - \lambda_t \left[ c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} - (1+r_t)b_t - t_t \right] \right. \quad (2.4)$$

The maximization results in the following first order conditions [1]:

$$k_{t+1}\left\{-\lambda_t + \beta\left[\lambda_{t+1}\left(f'(k_{t+1}) + (1-\delta)\right)\right]\right\} = 0 \quad (2.5)$$

$$M_{t+1}\left\{\beta\left[U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left(\frac{1}{c_{t+1}P_{t+1}}\right) + \lambda_{t+1}\left(\frac{1}{P_{t+1}}\right)\right] - \lambda_t\left(\frac{1}{P_t}\right)\right\} = 0 \quad (2.6)$$

$$c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} - (1+r_t)b_t - t_t = 0 \quad (2.7)$$

$$b_{t+1}\left\{\beta\lambda_{t+1}(1+r_{t+1}) - \lambda_t\right\} = 0 \quad (2.8)$$

$$c_t\left\{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right)\left|\frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t}\right| - \lambda_t\right\} = 0 \quad (2.9)$$

It is important to note that when the optimization with respect to consumption is done, the effect of both the consumption itself, and the rate at which cash can be exchanged into goods need to be taken into account. [2]

[1] To see the full derivation refer to Appendix B, section 1

[2] If $z = f\left(x(t), y(t)\right)$, then: $\frac{dz}{dt} = \frac{\partial z}{\partial x}\left(\frac{dx}{dt}\right) + \frac{\partial z}{\partial y}\left(\frac{dy}{dt}\right)$
So if $U = f\left(c_t, e_t(c_t)\right)$, then: $\frac{dU}{dc_t} = \frac{\partial U}{\partial c_t}\left(\frac{dc_t}{dc_t}\right) + \frac{\partial U}{\partial e_t}\left(\frac{de_t}{dc_t}\right) = \frac{\partial U}{\partial c_t} + \frac{\partial U}{\partial\left(\frac{M_t}{c_t P_t}\right)}\left(-\frac{M_t}{(c_t)^2 P_t}\right)$

The first order conditions are taken along with the transversality conditions to find the long run equilibrium. This set of equations characterize the optimal path of the dynamic model which allow identifying the paths that make the solutions, expressed as Euler equations, stable in an infinite time horizon.

$$\begin{cases} lim_{t\to\infty}\left(\beta\lambda_t\left(k_{t+1}\right)\right)=0 \\ lim_{t\to\infty}\left(\beta\lambda_t\left(\frac{M_{t+1}}{P_t}\right)\right)=0 \\ lim_{t\to\infty}\left(\beta\lambda_t\left(b_{t+1}\right)\right)=0 \end{cases} \tag{2.10}$$

The transversality conditions indicate that the individual has a higher valuation for the assets that are held in the near future. When the assets are located too far in time, the discounting process along with the multiplier diminish their value, therefore their weight within the optimization process. This mechanism implies that although the individual maximizes across all the inter-temporal horizon, there is a preference for assets that have returns that are closer to the present. The former conditions allow achieving stability in the steady state since the present bias will motivate the individual to distribute consumption close to the actual period, which allows for convergence in the long run as the magnitudes of the choice variables will be held constant across time once the equilibrium is achieved.

Moreover, is possible to simplify the optimization by excluding corner solutions. The reduction can be made by assuming that any equilibrium where at least one of the decision variables is zero is not relevant to the problem in question, as the individual values diversity on the allocation choices. The simplification allows setting a further restriction on the optimization problem so that the outcome is restricted to internal solutions only.

$$c_t>0, M_{t+1}>0, k_{t+1}>0, b_{t+1}>0 \tag{2.11}$$

The previous conditions can be used to derive the Euler equations that characterize the solutions in equilibrium [3]:

The Euler equation for capital:

$$\frac{\lambda_t}{\beta\lambda_{t+1}} = \frac{U_1'\left(c_t, \frac{M_t}{c_tP_t}\right) - U_2'\left(c_t, \frac{M_t}{c_tP_t}\right)\left|\frac{d\left(\frac{M_t}{c_tP_t}\right)}{dc_t}\right|}{\beta\left(U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|\right)}$$
$$= f'(k_{t+1}) + (1-\delta) = 1 + r_{t+1} \quad (2.12)$$

The money demand function:

$$\frac{U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{\left[U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|\right](c_{t+1})} = i_{t+1} \quad (2.13)$$

The Euler equation for bonds:

$$\frac{U_1'\left(c_t, \frac{M_t}{c_tP_t}\right) - U_2'\left(c_t, \frac{M_t}{c_tP_t}\right)\left|\frac{d\left(\frac{M_t}{c_tP_t}\right)}{dc_t}\right|}{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|} = \beta(1 + r_{t+1}) \quad (2.14)$$

Now the government's side of the economy must be analyzed to find the equilibrium values that clear the market in the long run. This budget constraint consists of a series of assets and liabilities that limit the government's income and therefore its expenditure.

To begin, it is possible to represent the aggregate budget constraint in nominal terms:

$$(M_t - M_{t-1}) + B_t = T_{t-1} + (1 + i_{t-1})B_{t-1} \quad (2.15)$$

The right-hand side of the condition shows the government's sources of income, where the inter-period difference of money flows in the economy corresponds to the amount of expenditure that is financed via seigniorage, and the second source of income which

[3] To see the full derivation refer to Appendix B, section 2

corresponds to the money received when emitting government bonds that are bought by the households. Moreover, the left-hand side of the equation shows the liabilities which are represented by the interest rate due on the bonds sold during the previous period and the transfers made to the individuals in the current period.

Rewriting equation 2.15 in real terms:

$$\frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} + b_t = t_{t-1} + (1 + i_{t-1})\frac{B_{t-1}}{P_t} \tag{2.16}$$

The preceding equation can be simplified even more by using the Fischer equation and iterated for one period:

$$\frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} + b_{t+1} = t_t + (1 + r_t)b_t \tag{2.17}$$

The existence of internal solutions allows to take the individual's budget constraint stated in equation 2.2 with equality, which along with equation 2.17 leads to the joint budget constraint of the economy, this expression aligns the decisions of the government and the individuals in equilibrium. This result will be used afterward to determine consumption, capital accumulation, and production in the steady state.

$$c_t + k_t = f(k_t) + (1 - \delta)k_t \tag{2.18}$$

Finally, the steady state can be calculated. If this point in time has been reached, it can be assumed that there are no more shifts in the outcomes of the economy. At this point, capital accumulation, production, and consumption are fixed in their long-run efficient levels (noted by the subscript $ss$). Therefore, it is possible to assume the following:

$$\begin{cases} t_t = t_{ss} & \lambda_t = \lambda_{t+1} = \lambda_{ss} \\ \pi_t = \pi_{ss} & k_t = k_{t+1} = k_{ss} \end{cases} \tag{2.19}$$

Applying these assumptions into the Euler equation for capital, written in 2.12:

$$\frac{1}{\beta} = f'(k_{ss}) + (1 - \delta) \tag{2.20}$$

Assuming a discrete discount factor [4], the steady state condition can be rewritten as:

$$(1+\rho) = f'(k_{ss}) + (1-\delta) \tag{2.21}$$

Which can be further simplified to:

$$(\rho+\delta) = f'(k_{ss}) \tag{2.22}$$

This last equation represents the condition from which the capital on the steady state can be derived. It also shows how in equilibrium, the marginal product of capital should be large enough to cover the depreciation rate and the inter-temporal converting factor, which is analogous to the opportunity cost of taking cash holdings on into the future via capital instead of investment in bonds.

## 2.2 Illustration: Assumption of Specific Functional Forms

To further explore the solutions obtained from the modified model, an illustration of the equilibrium using some specific functional forms is provided. Equation 2.23 characterizes an additive utility function, whose first argument makes it concave and increasing for all the values of consumption. The second argument is a Sigmoid function with an attributed weight of $\theta$, which makes utility concave for the conversion factor of money into consumption goods, and also reaches a maximum contribution to the overall utility when $e_t$ is equal to one[5]. On the other hand, equation 2.24 defines a neoclassical one argument Cobb-Douglas production function, whose marginal productivity serves to determine capital accumulation rates.

$$U\left(c_t, \frac{M_t}{P_t}\right) = \sqrt{c_t} + \theta\left(\frac{M_t}{c_tP_t} \cdot \frac{1}{\sqrt{\left(\frac{M_t}{c_tP_t}\right)^2 + \gamma}}\right) \quad for \;\; (0 \leq \theta \leq 1) \;\; \wedge \;\; (0 \leq \gamma \leq 0.01) \tag{2.23}$$

[4]Meaning that $\beta = \frac{1}{1+\rho}$

[5]The reasoning behind this assumption is explained in detail in chapter 3. For some other functional forms that satisfy these characteristics, see Appendix C.

$$y = f(k_t) = Ak^{\alpha} \quad for \ \ (0 \leq \alpha \leq 1) \tag{2.24}$$

The marginal utilities with respect to consumption and the conversion rate are respectively given by:

$$U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) = \frac{1}{2\sqrt{c_t}} - \frac{\theta\left(\frac{M_t}{P_t} \cdot \gamma\right)}{\left[\left(\frac{M_t}{P_t}\right)^2 + \gamma c_t^2\right]\sqrt{\left(\frac{M_t}{c_t P_t}\right)^2 + \gamma}} \tag{2.25}$$

$$U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) = \frac{\theta\gamma}{\left(\gamma + \left(\frac{M_t}{c_t P_t}\right)^2\right)^{\frac{3}{2}}} \tag{2.26}$$

From the iteration of equation 2.13 to the previous period, the use of the marginal utilities in equations 2.25 and 2.26, the following money demand function is obtained, after some algebraic manipulation:

$$\frac{2\gamma c_t^{\frac{7}{2}}}{\left(\left(\frac{M_t}{P_t}\right)^2 + \gamma c_t^2\right)^{\frac{3}{2}} - 2\left(\frac{M_t}{P_t}\right)\gamma c_t^{\frac{3}{2}}} = i_t \tag{2.27}$$

As the variable for the real money influxes in equilibrium cannot be isolated entirely, the implicit function theorem is used to calculate the comparative statics:[6]

$$\frac{\partial\left(\frac{M_t}{P_t}\right)}{\partial\left(c_t\right)} = \frac{-\left(\frac{\gamma c_t^{\frac{5}{2}}\left[7\left(\frac{M_t}{P_t}\right)^2\sqrt{\left(\frac{M_t}{P_t}\right)^2 + \gamma c_t^2} + (\gamma c_t^2)\sqrt{\left(\frac{M_t}{P_t}\right)^2 + \gamma c_t^2} - 8\left(\frac{M_t}{P_t}\right)\gamma c_t^{\frac{3}{2}}\right]}{\left[\left(\left(\frac{M_t}{P_t}\right)^2 + \gamma\, c_t^2\right)^{(3/2)} - 2\left(\frac{M_t}{P_t}\right)\gamma\, c_t^{\left(\frac{3}{2}\right)}\right]^2}\right)}{-\left(\frac{2c_t^{\frac{7}{2}}\gamma\left[3\left(\frac{M_t}{P_t}\right)\sqrt{c_t^2\gamma + \left(\frac{M_t}{P_t}\right)^2} - 2c_t^{\frac{3}{2}}\gamma\right]}{\left[\left(c_t^2\,\gamma + \left(\frac{M_t}{P_t}\right)^2\right)^{\left(\frac{3}{2}\right)} - 2\,c_t^{\left(\frac{3}{2}\right)}\gamma\left(\frac{M_t}{P_t}\right)\right]^2}\right)} \geq 0 \tag{2.28}$$

[6]The verification of inequalities in equations 2.28 and 2.29 was done using Matlab.

$$\frac{\partial\left(\frac{M_t}{P_t}\right)}{\partial\left(i_t\right)} = \frac{-\left(-1\right)}{-\left(\frac{2c_t^{\frac{7}{2}}\gamma\left[3\left(\frac{M_t}{P_t}\right)\sqrt{c_t^2 k+\left(\frac{M_t}{P_t}\right)^2}-2c_t^{\frac{3}{2}}\gamma\right]}{\left[\left(c_t^2\,\gamma+\left(\frac{M_t}{P_t}\right)^2\right)^{\left(\frac{3}{2}\right)}-2\,c_t^{\left(\frac{3}{2}\right)}\,\gamma\left(\frac{M_t}{P_t}\right)\right]^2}\right)} \leq 0 \tag{2.29}$$

The production function in equation 2.24 along with the steady state condition in equation 2.22 yields the steady state capital accumulation level:

$$k_{ss} = \left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} \tag{2.30}$$

The level of production in equilibrium is given by the capital accumulation in the steady state from equation 2.30 and the production function in equation 2.24:

$$y_{ss} = A\left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{2.31}$$

The steady state consumption level follows from the joint budget restrictions in equation 2.18 and the steady state capital in equation 2.30:

$$c_{ss} = A\left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} \tag{2.32}$$

Taking the steady state capital and and consumption levels into the joint budget restrictions in equation 2.18 yields:

$$\begin{aligned} c_{ss} + k_{ss} &= A k_{ss}^{\alpha} + (1-\delta)k_{ss} \\ \underbrace{A k_{ss}^{\alpha}}_{Production} &= \underbrace{c_{ss}}_{Consumption} + \underbrace{\delta k_{ss}}_{Investment} \end{aligned} \tag{2.33}$$

This relationship verifies the fulfillment of the fundamental macroeconomic identity, for a closed economy. It shows that from an expenditure perspective, the total economic activity is given by the sum of all the final demands of products in the economy. That is, the total expenditure on final goods and services (consumption and investment in this case).

The previous results illustrate that despite the modifications made the equilibrium still holds its neutrality properties. This is evident, as all the changes in the money availability ($M_t, M_{t+1}, M_{t+2}$, etc.) do not affect any of the real variables in the steady state ($y_t,\ c_t\ and\ K_t$). Hence, the classical dichotomy is accomplished within this setting, allowing for the real and nominal variables to be analyzed separately. This result depicts how changes in the way money is valued within the utility function, only have potential effects over the decisions in the short and medium run (which are the time horizons in which monetary flows are relevant for the outcomes of the economy), leaving unaltered the steady state equilibrium. As a result, the seigniorage conducted by the government does not affect the real economy, causing only a proportional rise in prices. Inflation affects choices in the short run, via two channels: the decrease of the exchange conversion rate of money into consumption goods and the decrease in the purchasing capacity of an individual with a given level of available resources (via the budget constraint).

Moreover, the outcomes also entail super-neutrality, as neither the inflation rate nor the growth rate of the money supply affect any of the real variables in equilibrium. Super-neutrality is only achieved in the long run, where prices are still proportional to the nominal money supply, not only in response to the permanent changes in the money supply but also in response to the permanent changes on its growth rate. This result is relevant for monetary policy formulation, as it states that modifications in the rate of monetary emission do not modify the capital ratio of the economy and along with it the growth path of the long run per capita GDP.

# Chapter 3

# Evaluation and Discussion

## 3.1 Theoretical Implications of the Results

Once the setting of the model has been derived, some intuitive and theoretical motivations need to be made. The objective of the following section is to analyze the implications of the results obtained in the modified model while keeping the solutions attained by the Sidrauski model as a benchmark.

On the first place, it is useful to understand more in-depth the assumptions that underlie the characterization of the utility function. The acquisition of a good or service by an individual could be easily made through an exchange with the seller of the product. During such undertaking, the consumer uses the money holdings he owns to pay the price that the producer demands, which entitles him to the ownership and posterior consumption of the good. If it were the case that the individual did not have the cash available, but still wanted to acquire the product, he would have to devise a contractual mechanism with the seller which allowed him to commit to a payment in the future while still being able to receive the good in the current period. Such contract is costly for both the buyer and the seller. For the manufacturer, postponing the payments to the future implies a degree of uncertainty in the revenues which constitutes a factor of disutility given risk-averse preferences. Moreover, the future payment needs to cover the opportunity cost of the investments that the producer could have done with the revenues of the sale in the case he received them immediately. Devising a contract incurs a cost for the individual. Making such an agreement legally binding requires the use of particular institutions and proceedings that impose further expenses in terms of realization and effort. All the previously mentioned expenses could be interpreted as transaction costs, which are rendered irrelevant with the use of readily available money as payment. Hence, the role of money as a medium of exchange aids in eliminating transaction inefficiencies that emerge when both parties bargain in a cash-absent setting. Money saves the individual the cost of having to acquire his desired consumption employing inconvenient purchases (which are done through contracts or barter) and gives him the possibility to achieve his preferred consumption level more conveniently and efficiently.

The model setup attests to the presence of an opportunity cost related to cash holdings. On the one hand, cash does not generate any return; as a matter of fact, it might even generate some nominal losses in the presence of inflation. The loss materializes when the part of the individual's wealth that is held as cash cannot be simultaneously invested in any other productive holdings, such as bonds or productive capital accumulation. On the other hand, money allows the individual to avoid inconvenient purchases and avert the disutility associated with them. Consequently, the effects of the income burden and the indirect utility gains, pose a trade-off between money and transaction costs which are visible when analyzing the utility effects of the monetary holdings in the margin.

The differentiable aspects of the utility function can be characterized as follows:

$$U(X) = U(c_t, e_t)|\ \forall\ X \geq 0\ \in \mathbb{R} : \begin{cases} \frac{\partial U_1(X)}{\partial X} \geq 0 \\ \frac{\partial U^2(X)}{\partial X^2} \leq 0 \end{cases} \tag{3.1}$$

The first order derivative is non-negative for both of the utility arguments. For consumption, it is strictly positive, which reflects the assumption of non-satiation. Meaning that the individual would always prefer to consume more whenever possible, as a larger, more abundant basket would provide more utility. In the case of the exchange conversion rate of money into consumption goods, the derivative is positive for $0 \leq e_t \leq 1$ and is zero thereafter[1]. Hence, there is a limited range for which an increase in the conversion factor can raise utility, as a more substantial fraction of the preferred basket is acquired utilizing convenient purchases, which represents a saving regarding transaction costs for the individual. Having $e_t = 1$ implies that $M_t = c_t P_t$, in such a case, money holdings are exactly enough to acquire the whole nominal value of the desired consumption conveniently

[1]In the case of the functional form evaluated in section 2.2, an approximation to this particularity is made by choosing a function whose values are very close to zero (but not strictly zero) for $e_t \geq 1$

by using money. There is satiation for larger values of $e_t$, where money holdings do not provide any additional marginal utility as the individual has already afforded all of his optimal consumption in the most convenient way possible. The limited contribution of marginal utility leads to the individual doing his valuation in terms of money holdings, relative to the nominal value of the basket of goods that he is willing to acquire. Thus, utility increases in cash holdings, but the effect is restricted to the extent to which they can be used to afford the desired consumption bundle.

The marginal effects on utility are visible when analyzing the derivative of utility with respect to consumption more in detail:

$$\frac{\partial U(c_t, e_t(c_t))}{\partial c_t} = \underbrace{\frac{\partial U(c_t, e_t(c_t))}{\partial c_t}}_{a>0} + \overbrace{\underbrace{\frac{\partial U(c_t, e_t(c_t))}{\partial e_t(c_t)}}_{c\geq 0} \underbrace{\left(\frac{de_t(c_t)}{dc_t}\right)}_{d<0}}^{b\leq 0} > 0 \tag{3.2}$$

The part of the derivative labeled as $a$ is positive. It represents the direct utility of consumption, and its sign is a reflection of the previously explained non-satiation assumption and how utility increases when being entitled to a broader basket of goods. The part labeled as $b$ is negative, which could be interpreted as the disutility of having to face transaction costs. If money holdings are kept constant, there is a loss in utility from smooth transactions as now a smaller fraction of the optimal consumption can be acquired via money holdings, represented in a lower exchange factor $e_t$. Therefore, a larger consumption level has also a negative effect in *ceteris paribus* conditions, as the acquisition of this additional part of the basket represents a burden since the individual has to engage in effort-demanding agreements to attain it. Part $c$ of this factor is the marginal utility from the transactions alone, which depicts how much utility changes with $e_t$, this effect is non-negative as addressed previously. Part $d$ represents explicitly the marginal change of transactions covered by cash due to an increase in consumption, or in other words, how much the exchange rate $e_t$ changes because of the change in consumption.

These contrary effects, lead to an intra-temporal trade-off between the increase of consumption and the additional transaction costs that arise from having to make a larger fraction of inconvenient purchases. Nonetheless, under this setting, the effect of additional consumption is always positive. It means that the positive effect of marginal consumption is always larger than the disutility generated by the decrease in the conversion factor due to the change of preferences towards a larger basket. Consequently, the individual would always be inclined to increase his consumption if possible, despite the negative consequences that such a shift might represent for some of the arguments at the margin. The utility function in the limit behaves as follows:

$$\lim_{c_t \to \infty} U(c_t, e_t(c_t)) = \infty \tag{3.3}$$

The second order derivative is assumed non-positive for both of its arguments. Such a condition implies that the gains in utility from additional consumption are lower for larger bundles, where the extra good is less of an increase with respect to the whole basket. The gains in utility are also decreasing the more extensive the fraction of the desired consumption is acquired through convenient purchases (as $e_t$ approaches to 1) and are zero afterward, which makes the second order derivative negative between $0 \leq e_t \leq 1$ and zero thereafter. The interaction of first and second order conditions leads to the utility function being concave in both of its components.

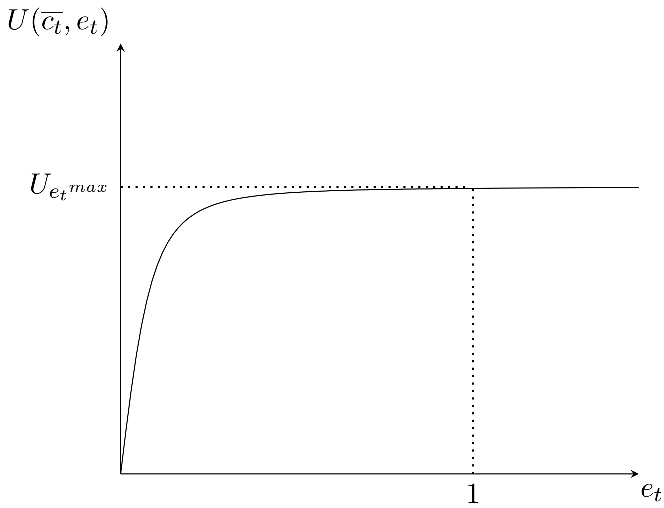


FIGURE 3.1: One Dimensional Limit of the Utility Function

Figure 3.1 graphically represents the relation between the utility and the conversion factor when keeping consumption constant [2]. After $e_t \geq 1$ there are no additional utility gains form the money holdings, therefore an optimizing behavior implies:

$$0 < e_t^{Optimal} \leq 1 \tag{3.4}$$

[2]Note that this analysis can only be done for $\overline{c_t}$ and not for $\overline{e_t}$ as $e_t$ also changes with $c_t$.

When $e_t = 1$, money holdings are sufficient enough to acquire all the the nominal value of the consumption bundle through smooth purchases. Ergo, when $e_t = 1$: $U(\overline{c_t}, e_t) = U_{e_t{}^{max}}$. Which implies that for a given level of prices and consumption, there is a maximum point up to which money holdings can ease transactions contributing to utility. Hence:

$$\lim_{e_t \to \infty} U(c_t, e_t(c_t)) = U_{e_t{}^{max}} \tag{3.5}$$

The insights on the utility function characterize the intra-period dynamics. To better understand how the individual evaluates his decision making for the inter-period setting, the equations that arise from the first order maximization conditions should be analyzed. The Euler equation for capital in 2.12 can be rewritten as:

$$\underbrace{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right)\left|\frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t}\right|}_{a}$$
$$= \underbrace{\beta\left(\overbrace{f'(k_{t+1}) + (1-\delta)}^{b}\overbrace{\left[U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|\right]}^{c}\right)}_{d} \tag{3.6}$$

Here the individual evaluates the most beneficial way of allocating his inter-temporal consumption. In the optimum, he should be indifferent between transferring utility across periods by deciding to consume more or less at a given moment in time, and transferring wealth into the future by means of capital accumulation. Argument $a$ of this equation shows the marginal utility from consuming one additional unit in the current period and the respective dis-utility that arises from having to purchase that additional consumption trough inconvenient transactions (as money holdings are kept constant). Argument $c$ shows the same relation for the future period. Part $b$, refers to the net rate of return of physical capital: it accounts for what the capital investments in the current period will yield when used in production on the next period ($f'(k_{t+1})$), as well as the compensation for the depreciation of the capital stock when passing from one moment in time to another. In the end, part $d$ reflects how if the extra consumption in the current period is not purchased, its value represents “savings” that can be used to get some additional marginal utility form consumption in the next period, while the resources to do so are transferred to the future via capital accumulation. In order for both time dimensions to be comparable, the future utility is evaluated in terms of its present value using the discount factor $\beta$.

From the Euler equation for capital in 2.12 it also follows:

$$f'(k_{t+1}) + (1 - \delta) = 1 + r_{t+1} \tag{3.7}$$

Hence, when deciding for a mean to transfer the savings arising when abstaining from consumption in the current period to the future, the individual is indifferent between using capital or government bonds, as both yield the same net return in equilibrium. The rate of return of bonds is expressed in real terms so that it can be commensurable with consumption and capital. The same relation could also be expressed using the nominal rate of return, but in order to be comparable to the returns of capital (which are given in real terms), it should also account for price inflation not to overestimate solely for nominal increases.

A higher interest rate implies a higher opportunity cost of holding money, therefore a lower desired quantity of cash in equilibrium. Larger rates make it more costly to acquire money, as the realized returns individuals cease to receive are greater. Moreover, an increase in the size of the desired consumption motivates the individual to acquire more cash, as it alleviates the dis-utility form having to meet the new level of consumption with inconvenient purchases. This effect increases the money demand for a given interest rate and price level. A positive relationship between consumption and production arises, as a larger income from production leads to a higher desired overall consumption. Hence, it is possible to state that an increase in production raises the money demand in the same way as consumption does. Money demand also rises when there is an increase in prices, as inflation makes it necessary to have more money holdings to purchase the same consumption basket.

If $D\left(\frac{M_{t+1}}{P_{t+1}}, c_{t+1}\right)$ denotes the money demand in real terms, due to the positive relationship between consumption and production it is analogous to $D\left(\frac{M_{t+1}}{P_{t+1}}, y_{t+1}\right)$. Consequently, the properties of the money demand are summarized as follows:

$$D\left(\frac{M_{t+1}}{P_{t+1}}, y_{t+1}\right) \mid \forall\ M_{t+1}\ \wedge\ P_{t+1}\ \wedge\ y_{t+1} \geq 0\ \in \mathbb{R}: \begin{cases} D_1'\left(\frac{M_{t+1}}{P_{t+1}}, y_{t+1}\right) \leq 0 \\ D_2'\left(\frac{M_{t+1}}{P_{t+1}}, y_{t+1}\right) \geq 0 \end{cases} \tag{3.8}$$

Such properties are consistent with other theoretical approximations in the literature, such as that of the Baumol-Tobin optimal cash balance optimization (Tobin [1989]). This finding implies that despite the fact that in the modified version of the model, money is valued for its properties as a medium of exchange, the dynamics behind the monetary demand move in the same direction of the existing theoretical and empirical findings.

## 3.2 Model Comparison

This last section aims to expressly and succinctly state the ways in which the outcomes obtained by both the modified and the Sidrauski model are comparable to each other. As hinted by the analysis above, overall most of the results end up being in the same direction, so that the conclusions of the modified model are in line with the original analysis. Changes in the way money is valued by the individual only seem to have an effect in the short run, as the steady-state outcomes depend mainly on the production capacity of the economy that is modeled via the production function. The fact that long-run outcomes do not change after modifications in the way money holdings enter the utility function contributes to support the evidence for money neutrality and super-neutrality, as monetary flows are a nominal variable that does not have an effect on the real steady-state outcomes such as production, capital accumulation, and consumption.

In the short run, both models result in an equilibrium where the two available investment vehicles -government bonds and physical capital- should have the same real returns so that the individual is indifferent between using either of them to transfer his savings when abstaining from consumption to the future periods. The main difference relies on the effects on utility from consumption and money holdings in the margin. When comparing the Euler equations for capital in the modified model 2.12 and in the original model A.17, it is visible how setting the nominal value of the consumption basket as a referent for evaluating the optimal amount of cash holdings affects the way in which the individual values additional consumption. Since now the conversion factor of money into goods ($e_t$) is a function of consumption as well, there is an added trade-off from increasing consumption when keeping money holdings constant, that is reflected in the additional negative argument it the marginal utility. The trade-off arises because a larger desired basket implies that a greater fraction of the purchases should be made inconveniently via effort-demanding arrangements. As a consequence, when evaluating the inter-temporal distribution of consumption, there is an additional marginal dis-utility form increasing consumption that should also be covered when transferring wealth across periods.

The new argument on the marginal utility as well as the evaluation of the nominal money influxes not in real terms as in the Sidrauski model, but in terms of the size of the consumption bundle, are what captures the role of money as a medium of exchange. There is an additional value to money in this scenario, as it is also assessed as a source of cost and effort reduction due to its property of easing transactions. This change in valuation potentially causes the individual's choices in the long run to differ, as now more effects are taken into account when deciding the asset inter-temporal allocations.

In line with the results of neutrality and super-neutrality, the change in the money valuation affects the money demand. Both demands are in line with the theoretical and empirical relations found in literature such as the positive dependence on income and the negative dependence on the interest rate. The differences are visible when looking at the money demand in the original model (Equation A.19 on appendix A) in comparison to the new demand (Equation 2.13). Now, the individual takes into account the marginal utility form money holdings but with respect to the size of the basket that he wants to buy, showing how cash is only needed to the extent up to which it can be transformed into consumption. In this order of ideas, it is noticeable that the money demand from the modified model reacts to shocks in the same way as the benchmark case, but it is less sensitive to such changes as it takes the size of the desired consumption bundle as a reference.

# Chapter 4

# Conclusion

Overall, the proposed framework aims to analyze the effects of a modification in the way money flows are evaluated by an individual, by means of the utility function, in the context of the Sidrauski: money in utility model. The role of money as a medium of exchange is modeled, by introducing a conversion factor that captures the fraction of the desired optimal consumption that is purchased conveniently using cash flows ($e_t$). The original model's findings of neutrality and super-neutrality in equilibrium prove to be robust to the proposed changes. Money neutrality is validated in two ways: (i) In the fact that changes in the nominal variables do not affect any of the real outcomes and (ii) as alterations in the valuation of money availability appear to have implications exclusively in the short run, whereas the long run equilibrium is held intact. This absence of change occurs since the steady state depends solely on the production capacity of the economy and not on the preferences of the individuals.

Despite the long run outcome being unchanged, potential adjustments in the short run are relevant to inquire into, since the trajectory towards the steady-state determines the intermediate outcomes of the economy. The transition dynamics are especially important from the perspective of policy formulation as they affect the welfare of the individuals in the near future. Moreover, having an understanding of the individual's attitudes towards money holdings proves to be notably useful when formulating monetary policies particularly, because the money supply reactions -when responding to contingencies related to the economic cycle- determine characteristics of the financial environment. Monetary policy with a comprehensive understanding of the preferences of the individuals, as well as their mechanisms for internalizing the functions of money, is more likely to guarantee price stability, by minimizing the misalignments of money demand and supply, hence optimizing the path of convergence towards the steady state.

Money is an element present in most of the transactions of the modern societies. Therefore its role in determining the directions of the economic outcomes cannot be neglected. More so, in economies that are far from reaching their productive potential or are transitioning to new levels of productivity. The proposed modification sensibly addresses the main criticisms with respect to the intrinsic value attributed to money holdings in the formulation original model, making the understanding of the economy more realistic and contributing to a deeper comprehension of the reasons behind individual choices. Limitations of the conclusions in the present study are related to its theoretical nature since for reality to be modeled many simplifying assumptions need to be made. Assumptions forestall the comprehension of all the intricate details of the economic reality, but also allow for the isolation of effects facilitating their independent examination.

To further explore the implications of variations in money demand and supply, the conclusions presented in this paper can be tested theoretically by relaxing some of the other assumptions made in the model. Interesting results might be achieved by allowing for the presence of uncertainty by including variables that evolve according to stochastic processes. Furthermore, some insights can also be gained from changing the types of expectations, or diverging the focus towards broader monetary aggregates, by including the access to money from bank accounts by the use of readily available electronic payments. Additional verification of the theoretical predictions can be done by using data to appraise the predictive capacity of the model as well as its degree of descriptiveness from historical observations. Finally, controlled behavioral studies can be used to farther scrutinize the way in which individuals value the different functions of money and their implications on the reasoning behind economic decisions.

# Appendix A

# Benchmark: The Sidrauski, Money in Utility Model

The model considers a representative individual that has the following inter-temporal optimization problem in a discrete time setting.

$$Max_{(c_t, M_{t+1}, k_{t+1}, b_{t+1})_{t=0}^{\infty}} : \sum_{t=0}^{\infty} \beta^t U\left(c_t, \frac{M_t}{P_t}\right) \tag{A.1}$$

$$s.t. : c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} \leqslant f(k_t) + (1-\delta)k_t + \frac{M_t}{P_t} + (1+r_t)b_t + t_t \tag{A.2}$$

Meaning that he maximizes the present value of his utility, which is given by a function that is concave and positive in two arguments: consumption and real money holdings. In this setting, the individual has the opportunity to invest in zero-risk government bonds, that gain a real interest rate $r_t$ each period, but also in capital that later is used for production and depreciates every period at a rate $\delta$.These investments allow him to finance his future and current consumption as well as his money holdings in the form of cash. Finally, the government has a direct effect on the budget constraint of the individual as it can increase his endowments through a positive lump-sum transfer (i.e., subsidy) when $t_t < 0$ or a lump sum tax when $t_t > 0$.

The setup of the optimization leads to the following nonlinear programming:

$$\Gamma = \sum_{t=0}^{\infty} \beta^t \left\{ U \left( c_t, \frac{M_t}{P_t} \right) \right.$$
$$\left. - \lambda_t \left[ c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} - (1+r_t)b_t - t_t \right] \right\} \tag{A.3}$$

The Karush-Khun-Tucker method leads to the following first-order conditions when optimizing with respect to each of the decision variables:

$$k_{t+1} \left\{ -\lambda_t + \beta \left[ \lambda_{t+1} \left( f'(k_{t+1}) + (1-\delta) \right) \right] \right\} = 0 \tag{A.4}$$

$$M_{t+1} \left\{ \beta \left[ U_2' \left( c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}} \right) \frac{1}{P_{t+1}} + \lambda_{t+1} \left( \frac{1}{P_{t+1}} \right) \right] - \lambda_t \left( \frac{1}{P_t} \right) \right\} = 0 \tag{A.5}$$

$$b_{t+1} \left\{ \beta \left[ \lambda_{t+1}(1+r_{t+1}) \right] - \lambda_t \right\} = 0 \tag{A.6}$$

$$c_t \left\{ U_1' \left( c_t, \frac{M_t}{c_t P_t} \right) - \lambda_t \right\} = 0 \tag{A.7}$$

$$c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} - (1+r_t)b_t - t_t = 0 \tag{A.8}$$

It is possible to simplify the optimization by excluding corner solutions. This can be done by assuming that any equilibrium where at least one of the decision variables is zero is not relevant to the problem in question, as the individual values the diversity on his allocation choices. This allows setting a further restriction on the optimization problem so that the outcome is restricted to internal solutions only.

$$c_t > 0, M_{t+1} > 0, k_{t+1} > 0, b_{t+1} > 0 \tag{A.9}$$

Using the the previous conditions, along with equations A.1 to A.8, the problem gets reduced to a 5*4 linear equation system that can be expressed as follows:

$$\lambda_t = U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) \tag{A.10}$$

$$\lambda_t = \beta \left[\lambda_{t+1} \left(f'(k_{t+1}) + (1-\delta)\right)\right] \tag{A.11}$$

$$\lambda_t \left(\frac{1}{P_t}\right) = \beta \left[U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) \frac{1}{P_{t+1}} + \lambda_{t+1} \left(\frac{1}{P_{t+1}}\right)\right] \tag{A.12}$$

$$\lambda_t = \beta \left[\lambda_{t+1}(1 + r_{t+1})\right] \tag{A.13}$$

The transversality conditions used to characterize the optimal path of the dynamic model, allow identifying the paths that make the solutions, expressed in the way of Euler equations, stable in the long run.

$$lim_{t \to \infty} \left(\beta \lambda_t \left(\frac{M_{t+1}}{P_t}\right)\right) = 0 \tag{A.14}$$

$$lim_{t \to \infty} \left(\beta \lambda_t \left(k_{t+1}\right)\right) = 0 \tag{A.15}$$

$$lim_{t \to \infty} \left(\beta \lambda_t \left(b_{t+1}\right)\right) = 0 \tag{A.16}$$

The first optimal condition is the Euler equation for capital which characterizes the trade-off between consumption and capital accumulation.

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = \frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right)}{\beta U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)} = f'(k_{t+1}) + (1-\delta) = 1 + r_{t+1} \tag{A.17}$$

The second optimal condition is the Euler equation for bonds which characterizes the trade-off between consumption and investment on government bonds.

$$\frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right)}{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)} = \beta(1 + r_{t+1}) \tag{A.18}$$

Then, it is possible to find the money demand function, which shows the preferences of the individual regarding his money holdings with respect to his consumption in the future and the interest rate, which represents the opportunity cost of the cash owned.

$$\frac{U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)}{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)} = i_{t+1} \tag{A.19}$$

The government on its side has a budget constraint that limits the extent up to which the state's liabilities, which are represented by the interest rate due on its bonds and the transfers made to the individuals, can be financed with seigniorage and bond emission.

$$\frac{M_{t+1}}{P_{t+1}} - \frac{M_t}{P_t} + b_{t+1} = (1 + r_{t+1}) b_t + t_t \tag{A.20}$$

The former equation, along with the budget constraint of the individual stated in equation A.2 and the internal solution restrictions from equation A.9, lead to the following joint budget constraint:

$$c_t + k_t = f(k_t) + (1 - \delta) k_t \tag{A.21}$$

Lastly, the condition for the steady state can be found from the Euler equation for capital accumulation A.17, after assuming that in the long run the multipliers, as well as the government transfers, the inflation, and the capital are on their steady-state levels.

$$\frac{1}{\beta} = f'(k_{ss}) + (1 - \delta) \tag{A.22}$$

# Appendix B

# Mathematical Specificities of the Modified Model

## B.1 Derivation of the First Order Conditions

Opening the sum in equation 2.4 by means of iterations:

$$\Gamma = \beta^t \left\{ U\left(c_t, \frac{M_t}{c_t P_t}\right) - \lambda_t \left[ c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} \right.\right.$$
$$- (1+r_t)b_t - t_t + \beta^{t+1} \left\{ U\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - \lambda_{t+1} \left[ c_{t+1} + k_{t+2} + \frac{M_{t+2}}{P_{t+1}} + b_{t+2} \right.\right.$$
$$- f(k_{t+1}) - (1-\delta)k_{t+1} - \frac{M_{t+1}}{P_{t+1}} - (1+r_{t+1})b_{t+1} - t_{t+1} + \beta^{t+2}\{...\} + \beta^{t+3}\{...\} + ... \tag{B.1}$$

Taking the derivatives to find the first order conditions:

$$\frac{\partial \Gamma}{\partial k_{t+1}} = -\lambda_t + \beta\lambda_{t+1}\left[f'(k_{t+1}) + (1-\delta)\right] \leq 0 \tag{B.2}$$

$$k_{t+1}\left\{-\lambda_t + \beta\left[\lambda_{t+1}\left(f'(k_{t+1}) + (1-\delta)\right)\right]\right\} = 0 \tag{B.3}$$

$$\frac{\partial \Gamma}{\partial M_{t+1}} = -\lambda_t \left(\frac{1}{P_t}\right) + \beta \left[ U_2'\!\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) \left(\frac{1}{c_{t+1}P_{t+1}}\right) + \lambda_{t+1}\left(\frac{1}{P_{t+1}}\right)\right] \tag{B.4}$$

$$M_{t+1}\left\{\beta \left[ U_2'\!\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) \left(\frac{1}{c_{t+1}P_{t+1}}\right) + \lambda_{t+1}\left(\frac{1}{P_{t+1}}\right)\right] - \lambda_t \left(\frac{1}{P_t}\right)\right\} = 0 \tag{B.5}$$

$$\frac{\partial \Gamma}{\partial \lambda_t} = c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} - (1+r_t)b_t - t_t = 0 \tag{B.6}$$

$$c_t + k_{t+1} + \frac{M_{t+1}}{P_t} + b_{t+1} - f(k_t) - (1-\delta)k_t - \frac{M_t}{P_t} - (1+r_t)b_t - t_t = 0 \tag{B.7}$$

$$\frac{\partial \Gamma}{\partial b_{t+1}} = -\lambda_t + \beta\left[\lambda_{t+1}(1+r_{t+1})\right] \leq 0 \tag{B.8}$$

$$b_{t+1}\left\{\beta\lambda_{t+1}(1+r_{t+1}) - \lambda_t\right\} = 0 \tag{B.9}$$

$$\frac{\partial \Gamma}{\partial c_t} = U_1'\!\left(c_t, \frac{M_t}{c_tP_t}\right) - U_2'\!\left(c_t, \frac{M_t}{c_tP_t}\right)\left|\frac{d\left(\frac{M_t}{c_tP_t}\right)}{dc_t}\right| - \lambda_t \tag{B.10}$$

$$c_t\left\{U_1'\!\left(c_t, \frac{M_t}{c_tP_t}\right) - U_2'\!\left(c_t, \frac{M_t}{c_tP_t}\right)\left|\frac{d\left(\frac{M_t}{c_tP_t}\right)}{dc_t}\right| - \lambda_t\right\} = 0 \tag{B.11}$$

## B.2 Derivation of the Euler Equations

Using the internal solution restrictions shown in equation 2.11, along with equations 2.4 to 2.9, the problem gets reduced to a 5*4 linear equation system that can be expressed as follows:

$$\lambda_t = U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right) \left| \frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t} \right| \tag{B.12}$$

$$\lambda_t \left(\frac{1}{P_t}\right) = \beta \left[ U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) \left(\frac{1}{c_{t+1} P_{t+1}}\right) + \lambda_{t+1} \left(\frac{1}{P_{t+1}}\right) \right] \tag{B.13}$$

$$\lambda_t = \beta \lambda_{t+1}(1 + r_{t+1}) \tag{B.14}$$

$$\lambda_t = \beta\ \lambda_{t+1}\big(f'(k_{t+1}) + (1-\delta)\big) \tag{B.15}$$

From conditions B.14 and B.15

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = f(k_{t+1})' + (1-\delta) = (1 + r_{t+1}) \tag{B.16}$$

From condition B.12 and its iteration for one period:

$$\lambda_{t+1} = U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) \left| \frac{d\left(\frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)}{dc_{t+1}} \right| \tag{B.17}$$

From equations B.16 and B.17:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = \frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right) \left| \frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t} \right|}{\beta\left(\lambda_{t+1} + U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) \left| \frac{d\left(\frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)}{dc_{t+1}} \right|\right)} \tag{B.18}$$

From equations B.17 and B.18

$$\frac{\lambda_t}{\beta\lambda_{t+1}} = \frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right)\left|\frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t}\right|}{\beta\left(\lambda_{t+1} + U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|\right)} =$$

$$f(k_{t+1})' + (1-\delta) = (1 + r_{t+1}) \quad \text{(B.19)}$$

From conditions B.12 and B.13:

$$\frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right)\left|\frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t}\right|}{P_t} =$$

$$\beta\left[\frac{U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{c_{t+1}P_{t+1}}\right] + \beta\left[\frac{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|}{P_{t+1}}\right] \quad \text{(B.20)}$$

$$\frac{\frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right)\left|\frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t}\right|}{P_t}}{\beta\left[\frac{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|}{P_{t+1}}\right]} =$$

$$\frac{\beta\left[\frac{U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{c_{t+1}P_{t+1}}\right]}{\beta\left[\frac{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_t P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\right)}{dc_{t+1}}\right|}{P_{t+1}}\right]} + 1 \quad \text{(B.21)}$$

Rewriting equation B.21:

$$\frac{U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{U_1'\Big(c_{t+1}, \frac{M_{+1t}}{c_{t+1}P_{t+1}}\Big) - U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)\left|\frac{d\Big(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{dc_{t+1}}\right|(c_{t+1})} = \frac{U_1'\Big(c_t, \frac{M_t}{c_tP_t}\Big) - U_2'\Big(c_t, \frac{M_t}{c_tP_t}\Big)\left|\frac{d\Big(\frac{M_t}{c_tP_t}\Big)}{dc_t}\right|}{\beta\left[U_1'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big) - U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)\left|\frac{d\Big(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{dc_{t+1}}\right|\right]}\left(\frac{P_{t+1}}{P_t}\right) - 1 \quad \text{(B.22)}$$

Using equations B.19 and B.22:

$$\frac{U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{U_1'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big) - U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)\left|\frac{d\Big(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{dc_{t+1}}\right|(c_{t+1})} = \left[(1 + r_{t+1})\left(\frac{P_{t+1}}{P_t}\right)\right] - 1 \quad \text{(B.23)}$$

This equality can be further simplified in terms of inflation and the real interest rate [1]:

$$\frac{U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{U_1'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big) - U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)\left|\frac{d\Big(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{dc_{t+1}}\right|(c_{t+1})} = (1 + r_{t+1})\,(\pi_{t+1} + 1) - 1 \quad \text{(B.24)}$$

Leading to the final, most simplified version which is now in terms of the nominal interest rate and the inflation rate, which yields the money demand function that can be written as follows:

$$\frac{U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{U_1'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big) - U_2'\Big(c_{t+1}, \frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)\left|\frac{d\Big(\frac{M_{t+1}}{c_{t+1}P_{t+1}}\Big)}{dc_{t+1}}\right|(c_{t+1})} = i_{t+1} \quad \text{(B.25)}$$

From the conditions B.12 and B.14, the Euler equation for bonds can be derived:

[1] This is done by using the Fischer equation: $(1 + i_{t+1}) = (1 + r_{t+1})(1 + \pi_{t+1})$ and the definition of inflation as $\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t}$, which can be rewritten as $\pi_{t+1} + 1 = \frac{P_{t+1}}{P_t}$

$$\frac{U_1'\left(c_t, \frac{M_t}{c_t P_t}\right) - U_2'\left(c_t, \frac{M_t}{c_t P_t}\right)\left|\frac{d\left(\frac{M_t}{c_t P_t}\right)}{dc_t}\right|}{U_1'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right) - U_2'\left(c_{t+1}, \frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)\left|\frac{d\left(\frac{M_{t+1}}{c_{t+1} P_{t+1}}\right)}{dc_{t+1}}\right|} = \beta(1 + r_{t+1}) \tag{B.26}$$

# Appendix C

# Alternative Utility Functions

Some alternative functional forms that satisfy the conditions for the modified model are presented in this section.

1. Constant relative risk aversion for consumption and a function defined by parts for the conversion factor:

$$U\left(c_t, \frac{M_t}{P_t}\right) = \frac{c_t^{1-\theta}}{1-\theta} + \begin{cases} sin\left[\frac{\pi}{2}\left(\frac{M_t}{c_t P_t}\right)\right] & if \;\; 0 \leq \frac{M_t}{c_t P_t} < 1 \\ 1 & if \;\; \frac{M_t}{c_t P_t} \geq 1 \end{cases} \quad for \;\; (0 \leq \theta \leq 1) \tag{C.1}$$

2. Additive equal weights for a constant relative risk aversion for consumption and an exponential utility function for the conversion factor:

$$U\left(c_t, \frac{M_t}{P_t}\right) = \frac{1}{2}\left(\frac{c_t^{1-\theta}}{1-\theta}\right) + \frac{1}{2}\left[1 - e^{-\alpha\left(\frac{M_t}{c_t P_t}\right)}\right] \quad for \;\; (0 \leq \theta \leq 1) \;\; \wedge \;\; (\alpha \geq 7) \tag{C.2}$$

3. Constant relative risk aversion for consumption and a rational function for the conversion factor:

$$U\left(c_t, \frac{M_t}{P_t}\right) = \frac{c_t^{1-\theta}}{1-\theta} + \frac{\left(\frac{M_t}{c_t P_t}\right)}{\left(\frac{M_t}{c_t P_t}\right) + \kappa} \quad for \;\; (0 \leq \theta \leq 1) \;\; \wedge \;\; (\kappa \geq 0.01) \tag{C.3}$$

4. Constant relative risk aversion for consumption and a hyperbolic tangent function for the conversion factor:

$$U\left(c_t, \frac{M_t}{P_t}\right) = \frac{c_t^{1-\theta}}{1-\theta} + \frac{\left[e^{\alpha\left(\frac{M_t}{c_t P_t}\right)} - e^{-\alpha\left(\frac{M_t}{c_t P_t}\right)}\right]}{\left[e^{\alpha\left(\frac{M_t}{c_t P_t}\right)} + e^{-\alpha\left(\frac{M_t}{c_t P_t}\right)}\right]} \quad for \;\; (0 \leq \theta \leq 1) \;\; \wedge \;\; (\alpha \geq 4.15) \tag{C.4}$$

5. Constant relative risk aversion for consumption and a logistic function for the conversion factor:

$$U\left(c_t, \frac{M_t}{P_t}\right) = \frac{c_t^{1-\theta}}{1-\theta} + \left(\frac{1}{1+e^{-\kappa\left(\frac{M_t}{c_t P_t}\right)}} - \frac{1}{2}\right) \quad for \;\; (0 \leq \theta \leq 1) \;\; \wedge \;\; (\kappa \geq 10) \tag{C.5}$$

# Bibliography


D. B. Barro, R. J. Gordon. A positive theory of monetary policy in a natural rate model. *Journal of political economy*, pages 589–610, 1983. URL https://www.journals.uchicago.edu/doi/abs/10.1086/261167.

D. B. Barro, R. J. Gordon. Rules, discretion and reputation in a model of monetary policy. *Journal of monetary economics*, pages 101–121, 1984. URL https://www.sciencedirect.com/science/article/pii/030439328390051X.

R. J. Barro. Reputation in a model of monetary policy with incomplete information. *Journal of monetary economics*, pages 3–20, 1986. URL https://www.sciencedirect.com/science/article/pii/0304393286900036.

F. Cagan. The monetary dynamics of hyperinflation; 1 general monetary characteristics of hyperinflation in :studies in the quality theory of money. *Studies in the Quality Theory of Money, Edited by Milton Friedman*, pages 25–117, 1956. URL https://scholar.google.nl/scholar?hl=en&as_sdt=0%2C5&q=The+Monetary+Dynamics+cagan&btnG=.

R. Clower. A reconsideration of the microfoundations of monetary theory. *Economic Inquiry*, pages 1–8, 1967. URL https://onlinelibrary.wiley.com/doi/full/10.1111/j.1465-7295.1967.tb01171.x.

J. Hicks. "is-lm": An explanation. *Journal of Post Keynesian Economics*, pages 139–154, 1980. URL https://www.jstor.org/stable/4537583?seq=1#page_scan_tab_contents.

W.S. Jevons. General equilibrium models of monetary economies. *Science Direct*, pages 55–65, 1989. URL https://www.sciencedirect.com/science/article/pii/B9780126639704500098.

Solow R. M. Orphanides, A. Money, inflation and growth. *Handbook of monetary economics*, pages 223–261, 2007. URL https://www.sciencedirect.com/science/article/pii/S1573449805800098.

R Reis. The analytics of monetary non-neutrality in the sidrauski model. *Economics Letters*, pages 129–135, 2007. URL https://www.sciencedirect.com/science/article/pii/S0165176506002801.

J. A. Ritter. The transition from barter to fiat money. *The American Economic Review*, pages 134–149, 1995. URL http://www.jstor.org/stable/pdf/2118000.pdf?refreqid=excelsior%3A3c3c9ebf922a1a5d1403b2431d730e8a.

M. Sidrauski. Rational choice and patterns of growth in a monetary economy. *The American Economic Review*, pages 534–544, 1967. URL http://www.jstor.org/tc/accept?origin=/stable/pdf/1821653.pdf?refreqid=excelsior%3Aa7979aeaa5677c5667094d3fc3b8166e.

J. B. Taylor. The monetary transmission mechanism: an empirical framework. *Journal of Economic Perspectives*, pages 11–26, 1995. URL https://www.aeaweb.org/articles?id=10.1257/jep.9.4.11.

J. Tobin. The optimal cash balance proposition: Maurice allais' priority. *Journal of Economic Literature*, pages 1160–1162, 1989. URL http://www.jstor.org/stable/2726778?seq=1#page_scan_tab_contents.