Biased Technological Change, Human Capital

and Factor Shares.

Hernando Zuleta[1]

April 26, 2007

Abstract

We propose a one-good model where technological change is factor saving and costly. We consider a production function with two reproducible factors: physical capital and human capital, and one not reproducible factor. The main predictions of the model are the following: (a) The elasticity of output with respect to the reproducible factors depends on the factor abundance of the economies. (b) The income share of reproducible factors increases with the stage of development. (c) Depending on the initial conditions, in some economies the production function converges to AK, while in other economies long-run growth is zero. (d) The share of human factors (raw labor and human capital) converges to a positive number lower than one. Along the transition it may decrease, increase or remain constant.

Journal of Economic Literature classification: 011, 031, 033.

Keywords: endogenous growth, human capital, factor using and factor saving innovations, factor income shares.

1   Introduction

The works by Cobb and Douglas (1928) and Kaldor (1961) created a paradigm for macroeconomics. The idea that labor income share does not decrease or increase with time or with the stage of development have had important implications in macroeconomics and growth theory. Considering an aggregate production function, if factor income shares are constant and the price of each factor is determined by its marginal productivity then the elasticity of output with respect to each factor is also constant. In other words the constancy of factor shares implies that the Cobb-Douglas is a good approximation for the aggregate production function.

Subscribing to this paradigm, almost all of the literature on economic growth accounting assumes that the elasticity of output with respect to capital (and labor) is constant (see Easterly and Levine, 2002; Young, 1994 and Solow, 1957, among others). However, economic growth models of biased innovations predict a positive correlation between capital abundance and capital income share[1]. Indeed, if factor prices are determined by marginal productivity of factors then labor saving innovations reduce the income share of workers and increase the capital income share. In more general terms, the income share of not reproducible factors decreases with the stage of development while the income share of reproducible factors increases. Therefore, the theoretical argument described above implies that the income share of capital should be positively correlated

---

[1] Studies of factor saving innovations and economic growth have constantly grown in number. Some examples are Zeira (1998), Peretto and Seater (2006) and Zuleta (2007), among others.

with the stage of development.[2]

We propose a model of factor saving innovations to explain the absence of a secular trend in labor shares. We consider a model with two reproducible factors: human capital (H) and physical capital (K) and one not reproducible factor (L). Assuming that any technology can be obtained paying a cost, capital abundant economies have more incentives to adopt capital intensive technologies. In the same way, in countries where the capital intensity of the technology is higher agents have more incentives to save. This produces a virtuous circle driving capital abundant economies to long run growth.

We consider a one good economy and assume a set of Cobb-Douglas production functions of the form $Y = AK^{\alpha}H^{\beta}L^{1-\alpha-\beta}$. Thus we can rewrite the production function defining y as output per unit of not reproducible factor,

$$y = \frac{Y}{L} = Ak^{\alpha}h^{\beta}$$

L can be understood as a combination of not reproducible factors, i.e., raw labor and land. Therefore an increase in α (β) is physical capital-using (human capital-using) , labor saving and land saving technological change.

The main results of the model are the following: (i) The elasticity of output with respect to reproducible factors depends on the factor abundance of the economies. (ii) The income share of reproducible factors increases with the stage

---

[2]There are two other theoretical reasons why the elasticity of output with respect to reproducible factors, namely, physical capital and human capital, should be positively correlated with the stage of development: the Hecksher-Ohlin theory of International Trade and the theory of International Capital Flows .

of development. (iii) Depending on the initial conditions, in some economies the production function converges to AK while in other economies long-run growth is zero. (iv) The income share of reproducible factors converges to a positive number lower than one and along the transition it may decrease, increase or remain constant.

Three pieces of empirical evidence motivate our work:

1. In the field of empirical economic growth, Durlauf and Johnson (1995) and Duffy and Papageorgiou (2000) find that as economies grow their technologies become more intensive in reproducible factors, that is, the elasticity of output with respect to reproducible factors is higher in rich economies.

2. With regard to the behavior of factor income shares, we know that: (i) In developed countries the share of agriculture in total output is usually smaller than it is in developing countries. By the same token, the share of agriculture in total output is reduced as economies grow. Since land is a major input in agriculture but not in other sectors these facts suggest that land income share may decrease with the stage of development. Consistently, from 1870 to 1990 in the United States the share of land in Net National Product has been continuously reduced (see Rhee, 1991 and Hansen and Prescott, 2002). (ii) Over the past 60 years, the US relative supply of skilled work has increased rapidly. However, there has not been a downward trend for the returns to college education. On the contrary, over this period, the college premium has increased (see Krueger,

3

1999; Krusell et.al., 2000 and Acemoglu 2002). Moreover, according to Gottshalk (1997), real wages of unskilled labor significantly fell during the past few decades (iii) In addition, it has been argued that labor income share does not decrease or increase with development (Gollin, 2002). However, the standard measure of labor income share includes skilled and unskilled labor income share, that is, it includes human capital. In the same way, the standard measure of capital income share includes land income share. Therefore, it seems that the income share of not reproducible factors (land and unskilled labor) has decreased, while the income share of reproducible factors has increased during the 20th century.

3. Blanchard (1998) notes that since the early 80's a dramatic decrease in labor income share has occurred in Europe (5 to 10 percentage points of GDP) and suggests that this decline could be explained by non-neutral changes in technology.

There are several models of biased technological change where labor share are relatively constant.[3] Boldrin and Levin (2002) build a model where the production function is Leontief so, factor prices are determined by opportunity cost and not by marginal productivity. Zuleta and Young (2006) and Zeira (2006) consider models with two final goods and assume that labor saving innovation can only be implemented in the capital intensive sector. In this setting, the effect of labor saving innovations is compensated in the aggregate with the in-

---

[3]Kennedy (1964), Samuelson (1965) and Drandakis and Phelps (1966) are pioneers in this literature.

crease in the share of the labor intensive sector. [4] To the best of our knowledge this is the first model where physical capital using innovations are accompanied with human capital using innovations in such a way that the labor income share does not present a clear decreasing trend.

In then next section we present the model. Then we conclude in the last section.

2   The Model

2.1   The cost of changing technology

We assume that there are different qualities of physical and human capital. Any type of physical capital embodies a technology $(\alpha, \beta.)$ Capital types that embody more capital intensive technologies are more costly. In particular, we assume that for 1 unit of output devoted to build capital goods of type α, β the number of capital goods is given by $K_{\alpha,\beta} = 1 + [\ln(\Psi_\alpha - \alpha)]\Phi + [\ln(\Psi_\beta - \beta)]\Phi$ where Φ is a measure for scale, Ψ_α and Ψ_β are technological parameters such that $\Psi_\alpha \leq 1$ and $\Psi_\beta \leq 1$. We also assume that each plant can only operate with one technology so, only one type of capital is used at a time and we can drop the subindex α, β.

For simplicity, we choose not reproducible factors as a measure of scale, so if

---

[4] While this paper examines the dynamics of labor share in response to technical change, various theoretical and empirical research has explored the non-technical determinants of labor share, e.g., Gomme and Greenwood (1995) and Boldrin and Horvath (1995) (unemployment insurance/labor contracts); Blanchard (1997), Blanchard and Wolfers (2000), Bentolila and Saint-Paul (2003) and Kessing (2003) (labor adjustment costs and bargaining power); Bertola (1993) (fiscal policy); and Ambler and Cardia (1998) (monopolistic competition).

L_i is the amount of not reproducible factors used by the firm i then for 1 unit of output devoted to build capital goods of type α, β the stock of capital is given by[5],

$$K_i = 1 + L_i \left[\ln(\Psi_\alpha - \alpha_i)\right] + L_i \left[\ln(\Psi_\beta - \beta_i)\right]$$

Following Barro and Sala-i-Martin (1995) we also assume that physical and human capital are produced with the same technology.[6] Therefore, the total amount of assets of the firm a_i can be written as

$$a_i = K_i + H_i - L_i \left[\ln(\Psi_\alpha - \alpha)\right] - L_i \left[\ln(\Psi_\beta - \beta)\right] \tag{1}$$

Defining $a_{K,i} = K_i - L_i \left[\ln(\Psi_\alpha - \alpha_i)\right]$ and $a_{H,i} = H_i - L_i \left[\ln(\Psi_\beta - \beta_i)\right]$ the output produced by a firm i using K units of capital of type α_i, β_i can be written as

$$Y_i = A(a_K + \left[\ln(\Psi_\alpha - \alpha_i)\right] L_i)^{\alpha_i} (a_H + \left[\ln(\Psi_\beta - \beta_i)\right] L_i)^{\beta_i} L_i^{1-\alpha_i-\beta_i} \tag{2}$$

In other words, we assume that the production technology in each period is chosen from a time invariant set of technologies where a more superior technology also entails larger costs on acquiring capital. We are aware of the fact that new things are invented each year and many of the technologies we use nowadays were recently created. However, the aim of the paper is not to ex-

---

[5] This function is chosen because of its tractability and the main results of the model do not depend on such an assumption. See Zeira (2005) or Peretto and Seater (2006) for different costs functions.

[6] This simplifying assumption does not affect the results of the model.

plain the process of creation of new technologies but to explain (i) why some societies adopt capital intensive techniques of production while others use labor intensive technologies and (ii) what are the effects of these differences in terms of functional distribution of income and economic growth.[7]

Markets are competitive so firms choose the technology in order to maximize output,

$$\max_{\alpha_i,\beta_i} Y_i \quad s.t \quad \alpha_i \geq \alpha_0 \quad \text{and} \quad \beta_i \geq \beta_{0.}$$

As a result, in the interior solution, the technology is given by (complete derivation in the Appendix 4.1),

$$\alpha_i = \Psi_\alpha \frac{\frac{K_i}{L_i} \ln\left(\frac{K_i}{L_i}\right)}{1 + \frac{K_i}{L_i} \ln\left(\frac{K_i}{L_i}\right)} \quad \text{and} \quad \beta_i = \Psi_\beta \frac{\frac{H_i}{L_i} \ln\left(\frac{H_i}{L_i}\right)}{1 + \frac{H_i}{L_i} \ln\left(\frac{H_i}{L_i}\right)}$$

Note that, holding the rest constant, any increase in the size of the firm affects K_i, H_i and L_i in the same proportions, so the equilibrium levels of α and β are independent of the size of the firm. If all firms use the same technology and face the same market prices then for any pair of firms i and j, $\frac{K_i}{L_i} = \frac{K_i}{L_j} = \frac{K}{L}$, where $\frac{K}{L}$ is capital per unit of not reproducible factors. Similarly, for any pair of firms i and j, $\frac{H_i}{L_i} = \frac{H_j}{L_j} = \frac{H}{L}$, where $\frac{H}{L}$ is human capital per unit of not reproducible factors.

---

[7] An extension of the model where technologies are created with a probability $p(\rho s)$ such that $0 \leq \rho \leq 1$ and $p'(\rho s) > 0$ would deliver the same predictions of the model we are presenting.

Therefore, the equilibrium technology (common for every firm) is given by,

$$\alpha_t = \max\left\{\alpha_0, \Psi_\alpha \frac{k_t \ln k_t}{k_t \ln k_t + 1}\right\} \quad \text{and} \quad \beta_t = \max\left\{\beta_0, \Psi_\beta \frac{h_t \ln h_t}{h_t \ln h_t + 1}\right\}$$

Note that under this setting the decentralized solution is not optimal. Indeed, if the production function is Cobb-Douglas and markets are competitive then the wage is given by $w = (1 - \alpha)Ak^\alpha$. Therefore, if make zero profits, then the interest rate must be lower than the marginal productivity of assets $r < \frac{\partial Y}{\partial a}$ because firms must pay for the technology. This fact may have interesting implications for optimal taxation. However, we want to focus on the effects that factor saving innovations have on the behavior of factors shares and the simplest way to present this idea is solving the planner problem.

2.2 The dynamic problem.

For the dynamic problem we assume constant population, homogenous agents, infinite horizon and logarithmic utility function.

To obtain the dynamic restrictions of the problem we differentiate equation 1,

$$\dot{a}_K + \dot{a}_H = \dot{a} = \dot{k} + \dot{h} + \frac{\dot{\alpha}}{\Psi_\alpha - \alpha} + \frac{\dot{\beta}}{\Psi_\beta - \beta}$$

Therefore, the restrictions can be expressed in the following way:

$$\dot{k_t} = u_t \left(Ak_t^{\alpha_t} h_t^{\beta_t} - c_t \right) \tag{3}$$

$$\dot{h_t} = a_t \left(Ak_t^{\alpha_t} h_t^{\beta_t} - c_t \right) \tag{4}$$

$$\dot{\beta_t} = b_t \left(\Psi_\beta - \beta_t \right) \left(Ak_t^{\alpha_t} h_t^{\beta_t} - c_t \right) \tag{5}$$

$$\dot{\alpha_t} = z_t \left(\Psi_\alpha - \alpha_t \right) \left(Ak_t^{\alpha_t} h_t^{\beta_t} - c_t \right) \tag{6}$$

where u_t is the share of savings devoted to increase the stock of physical capital, a_t is the share of savings devoted to increase human capital, b_t is the share of savings devoted to human capital using technological changes; z_t is the share of savings devoted to physical capital using technologies changes and $u_t + a_t + b_t + z_t = 1$.

The planner problem is the standard one: maximize the present discounted utility of the representative agent subject to the restrictions,

$$Max \int \log c_t e^{-\rho t} dt$$

$$s.t. \quad (3), (4), (5), (6)$$

$$\beta_t \geq \beta_0$$

$$\alpha_t \geq \alpha_0 \tag{7}$$

9

and the transversality conditions are

$$\lim_{t\to\infty} \frac{e^{-\rho t}}{c_t} k_t = 0$$

$$\lim_{t\to\infty} \frac{e^{-\rho t}}{c_t} h_t = 0$$

where β_0 and α_0 are the initial technologies.

From the first order conditions it follows that for the interior solution,

$$\frac{\dot{c}_t}{c_t} = \alpha_t A k_t^{\alpha_t - 1} h_t^{\beta_t} - \rho = \beta_t A k_t^{\alpha_t} h_t^{\beta_t - 1} - \rho \tag{8}$$

and

$$\alpha_t = \max\left\{\alpha_0, \Psi_\alpha \frac{k_t \ln k_t}{k_t \ln k_t + 1}\right\} \tag{9}$$

$$\beta_t = \max\left\{\beta_0, \Psi_\beta \frac{h_t \ln h_t}{h_t \ln h_t + 1}\right\} \tag{10}$$

From equation 8 it follows that, in the interior solution,

$$\frac{k_t}{h_t} = \frac{\alpha_t}{\beta_t} \tag{11}$$

Therefore, the production function can be rewritten as $A(\frac{\beta_t}{\alpha_t})^{\beta_t} k_t^{\alpha_t + \beta_t}$ and the optimal consumption growth rate is

$$\frac{\dot{c}_t}{c_t} = (\alpha_t)^{1-\beta_t} (\beta_t)^{\beta_t} A k_t^{\alpha_t + \beta_t - 1} - \rho \tag{12}$$

10

Equation 12 is the consumption growth rate of any model where physical and human capital are produced with the same technology (see Barro and Sala-i-Martin, 1995). However, in our setting α and β are variables not parameters, and grow with the economy. For this reason the consumption growth rate may not decrease as the economy accumulates assets. Indeed, the growth rate of consumption is increasing whenever $k \geq 1$ (the proof is presented in the Appendix 4.3).

From equations 9, 10 and 11, in the interior solution the variables h, α and β can be expressed as functions of k

$h_t = h(k_t)$, $\alpha_t = \alpha(k_t)$, $\beta_t = \beta(k_t)$ where $h'(k_t) \geq 0$, $\alpha'(k_t) \geq 0$ and $\beta'(k_t) \geq 0$.

Therefore, we can define k_{min} as follows:

Definition 1 k_{min} *is the capital per unit of not reproducible factors such that the optimal consumption growth rate is zero, that is,*

$$(\alpha(k_{min}))^{1-\beta(k_{min})} (\beta(k_{min}))^{\beta(k_{min})} k^{\alpha(k_{min})+\beta(k_{min})-1} = \frac{\rho}{A}.$$

Now, given that the growth rate of consumption is increasing whenever $k \geq 1$ and $h'(k_t) \geq 0$, $\alpha'(k_t) \geq 0$ and $\beta'(k_t) \geq 0$, from definition 1 it follows that:

1. If $k > k_{min}$ then the optimal consumption growth rate is positive.

2. If $k < k_{min}$ then the optimal consumption growth rate is negative.

Therefore, for economies where reproducible factors are relatively scarce it can be optimal to consume the entire output. On the other hand, if the share

11

of reproducible factors is high then consumption growth rate is positive. As we show below, two candidates for optimal path may arise: one with a small amount of reproducible factors in the long-run and another with an infinite amount of reproducible factors.

2.3   Long run

There are two basic types of long-run equilibria that the model can support. First, there is a neoclassical steady-state where $\alpha = \alpha_0$, $\beta = \beta_0$ and $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = 0$. The second one is a Balanced Growth Path (BGP) where $\alpha + \beta = 1$ and $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = 0$.

2.3.1   Steady State

Consider an initial technology α_0, β_0 such that

$$(\alpha_0)^{1-\beta_0} (\beta_0)^{\beta_0} < \frac{\rho}{A} \quad \text{and} \quad (\alpha_0)^{\alpha_0} (\beta_0)^{1-\alpha_0} < \frac{\rho}{A}$$

In this case, the steady state physical and human capital per unit of not reproducible factors are

$$k^* = \left(\frac{(\alpha_0)^{1-\beta_0} (\beta_0)^{\beta_0} A}{\rho} \right)^{\frac{1}{1-\alpha_0-\beta_0}}$$

$$h^* = \left(\frac{(\alpha_0)^{\alpha_0} (\beta_0)^{1-\alpha_0} A}{\rho} \right)^{\frac{1}{1-\alpha_0-\beta_0}}$$

Note that $k^* < 1$ and $h^* < 1$. Therefore, from equations 9 and 10 it follows that

if $k \leq k^*$ and $h \leq h^*$ then $\alpha = \alpha_0$ and $\beta = \beta_0$. Therefore, economies that are abundant in not reproducible factors are likely to converge to a steady state.

2.3.2 Balanced Growth Path

From equations 9, 10 and 12 it follows that,

$$\lim_{k \to \infty} \frac{\dot{c}_t}{c_t} = (\Psi_\alpha)^{1-\Psi_\beta} (\Psi_\beta)^{\Psi_\beta} Ak_t^{\Psi_\alpha + \Psi_\beta - 1} - \rho \tag{13}$$

Form equations 9 and 13 it follows that:

1. If $\Psi_\alpha + \Psi_\beta \leq 1$ and $(\Psi_\alpha)^{1-\Psi_\beta} (\Psi_\beta)^{\Psi_\beta} A < 2\rho$ then the problem has a finite value solution (proof in the Appendix 4.4)

2. If $\Psi_\alpha + \Psi_\beta \geq 1$ and $(\Psi_\alpha)^{1-\Psi_\beta} (\Psi_\beta)^{\Psi_\beta} > \frac{\rho}{A}$ then capital abundant economies, that is economies where $k > k_{min}$, converge to a BGP (proof in the Appendix 4.5).

From results 1 and 2 it follows that if $\Psi_\alpha + \Psi_\beta = 1$ then the dynamic problem can be solved and capital abundant economies converge to a Balanced Growth Path while poor economies converge to a steady state.

Now, in the Balanced Growth Path the growth rate of human and physical capital must be the same, $\frac{\dot{k}_t}{k_t} = \frac{\dot{h}_t}{h_t}$, so

$$u_t \left(\left(\frac{\beta}{\alpha} \right)^\beta Ak_t^{\alpha_t + \beta_t - 1} - \frac{c_t}{k_t} \right) = a_t \left(\left(\frac{\beta}{\alpha} \right)^{\beta-1} Ak_t^{\alpha_t + \beta_t - 1} - \frac{c_t}{k_t} \frac{\alpha}{\beta} \right)$$

and

$$\frac{u_t}{a_t} = \frac{\alpha}{\beta}$$

Using the First Order Conditions we find that $\lim_{k\to\infty} b_t = \lim_{h\to\infty} z_t = 0$, so $\lim_{k\to\infty} (a_t + u_t) = 1$ (proof is in the Appendix 4.2) so,

$$\lim_{k\to\infty} u_t = \Psi_\alpha \quad \text{and} \quad \lim_{h\to\infty} a_t = \Psi_\beta$$

Therefore, in the Balanced Growth Path,

$$\left(\frac{c}{k}\right)^* = \lim_{k\to\infty} \frac{c}{k} = \frac{\rho}{\Psi_\alpha} \tag{14}$$

$$\left(\frac{c}{h}\right)^* = \lim_{h\to\infty} \frac{c}{h} = \frac{\rho}{\Psi_\beta} \tag{15}$$

Note that equations 14 and 15 imply that $\lim_{k\to\infty} \frac{c}{h+k} = \rho$. Therefore in the BGP the ratio consumption assets is equal to the discount rate as derived in several endogenous growth models.

2.3.3   Transition

We already showed that depending on the parameters $\Psi_\alpha, \Psi_\beta, A$ and ρ there can be long run growth. Similarly, depending on the initial conditions some economies can be trapped in a steady state. In this section assume $\Psi_\alpha + \Psi_\beta = 1$ and characterize the behavior of some of the main variables along the transition and identify the conditions under which an economy converges to a steady state

14

and the conditions under which it converges to a Balanced Growth Path.

Using equations 3 and 12 we can characterize the behavior of the consumption capital ratio $\frac{c}{k}$,

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{k}_t}{k_t} = (\alpha_t)^{1-\beta_t}(\beta_t)^{\beta_t} A k_t^{\alpha_t+\beta_t-1} - \rho - u_t\left(A k_t^{\alpha_t+\beta_t-1} - \frac{c_t}{k_t}\right) \qquad (16)$$

From equation 16 it follows that $\frac{\dot{k}_t}{k_t} \geq \frac{\dot{c}_t}{c_t}$. To see why suppose that $\frac{\dot{c}_t}{c_t} > \frac{\dot{k}_t}{k_t}$. If this is the case then $\frac{c_t}{k_t} > \frac{\rho}{u_t} + A\left(1 - \frac{(\alpha_t)^{1-\beta_t}(\beta_t)^{\beta_t}}{u_t}\right)$ so $\frac{c_t}{k_t} > \left(\frac{c}{k}\right)^*$. But note that $A\left(1 - (\alpha_t)^{1-\beta_t}(\beta_t)^{\beta_t}\right)$ decreases and u_t grows as the economy accumulates capital, so $\frac{\rho}{u_t} + A\left(1 - \frac{(\alpha_t)^{1-\beta_t}(\beta_t)^{\beta_t}}{u_t}\right)$ decreases as the economy grows. Therefore, if $\frac{\dot{c}_t}{c_t} > \frac{\dot{k}_t}{k_t}$ then $\frac{\dot{c}_t}{c_t} - \frac{\dot{k}_t}{k_t}$ grows with time and the economy cannot converge to a BGP. Moreover, if $\frac{\dot{c}_t}{c_t} > \frac{\dot{k}_t}{k_t}$ then $\frac{c_t}{k_t}$ converges to infinity which is not feasible.

Proposition 2 *Define* $\tilde{k} = \left(\frac{\rho}{\Psi_\alpha} k_{\min} \frac{1}{(\alpha_0)^{1-\beta_0}(\beta_0)^{\beta_0}}\right)^{\frac{1}{\alpha_0+\beta_0}}$, *if* $k_0 < \tilde{k}$ *and* $k_0 < k_{\min}$ *then the economy converges optimally to a steady sate.*

Proof. Suppose not, that is, there exists a k_0 such that $k_0 < \tilde{k}$, $k_0 < k_{\min}$ and the economy presents long-run growth.

1. In order to have capital accumulation or technological change consumption must satisfy $c_0 < Y_0$.

2. In the interior solution, the consumption-capital ratio decreases as the stock of capital grows and converges to $\frac{\rho}{\Psi_\alpha}$ as capital goes to infinity. Moreover as long as $k < k_m$ the growth rate of consumption is negative. Therefore, for any t such that $k_t \leq k_m$ it must be true that $c_t < c_0$.

3. Since the consumption-capital ratio decreases with time and converges to ρ in the long-run then in the optimal path $c_t > \frac{\rho}{\Psi_\alpha} k_t$ for any $t < \infty$.

From 2 and 3 it follows that given k_0, if there is an optimal path with long-run growth then $c_0 > \frac{\rho}{\Psi_\alpha} k_{\min}$.

From 1, 2 and 3 it follows that output at period zero must be higher than $\frac{\rho}{\Psi_\alpha} k_{\min}$, namely,

$(\alpha_0)^{1-\beta_0} (\beta_0)^{\beta_0} A k_0^{\alpha_0 + \beta_0} > \frac{\rho}{\Psi_\alpha} k_{\min}$ so

$$k_0 > \left(\frac{\rho}{\Psi_\alpha} k_{\min} \frac{1}{(\alpha_0)^{1-\beta_0} (\beta_0)^{\beta_0}} \right)^{\frac{1}{\alpha_0 + \beta_0}}$$

which contradicts $k_0 < \tilde{k}$ ∎

Proposition 3 *If $k_0 > k_m$ then the economy presents long-run growth.*

The proof is straightforward. When $k > k_m$ the marginal productivity of savings is higher than the discount rate. Therefore, savings are used to increase K, H, α and β and the consumption growth rate is positive.

2.4 The Behavior of Factor Shares

In the previous sections we describe the main results of the model and characterize the long run equilibrium. We find that the share of reproducible factors depends on the relative abundance of the factors. In this section we explore the possible equilibrium paths of the labor income share. As we stated before, the standard measure of labor income includes raw labor income and human capital income and the standard measure of capital income includes physical

16

capital income and land income. In order to describe the behavior of the standard measures of factor shares we need to rewrite the production function in the following way

$$Y = AK^\alpha H^\beta N^\gamma l^{1-\alpha-\beta-\gamma}$$

where N is land, l is raw labor, γ is land income share, $1 - \alpha - \beta - \gamma$ is raw labor income share and $N^\gamma l^{1-\alpha-\beta-\gamma} = L^{1-\alpha-\beta}$. Therefore, the share of human factors is $1 - \gamma - \alpha$ and the share of non human factors is $\gamma + \alpha$.

From section 2 we obtain the growth rates of α and β. Differentiating equations 9 and 10 we find

$$\frac{\dot{\alpha}_t}{\alpha_t} = (\Psi_\alpha - \alpha_t)\left(\frac{1 + \ln k_t}{\ln k_t}\right)\frac{\dot{k}_t}{k_t} \tag{17}$$

$$\frac{\dot{\beta}_t}{\beta_t} = (\Psi_\beta - \beta_t)\frac{1 + \ln h_t}{h_t \ln h_t + 1}\frac{\dot{h}_t}{h_t} \tag{18}$$

Therefore, as long as the economy accumulates assets the share of physical and human capital in the production function (α and β) grows. This implies that along the transition the economy undertakes technological changes that are human capital using, physical capital using, raw labor saving and land saving. Under these conditions the share of human factors $(1 - \gamma - \alpha)$ remains constant if labor saving innovations are always human capital using and land saving innovations are always physical capital using, that is, $\dot{\gamma} = -\dot{\alpha}$.

In real life, innovations can be physical capital using and raw labor saving or human capital using and land saving. Indeed, human factors shares move up and down (see Bentolila and Saint Paul, 2003 or Young, 2005) However, in

the long run technologies are likely to become more intensive in reproducible factors.

In our model, as long as $\Psi_\alpha > 0$ and $\Psi_\beta > 0$ in the long run both labor income share and capital income share converge to a positive number. The behavior of these shares along the transition depends on the parameters Ψ_β and Ψ_β and on the effects that innovations have on the share of land γ. In any case, this model is consistent with the empirical evidence regarding the behavior of factor shares.

3 Conclusions

We present a one good model of economic growth with two reproducible factors where technological change is factor saving and factor shares are determined by technology. Assuming that technologies can be changed paying a cost we find that agents in capital abundant economies are more likely to adopt capital intensive technologies than agents in poor economies. As a result, the elasticity of output with respect to reproducible factors depends on the relative abundance factors. We also show that capital abundance stimulates innovations that save not reproducible factors and that savings are higher in economies where the technology is more capital intensive For this reason, rich economies may achieve long-run growth while poor economies may converge to a steady state.

Since factor prices are given by marginal productivity, as economies grow, the income share of the reproducible factor grows while the income share of not

reproducible factors decreases. This prediction is consistent with the old result of constant labor income share. Indeed, human capital accumulation stimulates human capital-using innovations and increases human capital income share. The increase in human capital income share can counterweight the reduction in raw labor income share in such a way that total labor income share (including remuneration for human capital) remain constant. The same logic can be applied to land and physical capital.

References

[1] Ambler, S. and Cardia, E., 1998. The Cyclical Behavior of Wages and Profits under Imperfect Competition. Canadian Journal of Economics 31 (1), 148-164.

[2] Barro, R. J. and Sala-i-Martin, X., 1995. Economic Growth. McGraw-Hill Advanced Series in Economics.

[3] Bentolila, S. and Saint-Paul, G., 2003. Explaining Movements in the Labor Share. Contributions to Macroeconomics 3 (1), 1103-1103

[4] Bertola, G., 1993. Factor Shares and Savings in Endogenous Growth. The American Economic Review 83 (5), 1184-1198.

[5] Blanchard, O. J., 1997. The Medium Run. Brookings Papers on Economic Activity 2, 89-158.

[6] Blanchard, O. J. and Wolfers, J., 2000. The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence. Economic Journal 110 (462), 1-33.

[7] Boldrin, M. and Horvath, M., 1995. Labor Contracts and Business Cycles. Journal of Political Economy 103 (5), 972-1004.

[8] Boldrin, M. and Levine, D. K., 2002 Factor Saving Innovation. Journal of Economic Theory 105 (1), 18-41

[9] Cobb, C. and Douglas, P.H., 1928. A theory of Production. American Economic Review 18, 139–165.

[10] Duffy, J.. and Papageorgiou, C., 2000. A Cross-Country Empirical Investigation of the Aggregate Production Function Specification. Journal of Economic Growth 5 (1), 87-120.

[11] Durlauf, S. and Johnson, P., 1995. Multiple Regimes and Cross-Country Growth Behavior. Journal of Applied Econometrics 10 (4), 365-384.

[12] Drandakis, E. M. and Phelps, E. S., 1966 A Model of Induced Invention, Growth and Distribution." Economic Journal 76, 823-840.

[13] Easterly, W. and Levine, R., 2001 It's Not Factor Accumulation: Stylized Facts and Growth Models. World Bank Economic Review 15 (2), 177-219.

[14] Gollin, D., 2002. Getting Income Shares Right. Journal of Political Economy 110 (2), 458-474.

[15] Gomme, P. and Greenwood, J., 1995. On the Cyclical Allocation of Risk. Journal of Economic Dynamics and Control 19, 91-124.

[16] Gottschalk, P., 1997, Inequality, Income Growth and Mobility: The Basic Facts, Journal of Economic Perspectives 11(2), 21-40.

[17] Hansen, G. and Prescott, E., 2002. Malthus to Solow. American Economic Review 92 (4), 1205-17.

[18] Heckscher, E. F., 1919. The Effect of Foreign Trade on the Distribution of Income. Ekonomisk Tidskrift, 1919.

[19] Kennedy, C., 1964. Induced Bias in Innovation and the Theory of Distribution. Economic Journal 74, pp. 541–7.

[20] Krueger, A.,1999. Measuring Labor's Share" American Economic Review 89, (2), 45-21.

[21] Krusell, P. Ohanian, L. Ríos-Rull, J. V. Violante, G., 2000. Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. Econometrica 68 (5), 1029-53.

[22] Ohlin, B., 1933. Interregional and International Trade. Cambridge: Harvard University Press.

[23] Peretto, P. and Seater, J., 2006 Augmentation or Elimination? Working Paper http://ideas.repec.org/p/deg/conpap/c011_060.html

[24] Rhee, C., 1991. Dynamic Inefficiency in an Economy with Land. Review of Economic Studies 58 (4), 791-797.

[25] Samuelson, P. A., 1965. A Theory of Induced Innovation on Kennedy-von Weisacker Lines. Review of Economics and Statistics 47 (4), 343-356.

[26] Solow, R., 1957. Technical Change and the Aggregate Production Function. Review of Economics and Statistics 39, 312-320.

[27] Young, A., 1995. The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience. Quarterly Journal of Economics 110: 641-680.

[28] Young, A. T., 2005. One of the Things We Know that Ain't So: Why US Labor's Share is not Relatively Stable. Working Paper, January 2005, http://ssrn.com/abstract=650783.

[29] Zeira, J., 1998. Workers, Machines and Economic Growth. Quarterly Journal of Economics 113 (4), 1091-1117.

[30] Zeira, J., 2005. Machines as Engines of Growth. CEPR 2005, Discussion Papers 5429.

[31] Zuleta, H., 2006 "Factor Saving Innovations and Factor Income Shares. Working Paper, http://www.dotec-colombia.org/articulo.php?id=2778

[32] Zuleta, H. and Young, A., 2006. Labor's Shares – Aggregate and Industry: Accounting for Both in a Model of Development with Induced Innovation. Working Paper, http://papers.ssrn.com/abstract=880062

4 Appendix

4.1 Choosing Factor Shares

$$\max_{\alpha_i,\beta_i} Y_i \quad \text{s.t.} \quad \alpha_i \geq \alpha_0 \ \text{ and } \ \beta_i \geq \beta_0$$

F.O.C.

$$\frac{\partial Y_i}{\partial \alpha_i} = -\frac{\alpha_i}{\Psi_\alpha - \alpha_i} A(K_i)^{\alpha_i-1}(H_i)^{\beta_i} L_i^{1-\alpha_i-\beta_i} + A(K_i)^{\alpha_i}(H_i)^{\beta_i} L_i^{1-\alpha_i-\beta_i} \ln k_i + \phi_\alpha = 0$$

$$\frac{\partial Y_i}{\partial \alpha_i} = -\frac{\beta_i}{\Psi_\beta - \beta_i} A(K_i)^{\alpha_i}(H_i)^{\beta_i-1} L_i^{1-\alpha_i-\beta_i} + A(K_i)^{\alpha_i}(H_i)^{\beta_i} L_i^{1-\alpha_i-\beta_i} \ln h_i + \phi_\beta = 0$$

Therefore, the equilibrium technology (common for every firm) is given by,

$$\alpha_t = \max\left\{\alpha_0, \Psi_\alpha \frac{k_t \ln k_t}{k_t \ln k_t + 1}\right\} \quad \text{and} \quad \beta_t = \max\left\{\beta_0, \Psi_\beta \frac{h_t \ln h_t}{h_t \ln h_t + 1}\right\}$$

4.2 Savings and assets

Differentiating equations 9, 10 and 11

$$\dot{\alpha}_t = \Psi_\alpha \frac{1 + \ln k_t}{(k_t \ln k_t + 1)^2} \dot{k}_t \tag{19}$$

$$\dot{\beta}_t = \Psi_\beta \frac{h_t \ln h_t}{(h_t \ln h_t + 1)^2} \dot{h}_t \tag{20}$$

$$\frac{\dot{k}_t}{k_t} - \frac{\dot{h}_t}{h_t} = \frac{\dot{\alpha}_t}{\alpha_t} - \frac{\dot{\beta}_t}{\beta_t} \tag{21}$$

Combining equations 19, 20 and 21 with 3, 4, 5 and 6

$$z_t = \frac{\Psi_\alpha}{(1 - \alpha_t)} \frac{1 + \ln k_t}{(k_t \ln k_t + 1)^2} u_t \tag{22}$$

$$b_t = \frac{\Psi_\beta}{(1 - \beta_t)} \frac{1 + \ln h_t}{h_t \ln h_t + 1} a_t \tag{23}$$

$$\frac{u_t}{k_t} - \frac{a_t}{h_t} = \frac{z_t}{\alpha_t}(1 - \alpha_t) - \frac{b_t}{\beta_t}(1 - \beta_t) \tag{24}$$

Combining equations 22 and 9 $z_t = \frac{\Psi_\alpha}{1 + k_t \ln k_t (1 - \Psi_\alpha)} \frac{1 + \ln k_t}{(k_t \ln k_t + 1)^2} u_t$. Therefore,

$$\lim_{k \to \infty} z_t = 0.$$

Similarly, combining equations 23 and 10 $b_t = \frac{\Psi_\beta}{1 + h_t \ln h_t (1 - \Psi_\alpha)} \frac{1 + \ln h_t}{(h_t \ln h_t + 1)^2} a_t$.

Therefore, $\lim_{h \to \infty} b_t = 0.$

Therefore, from equation 24 it follows that in the long run $\frac{u_t}{k_t} = \frac{a_t}{h_t}$.

Finally, $\frac{h_t}{k_t} = \frac{\beta_t}{\alpha_t}$. so

$$\lim_{k \to \infty} u_t = \Psi_\alpha \quad \text{and} \quad \lim_{h \to \infty} a_t = \Psi_\beta$$

4.3 The growth rate of consumption increases as the amount of reproducible factors grow.

Define $r = (\alpha_t)^{1 - \beta_t} (\beta_t)^{\beta_t} A k_t^{\alpha_t + \beta_t - 1}$

$$\log r = (1 - \beta) \log (\alpha) + \beta \log(\beta) + \log(A) + (\alpha + \beta - 1) \log k$$

combining with equation 11 and rearranging,

$$\log r = \log(\alpha) + \beta \log(h) + \log(A) + (\alpha - 1)\log k$$

Differentiating,

$$\frac{\dot{r}}{r} = \frac{\dot{\alpha}}{\alpha} + \beta\frac{\dot{h}}{h} + \dot{\beta}\log(h) + \dot{\alpha}\log k - (1-\alpha)\frac{\dot{k}}{k}$$

Combining with equation 17 and rearranging,

$$\frac{\dot{r}}{r} = \left(\Psi_\alpha \frac{(1+\ln k)k}{k\ln k + 1}\left(\frac{1}{\alpha} + \ln k\right) - 1\right)(1-\alpha)\frac{\dot{k}}{k} + \beta\frac{\dot{h}}{h} + \dot{\beta}\log(h_t)$$

Combining with equation 11 and rearranging,

$$\frac{\dot{r}}{r} = \left(\frac{1+\alpha\ln k\,(1+\ln k)}{\ln k}\right)(1-\alpha)\frac{\dot{k}}{k} + \beta\frac{\dot{h}}{h} + \dot{\beta}\log(h_t)$$

Therefore, if $k > 1$ and $\frac{\dot{k}}{k} > 0$ then $\frac{\dot{r}}{r} > 0$.

4.4 If $\Psi_\alpha + \Psi_\beta \leq 1$ and $(\Psi_\alpha)^{1-\Psi_\beta}(\Psi_\beta)^{\Psi_\beta}A < 2\rho$ then the problem has a finite-value solution.

The maximized Hamiltonian is given by $H^0(a_t, \lambda_t) = u(c_t^*)e^{-\rho t} + \lambda_t\left(Y_t - c_t^*\right)$.

Therefore, we have to prove that $\lim_{t\to\infty} u(c_t^*)e^{-\rho t} = 0$ To simplify notation we drop the index *.

Note that $\lim_{t\to\infty} u(c_t) = \infty$ and $\lim_{t\to\infty} e^{-\rho t} = 0$, so in order to find the limit we

25

differentiate the expression $u(c_t)e^{-\rho t}$: $\frac{U'(c_t)}{U(c_t)}\dot{c}_t - \rho$

Recall that the log utility function is a special case of the more general function CRRA, $\frac{c^{1-\sigma}}{1-\sigma}$. Indeed, $\lim\limits_{\sigma \to 1} \frac{c^{1-\sigma}}{1-\sigma} = \log c$ [8], so

$$\frac{U'(c_t)}{U(c_t)}\dot{c}_t - \rho = \frac{1}{\sigma}\frac{\dot{c}_t}{c_t} - \rho$$

Now, we use the log utility function, so $\sigma = 1$ and $\frac{U'(c_t)}{U(c_t)}\dot{c}_t - \rho = \frac{\dot{c}_t}{c_t} - \rho$.

From equation 8 $\frac{\dot{c}_t}{c_t} = (\alpha_t)^{1-\beta_t}(\beta_t)^{\beta_t}Ak_t^{\alpha_t + \beta_t - 1} - \rho$

$$\frac{U'(c_t)}{U(c_t)}\dot{c}_t - \rho = (\alpha_t)^{1-\beta_t}(\beta_t)^{\beta_t}Ak_t^{\alpha_t + \beta_t - 1} - 2\rho$$

and if $\Psi_\alpha + \Psi_\beta \leq 1$ and $(\Psi_\alpha)^{1-\Psi_\beta}(\Psi_\beta)^{\Psi_\beta}A < 2\rho$ then

$$\lim_{k \to \infty}\left(\frac{U'(c_t)}{U(c_t)}\dot{c}_t - \rho\right) \leq (\Psi_\alpha)^{1-\Psi_\beta}(\Psi_\beta)^{\Psi_\beta}A - 2\rho$$

Therefore, if $(\Psi_\alpha)^{1-\Psi_\beta}(\Psi_\beta)^{\Psi_\beta}A < 2\rho$ then $\lim\limits_{t \to \infty}\left(\frac{U'(c_t)}{U(c_t)}\dot{c}_t - \rho\right) < 0$ and $\lim\limits_{t \to \infty} u(c_t)e^{-\rho t} = 0$.

4.5   If $\Psi_\alpha + \Psi_\beta \geq 1$ and $(\Psi_\alpha)^{1-\Psi_\beta}(\Psi_\beta)^{\Psi_\beta} > \frac{\rho}{A}$ then capital abundant economies converge to a BGP.

Define the function $f(k) = A(\alpha(k))^{1-\beta(k)}(\beta(k))^{\beta(k)}k^{\alpha(k)+\beta(k)-1} - \rho$

1. If $k > 1$ then $f(k)$ is strictly increasing in k.

2. If $\Psi_\alpha + \Psi_\beta \geq 1$ then $\lim\limits_{k \to \infty} f(k) = (\Psi_\alpha)^{1-\Psi_\beta}(\Psi_\beta)^{\Psi_\beta}A - \rho$. Therefore, if

---

[8] To show that the utility function converges to the logarithmic function as $\sigma \to 1$ we make use of L'Hospital's rule. As $\sigma \to 1$, both the numerator and denominator of the function approach zero. We differentiate both the numerator and the denominator with respect to σ and then take the limit of the derivatives' ratio as $\sigma \to 1$.

$(\Psi_\alpha)^{1-\Psi_\beta} (\Psi_\beta)^{\Psi_\beta} > \frac{\rho}{A}$ then $\lim\limits_{k\to\infty} f(k) > 0$.

From 1 and 2, there exists a $\tilde{k}$ such that $f(k) > 0$ for any $k > \tilde{k}$.

Suppose not, then for any k there exists a finite number M such that $M > k$ and $f(M) \le 0$.

$f(k)$ is strictly increasing in k, so $f(M) > f(k)$. Therefore, for any k, $f(k) \le 0$ and $\sup f(k) \le 0$.

Finally, $\lim\limits_{k\to\infty} f(k) = \sup f(k)$, so $\lim\limits_{k\to\infty} f(k) \le 0$ which contradicts $\lim\limits_{k\to\infty} f(k) > 0$.